Impact of ^{6}Li properties on reactions of astrophysical interest and universal behavior

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[PRL 129, 042503 (2022) & PRC 109, L061601 (2024)]

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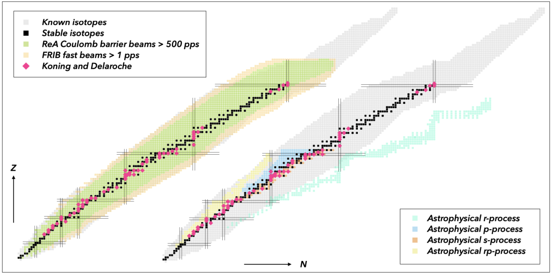
Need for accurate prediction of properties of exotic nuclei to refine our understanding of nucleosynthesis processes



[Hebborn *et al.* JPG **50** 060501 (2023)]

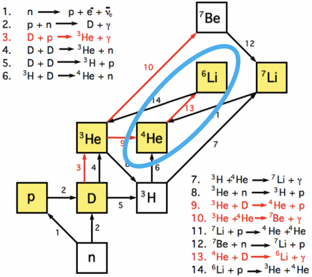
This talk : Can we predict accurately (α-)reaction of astro. interest ?

Unstable nuclei are studied through nuclear reactions
→ Can we improve their analysis ?

Are there universal features in nuclei ?

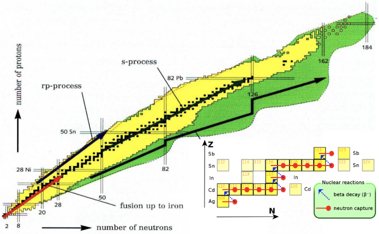
Various α-induced reactions play a key role in astrophysics

Big bang nucleosynthesis



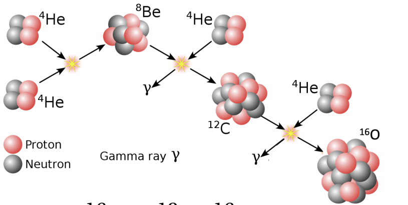
$\alpha(d,\gamma)^6$**Li** : ^{6}Li abundance

slow s-process



^{13}**C**$(\alpha,n)^{16}$**O** : major n source

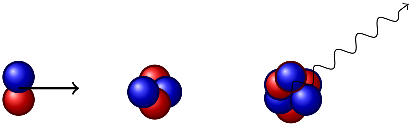
Helium burning



^{12}**C**$(\alpha,\gamma)^{16}$**O** : ^{12}C/^{16}O abundances

Reactions at low energy are difficult to measure as the two charged nuclei repulse each other

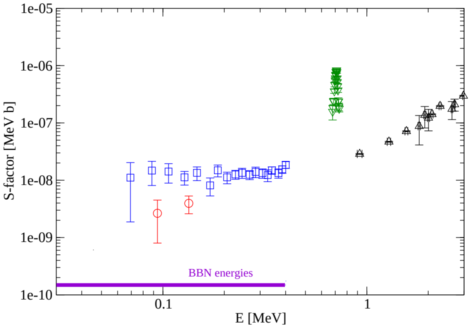
$\alpha(d,\gamma)^6\text{Li}$



very low cross section

= low reaction probability

$$\sigma(E) = \frac{\exp[-2\pi\eta]}{E}\,\boxed{\text{S(E)}}$$

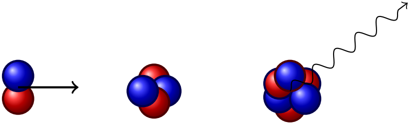


Data : [Anders *et al.* (LUNA) PRL **113** 042501 (2014)] [Kiener *et al.* PRC **44** 2196 (1991)] [Mohr *et al.* **50** 1543 (1994)] [Robertson *et al.* PRL **47** 1867 (1981)]

