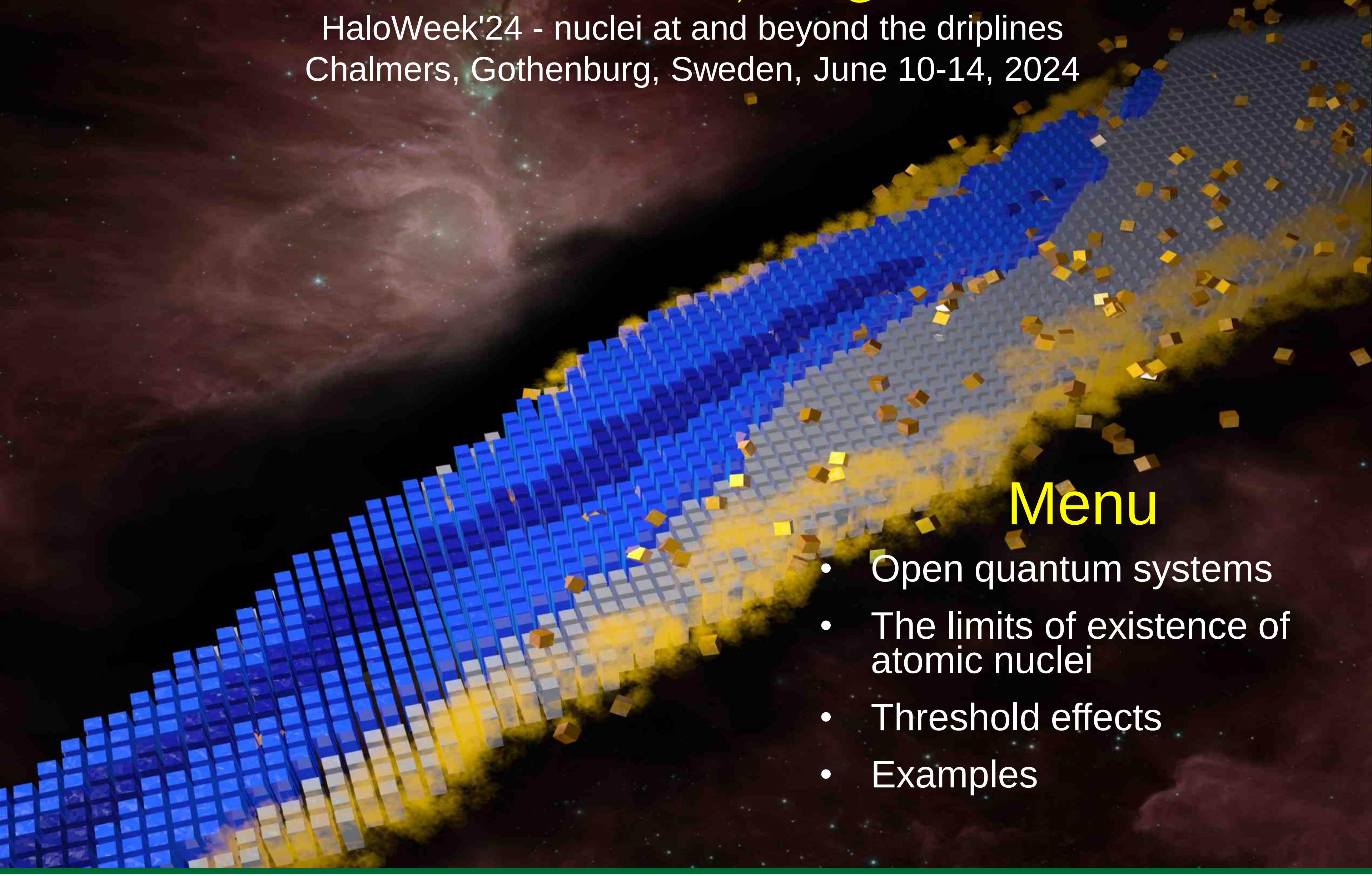
Threshold Phenomena: Exploring the Proton-Rich Nuclear Purgatory

Witold Nazarewicz, FRIB@MSU

HaloWeek'24 - nuclei at and beyond the driplines
Chalmers, Gothenburg, Sweden, June 10-14, 2024



Menu

- Open quantum systems
- The limits of existence of atomic nuclei
- Threshold effects
- Examples

Basic concepts

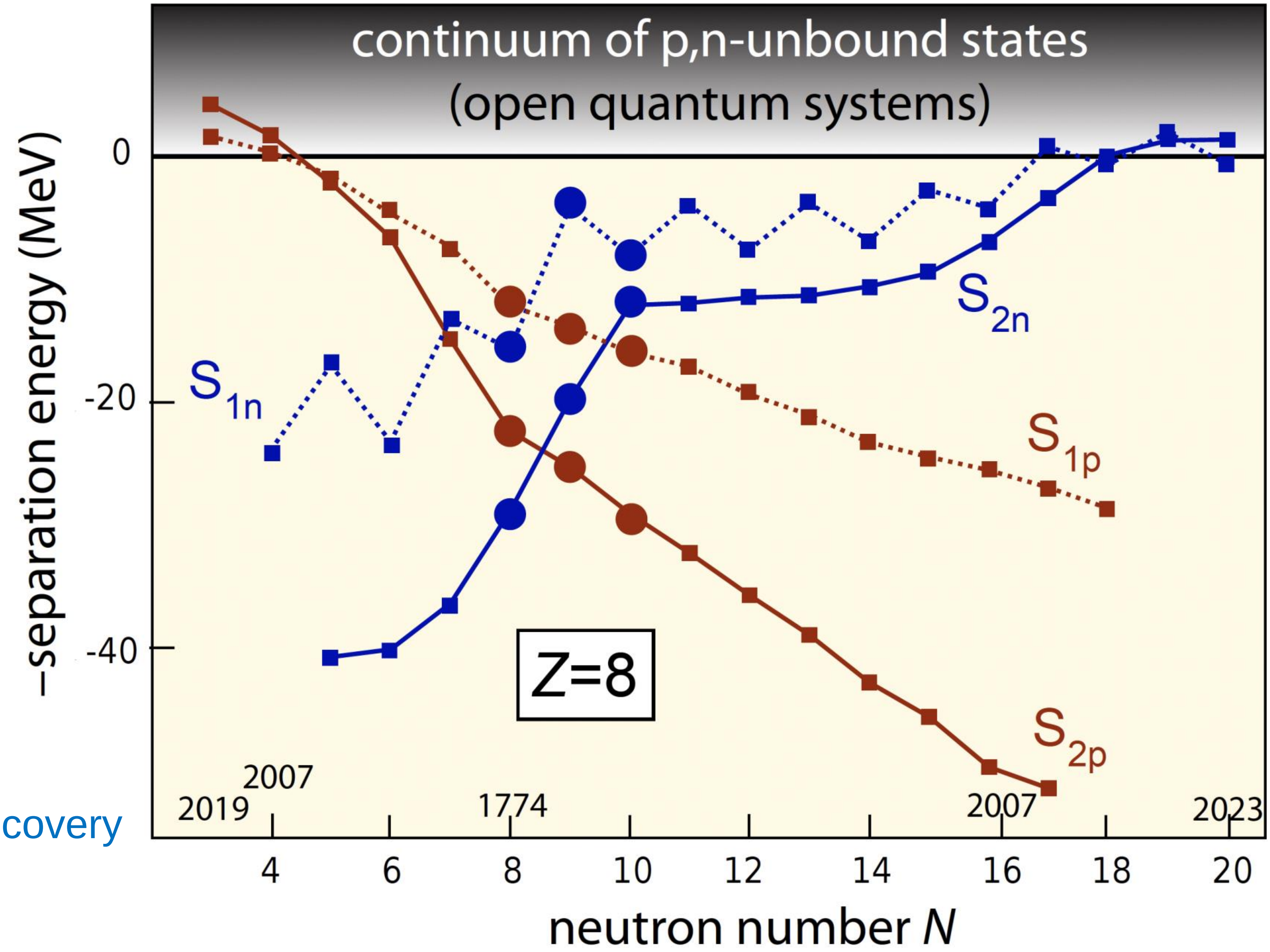
Recent reviews on proton-unstable nuclei

- Blank and Page, Springer Handbook of Nuclear Physics, 2022
- Pfützner and Mazzocchi, Springer Handbook of Nuclear Physics, 2022
- Pfützner, Mukha, and Wang, Prog. Part. Nucl. Phys. 2023

Questions:

