$p \to \pi^0 e^+$

$p \to \gamma e^+$

Baryon-number violation from the bottom up

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Standard Model and Beyond 2024 & Gordon Godfrey Workshop

Based mostly on 2312.13361 with Arnau Bas i Beneito, Juan Herrero-Garcia, Arcadi Santamaria and Michael A Schmidt

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The bottom-up approach has received a lot of attention recently

FitMaker: Ellis et al. 2012.02779
SMEFiT: Giani et al. 2302.06660
...

Measurement

↓

Flavio: Straub 1810.08132
DsixTools: Celis et al. 1704.04504
...

EFT

Matchete: Fuentes-Martín et al. 2212.04510
MatchmakerEFT: Carmona et al. 2112.10787

Predictions

↓

Granada dictionary: De Blas et al. 1711.10391
SOLD: Guedes et al. 2303.16965
JG, et al. (one-loop dictionary): 2412.01759
...

De Gouvea, Jenkins 0708.1344
...

Simplified models

↓

Theoretical framework

$R_{K^{(*)}}$

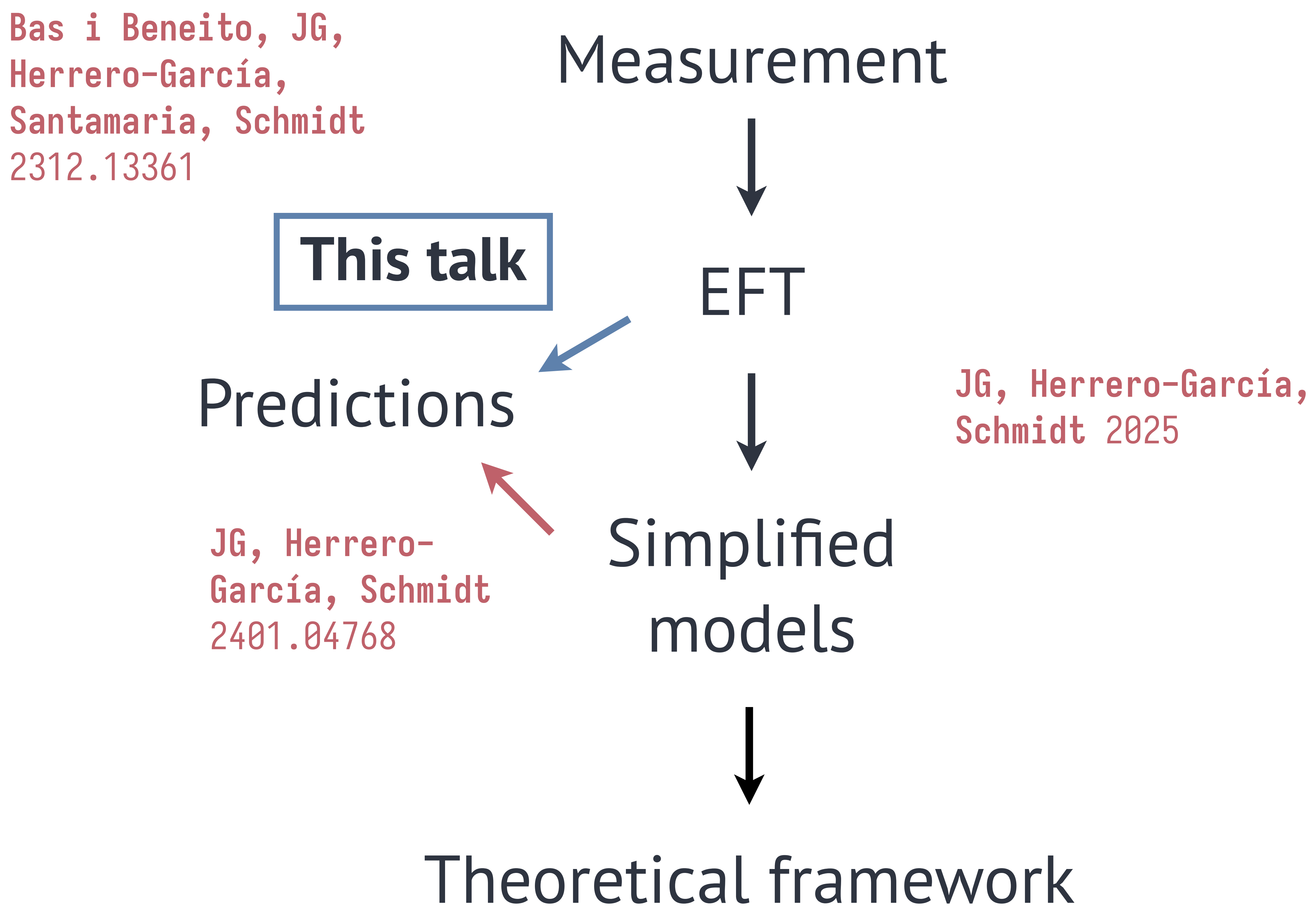
$(b^\dagger \bar{\sigma}^\mu s)(\mu^\dagger \bar{\sigma}_\mu \mu)$

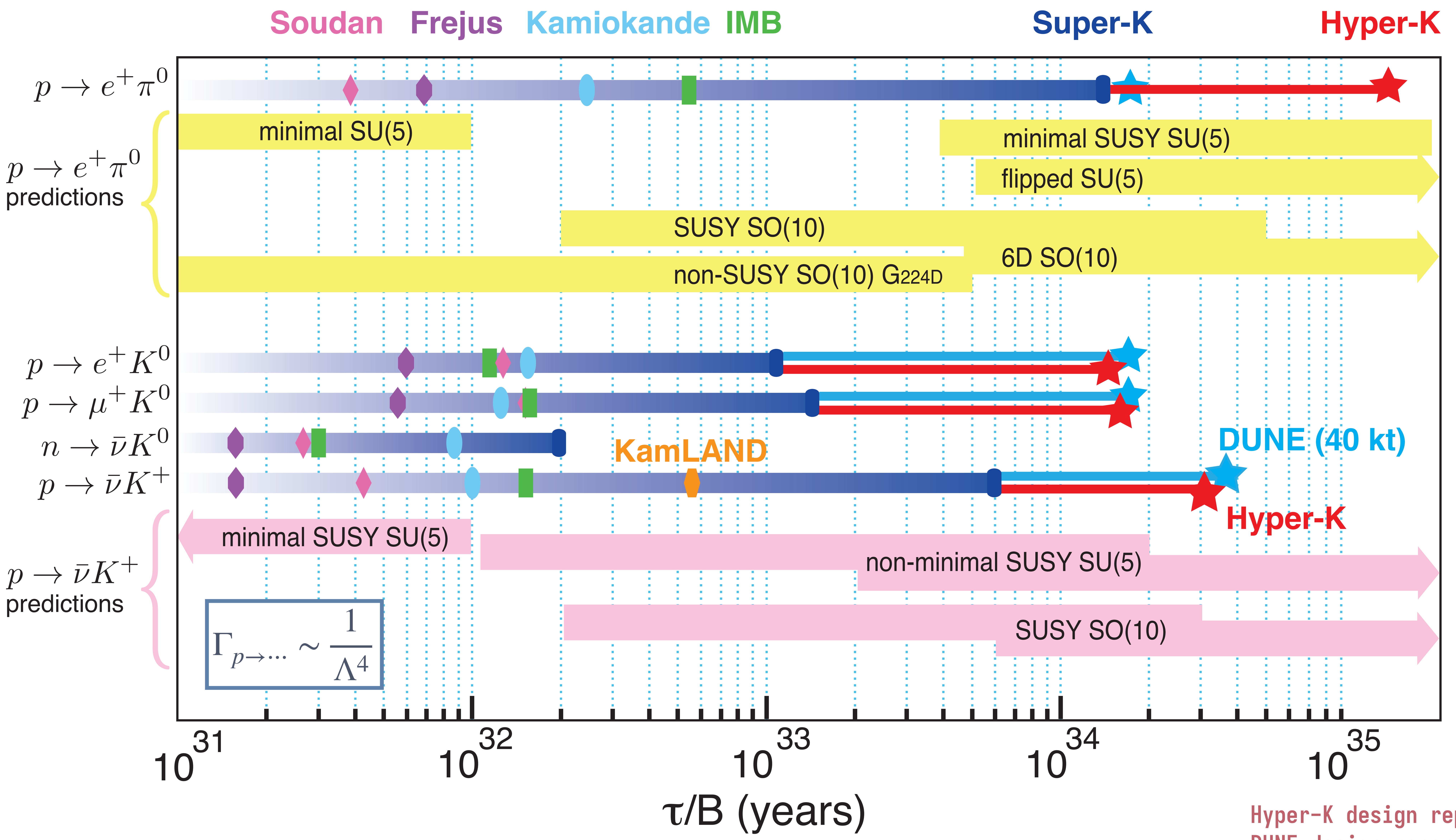
$(Q^\dagger \bar{\sigma}^\mu Q)(L^\dagger \bar{\sigma}_\mu L)$

$U_1^\mu \sim (\mathbf{3}, \mathbf{1}, 2/3)$

$\mathrm{SU}(4) \to \mathrm{SU}(3) \times \mathrm{U}(1)$

If we saw proton decay, how could we pin down the underlying model?





Hyper-K design report 1805.04163
DUNE design report II 2002.03005
Nath, Perez (review) hep-ph/0601023

The SMEFT predicts L and B violation

Kobach 1604.05726
Grzadkowski et al. 1008.4884
Liao, Ma 1901.10302
JG, Dutka 2211.02054

$$\mathscr{L} = \mathscr{L}_{d\leq 4} + \sum_{p,q} \frac{c_{pq}^{(5)}}{\Lambda}(L_p L_q)HH + \sum_{i=1}^{4} \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{d=6} + \sum_{i=1}^{6} \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_i^{d=7} + \cdots$$

$\Delta B = \Delta L = 1$

$$\mathcal{O}_{qqql} = (Q^i Q^j)(Q^l L^k)\epsilon_{ik}\epsilon_{jl}$$
$$\mathcal{O}_{qque} = (Q^i Q^j)(\bar{u}^\dagger \bar{e}^\dagger)\epsilon_{ij}$$
$$\mathcal{O}_{duue} = (\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}^\dagger)$$
$$\mathcal{O}_{duql} = (\bar{d}^\dagger \bar{u}^\dagger)(Q^i L^j)\epsilon_{ij}$$

$d = 6$

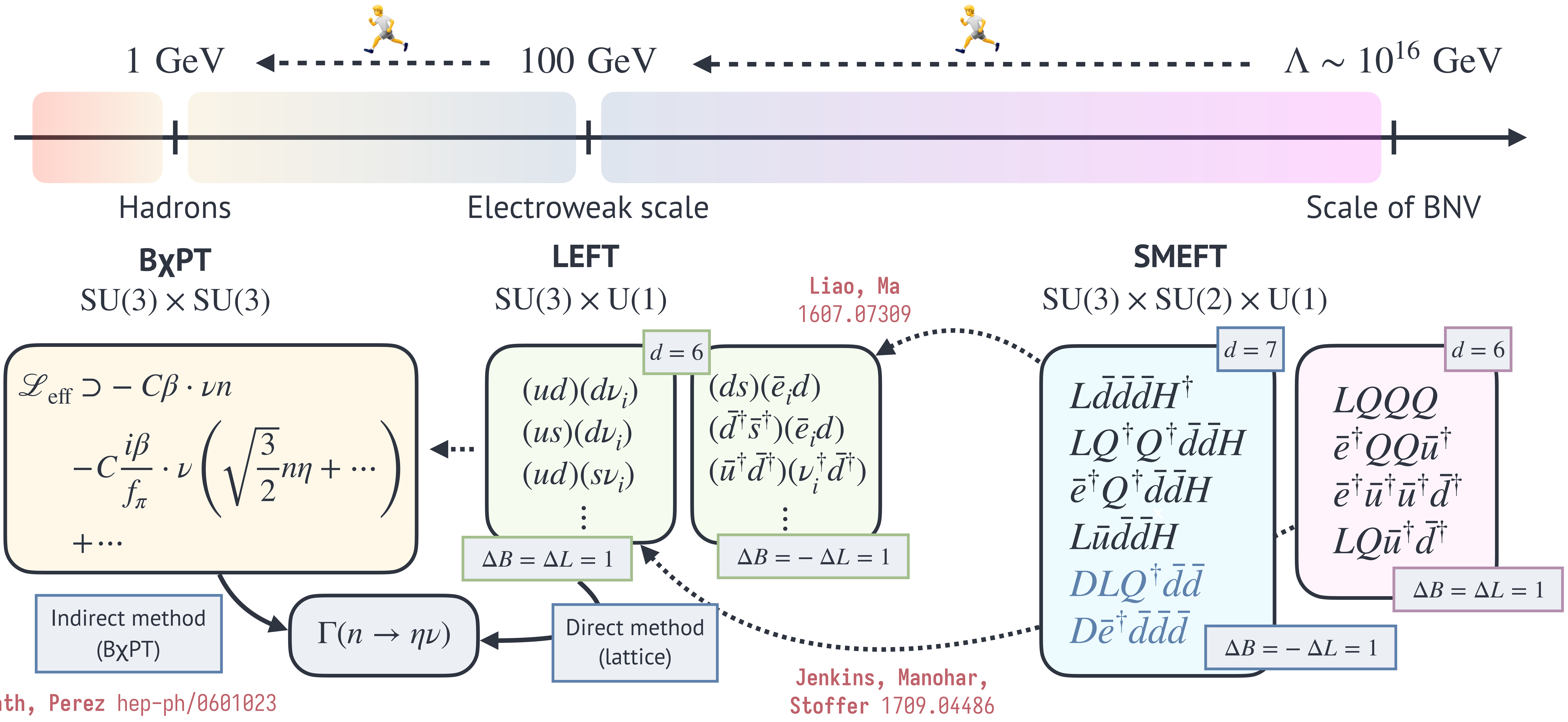
$\Delta B = -\Delta L = 1$

$$\mathcal{O}_{\bar{l}dddH} = (L^\dagger \bar{d}^\dagger)(\bar{d}^\dagger \bar{d}^\dagger)H$$
$$\mathcal{O}_{\bar{l}dqq\tilde{H}} = (L^\dagger \bar{d}^\dagger)(QQ^i)H_i^\dagger \qquad \mathcal{O}_{\bar{l}qdDd} = (L^\dagger \bar{\sigma}_\mu Q)(\bar{d}^\dagger iD^\mu \bar{d}^\dagger)$$
$$\mathcal{O}_{\bar{e}qdd\tilde{H}} = (\bar{e}Q^i)(\bar{d}^\dagger \bar{d}^\dagger)H_i^\dagger \qquad \mathcal{O}_{\bar{e}dddD} = (\bar{e}\sigma_\mu \bar{d}^\dagger)(\bar{d}^\dagger iD^\mu \bar{d}^\dagger)$$
$$\mathcal{O}_{\bar{l}dud\tilde{H}} = (L^\dagger \bar{d}^\dagger)(\bar{u}^\dagger \bar{d}^\dagger)\tilde{H}$$

$d = 7$

$$H \sim (\mathbf{1}, \mathbf{2}, \tfrac{1}{2}), \quad Q \sim (\mathbf{3}, \mathbf{2}, \tfrac{1}{6}), \quad \bar{u} \sim (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}), \quad \bar{d} \sim (\bar{\mathbf{3}}, \mathbf{1}, \tfrac{1}{3}), \quad L \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}), \quad \bar{e} \sim (\mathbf{1}, \mathbf{1}, 1)$$

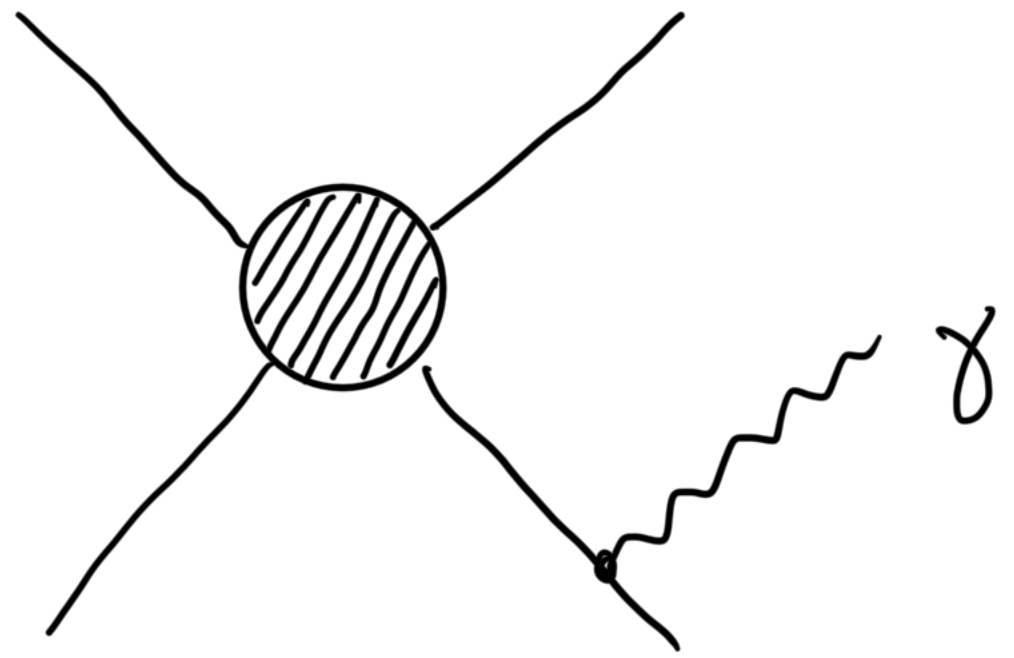
We use a ladder of EFTs to calculate decay rates



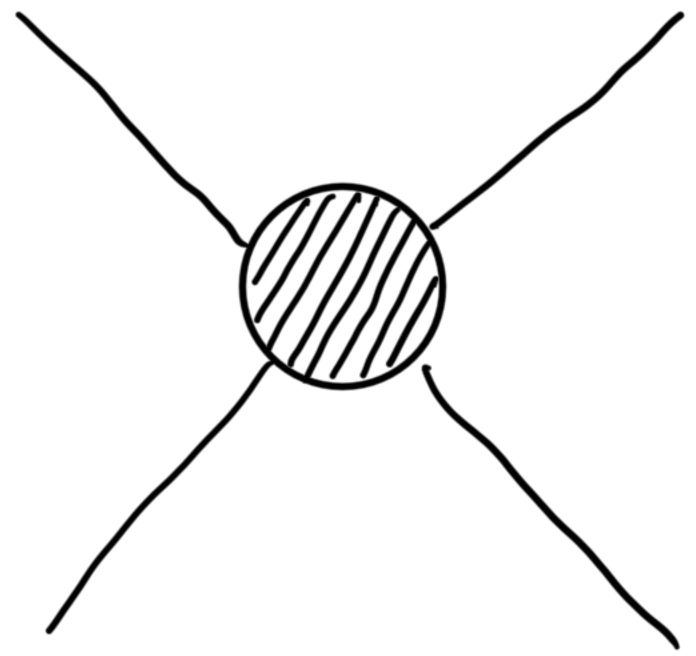
Just a handful of two-body decays could be leading signals

