Gravitational waves from Nnaturalness

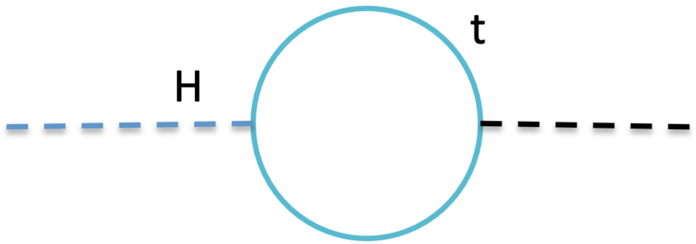
arXiv: 2310.xxxx [hep-ph]
Mudit Rai
Texas A&M University
Collaborators : Brian Batell, Akshay Ghalsasi, Matthew Low

Motivation

- Naturalness : EFT should not be extremely sensitive to the structure of the underlying UV theory.

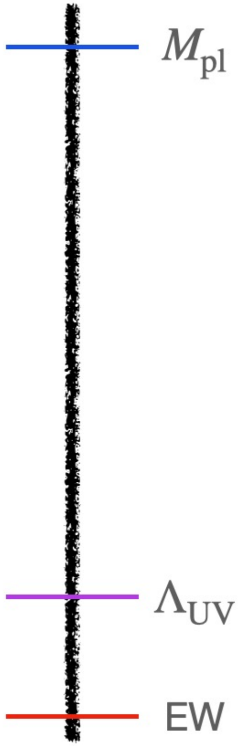
- 1-loop corrections from SM fields to the Higgs mass give quadratic corrections :

$$m_H^2 \propto \Lambda_{\mathrm{UV}}^2$$



- No evidence of new physics at TeV scale.

- If new physics should enter at a new scale, Λ_{UV}, say at O(10) TeV then we need to explain this small hierarchy.



https://arxiv.org/pdf/1501.01035.pdf
https://arxiv.org/pdf/2205.05708.pdf

Nnaturalness(Arkani-Hamed et.al. 2016)

- Nnaturalness is a model which consists of a large N copies of the Standard Model with varying values of the Higgs mass parameter.
- This allows the lightest of these sectors to have a weak scale parametrically lighter than the cutoff.
- Our SM is to be identified with the sector having the smallest (negative) squared Higgs mass.
- Finally, a light `reheaton' can naturally transfer most of its energy to our sector and only slight fractional energy densities to the other sectors.
- E.g., for $N = 10^4$, we will have quantum gravity becoming strong at 10^{16} GeV, and something like SUSY enters at $\Lambda_H = 10\,TeV$ to cut off the Higgs sector.
- We investigate probing Nnaturalness through gravitational wave (GW) signatures.

Nnaturalness : Model

- Consider N copies of SM, with the Higgs squared mass parameters assumed to vary uniformly from one sector to another, (i=0 corresponds to our SM)

$$-\Lambda_{\rm UV}^2 < m_{H,i}^2 < \Lambda_{\rm UV}^2$$

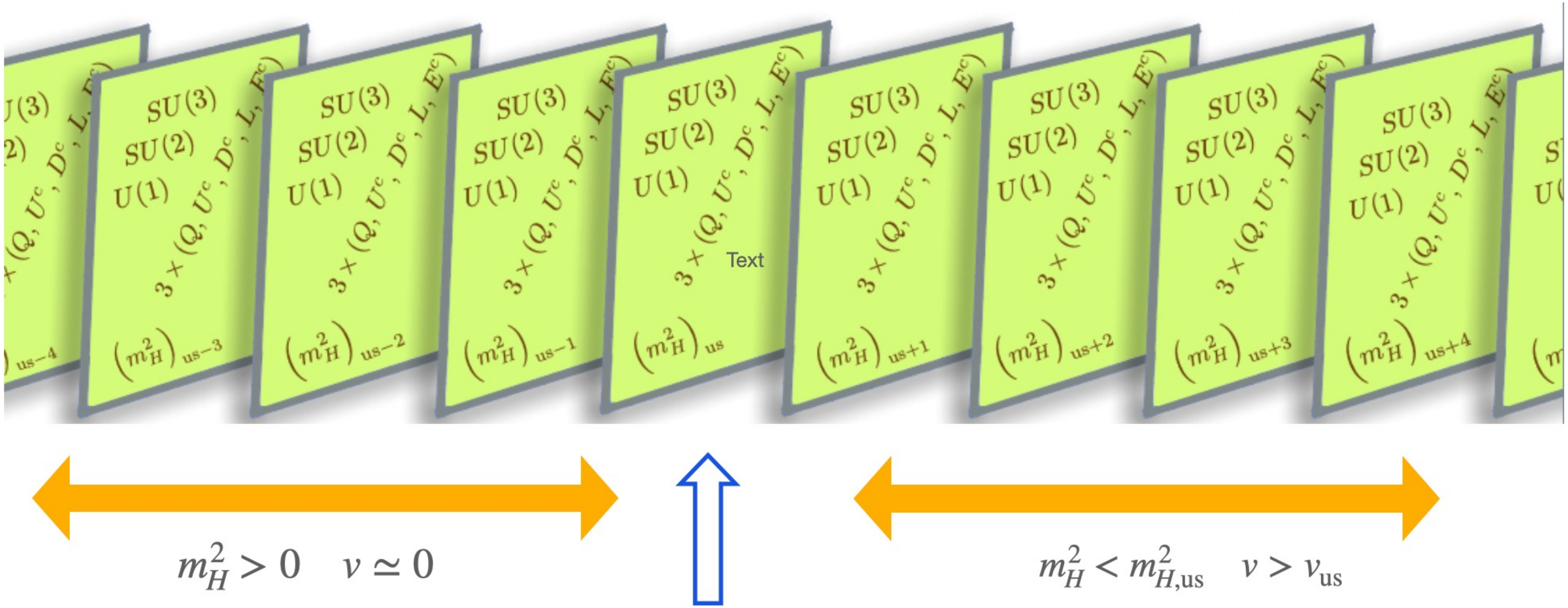
$$m_{H,i}^2 = -\frac{\Lambda_H^2}{N}(2i + r) = -(88~{\rm GeV})^2~(1 + \frac{2i}{r})$$

- We consider a scalar reheaton, which dominate the energy density of the universe at some time following inflation,

$$\mathcal{L}_\phi \supset -a\phi \sum_i |H_i|^2 - \frac{1}{2} m_\phi^2 \phi^2$$

arXiv:1607.06821 [hep-ph]