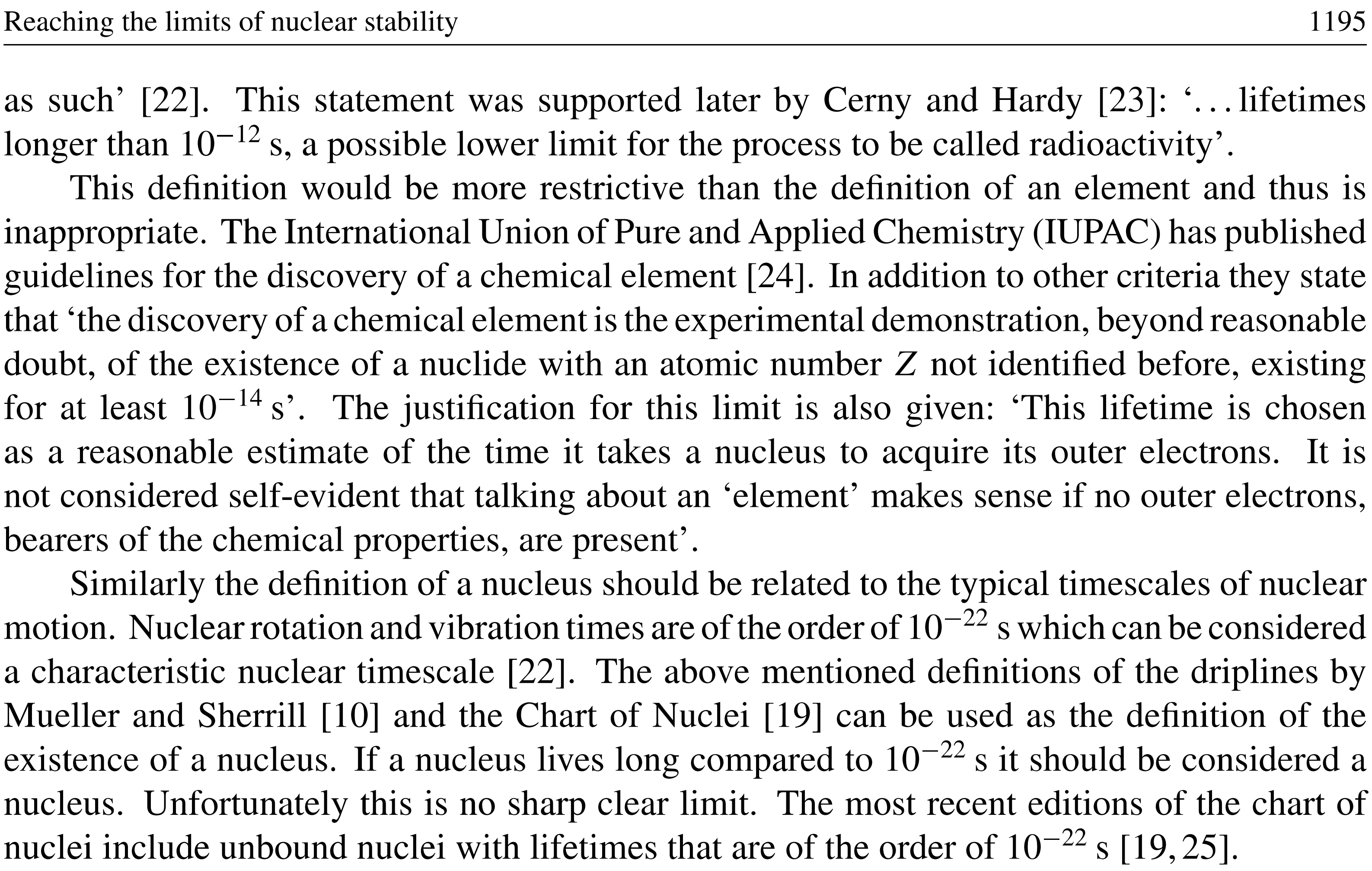
- *What is the nature of near-threshold states?*
- *What is the structure of the many-body continuum in the presence of several thresholds?*
- *At what point are states dissolved into the scattering continuum?*
- *What can we learn from two-nucleon decays?*
- *Can we reach the onset of the non-exponential decay by studying broad proton resonances?*

Charity **Kondo**



Discovery

Thoennessen, Rep. Prog. Phys. **67** (2004) 1187–1232



Reaching the limits of nuclear stability 1195

as such' [22]. This statement was supported later by Cerny and Hardy [23]: '…lifetimes longer than 10^{-12} s, a possible lower limit for the process to be called radioactivity'.

This definition would be more restrictive than the definition of an element and thus is inappropriate. The International Union of Pure and Applied Chemistry (IUPAC) has published guidelines for the discovery of a chemical element [24]. In addition to other criteria they state that 'the discovery of a chemical element is the experimental demonstration, beyond reasonable doubt, of the existence of a nuclide with an atomic number Z not identified before, existing for at least 10^{-14} s'. The justification for this limit is also given: 'This lifetime is chosen as a reasonable estimate of the time it takes a nucleus to acquire its outer electrons. It is not considered self-evident that talking about an 'element' makes sense if no outer electrons, bearers of the chemical properties, are present'.

Similarly the definition of a nucleus should be related to the typical timescales of nuclear motion. Nuclear rotation and vibration times are of the order of 10^{-22} s which can be considered a characteristic nuclear timescale [22]. The above mentioned definitions of the driplines by Mueller and Sherrill [10] and the Chart of Nuclei [19] can be used as the definition of the existence of a nucleus. If a nucleus lives long compared to 10^{-22} s it should be considered a nucleus. Unfortunately this is no sharp clear limit. The most recent editions of the chart of nuclei include unbound nuclei with lifetimes that are of the order of 10^{-22} s [19, 25].

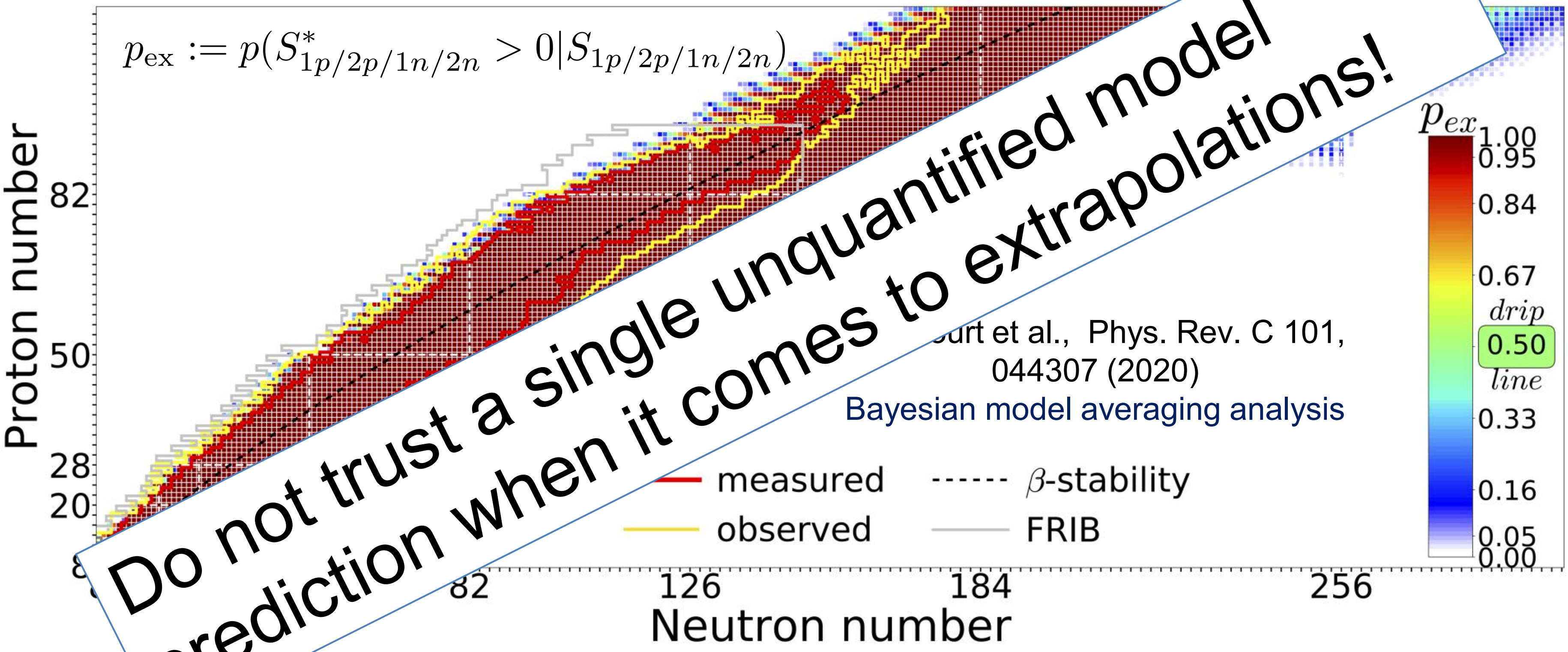
$$T_{s.p.} \text{ » } 3\times10^{-22}\,\text{sec} = 3\,\text{babysec}$$

A typical time associated with the s.p. nucleonic motion

For a nuclear state:

$$T_{1/2} >> T_{s.p.}$$

Nuclear landscape and particle drip lines



Do not trust a single unquantified model prediction when it comes to extrapolations!

...urt et al., Phys. Rev. C 101, 044307 (2020)

Bayesian model averaging analysis

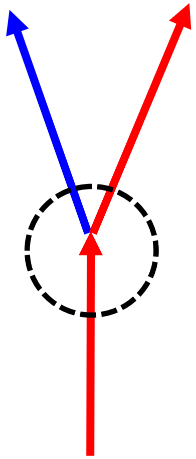
Proton drip line and beyond: Bayesian analysis of proton-emitting nuclei

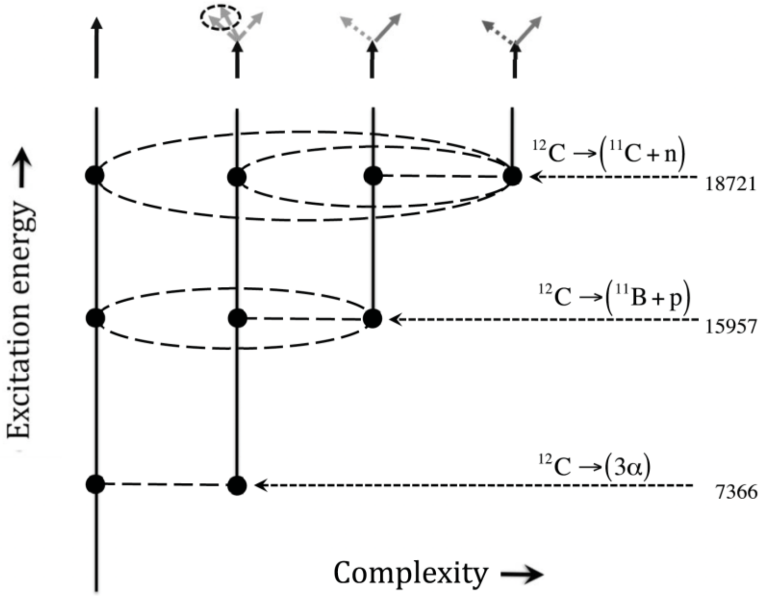
Neufcourt et al., Phys. Rev. C 101, 014319 (2020)

The precise position of thresholds essential for quantitative modeling.

