Effective field theory of weakly bound two-neutron halo nuclei

Dam Thanh Son (University of Chicago)

HaloWeek'24: Nuclei at and near the driplines
Chalmers University, 11 June 2024

References

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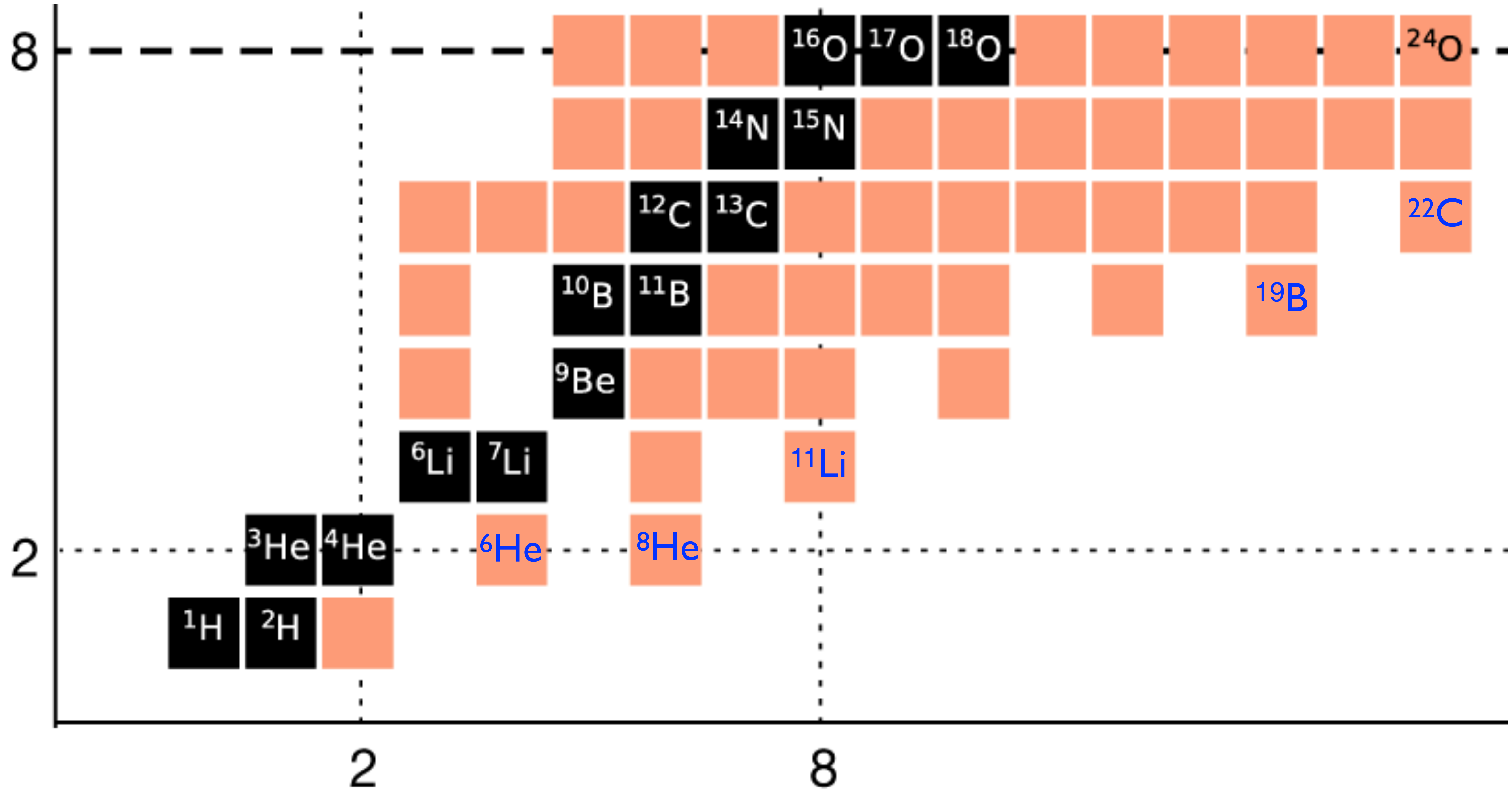
Davi Costa, Masaru Hongo, DTS, to appear



Tsunoda et al. Nature 587, 66 (2020)

Zeldovich's 1960 paper

SOVIET PHYSICS JETP VOLUME 11, NUMBER 4 OCTOBER, 1960

THE EXISTENCE OF NEW ISOTOPES OF LIGHT NUCLEI AND THE EQUATION OF STATE OF NEUTRONS

Ya. B. ZEL'DOVICH

Submitted to JETP editor October 22, 1959

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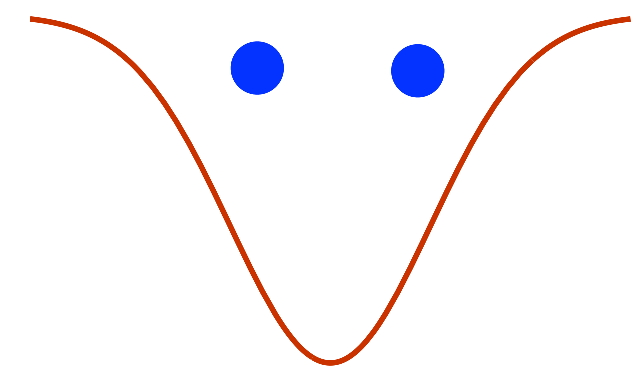
- One attractive model of two-neutron Borromean nuclei: Efimov effect
- large neutron-neutron, core-neutron scattering lengths, modeled by zero-range interaction
- Is Efimov effect necessary?

