

Astrometry, gravitational waves and synergies with Pulsar Timing Arrays

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*Based on work in progress with Marisol Cruz,
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Outline

PTA SGWB detection

SGWB and Astrometry

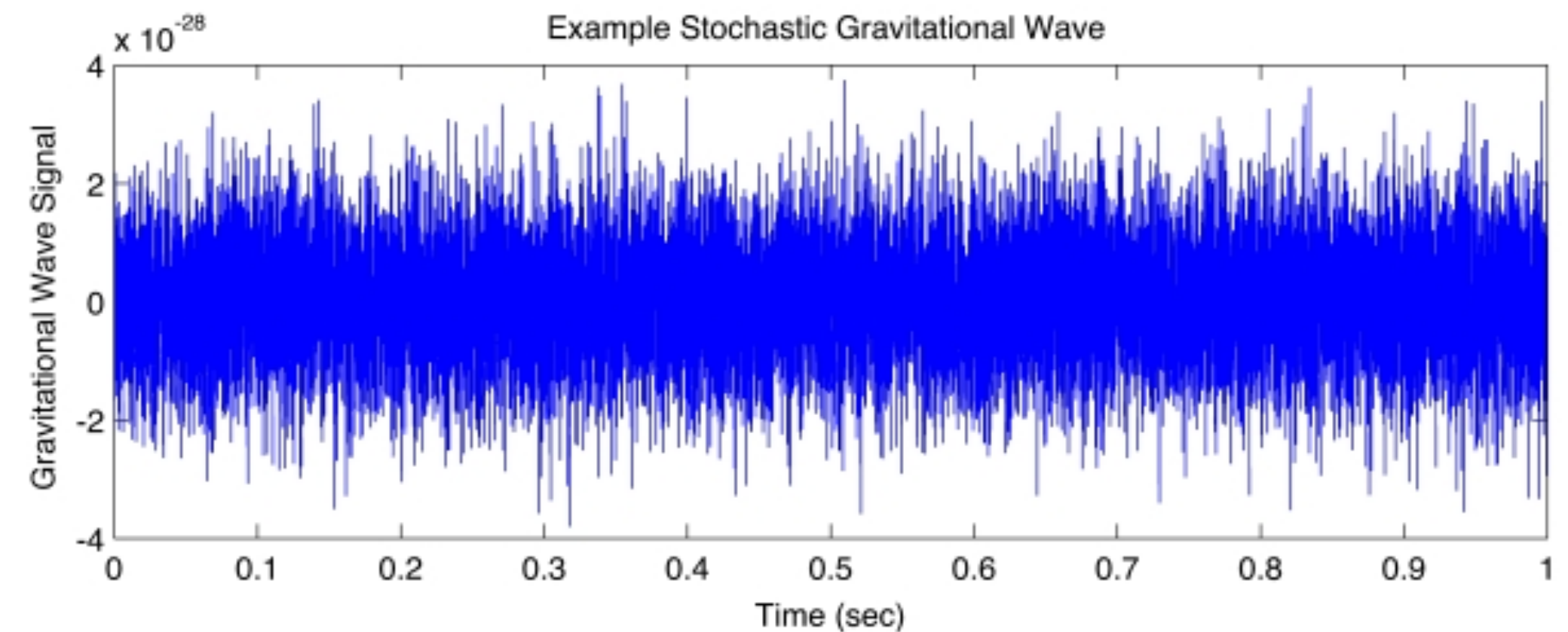
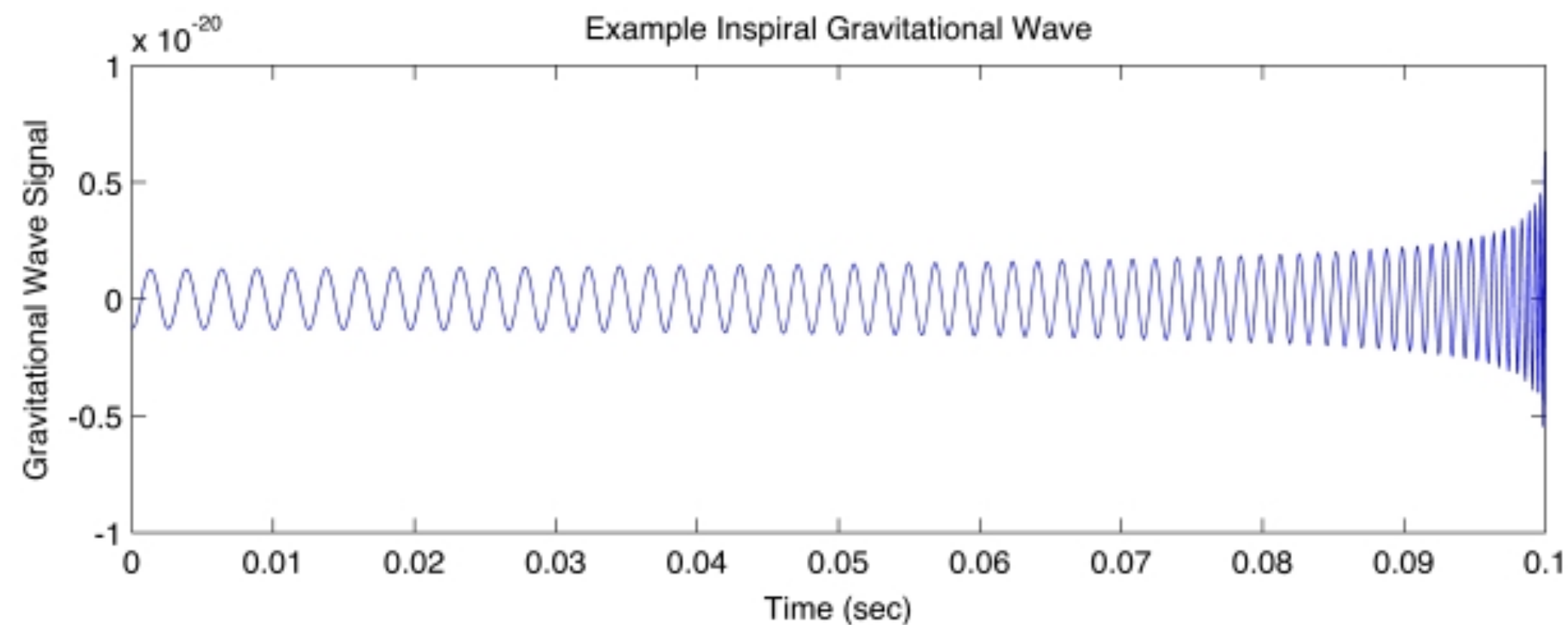
Forecasts with PTA + Astrometry

Summary

SGWB

Stochastic Gravitational Wave Background

Independent sources which are not individually resolvable lead to an incoherent superposition, large number of such sources \rightarrow SGWB



[Images: A. Stuver/LIGO]

Events can be individually detected e.g. by matched filtering

Looks like noise!

SGWB detection

The cross-correlation method

Suppose we have the data from two different detectors

$$d_I = h_I + n_I$$

$$d_J = h_J + n_J$$

Now, if we build the cross-correlation

$$C_{IJ} \equiv d_I d_J = h_I h_J + n_I h_J + n_J h_I + n_I n_J$$

$$\langle C_{IJ} \rangle = \langle h_I h_J \rangle + \langle n_I h_J + n_J h_I \rangle + \langle n_I n_J \rangle$$

THE SPECTRUM OF GRAVITATIONAL WAVES

PPTA, EPTA, NANOGrav, InPTA, CPTA, SKA (2030s)