Nnaturalness



$m_H^2 > 0 \quad v \simeq 0$

$m_H^2 < m_{H,\text{us}}^2 \quad v > v_{\text{us}}$

$m_{H,\text{us}}^2 = -(88\ \text{GeV})^2 \quad v_{\text{us}} = 246\ \text{GeV}$

$v_i \sim \frac{y_t \Lambda_{QCD,i}^3}{m_{H,i}^2} \ll \Lambda_{QCD,i} \sim O(100\ MeV)$

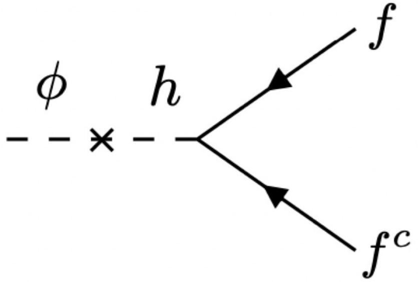
$v_{i\geq 0}^2 = -(m_{H_i}^2)/\lambda = v^2(2i/r+1)$

arXiv:1607.06821 [hep-ph]

Reheaton decays

- For SM like sectors, reheaton decays to all kinematically available final states via it's mixing with the Higgs h_i

$$\Gamma_{\phi\to\{f_i\}} = \theta_i^2\, \Gamma_{h_i\to\{f_i\}}(m_\phi) \propto 1/m_{h_i}^2 \sim \frac{1}{i}$$

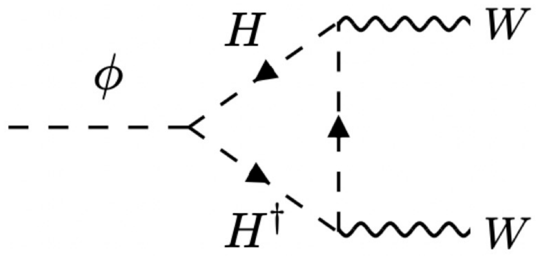


- Pre-dominant decay is to our SM (lightest higgs)

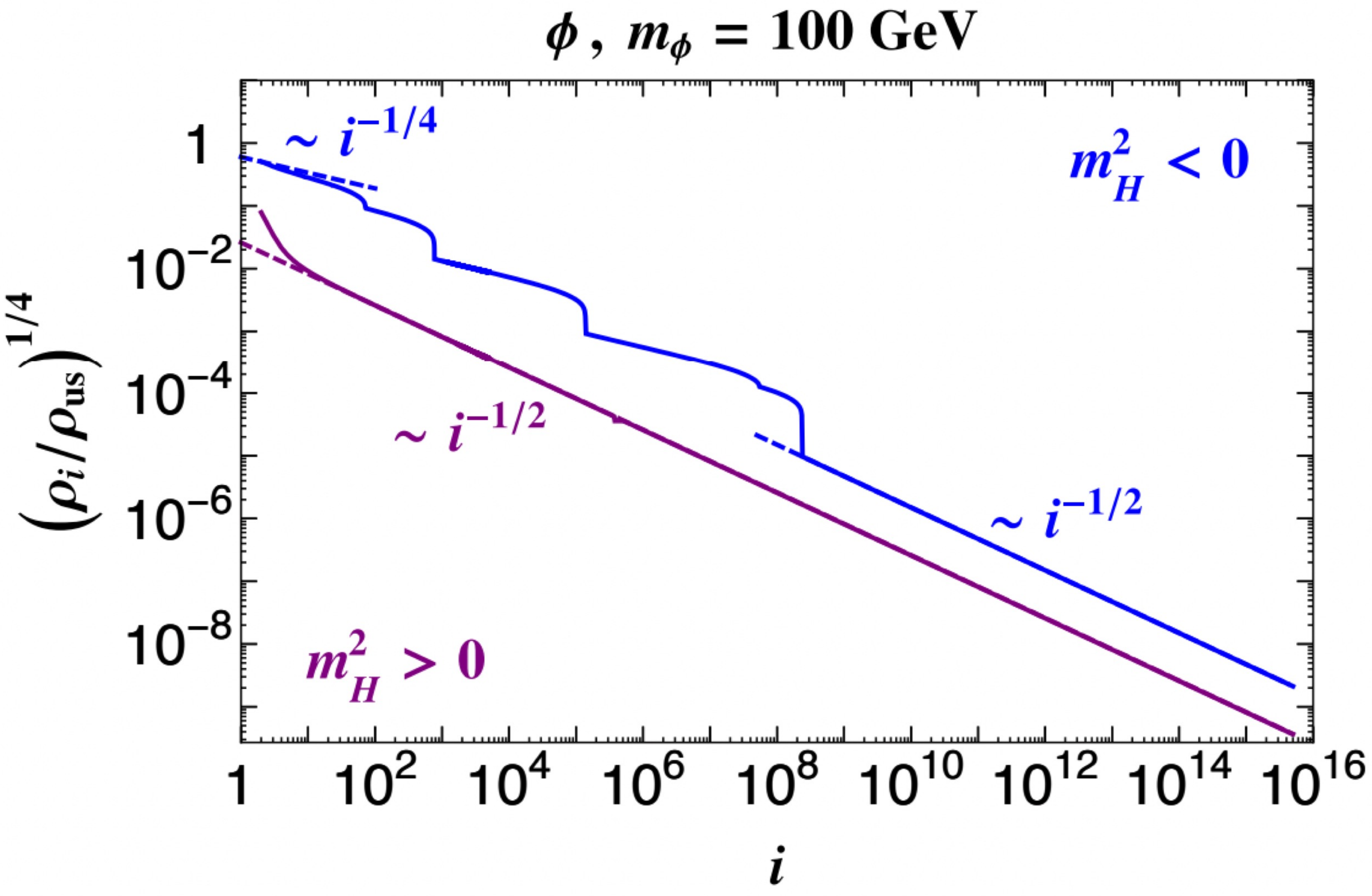
$$\rho_i/\rho_{\rm SM} \approx \Gamma_i/\Gamma_{\rm SM}. < 1$$

- In exotic sectors, no Higgs-scalar mass mixing in the absence of vev leads to decays via off shell exotic Higgs,

$$\Gamma_i \sim \frac{1}{m_{H,i}^4} \sim \frac{1}{i^2}$$



Reheaton decay : recap



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First exotic sector (i=-1) : mass spectrum

- Quark condensates form and lead to spontaneous breaking of the electroweak (SU2) symmetry at around $T_{-1,crit} \sim 85\ MeV$,

$$< q_L q_R > \simeq 4\pi f_{\pi,-1}^3 \ \rightarrow \ v_{-1} \simeq \frac{f_\pi^3}{m_{H,-1}^2}$$