Decay mode	Limit [10^{34} yr]	Hyper-K [10^{34} yr]	ΔI	ΔS	ΔL	
Proton channels						
$p \to \pi^0 e^+$	2.4	3.2	$-\frac{1}{2}$	0	-1	
$p \to \pi^0 \mu^+$	1.6	7.7	$-\frac{1}{2}$	0	-1	1
$p \to \eta^0 e^+$	1.0	4.3	$-\frac{1}{2}$	0	-1	
$p \to \eta^0 \mu^+$	0.47	4.9	$-\frac{1}{2}$	0	-1	
$p \to \pi^+ \nu_r$	0.039	—	$\frac{1}{2}$	0	± 1	2
$p \to K^0 e^+$	0.10	—	-1	1	-1	3
$p \to K^0 \mu^+$	0.16	—	-1	1	-1	
$p \to K^+ \nu_r$	0.59	3.2	0	1	± 1	4
$p \to \bar{K}^0 e^+$	0.10	—	0	-1	-1	5
$p \to \bar{K}^0 \mu^+$	0.16	—	0	-1	-1	
Neutron channels						
$n \to \pi^0 \nu_r$	0.11	—	$\frac{1}{2}$	0	± 1	
$n \to \eta^0 \nu_r$	0.016	—	$\frac{1}{2}$	0	± 1	
$n \to \pi^- e^+$	0.53	2.0	$-\frac{1}{2}$	0	-1	
$n \to \pi^- \mu^+$	0.35	1.8	$-\frac{1}{2}$	0	-1	
$n \to \pi^+ e^-$	0.0065	—	$\frac{3}{2}$	0	1	6
$n \to \pi^+ \mu^-$	0.0049	—	$\frac{3}{2}$	0	1	
$n \to K^+ e^-$	0.0032	1.0	1	1	1	7
$n \to K^+ \mu^-$	0.0057	—	1	1	1	
$n \to K^0 \nu_r$	0.013	—	0	1	± 1	
$n \to K^- e^+$	0.0017	—	0	-1	-1	
$n \to \bar{K}^0 \nu_r$	0.013	—	1	-1	± 1	

Decays to (anti-)leptons are generated at dimension (6) 7 in the SMEFT



$p \to e^+ \gamma$
$n \to \nu \gamma$



$p \to \rho^0 e^+$

Hyper-K estimates: 1805.04163

Leading signals predicted by operators are fixed by symmetries

Decay mode	Limit [10^{34} yr]	Hyper-K [10^{34} yr]	ΔI	ΔS	ΔL	
Proton channels						
$p \to \pi^0 e^+$	2.4	3.2	$-\frac{1}{2}$	0	-1	
$p \to \pi^0 \mu^+$	1.6	7.7	$-\frac{1}{2}$	0	-1	
$p \to \eta^0 e^+$	1.0	4.3	$-\frac{1}{2}$	0	-1	1
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$p \to K^0 e^+$	0.10	—	-1	1	-1	3
$p \to K^0 \mu^+$	0.16	—	-1	1	-1	
$p \to K^+ \nu_r$	0.59	3.2	0	1	± 1	4
$p \to \bar{K}^0 e^+$	0.10	—	0	-1	-1	5
$p \to \bar{K}^0 \mu^+$	0.16	—	0	-1	-1	
Neutron channels						
$n \to \pi^0 \nu_r$	0.11	—	$\frac{1}{2}$	0	± 1	
$n \to \eta^0 \nu_r$	0.016	—	$\frac{1}{2}$	0	± 1	
$n \to \pi^- e^+$	0.53	2.0	$-\frac{1}{2}$	0	-1	
$n \to \pi^- \mu^+$	0.35	1.8	$-\frac{1}{2}$	0	-1	
$n \to \pi^+ e^-$	0.0065	—	$\frac{3}{2}$	0	1	6
$n \to \pi^+ \mu^-$	0.0049	—	$\frac{3}{2}$	0	1	
$n \to K^+ e^-$	0.0032	1.0	1	1	1	7
$n \to K^+ \mu^-$	0.0057	—	1	1	1	
$n \to K^0 \nu_r$	0.013	—	0	1	± 1	
$n \to K^- e^+$	0.0017	—	0	-1	-1	
$n \to \bar{K}^0 \nu_r$	0.013	—	1	-1	± 1	

Decays to (anti-)leptons are generated at dimension (6) 7 in the SMEFT

	Name [42]	Operator	Flavour	ΔI	ΔS
	$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$	$\frac{1}{2}$	0
	$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$	0	1
	$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$(\mathbf{8},\mathbf{1})$	0	1
	$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$(\mathbf{8},\mathbf{1})$	$-\frac{1}{2}$	0
	$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$(\mathbf{8},\mathbf{1})$	-1	1
	$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	$-\frac{1}{2}$	0
	$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	-1	1
	$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	$-\frac{1}{2}$	0
	$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	-1	1
	$[\mathcal{O}^{S,RL}_{dud}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	$\frac{1}{2}$	0
	$[\mathcal{O}^{S,RL}_{dud}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
	$[\mathcal{O}^{S,RL}_{dud}]_{112r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(s\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
⛔	$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$	$(\bar{d}^\dagger \bar{s}^\dagger)(u\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
	$[\mathcal{O}^{S,RR}_{duu}]_{111r}$	$(\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$(\mathbf{1},\mathbf{8})$	$-\frac{1}{2}$	0
	$[\mathcal{O}^{S,RR}_{duu}]_{211r}$	$(\bar{s}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$(\mathbf{1},\mathbf{8})$	-1	1

$$\Delta B = \Delta L = -1$$

Generated at dimension-6 in the SMEFT

Name [42]	Operator	Flavour	ΔI	ΔS	
$[\mathcal{O}^{S,LL}_{ddd}]_{[12]r1}$	$(ds)(\bar{e}_r d)$	$(\mathbf{8},\mathbf{1})$	1	1	⛔
$[\mathcal{O}^{S,LR}_{udd}]_{11r1}$	$(ud)(\nu_r^\dagger \bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	$\frac{1}{2}$	0	
$[\mathcal{O}^{S,LR}_{udd}]_{12r1}$	$(us)(\nu_r^\dagger \bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	0	1	
$[\mathcal{O}^{S,LR}_{udd}]_{11r2}$	$(ud)(\nu_r^\dagger \bar{s}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	0	1	
$[\mathcal{O}^{S,LR}_{ddu}]_{[12]r1}$	$(ds)(\nu_r^\dagger \bar{u}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	0	1	⛔
$[\mathcal{O}^{S,LR}_{ddd}]_{[12]r1}$	$(ds)(e_r^\dagger \bar{d}^\dagger)$	$(\bar{\mathbf{3}},\mathbf{3})$	1	1	
$[\mathcal{O}^{S,RL}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger \bar{s}^\dagger)(\bar{e}_r d)$	$(\mathbf{3},\bar{\mathbf{3}})$	1	1	
$[\mathcal{O}^{S,RR}_{udd}]_{11r1}$	$(\bar{u}^\dagger \bar{d}^\dagger)(\nu_r^\dagger \bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$	$\frac{1}{2}$	0	
$[\mathcal{O}^{S,RR}_{udd}]_{12r1}$	$(\bar{u}^\dagger \bar{s}^\dagger)(\nu_r^\dagger \bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$	0	1	
$[\mathcal{O}^{S,RR}_{udd}]_{11r2}$	$(\bar{u}^\dagger \bar{d}^\dagger)(\nu_r^\dagger \bar{s}^\dagger)$	$(\mathbf{1},\mathbf{8})$	0	1	
$[\mathcal{O}^{S,RR}_{ddd}]_{[12]r1}$	$(\bar{d}^\dagger \bar{s}^\dagger)(e_r^\dagger \bar{d}^\dagger)$	$(\mathbf{1},\mathbf{8})$	1	1	

$$\Delta B = -\Delta L = -1$$

Generated at dimension-7 in the SMEFT

[42]: Jenkins, Manohar, Stoffer 1709.04486

Some LEFT operators are only generated above dimension-7 at tree level

	Name	SMEFT matching
$B-L=0$	$[\mathcal{O}^{S,LL}_{udd}]_{pqrs}$	$V_{qq'}V_{rr'}(C_{qqql,r'q'ps} - C_{qqql,q'r'ps} + C_{qqql,q'pr's})$
	$[\mathcal{O}^{S,LL}_{duu}]_{pqrs}$	$V_{pp'}(C_{qqql,rqp's} - C_{qqql,qrp's} + C_{qqql,qp'rs})$
	$[\mathcal{O}^{S,LR}_{duu}]_{pqrs}$	$-V_{pp'}(C_{qque,p'qrs} + C_{qque,qp'rs})$
	$[\mathcal{O}^{S,RL}_{duu}]_{pqrs}$	$C_{duql,pqrs}$
	$[\mathcal{O}^{S,RL}_{dud}]_{pqrs}$	$-V_{rr'}C_{duql,pqr's}$
	$[\mathcal{O}^{S,RL}_{ddu}]_{pqrs}$	$(C_{ddqlHH,pqrs} - C_{ddqlHH,qprs})\frac{v^2}{2\Lambda^2}$
	$[\mathcal{O}^{S,RR}_{duu}]_{pqrs}$	$C_{duue,pqrs}$
$B-L=2$	$[\mathcal{O}^{S,LL}_{ddd}]_{pqrs}$	$V_{ss'}V_{pp'}V_{qq'}(C_{eqqqHHH,rs'p'q'} - C_{eqqqHHH,rs'q'p'})\frac{v^3}{2\sqrt{2}\Lambda^3}$
	$[\mathcal{O}^{S,LR}_{udd}]_{pqrs}$	$-V_{qq'}C_{\bar{l}dqq\tilde{H},rspq'}\frac{v}{\sqrt{2}\Lambda}$
	$[\mathcal{O}^{S,LR}_{ddu}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{luqqHHH,rsp'q'} - C_{luqqHHH,rsq'p'})\frac{v^3}{2\sqrt{2}\Lambda^3}$
	$[\mathcal{O}^{S,LR}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{\bar{l}dqq\tilde{H},rsq'p'} - C_{\bar{l}dqq\tilde{H},rsp'q'})\frac{v}{2\sqrt{2}\Lambda}$
	$[\mathcal{O}^{S,RL}_{ddd}]_{pqrs}$	$V_{ss'}(C_{\bar{e}qdd\tilde{H},rs'qp} - C_{\bar{e}qdd\tilde{H},rs'pq})\frac{v}{\sqrt{2}\Lambda}$
	$[\mathcal{O}^{S,RR}_{udd}]_{pqrs}$	$C_{\bar{l}dud\tilde{H},rspq}\frac{v}{\sqrt{2}\Lambda}$
	$[\mathcal{O}^{S,RR}_{ddd}]_{pqrs}$	$C_{\bar{l}dddH,rspq}\frac{v}{\sqrt{2}\Lambda}$
	$[\mathcal{O}^{V,RL}_{ddu}]_{pqrs}$	$-C_{\bar{l}qdDd,srpq}$
	$[\mathcal{O}^{V,RL}_{ddd}]_{pqrs}$	$-V_{rr'}C_{\bar{l}qdDd,sr'pq}$
	$[\mathcal{O}^{V,RR}_{ddd}]_{pqrs}$	$-C_{\bar{e}dddD,srpq}$
	$[\mathcal{O}^{V,LL}_{ddu}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{qqlqHHD,p'q'rs} + C_{qqlqHHD,q'p'rs})\frac{v^2}{4\Lambda^2}$
	$[\mathcal{O}^{V,LL}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}V_{ss'}(C_{qqlqHHD,p'q'rs'} + C_{qqlqHHD,q'p'rs'})\frac{v^2}{4\Lambda^2}$
	$[\mathcal{O}^{V,LR}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{qqedHHD,p'q'rs} + C_{qqedHHD,q'p'rs})\frac{v^2}{4\Lambda^2}$
	$[\mathcal{O}^{V,RL}_{dud}]_{pqrs}$	$-V_{ss'}C^{*}_{udqlHHD,qps'r}\frac{v^2}{2\Lambda^2}$

Liao, Ma 1607.07309
Jenkins, Manohar, Stoffer 1709.04486

Gisbert, et al 2409.00218
Heeck, Watkins 2405.18478
Beneke, Finauri, Petrov 2404.09642

Flavour!

$$(\bar{d}_p^\dagger \bar{d}_q^\dagger)(Q_r^i L_s^j) H^k H^l \epsilon_{ik}\epsilon_{jl}$$

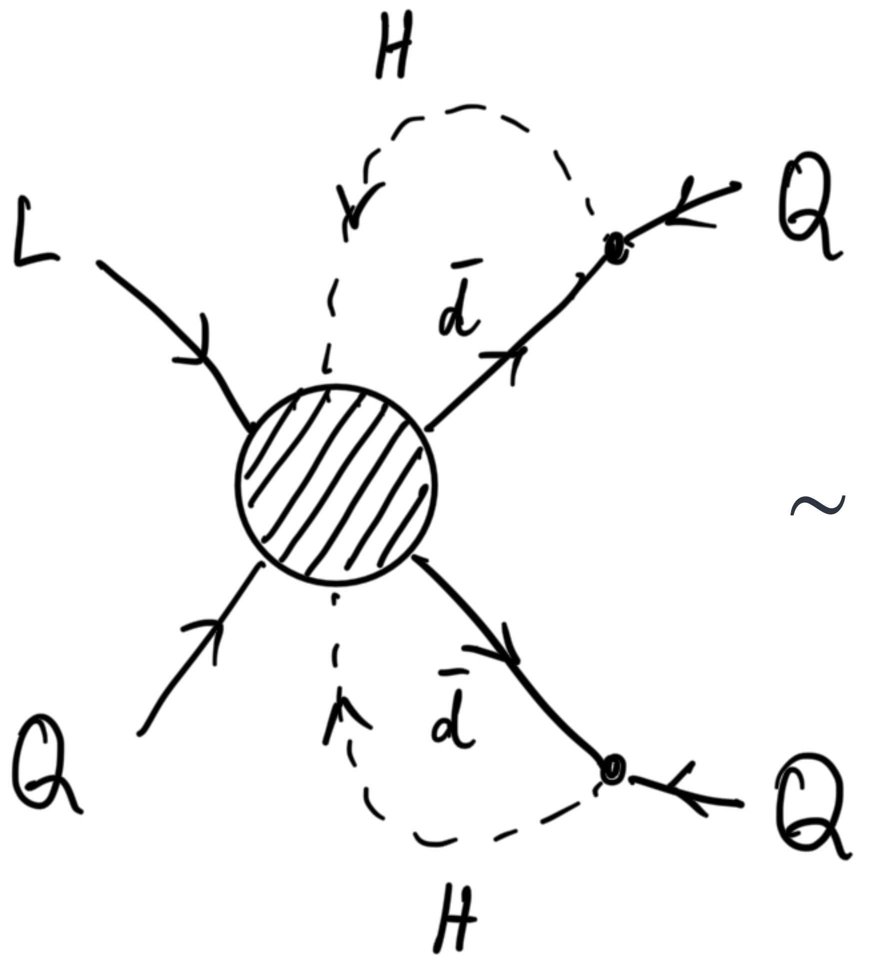
$$(\bar{e}_p^\dagger Q_{qi}^\dagger)(Q_{rj}^\dagger Q_{sk}^\dagger) H^i H^j H^k$$

$$(L_p^i \bar{u}_q)(Q_{rj}^\dagger Q_{sk}^\dagger) H^{i'} H^j H^k \epsilon_{ii'}$$

$$(Q_p^i i D^\mu Q_q^j)(\bar{e}_r \sigma^\mu \bar{d}_s^\dagger)\tilde{H}^k \tilde{H}^l \epsilon_{ik}\epsilon_{jl}$$

$$(Q_p^i i D^\mu Q_q^j)(L_r^\dagger \bar{\sigma}^\mu Q_s)\tilde{H}^k \tilde{H}^l \epsilon_{ik}\epsilon_{jl}$$

$$(\bar{u}_p i D^\mu \bar{d}_q)(Q_{ri}^\dagger \bar{\sigma}^\mu L_s^j) H^i H^k \epsilon_{jk}$$



$$\sim \left(\frac{1}{16\pi^2}\right)^2 \frac{y_d^2}{\Lambda^2}$$

JG, Herrero-García, Schmidt 2401.04768

Loop-induced nucleon decays often dominate because

$$\frac{v}{\Lambda} \ll \frac{1}{16\pi^2}$$

Difficult to imagine leading effects in these LEFT operators

Some LEFT … dimension-7 at tree level

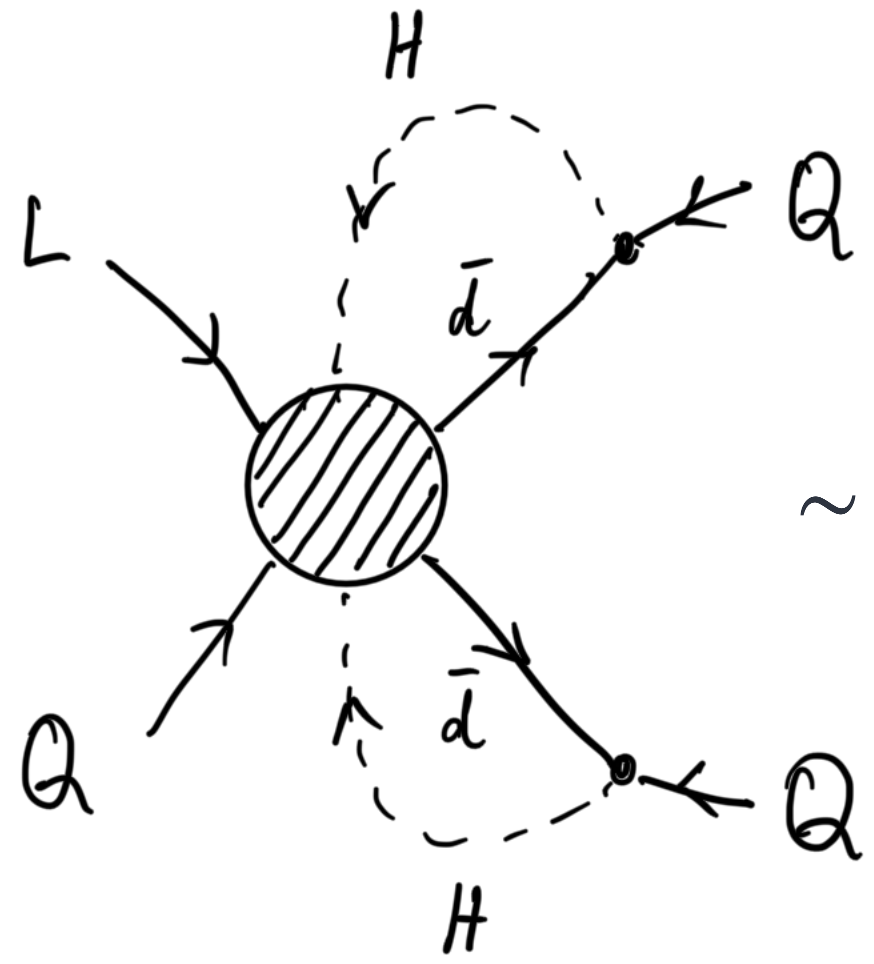
Lower limit [GeV]	Process	LEFT coefficient
$4.2 \cdot 10^{15} \cdot \sqrt{\lvert [C_{qque}]_{1111} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,LR}_{duu}]_{1111}$
$3.0 \cdot 10^{15} \cdot \sqrt{\lvert [C_{duql}]_{1111} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,RL}_{duu}]_{1111}$
$2.9 \cdot 10^{15} \cdot \sqrt{\lvert [C_{duue}]_{1111} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,RR}_{duu}]_{1111}$
$2.9 \cdot 10^{15} \cdot \sqrt{\lvert [C_{qqql}]_{1111} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,LL}_{duu}]_{1111}$
$2.0 \cdot 10^{15} \cdot \sqrt{\lvert [C_{qque}]_{1211} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,LR}_{duu}]_{1111}$
$1.7 \cdot 10^{15} \cdot \sqrt{\lvert [C_{qqql}]_{1121} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$1.4 \cdot 10^{15} \cdot \sqrt{\lvert [C_{qqql}]_{1211} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,LL}_{duu}]_{1111}$
$1.3 \cdot 10^{15} \cdot \sqrt{\lvert [C_{duql}]_{1121} \rvert}$	$p \to K^+ \nu$	$[C^{S,RL}_{dud}]_{1121}$
$7.8 \cdot 10^{14} \cdot \sqrt{\lvert [C_{duql}]_{2111} \rvert}$	$p \to K^+ \nu$	$[C^{S,RL}_{dud}]_{2111}$
$5.9 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{2121} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$5.1 \cdot 10^{14} \cdot \sqrt{\lvert [C_{duue}]_{2111} \rvert}$	$p \to K^0 e^+$	$[C^{S,RR}_{duu}]_{2111}$
$3.7 \cdot 10^{14} \cdot \sqrt{\lvert [C_{duql}]_{2121} \rvert}$	$p \to K^+ \nu$	$[C^{S,RL}_{dud}]_{2111}$
$3.6 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{1131} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$2.7 \cdot 10^{14} \cdot \sqrt{\lvert [C_{duql}]_{1131} \rvert}$	$p \to K^+ \nu$	$[C^{S,RL}_{dud}]_{1121}$
$2.6 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qque}]_{1311} \rvert}$	$p \to \pi^0 e^+$	$[C^{S,LR}_{duu}]_{1111}$
$2.5 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{1311} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$1.7 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{2311} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$1.2 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{1231} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$1.2 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{1321} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$1.2 \cdot 10^{14} \cdot \sqrt{\lvert [C_{qqql}]_{2131} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$
$4.7 \cdot 10^{13} \cdot \sqrt{\lvert [C_{duql}]_{2131} \rvert}$	$p \to K^+ \nu$	$[C^{S,RL}_{dud}]_{2111}$
$1.5 \cdot 10^{13} \cdot \sqrt{\lvert [C_{qqql}]_{3131} \rvert}$	$p \to K^+ \nu$	$[C^{S,LL}_{udd}]_{1121}$

