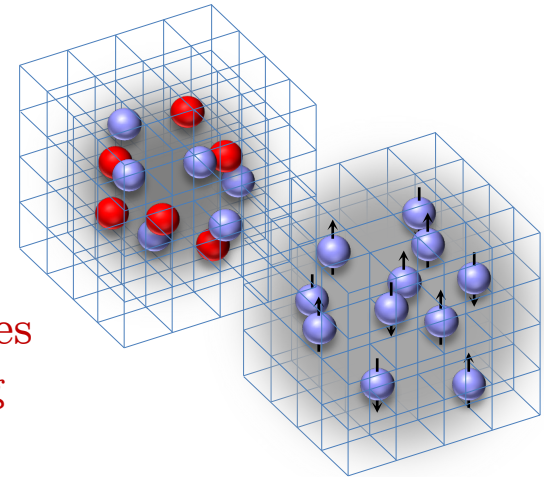


Lattice calculations of nuclear clustering

Dean Lee
Facility for Rare Isotope Beams
Michigan State University
Nuclear Lattice EFT Collaboration

HaloWeek'24 – nuclei at and beyond the driplines
Chalmers University of Technology, Gothenburg
June 9 – 14, 2024



MICHIGAN STATE
UNIVERSITY

NUCLEI
Nuclear Computational Low-Energy Initiative
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Outline

Lattice effective field theory

Wavefunction matching

Pinhole algorithm

Relativistic heavy ion collisions

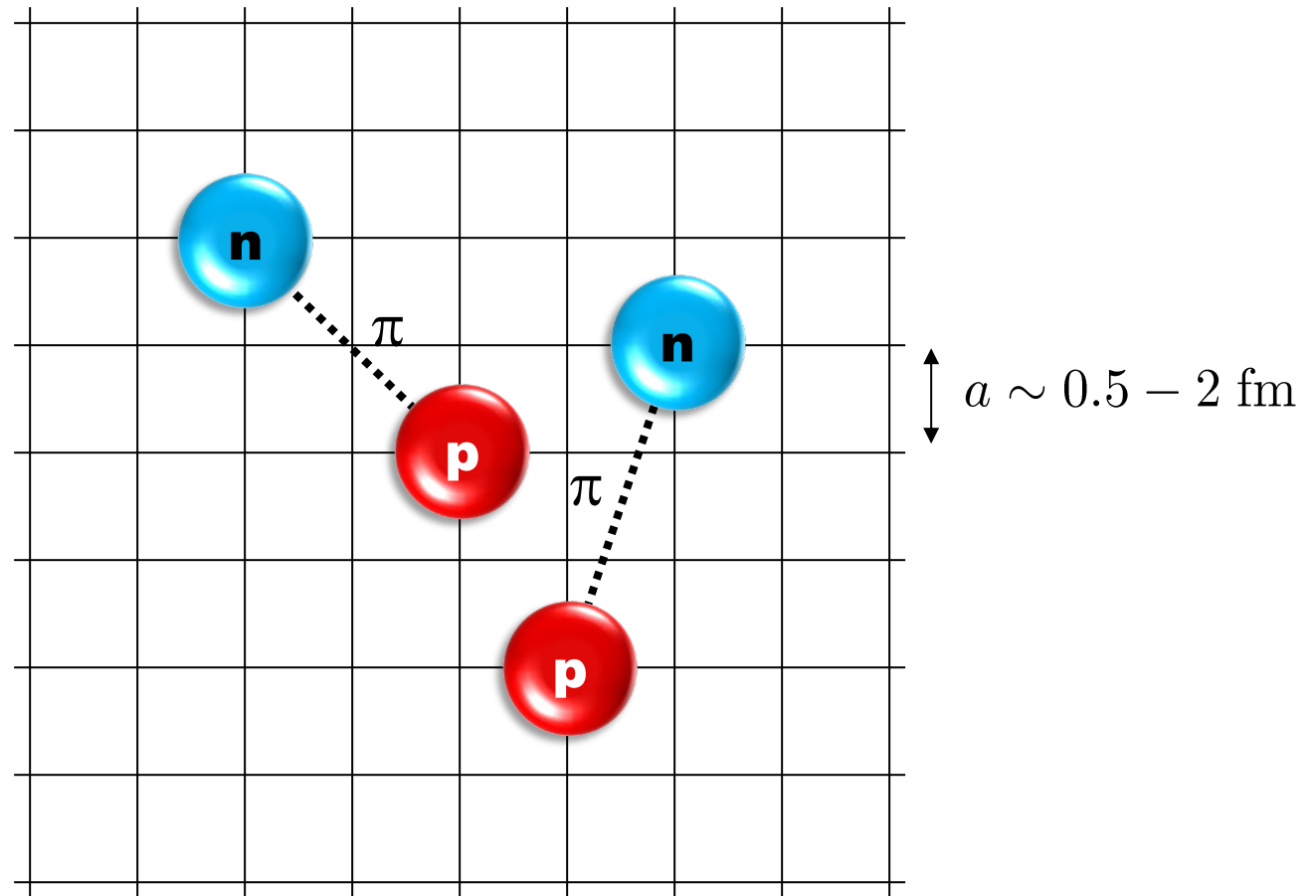
Asymptotic normalization coefficients

Isotopes of beryllium

Isotopes of carbon and oxygen

Summary and outlook

Lattice effective field theory

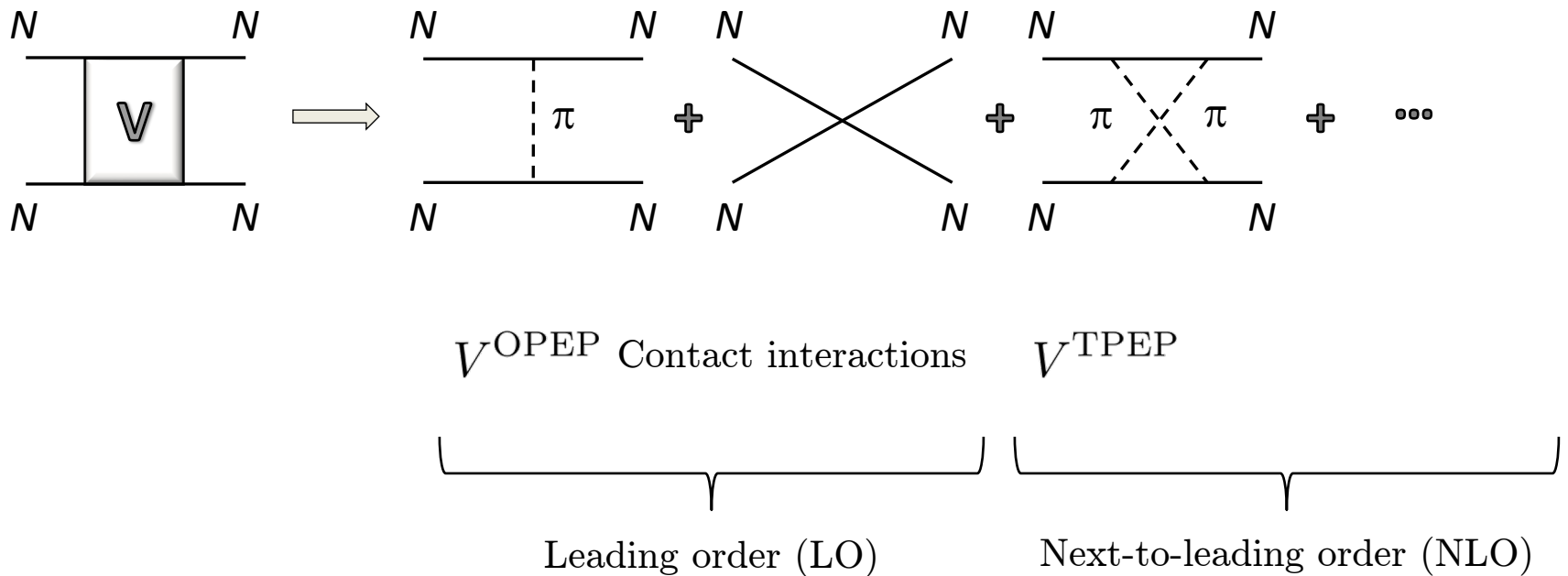


D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

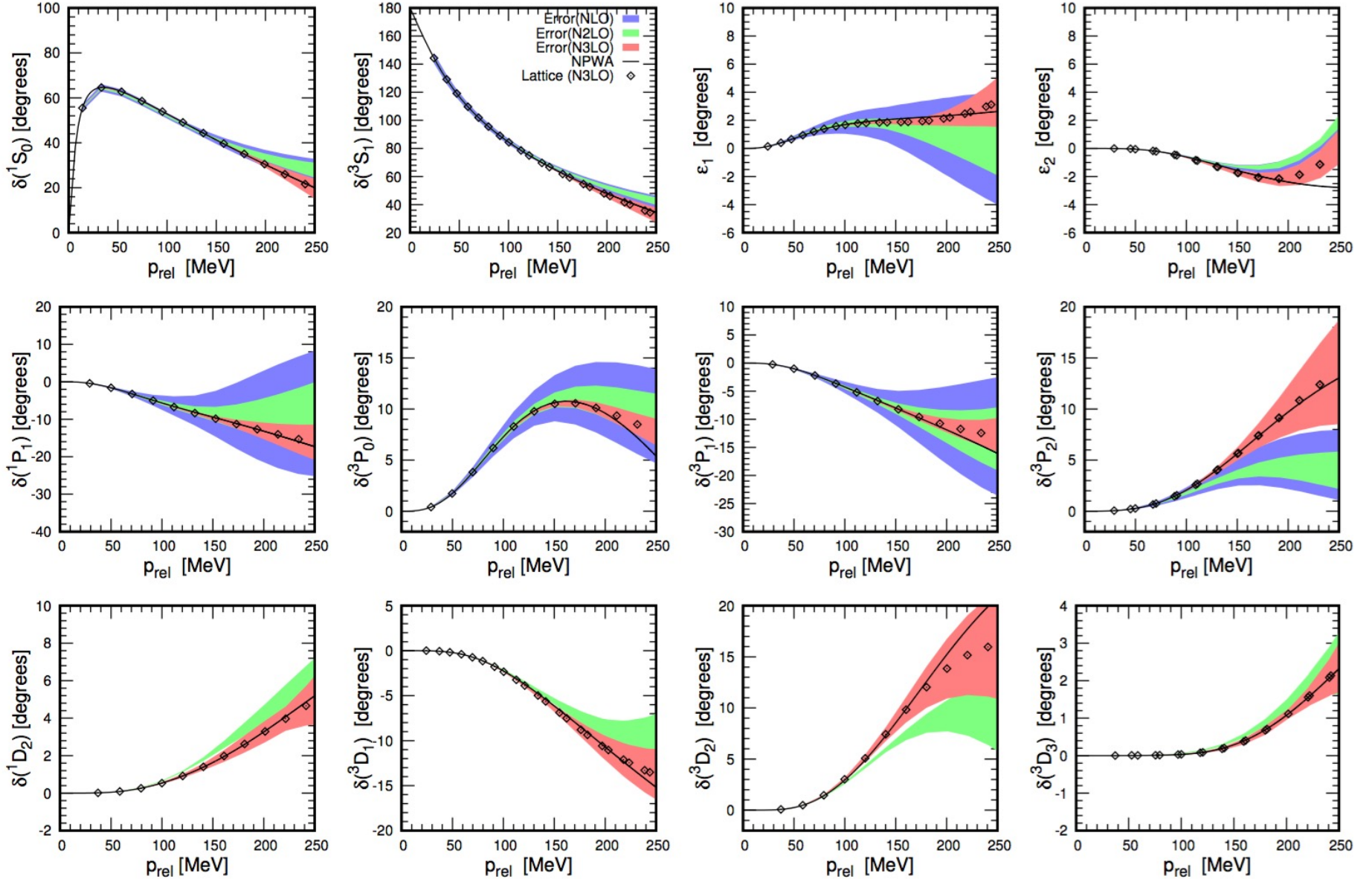
Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

Chiral effective field theory

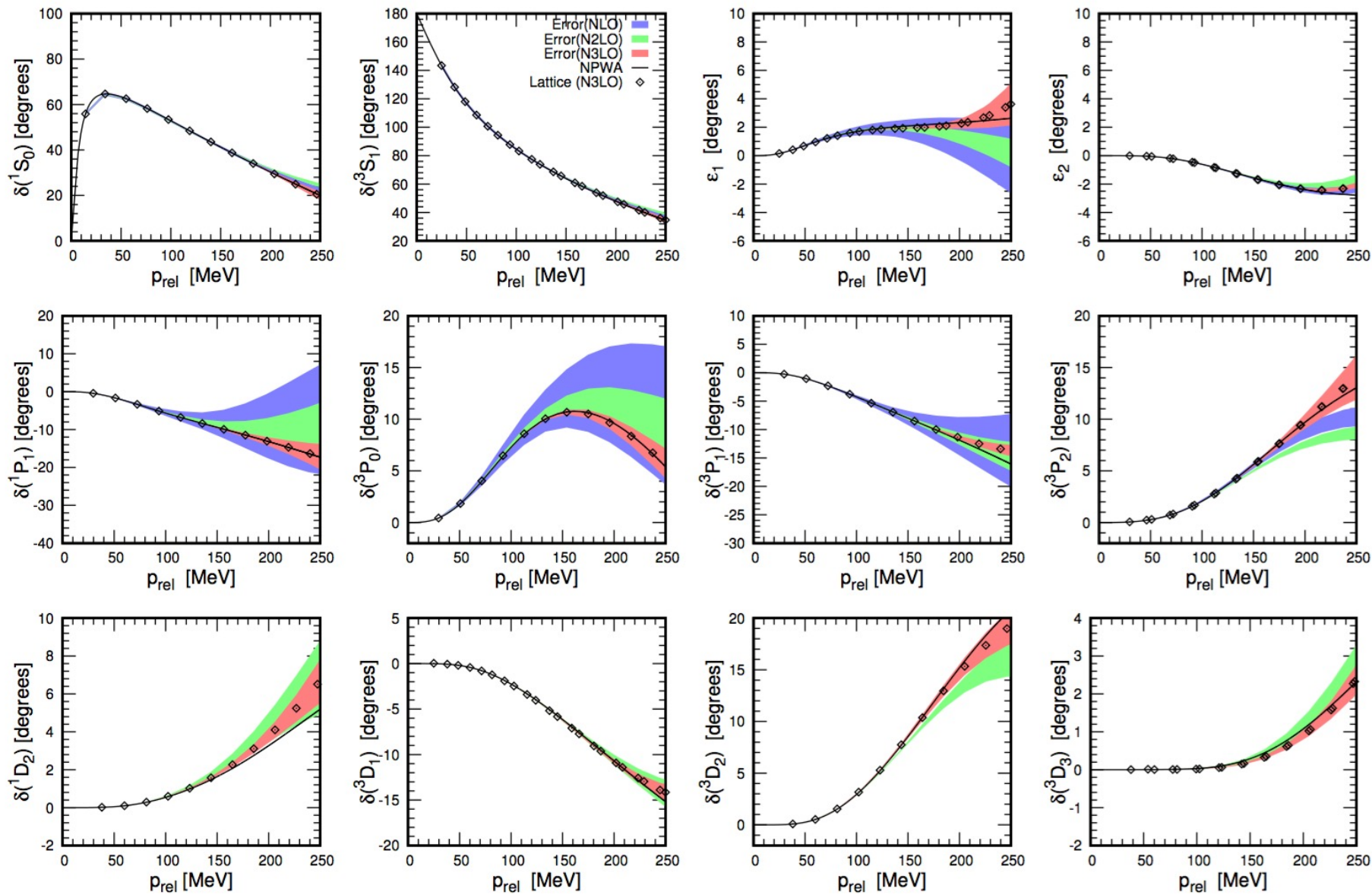
Construct the effective potential order by order