- 3 goldstones of $SU(6)_L \times SU(6)_R \rightarrow SU(6)_V$ gets eaten by W and Z

$$m_{W,Z} \simeq f_{\pi,-1} \simeq \mathcal{O}(10\mathrm{MeV})$$

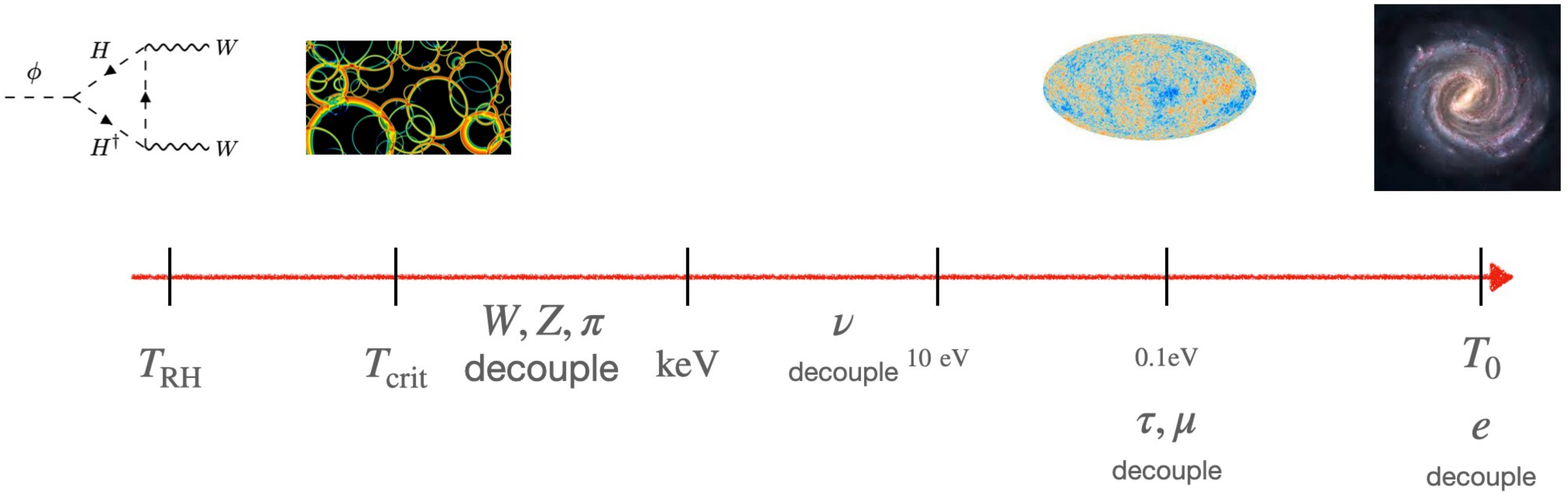
- The rest 32 gets their masses from the explicit symmetry breaking via Yukawas

$$m_{\pi,-1} \simeq \mathcal{O}(\mathrm{keV} - \mathrm{MeV})$$

- Leptons are the lightest

$$m_e \simeq \mathcal{O}(10^{-4}\mathrm{eV}),\ m_\mu \simeq \mathcal{O}(10^{-2}\mathrm{eV}),\ m_\tau \simeq \mathcal{O}(10^{-1}\mathrm{eV})$$

Cosmology of exotic sector



- As the exotic sector cools and its temperature reaches $\Lambda_{QCD,-1}{\sim}100\ MeV$, there's a FOPT due to quark condensates.

- Thus, near recombination(T~0.3 eV) the exotic sector relativistic species comprise photons, neutrinos, and electrons.

ΔN_{eff} in Nnaturalness

- Phenomenologically, Nnaturalness can only be probed via its effects on ΔN_{eff} during recombination.
- For our benchmarks, we use the experimental bound from CMB,

$$\Delta N_{\rm eff}^{\rm CMB} \lesssim 0.7 \qquad \text{(Planck + SH0ES)}$$

- SM like sectors have contributions via free streaming neutrinos

$$\Delta N_{{\rm eff},i>0}^{\rm CMB} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left[\frac{g_{*\rho,{\rm SM}}^{\rm RH}}{2}\right]\left[\frac{g_{*s,{\rm SM}}^{\rm CMB}}{g_{*s,{\rm SM}}^{\rm RH}}\right]^{4/3}\sum_{i>0}\left[\frac{g_{*\rho,i}^{\rm CMB}}{g_{*\rho,i}^{\rm RH}}\right]\left[\frac{g_{*s,i}^{\rm RH}}{g_{*s,i}^{\rm CMB}}\right]^{4/3}\frac{\Gamma_i}{\Gamma_{\rm SM}}$$

- Exotic sectors contribute via free streaming neutrinos and interacting radiation

$$\Delta N_{{\rm eff},-1}^{\rm CMB} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left[\frac{g_{*\rho,{\rm SM}}^{\rm RH}}{2}\right]\left[\frac{g_{*s,{\rm SM}}^{\rm CMB}}{g_{*s,{\rm SM}}^{\rm RH}}\right]^{4/3}\left[\frac{g_{*\rho,-1}^{\rm CMB}}{g_{*\rho,-1}^{\rm RH}}\right]\left[\frac{g_{*s,-1}^{\rm RH}}{g_{*s,-1}^{\rm CMB}}\right]^{4/3} D_{s,-1}^{4/3}\,\frac{\Gamma_{-1}}{\Gamma_{\rm SM}},$$

Gravitational Wave signal

- Cosmological FOPT leads to production of stochastic GW via :
  1. Collisions of bubble walls
  2. Production of sound waves from plasma surrounding the bubbles
  3. Hydrodynamic turbulence in the plasma[1]
- Differential GW density parameter characterizes them :

$$\Omega_{GW} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\log f}, \quad \rho_c = 3M_{pl}^2 H^2$$