Core-neutron s-wave resonance needed?



- Two particles with zero-range resonant interaction, in Gaussian potential of an infinitely massive core:

$$H = -\frac{1}{2}(\nabla_1^2 + \nabla_2^2) - V_0(e^{-r_1^2/2} + e^{-r_2^2/2}) - c_0\delta(\vec{r}_1 - \vec{r}_2)$$

- core-neutron scattering length diverges when $V_0 = V_0^{\rm cn} = 0.671$
- At which $V_0^{\rm 3body}$ 3-body bound state first appears? How close this value is to $V_0^{\rm cn}$?

Variational calculation

- $$\psi(\vec{r}_1, \vec{r}_2) = \frac{e^{-\alpha(r_1^2+r_2^2)}}{|\vec{r}_1 - \vec{r}_2|}$$

  satisfies Bethe-Peierls boundary condition

- variational bound $V_0^{\rm 3body} \leq 0.417 < \frac{2}{3} V_0^{\rm cn}$
- better variational ansatz: $V_0^{\rm 3body} \leq 0.3285 < \frac{1}{2} V_0^{\rm cn}$
- 3-body bound state appears long before 2-body one

Two regimes

- When the core-neutron scattering length is large: Efimov effect
- But 3-body bound state can exist without the Efimov effect

strength of core-n attraction

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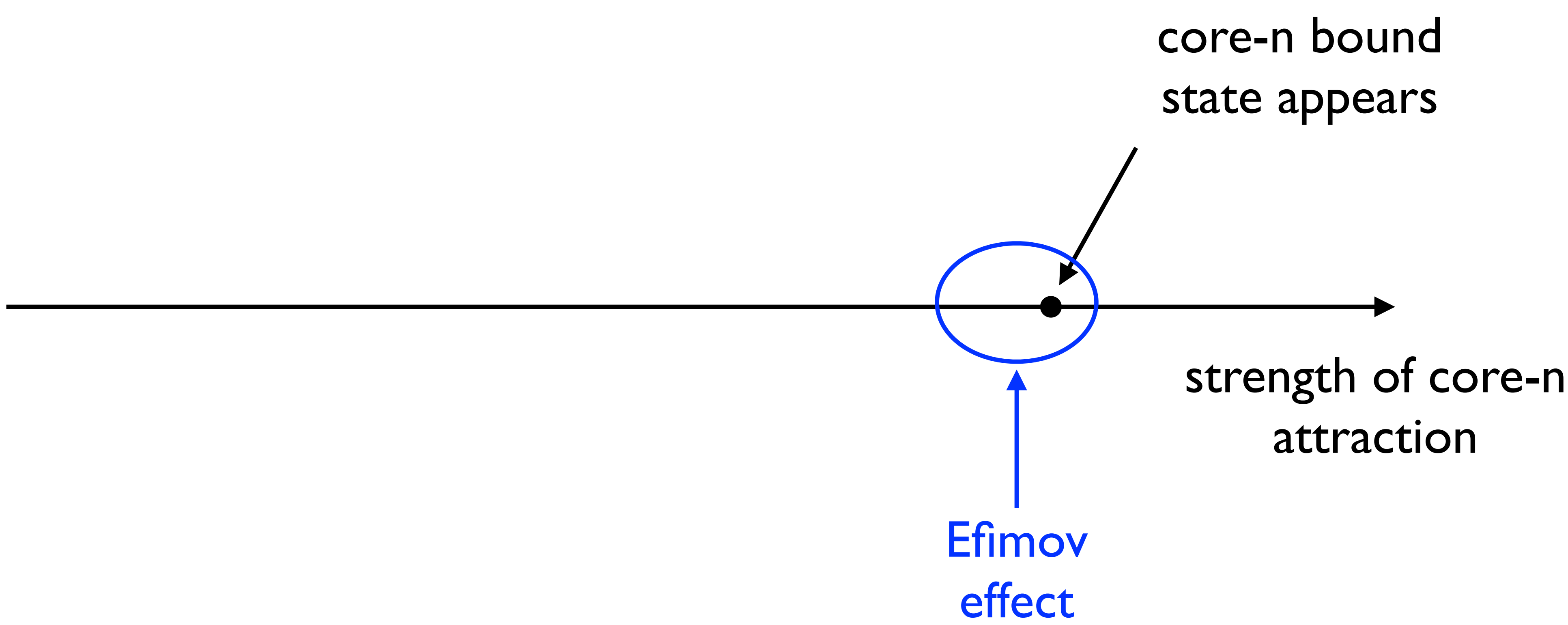