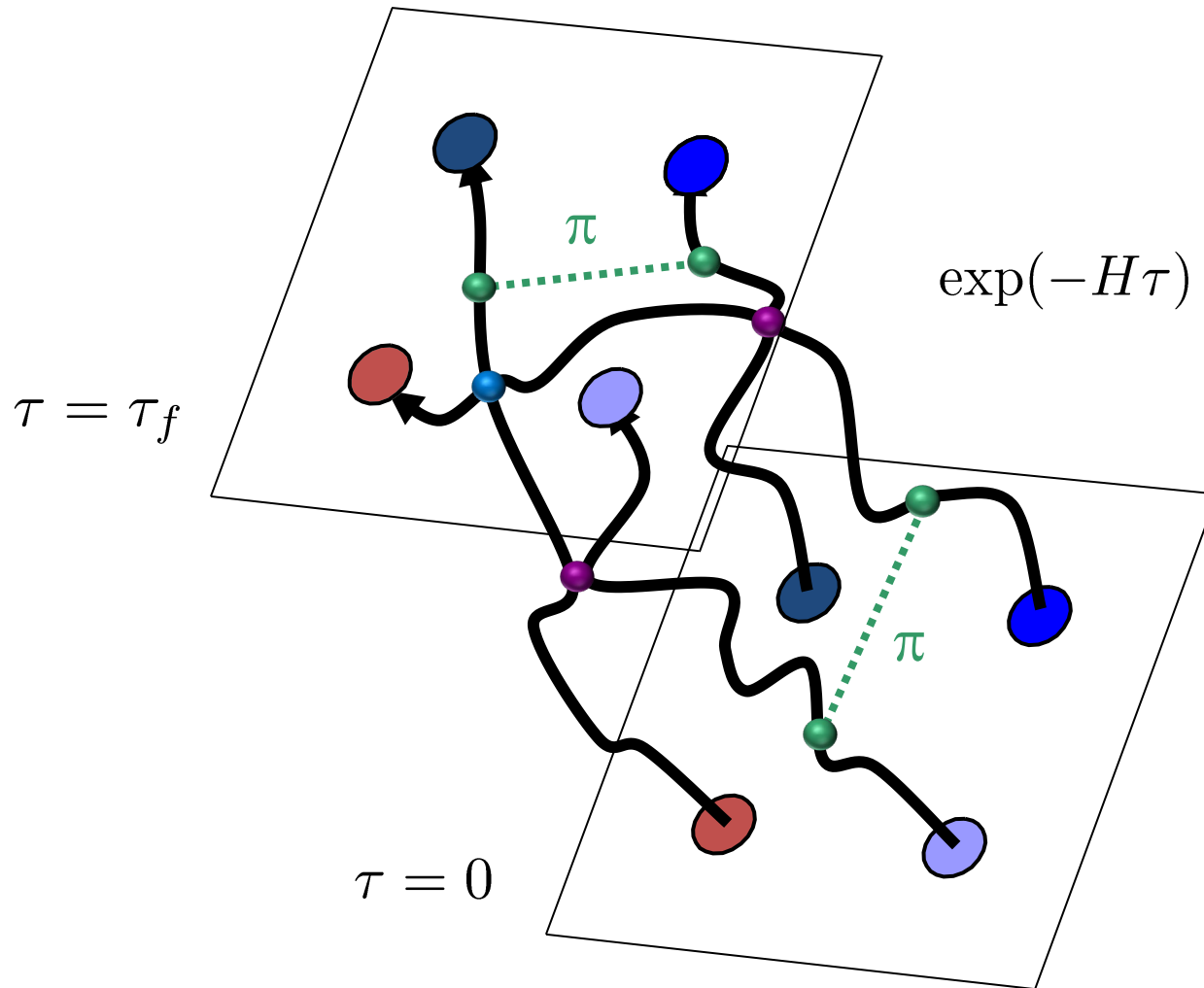
$a = 1.315$ fm



$a = 0.987 \text{ fm}$



Euclidean time projection

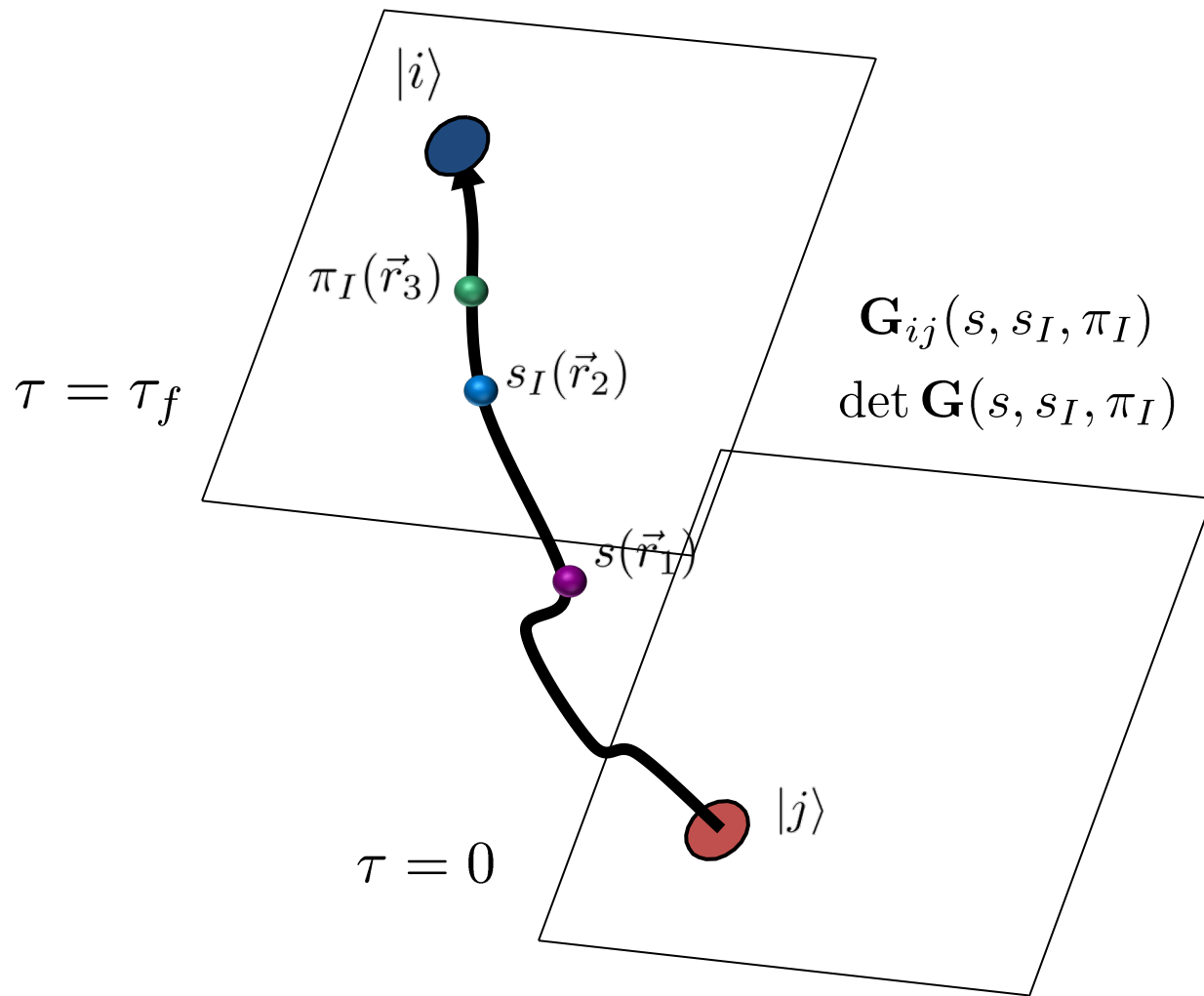


Auxiliary field method

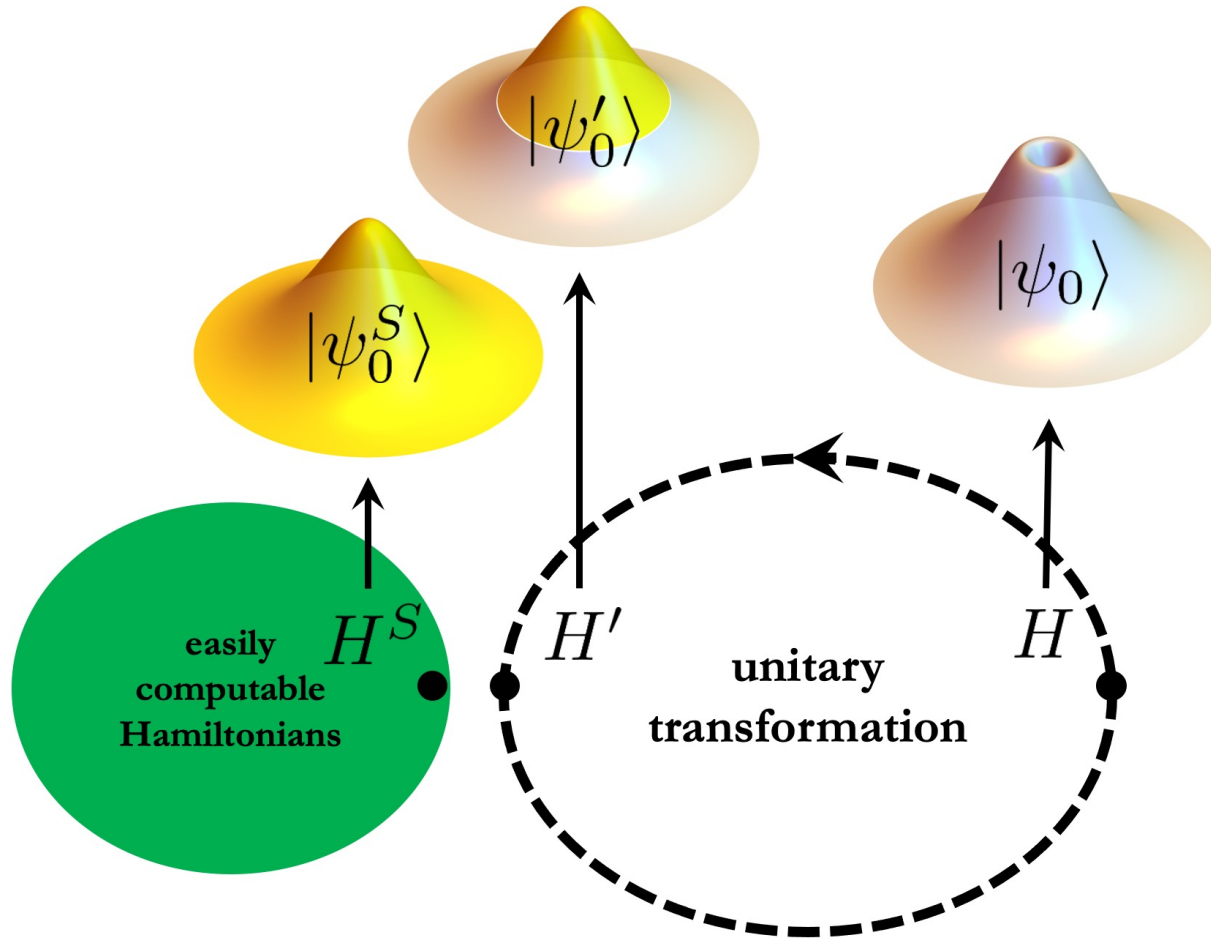
We can write exponentials of the interaction using a Gaussian integral identity

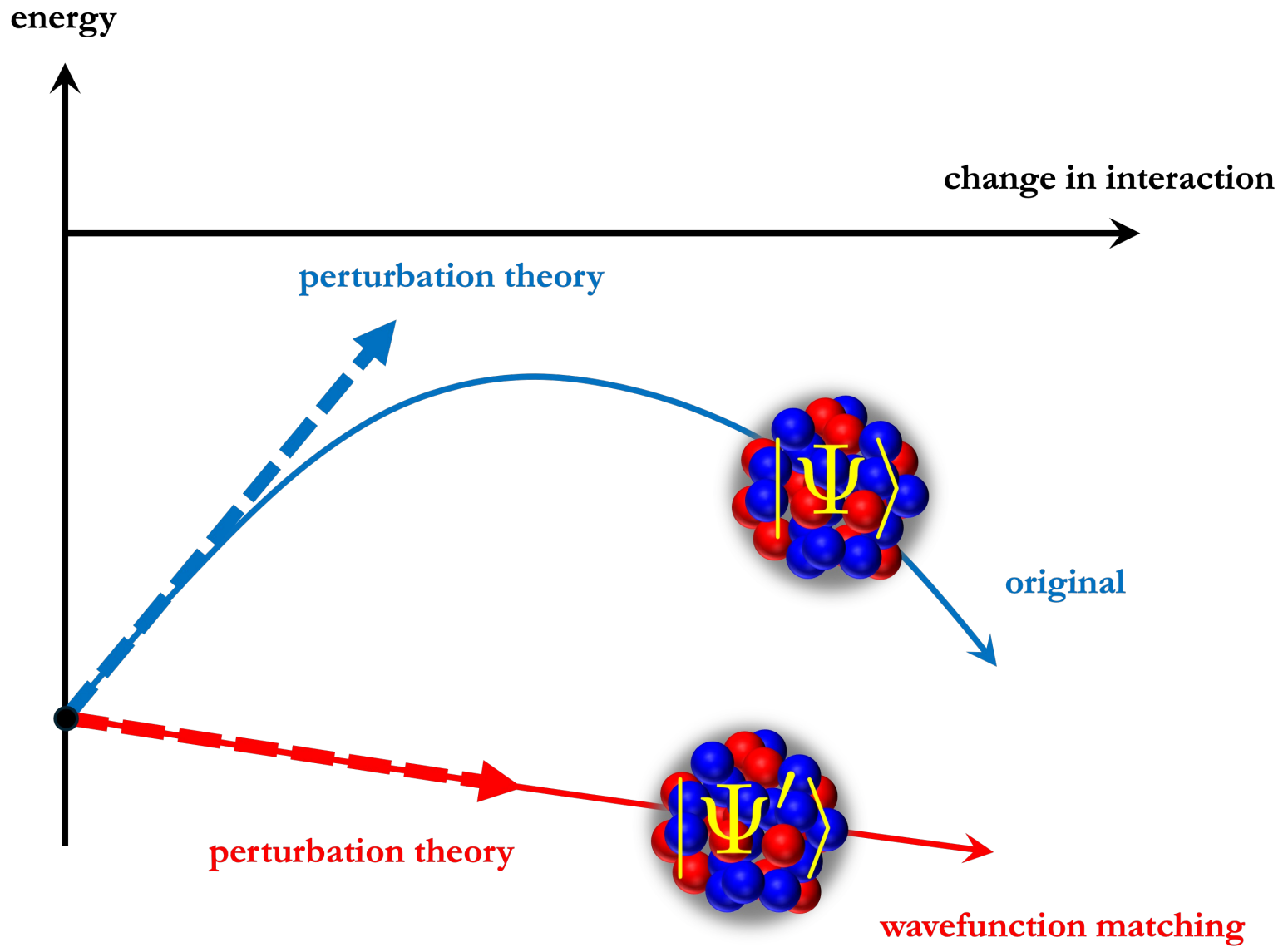
$$\begin{aligned} & \exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \diagdown \quad (N^\dagger N)^2 \\ & = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \diagup \quad s N^\dagger N \end{aligned}$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

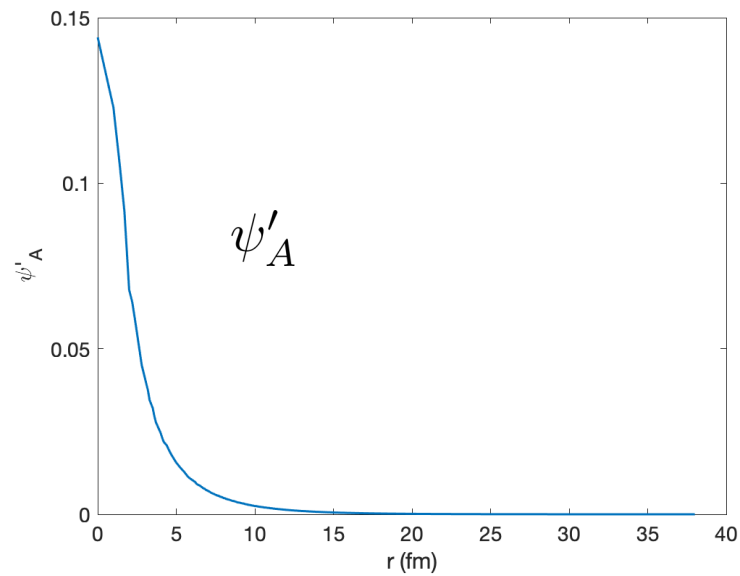
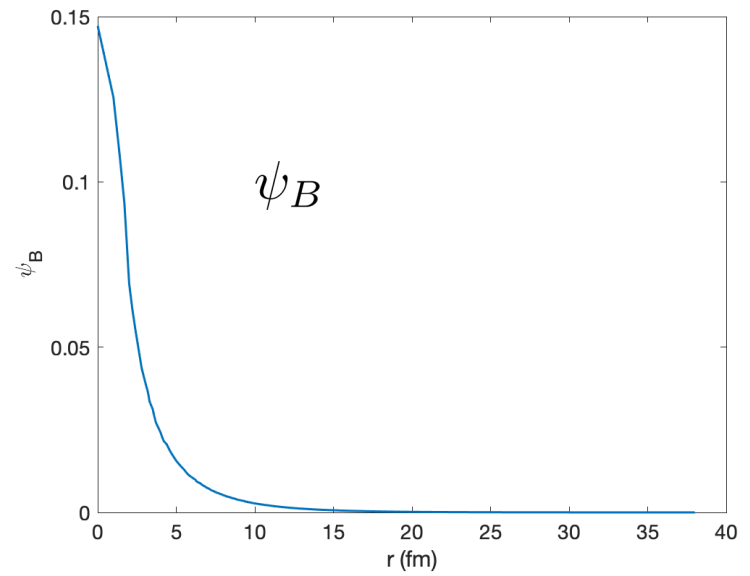
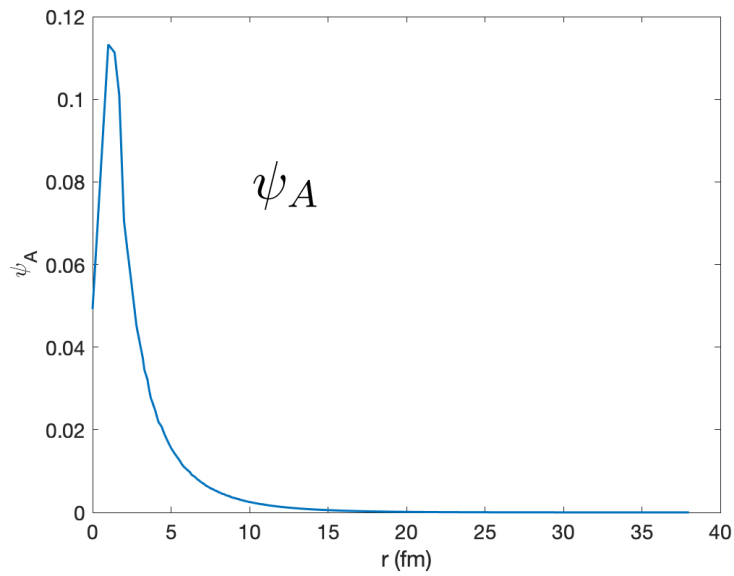