	Name	SM…
$B-L=0$	$[\mathcal{O}^{S,LL}_{udd}]_{pqrs}$	$V_{qq'}V_{rr'}(C_{qqql,r'q}$ …
	$[\mathcal{O}^{S,LL}_{duu}]_{pqrs}$	$V_{pp'}(C_{qqql,rqp'}$ …
	$[\mathcal{O}^{S,LR}_{duu}]_{pqrs}$	$-V_{pp'}(C$ …
	$[\mathcal{O}^{S,RL}_{duu}]_{pqrs}$	
	$[\mathcal{O}^{S,RL}_{dud}]_{pqrs}$	…
	$[\mathcal{O}^{S,RL}_{ddu}]_{pqrs}$	$(C_{ddqlHH}$ …
	$[\mathcal{O}^{S,RR}_{duu}]_{pqrs}$	
$B-L=2$	$[\mathcal{O}^{S,LL}_{ddd}]_{pqrs}$	$V_{ss'}V_{pp'}V_{qq'}(C_{eqqqH}$ …
	$[\mathcal{O}^{S,LR}_{udd}]_{pqrs}$	$-V$ …
	$[\mathcal{O}^{S,LR}_{ddu}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{luqqHH}$ …
	$[\mathcal{O}^{S,LR}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{\bar{l}dq}$ …
	$[\mathcal{O}^{S,RL}_{ddd}]_{pqrs}$	$V_{ss'}(C_{\bar{e}qdd}$ …
	$[\mathcal{O}^{S,RR}_{udd}]_{pqrs}$	…
	$[\mathcal{O}^{S,RR}_{ddd}]_{pqrs}$	…
	$[\mathcal{O}^{V,RL}_{ddu}]_{pqrs}$	
	$[\mathcal{O}^{V,RL}_{ddd}]_{pqrs}$	− …
	$[\mathcal{O}^{V,RR}_{ddd}]_{pqrs}$	
	$[\mathcal{O}^{V,LL}_{ddu}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{qqlqH}$ …
	$[\mathcal{O}^{V,LL}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}V_{ss'}(C_{qqlqH}$ …
	$[\mathcal{O}^{V,LR}_{ddd}]_{pqrs}$	$V_{pp'}V_{qq'}(C_{qqedH}$ …
	$[\mathcal{O}^{V,RL}_{dud}]_{pqrs}$	$-V_{ss}$ …

…486

…our!

…2



$$\sim \left(\frac{1}{16\pi^2}\right)^2 \frac{y_d^2}{\Lambda^2}$$

JG, Herrero-García, Schmidt 2401.04768

Loop-induced nucleon decays often dominate because

$$\frac{v}{\Lambda} \ll \frac{1}{16\pi^2}$$

Difficult to imagine leading effects in these LEFT operators

We calculate decay rates using BχPT

Aoki, et al. 1705.01338
Yoo et al. 2111.01608

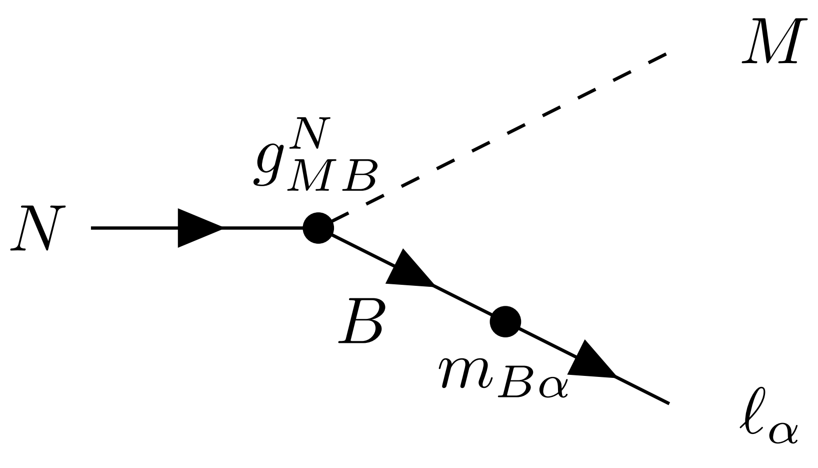
α, β are dominant source of uncertainty in our calculations

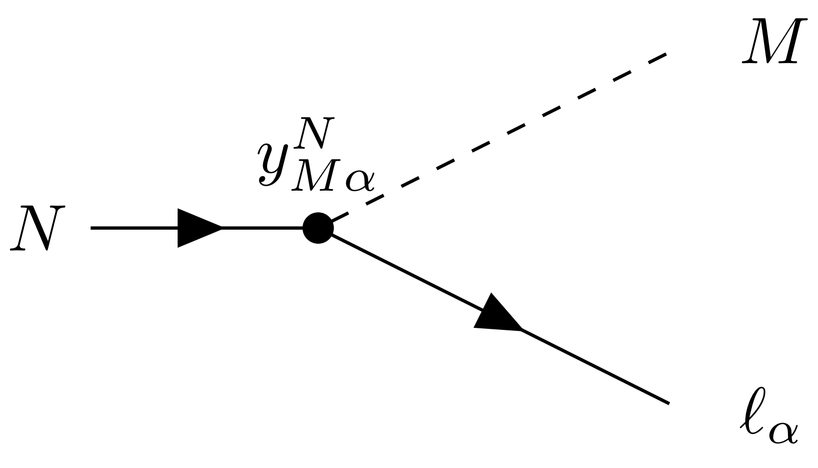
$$\langle 0 | \epsilon^{abc} (\bar{u}_a^\dagger \bar{d}_b^\dagger) u_c | p^{(s)} \rangle = \alpha P_L u_p^{(s)}$$

$$\langle 0 | \epsilon^{abc} (u_a d_b) u_c | p^{(s)} \rangle = \beta P_L u_p^{(s)}$$

$$\mathscr{L} = g_{MB}^N \bar{B} \gamma^\mu \gamma_5 N \partial_\mu M + m_{B\alpha} \bar{\ell}_\alpha B + i y_{M\alpha}^N \bar{\ell}_\alpha N M$$

$\Delta B = 0$ $\qquad$ $\Delta B = -1$





$$\Gamma_N^{(6)} = c_i^* \kappa_{ij} c_j \cdot \frac{m_N^5}{\Lambda^4} \qquad \Gamma_N^{(7)} = c_i^* \kappa_{ij} c_j \cdot \frac{m_N^7}{\Lambda^6}$$

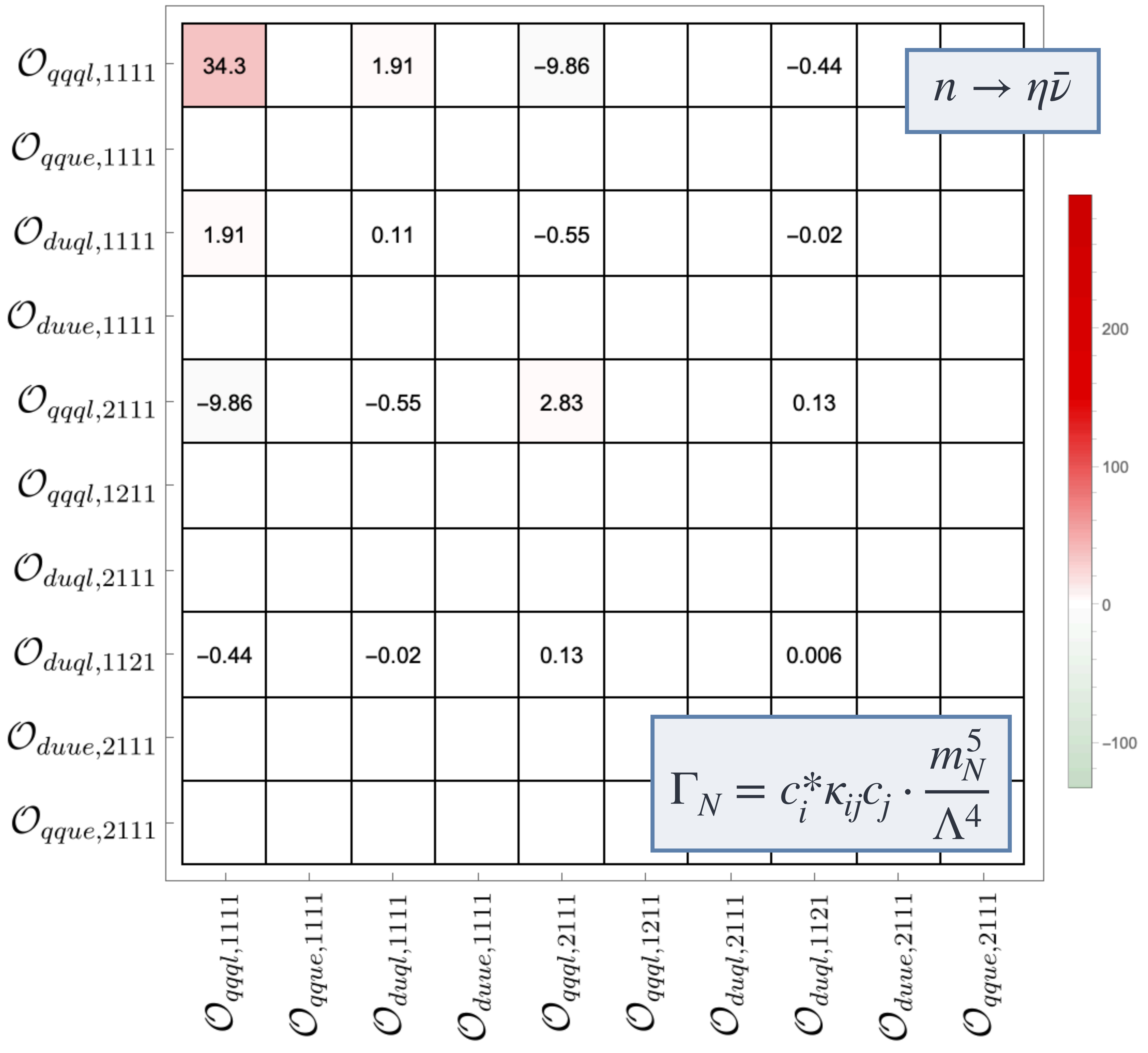
First-time calculation of dim-7 nucleon decay rates using the chiral-Lagrangian method

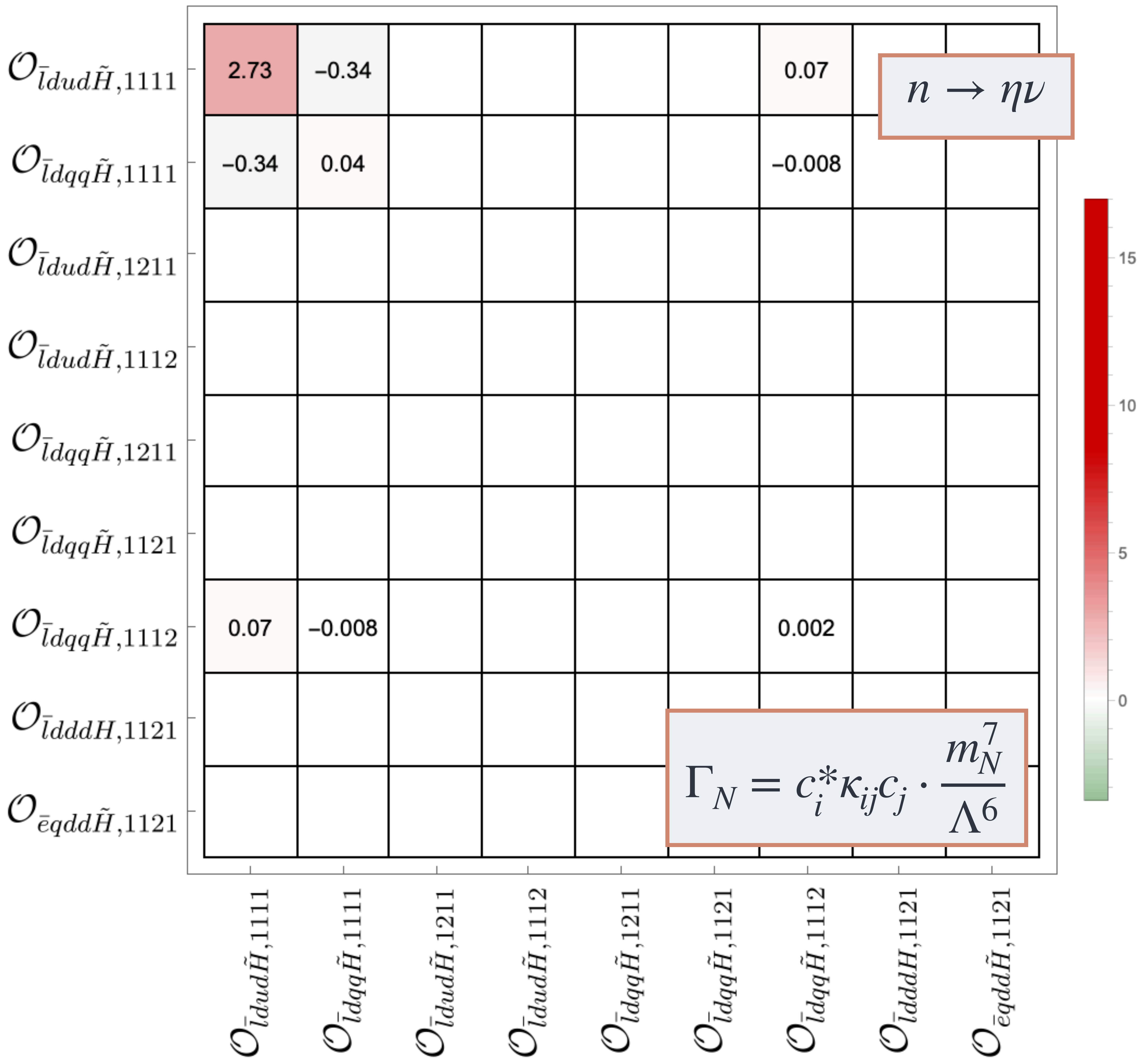
A Matching to BχPT of BNV dimension-6 LEFT operators

Name	LEFT	Flavour/BχPT
$[\mathcal{O}_{udd}^{S,LL}]_{rstu}$	$(u_r d_s)(d_t \nu_u)$	$(\mathbf{8}, \mathbf{1})$
$[\mathcal{O}_{udd}^{S,LL}]_{111r}$	$(ud)(d\nu_r)$	$-\beta \overline{\nu_{Lr}^c} \mathrm{tr}(\xi B \xi^\dagger P_{32}) \supset -\beta \overline{\nu_{Lr}^c} n - \frac{i\beta}{f_\pi} \overline{\nu_{Lr}^c} \left(\sqrt{\frac{3}{2}} n\eta - \frac{1}{\sqrt{2}} n\pi^0 + p\pi^- \right)$
$[\mathcal{O}_{udd}^{S,LL}]_{121r}$	$(us)(d\nu_r)$	$-\beta \nu_{Lr}^c \mathrm{tr}(\xi B \xi^\dagger P_{22}) \supset -\beta \nu_{Lr}^c \left(-\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} \right) - \frac{i\beta}{f_\pi} \nu_{Lr}^c n K^0$
$[\mathcal{O}_{udd}^{S,LL}]_{112r}$	$(ud)(s\nu_r)$	$-\beta \overline{\nu_{Lr}^c} \mathrm{tr}(\xi B \xi^\dagger P_{33}) \supset \beta \sqrt{\frac{2}{3}} \overline{\nu_{Lr}^c} \Lambda^0 - \frac{i\beta}{f_\pi} \overline{\nu_{Lr}^c} (n \bar{K}^0 + pK^-)$
$[\mathcal{O}_{duu}^{S,LL}]_{rstu}$	$(d_r u_s)(u_t e_u)$	$(\mathbf{8}, \mathbf{1})$
$[\mathcal{O}_{duu}^{S,LL}]_{111r}$	$(du)(ue_r)$	$-\beta \overline{e_{Lr}^c} \mathrm{tr}(\xi B \xi^\dagger \tilde{P}_{31}) \supset \beta \overline{e_{Lr}^c} p + \frac{i\beta}{f_\pi} \overline{e_{Lr}^c} \left(\sqrt{\frac{3}{2}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,LL}]_{211r}$	$(su)(ue_r)$	$-\beta \overline{e_{Lr}^c} \mathrm{tr}(\xi B \xi^\dagger P_{21}) \supset -\beta \overline{e_{Lr}^c} \Sigma^+ + \frac{i\beta}{f_\pi} \overline{e_{Lr}^c} p \bar{K}^0$
$[\mathcal{O}_{uud}^{S,LR}]_{[rs]tu}$	$(u_r u_s)(\bar{d}_t^\dagger \bar{e}_u^\dagger)$	—
$[\mathcal{O}_{duu}^{S,LR}]_{rstu}$	$(d_r u_s)(\bar{u}_t^\dagger \bar{e}_u^\dagger)$	$(\bar{\mathbf{3}}, \mathbf{3})$
$[\mathcal{O}_{duu}^{S,LR}]_{111r}$	$(du)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\alpha \overline{e_{Rr}^c} \mathrm{tr}(\xi^\dagger B \xi^\dagger \tilde{P}_{31}) \supset -\alpha \overline{e_{Rr}^c} p + \frac{i\alpha}{f_\pi} \overline{e_{Rr}^c} \left(-\frac{1}{\sqrt{6}} p\eta + \frac{1}{\sqrt{2}} p\pi^0 + n\pi^+ \right)$
$[\mathcal{O}_{duu}^{S,LR}]_{211r}$	$(su)(\bar{u}^\dagger \bar{e}_r^\dagger)$	$\longrightarrow$ $\alpha \overline{e_{Rr}^c} \mathrm{tr}(\xi^\dagger B \xi^\dagger P_{21}) \supset \alpha \overline{e_{Rr}^c} \Sigma^+ - \frac{i\alpha}{f_\pi} \overline{e_{Rr}^c} p \bar{K}^0$
$[\mathcal{O}_{uud}^{S,RL}]_{[rs]tu}$	$(\bar{u}_r^\dagger \bar{u}_s^\dagger)(d_t e_u)$	—
$[\mathcal{O}_{duu}^{S,RL}]_{rstu}$	$(\bar{d}_r^\dagger \bar{u}_s^\dagger)(u_t e_u)$	$(\mathbf{3}, \bar{\mathbf{3}})$