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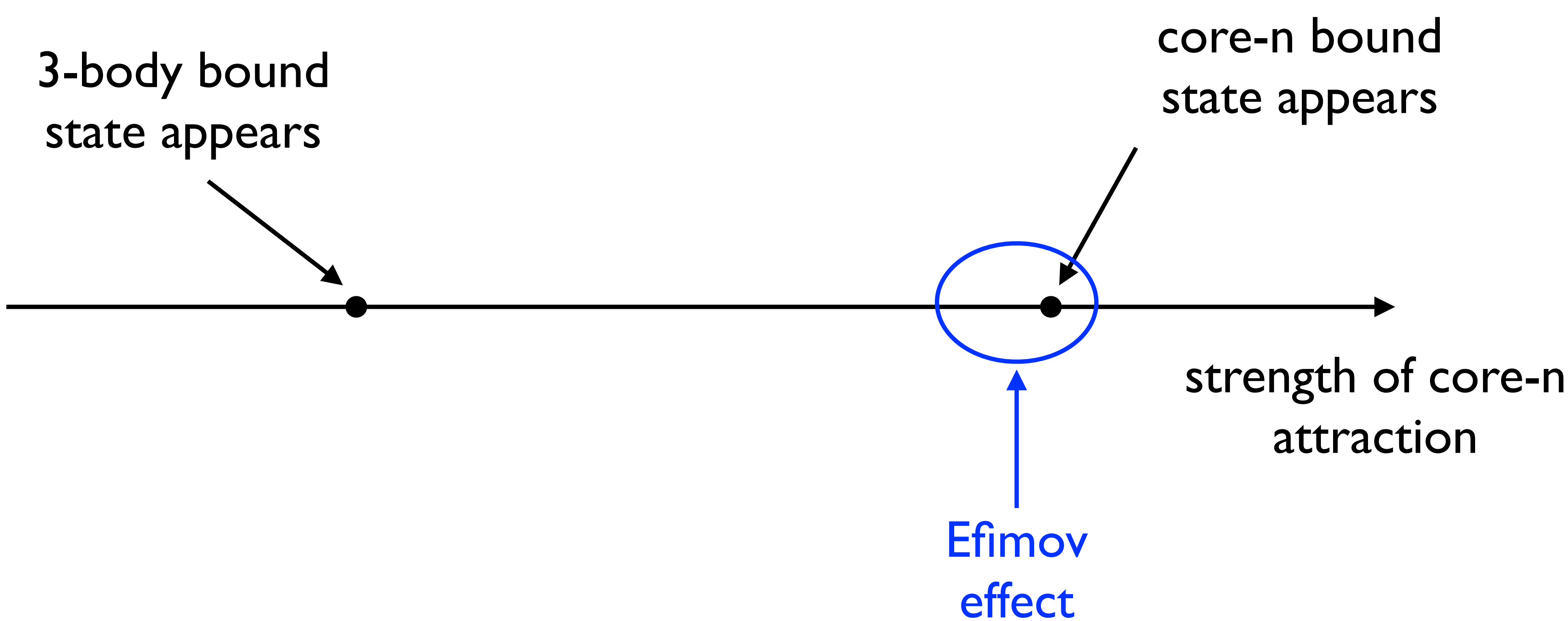

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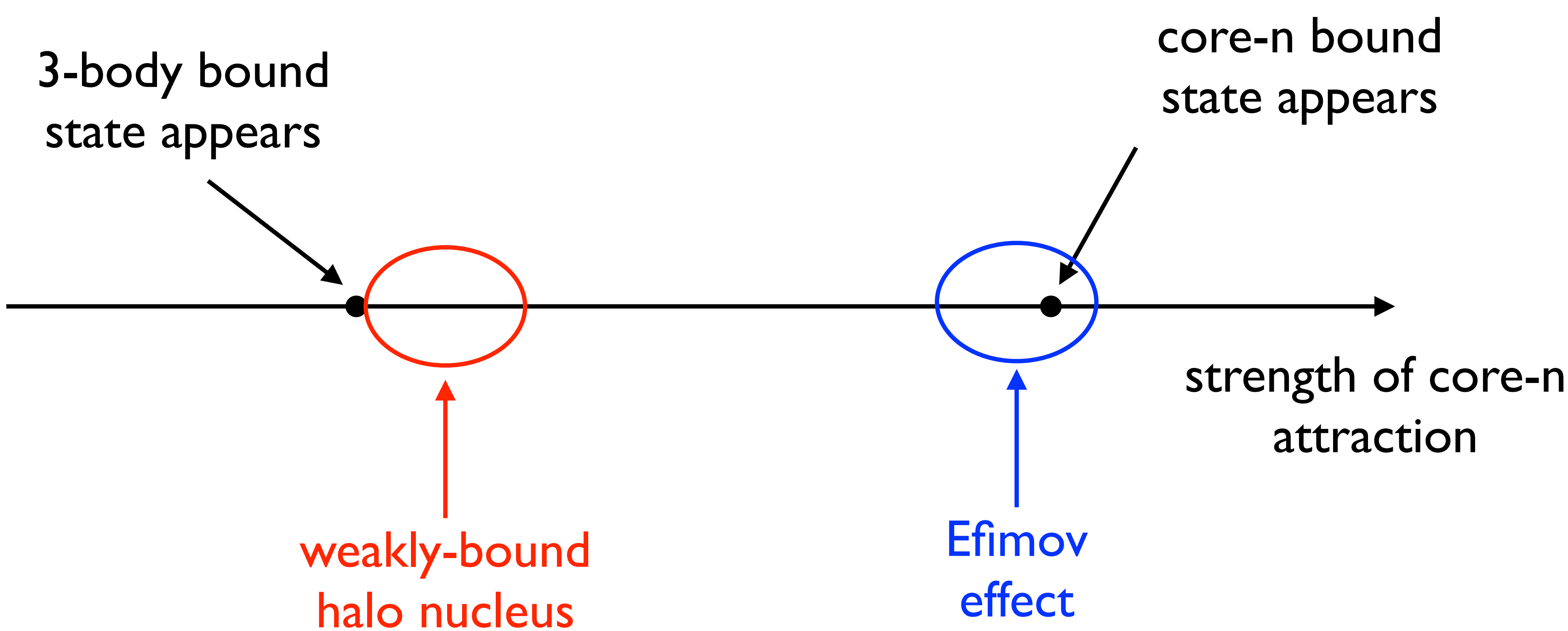