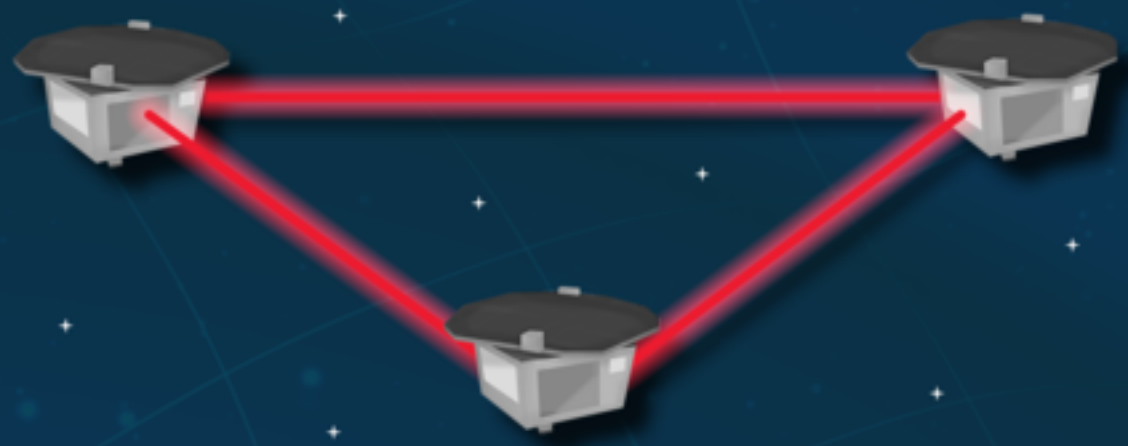


Observatories & experiments

Ground-based experiment



Space-based observatory



Pulsar timing array



Cosmic microwave background polarisation



Timescales

milliseconds

seconds

hours

years

billions of years

Frequency (Hz)