Threshold phenomena

- The threshold is a branching point (hence, nonanalytic behavior).
- The threshold effects are rooted in the *unitarity of scattering matrix* and the resulting *flux conservation*.
- If a new channel opens, a redistribution of the flux in other open channels appears, i.e., a modification of their reaction cross-sections.
- The shape of the near-threshold cusp depends strongly on the orbital angular momentum.





The multichannel representation of ^{12}C

With the increasing excitation energy, subsequent decay channels open at threshold energies Q_n, leading to a complex *multichannel network of couplings*. When a new channel opens up at the threshold Q_i, the unitarity imposes the appearance of new channel couplings; hence, a *modification of all eigenfunctions*.

Real-energy picture: change of asymptotics

E.P. Wigner, Phys. Rev. 73, 1002 (1948), the Wigner cusp
G. Breit, Phys. Rev. 107, 923 (1957)
A.I. Baz', JETP 33, 923 (1957)
D.R. Inglis, Nucl. Phys. 30, 1 (1962)
F.C. Barker, Proc. Phys. Soc. 84, 681 (1964)
A.M. Lane, Phys. Lett. 33B, 274 (1970)
S.N. Abramovich et al.,Part. and Nucl. 23, 305 (1992).

R-matrix approach:
anomaly is a result of different asymptotic conditions below and above the threshold

A characteristic behavior (a cusp) of scattering and reaction cross sections of *neutral* particles in the vicinity of a reaction threshold (Wigner threshold law)

$$\sigma(i \to j) \sim k_j^{2\ell_j+1} \sim E_j^{\ell_j+\frac{1}{2}}$$

Endoergic reaction with the production of slow neutral particles

$$\sigma(i \to j) \sim k_i^{2\ell_i-1} \sim E_i^{\ell_i-\frac{1}{2}}$$

Exoergic reaction (e.g., the absorption of slow neutrons by nuclei)

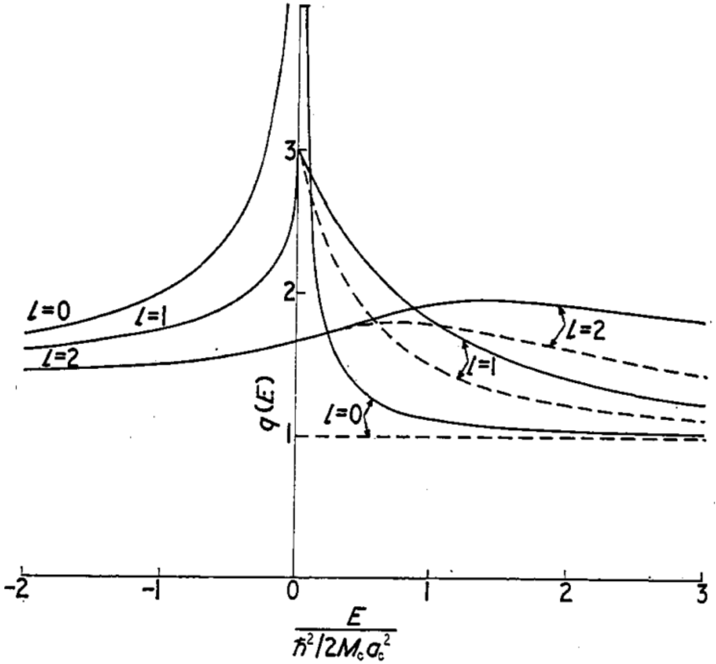
For charged particles, the angular momentum dependence is smooth.
Such a behavior is seen in all related quantities (scattering matrix, spectroscopic factors,…)

Barker, Proc. Phys. Soc. 84, 681 (1964)
R-matrix in the Lane-Thomas formulation. One level approximation

$$g_\ell(E) = \frac{1}{\pi}\frac{d\delta_\ell(E)}{dE}$$

This relation connects the Wigner cusp phenomenon with the appearance of threshold resonances and anti-bound (or virtual) states.

neutral particles



charged particles

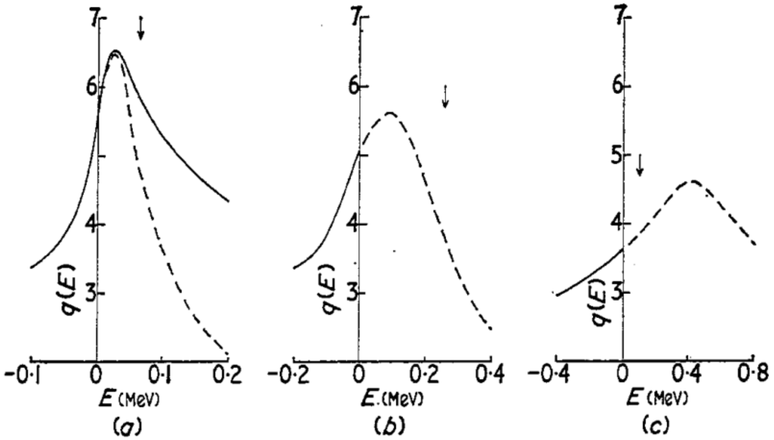
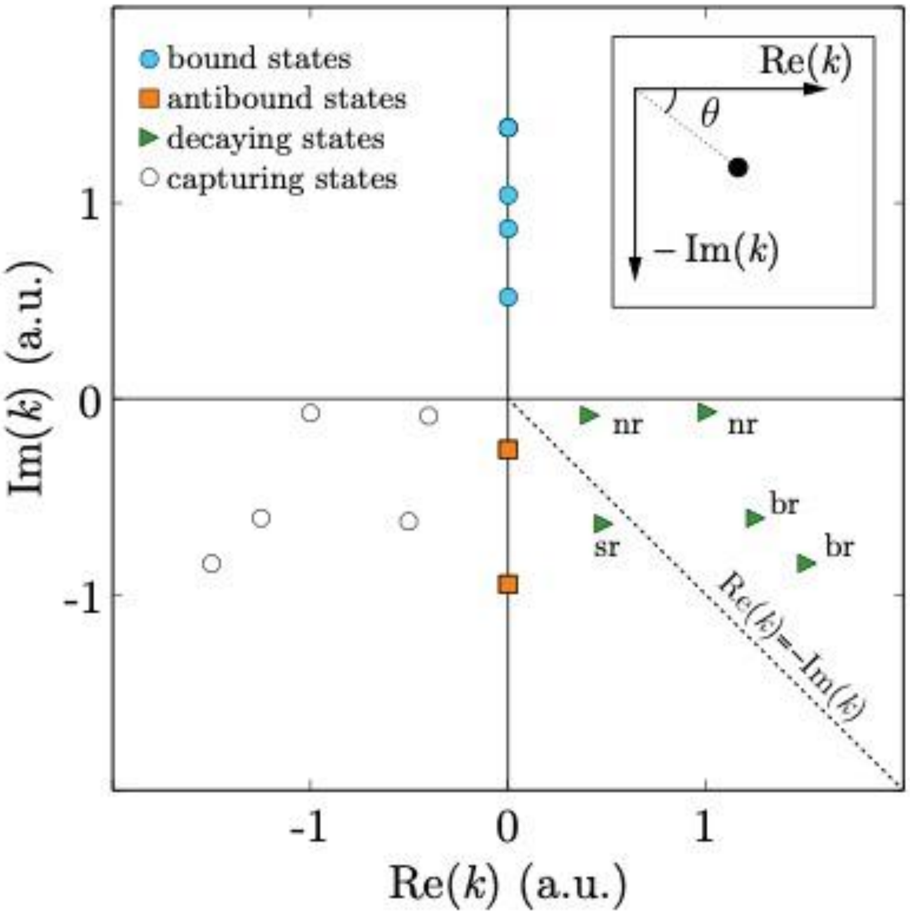


Figure 2. Enhancement factors for channels (*a*) ^{3}H + d, (*b*) ^{3}He + d, (*c*) ^{4}He + ^{4}He, all with $l = 0$ and with values of a_c and $\gamma_{\lambda c}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_1(E)$. Arrows indicate energies of observed levels of ^{5}He, ^{5}Li and ^{8}Be.

Large enhancement factor for the probability of finding the eigenenergy around the threshold

Proton emitters are decaying resonances. They are represented by resonant states in the complex-*k* plane

$$\tilde{E} = E_r - i\Gamma/2$$



- Resonant states with Re(E)>0 and small Γ can be associated with narrow resonances (nr). Proton emitters in heavy nuclei belong to this class.
- Many prompt proton emitters in light nuclei are broad resonances (br)
- The antibound proton states do not exist because of the Coulomb force.
- The subthreshold resonant states lie on the second Riemann energy sheet. They have Re(E) < 0 and Γε 0.
- Low-momentum threshold resonant states result in the low-energy cross-section enhancement. These poles should be viewed as *scattering features* rather than physical states of the system.