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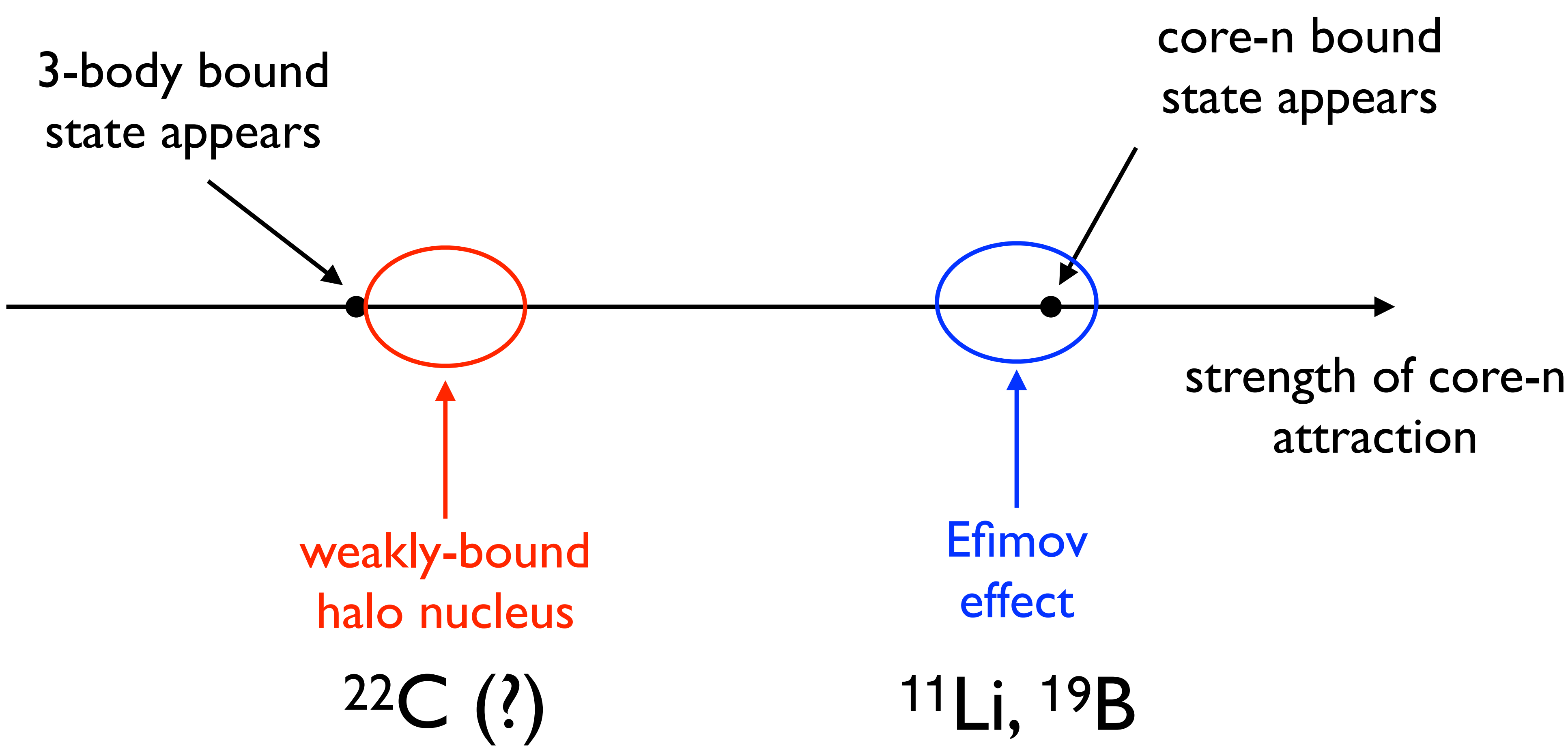


Examples

- ^{22}C has large matter radius Togano et al 2016 → small binding energy
- $|a(n^{20}C)| < 2.8$ fm Mosby et al 2013
- Hypertriton Λpn: total binding energy 2.35 MeV, $a_{pn} \approx 5.4$ fm
- but most estimate for the Λn scattering length is < 3 fm, and typically $|a| < r_{\text{eff}}$

Two fine tunings

- Weakly bound 2-neutron halos with two small energy scales:
- neutron-neutron virtual energy

$$a \approx -19 \text{ fm} \qquad \epsilon_n = \frac{\hbar^2}{m_n a^2} \approx 0.12 \text{ MeV}$$