100

1

10^{-2}

10^{-4}

10^{-6}

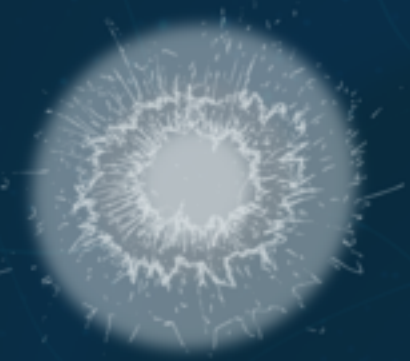
10^{-8}

10^{-16}

Cosmic fluctuations in the early Universe

BICEP/Keck, CMB-S4, LiteBIRD

Cosmic sources



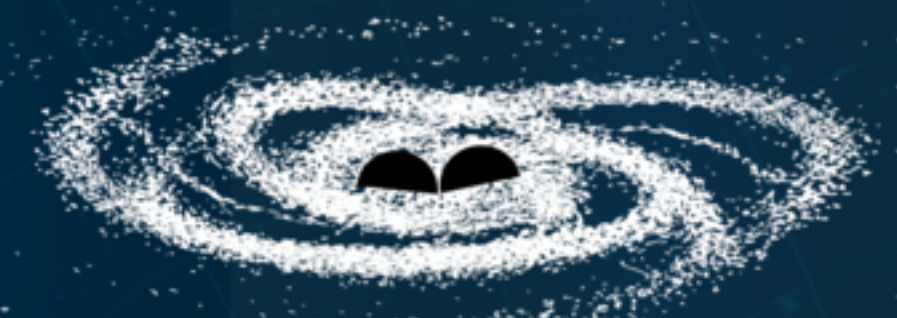
Supernova



Pulsar



Compact object falling onto a supermassive black hole



Merging supermassive black holes



Merging neutron stars in other galaxies



Merging stellar-mass black holes in other galaxies

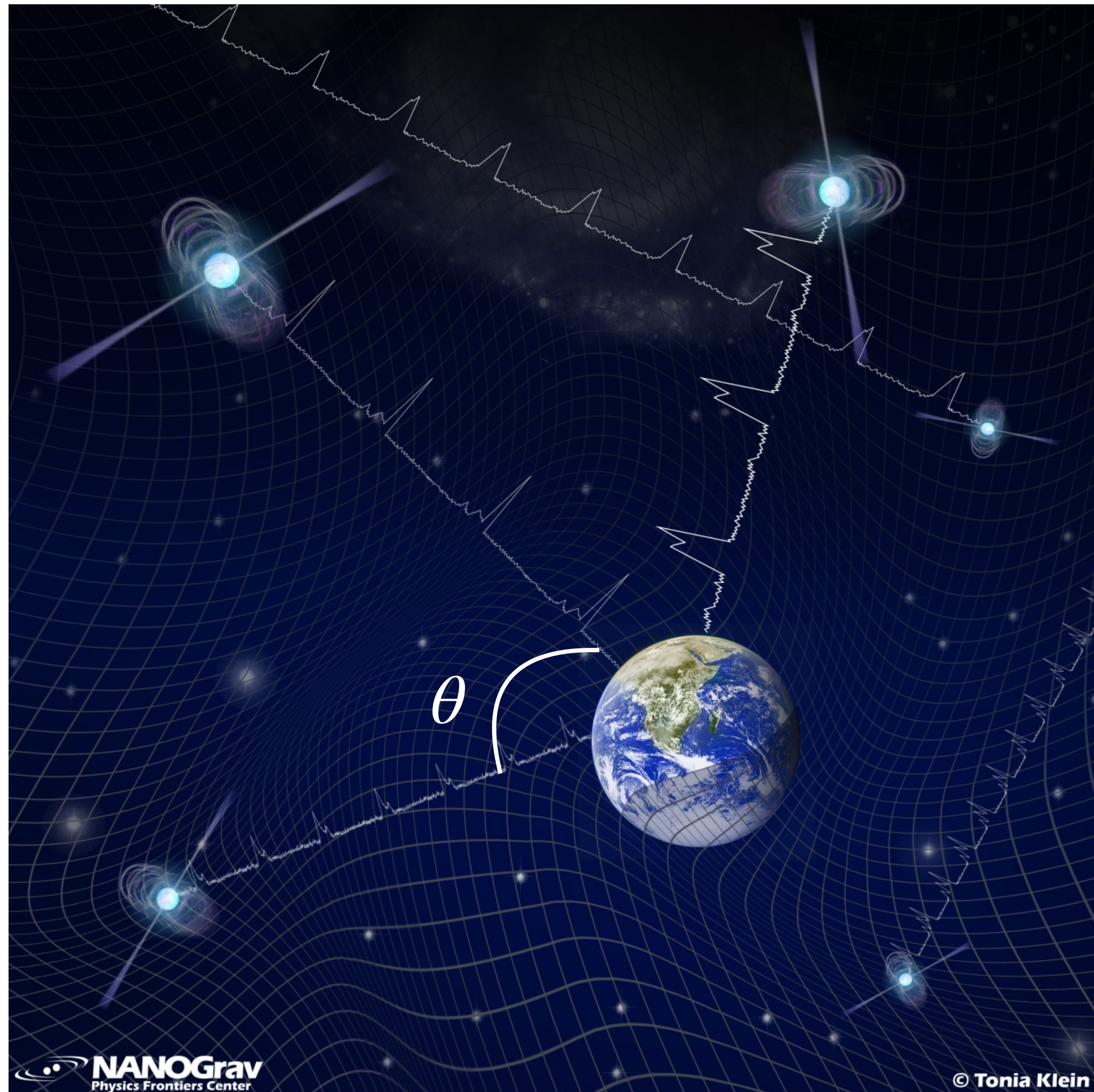


Merging white dwarfs in our Galaxy

#lisa



SGWB and PTAs



GW induce correlated deviations from expected time of arrival across pulsars

$$\langle d_I(f)d_J(f) \rangle \propto \gamma_{IJ}(f)S_h(f)$$

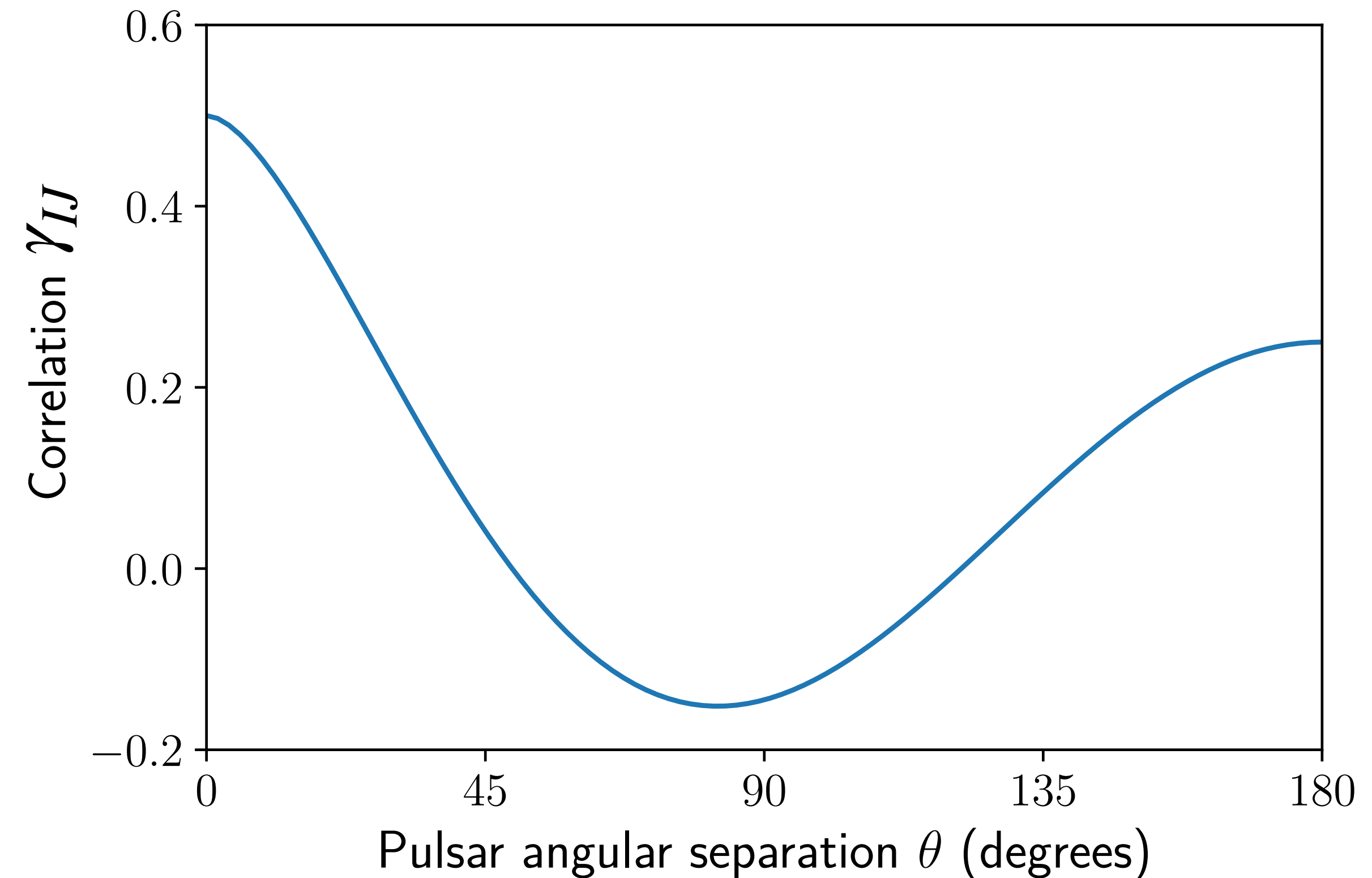
Geometric factor that depends on the relative positions of the 2 pulsars

SGWB detection

Overlap function: Pulsar Timing

The Hellings-Downs curve: PTA response to isotropic SGWB

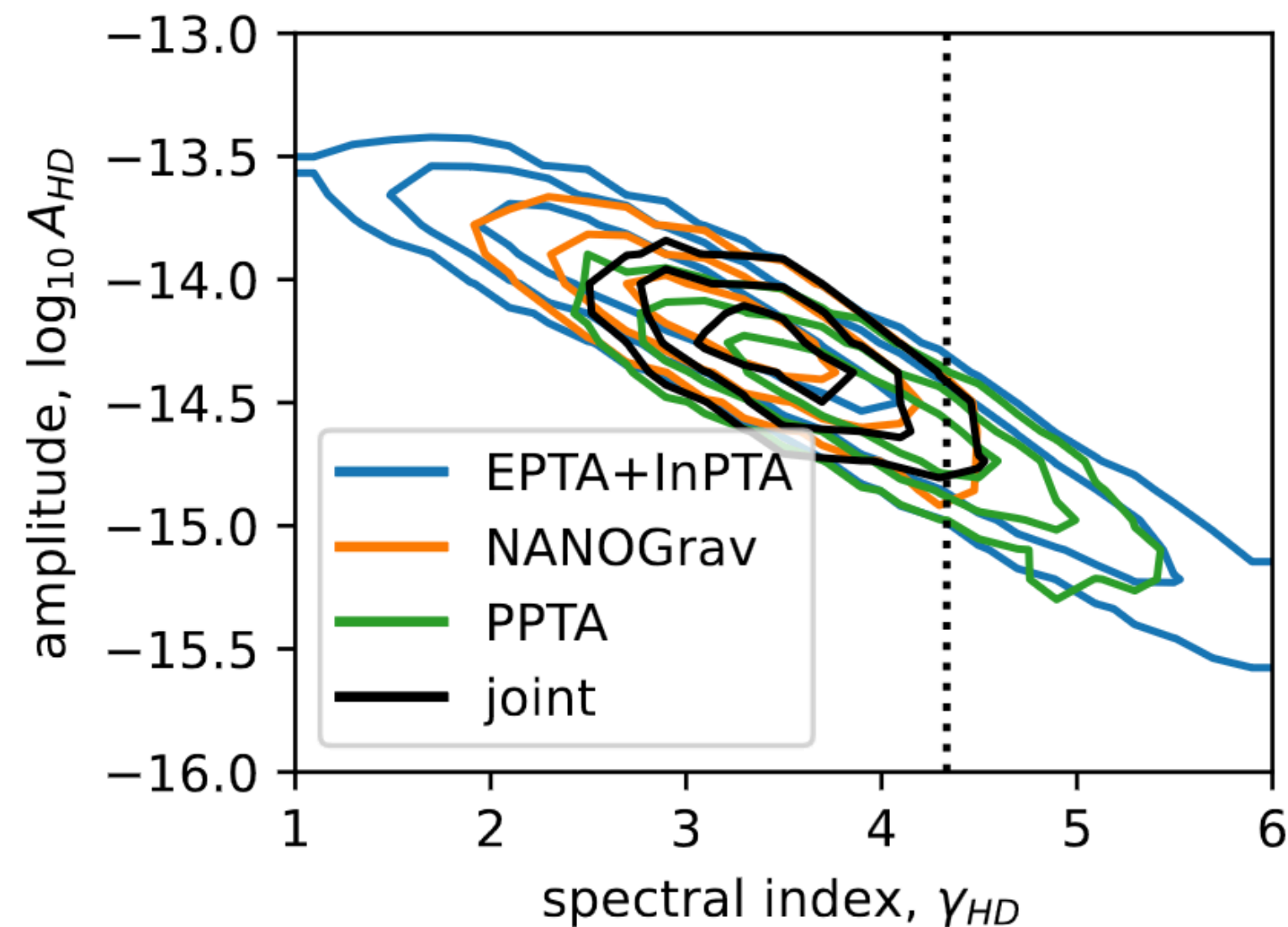
$$\gamma(\theta_{ij}) = 1 + \frac{\cos \theta_{ij}}{3} + 2(1 - \cos \theta_{ij}) \ln \left(\frac{1 - \cos \theta_{ij}}{2} \right)$$