Resonant states of the *NN* system

T=0

- *np*: bound state (deuteron), k=+i0.2315 fm^{-1}

T=1

- *np:* virtual state, k=–i0.044 fm^{-1}
- *nn*: virtual state, k=–i0.0559(33) fm^{-1} [Babanko and Petrov, Phys. At. Nucl. 76, 684 (2013)]
- *pp*: threshold resonant state, k=(0.0647–i0.0870) fm^{-1} [Kok, Phys. Rev. Lett. 45, 427 (1980)]

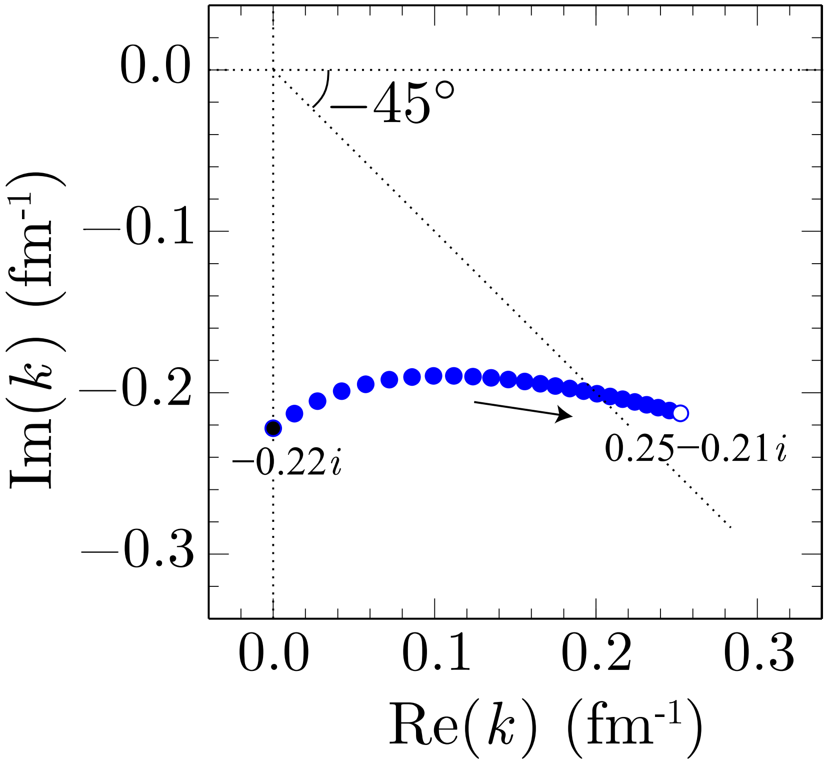
For the *4n* system, see Deltuva and Lazauskas, Phys. Rev. C 100, 044002 (2019)

Nakamura, Hiyama,…

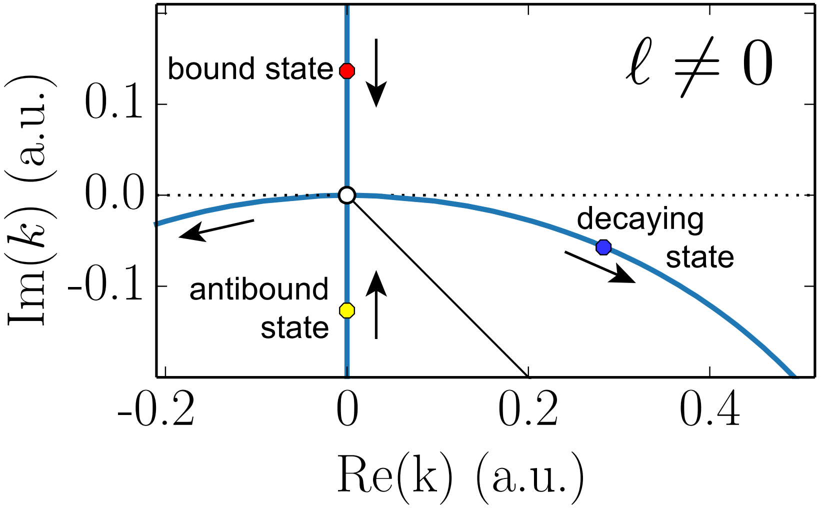
$\ell = 0$ component present

$\ell > 0$

Generic behavior: W. Domcke, J. Phys. B 14, 4889 (1981)



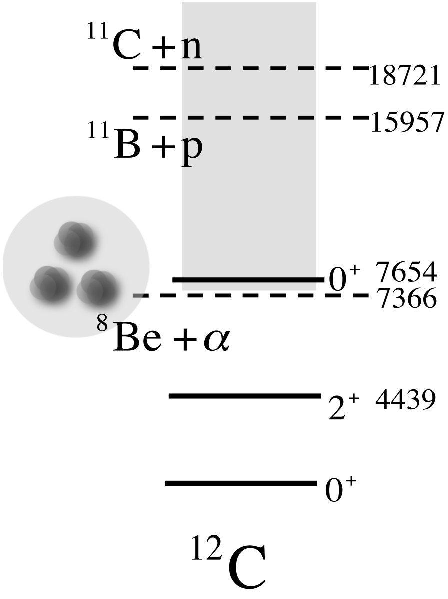
Bound state becomes virtual state after crossing the threshold

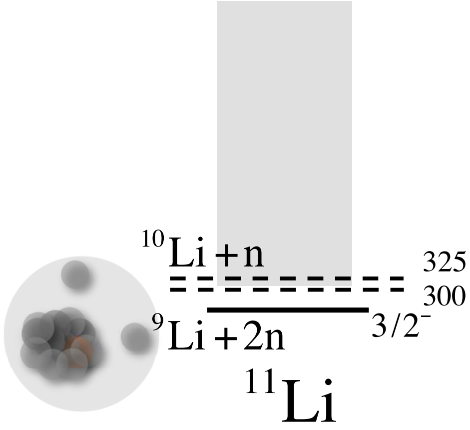


X. Mao et al., Phys. Rev. A 98, 062515 (2018)

Bound state and virtual state coalesce at the threshold forming an exceptional point (a double pole). As the potential strength decreases, two Gamow resonant states are born: one decaying and one capturing.

The origin of nuclear clustering: collective effect due to the continuum coupling





J. Okołowicz, M. Płoszajczak, WN,

- Fortschr. Phys. 61, 66 (2013)
- Prog. Theor. Phys. Supplement **196**, 230 (2012)
- Acta. Phys. Pol. 45, 331 (2014)

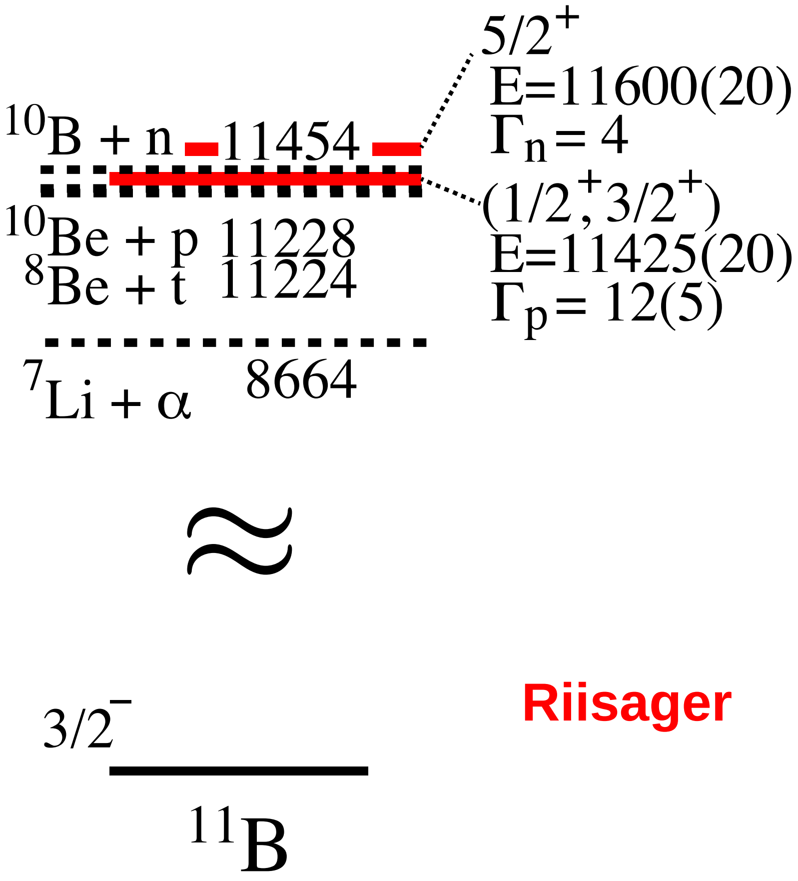
- Proximity of the threshold induces the collective mixing of shell-model eigenstates resulting in a single aligned eigenstate of the open quantum system Hamiltonian
- The presence of charged-cluster states near the respective charged-cluster emission threshold is a signature of a profound change of the near-threshold shell model wave function and the direct manifestation of continuum-coupling correlations.

Many experimental examples!

Near-Threshold Proton-Emitting Resonance in ^{11}B

- Evidence of a near-threshold resonance in ^{11}B, Ayyad et al., PRL 123, 082501 (2019); Phys. Rev. Lett. 129, 012501 (2022)
- Observation of a near-threshold proton resonance in ^{11}B, Lopez-Saavedra et al., Phys. Rev. Lett. 129, 012502 (2022)

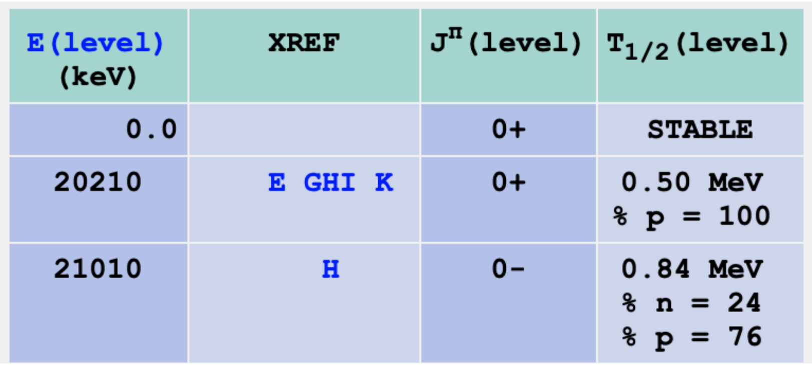
- J. Okołowicz et al. Phys. Rev. Lett. 124, 042502 (2020)
- A. Volya, Europhys. Lett. 130, 12001 (2020)
- W. Elkamhawy et al., Phys. Lett. B 821,136610 (2021)
- J. Okołowicz et al., J. Phys. G 49, 10LT01 (2022)



Riisager

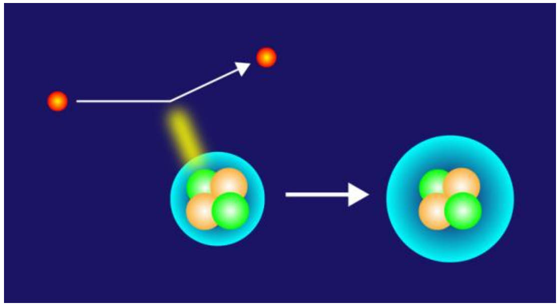
- Two close lying resonances: proton and neutron threshold-aligned states.
- Very important states!
- Note: cross section for neutron capture on ^{10}B is 3600 b!