- 2-neutron separation energy

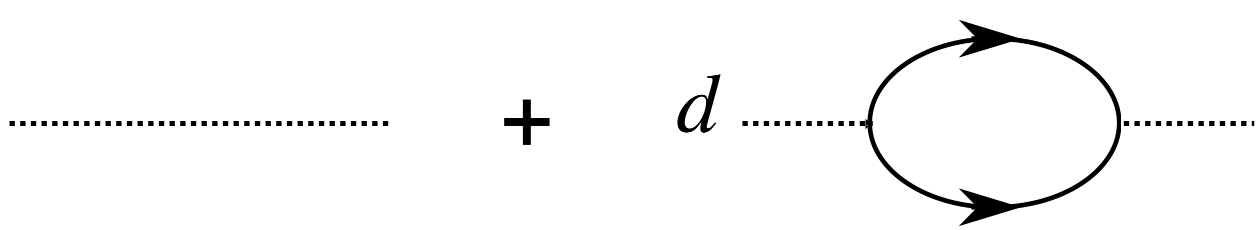
$$B(^{22}\text{C}) \sim 0.1 \text{ MeV}$$

- Appropriate approach: effective field theory (if no other small energy scale)

Neutrons sector

- $\mathscr{L}_{\text{neutron}} = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - c_0\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow$
- Introducing auxiliary field d ("dimer")
- $\mathscr{L}_{\text{neutron}} = i\psi^\dagger\left(\partial_t + \frac{\nabla^2}{2m}\right)\psi - \psi_\uparrow^\dagger\psi_\downarrow^\dagger d - d^\dagger\psi_\downarrow\psi_\uparrow + \frac{d^\dagger d}{c_0}$
- At unitarity: dimer propagator is scale invariant

$$G_d(\omega, \mathbf{p}) = \frac{1}{\sqrt{\frac{p^2}{4m} - \omega}}$$



Nonrelativistic power counting

- Set $m = 1$ $\quad [x] = -1, \ [t] = -2$

$$\langle \psi(t, \vec{x}) \psi^\dagger(0, \vec{0}) \rangle \sim \frac{1}{t^{3/2}} \exp\left(\frac{ix^2}{2t} \right) \qquad [\psi] = \frac{3}{2}$$

- $$\langle d(t, \vec{x}) d^\dagger(0, \vec{0}) \rangle \sim \frac{1}{t^2} \exp\left(\frac{ix^2}{4t} \right) \qquad [d] = 2$$

Operator product expansion:

$$\psi(\vec{x}) \psi(\vec{0}) = \frac{d(\vec{0})}{|\vec{x}|} + \cdots$$

EFTof weakly-bound halo nuclei: degrees of freedom

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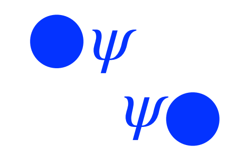

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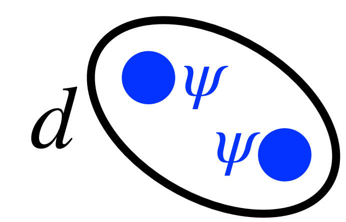

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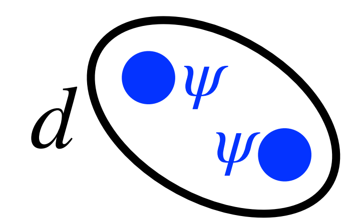

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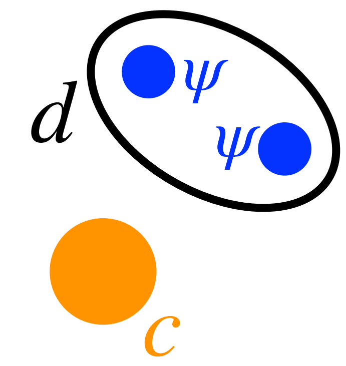

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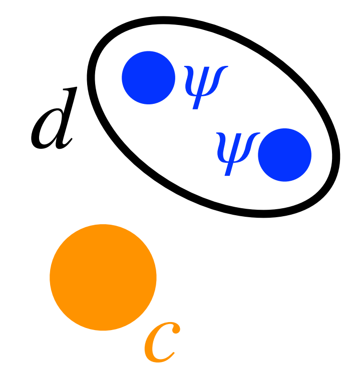

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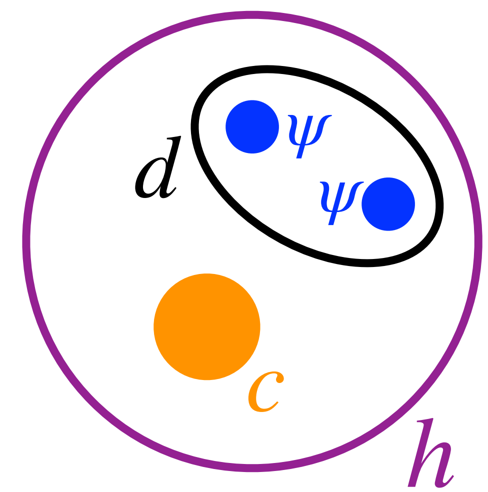

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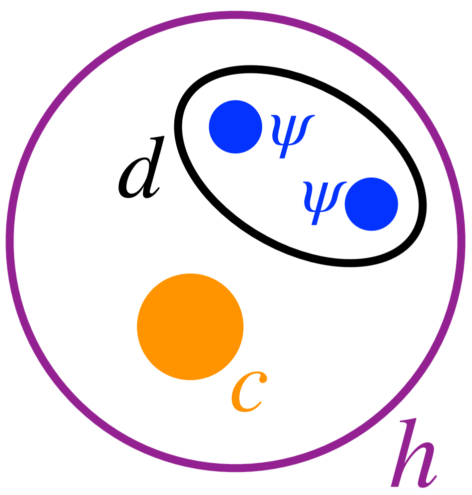
- Interaction: $h^\dagger d c + d^\dagger c^\dagger h$