Wavefunction matching



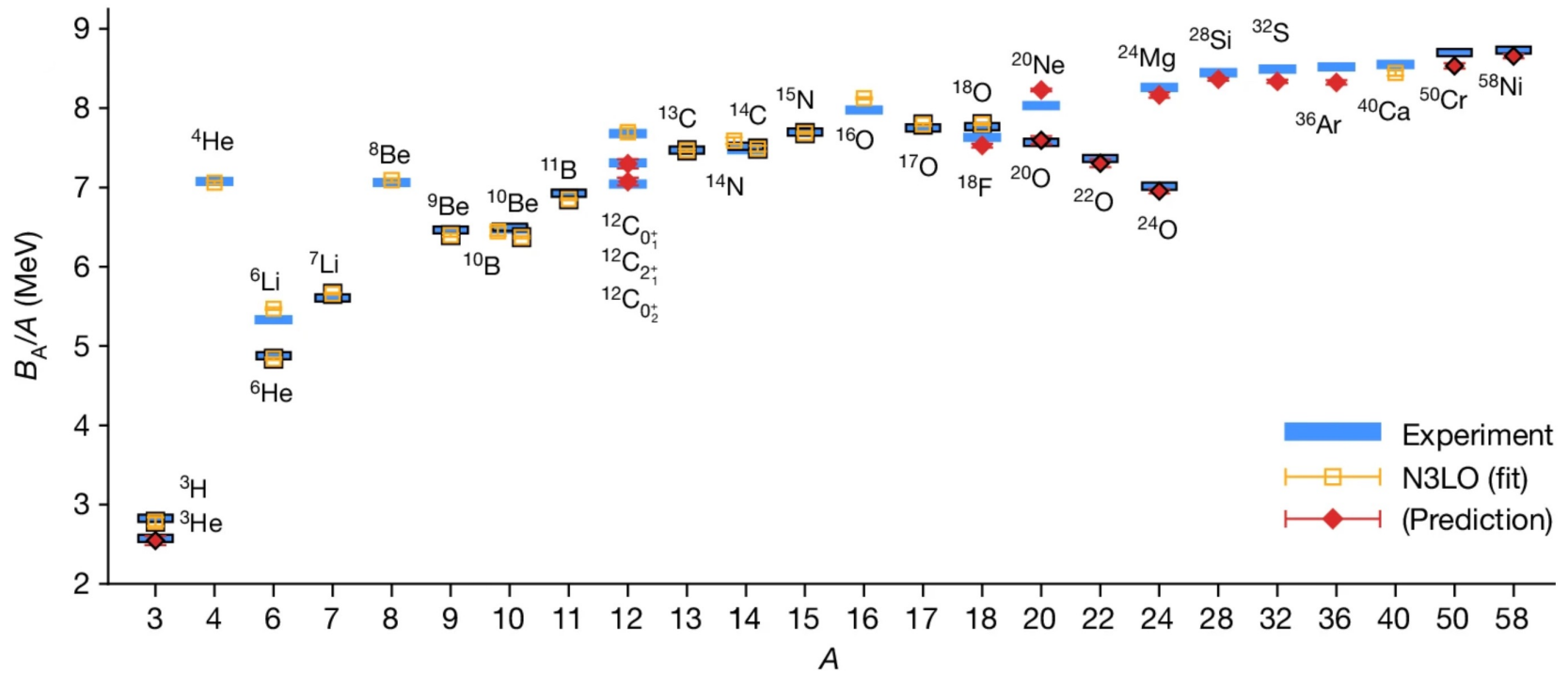


Ground state wavefunctions

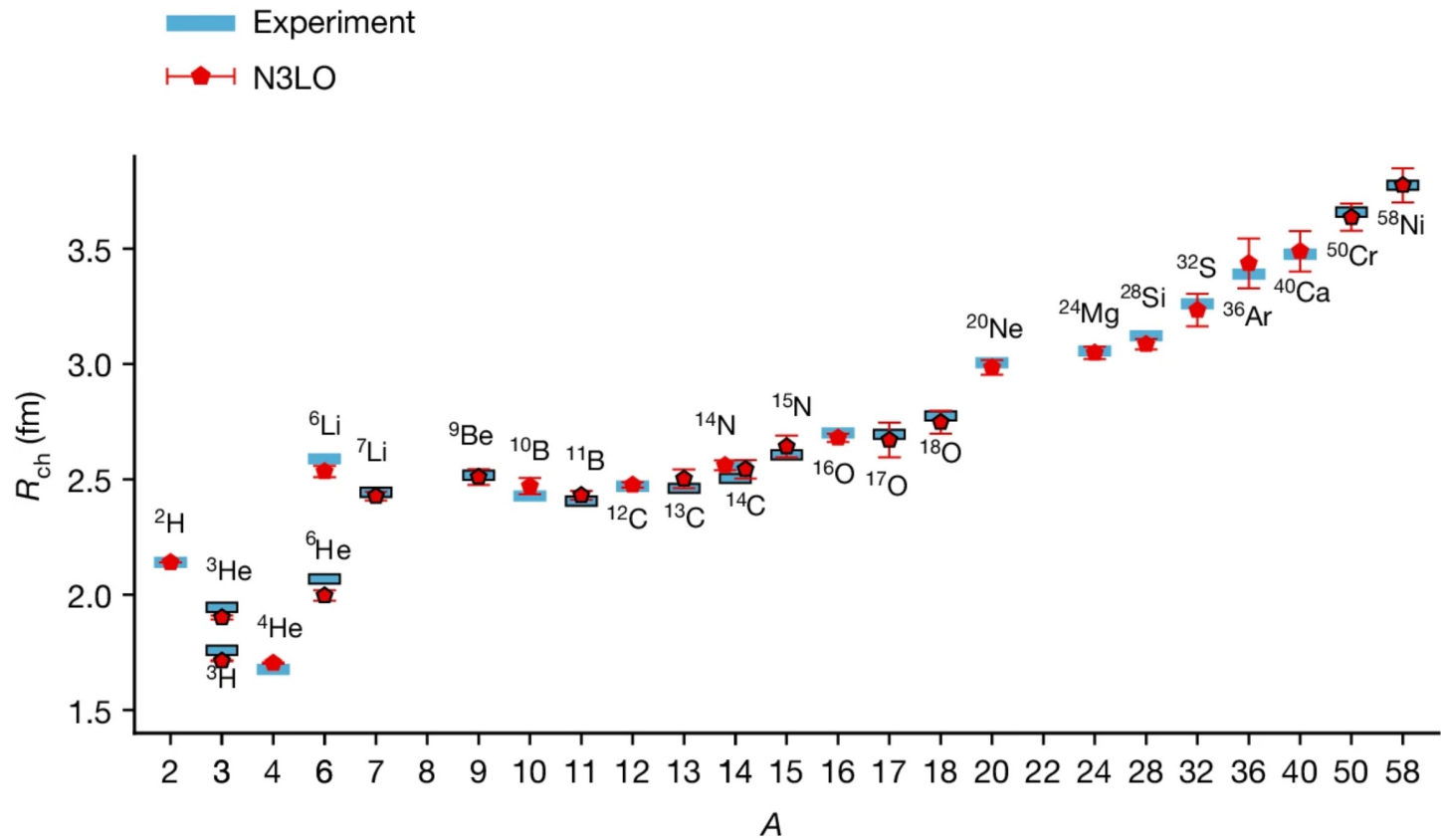


With wavefunction matching, we can now compute the eigenenergies starting from the eigenfunctions of H_B and using first-order perturbation theory.

$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle$ (MeV)	$\langle \psi_{B,n} H'_A \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