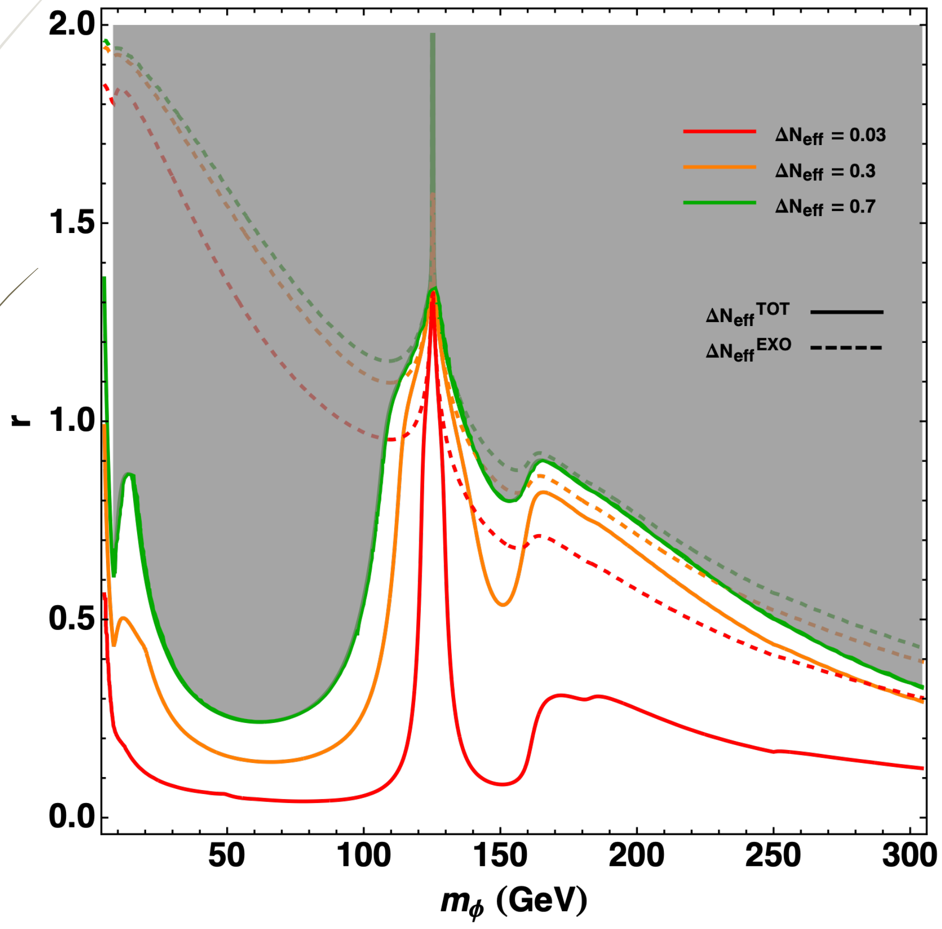
- Semi-analytical parametrizations can be used to describe them,

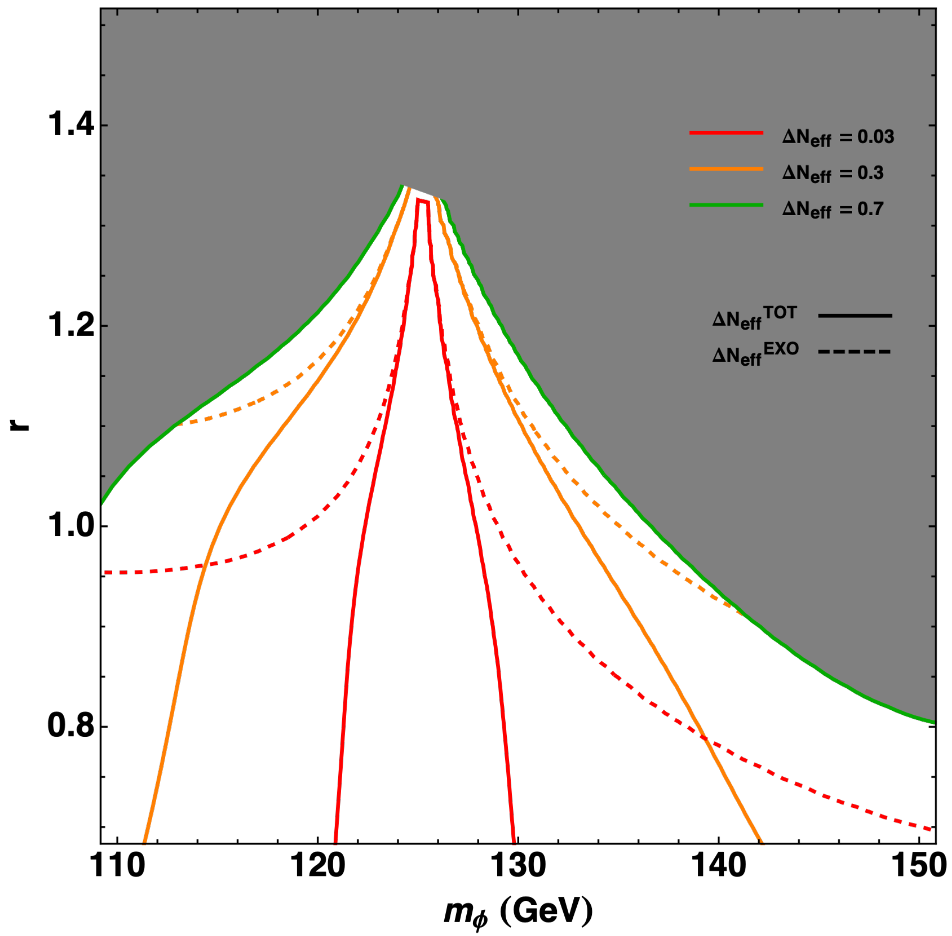
$$\Omega_{\rm GW}^{\rm em}(f_{\rm em}) = \sum_{I=\rm BW,SW} N_I\,\Delta_I(v_{\rm w})\left(\frac{\kappa_I(\alpha_{-1})\,\alpha_{\rm tot}}{1+\alpha_{\rm tot}}\right)^{p_I}\left(\frac{H}{\beta}\right)^{q_I} s_I(f_{\rm em}/f_{{\rm p},I}),$$

$$h^2\,\Omega_{\rm GW}^0(f) = h^2\mathcal{R}\,\Omega_{\rm GW}^{\rm em}\left(\frac{a_0}{a_{\rm perc}}f\right).$$