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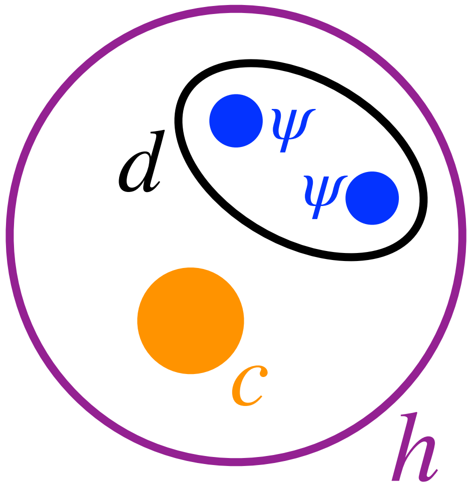
  - dimension: $\frac{3}{2}+\frac{3}{2}+2=5$: marginal



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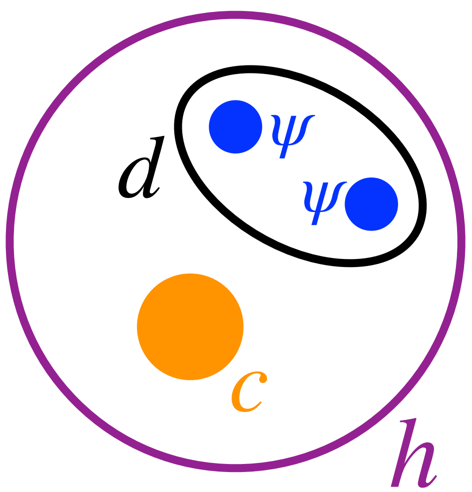
  - dimension: $\frac{3}{2} + \frac{3}{2} + 2 = 5$: marginal
  - leading-order EFT renormalizable



Effective Lagrangian

$$\mathscr{L} = h^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_h} + B\right)h + c^\dagger\left(\mathrm{i}\partial_t + \frac{\nabla^2}{2m_\phi}\right)c + g(h^\dagger cd + c^\dagger d^\dagger h)$$

$$+\mathscr{L}_{\text{neutron}}$$

Logarithmic running of g ($g \to 0$ in the IR, Landau pole in UV)

Universality?

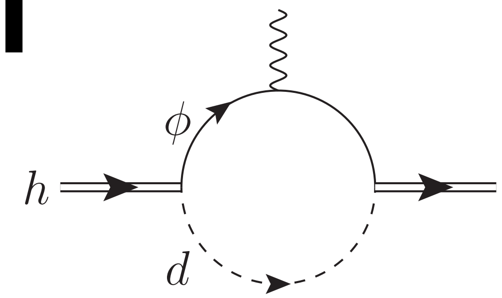
- Is the 3-body system universal?

  Can any physical quantity can be written as

$$O = B^{\Delta_O} F_O\left(\frac{B}{\epsilon_n}\right), \quad O(\omega) = B^{\Delta_O} F_O\left(\frac{\omega}{B}, \frac{B}{\epsilon_n}\right)$$

- Answer: almost, up to the logarithmically running coupling

Charge and matter radii



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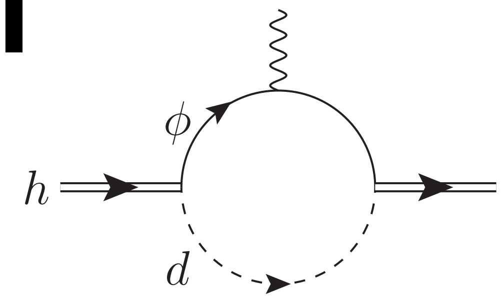

- Charge radius $$\langle r_c^2 \rangle = \frac{4}{\pi} \frac{A^{1/2}}{(A+2)^{5/2}} \frac{g^2}{B} f_c(\beta),$$

$$\beta = \sqrt{\frac{\epsilon_n}{B}}$$

$$A = A_{\text{core}}$$

$$f_c(\beta) = \frac{1}{1-\beta^2} - \frac{\beta \arccos \beta}{(1-\beta^2)^{3/2}}$$

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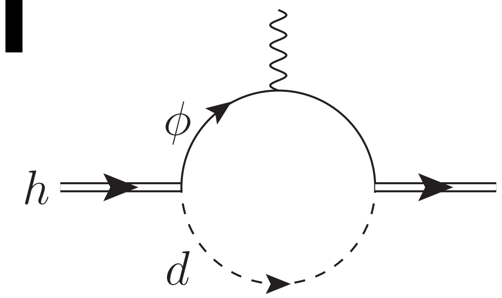

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$$f_n(\beta) = \frac{1}{\beta^3} \left[\pi - 2\beta + (\beta^2 - 2) \frac{\arccos \beta}{\sqrt{1-\beta^2}} \right]$$

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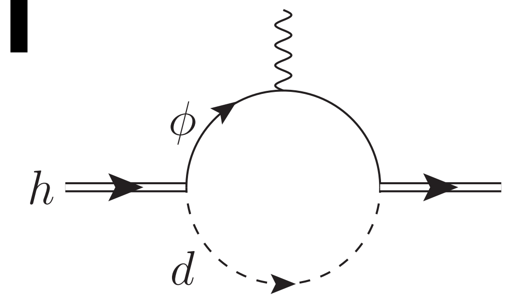