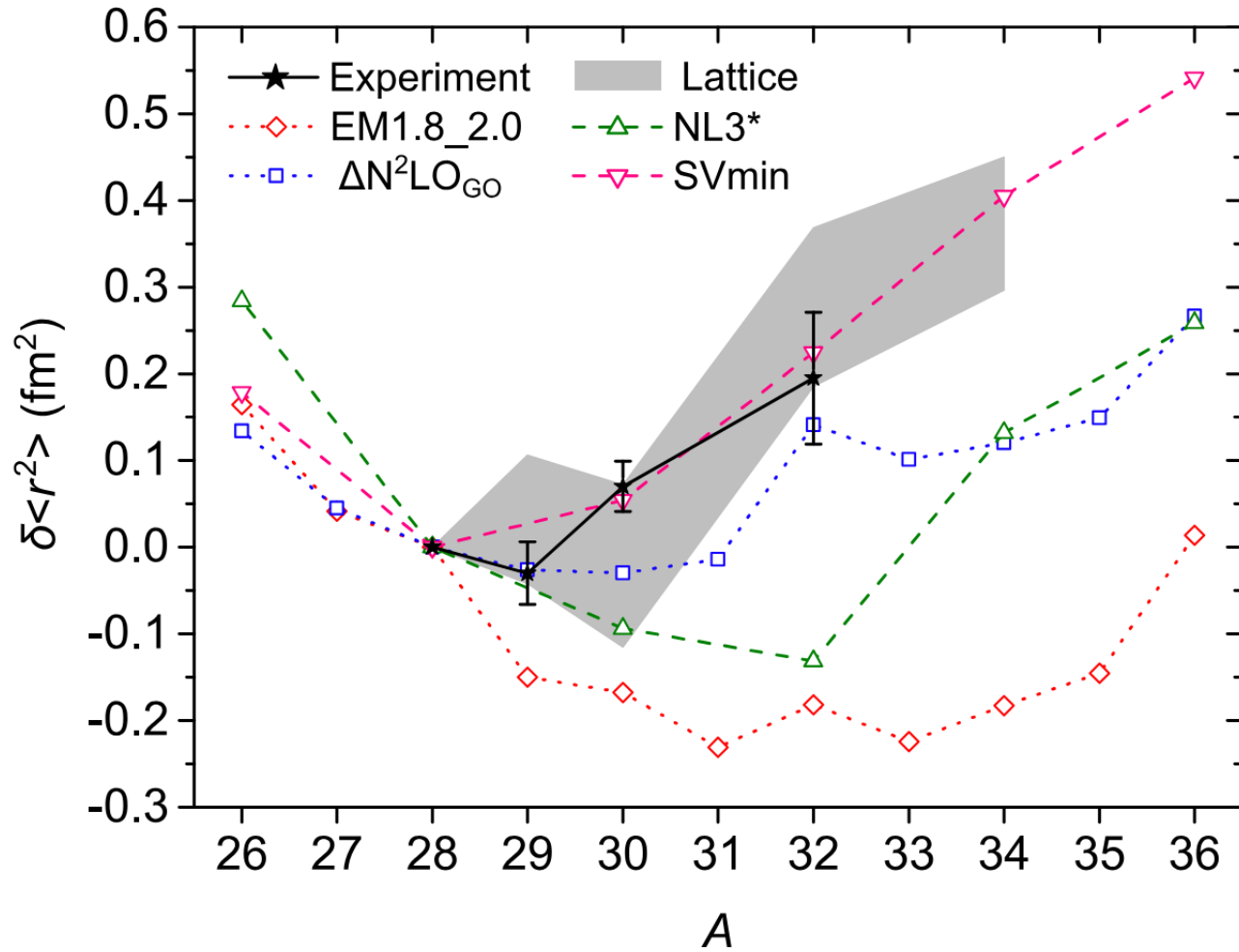


Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

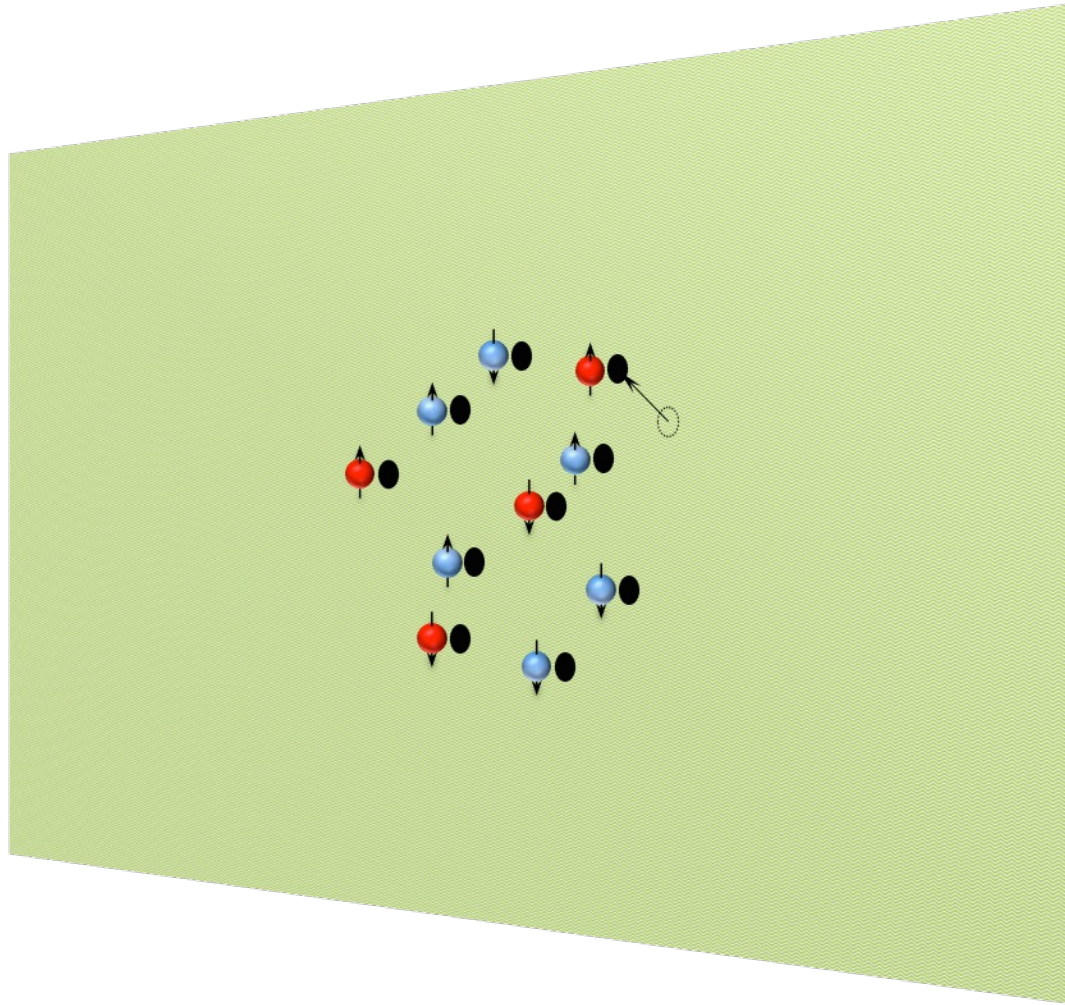
Silicon isotopes

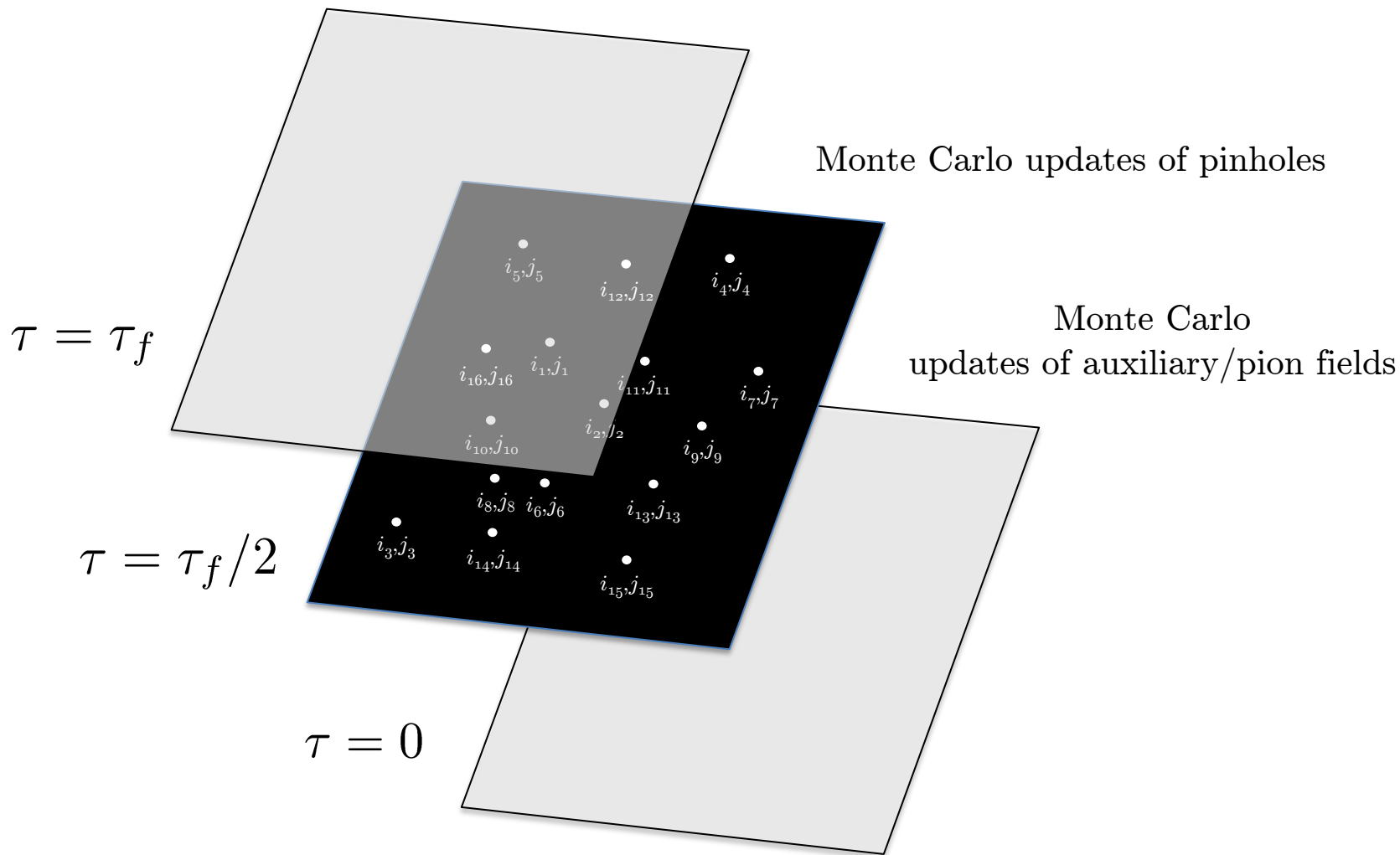


K. König et al., PRL 132, 162502 (2024)

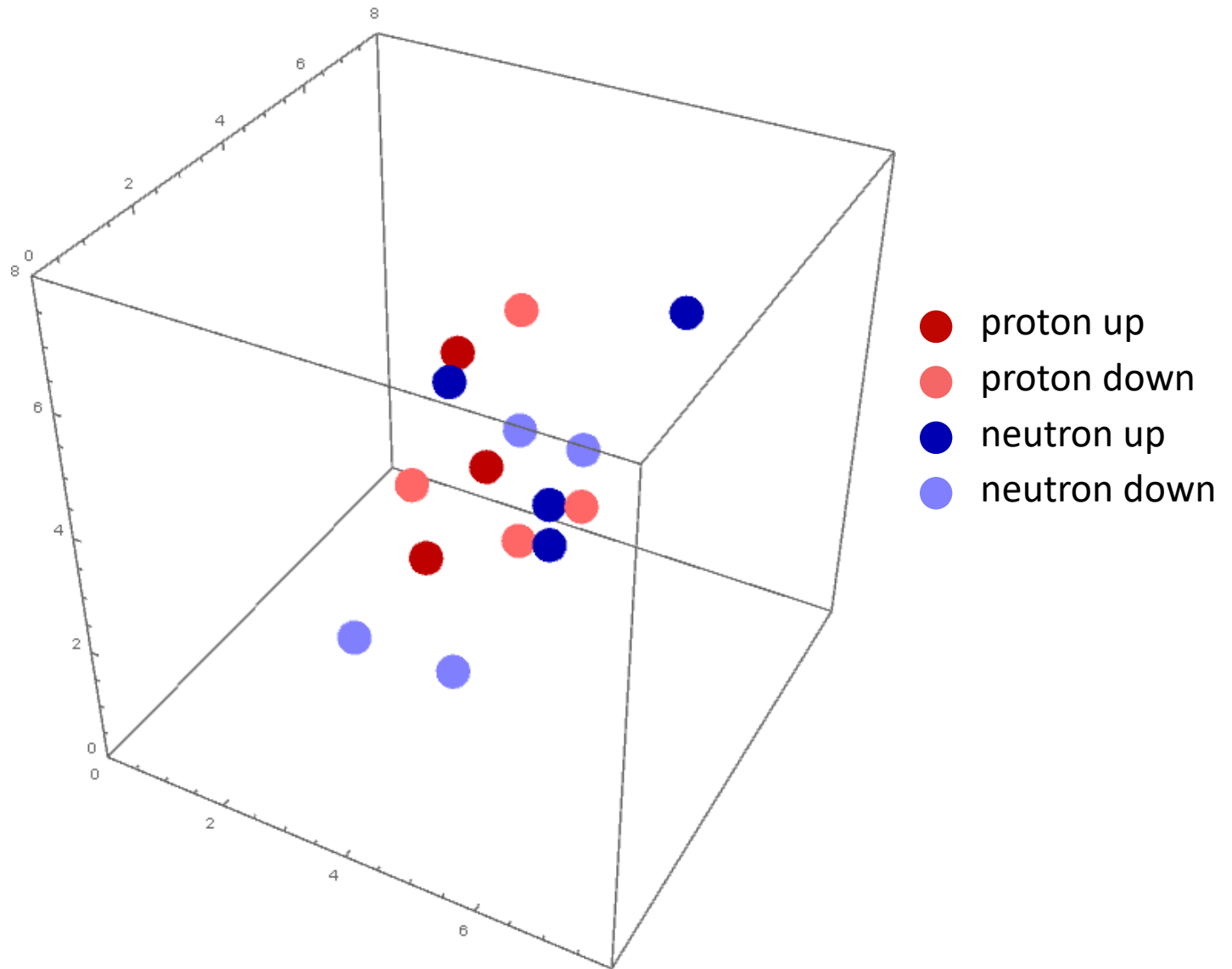
Lattice calculations by Y. Ma

Pinhole algorithm



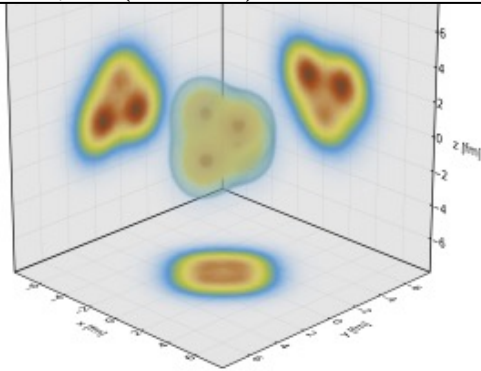


^{16}O

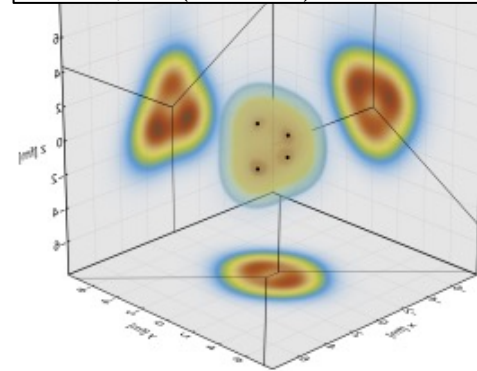


Relativistic heavy collisions: $^{16}\text{O}^{16}\text{O}$ versus $^{20}\text{Ne}^{20}\text{Ne}$

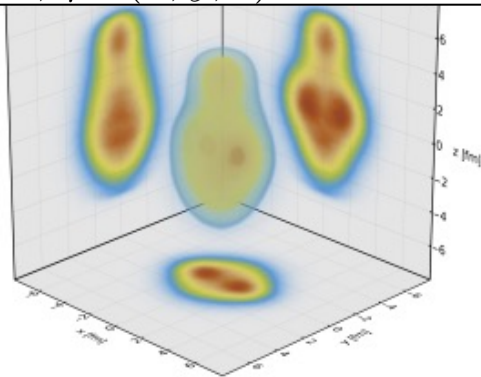
$^{16}\text{O}, \rho_m(x, y, z) - \text{NLEFT}$



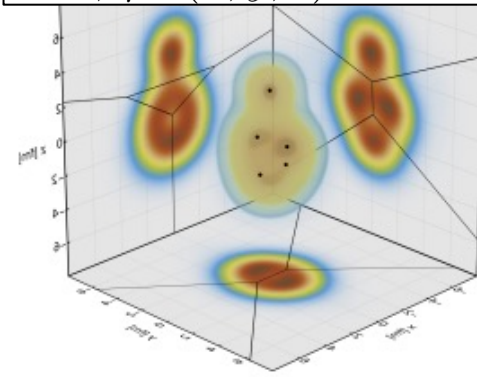
$^{16}\text{O}, \rho_m(x, y, z) - \text{PGCM}$



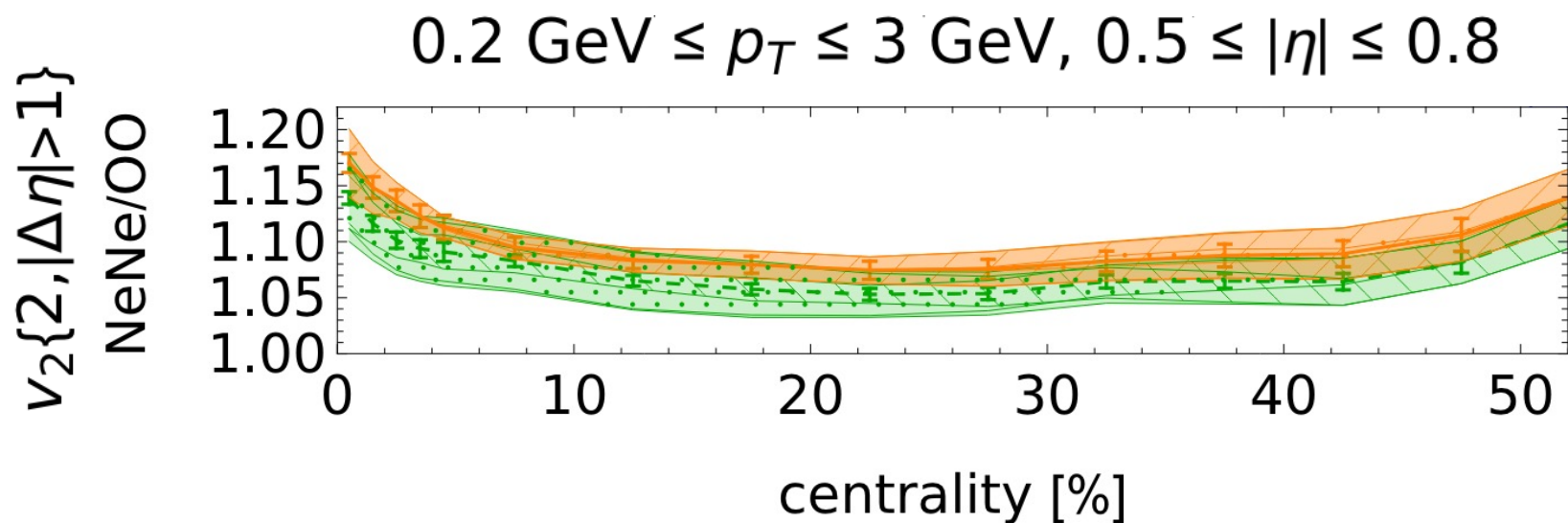
$^{20}\text{Ne}, \rho_m(x, y, z) - \text{NLEFT}$



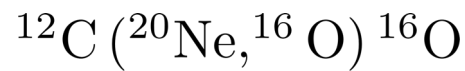
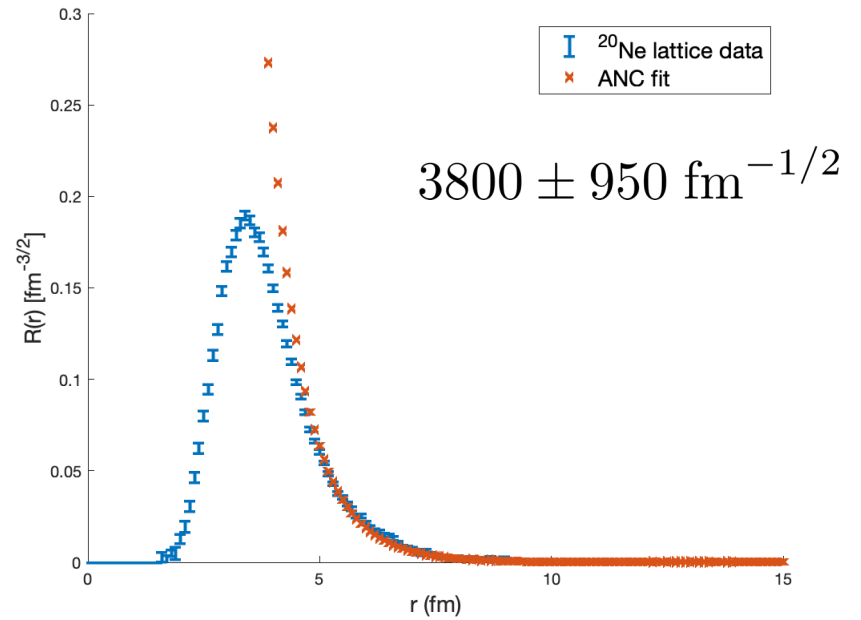
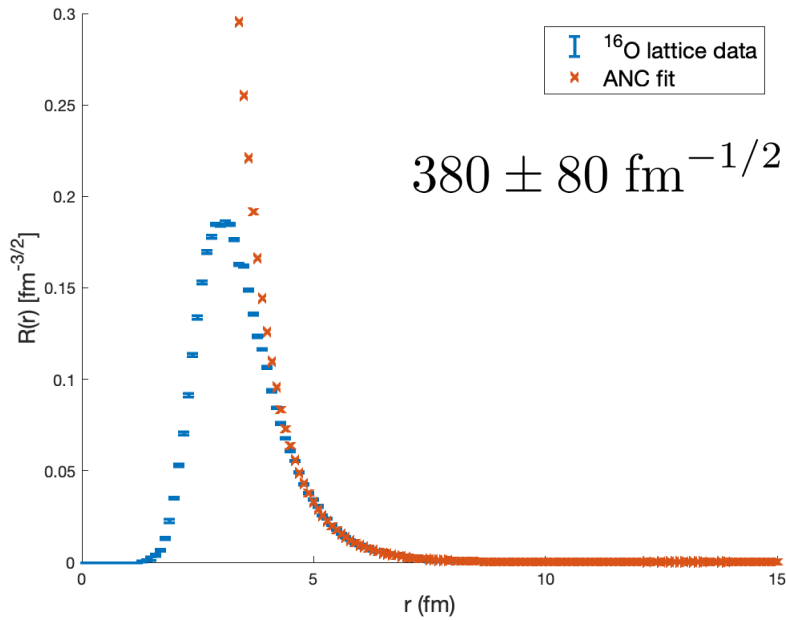
$^{20}\text{Ne}, \rho_m(x, y, z) - \text{PGCM}$



For the 1% most central events, the elliptic flow of $^{20}\text{Ne}^{20}\text{Ne}$ collisions is enhanced by as much as $1.170(8)\text{stat.}(30)\text{syst.}$ for NLEFT and $1.139(6)\text{stat.}(39)\text{syst.}$ for PGCM relative to $^{16}\text{O}^{16}\text{O}$ collisions.