1. neglected in this study
2. RH temperature of SM is fixed at 100 GeV

Cosmological Probes of NNaturalness





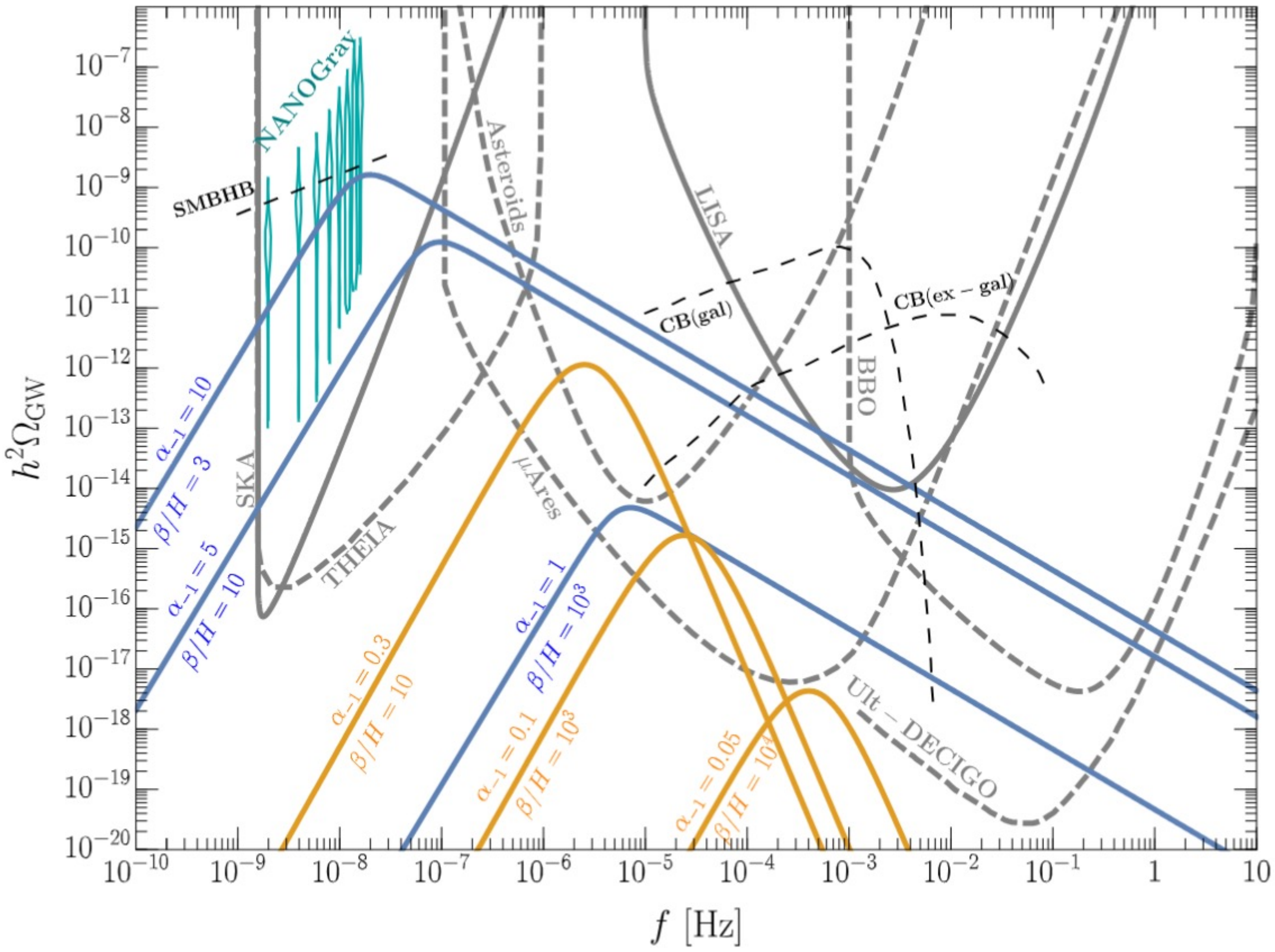
Two different scenarios

1. **Runaway phase transition** : $v_w = 1, \alpha_{-1} = 5$, $\kappa_{BW} = 1, \kappa_{SW} = 0.$
   - bubble walls overcome friction from plasma, accerelating to become ultra-relativistic
   - Gravitational waves are sourced by bubble wall collisions
   - α_{-1} should be large to create sufficient pressure resulting in acceralating bubble

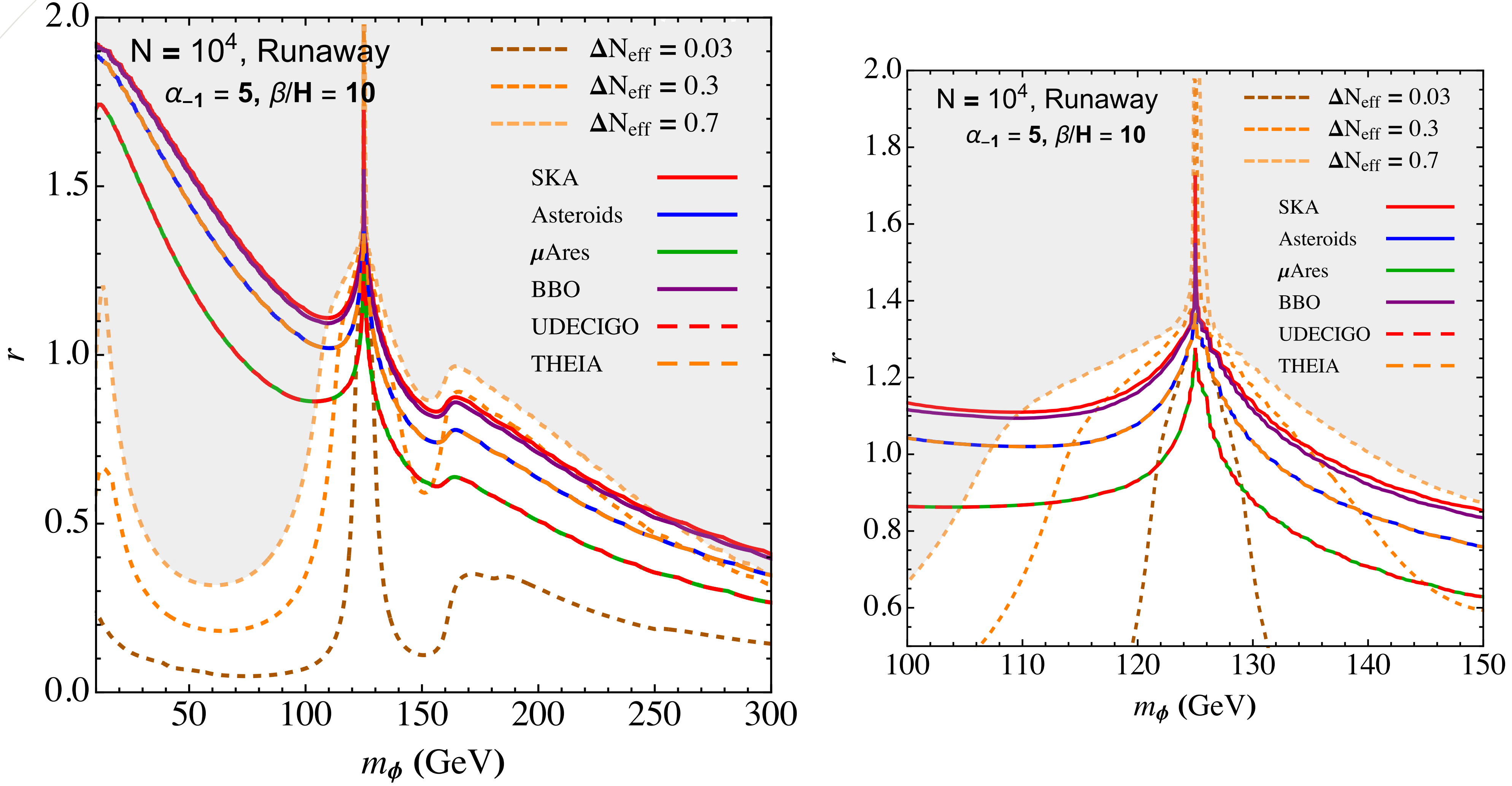
2. **Non-runaway phase transition :** $v_w = \frac{1}{\sqrt{3}}, \alpha_{-1} = 0.3, \kappa_{BW} = 0, \kappa_{SW} = 0.55$
   - Bubble walls cannot overcome friction due to plasma, transition energy gets dumped into coherent plasma motion
   - Gravitational waves are sourced by sound waves
   - Assumed local thermal equilibrium, and α_{-1} is bounded and small.