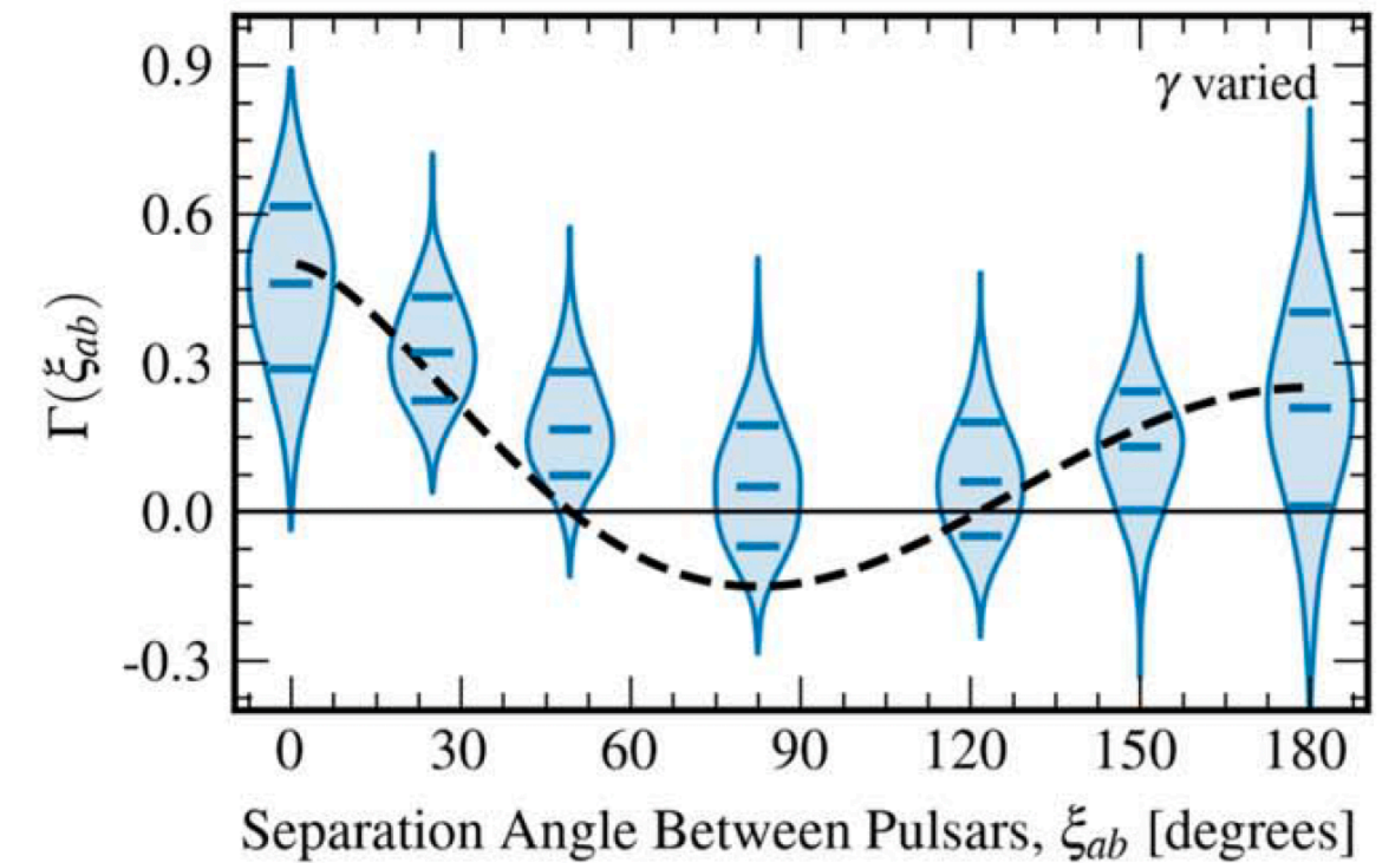
News from PTAs

Strong evidence for SGWB detected by
NANOGrav, EPTA, PPTA, InPTA, CPTA

HD correlations detected with $\sim 2 - 4\sigma$
significance



IPTA joint analysis, arxiv: 2309.00693



NANOGrav 15 year analysis

$$S_h(f) = \frac{A^2}{2f} \left(\frac{f}{f_{\text{ref}}} \right)^{2\alpha}, \quad \gamma = 3 - 2\alpha$$

What comes next?

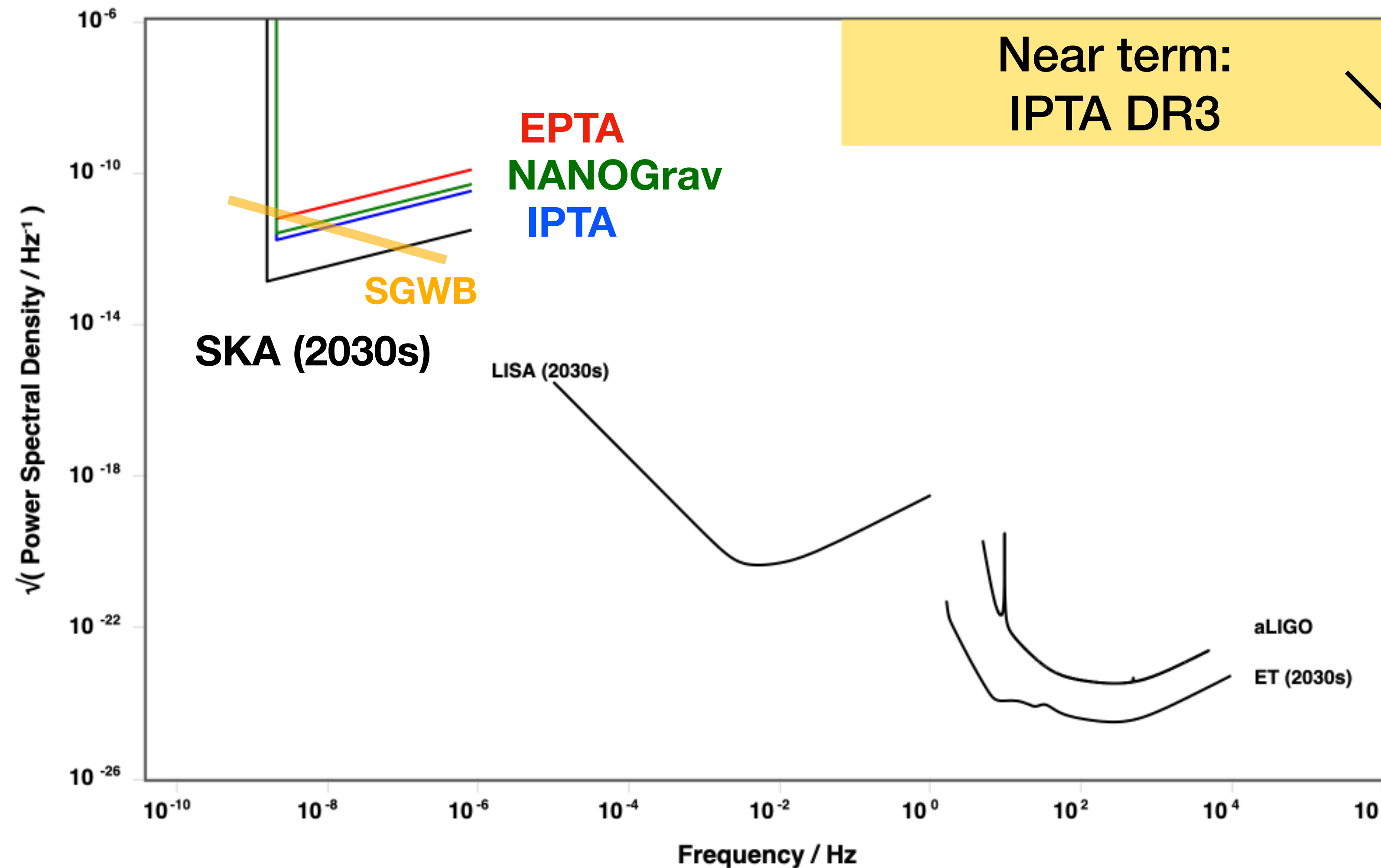
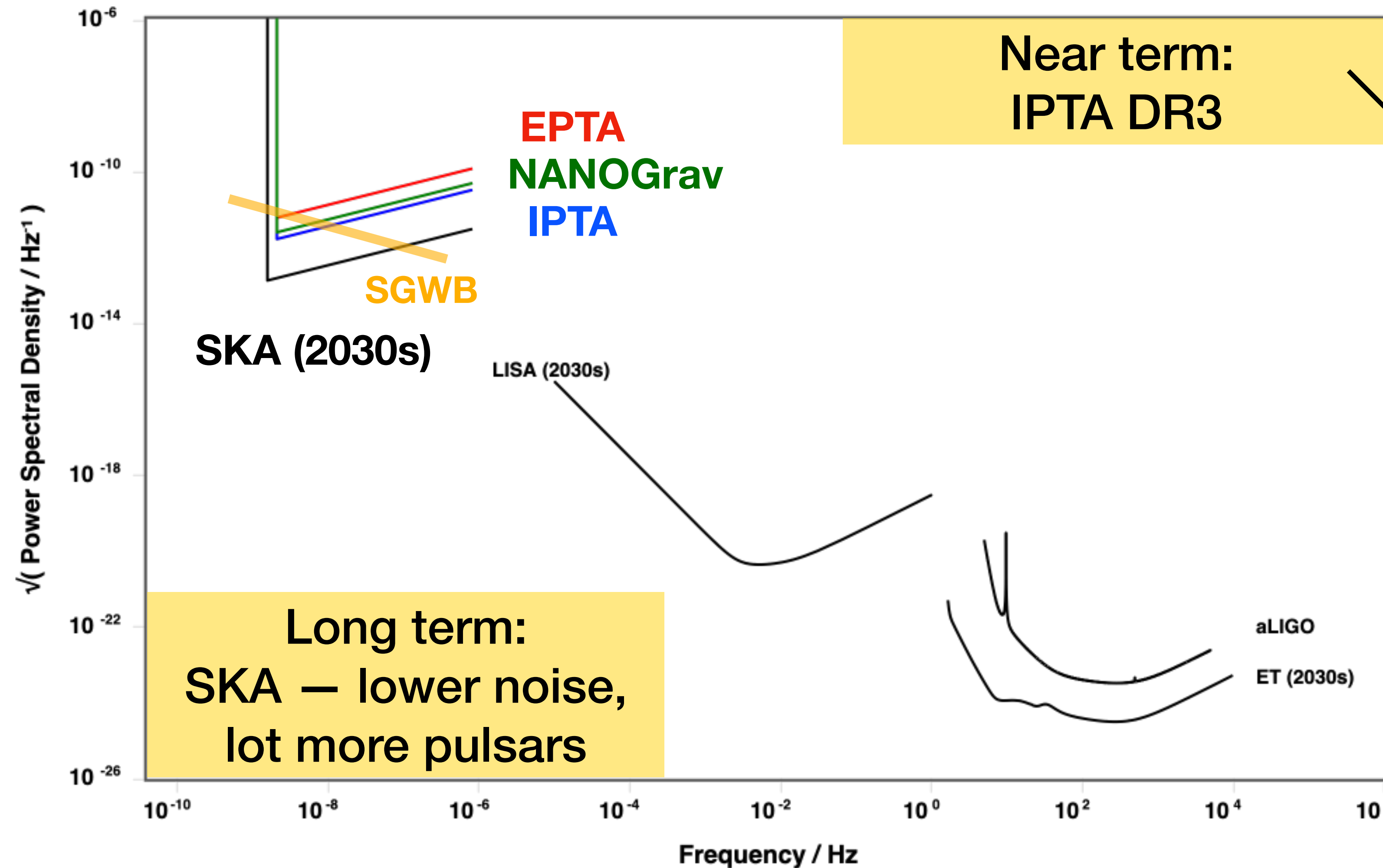