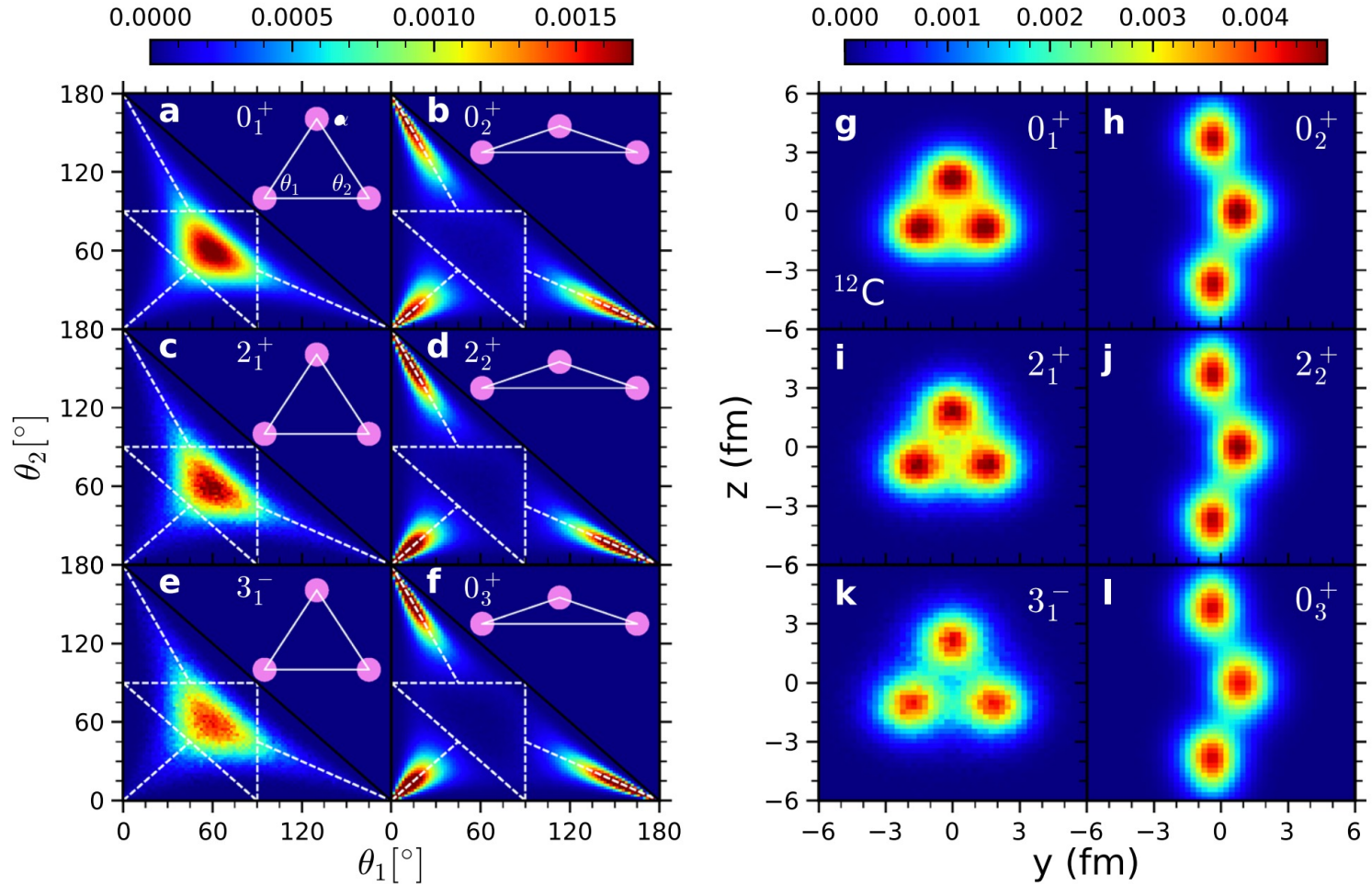


Asymptotic normalization coefficients

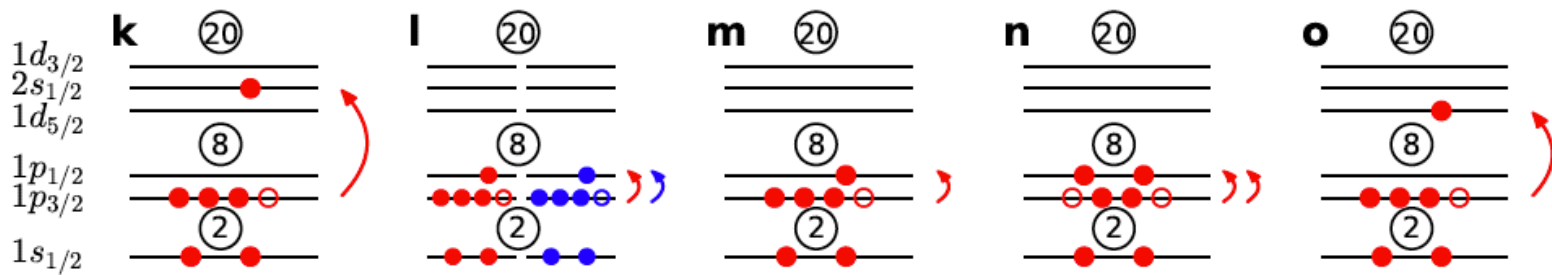
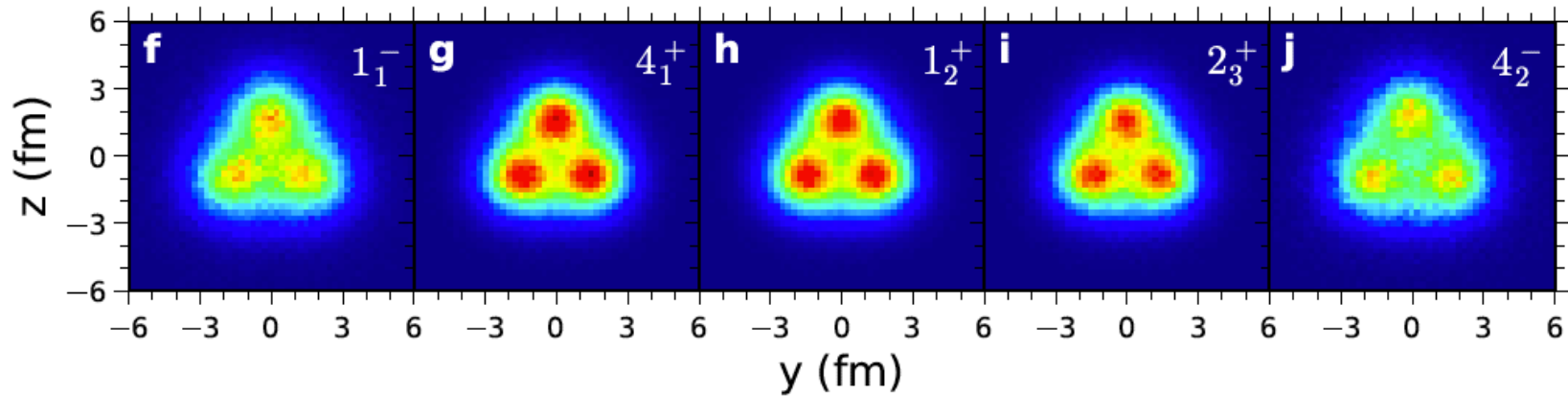


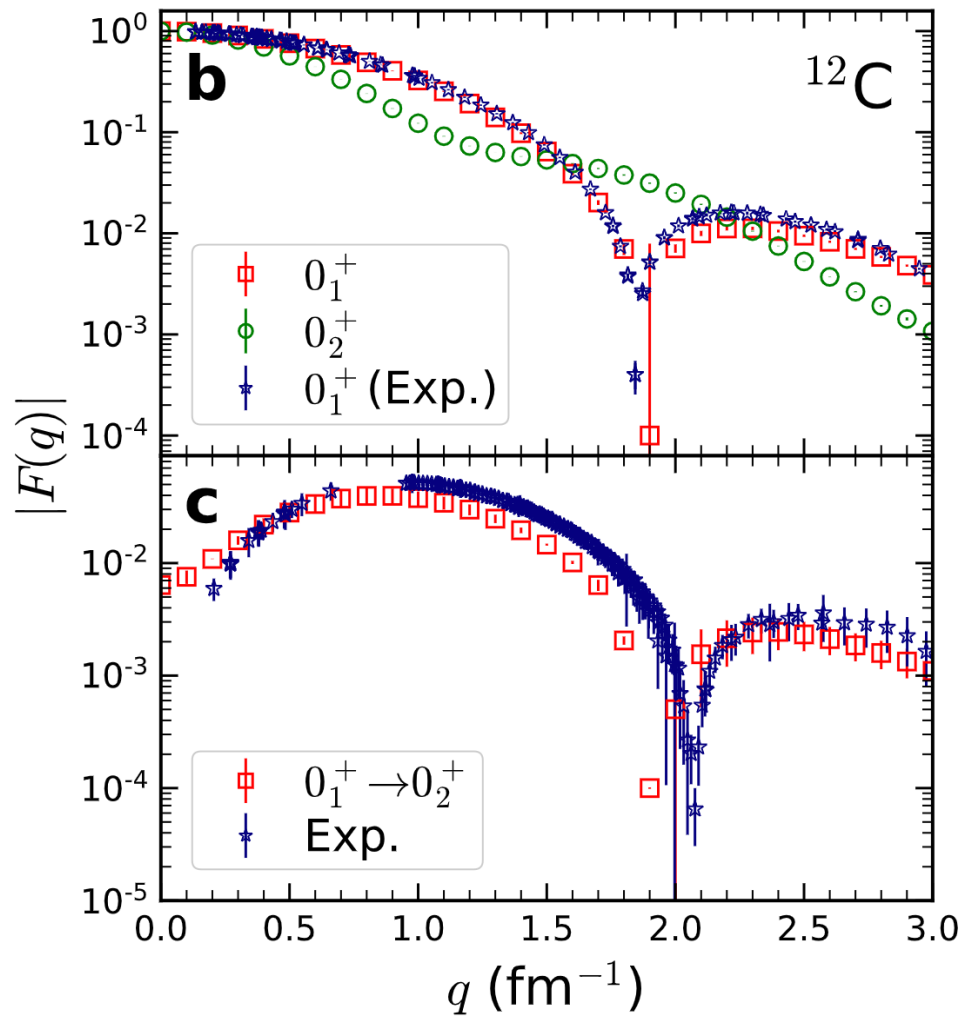
E. Harris et al., work in progress

Emergent geometry and duality of ^{12}C

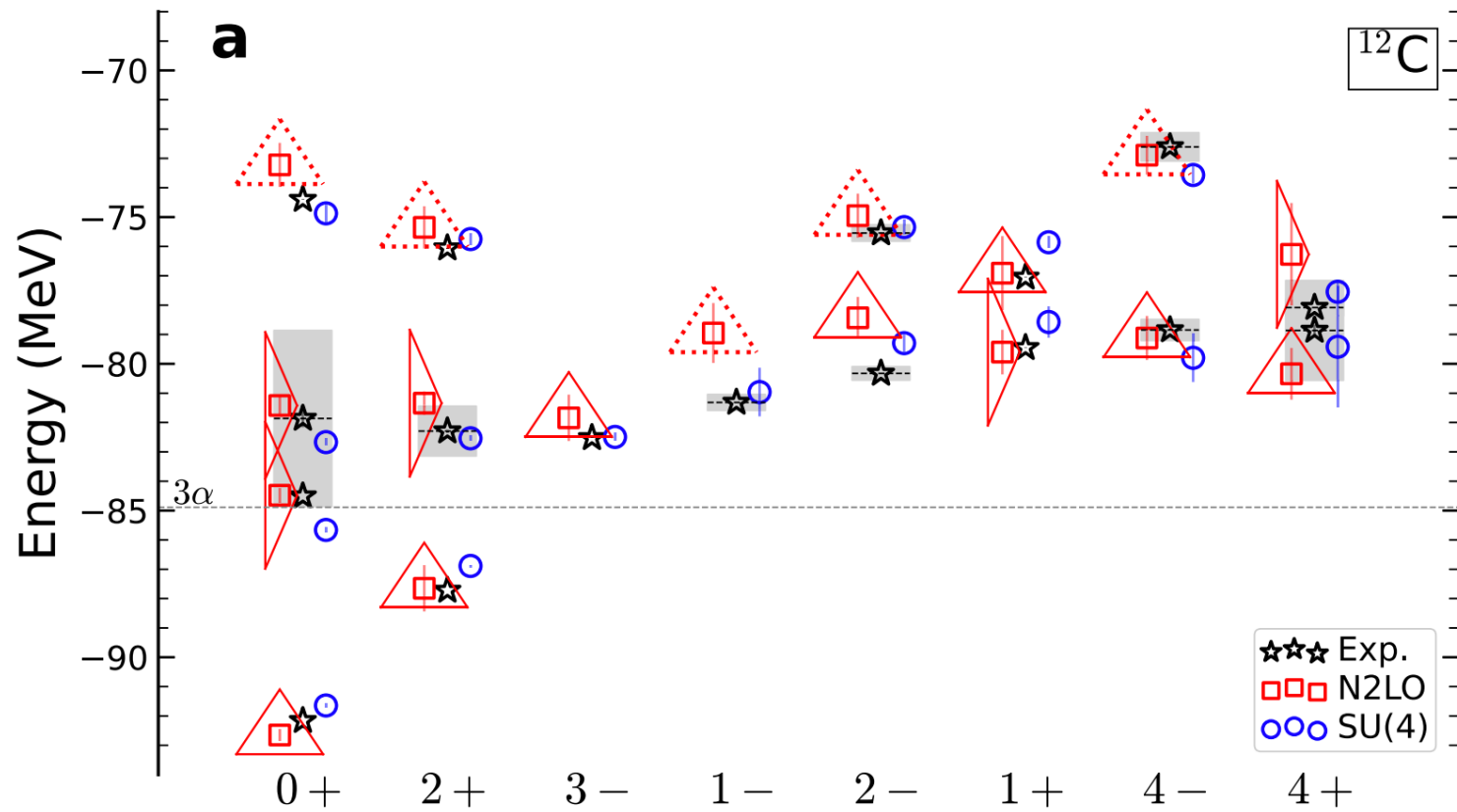


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)



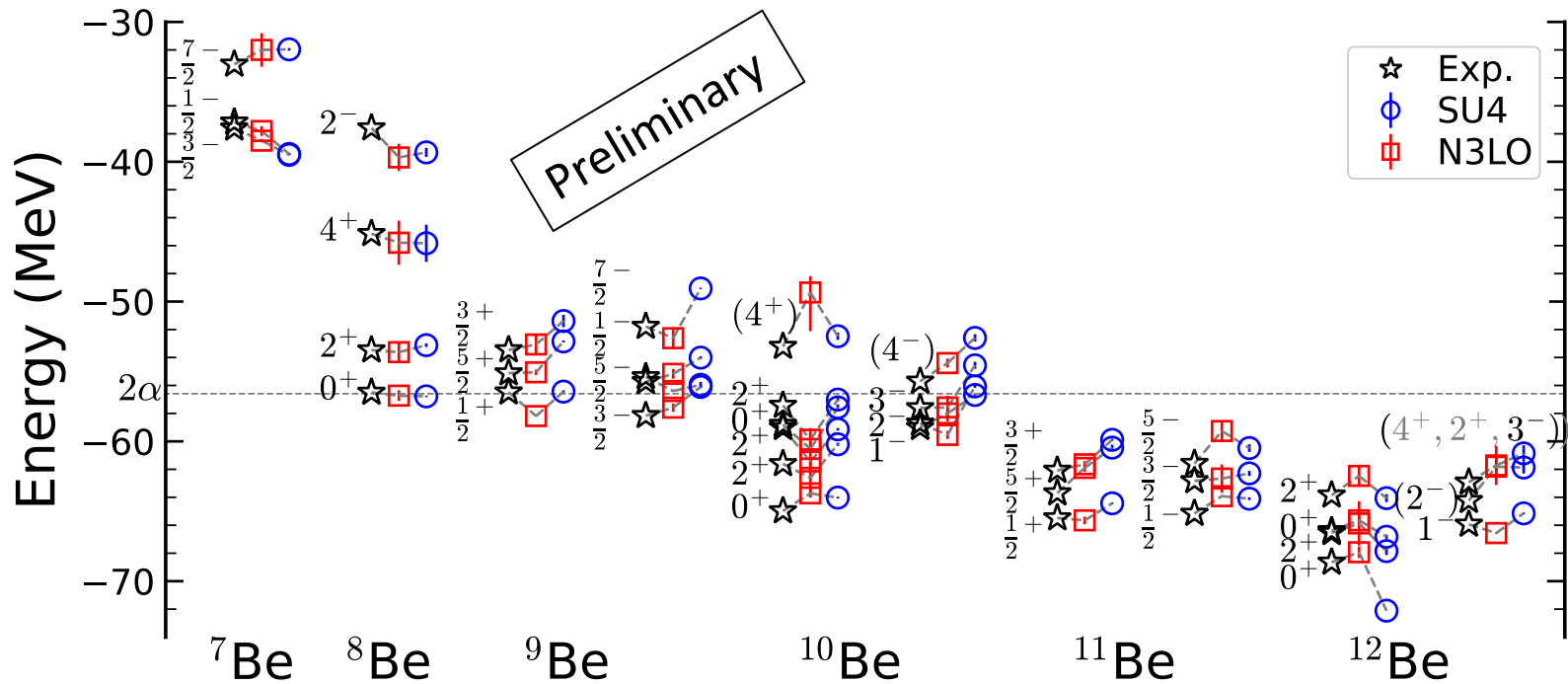


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)



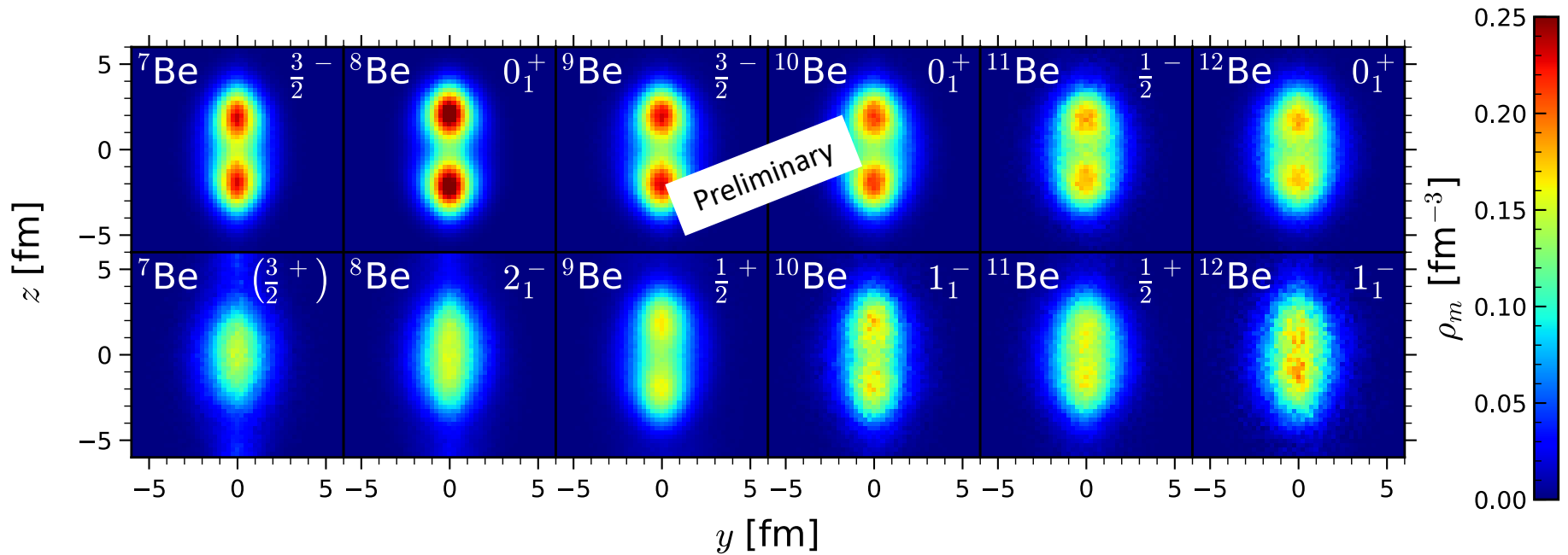
Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