- Rate of transition : $\beta/H \in [3, 10^4].$
- Effective models for QCD (NJL, PNJL, LSM) predict : $\alpha_{-1} \simeq 0.1, \frac{\beta}{H} \simeq 10^4$

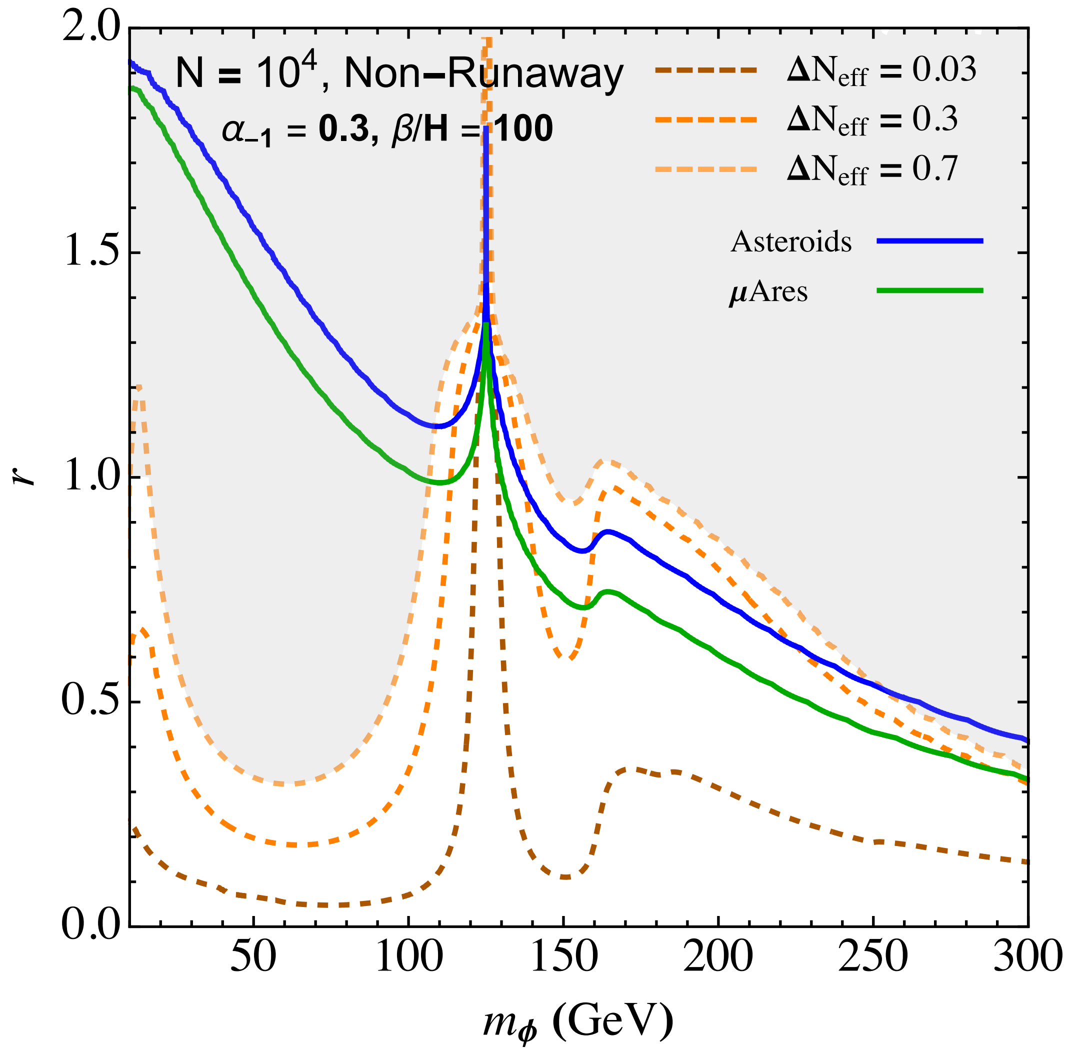
GW detection

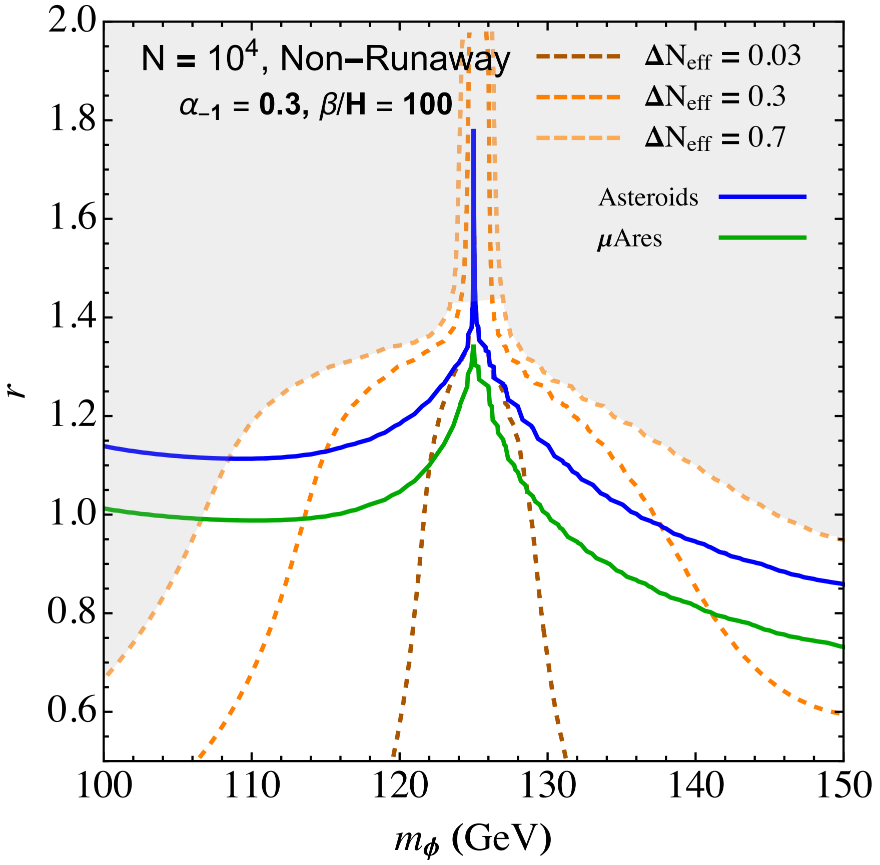


Mapping to Nnaturalness : Runaway



Mapping to Nnaturalness : non-runaway





Conclusion

- Nnaturalness addresses the small hierarchy puzzle associated with the higgs mass via N copies of SM with a range of Higgs mass squared values.
- The Higgs first exotic sector undergoes a QCD FOPT, with the GW peaked in the $\mathrm{n}Hz - \mu Hz$ frequency.
- Nnaturalness by itself cannot explain the recent observation of stochastic GW background observed recently by NANOGrav.
- We find however that a large parameter of Nnaturalness parameter space can be probed by future PTA experiments as well as planned (LISA, BBO) and proposed (μ-Ares) space based gravitational wave observatories.

THANK YOU!

BACKUP Slides

Reheaton decays : Exotic Sectors

- 1-loop decays to SU(2) gauge bosons are given as,

$$\Gamma_{\phi\to B_{-i}B_{-i}} = \frac{g'^4 a^2}{4096\pi^5 m_\phi}|\tau A_0\left(\tau\right)|^2$$

$$\Gamma_{\phi\to W^a_{-i}W^a_{-i}} = \frac{3g^4 a^2}{4096\pi^5 m_\phi}|\tau A_0\left(\tau\right)|^2, \qquad \tau = \frac{m_\phi^2}{4m_{H_i}^2},$$

where $A_0(\tau)$ is given by

$$A_0(\tau) = \tau^{-2}(f(\tau)-\tau),$$