→ **Need theory to guide the extrapolation**

Reactions involving light nuclei can be predicted using ab initio methods

$\alpha(d,\gamma)^6\text{Li}$



1) Use an accurate model

2) χ-EFT interactions (cf Dean's talk)

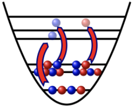
3) Have an estimate of model & input uncertainties

For a complete *ab initio* description, we need both structure... and dynamical clustered description

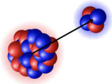
No core shell-model with continuum

[Navrátil, Quaglioni, Hupin, Romero-Redondo and Calci, Phys. Scr. **91**, 053002 (2016)]

$$\Psi = \sum_{\lambda} c_{\lambda} \Big|$$



$$\Big\rangle + \sum_{\nu} \int dr\, u_{\nu}(r) \Big|$$



$$\Big\rangle$$

Discrete structure information input

Continuous dynamical input (clustering/reactions)

$\oplus$ **Bound states, narrow resonances**

→ short-range

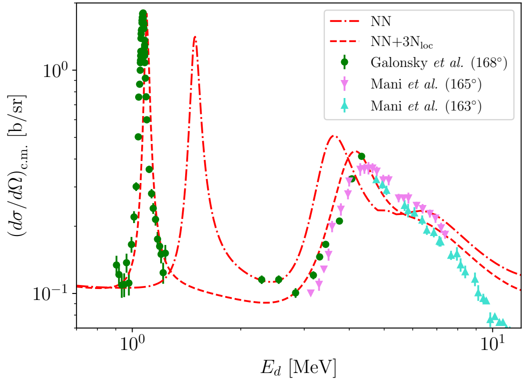
$\oplus$ **Bound & scattering states, reactions**

→ long-range

Ab initio predictions are accurate for α-d scattering

Convergence with 10 + & 5 – parity ^{6}Li states,
d g.s. + 8 d pseudostates (d breakup included)
at $N_{max} = 11$ using NN+3N forces

HPC at LLNL

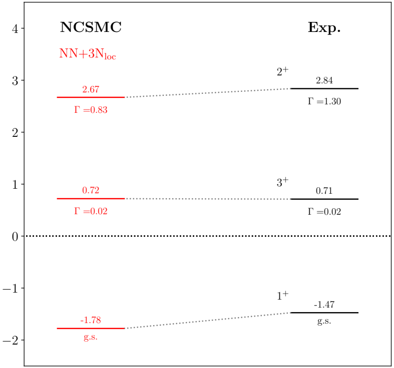


Importance of 3N (SRG-induced & chiral)

Ab initio predictions are accurate for ^{6}Li spectrum but... not perfect

Convergence with 10 + & 5 − parity ^{6}Li states, d g.s. + 8 d pseudostates (d breakup included) at $N_{max} = 11$

HPC at LLNL



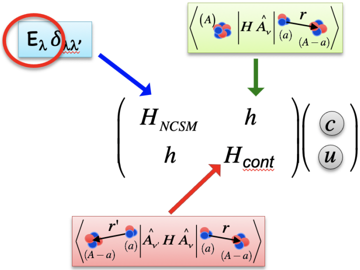
Accurate prediction of ^{4}He$(d,\gamma)^6$Li

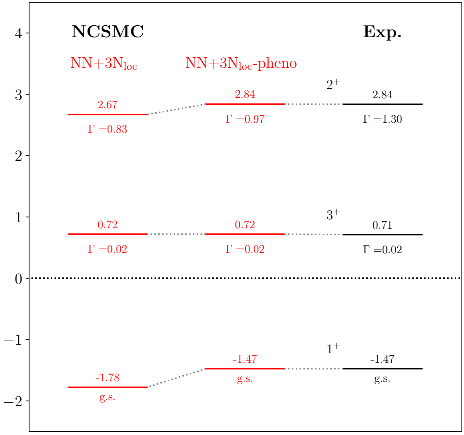
→ need to have the right ^{6}Li binding

Use of a phenomenological correction for the overbinding and the position of the 2^+ resonance