Isotopes of beryllium

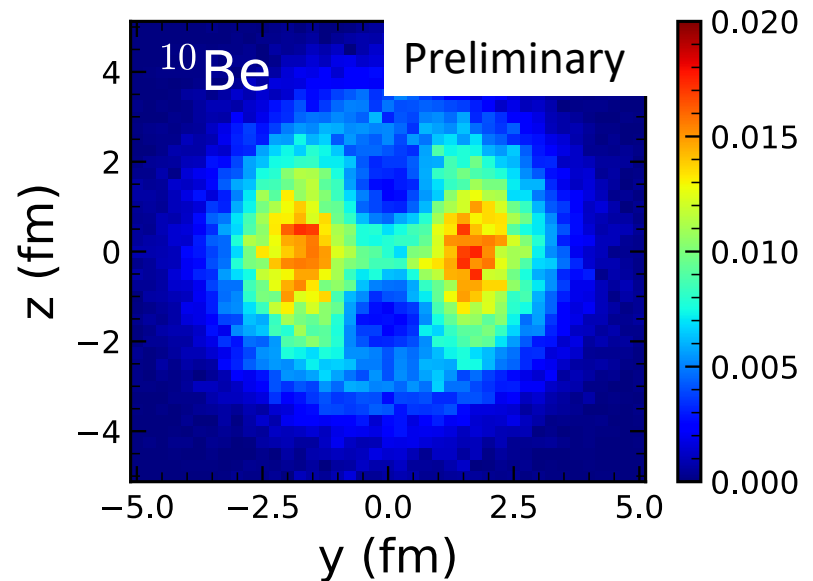
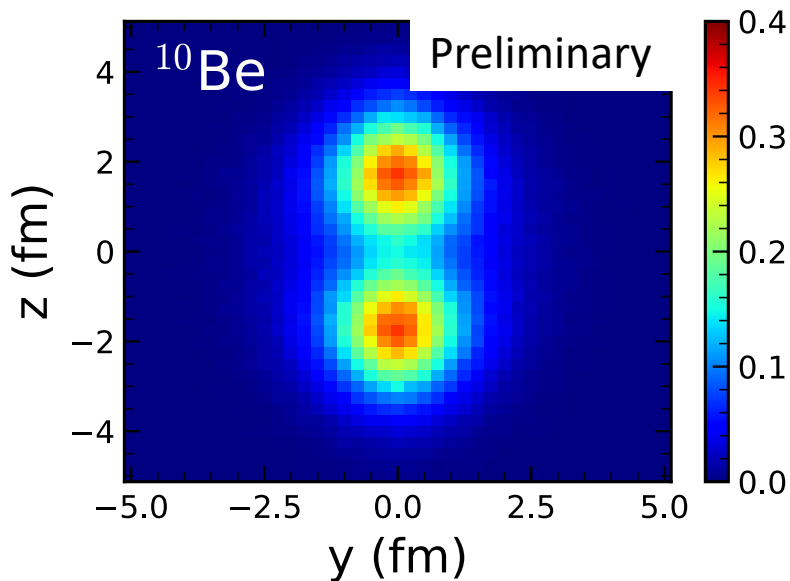


S. Shen, et al., work in progress

Isotopes of beryllium

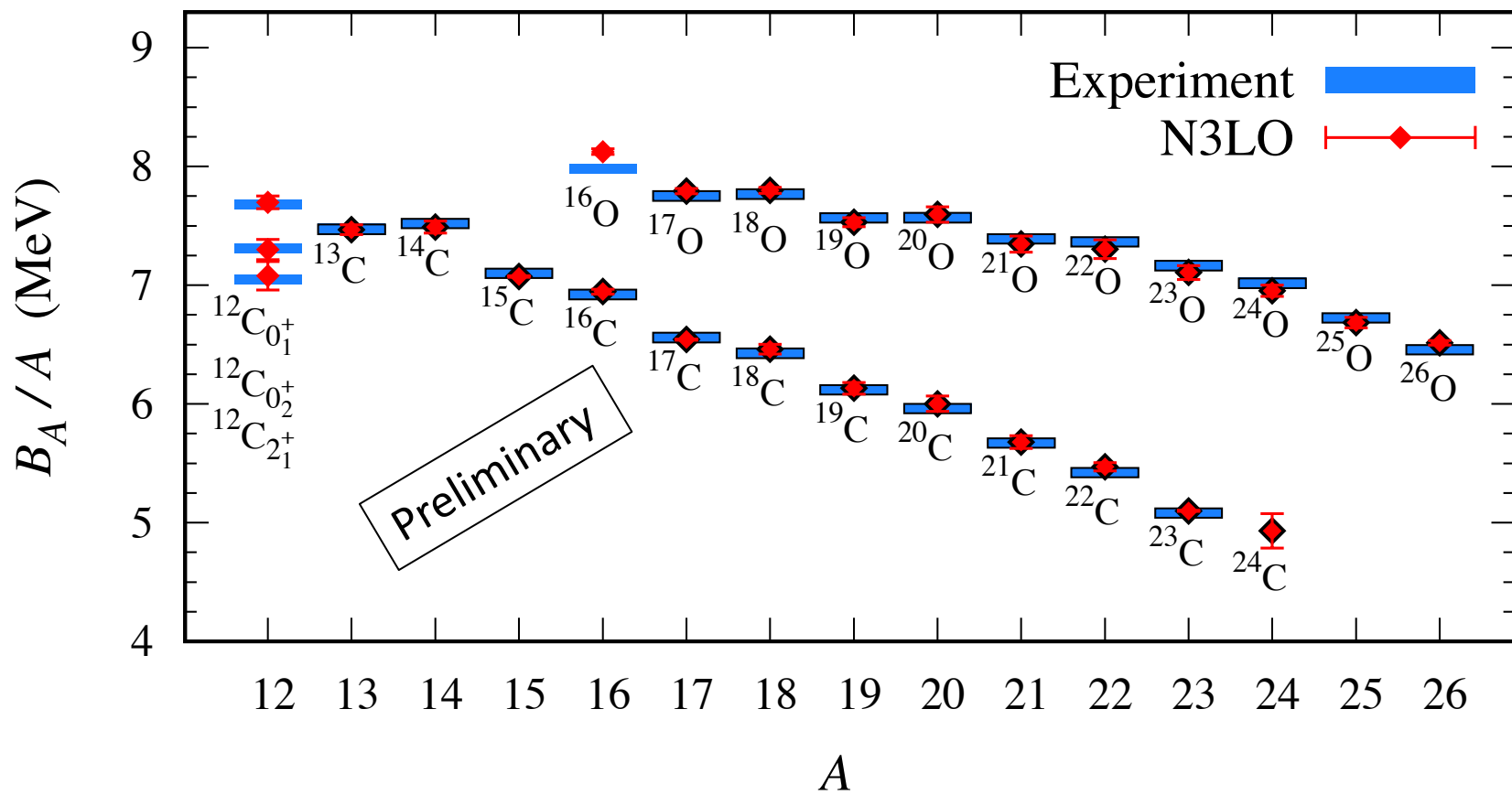


S. Shen, et al., work in progress



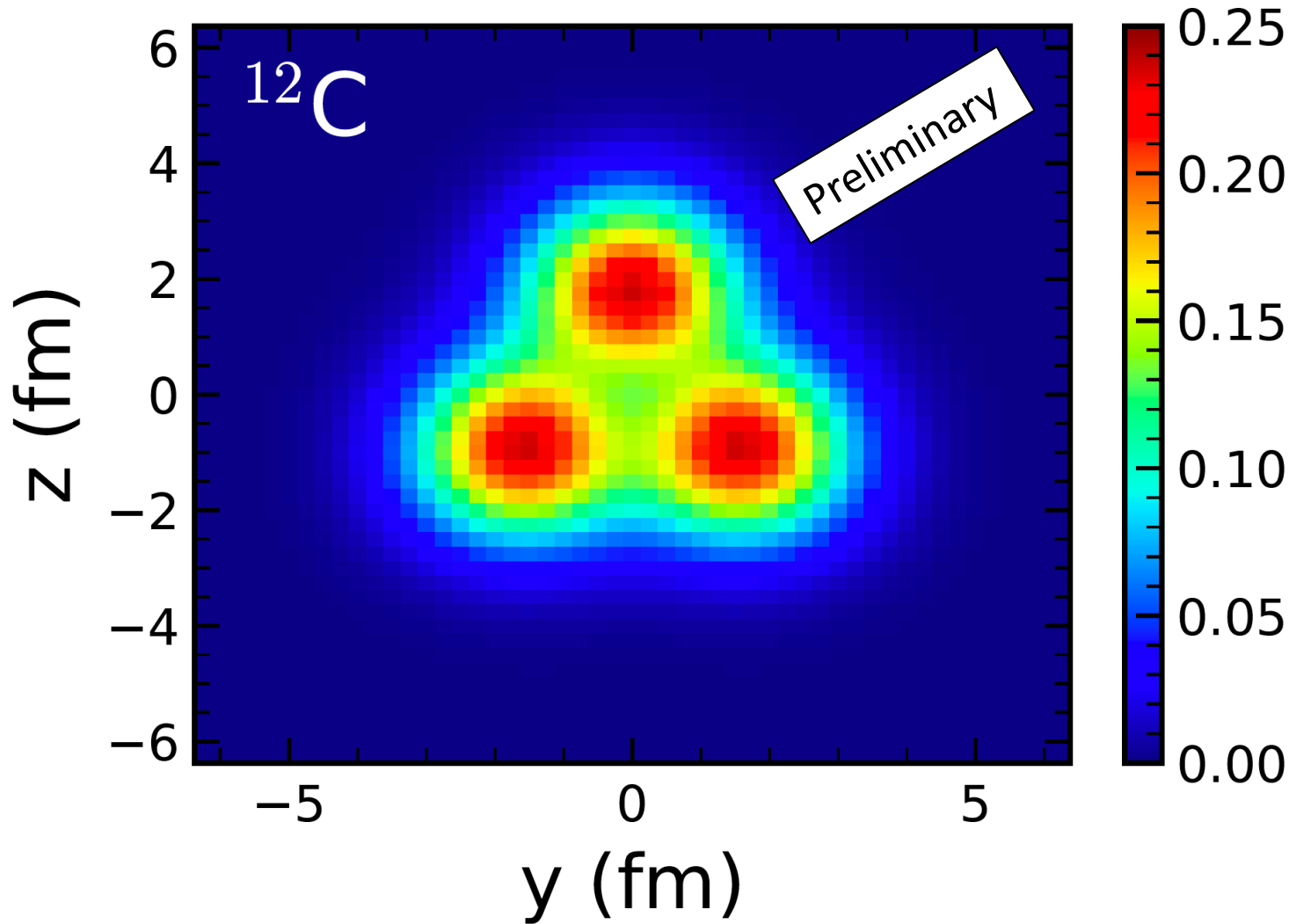
The left panel shows the intrinsic shape of the total nucleon density for ^{10}Be . The right panel shows the density distribution of the two neutrons furthest away from the protons in ^{10}Be .