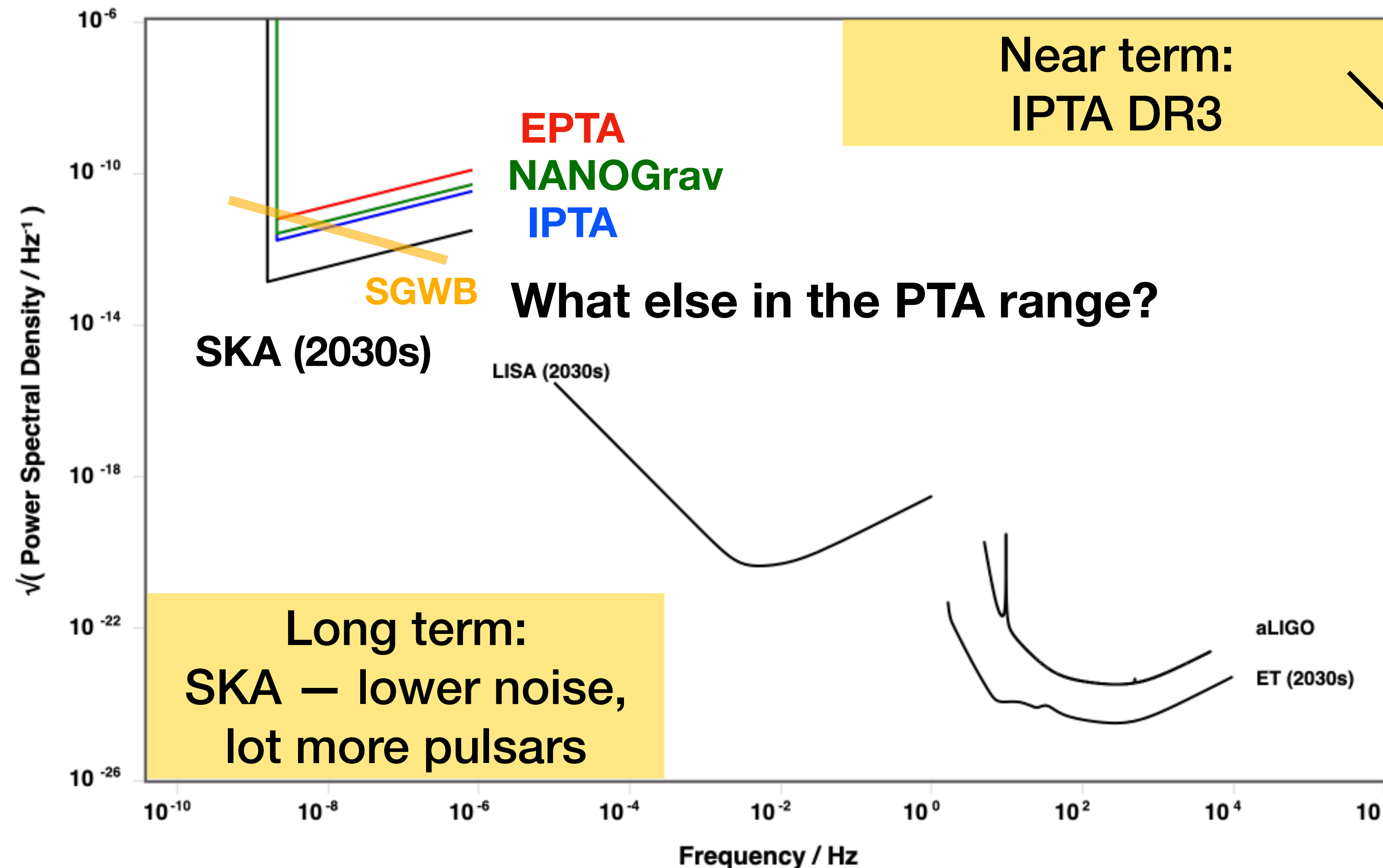
Image: GWplotter.com

What comes next?