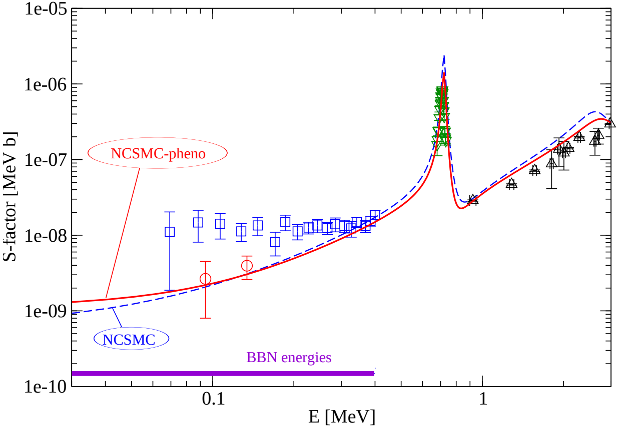
Phenomenological correction

→ adjustments of NCSM E_{1^+} and E_{2^+}





Ab initio prediction fills the experimental gap for $\alpha(d,\gamma)^6$Li

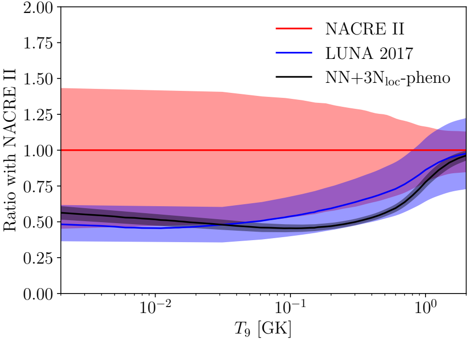


Excellent agreement with data : importance of E_{1^+} at low energies and E_{2^+} at higher energies

What is the uncertainty due to the choice of χ-EFT force & to the finite size of the basis ?

Ab initio-informed predictions reduce the uncertainties on the $\alpha(d,\gamma)^6$Li rate by an average factor 7

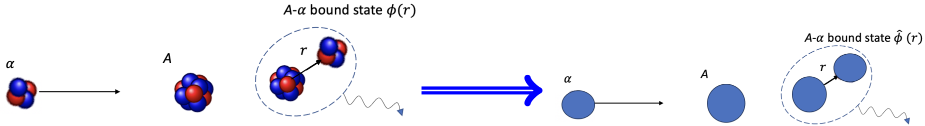
Comparison of two chiral forces and different N_{max}
→ Small uncertainties thanks to the adjustment of the ^{6}Li g.s. energy



[Hebborn, Hupin, Kravvaris, Quaglioni, Navrátil, Gysbers, Phys. Rev. Lett. **129**, 042503 (2022)]

→ **What about reactions involving heavier nuclei, e.g., ^{13}C$(\alpha,n)^{16}$O & ^{12}C$(\alpha,\gamma)^{16}$O ?**

For reactions involving heavier nuclei, one needs to make approximations



To make accurate reaction predictions :

1) Two-body model

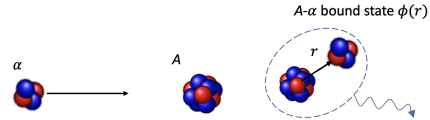
2) $A-\alpha$ **Interactions reproducing low-energy spectrum**

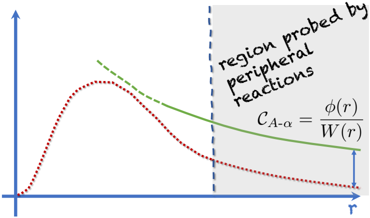
3) Have an estimate of model & input uncertainties

$A-\alpha$ interactions can be constrained using indirect reactions, e.g., (^{6}Li, d) transfer data