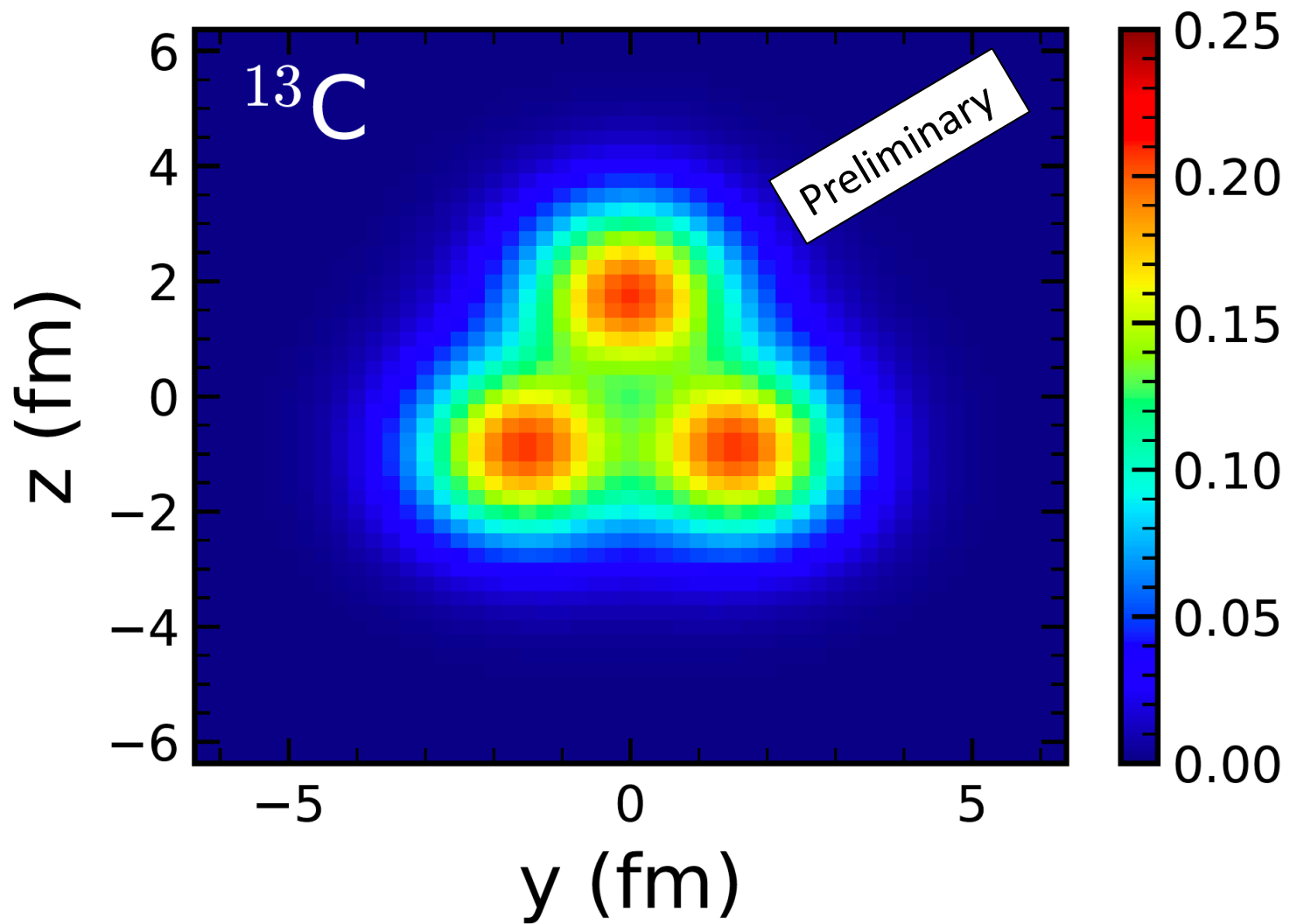
Isotopes of carbon and oxygen

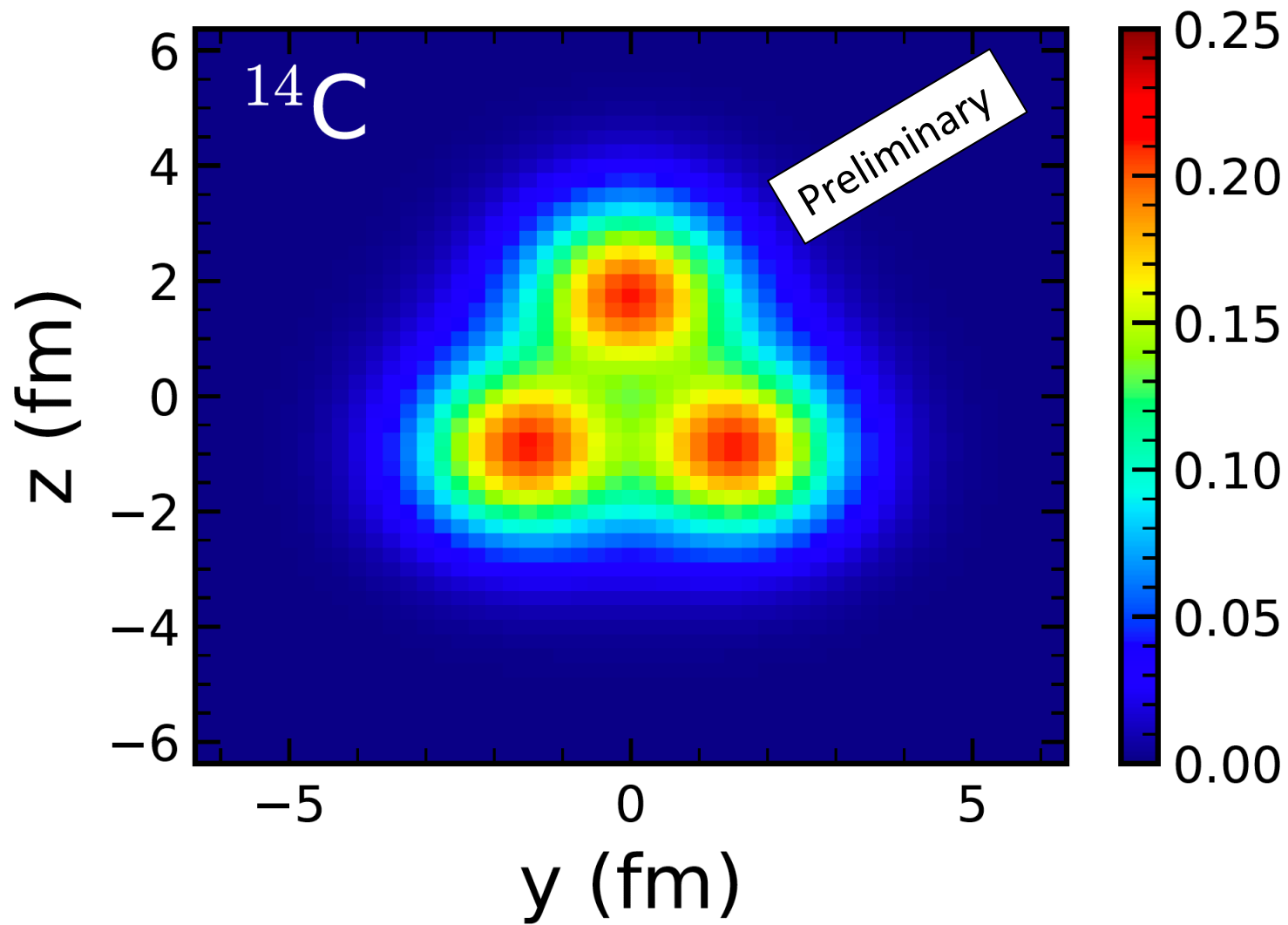


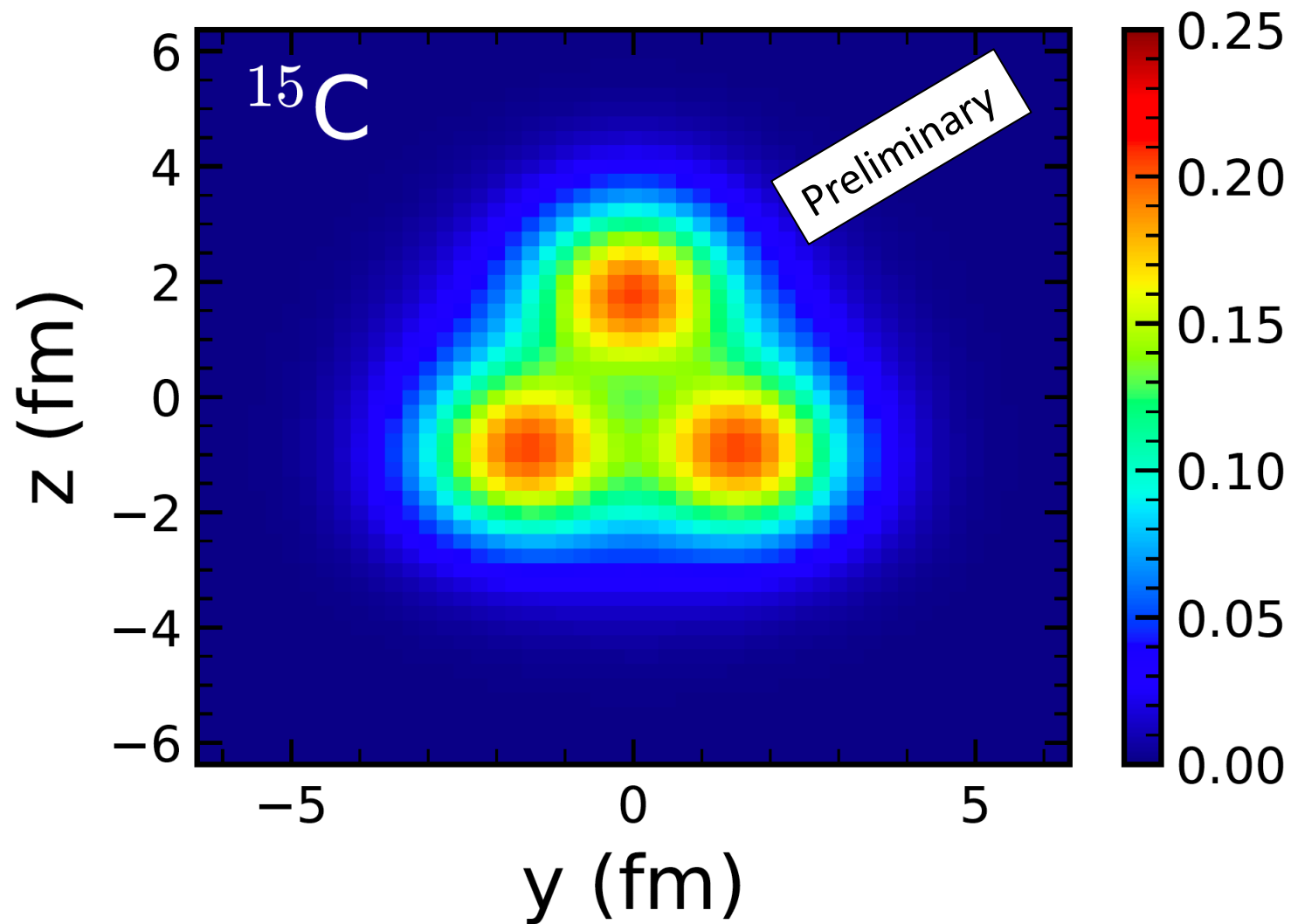
M. Kim, Song, Y. Kim, et al., work in progress