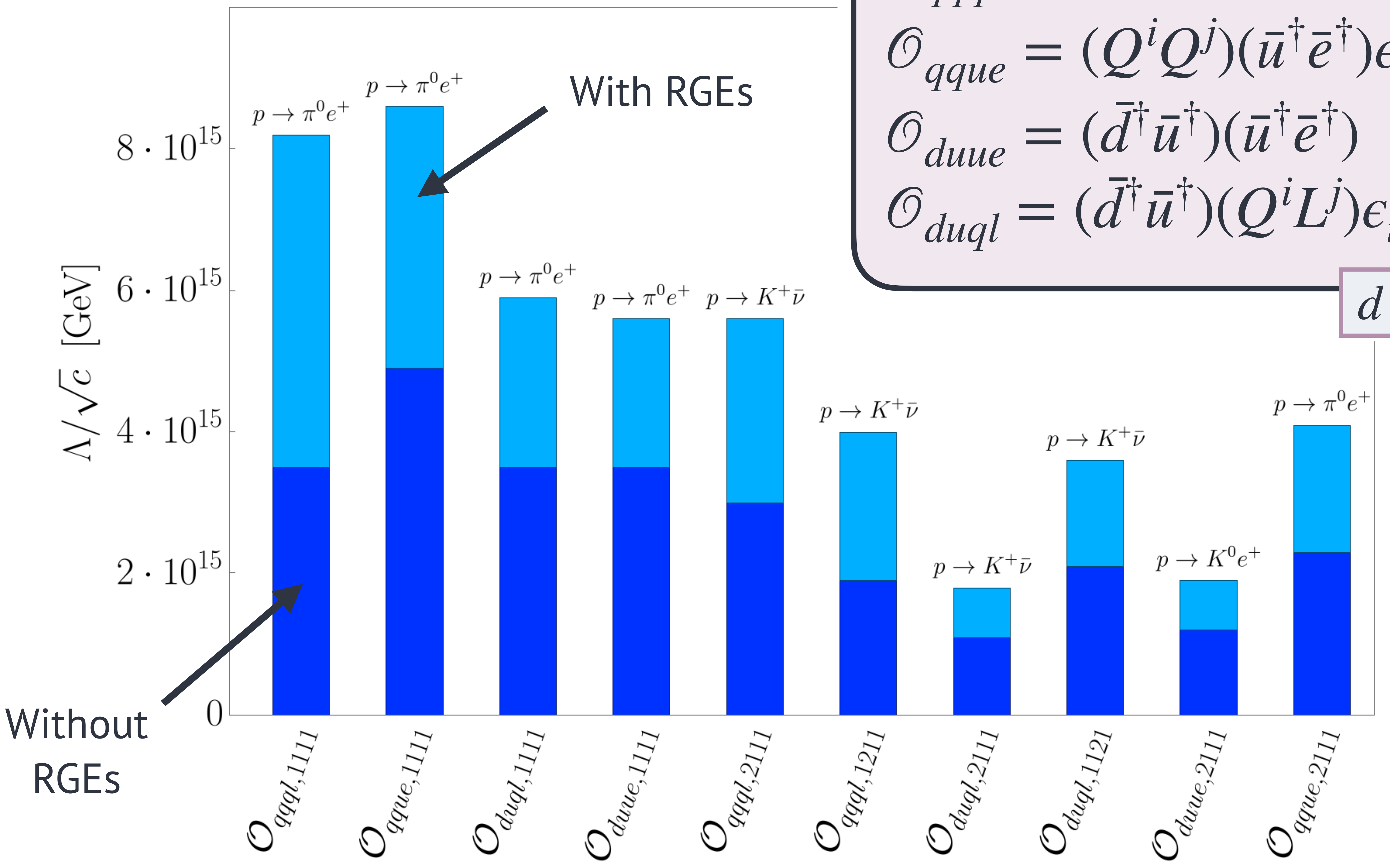
We package the decay rates into numerical matrices that are available online

https://zenodo.org/records/12664770





Running can lead to large enhancements in the limits derived



$\Delta B = \Delta L = 1$

$$\mathcal{O}_{qqql} = (Q^i Q^j)(Q^l L^k)\epsilon_{ik}\epsilon_{jl}$$
$$\mathcal{O}_{qque} = (Q^i Q^j)(\bar{u}^\dagger \bar{e}^\dagger)\epsilon_{ij}$$
$$\mathcal{O}_{duue} = (\bar{d}^\dagger \bar{u}^\dagger)(\bar{u}^\dagger \bar{e}^\dagger)$$
$$\mathcal{O}_{duql} = (\bar{d}^\dagger \bar{u}^\dagger)(Q^i L^j)\epsilon_{ij}$$

$d = 6$

- Assume **single-operator dominance**
- Running dominated by gauge interactions, can be large
  - Expressions look like
  $$16\pi^2 \mu \frac{d\mu}{dc_i} = -4g_3^2 c_i + \cdots$$
  - 1.6 – 2.3 factor enhancement
- Strongest lower limit
  $$\Lambda/\sqrt{c} > 2 \cdot 10^{15}\ \text{GeV}$$

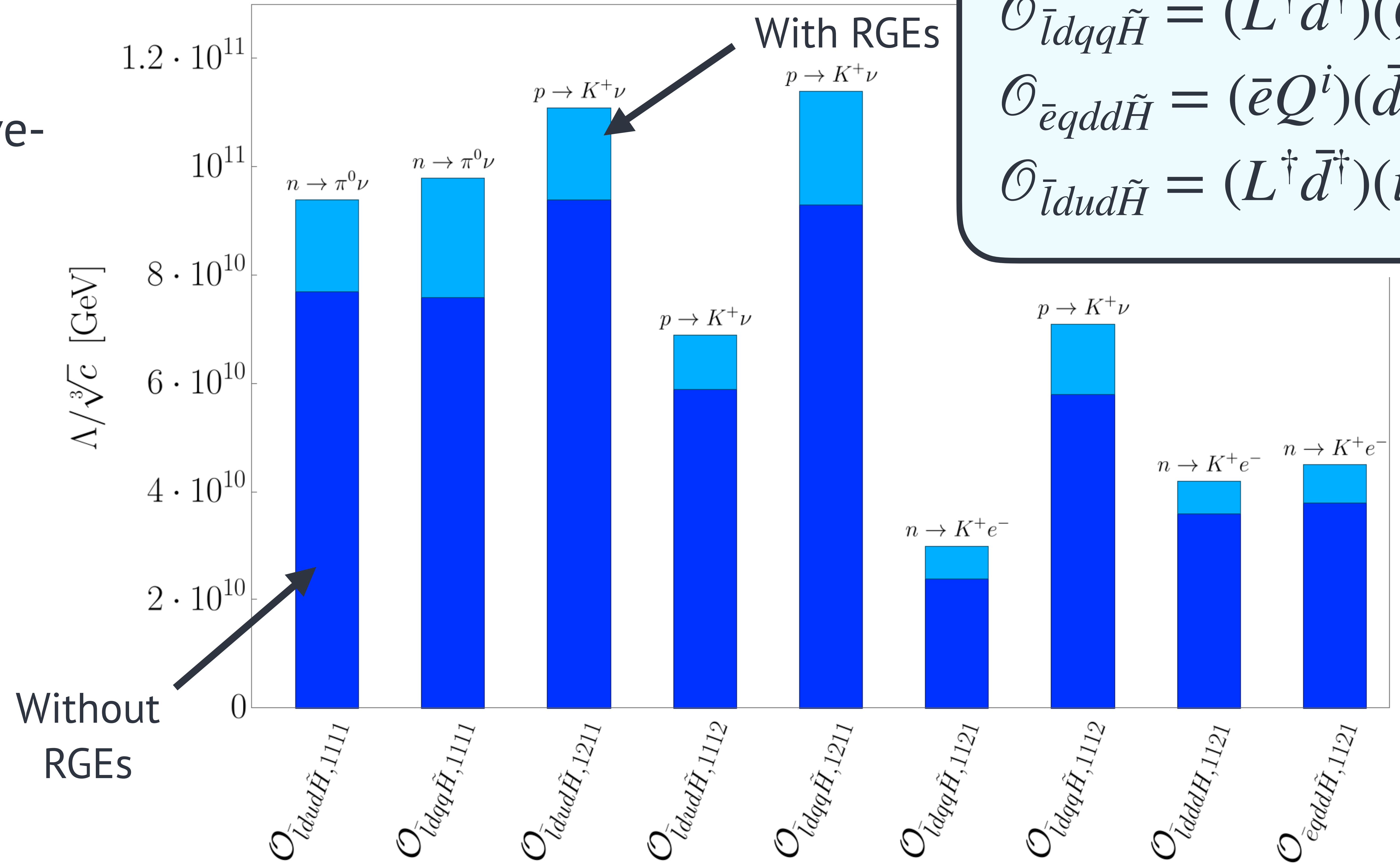
The effect is milder at dimension 7 because of an accidental cancellation

- Assume **single-operator dominance**
- Top-quark Yukawa relevant for Higgs wave-function renormalisation
- Expressions look like

$$16\pi^2 \mu \frac{d\mu}{dc_i} = (-4g_3^2 + y_t^2)c_i + \cdots$$

- 1.2 – 1.3 factor enhancement
- Strongest lower limit

$$\Lambda/\sqrt{c} > 2 \cdot 10^{10}\ \text{GeV}$$

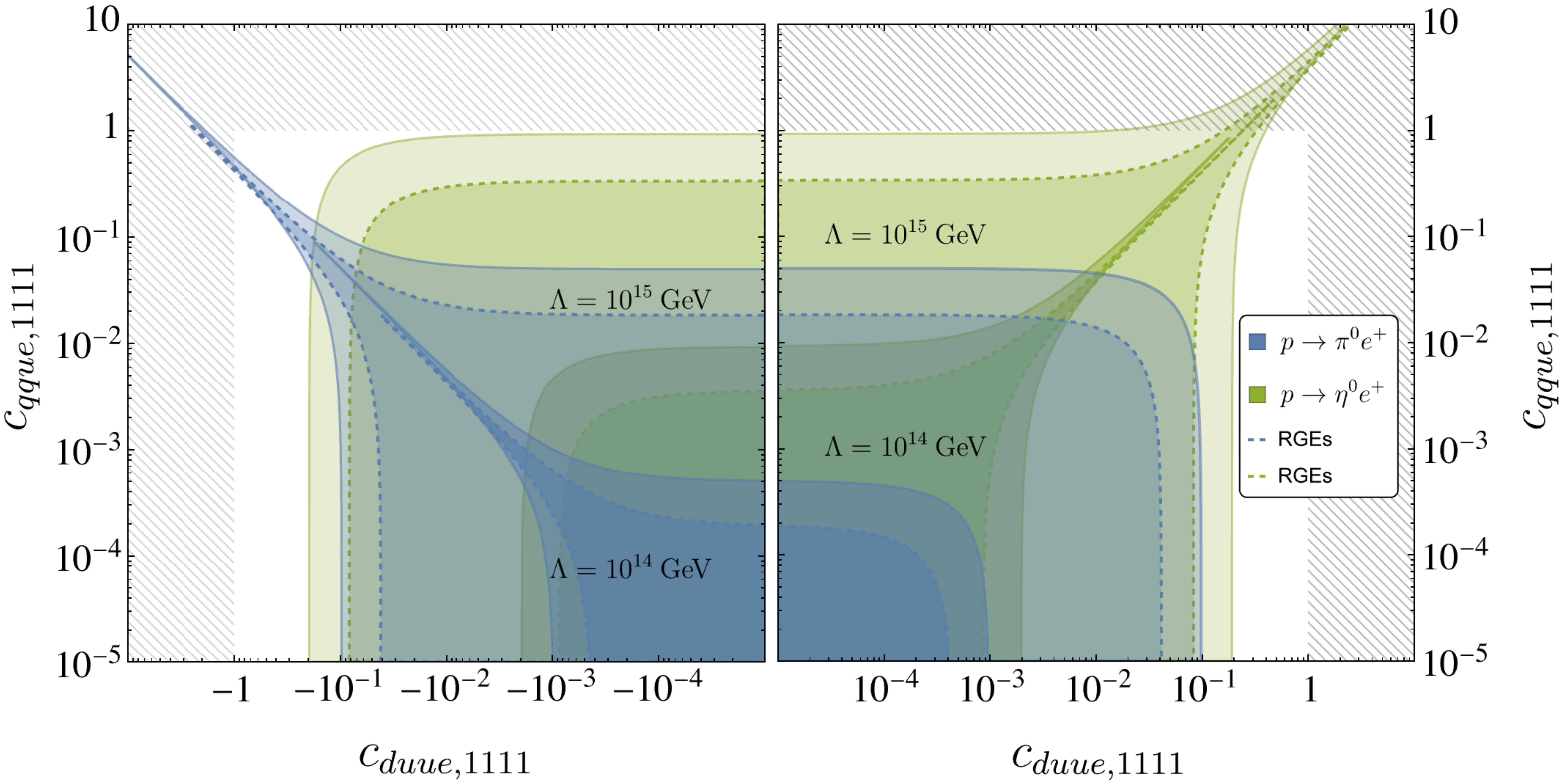


$\Delta B = -\Delta L = 1$

$$\mathcal{O}_{\bar{l}dddH} = (L^\dagger \bar{d}^\dagger)(\bar{d}^\dagger \bar{d}^\dagger)H$$
$$\mathcal{O}_{\bar{l}dqq\tilde{H}} = (L^\dagger \bar{d}^\dagger)(QQ^i)H_i^\dagger$$
$$\mathcal{O}_{\bar{e}qdd\tilde{H}} = (\bar{e}Q^i)(\bar{d}^\dagger \bar{d}^\dagger)H_i^\dagger$$
$$\mathcal{O}_{\bar{l}dud\tilde{H}} = (L^\dagger \bar{d}^\dagger)(\bar{u}^\dagger \bar{d}^\dagger)\tilde{H}$$

$d = 7$

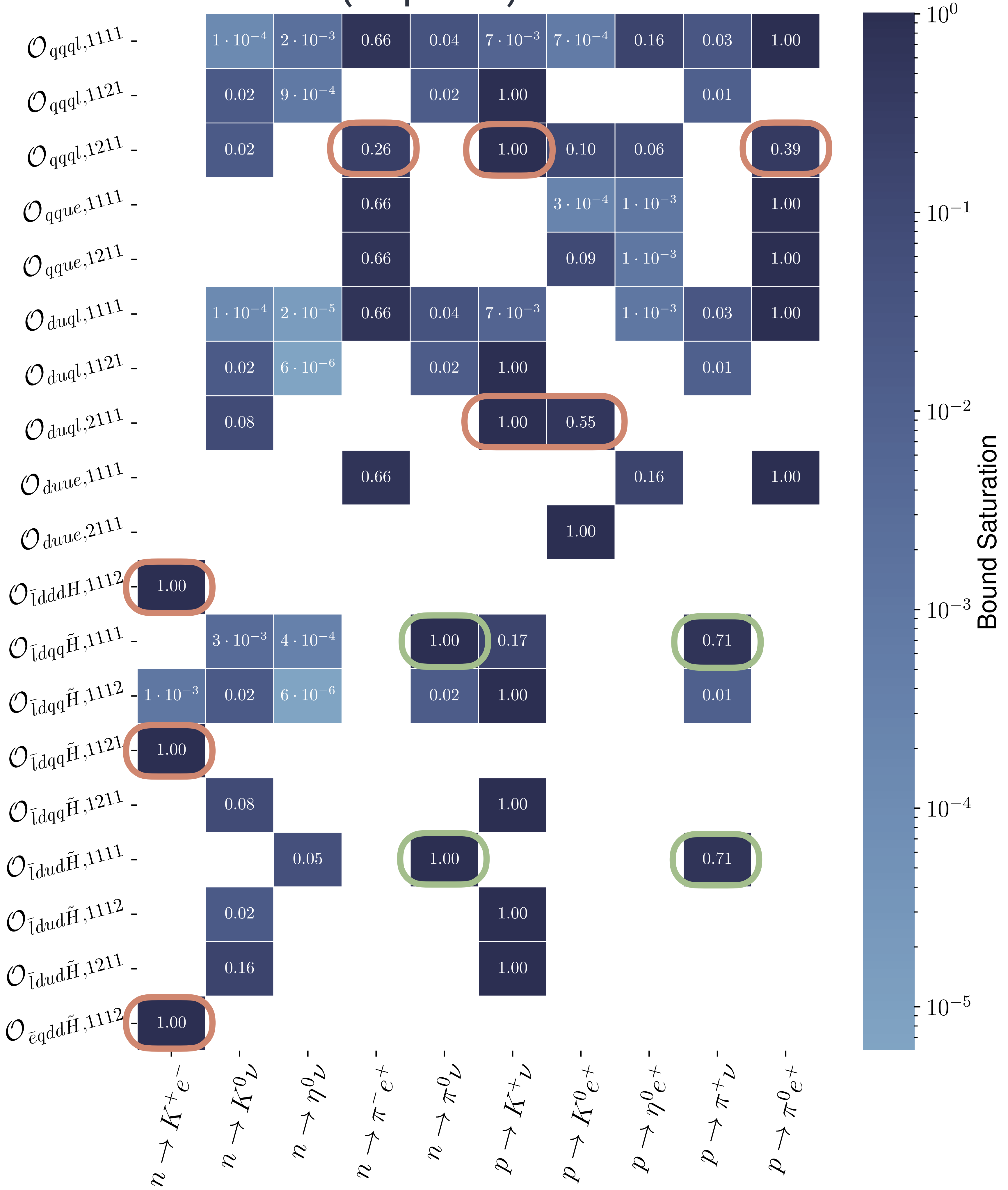
Pairs of non-zero Wilson coefficients show how different decay modes provide complementary constraints



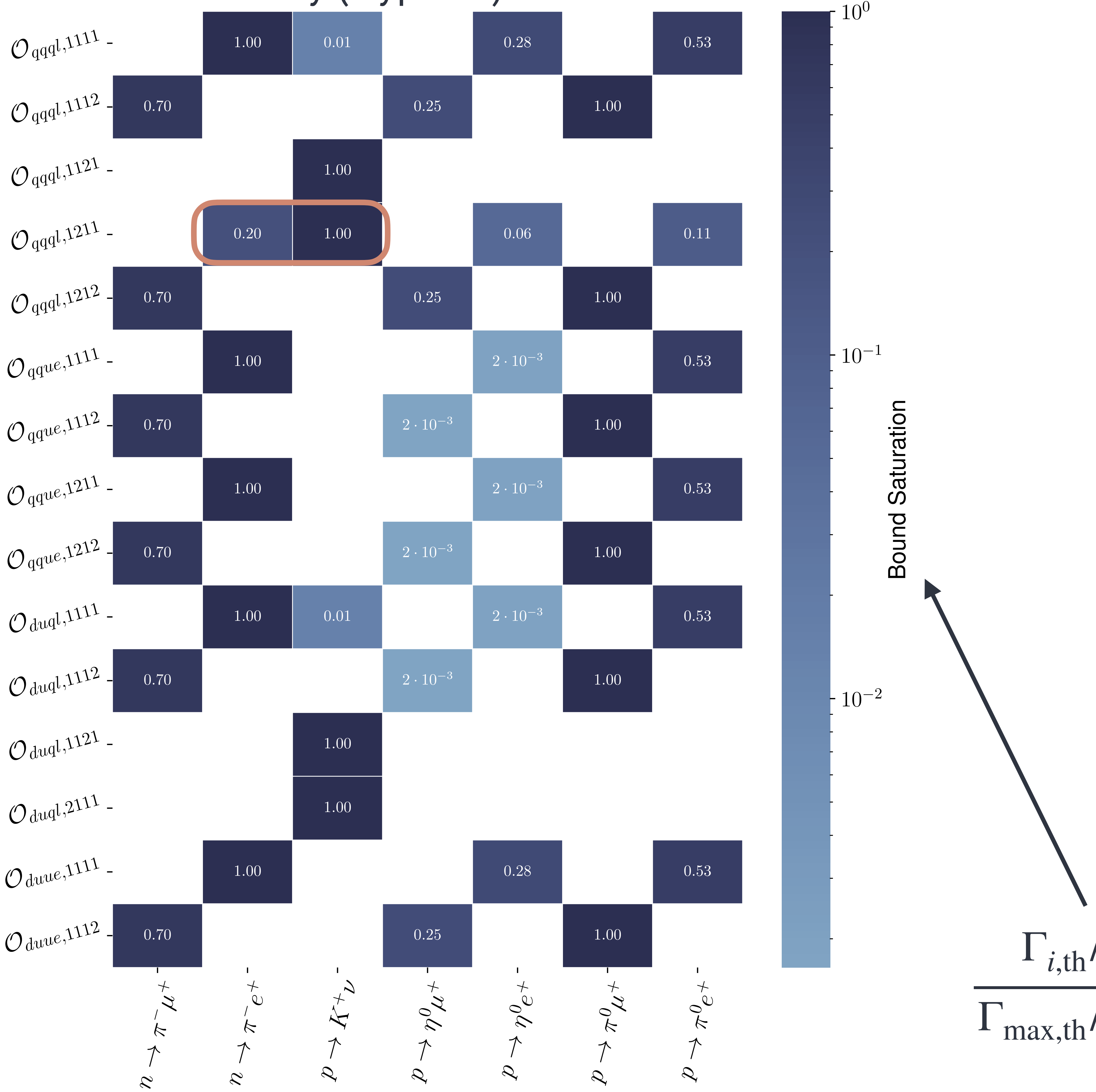
Several positive signals may allow us to exclude or determine if a single operator dominates

Recall uncertainties in α, β!