$$f(\tau) = \begin{cases} \arcsin^2(\sqrt{\tau}) & \tau \leq 1, \\ -\frac{1}{4}\left(\log\left(\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}}\right) - i\pi\right)^2 & \tau > 1. \end{cases}$$

Reheaton decays : Exotic Sectors

- For $m_\phi > 2m_{H_{-i}}$, the decay to 2 exotic Higgs is given as

$$\Gamma_{\phi\to H_{-i}H^\dagger_{-i}} = \frac{a^2}{8\pi m_\phi}\sqrt{1-\frac{4m^2_{H_{-i}}}{m^2_\phi}}.$$

- For $m_{H_{-i}} < m_\phi < 2m_{H_{-i}}$ the three body decay happens with one off-shell Higgs. Leading state is the one with top quark, given as

$$\Gamma_{\phi\to H_{-i}Q_{L,-i}\bar{t}_{R,-i}} = \frac{3y_t^2 a^2}{128\pi^3 m_\phi^3}\int_0^{(m_\phi - m_{H_{-i}})^2} ds \frac{s\lambda^{1/2}(m^2_\phi, m^2_{H_{-i}}, s)}{(s-m^2_{H_{-i}})^2 + (m_{H_{-i}}\Gamma_H)^2}$$

The 4 body decay with both off shell Higgs is not relevant for our parameter space

Cosmology of exotic sector

- Neutrinos typically decouple while mu, tau are still relativistic.

$$T_{-1,\nu\,\mathrm{dec}} \sim 50\,\mathrm{eV} - 200\,\mathrm{eV}$$

- Following neutrino decoupling, the taus, and subsequently the muons, annihilate and heat the photon bath relative to the neutrinos by a factor

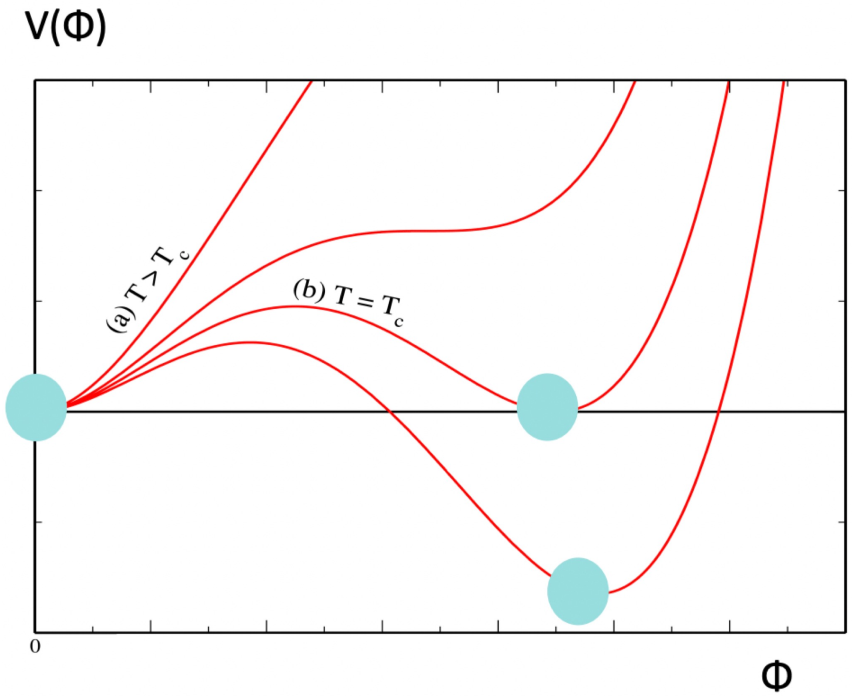
$$\frac{T^{\nu}_{-1}}{T_{-1}} = \left(\frac{11}{25}\right)^{1/3} \quad \text{for} \quad T_{-1} < m_{\mu_{-1}},$$

- Thus, near recombination the exotic sector relativistic species comprise photons, neutrinos, and electrons.