Coupled-channel no-core Gamow shell model description of the excited 0^+ state of the alpha particle, Michel, Płoszajczak, WN, Phys. Rev. Lett. 131, 242502 (2023)

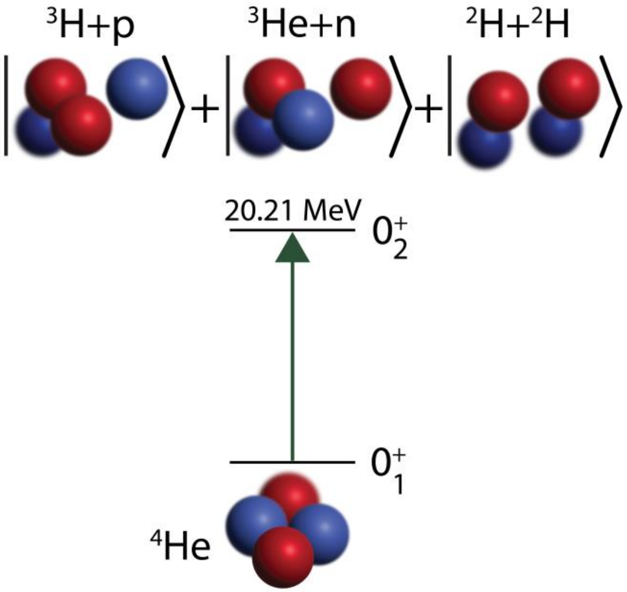


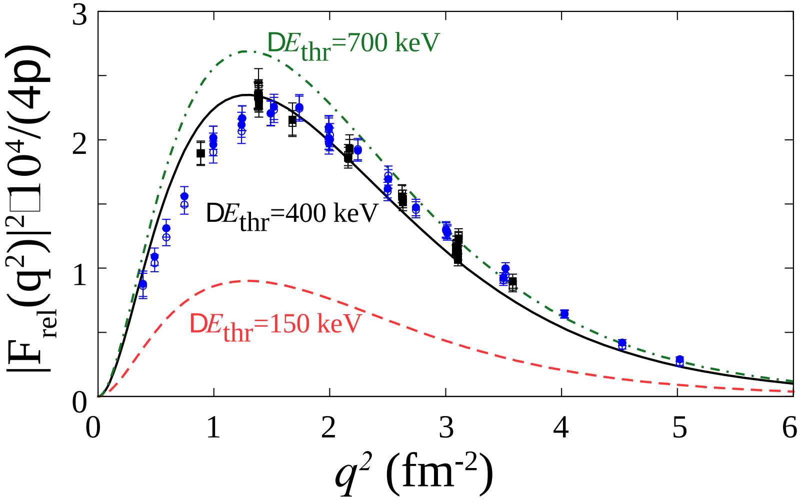
E(level) (keV)	XREF	J^π(level)	$T_{1/2}$(level)
0.0		0+	STABLE
20210	E GHI K	0+	0.50 MeV % p = 100
21010	H	0-	0.84 MeV % n = 24 % p = 76

Experiment: Monopole form factor
Phys. Rev. Lett. 130, 152502 (2023)



S_p=19.8 MeV, S_n=20.6 MeV ⇒ open quantum system description needed!





Very complex structure of the proton continuum!

Time-dependent framework

$$\Psi(t) = \int \sum_i a_i(E)\; e^{-i\frac{E}{\hbar}t}\; |E, c_i\rangle \;\mathrm{d}E$$

asymptotic
configuration

survival
amplitude

$$\mathcal{A}(t) = \langle\Psi(0)|\Psi(t)\rangle = \int_0^{+\infty} \rho(E) e^{-i\frac{E}{\hbar}t} \mathrm{d}E$$

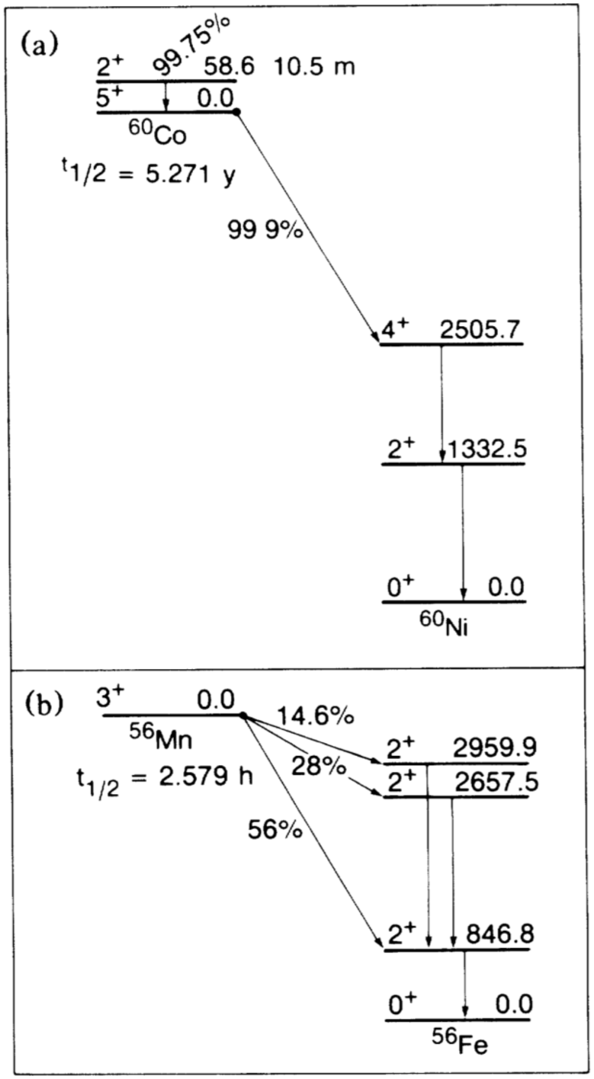
spectral function $\rho(E) = |\langle E|\Psi\rangle|^2$

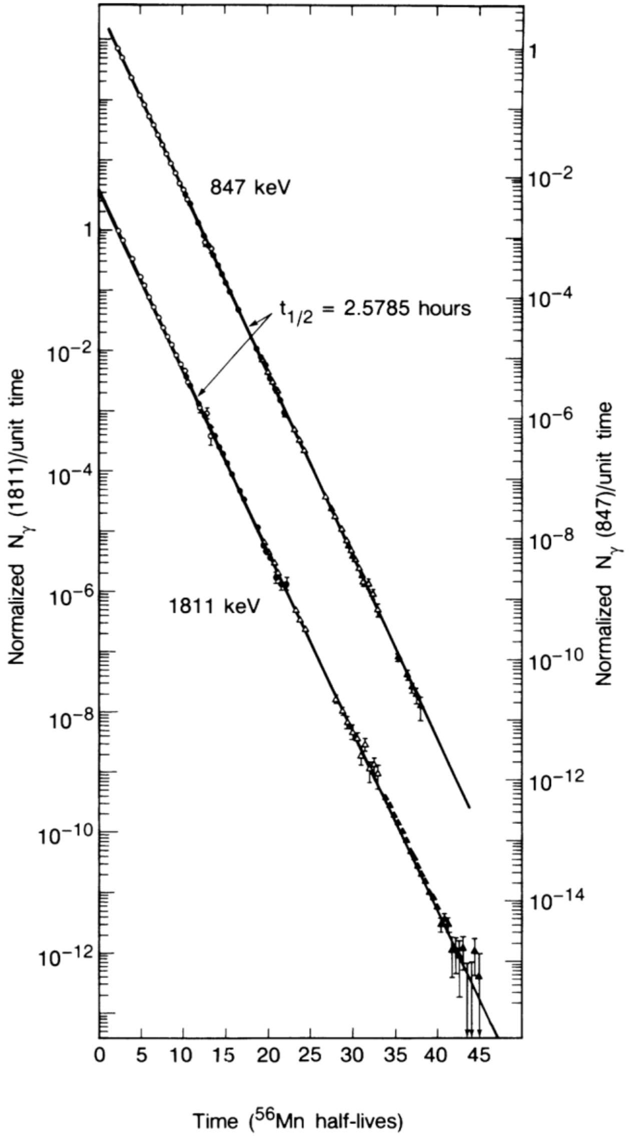
Fore a narrow, isolated resonance, the spectral function assumes the Breit-Wigner shape.