Current bounds (Super K)



Future sensitivity (Hyper K)



$$\frac{\Gamma_{i,\text{th}}/\Gamma_{i,\text{exp}}}{\Gamma_{\text{max,th}}/\Gamma_{\text{max,exp}}}$$

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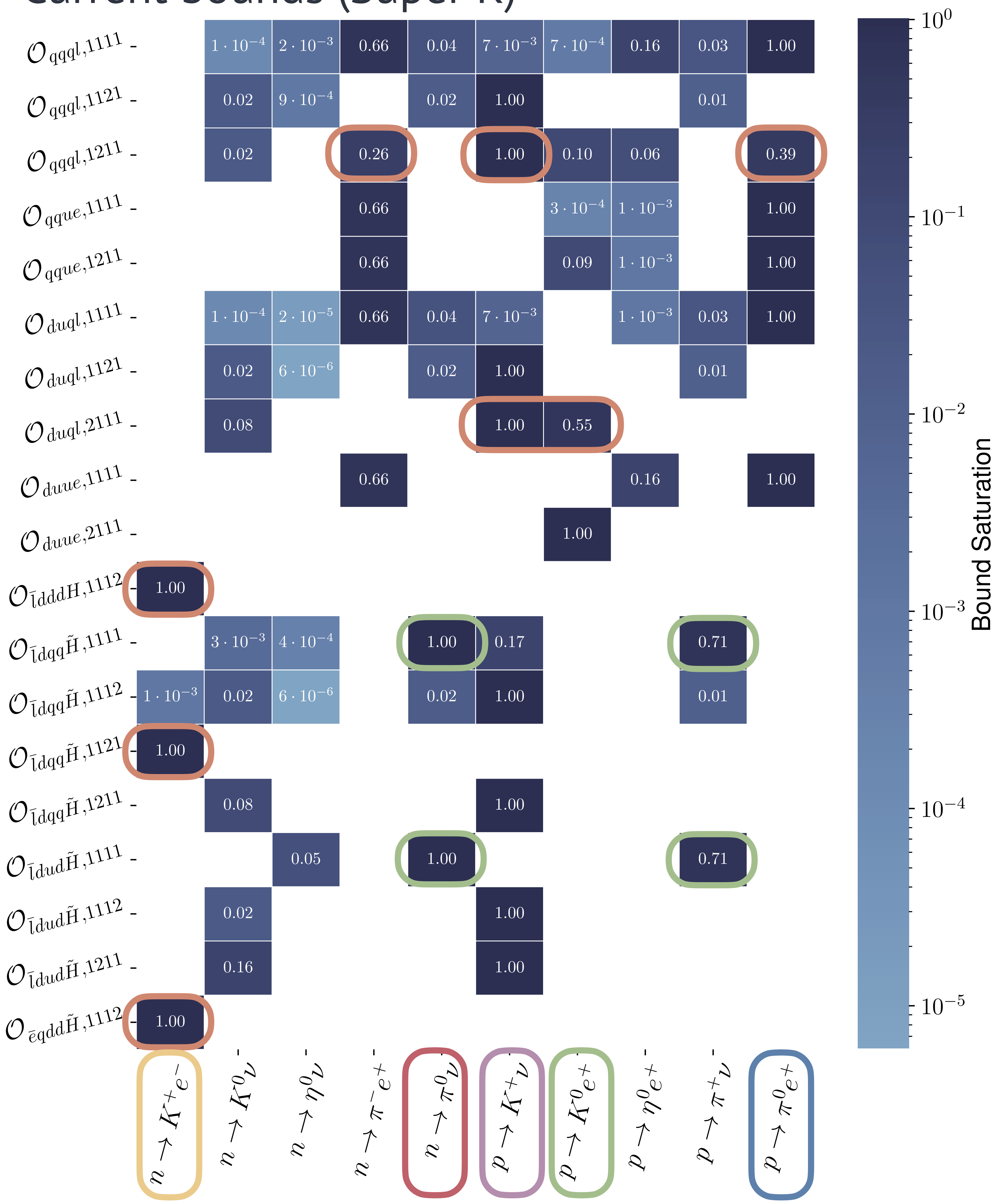
Current bounds (Super K)



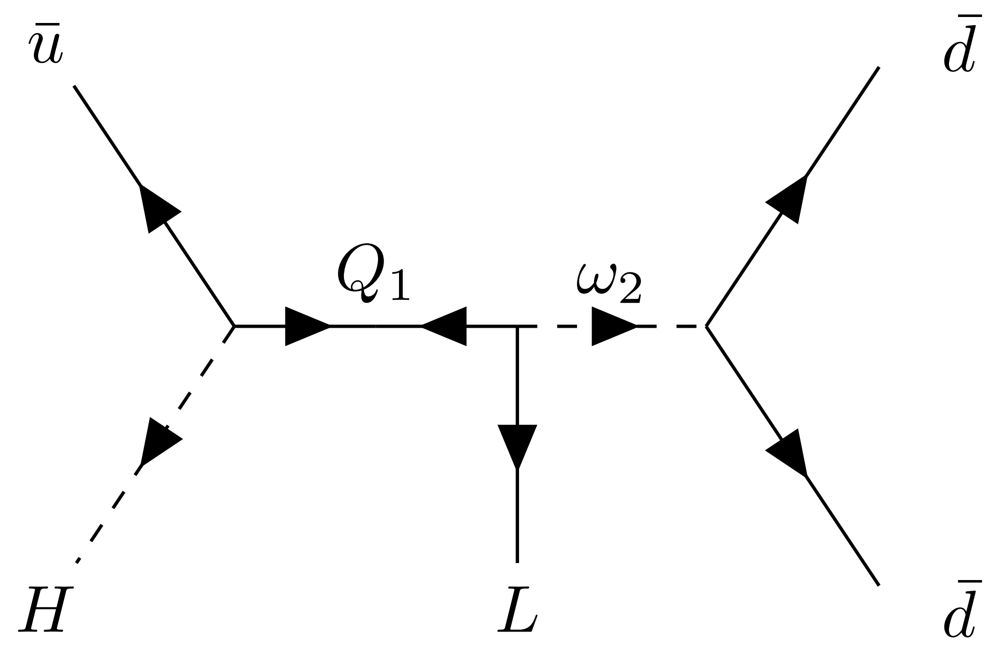
Decay mode	Limit [10^{34} yr]	Hyper-K [10^{34} yr]	ΔI	ΔS	ΔL	
Proton channels						
$p \to \pi^0 e^+$	2.4	3.2	$-\frac{1}{2}$	0	-1	
$p \to \pi^0 \mu^+$	1.6	7.7	$-\frac{1}{2}$	0	-1	1
$p \to \eta^0 e^+$	1.0	4.3	$-\frac{1}{2}$	0	-1	
$p \to \eta^0 \mu^+$	0.47	4.9	$-\frac{1}{2}$	0	-1	
$p \to \pi^+ \nu_r$	0.039	—	$\frac{1}{2}$	0	± 1	2
$p \to K^0 e^+$	0.10	—	-1	1	-1	3
$p \to K^0 \mu^+$	0.16	—	-1	1	-1	
$p \to K^+ \nu_r$	0.59	3.2	0	1	± 1	4
$p \to \bar{K}^0 e^+$	0.10	—	0	-1	-1	
$p \to \bar{K}^0 \mu^+$	0.16	—	0	-1	-1	
Neutron channels						
$n \to \pi^0 \nu_r$	0.11	—	$\frac{1}{2}$	0	± 1	
$n \to \eta^0 \nu_r$	0.016	—	$\frac{1}{2}$	0	± 1	
$n \to \pi^- e^+$	0.53	2.0	$-\frac{1}{2}$	0	-1	
$n \to \pi^- \mu^+$	0.35	1.8	$-\frac{1}{2}$	0	-1	
$n \to \pi^+ e^-$	0.0065	—	$\frac{3}{2}$	0	1	
$n \to \pi^+ \mu^-$	0.0049	—	$\frac{3}{2}$	0	1	
$n \to K^+ e^-$	0.0032	1.0	1	1	1	5
$n \to K^+ \mu^-$	0.0057	—	1	1	1	
$n \to K^0 \nu_r$	0.013	—	0	1	± 1	
$n \to K^- e^+$	0.0017	—	0	-1	-1	
$n \to \bar{K}^0 \nu_r$	0.013	—	1	-1	± 1	

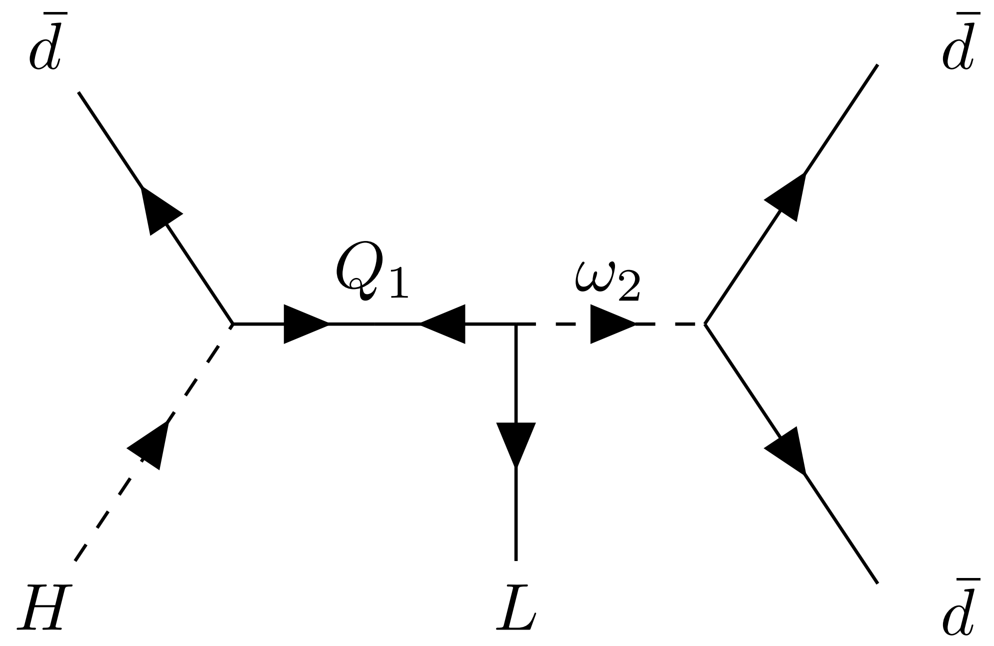
Decays to (anti-)leptons are generated at dimension (6) 7 in the SMEFT

Example UV model shows flavour is important

Introduce scalar LQ and vector-like fermion

$$\omega_2 \sim (\mathbf{3}, \mathbf{1}, \tfrac{2}{3}) \quad Q_1 + \bar{Q}_1^\dagger \sim (\mathbf{3}, \mathbf{2}, \tfrac{1}{6})$$



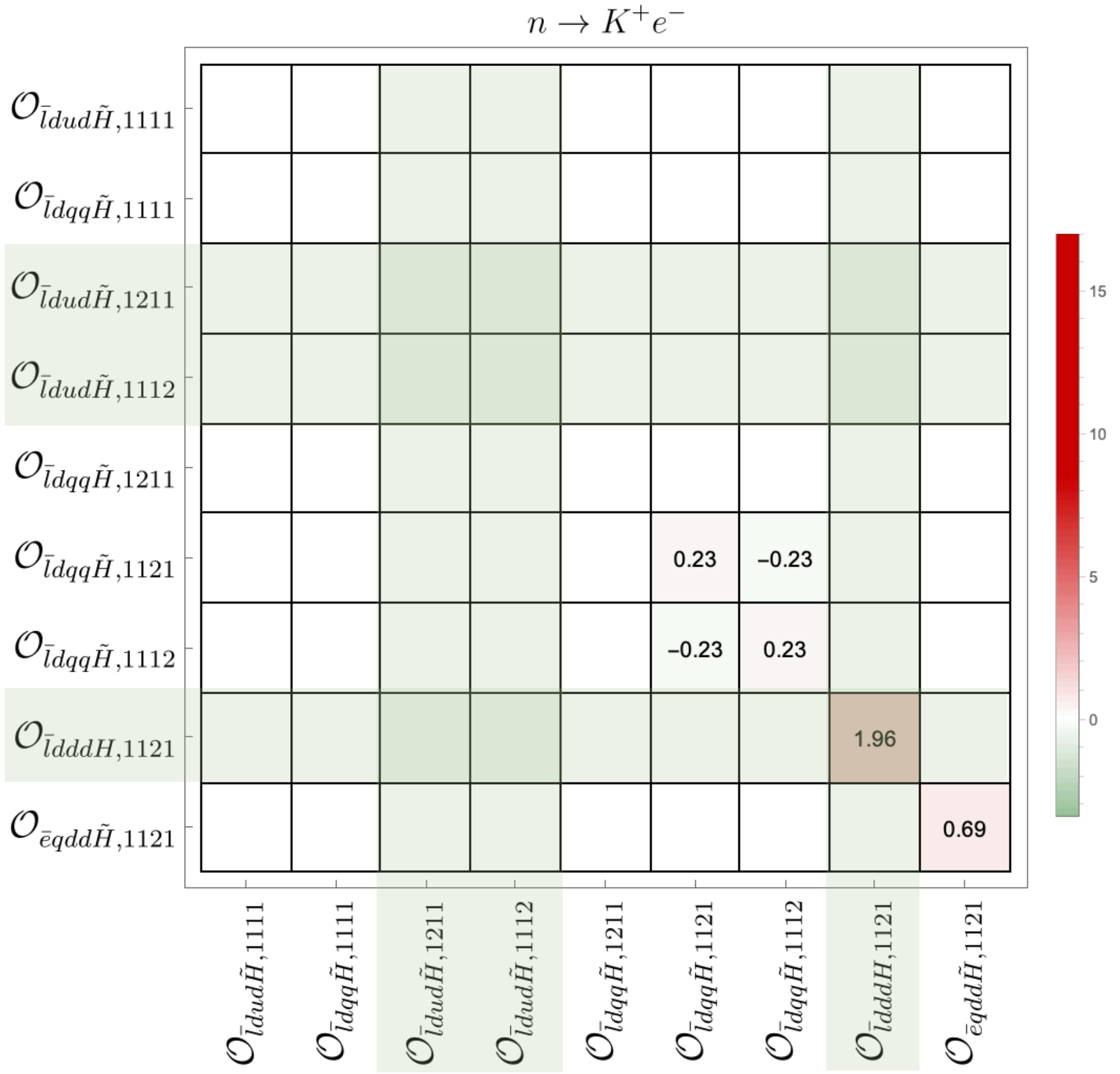


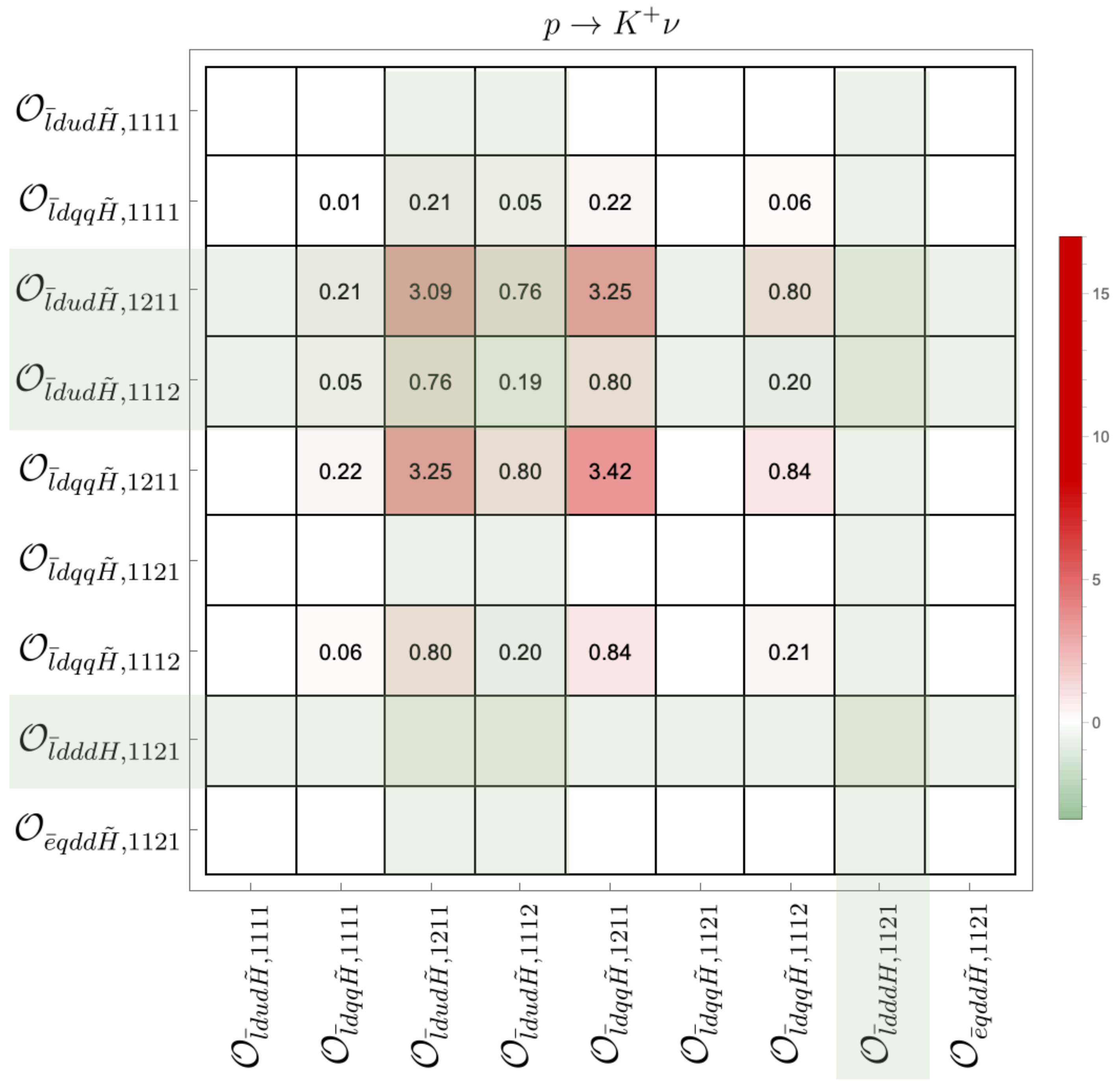
Model generates two $d = 7$ operators at tree level

$$\frac{c_{\bar{l}dddH}^{pqrs}}{\Lambda^3} = \frac{y_{dd}^{[rs]} y_{dH}^{q*} y_{LQ_1}^{p*}}{M_{\omega_2}^2 M_Q} \qquad \frac{c_{\bar{l}dud\tilde{H}}^{pqrs}}{\Lambda^3} = 2\frac{y_{dd}^{[qs]} y_{uH}^{r*} y_{LQ_1}^{p*}}{M_{\omega_2}^2 M_Q}$$