Improved constraints and increased HD correlation significance

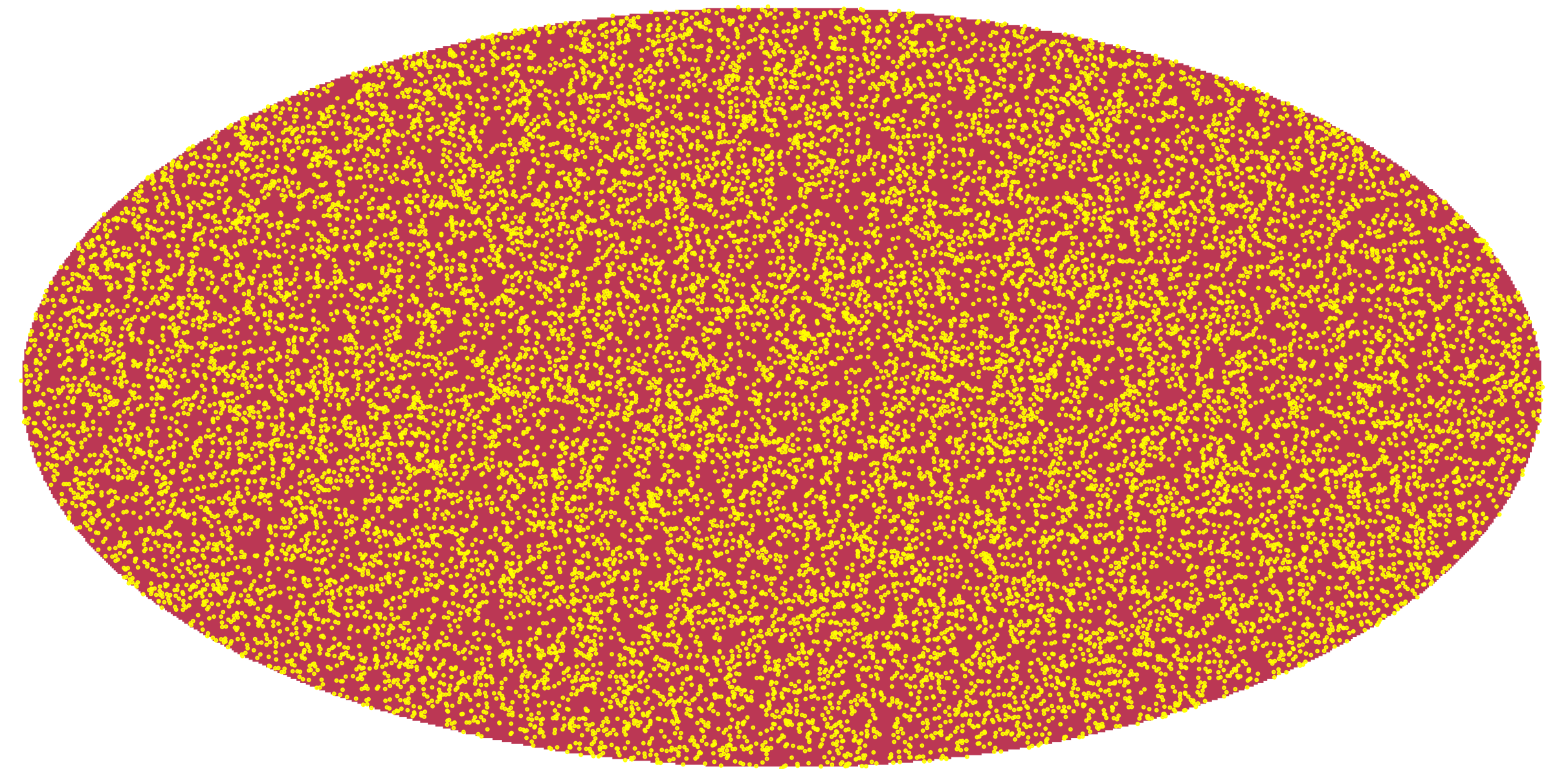
What comes next?



Astrometry and SGWB

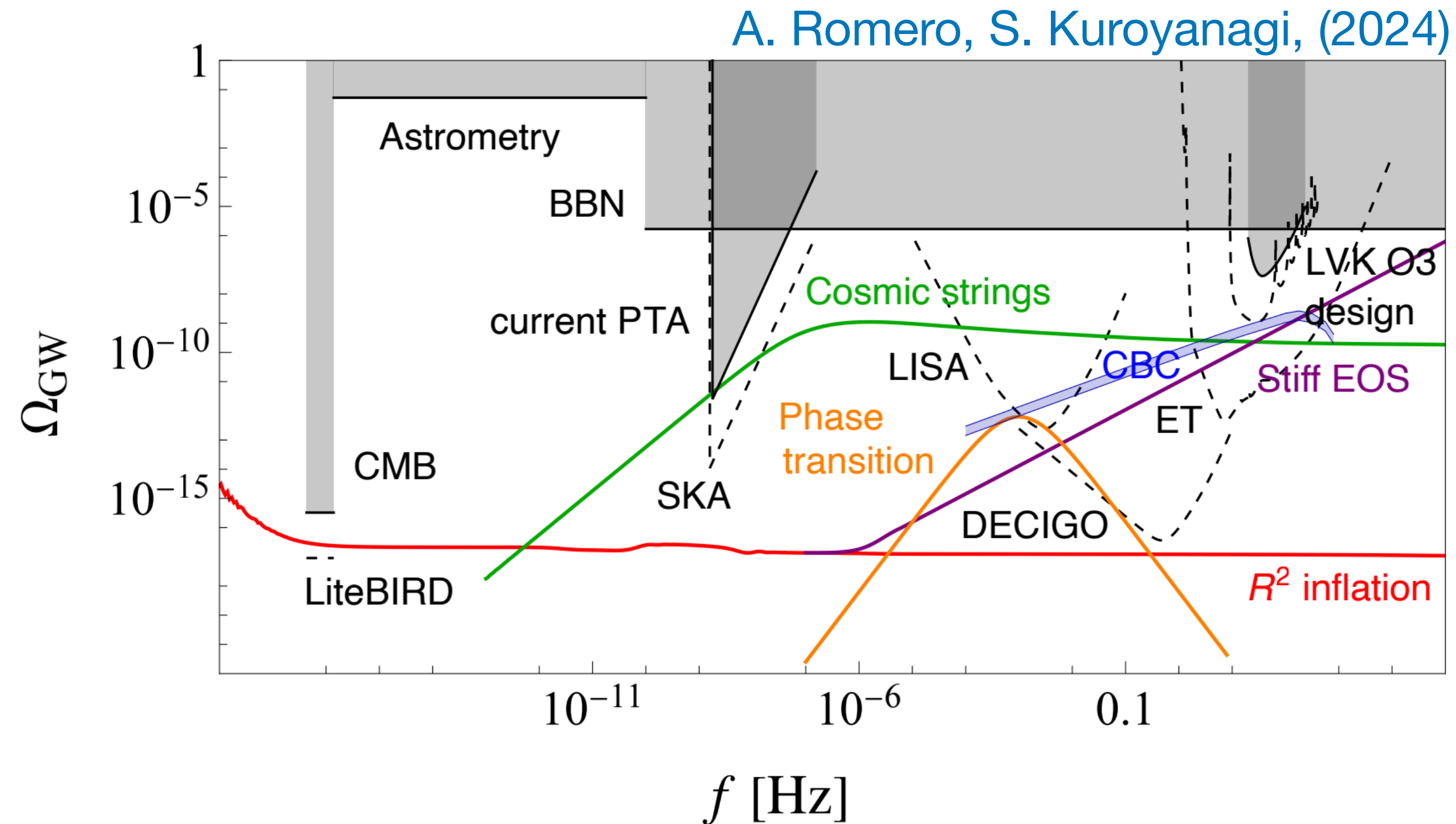
Precision astrometry with a large number of stars as a SGWB detector

[see [Book, Flanagan \(2010\)](#) for a review]



Gaia has $N \sim 10^9$ observed over 10 years with $\mathcal{O}(mas)$ precision. Already used to put constraints on low-frequency SGWB [[Darling et al. 2018](#); [Aoyama et al. 2021](#); [Jaraba et al. \(2023\)](#)]

Astrometry and SGWB



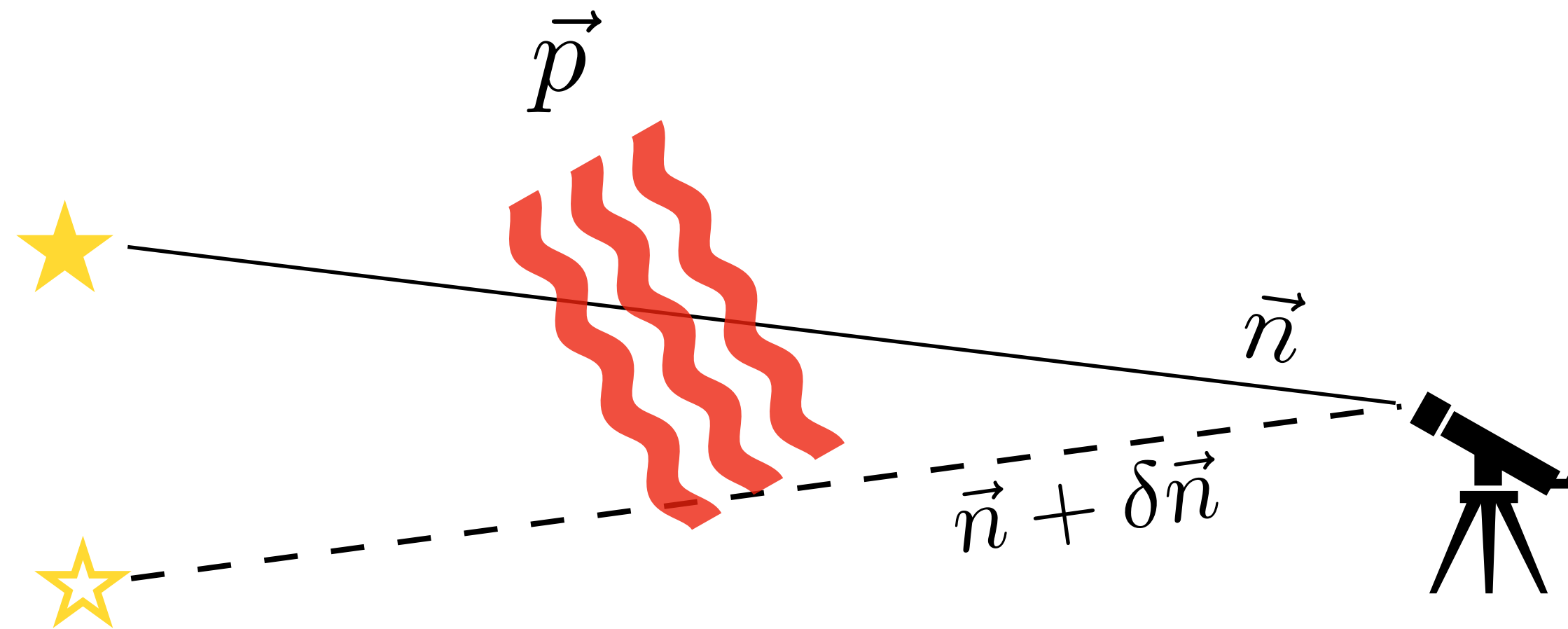
More Gaia data is coming in the next few years + future experiments (Roman, Theia...)