$$\tilde{E} = E_r - i\Gamma/2$$

$$|\mathcal{A}(t)|^2 \propto \exp(-\Gamma t)$$

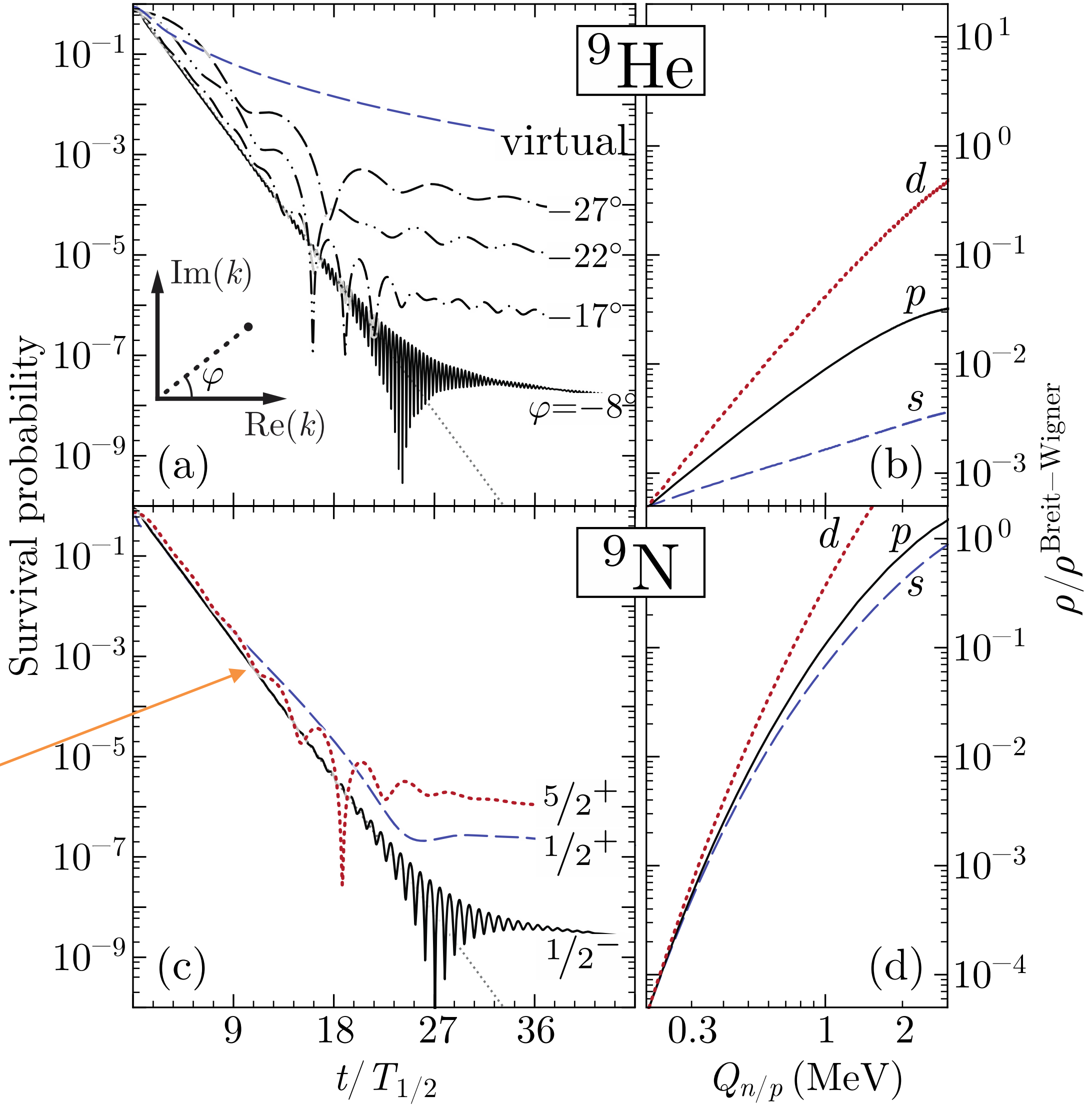
E. Norman et al., PRL 60, 2246 (1988) [^{60}Co and ^{56}Mn] and Phys. Lett. B 357, 521 (1995) [^{40}K]





The Decay law at long times

S.M. Wang, WN, A. Volya, Y.G. Ma, Phys. Rev. Res. 5, 023183 (2023)



resonant contribution

Proton emission in light and heavy nuclei

Light nuclei

- Small Coulomb barriers
- Broad resonances or scattering features; short lifetimes
- Spectral functions deviate from Breit-Wigner
- WKB description questionable
- Time-dependent description possible
- Low ℓ-wave dominance
- Prompt emission

Heavy nuclei

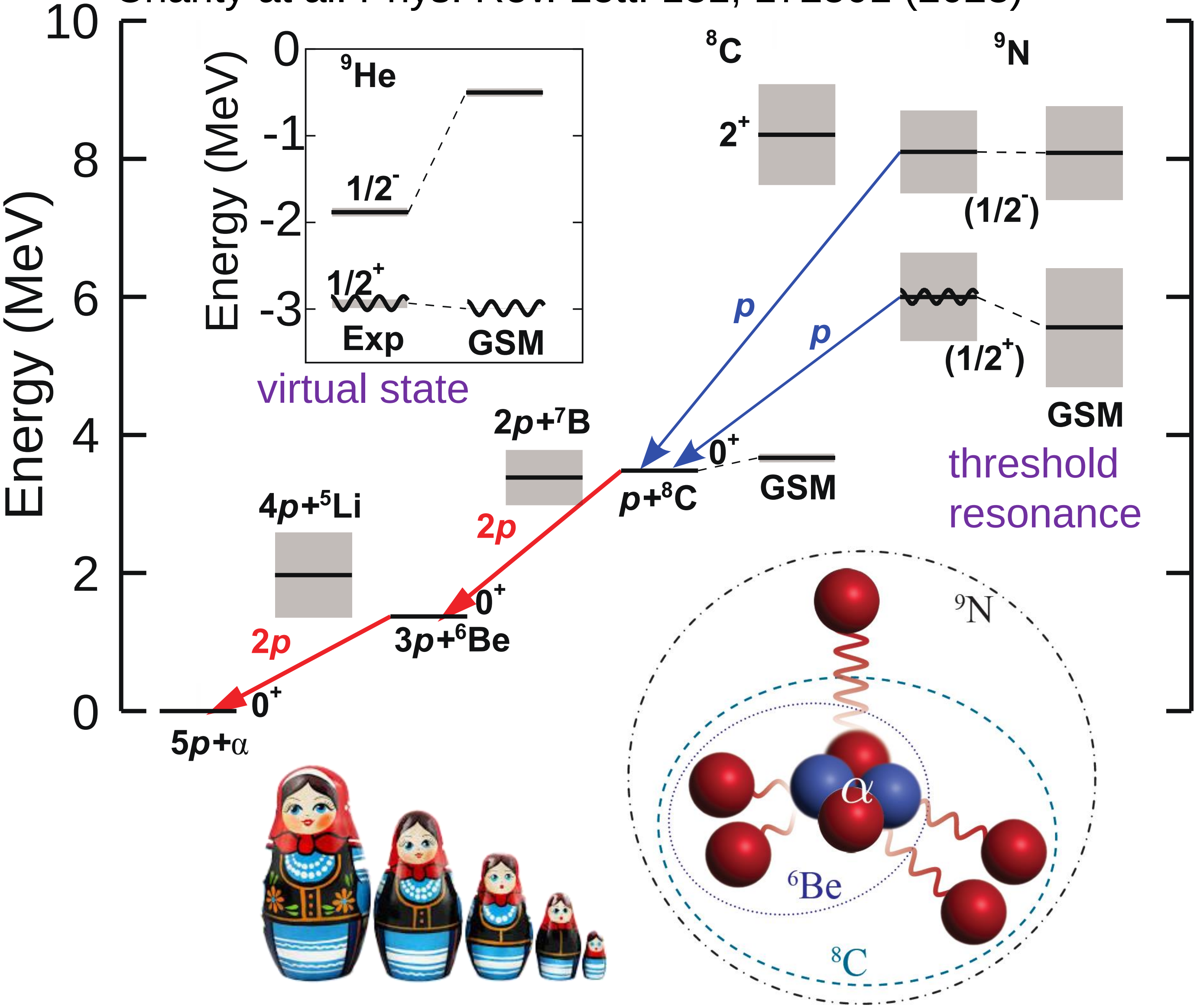
- Large Coulomb barriers
- Narrow resonances; long lifetimes
- Gamow states have Breit-Wigner spectral functions
- WKB description excellent
- Time-dependent description hardly applicable
- All ℓ-waves present: low and high
- Proton radioactivity

Lifetimes as short as ps can be considered as radioactivity [Cerny and Hardy, Annu. Rev. Nucl. Part. Sci. 27, 323 (1977)]

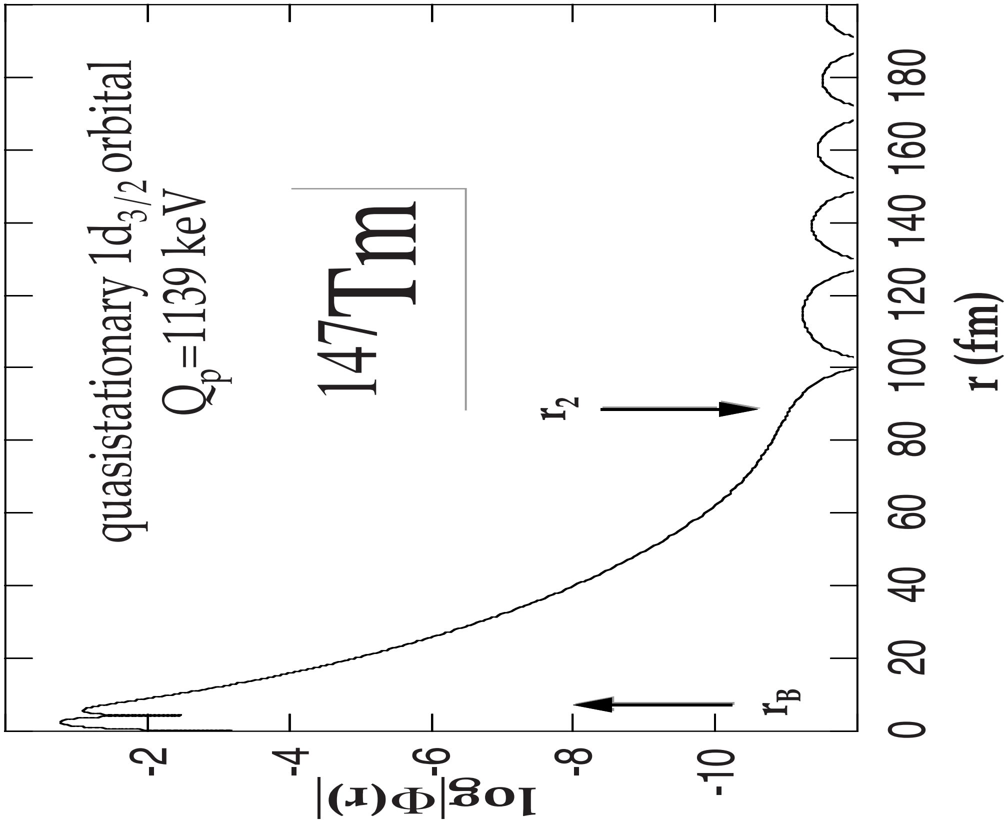
^{9}N: 5-proton emitter

Charity

Charity at al. Phys. Rev. Lett. 131, 172501 (2023)