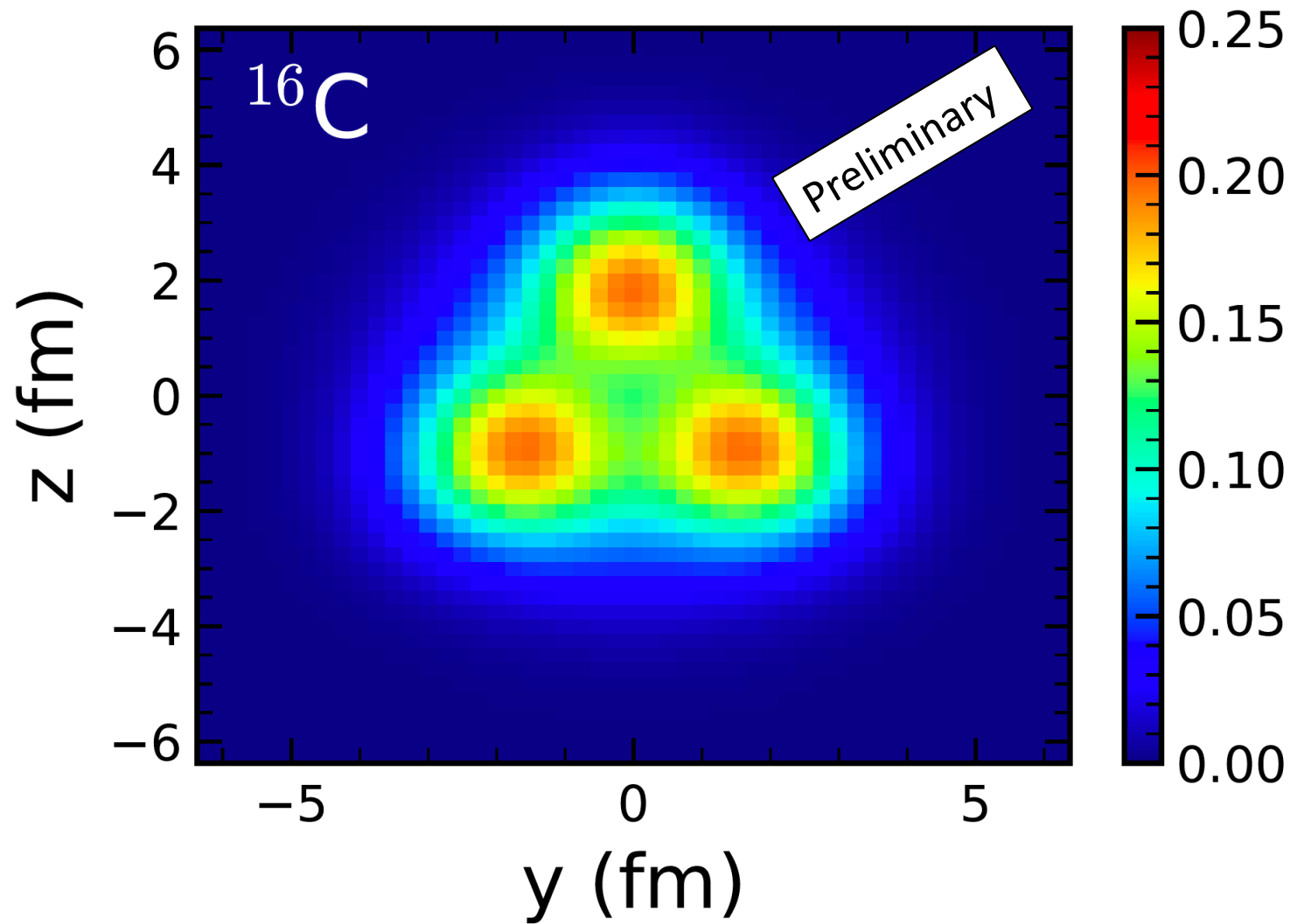
Isotopes of carbon

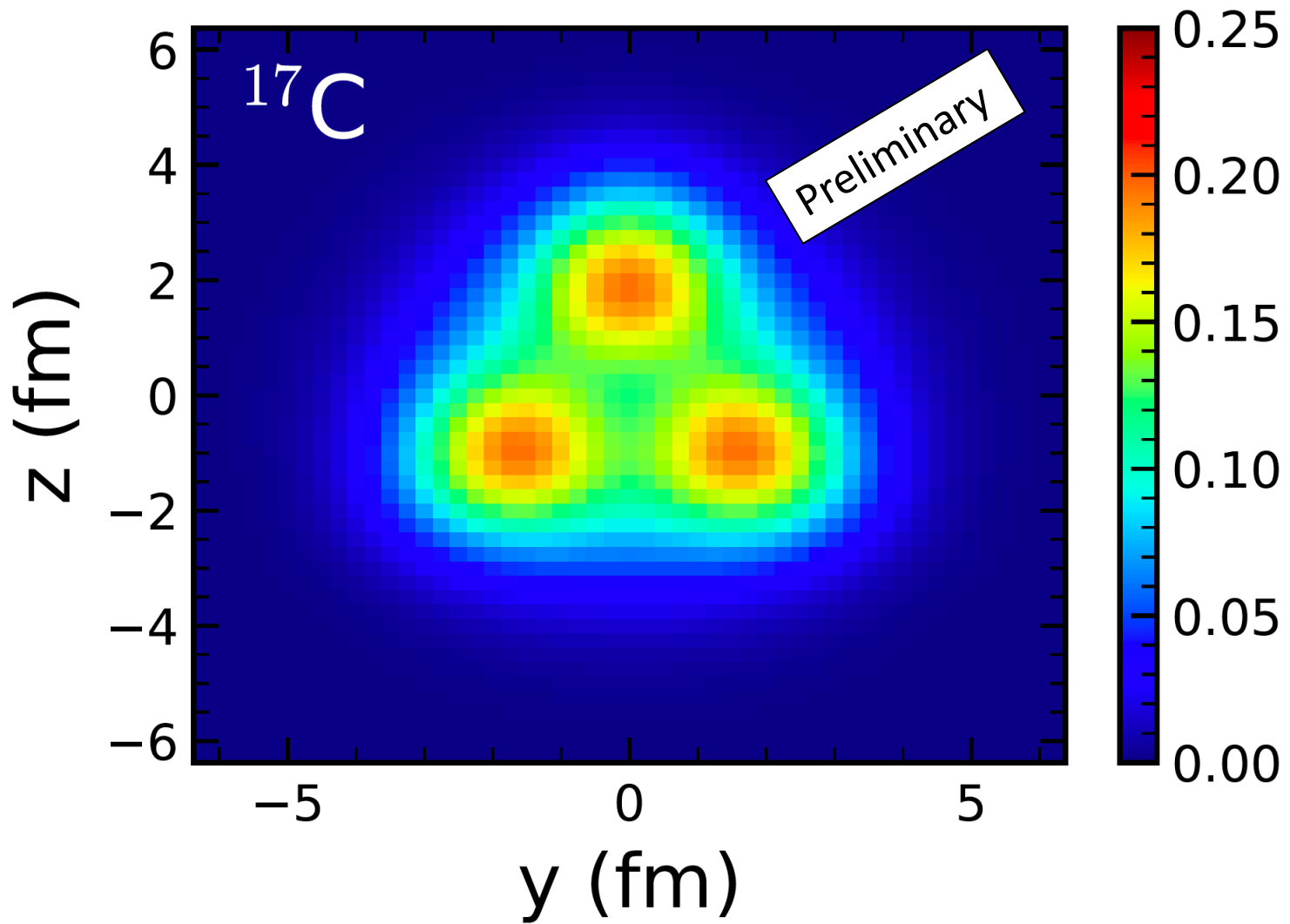


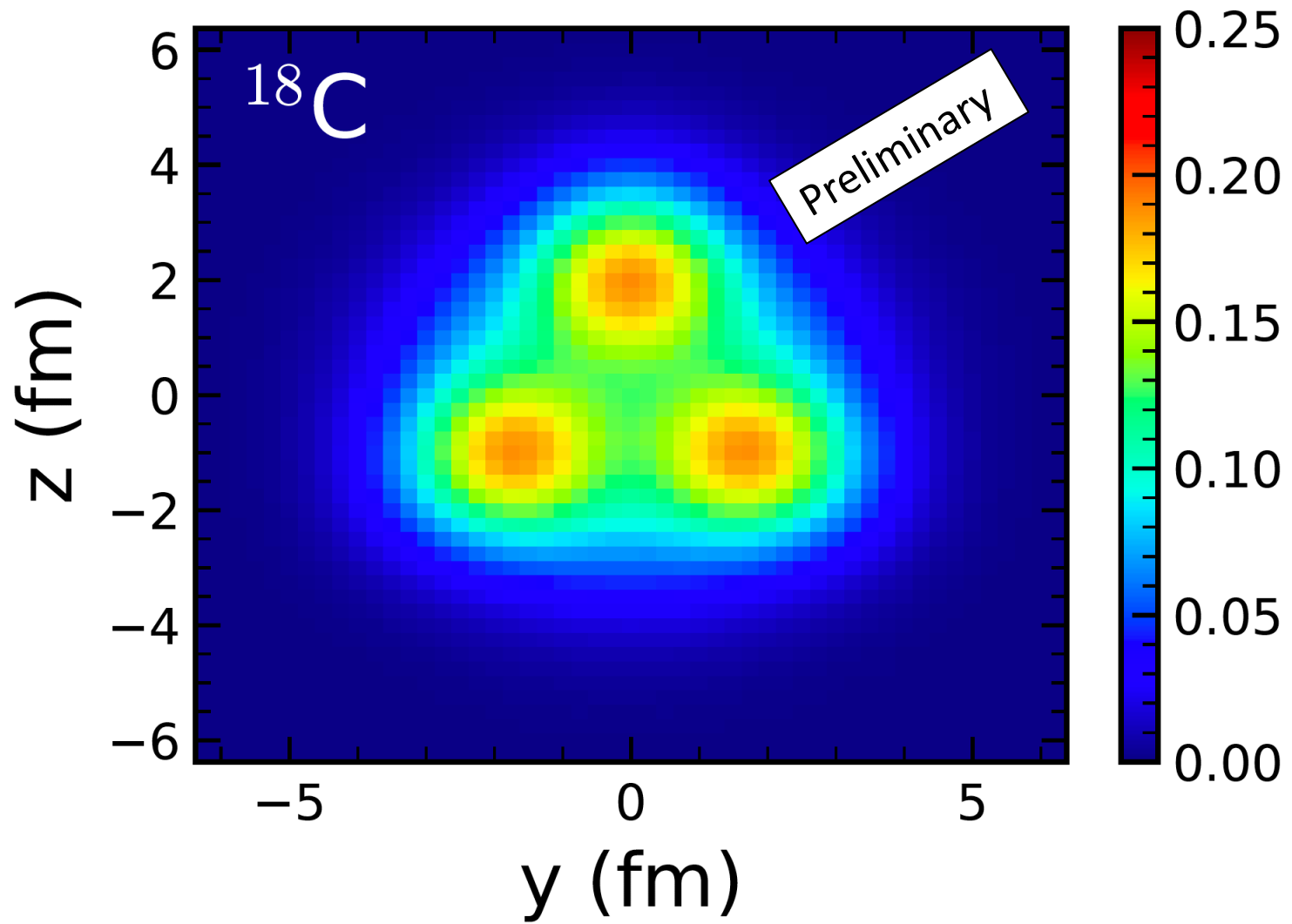


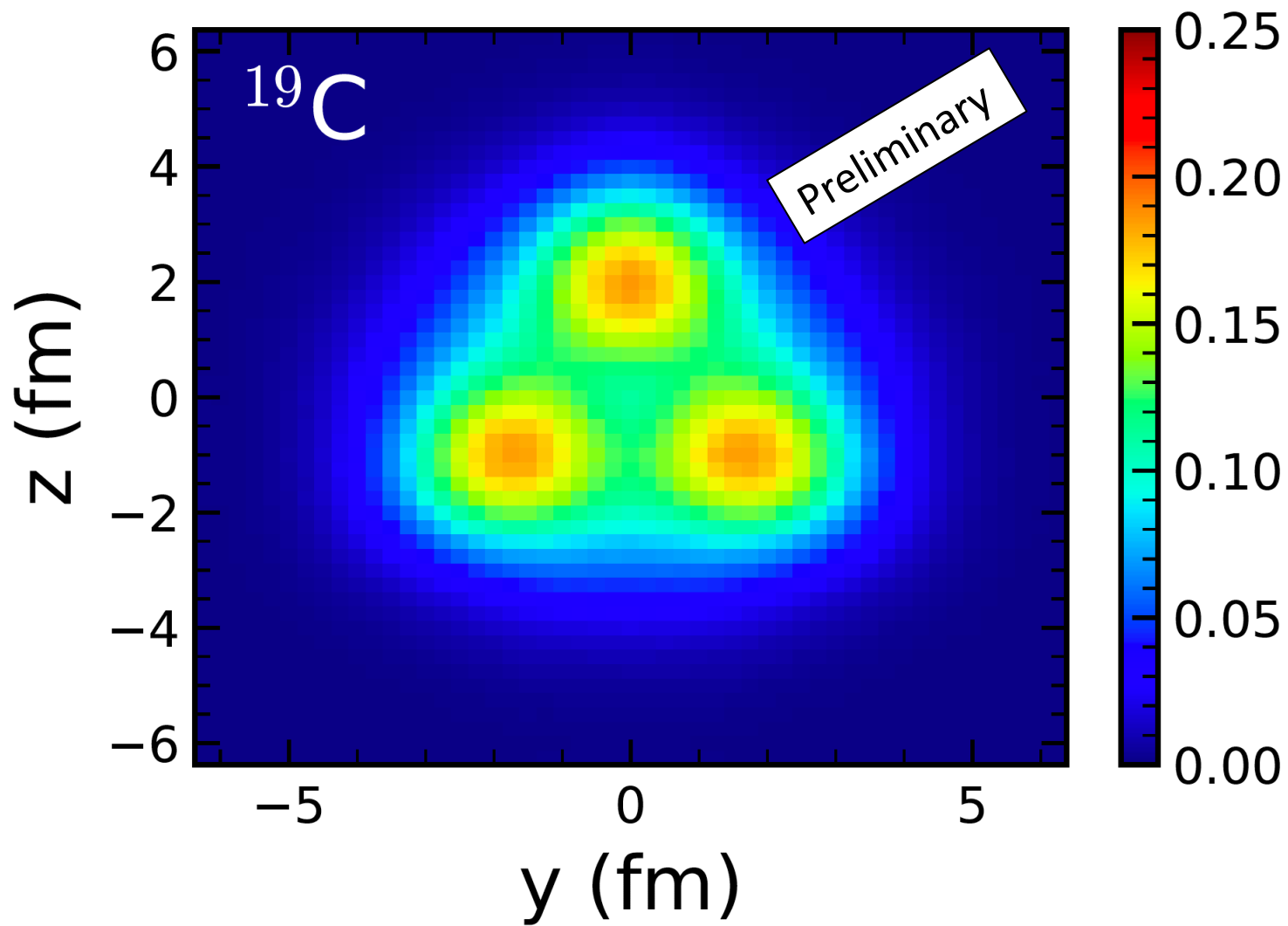


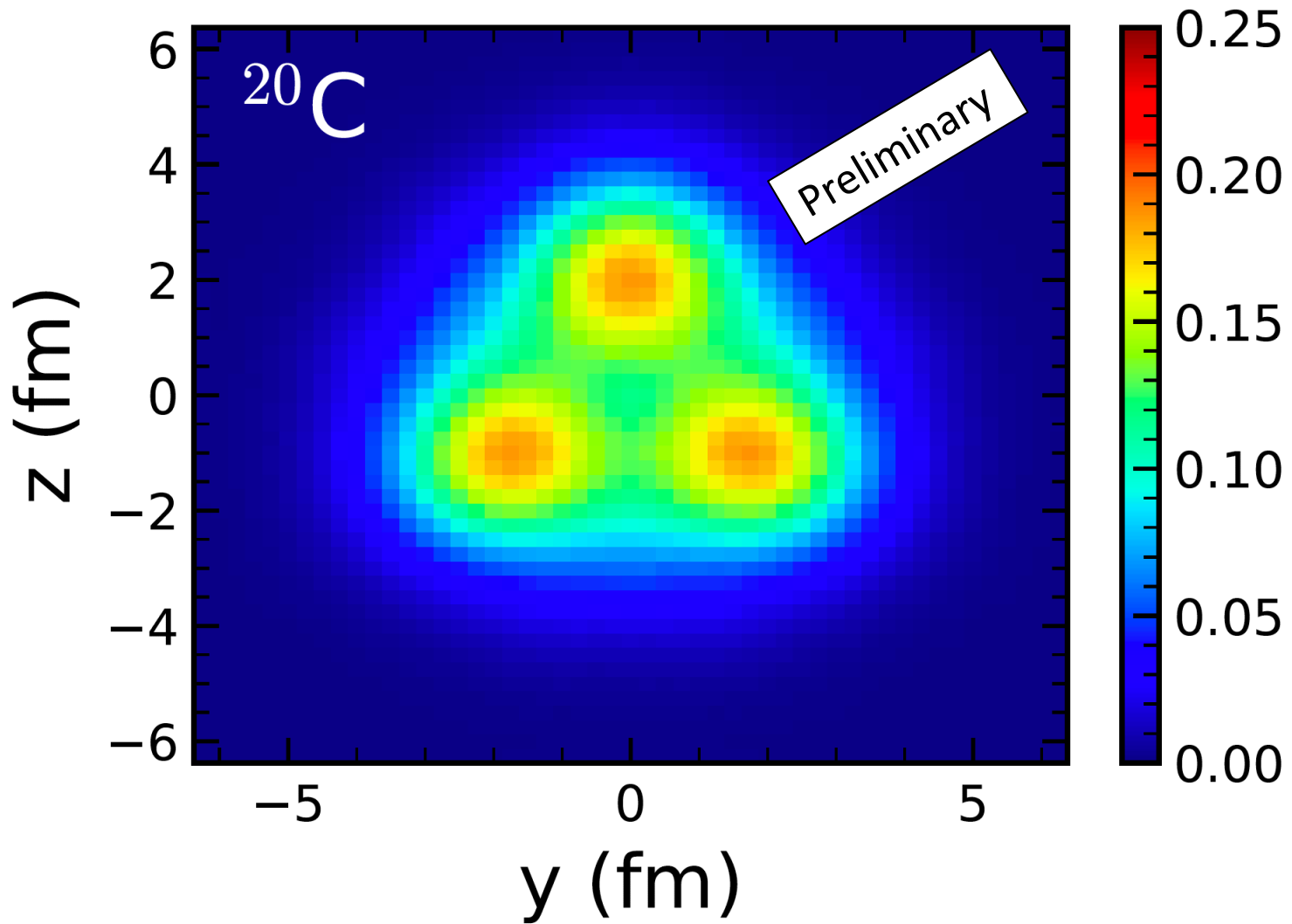




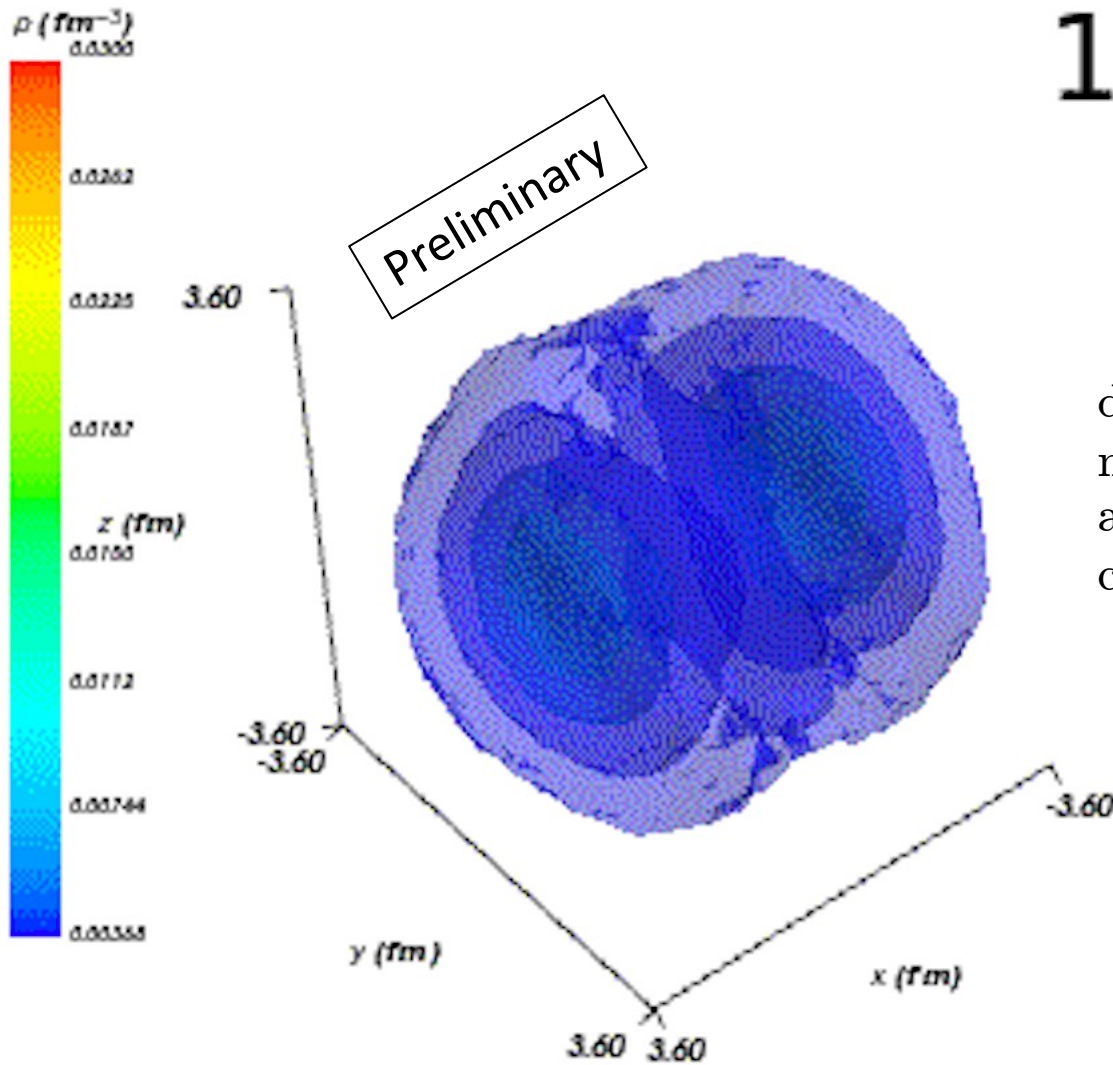






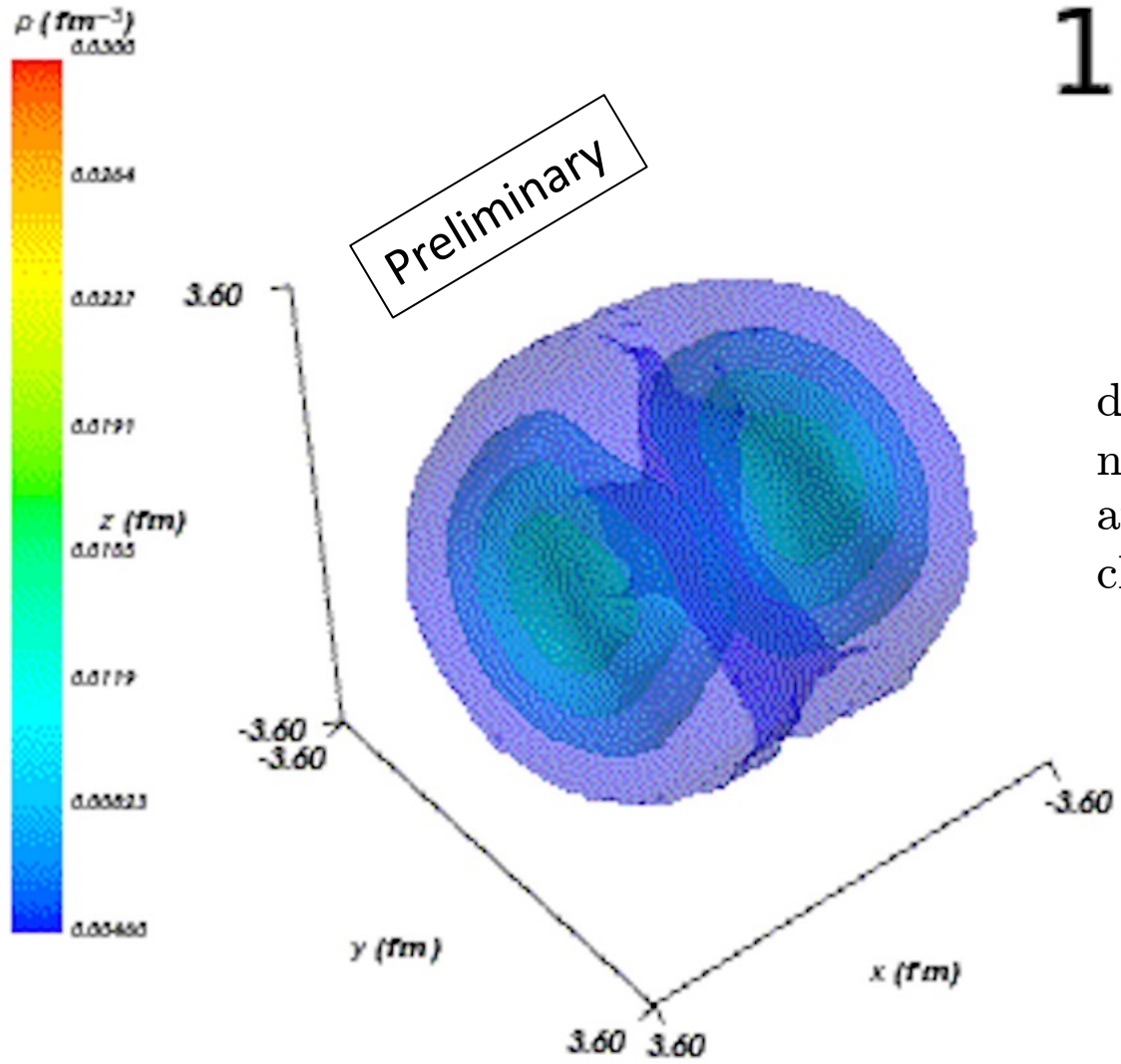


^{13}C



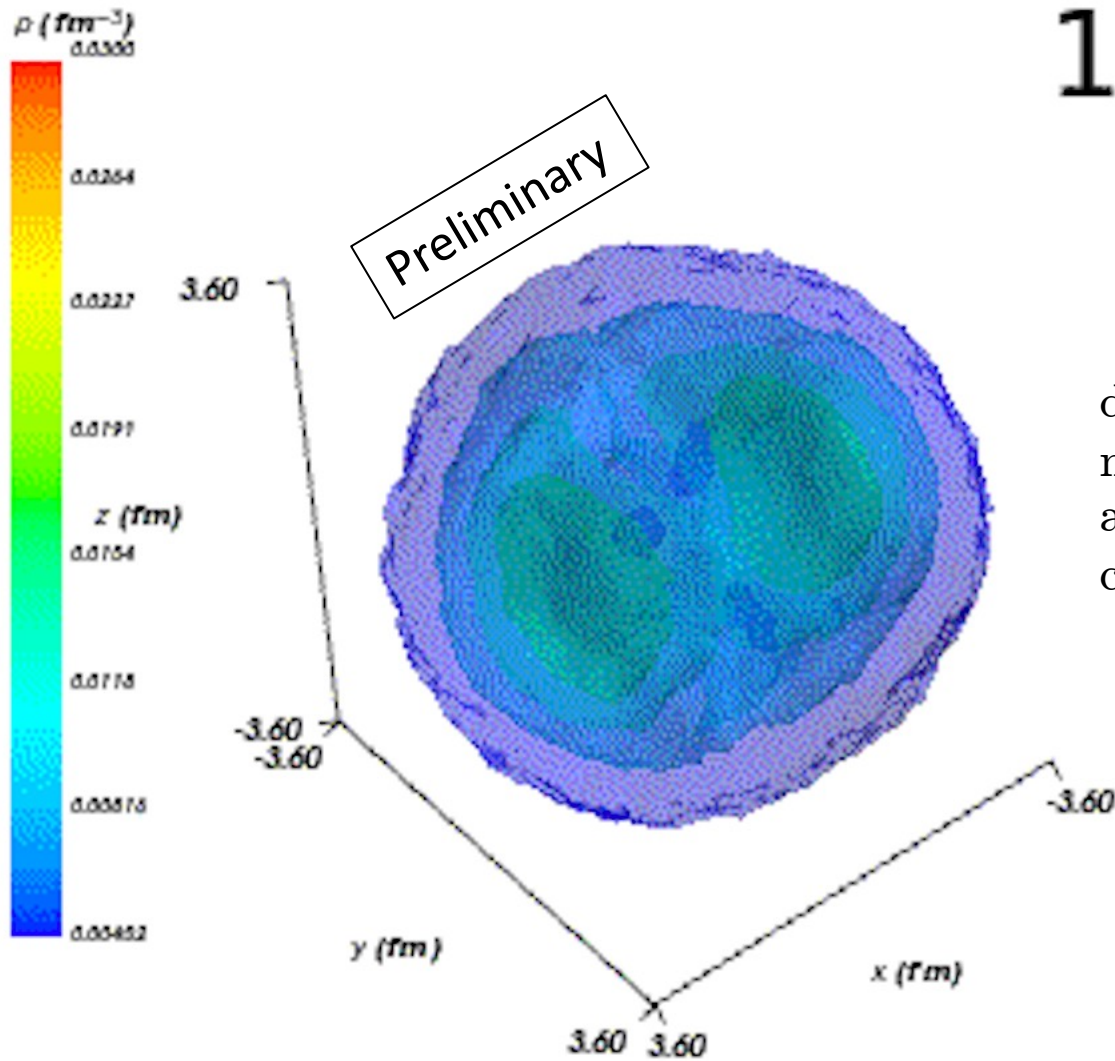
density of the
neutron furthest
away from its
closest proton

^{14}C



density of the two neutrons furthest away from their closest protons

^{15}C



density of the three
neutrons furthest
away from their
closest protons

Summary and outlook

Nuclear lattice effective field theory is being used to perform *ab initio* calculations of nuclear structure. Wavefunction matching allows for calculations using high-fidelity chiral effective field theory interactions. The pinhole algorithm allows for detailed studies of nuclear structure correlations, clusters, sizes, and shapes. We are exploring the properties of the beryllium, carbon, and oxygen isotopes.