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- Each contains g^2, but universal ratio

$$\frac{\langle r_m^2 \rangle}{\langle r_c^2 \rangle} = \frac{A}{2} \left[1 + \frac{f_n(\beta)}{f_c(\beta)} \right] = \begin{cases} \frac{2}{3} A & B \gg \epsilon_n \\ A & B \ll \epsilon_n \end{cases}$$

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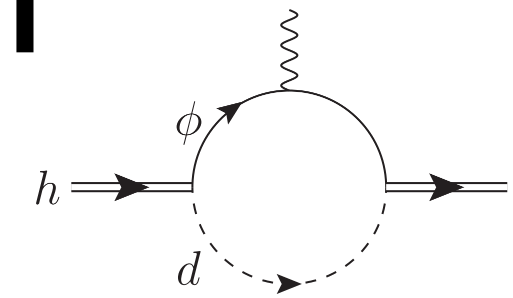

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Hongo, DTS 2201.09912
Naidon 2302.08716

E1 dipole strength function

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- can be mapped to current-current correlation

$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega}(\omega) \sim \frac{1}{\omega^2}\mathrm{Im}\,\langle JJ\rangle(\omega)$$

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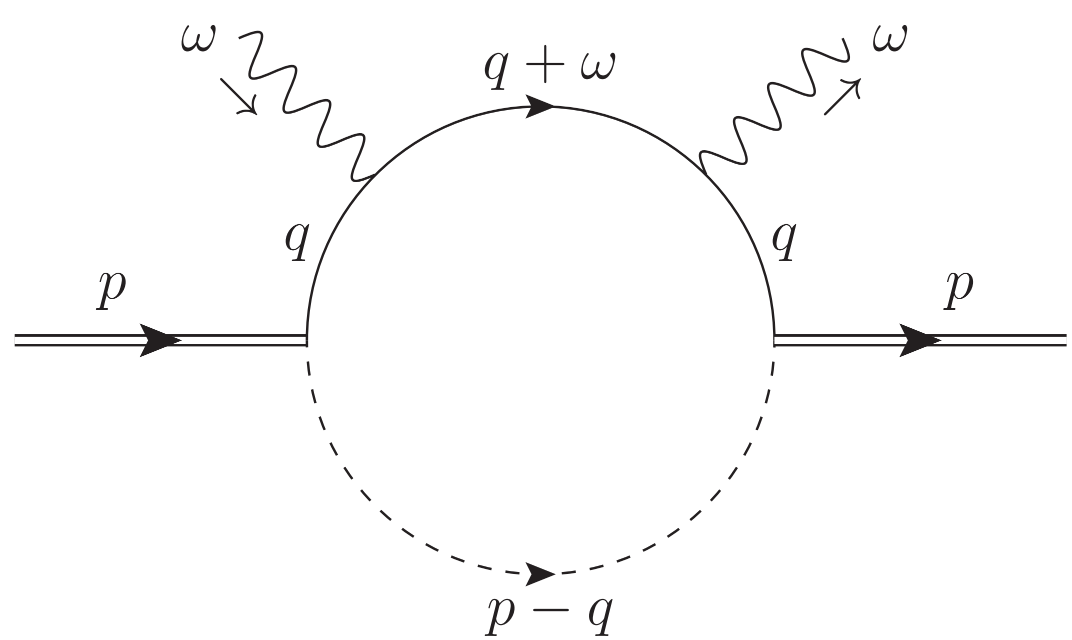


Dipole strength in the unitarity limit

- When neutrons are in the unitarity limit $a = \infty$

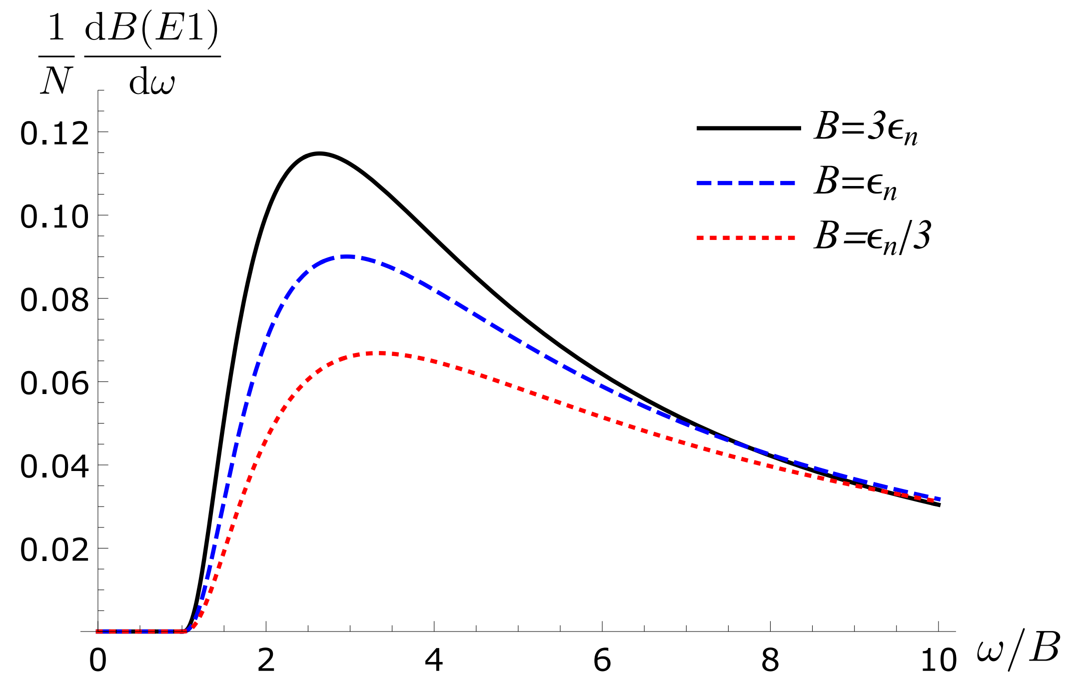
$$\frac{\mathrm{d}B(E1)}{\mathrm{d}\omega} \sim g^2 \frac{(\omega - B)^2}{\omega^4}$$