Heavy proton emitters
Proton radioactivity: a sensitive spectroscopic tool

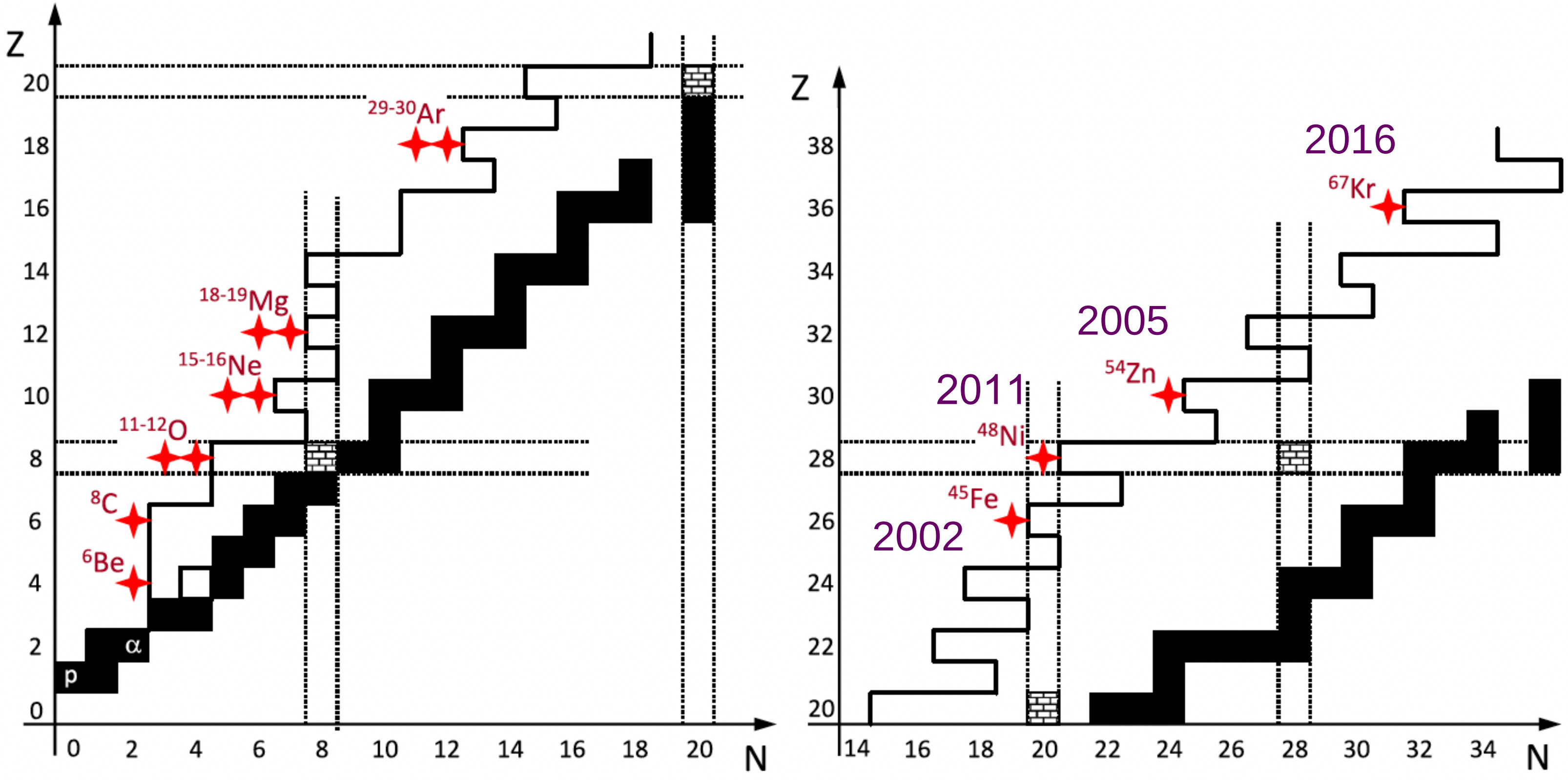


- Perfect examples of WKB tunneling (the proton has already been formed). Other approaches to narrow resonances (Gamow, R-matrix, DWBA, TPA…) give similar results
- Lifetimes (Geiger-Nuttall) *primarily* governed by *Q*, *Z*, and ℓ
- Deviations from Geiger-Nuttall uncover structural effects (e.g., deformation, pairing)

Two-proton emission

Moral

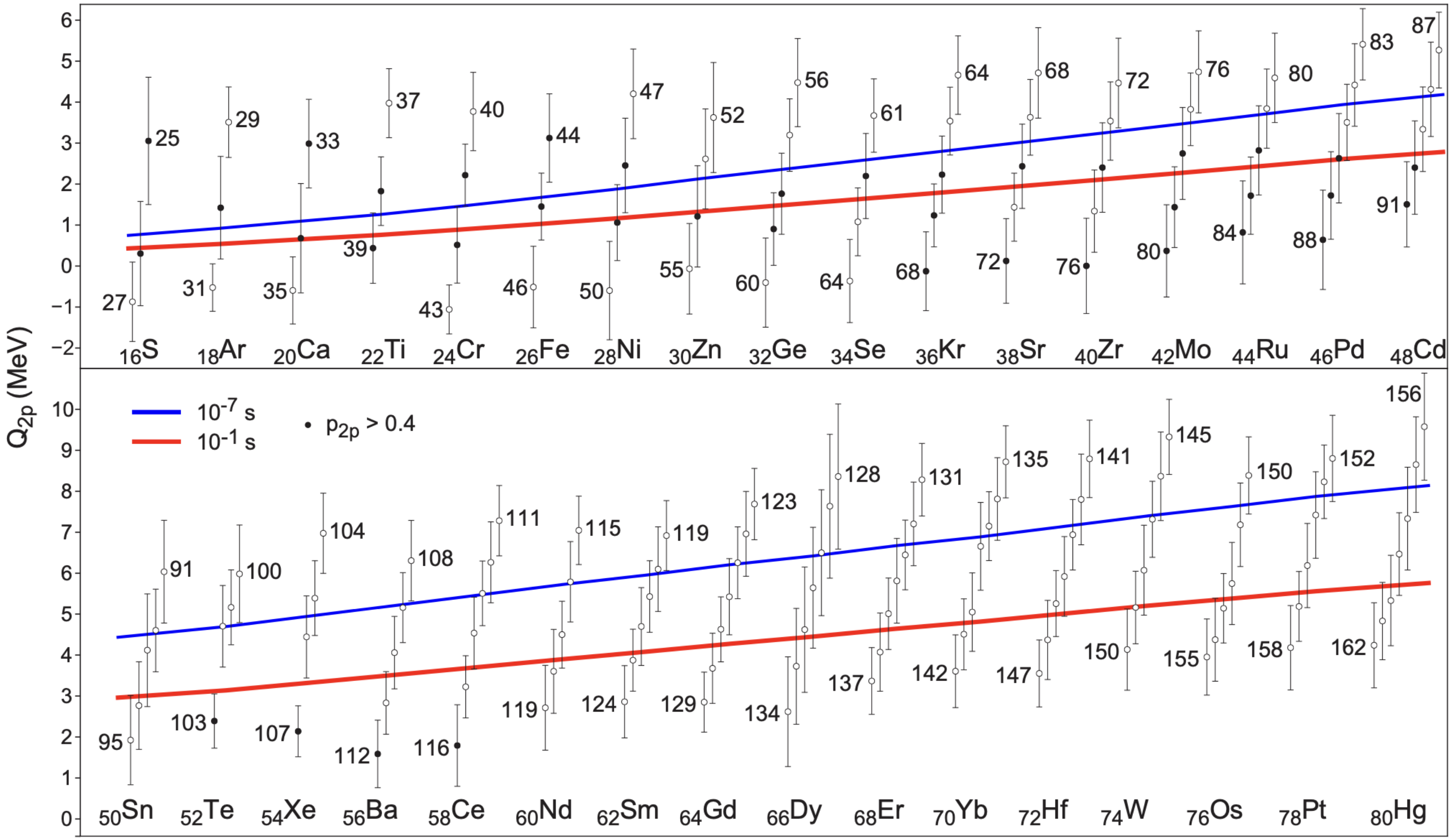
Two-proton emission and related phenomena, Pfützner, Mukha, and Wang, Prog. Part. Nucl. Phys. 2023.



Many instances of sequential and democratic decay
True (simultaneous) 2p emission observed!