$\mathcal{O}_{\bar{l}dddH}^{1211} : \; n \to K^+ e^-$

$\mathcal{O}_{\bar{l}dud\tilde{H}}^{1211,1112} : \; p \to K^+ \nu, \, n \to K^0 \nu$





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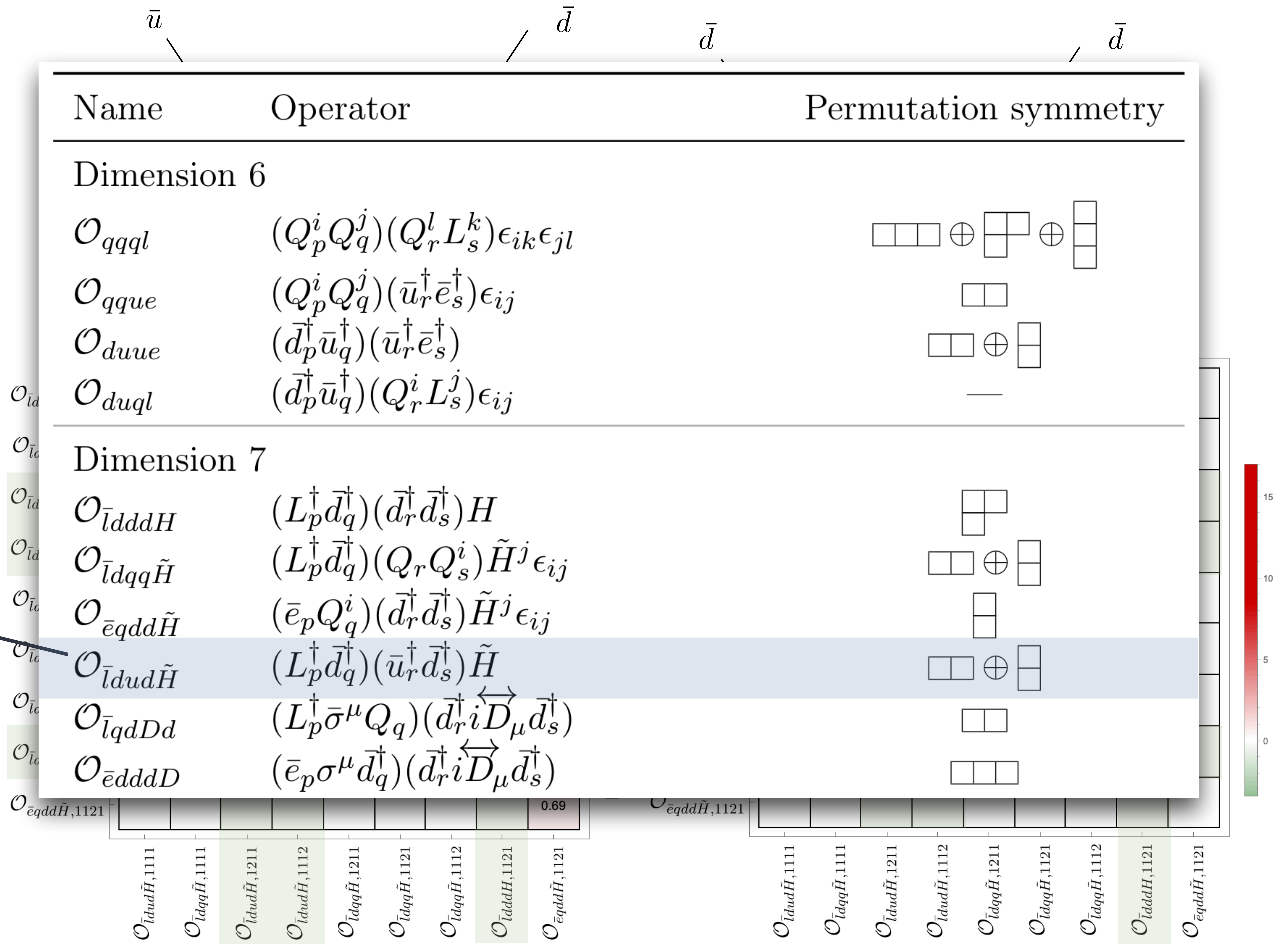
Model generates two $d = 7$ operators at tree level

$$\frac{c^{pqrs}_{\bar{l}dddH}}{\Lambda^3} = \frac{y^{[rs]}_{dd} y^{q*}_{dH} y^{p*}_{LQ_1}}{M^2_{\omega_2} M_Q} \qquad \frac{c^{pqrs}_{\bar{l}dud\tilde{H}}}{\Lambda^3} = 2\frac{y^{[qs]}_{dd} y^{r*}_{uH} y^{p*}_{LQ_1}}{M^2_{\omega_2} M_Q}$$

$\mathcal{O}^{1211}_{\bar{l}dddH} : \; n \to K^+ e^-$

$\mathcal{O}^{1211,1112}_{\bar{l}dud\tilde{H}} : \; p \to K^+ \nu, \, n \to K^0 \nu$



Name	Operator	Permutation symmetry
Dimension 6		
$\mathcal{O}_{qqql}$	$(Q^i_p Q^j_q)(Q^l_r L^k_s)\epsilon_{ik}\epsilon_{jl}$	$[3] \oplus [2,1] \oplus [1,1,1]$
$\mathcal{O}_{qque}$	$(Q^i_p Q^j_q)(\bar{u}^\dagger_r \bar{e}^\dagger_s)\epsilon_{ij}$	$[2]$
$\mathcal{O}_{duue}$	$(\bar{d}^\dagger_p \bar{u}^\dagger_q)(\bar{u}^\dagger_r \bar{e}^\dagger_s)$	$[2] \oplus [1,1]$
$\mathcal{O}_{duql}$	$(\bar{d}^\dagger_p \bar{u}^\dagger_q)(Q^i_r L^j_s)\epsilon_{ij}$	—
Dimension 7		
$\mathcal{O}_{\bar{l}dddH}$	$(L^\dagger_p \bar{d}^\dagger_q)(\bar{d}^\dagger_r \bar{d}^\dagger_s)H$	$[2,1]$
$\mathcal{O}_{\bar{l}dqq\tilde{H}}$	$(L^\dagger_p \bar{d}^\dagger_q)(Q_r Q^i_s)\tilde{H}^j \epsilon_{ij}$	$[2] \oplus [1,1]$
$\mathcal{O}_{\bar{e}qdd\tilde{H}}$	$(\bar{e}_p Q^i_q)(\bar{d}^\dagger_r \bar{d}^\dagger_s)\tilde{H}^j \epsilon_{ij}$	$[1,1]$
$\mathcal{O}_{\bar{l}dud\tilde{H}}$	$(L^\dagger_p \bar{d}^\dagger_q)(\bar{u}^\dagger_r \bar{d}^\dagger_s)\tilde{H}$	$[2] \oplus [1,1]$
$\mathcal{O}_{\bar{l}qdDd}$	$(L^\dagger_p \bar{\sigma}^\mu Q_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s)$	$[2]$
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}_p \sigma^\mu \bar{d}^\dagger_q)(\bar{d}^\dagger_r i\overleftrightarrow{D}_\mu \bar{d}^\dagger_s)$	$[3]$

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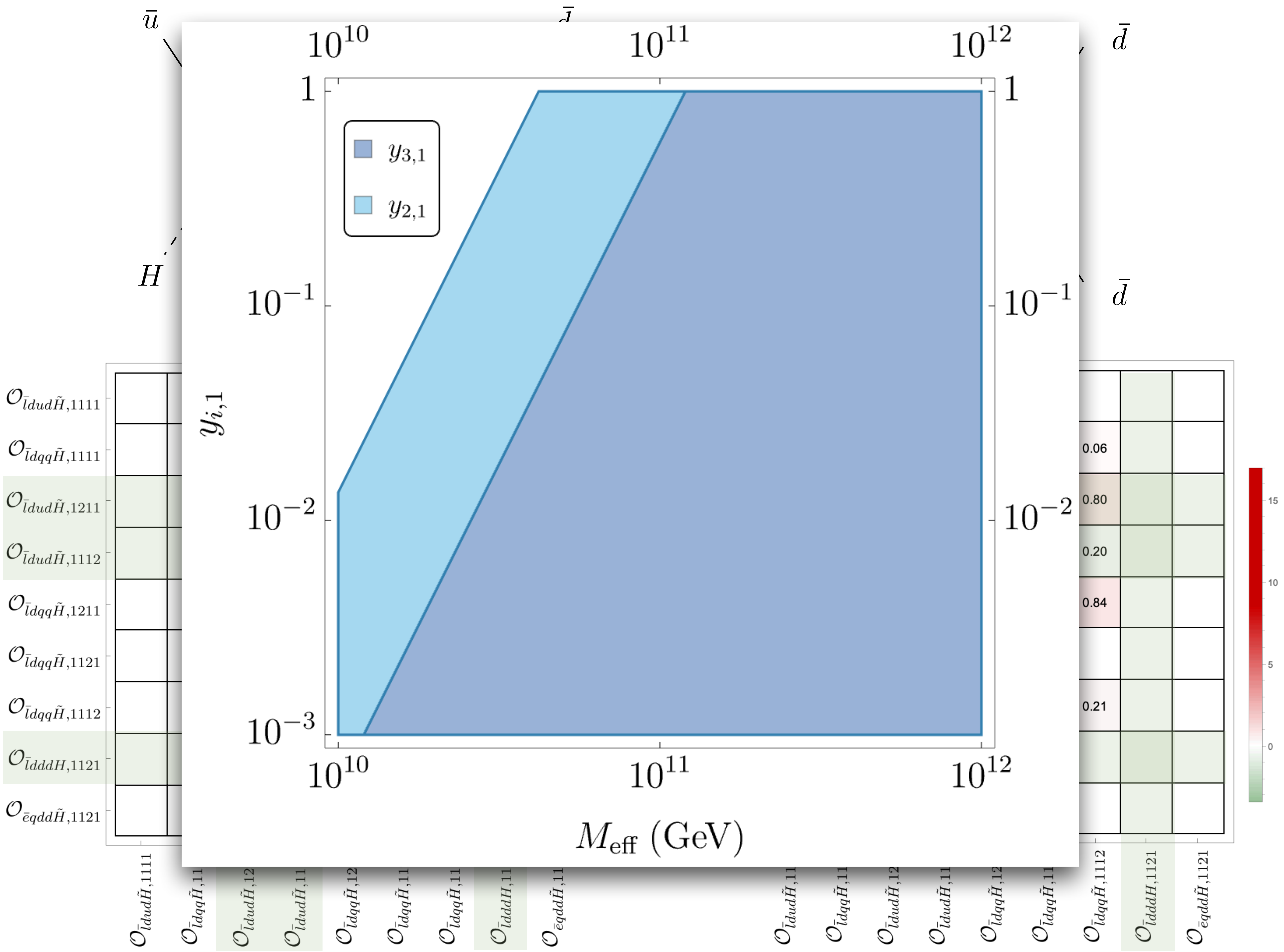
$$\frac{c_{\bar{l}dddH}^{pqrs}}{\Lambda^3} = \frac{y_{dd}^{[rs]} y_{dH}^{q*} y_{LQ_1}^{p*}}{M_{\omega_2}^2 M_Q} \qquad \frac{c_{\bar{l}dud\tilde{H}}^{pqrs}}{\Lambda^3} = 2\frac{y_{dd}^{[qs]} y_{uH}^{r*} y_{LQ_1}^{p*}}{M_{\omega_2}^2 M_Q}$$

$\mathcal{O}_{\bar{l}dddH}^{1211} : \; n \to K^+ e^-$

$\mathcal{O}_{\bar{l}dud\tilde{H}}^{1211,1112} : \; p \to K^+ \nu, \; n \to K^0 \nu$



Conclusions

- Depending on **symmetries**, dominant contributions from either $d = 6$ ($B - L = 0$) or $d = 7$ ($B - L = 2$)
- **RG corrections are important**, limits enhanced by up to factor of 2.3
- Complementary constraints **exclude flat directions**
- Several positive signals may allow us to **determine the origin of baryon-number violation**
- **Caution:** Uncertainty on hadronic inputs is large

$$p \to \pi^0 e^+$$

$$p \to \gamma e^+$$

$$n \to \pi^0$$

Thanks for your attention

$p \to \pi^0 e^+$

$p \to \gamma e^+$

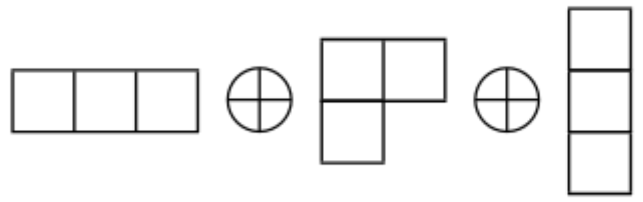
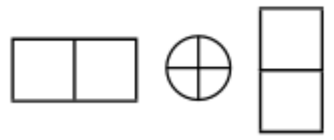
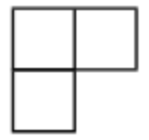
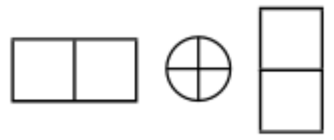
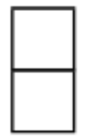
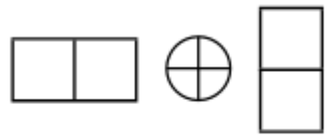
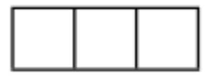
$n \to \pi^0$

$n \to$

p

n

Backup

Name	Operator	Permutation symmetry
Dimension 6		
$\mathcal{O}_{qqql}$	$(Q_p^i Q_q^j)(Q_r^l L_s^k)\epsilon_{ik}\epsilon_{jl}$	
$\mathcal{O}_{qque}$	$(Q_p^i Q_q^j)(\bar{u}_r^\dagger \bar{e}_s^\dagger)\epsilon_{ij}$	
$\mathcal{O}_{duue}$	$(\bar{d}_p^\dagger \bar{u}_q^\dagger)(\bar{u}_r^\dagger \bar{e}_s^\dagger)$	
$\mathcal{O}_{duql}$	$(\bar{d}_p^\dagger \bar{u}_q^\dagger)(Q_r^i L_s^j)\epsilon_{ij}$	—
Dimension 7		
$\mathcal{O}_{\bar{l}dddH}$	$(L_p^\dagger \bar{d}_q^\dagger)(\bar{d}_r^\dagger \bar{d}_s^\dagger)H$	
$\mathcal{O}_{\bar{l}dqq\tilde{H}}$	$(L_p^\dagger \bar{d}_q^\dagger)(Q_r Q_s^i)\tilde{H}^j\epsilon_{ij}$	
$\mathcal{O}_{\bar{e}qdd\tilde{H}}$	$(\bar{e}_p Q_q^i)(\bar{d}_r^\dagger \bar{d}_s^\dagger)\tilde{H}^j\epsilon_{ij}$	
$\mathcal{O}_{\bar{l}dud\tilde{H}}$	$(L_p^\dagger \bar{d}_q^\dagger)(\bar{u}_r^\dagger \bar{d}_s^\dagger)\tilde{H}$	
$\mathcal{O}_{\bar{l}qdDd}$	$(L_p^\dagger \bar{\sigma}^\mu Q_q)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger)$	
$\mathcal{O}_{\bar{e}dddD}$	$(\bar{e}_p \sigma^\mu \bar{d}_q^\dagger)(\bar{d}_r^\dagger i\overleftrightarrow{D}_\mu \bar{d}_s^\dagger)$	

Some two-body decays proceed through dimension-7 LEFT operators

Decay mode	Limit [10^{34} yr]	Hyper-K [10^{34} yr]	ΔI	ΔS	ΔL	
Proton channels						
$p \to \pi^0 e^+$	2.4	3.2	$-\frac{1}{2}$	0	-1	
$p \to \pi^0 \mu^+$	1.6	7.7	$-\frac{1}{2}$	0	-1	
$p \to \eta^0 e^+$	1.0	4.3	$-\frac{1}{2}$	0	-1	
$p \to \eta^0 \mu^+$	0.47	4.9	$-\frac{1}{2}$	0	-1	
$p \to \pi^+ \nu_r$	0.039	—	$\frac{1}{2}$	0	± 1	
$p \to K^0 e^+$	0.10	—	-1	1	-1	
$p \to K^0 \mu^+$	0.16	—	-1	1	-1	
$p \to K^+ \nu_r$	0.59	3.2	0	1	± 1	
$p \to \bar{K}^0 e^+$	0.10	—	0	-1	-1	5
$p \to \bar{K}^0 \mu^+$	0.16	—	0	-1	-1	
Neutron channels						
$n \to \pi^0 \nu_r$	0.11	—	$\frac{1}{2}$	0	± 1	
$n \to \eta^0 \nu_r$	0.016	—	$\frac{1}{2}$	0	± 1	
$n \to \pi^- e^+$	0.53	2.0	$-\frac{1}{2}$	0	-1	
$n \to \pi^- \mu^+$	0.35	1.8	$-\frac{1}{2}$	0	-1	
$n \to \pi^+ e^-$	0.0065	—	$\frac{3}{2}$	0	1	6
$n \to \pi^+ \mu^-$	0.0049	—	$\frac{3}{2}$	0	1	
$n \to K^+ e^-$	0.0032	1.0	1	1	1	
$n \to K^+ \mu^-$	0.0057	—	1	1	1	
$n \to K^0 \nu_r$	0.013	—	0	1	± 1	
$n \to K^- e^+$	0.0017	—	0	-1	-1	
$n \to \bar{K}^0 \nu_r$	0.013	—	1	-1	± 1	

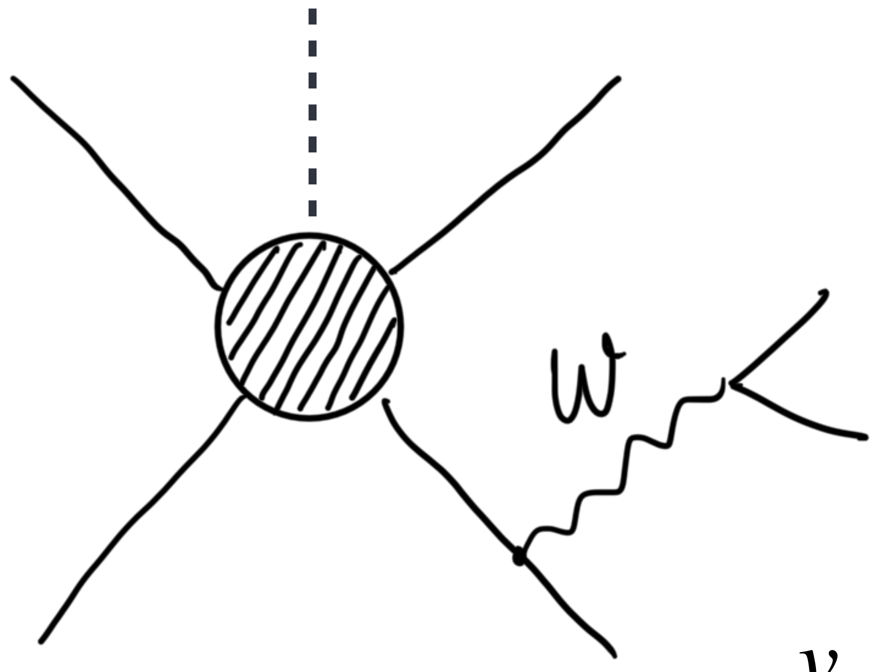
Decays to (anti-)leptons are generated at dimension (6) 7 in the SMEFT