Forecasts for Theia $\mathcal{O}(10^{-10})$, much better angular resolution and lot more stars [J. García-Bellido et al. 2021]

Astrometry and SGWB

GW induced deflection

GWs affect the observed position of the star



For distant sources, $D \gg \lambda_{\text{GW}}$

$$\delta n^i(t, \vec{n}) = \mathcal{R}_{ikl}(\vec{n}, \vec{p}) h_{ij}(t)|_{\text{earth}}, \quad \mathcal{R}_{ikl}(\vec{n}, \vec{p}) = \frac{n_k}{2} \left[\frac{(n_i + p_i)n_l}{1 + \vec{n} \cdot \vec{p}} - \delta_{il} \right]$$

[Book, Flanagan (2010)]

Astrometry and SGWB

Correlated deflections

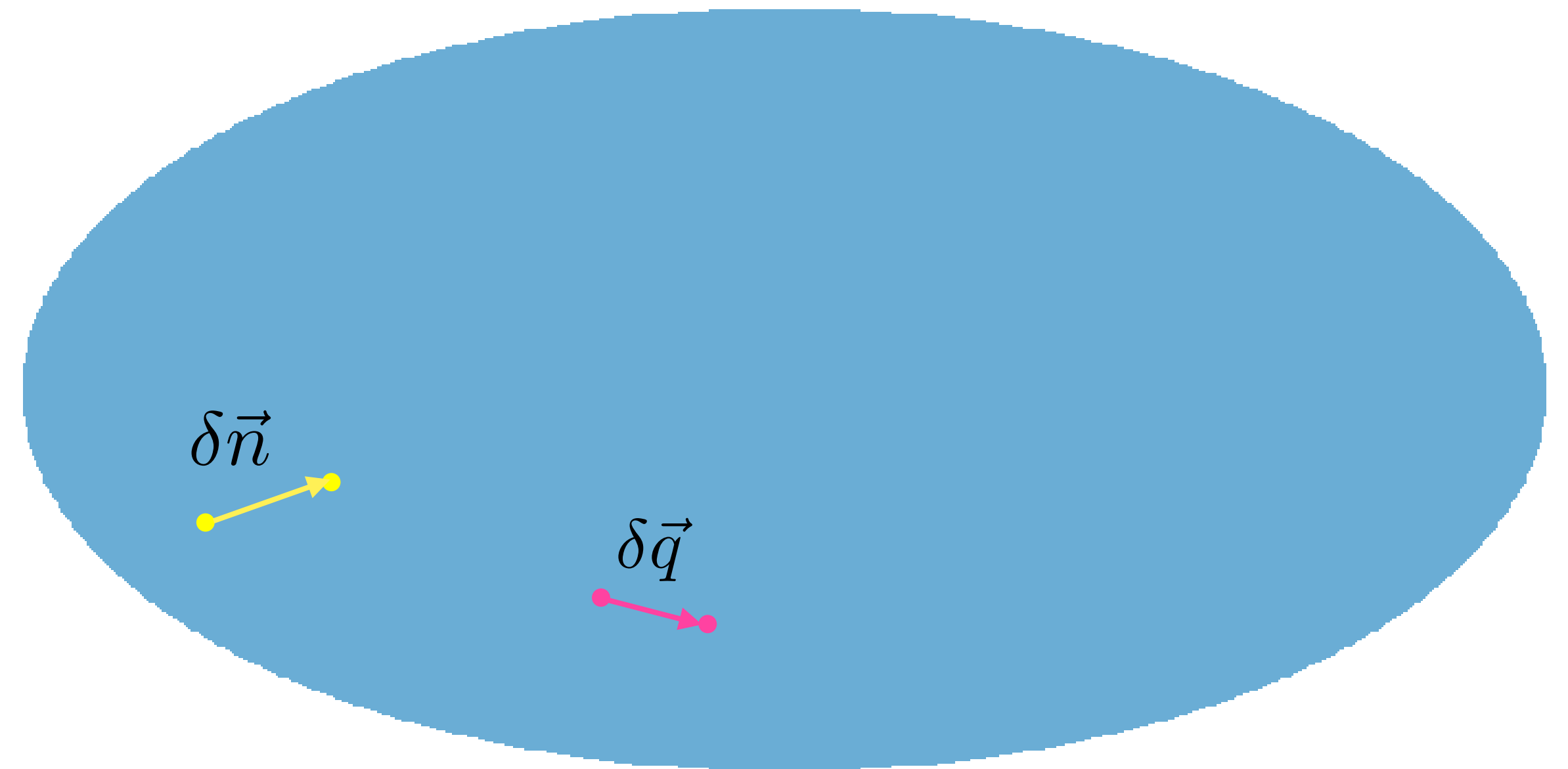
SGWB PSD

$$\langle \delta n^i \delta q^j \rangle \propto \int df S_h(f) H_{ij}^{(0)}(\vec{n}, \vec{q})$$

Geometry dependent correlation

$$H_{ij}^{(0)}(\vec{n}, \vec{q}) = \frac{\pi}{3(1-y)^2} (1 - 8y + 7y^2 - 6y^2 \ln y) \\ \times [(2 - 2y)\delta_{ij} - n_i n_j - q_i q_j - q_i n_j + (1 - 2y)q_j n_i]$$

$$y \equiv \frac{1 - \vec{n} \cdot \vec{q}}{2}$$



Astrometry x PTA

Cross-correlations

The angular deflections and timing residuals induced by the SGWB are correlated

$$K_i^{(0)}(\vec{n}, \vec{x}) = \frac{16\pi}{3} \frac{(1 - 2y) n_i - x_i}{4y(1 - y)} (2y - 2y^2 + 3y^2 \ln(y))$$

Pulsar direction Star direction

Astrometry x PTA

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The diagram shows two arrows originating from the vector variables in the equation above. One arrow points from \vec{n} to the text "Pulsar direction" below it. The other arrow points from \vec{x} to the text "Star direction" below it. Both arrows and the text labels are in red.

Can cross-correlating Astrometry with PTA help?

Astrometry x PTA

The setup

Forecasts with PTA at current sensitivity and an astrometric survey with 0.01 mas precision and 10^6 stars.

Joint Gaussian likelihood in the timing residuals and angular deflections.