Proton drip line and beyond: Bayesian analysis of proton-emitting nuclei

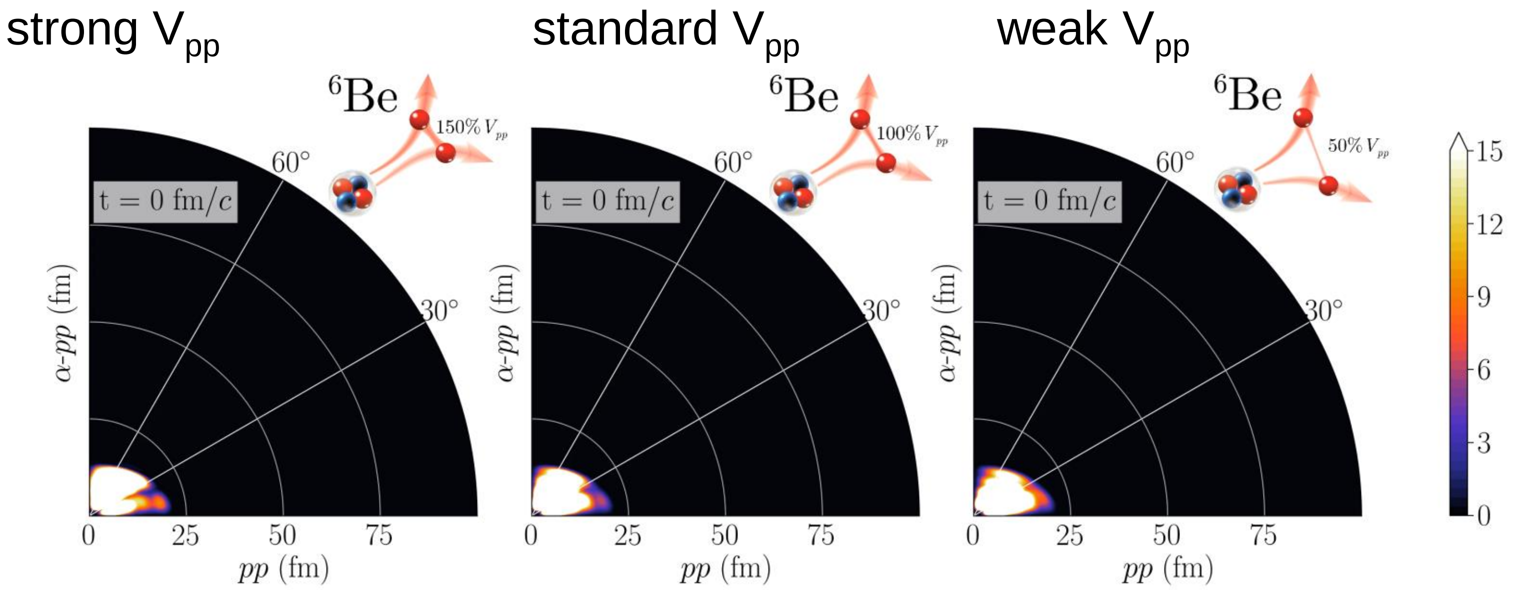
L. Neufcourt et al., Phys. Rev. C 101, 014319 (2020)

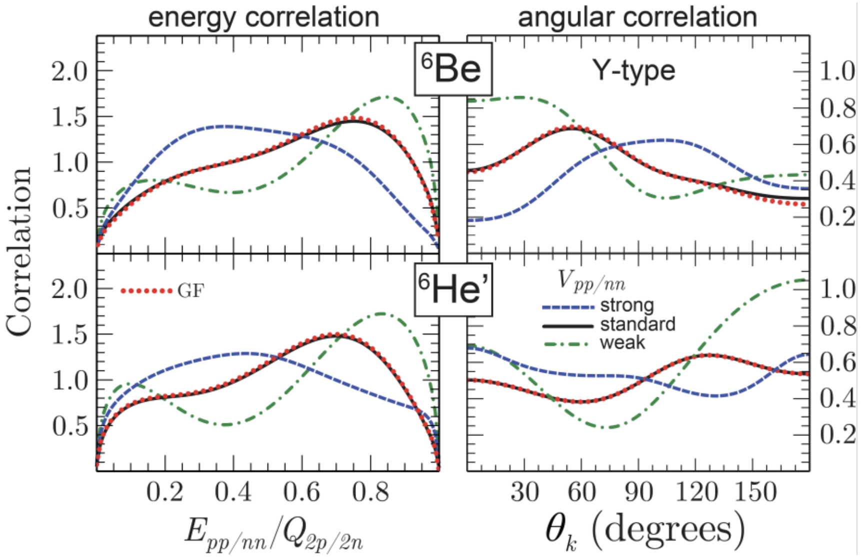


The most promising new candidates for the true 2p radioactivity are: ^{30}Ar, ^{34}Ca, ^{39}Ti, ^{42}Cr, ^{58}Ge, ^{62}Se, ^{66}Kr, ^{70}Sr, ^{74}Zr, ^{78}Mo, ^{82}Ru, ^{86}Pd, ^{90}Cd, and ^{103}Te. Competition between alpha and pp decay expected in ^{103}Te and ^{145}Hf. Many unresolved structural questions above ^{100}Sn.

Correlation studies for 2-nucleon emission

S. Wang and WN, Phys. Rev. Lett. 126, 142501 (2021)


strong V_{pp} standard V_{pp} weak V_{pp}

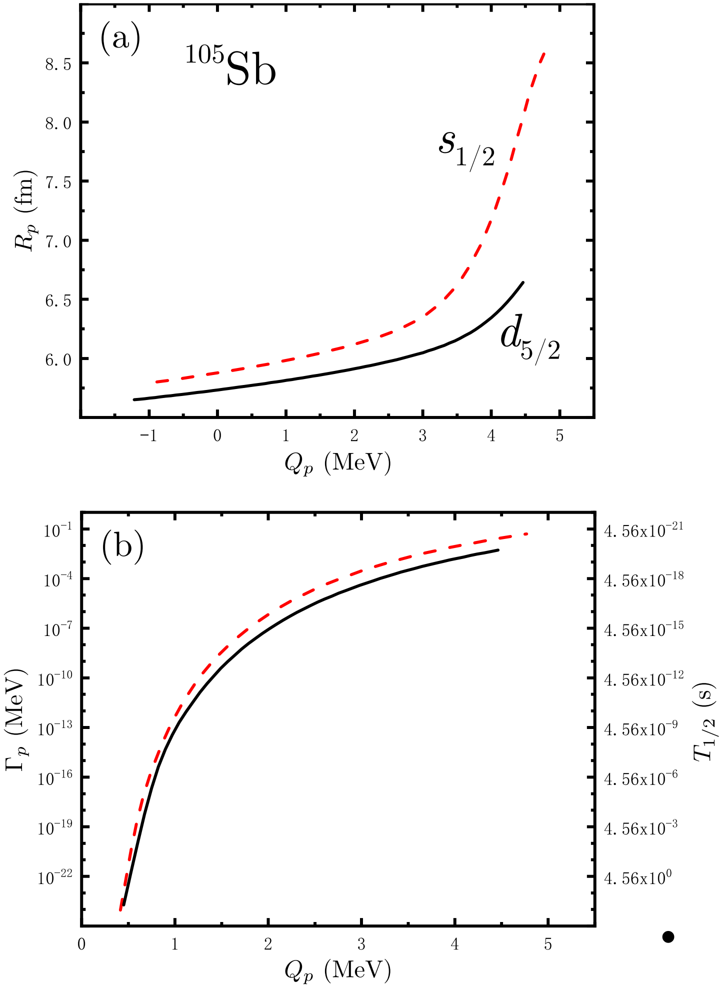


Time-dependent 3-body model

The energy and angular correlations in the Jacobi-Y angle between emitted nucleons strongly depend on the initial-state structure: the correlations between emitted nucleons provide invaluable information on the dinucleon structure in the initial state!

Charge radii of proton emitters

K. Lynch et al., ISOLDE (Sb chain)
L. V. Rodriguez et al, ISOLDE (Tm chain)



Y. Lin, S.M. Wang, and WN

Expectation value of an operator in a resonant state
T. Berggren, Phys. Lett. B 373, 1 (1996)

T. Myo and K Katō, Phys. Rev. C107, 014301 (2023)

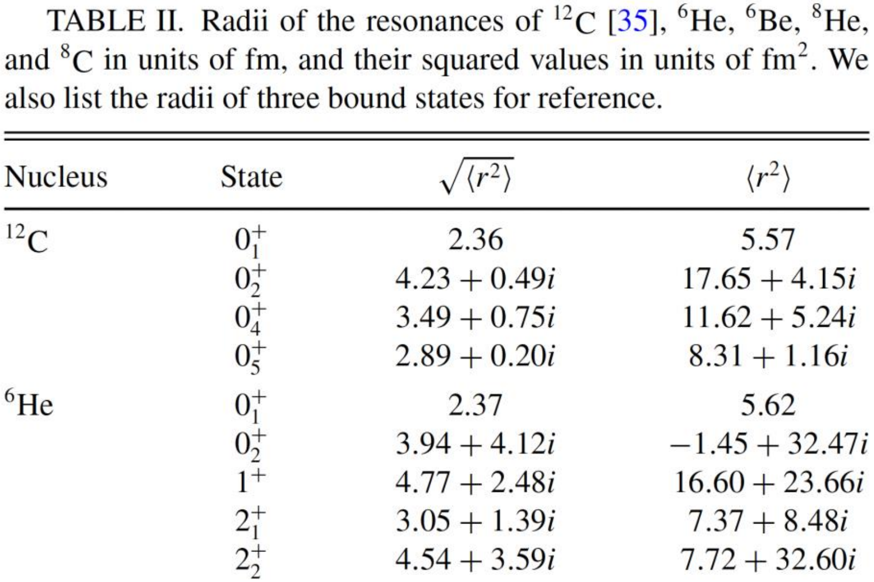


TABLE II. Radii of the resonances of ^{12}C [35], ^{6}He, ^{6}Be, ^{8}He, and ^{8}C in units of fm, and their squared values in units of fm^2. We also list the radii of three bound states for reference.

Nucleus	State	$\sqrt{\langle r^2\rangle}$	$\langle r^2\rangle$
^{12}C	0_1^+	2.36	5.57
	0_2^+	$4.23 + 0.49i$	$17.65 + 4.15i$
	0_4^+	$3.49 + 0.75i$	$11.62 + 5.24i$
	0_5^+	$2.89 + 0.20i$	$8.31 + 1.16i$
^{6}He	0_1^+	2.37	5.62
	0_2^+	$3.94 + 4.12i$	$-1.45 + 32.47i$
	1^+	$4.77 + 2.48i$	$16.60 + 23.66i$
	2_1^+	$3.05 + 1.39i$	$7.37 + 8.48i$
	2_2^+	$4.54 + 3.59i$	$7.72 + 32.60i$

G. Papadimitriou et al., Phys. Rev. C 84, 051304(R) (2011)

- Observables in resonant states acquire imaginary parts
- Visible impact of Q_p on charge radii

Summary

- Proton-unbound systems are many-body open quantum systems: many interdisciplinary avenues.
- In the area of proton emission, we have largely moved from the discovery phase to the era of using this process as a spectroscopic tool.
- Proton resonances are structurally different from neutron resonances (pp vs. nn, mirror nuclei).
- Two-proton correlation studies will bring a wealth of structural information.
- Proton-emitting states are important astrophysically
- It is a wonderful playground for nuclear theories

Thank you!