Result for dipole strength

$$\frac{dB(E1)}{d\omega} \sim g^2 \frac{(\omega - B)^2}{\omega^4} f_{E1}\left(\sqrt{\frac{\epsilon_{nn}}{\omega - B}}\right)$$

$$f_{E1}(x) = 1 - \frac{8}{3}x(1+x^2)^{3/2} + 4x^2\left(1 + \frac{2}{3}x^2\right)$$

M.Hongo, DTS 2201.09912



consistency check: sum rules

$$\int_0^\infty d\omega \, \frac{dB(E1)}{d\omega} = \frac{3}{4\pi} Z^2 e^2 \langle r_c^2 \rangle,$$

$$\int_0^\infty d\omega \, \omega \frac{dB(E1)}{d\omega} = \frac{3}{4\pi} Z^2 e^2 \frac{3}{A(A+2)},$$

Corrections to EFT

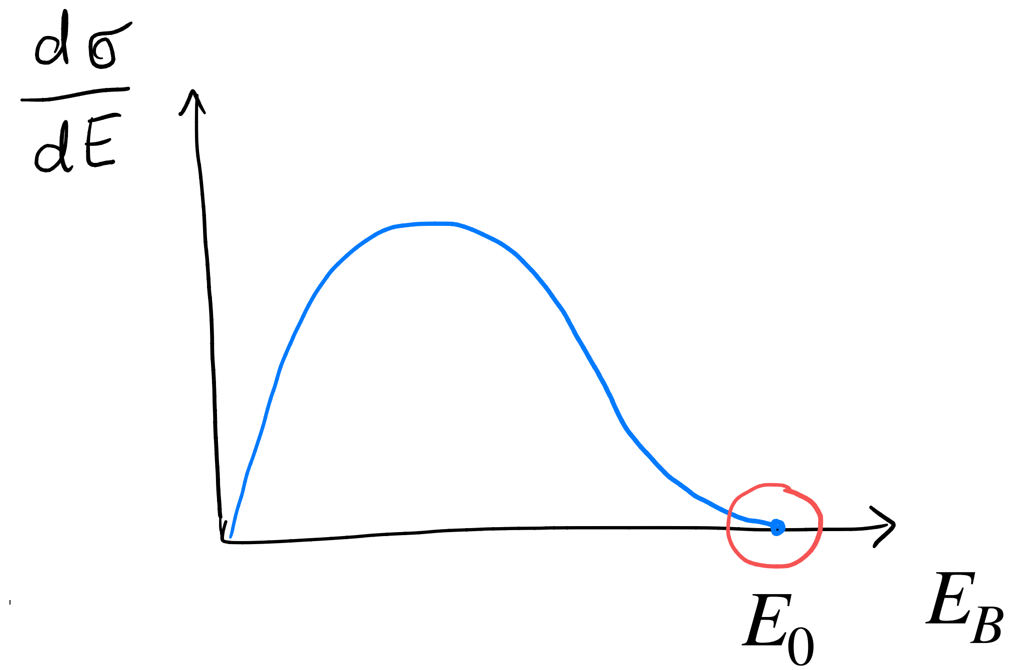
- Corrections to EFT: irrelevant terms EFT
- Two least irrelevant terms:
  - Effective range in *n*-*n* scattering: $r_0 d^\dagger \left(\mathrm{i}\partial_t - \frac{1}{4}\nabla^2\right) d$
  - *s*-wave core-neutron scattering $a_{cn}\phi^\dagger\psi^\dagger\psi\phi$
- One can develop perturbation theory in r_0 and a_{cn}

Conclusion

- Weakly bound two-neutron halo nuclei can be described by an EFT. Renormalizable with 1 log-running coupling
- Ratios of radii and shape of E1 dipole function are universal, analytically computable
- Corrections: *nn* effective range D.Costa, M.Hongo, DTS to appear, core-neutron scattering length
- Applications beside ^{22}C? ^{6}He? Cold atom realization?
- Unstable systems, e.g., ^{26}O, Λnn ?

H.Hammer, DTS PNAS 118 (2021) e2108716118 (arXiv:2103.12610)

$$A_1 + A_2 \to B + \underbrace{n + n + \cdots n}_{N}$$



$$\frac{d\sigma}{dE} \sim \left(E_{\rm rel}^{nn\cdots n}\right)^{\alpha}$$

$$0.1\,\text{MeV} \ll E_{\rm rel}^{nn\cdots n} \ll 5\,\text{MeV}$$

N	α	
2	-0.5	Watson-Migdal 1950's
3	1.77	
4	2.5	