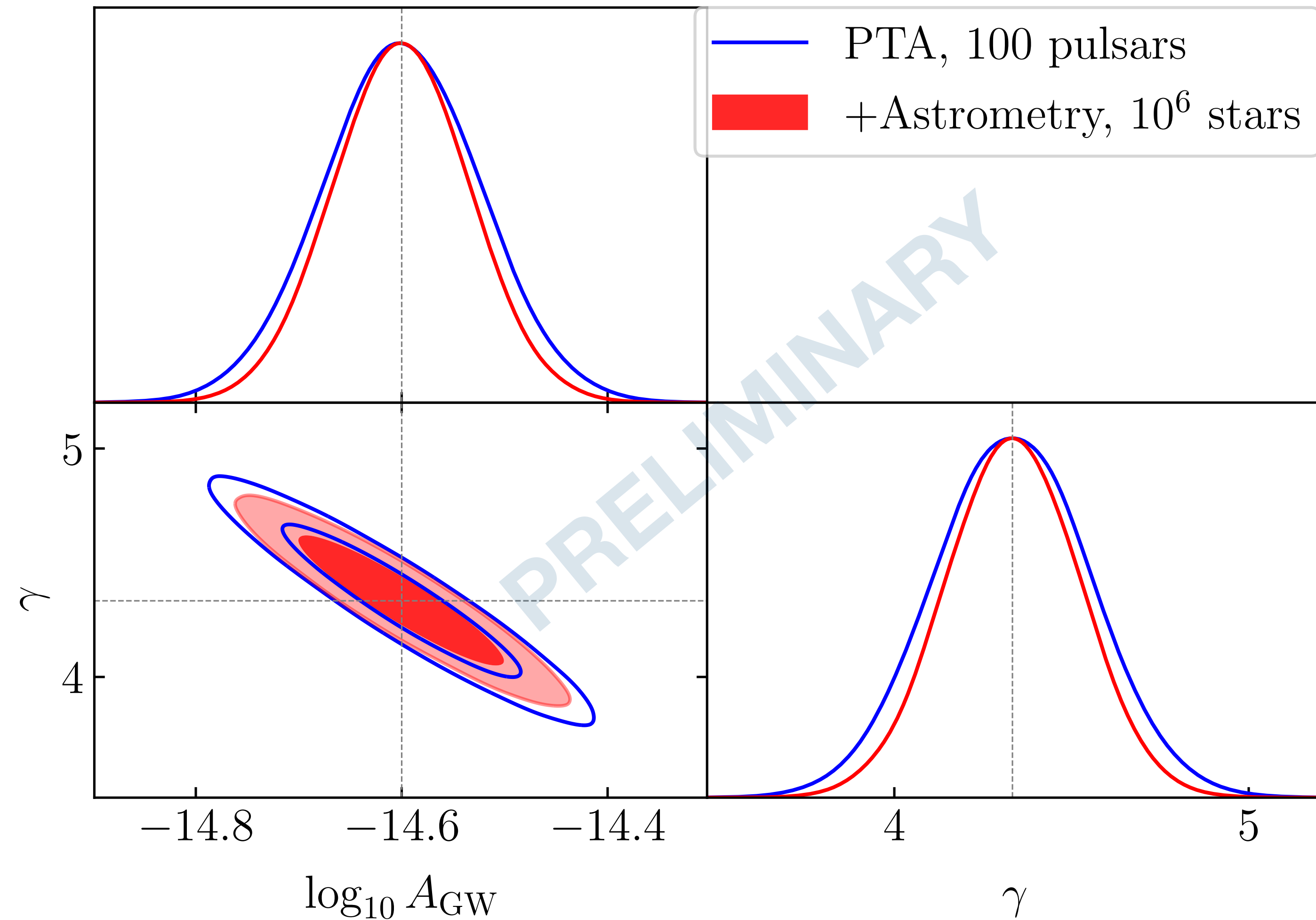
$$-\ln \mathcal{L} \sim (\delta t, \delta \vec{n}) \mathcal{C}^{-1} (\delta t, \delta \vec{n})^T$$

$$\mathcal{C} = \begin{array}{cc} \begin{array}{c} \text{PTA} \\ \boxed{C_{\delta t \delta t}} \end{array} & \begin{array}{c} \text{PTA x Astro} \\ \boxed{C_{\delta t \delta \vec{n}}} \end{array} \\ \begin{array}{c} \boxed{C_{\delta t \delta \vec{n}}^T} \\ \text{PTA x Astro} \end{array} & \begin{array}{c} \boxed{C_{\delta \vec{n} \delta \vec{n}}} \\ \text{Astro} \end{array} \end{array}$$

Astrometry x PTA

Power-law

~10 % improvement over current PTA constraints

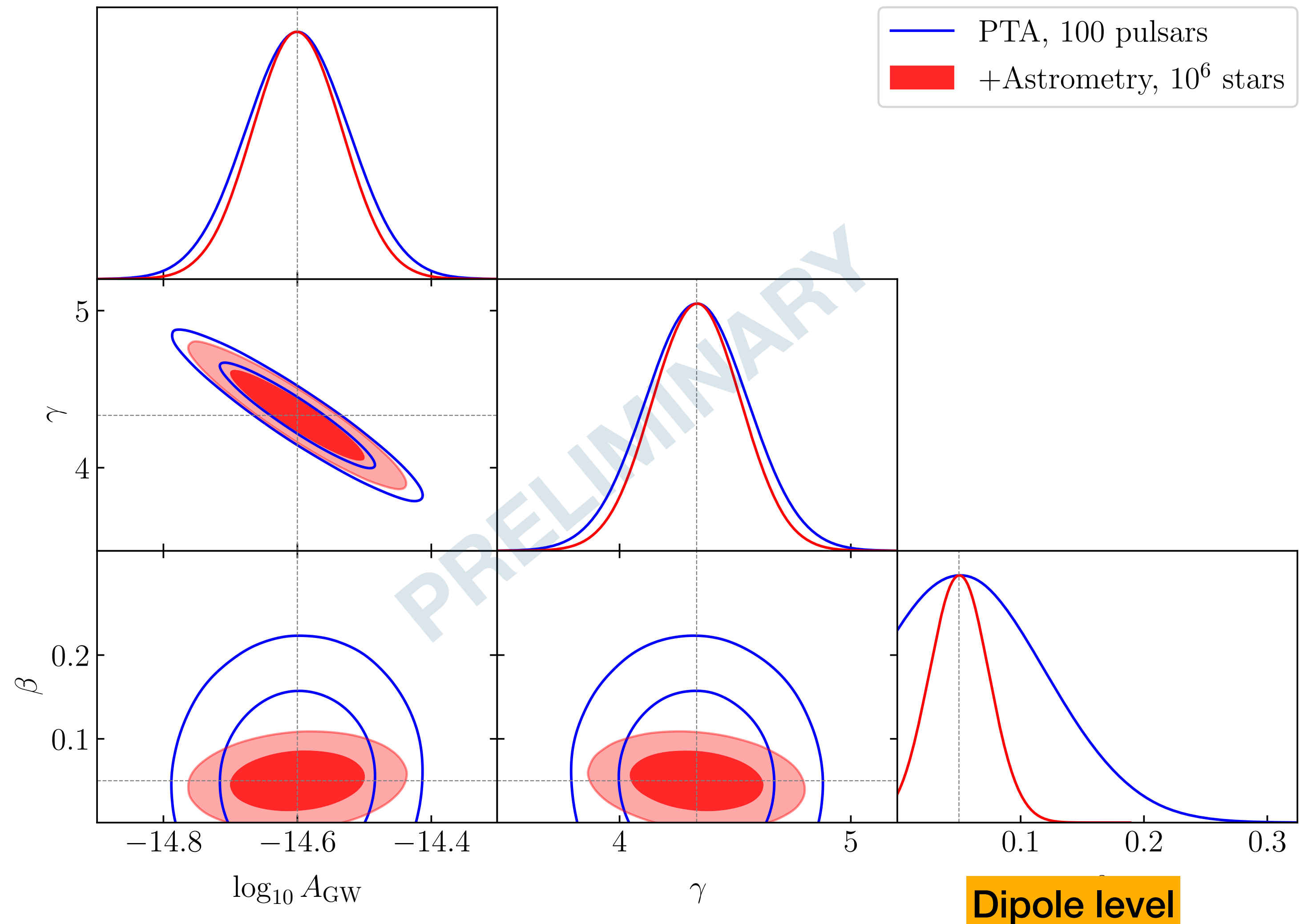


Astrometry x PTA

Dipole anisotropy

Minimum detectable dipole anisotropy relative to monopole ~ 0.05 .

Current PTA level ~ 0.1



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Astrometry offers a complementary probe of SGWB in the low-frequency range

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Future work: how to implement this in practice?

Thank you!