DoF during various epochs

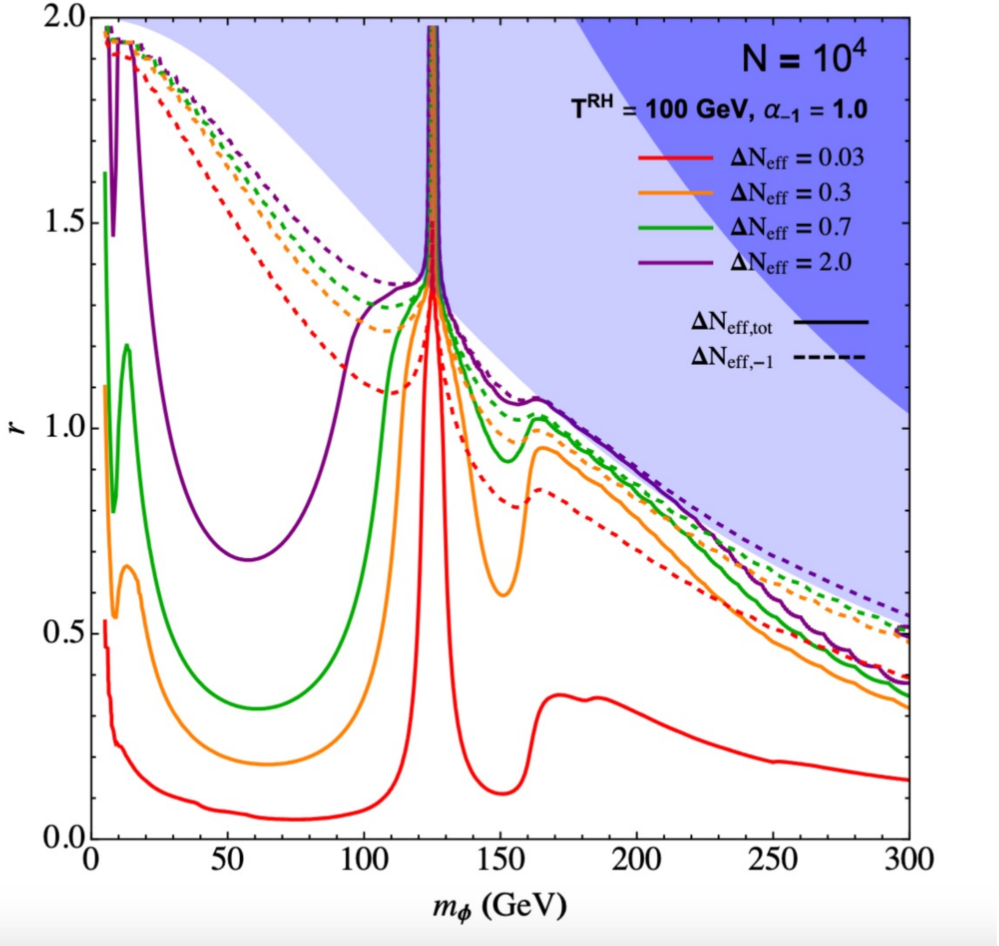
Epoch \ Sector	$g_{*\rho}$			g_{*s}		
	SM	$i=1$	$i=-1$	SM	$i=1$	$i=-1$
RH	≈ 100	≈ 100	≈ 100	≈ 100	≈ 100	≈ 100
perc	≈ 60	≈ 60	102.75	≈ 60	≈ 60	≈ 102.75
rh	≈ 60	≈ 60	58.75	≈ 60	≈ 60	≈ 58.75
CMB	3.36	2	12.4	3.91	3.91	12.8
0	2	2	7.25	3.91	3.91	7.81

FOPT : Parameters



- Tc – critical temperature
- Tn – nucleation temperature
- Tp - percolation temperature
- Strength of PT : $\alpha_{-1} = \frac{\Delta V}{\rho_{\mathrm{rad},-1}}$ $\alpha_{\mathrm{tot}} = \frac{\Delta V}{\rho_{\mathrm{rad,tot}}}$
- Rate of transition : $\frac{\beta}{H}$
- QCD with 3 or more massless quarks undergoes FOPT [Pisarski,Wilczek 1984]

https://osuhep.okstate.edu/images/seminar-slides/EWPT_Huang.pdf



The solid contours show the total ΔN_{eff} while the dashed contours show the contribution only from the exotic sectors. The blue shaded region shows where the decay $\phi \to H_{-1}H_{-1}$ is on-shell.

GW signal parametrisation

- GW today is redshifted, and is related to that at emission as,

$$h^2\,\Omega^0_{\rm GW}(f) = h^2\mathcal{R}\,\Omega^{\rm em}_{\rm GW}\left(\frac{a_0}{a_{\rm perc}}f\right).$$

- The following parameters go into expression for Ω_{GW}^{em}
- Normalization factors and exponents :

$$(N_{\rm BW}, N_{\rm SW}) = (1, 0.159) \qquad (p_{\rm BW}, p_{\rm SW}) = (2, 2) \qquad (q_{\rm BW}, q_{\rm SW}) = (2, 1)$$

- Potential suppression due to wall velocity :

$$(\Delta_{\rm BW}, \Delta_{\rm SW}) = (\tfrac{0.11 v_{\rm w}^3}{0.42+v_{\rm w}^3}, 1)$$

- Spectral shape function and peak frequencies :

$$s_{\rm BW}(x) = \frac{3.8\,x^{2.8}}{1+2.8\,x^{3.8}}, \qquad s_{\rm SW}(x) = x^3\left(\frac{7}{4+3\,x^2}\right)^{7/2},$$

$$f_{\rm p,BW} = 0.23\,\beta, \qquad f_{\rm p,SW} = 0.53\,\beta/v_{\rm w}.$$

Cosmological Constraints

- Pulsar Timing Array (PTA experiments - observing correlations in variations in arrival times of radio signals from pulsars)
  - Nanograv : Recent data probing GW signals at O(1-10) nHz frequencies
  - SKA (Square kilometer Array) :
- Space based interferometers:
  - BBO (Big Bang Observatory)
  - MuAres
  - LISA (Laser Interferometer Space Antenna)