Name	Ref. [56]	Operator	Flavour	Indices	ΔI	ΔS
$[\mathcal{O}^{V,RL}_{dud}]_{pqrs}$	$\mathcal{O}^*_{d\nu udD1}$	$(\bar{d}_p^\dagger i\overleftrightarrow{D}_\mu \bar{u}_q^\dagger)(\nu_r^\dagger \bar{\sigma}^\mu d_s)$	$(\mathbf{3}, \mathbf{6})$	$11r1$	$\frac{1}{2}$	0
				$21r1$	0	1
				$11r2$	0	1
$[\mathcal{O}^{V,LL}_{ddu}]_{\{pq\}rs}$	$\mathcal{O}^*_{u\nu dD1}$	$(d_p i\overleftrightarrow{D}_\mu d_q)(\nu_r^\dagger \bar{\sigma}^\mu u_s)$	$(\mathbf{10}, \mathbf{1})$	$11r1$	$\frac{1}{2}$	0
				$12r1$	0	1
$[\mathcal{O}^{V,RL}_{ddu}]_{\{pq\}rs}$	$\mathcal{O}^*_{u\nu dD2}$	$(\bar{d}_p^\dagger i\overleftrightarrow{D}_\mu \bar{d}_q^\dagger)(\nu_r^\dagger \bar{\sigma}^\mu u_s)$	$(\mathbf{3}, \mathbf{6})$	$11r1$	$\frac{1}{2}$	0
				$12r1$	0	1
$[\mathcal{O}^{V,LL}_{ddd}]^{\square\square\square}_{pqrs}$	$\mathcal{O}^*_{dedD1}$	$(d_p i\overleftrightarrow{D}_\mu d_q)(e_r^\dagger \bar{\sigma}^\mu d_s)$	$(\mathbf{10}, \mathbf{1})$	$111r$	$\frac{3}{2}$	0
				$121r$	1	1
$[\mathcal{O}^{V,RL}_{ddd}]_{\{pq\}rs}$	$\mathcal{O}^*_{dedD2}$	$(\bar{d}_p^\dagger i\overleftrightarrow{D}_\mu \bar{d}_q^\dagger)(e_r^\dagger \bar{\sigma}^\mu d_s)$	$(\mathbf{3}, \mathbf{6})$	$11r1$	$\frac{3}{2}$	0
				$12r1$	1	1
				$11r2$	1	1
$[\mathcal{O}^{V,LR}_{ddd}]_{\{pq\}rs}$	$\mathcal{O}^*_{dedD3}$	$(d_p i\overleftrightarrow{D}_\mu d_q)(\bar{e}_r \sigma^\mu \bar{d}_s^\dagger)$	$(\mathbf{6}, \mathbf{3})$	$11r1$	$\frac{3}{2}$	0
				$12r1$	1	1
				$11r2$	1	1
$[\mathcal{O}^{V,RR}_{ddd}]^{\square\square\square}_{pqrs}$	$\mathcal{O}^*_{dedD4}$	$(\bar{d}_p^\dagger i\overleftrightarrow{D}_\mu \bar{d}_q^\dagger)(\bar{e}_r \sigma^\mu \bar{d}_s^\dagger)$	$(\mathbf{1}, \mathbf{10})$	$111r$	$\frac{3}{2}$	0
				$121r$	1	1

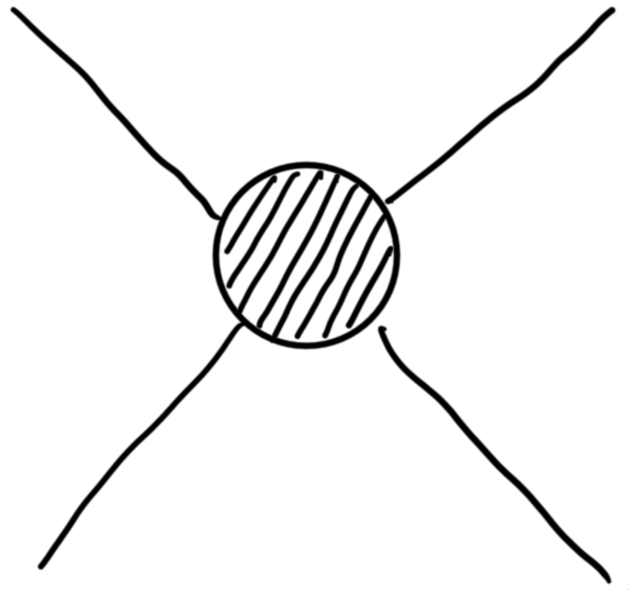
$$\Delta B = -\Delta L = -1$$

Dimension-7 LEFT operators generated at dimension-7 in the SMEFT

[56]: Liao, Ma, Wang 2005.08013



$$\mathscr{A}^{d=6} \sim C^{d=7}_{\text{SMEFT}} \cdot G_F \frac{v}{\Lambda}$$



$$\mathscr{A}^{d=7} \sim C^{d=7}_{\text{SMEFT}} \frac{p}{\Lambda}$$

$$\mathcal{O}_{\bar{l}qdDd} = (L^\dagger \bar{\sigma}_\mu Q)(\bar{d}^\dagger i D^\mu \bar{d}^\dagger)$$

$$\mathcal{O}_{\bar{e}dddD} = (\bar{e}\sigma_\mu \bar{d}^\dagger)(\bar{d}^\dagger i D^\mu \bar{d}^\dagger)$$

Direct and indirect methods of calculation roughly agree, huge improvement

- Uncertainties have improved a lot over the past decade!
- Direct method can give lifetimes 2-3x larger

  ⇒ 40–70% weaker constraints on the scale of $d = 6$ coeffs

  ⇒ 26–44% weaker constraints on the scale of $d = 7$ coeffs

Aoki, et al. 0806.1031

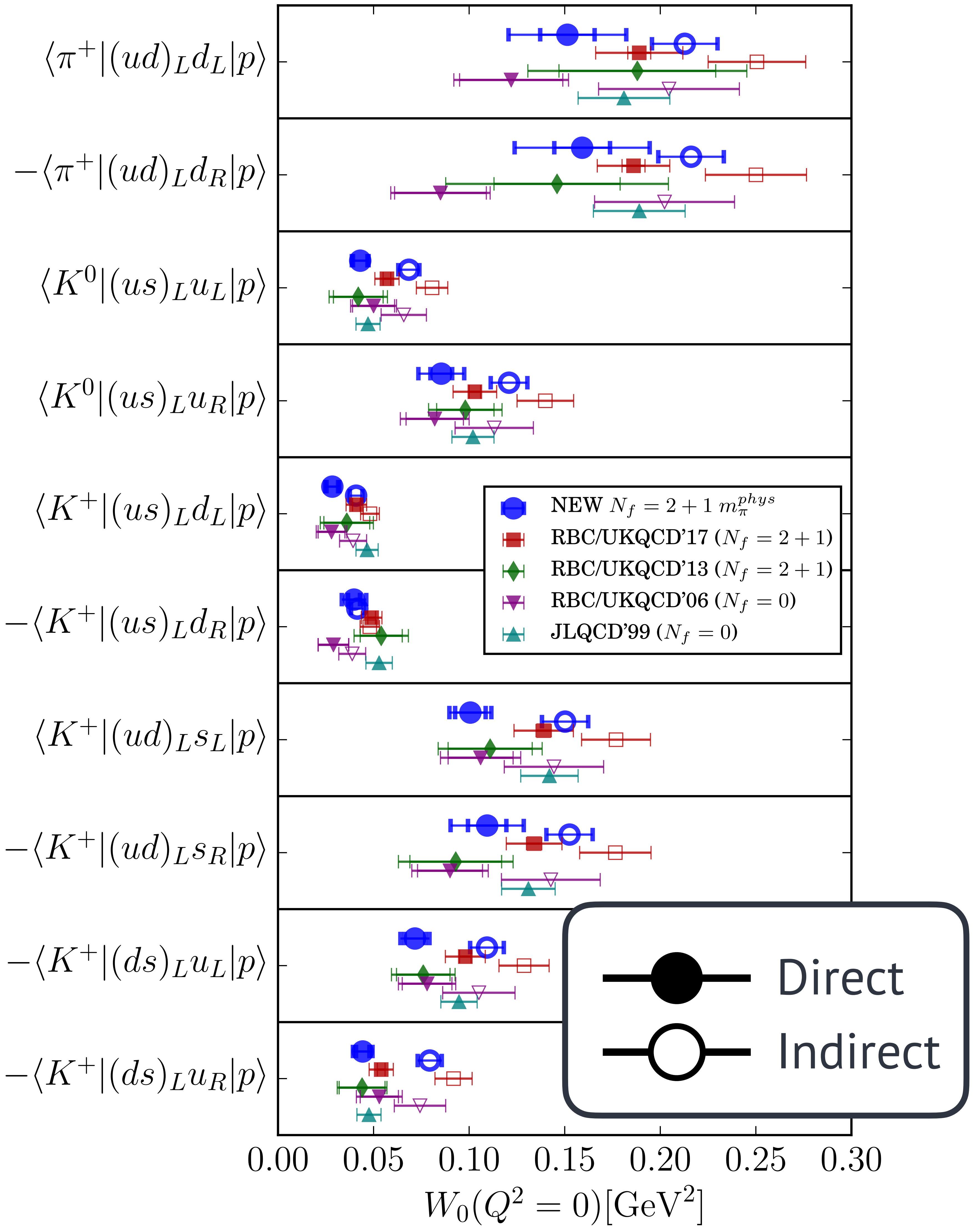
$$\alpha = -0.0112 \pm 0.0012_{(\mathrm{stat})} \pm 0.0022_{(\mathrm{syst})}\ \mathrm{GeV}^3$$
$$\beta = 0.0120 \pm 0.0013_{(\mathrm{stat})} \pm 0.0023_{(\mathrm{syst})}\ \mathrm{GeV}^3.$$

$\sim 22\,\%$ uncertainty

$\sim 9\,\%$ uncertainty

	24ID	32ID	cont.
α	$-0.0999(59)$	$-0.01106(39)$	$-0.01257(111)$
β	$0.01020(57)$	$0.01117(42)$	$0.01269(107)$

Yoo, et al. 2111.01608



Aoki et al. (JLQCD) hep-lat/9911026
Aoki, Dawson, Noaki, Soni hep-lat/0607002
Aoki, Izubuchi, Shintani, Soni 1705.01338
Yoo, et al. 2111.01608

We match onto the BχPT using operator symmetries

$\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \qquad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix}$$

$$\xi \equiv e^{iM/f_\pi} \to L\xi U^\dagger = U\xi R^\dagger \qquad B \to UBU^\dagger$$

$\xi B \xi \sim (\mathbf{3}, \bar{\mathbf{3}})$

$\xi^\dagger B \xi^\dagger \sim (\bar{\mathbf{3}}, \mathbf{3})$

$\xi B \xi^\dagger \sim (\mathbf{8}, \mathbf{1})$

$\xi^\dagger B \xi \sim (\mathbf{1}, \mathbf{8})$

$$[\xi B \xi^\dagger \nu_r]^k_l \sim (q_i q_j)(q_l \nu_r)\epsilon^{ijk} - \frac{1}{3}(q_i q_j)(q_m \nu_r)\epsilon^{ijm}\delta^k_l \supset [\mathcal{O}^{S,LL}_{udd}]_{111r},\ [\mathcal{O}^{S,LL}_{udd}]_{121r},\ [\mathcal{O}^{S,LL}_{udd}]_{112r}$$

Projection matrix P_{ij} necessary to pick out component corresponding to single operator

Name [42]	Operator	Flavour	ΔI	ΔS
$[\mathcal{O}^{S,LL}_{udd}]_{111r}$	$(ud)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$	$\frac{1}{2}$	0
$[\mathcal{O}^{S,LL}_{udd}]_{121r}$	$(us)(d\nu_r)$	$(\mathbf{8},\mathbf{1})$	0	1
$[\mathcal{O}^{S,LL}_{udd}]_{112r}$	$(ud)(s\nu_r)$	$(\mathbf{8},\mathbf{1})$	0	1
$[\mathcal{O}^{S,LL}_{duu}]_{111r}$	$(du)(ue_r)$	$(\mathbf{8},\mathbf{1})$	$-\frac{1}{2}$	0
$[\mathcal{O}^{S,LL}_{duu}]_{211r}$	$(su)(ue_r)$	$(\mathbf{8},\mathbf{1})$	-1	1
$[\mathcal{O}^{S,LR}_{duu}]_{111r}$	$(du)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\bar{\mathbf{3}},\mathbf{3})$	$-\frac{1}{2}$	0
$[\mathcal{O}^{S,LR}_{duu}]_{211r}$	$(su)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\bar{\mathbf{3}},\mathbf{3})$	-1	1
$[\mathcal{O}^{S,RL}_{duu}]_{111r}$	$(\bar{d}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	$-\frac{1}{2}$	0
$[\mathcal{O}^{S,RL}_{duu}]_{211r}$	$(\bar{s}^\dagger\bar{u}^\dagger)(ue_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	-1	1
$[\mathcal{O}^{S,RL}_{dud}]_{111r}$	$(\bar{d}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	$\frac{1}{2}$	0
$[\mathcal{O}^{S,RL}_{dud}]_{211r}$	$(\bar{s}^\dagger\bar{u}^\dagger)(d\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
$[\mathcal{O}^{S,RL}_{dud}]_{112r}$	$(\bar{d}^\dagger\bar{u}^\dagger)(s\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
$[\mathcal{O}^{S,RL}_{ddu}]_{[12]1r}$	$(\bar{d}^\dagger\bar{s}^\dagger)(u\nu_r)$	$(\mathbf{3},\bar{\mathbf{3}})$	0	1
$[\mathcal{O}^{S,RR}_{duu}]_{111r}$	$(\bar{d}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\mathbf{1},\mathbf{8})$	$-\frac{1}{2}$	0
$[\mathcal{O}^{S,RR}_{duu}]_{211r}$	$(\bar{s}^\dagger\bar{u}^\dagger)(\bar{u}^\dagger\bar{e}^\dagger_r)$	$(\mathbf{1},\mathbf{8})$	-1	1

RGEs

$$\dot{C}_{duue,prst} = \left(-4g_3^2 - 2g_1^2\right) C_{duue,prst} - \frac{20}{3} g_1^2 \, C_{duue,psrt}$$

$$\dot{C}_{duq\ell,prst} = \left(-4g_3^2 - \frac{9}{2}g_2^2 - \frac{11}{6}g_1^2\right) C_{duq\ell,prst}$$

$$\dot{C}_{qque,prst} = \left(-4g_3^2 - \frac{9}{2}g_2^2 - \frac{23}{6}g_1^2\right) C_{qque,prst}$$

$$\dot{C}_{qqq\ell,prst} = \left(-4g_3^2 - 3g_2^2 - \frac{1}{3}g_1^2\right) C_{qqq\ell,prst} \boxed{- 4g_2^2 \left(C_{qqq\ell,rpst} + C_{qqq\ell,srpt} + C_{qqq\ell,psrt}\right)}$$

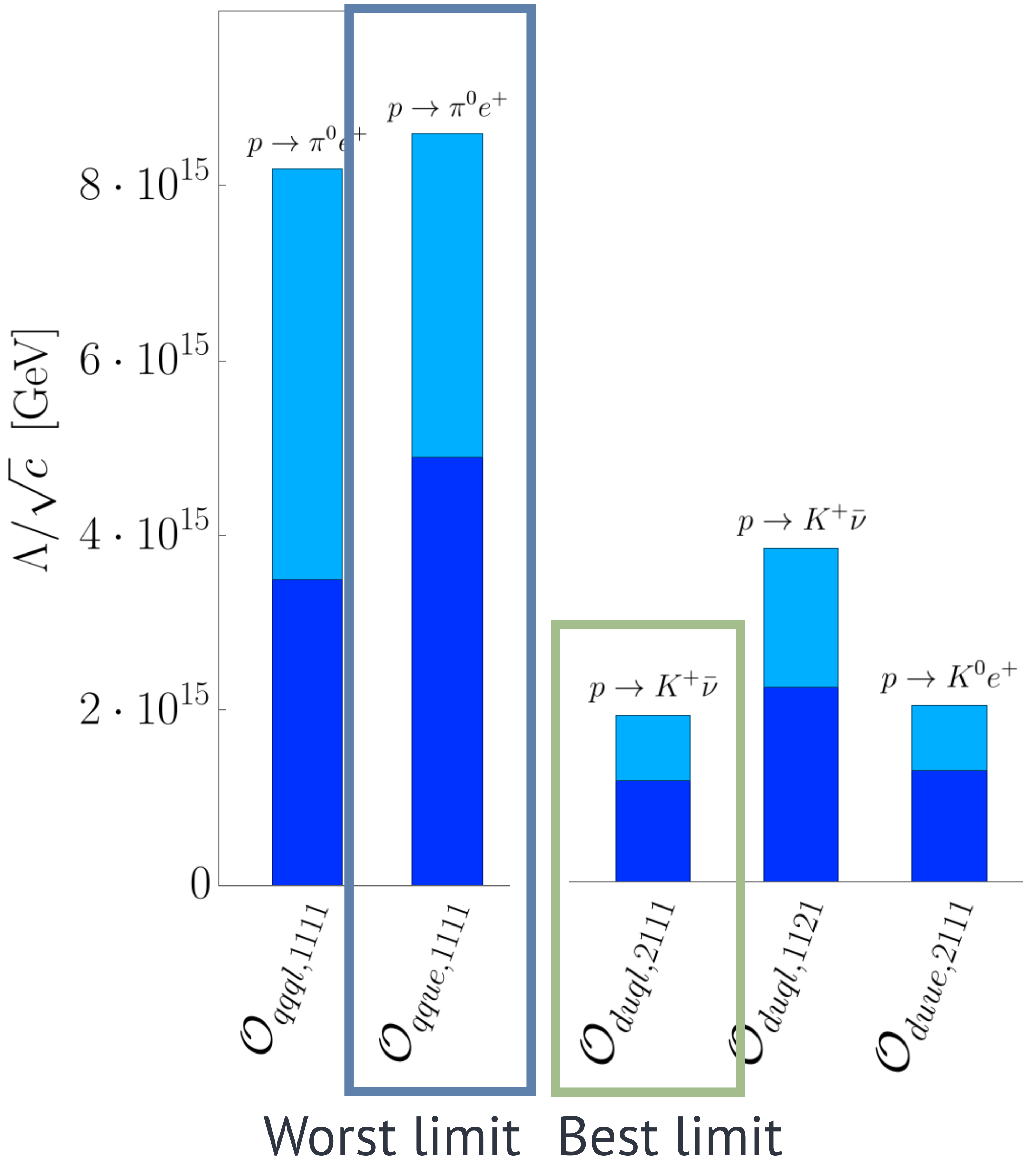
$$\dot{C}_{\bar{l}dud\tilde{H},prst} = \left(-4g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dud\tilde{H},prst} - \frac{10}{3} g_1^2 \, C_{\bar{l}dud\tilde{H},ptsr} \,,$$

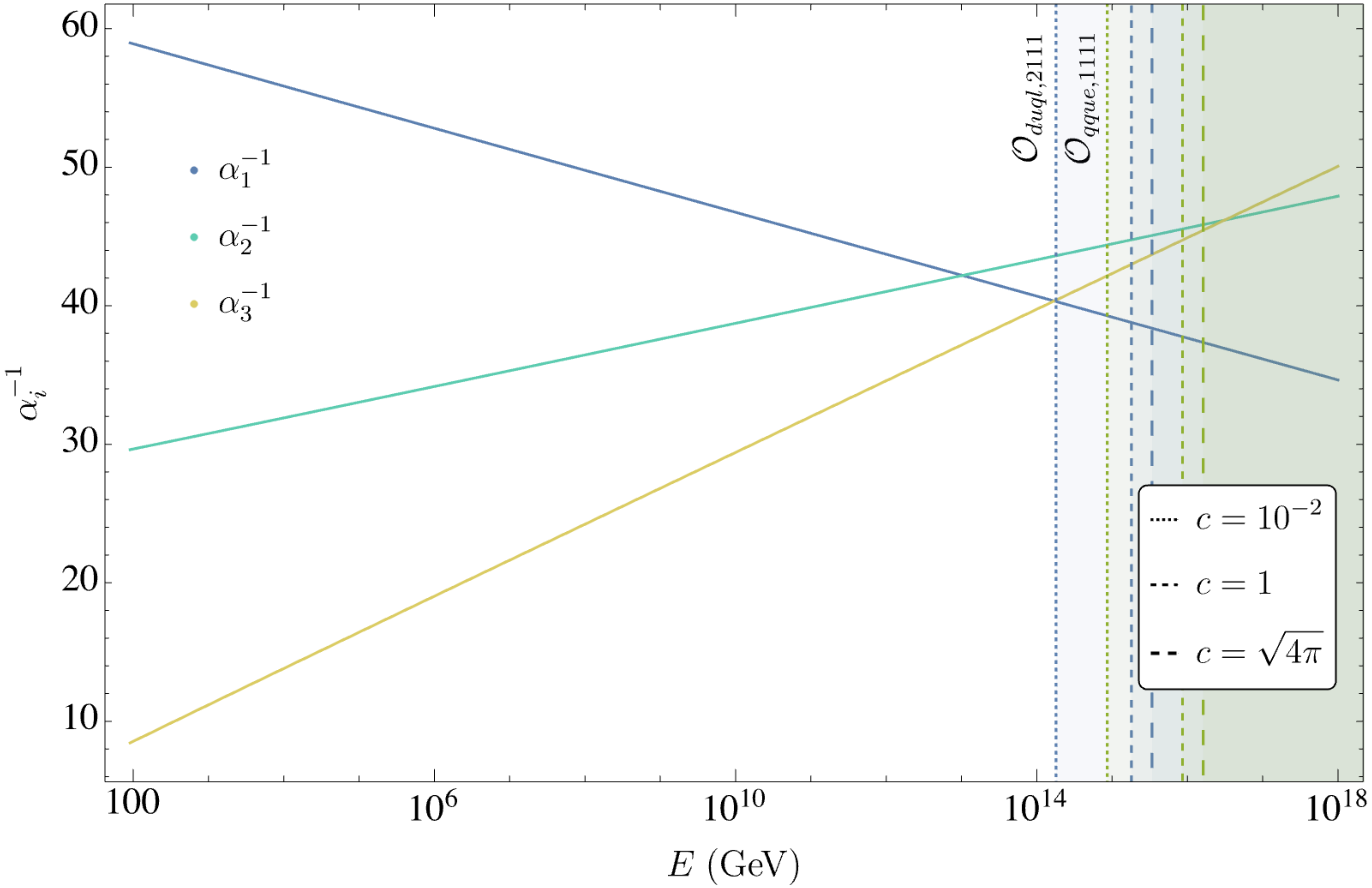
$$\dot{C}_{\bar{l}dddH,prst} = \left(- 4g_3^2 - \frac{9}{4}g_2^2 - \frac{13}{12}g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dddH,prst} \,,$$

$$\dot{C}_{\bar{e}qdd\tilde{H},prst} = \left(- 4g_3^2 - \frac{9}{4}g_2^2 + \frac{11}{12}g_1^2 \boxed{+ y_t^2}\right) C_{\bar{e}qdd\tilde{H},prst} \,,$$

$$\dot{C}_{\bar{l}dqq\tilde{H},prst} = \left(- 4g_3^2 - \frac{15}{4}g_2^2 - \frac{19}{12}g_1^2 \boxed{+ y_t^2}\right) C_{\bar{l}dqq\tilde{H},prst} - 3g_2^2 \, C_{\bar{l}dqq\tilde{H},prts} \,.$$

Limits compatible with gauge-coupling unification at the α_2, α_3 crossing for $c > 10^{-2}$





JG, Dutka, arXiv:2211.02054

A model with dim-7 proton decay: Low-scale Pati–Salam (1)

- At dimension 5, a set of Weinberg-like operators generate diquark couplings for χ

Field content	J	Number of operators
$\Phi^\dagger\Phi^\dagger\xi\xi$	0	$n_\xi(n_\xi+1)/2$
$\Phi^\dagger\chi^\dagger f_L\xi$	0	$n_\xi n_f$
$\Phi^\dagger\Phi\xi\xi$	0	$n_\xi(n_\xi+1)/2$
$\chi^\dagger\chi^\dagger f_L f_L$	0	$n_f(n_f+1)/2$
$\chi^\dagger\chi^\dagger f_R f_R$	0	$n_f(n_f+1)$
$\Phi\chi^\dagger f_L\xi$	0	$n_\xi n_f$
$\chi^\dagger\chi\xi\xi$	0	$n_\xi(n_\xi+1)/2$
$\chi\chi f_L f_L$	4	$n_f(n_f+1)/2$
$\chi\chi f_R f_R$	4	$n_f(n_f+1)/2$
$\Phi\Phi\xi\xi$	0	$n_\xi(n_\xi+1)/2$

Balanced antisymmetry

$$(\mathcal{O}_X)_{pq} = f_X^{\alpha i} f_X^{\beta j}\chi^{\gamma k}\chi^{\delta l}\epsilon_{\alpha\beta\gamma\delta}\epsilon_{ij}\epsilon_{kl}$$

$$\downarrow \langle\chi\rangle$$

$$\mathscr{L}_{\chi^d} \supset \frac{m_u}{v_R}\cdot\bar{\nu}\chi^{d\dagger}d_R + \sum_{X\in\{L,R\}}\frac{C_X v_R}{\sqrt{2}\Lambda}\cdot d_X u_X \chi^d + \text{h.c.}$$

$$\xi = \cos\theta N_L - \sin\theta\nu + \mathcal{O}(\mu/v_R)N_R^c$$

$$\sin\theta \approx \frac{m_u}{|y_R v_R|}$$

For now, just one generation

$$f_L = \begin{pmatrix} u_L & \nu_L \\ d_L & e_L \end{pmatrix} \sim (\mathbf{4},\mathbf{2},\mathbf{1})$$

$$f_R = \begin{pmatrix} u_R & \nu_R \\ d_R & e_R \end{pmatrix} \sim (\mathbf{4},\mathbf{1},\mathbf{2})$$

$$\chi = \begin{pmatrix} \chi^u & \chi^0 \\ \chi^d & \chi^- \end{pmatrix} \sim (\mathbf{4},\mathbf{1},\mathbf{2})$$

$$\xi \sim (\mathbf{1},\mathbf{1},\mathbf{1}) \quad \Phi \sim (\mathbf{1},\mathbf{2},\mathbf{2})$$

$$\mathrm{SU}(4)\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$$

$$\downarrow\langle\chi\rangle$$

$$\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y$$

JG, Dutka, arXiv:2211.02054

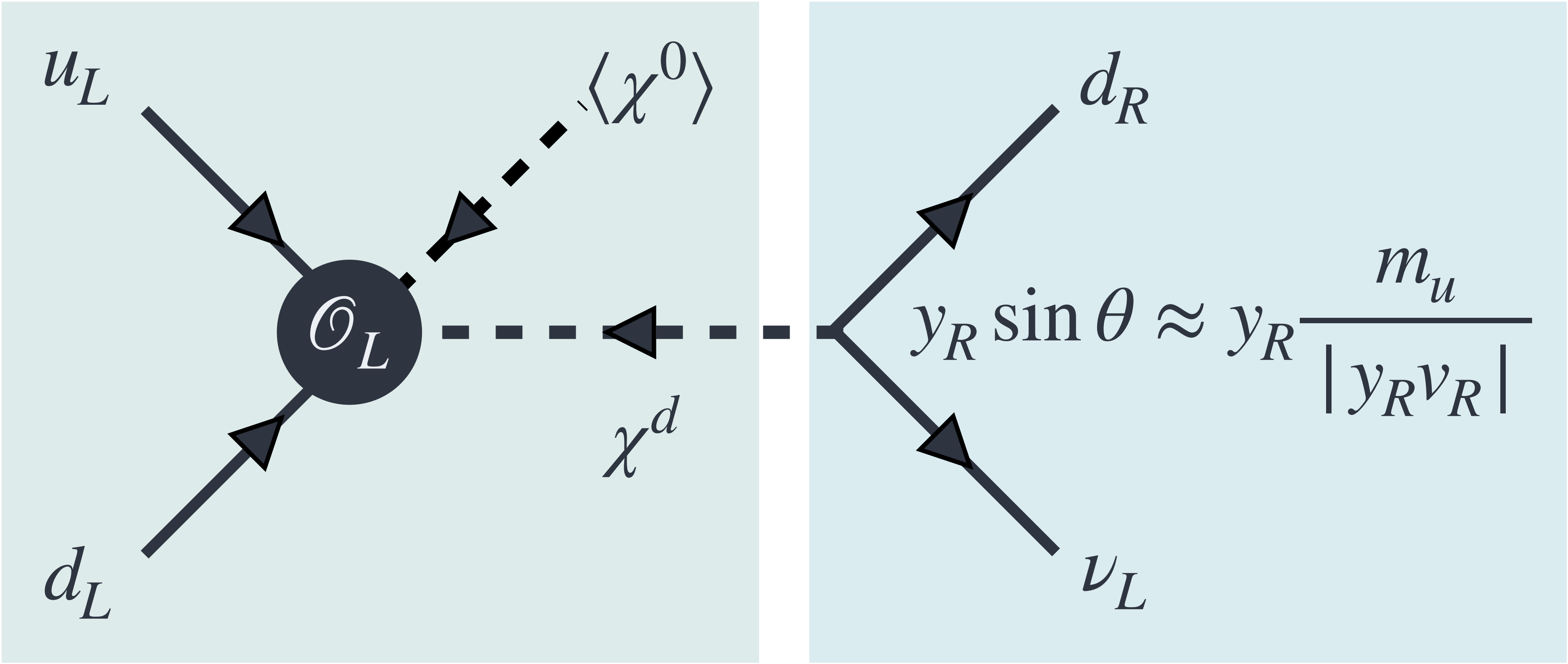
A model with dim-7 proton decay: Low-scale Pati–Salam (2)

- Integrating out χ^d from $\mathscr{L}_{\chi^d}$ gives two $B-L=2$ dimension-6 operators in the WET

$$\mathscr{L}_{\chi^d} \supset \frac{m_u}{v_R} \cdot \bar{\nu}\chi^{d\dagger} d_R + \sum_{X\in\{L,R\}} \frac{C_X v_R}{\sqrt{2}\Lambda} \cdot d_X u_X \chi^d + \text{ h.c.}$$

$$\mathscr{L}^{(6)}_{\text{eff}} \supset \sum_X \frac{C_X}{\Lambda} \frac{v_R}{\sqrt{2}} \frac{y_R \sin\theta}{M^2_{\chi^d}} \cdot (u_X d_X)(\bar{\nu}_L d_R) + \text{h.c.}$$

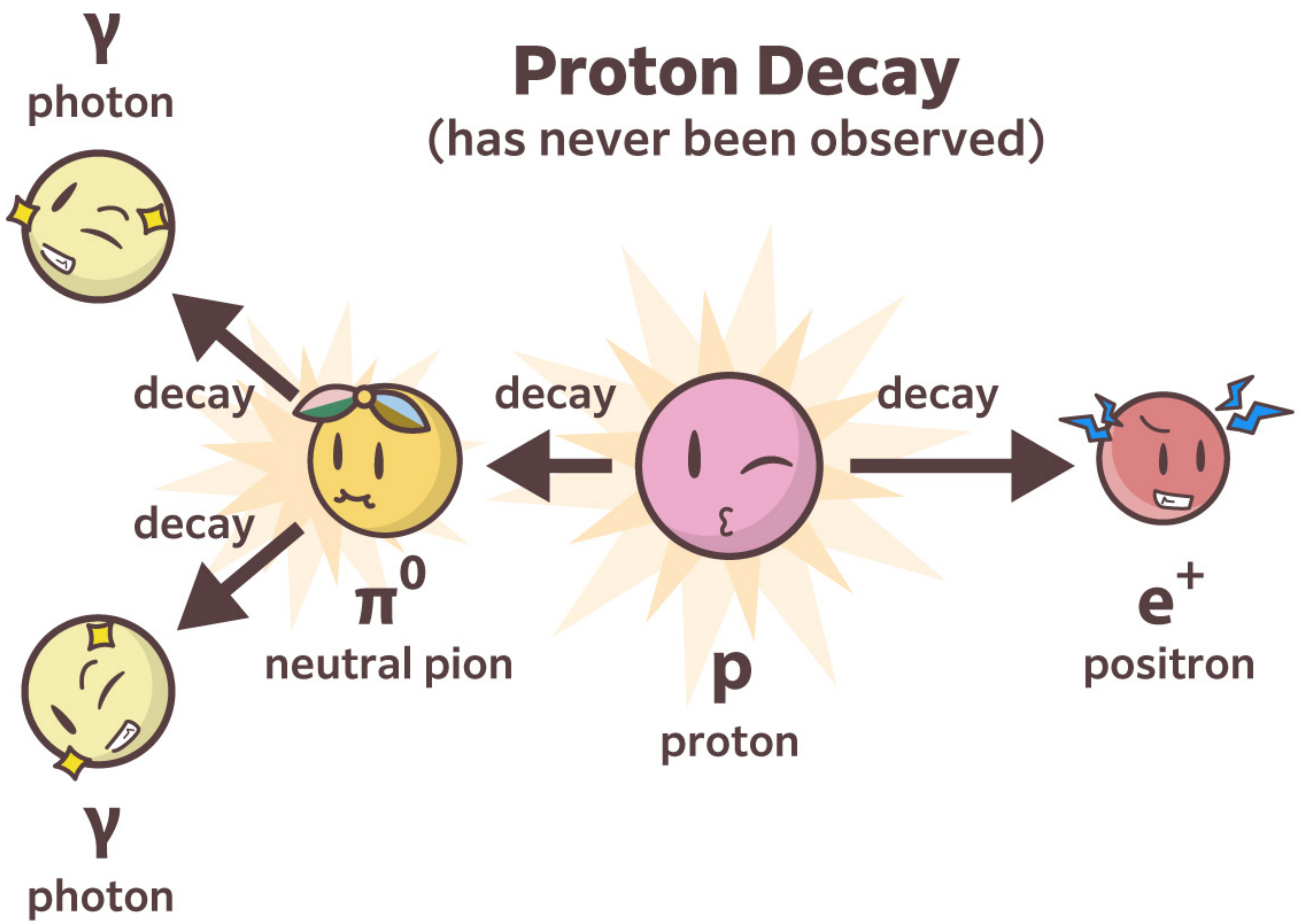
$$\sim \sum_X \frac{C_X}{\sqrt{2}} \frac{m_u}{\Lambda v_R^2} \cdot (u_X d_X)(\bar{\nu}_L d_R) + \text{h.c.} \quad \to \quad \mathscr{O}^{S,XR}_{udd}$$



WET basis: Jenkins, Manohar, Stoffera, arXiv:1709.04486

- Dimension-7 operators generated in the SMEFT: $(\bar{L}d_R)(u_R d_R)\tilde{H}$ and $(\bar{L}d_R)(QQ)\tilde{H}$

- Predict the dominant decay modes: $n \to \pi^0\nu$, $p \to \pi^+\nu$



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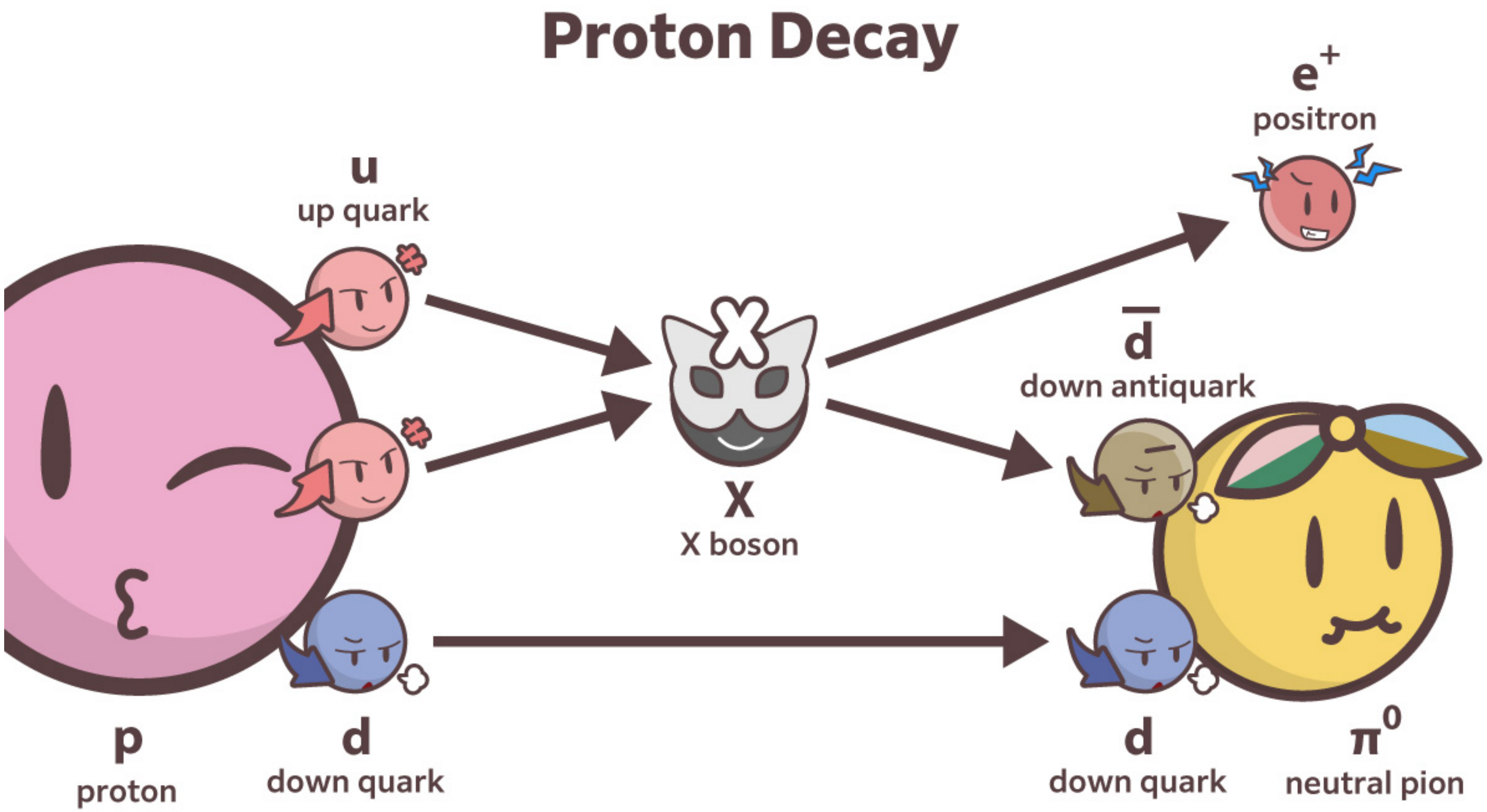

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