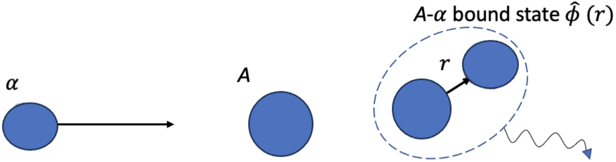
At $E \to 0$ MeV, non-resonant reactions are peripheral, they scale with the ANC^2 of subthreshold states

At low energies :





The cross section can be obtained in a two-body model

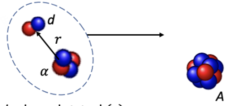


$$\sigma_{\alpha,\gamma} \approx C^2_{A-\alpha}\hat{\sigma}_{\alpha,\gamma}$$

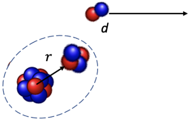
If one knows $C^2_{A-\alpha}$, one can determine accurately the rate at low E !

α-transfer (^{6}Li, d) around the Coulomb barrier are also peripheral and can be used to extract ANCs

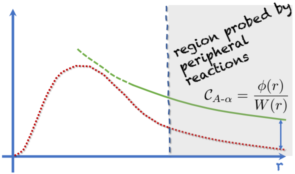
At low energies :



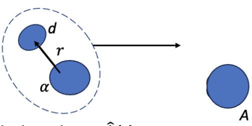
d-α bound state $\psi(r)$



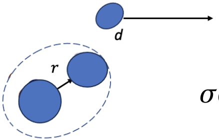
A-α bound state $\phi(r)$



The cross section can be obtained in a three-body model



d-α bound state $\hat{\psi}(r)$



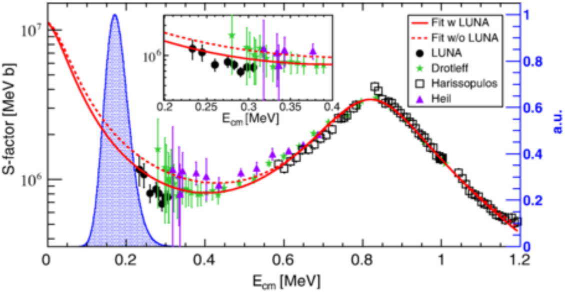
A-α bound state $\hat{\phi}(r)$

$$\sigma_{^6\mathrm{Li},d} \approx C^2_{\alpha-d} C^2_{A-\alpha} \hat{\sigma}^{DWBA}_{^6\mathrm{Li},d}$$

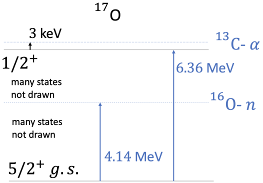
If one knows $C^2_{\alpha-d}$, one can determine $C^2_{A-\alpha}$ from (^{6}Li, d) data !

ANC method : [Tribble *et al.* Rep. Prog. Phys. **77**, 106901 (2014)]

The $^{13}C(\alpha,n)^{16}O$ S-factor has been measured underground and extrapolated to zero energies...



[Ciani *et al.* (LUNA collaboration) PRL **127**, 152701 (2021)]

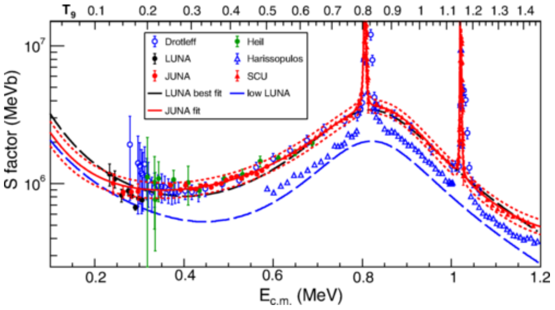


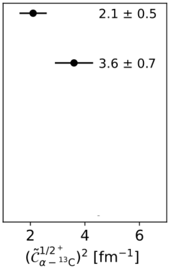
$(C^{1/2+}_{^{13}\mathrm{C}-\alpha})^2$ constrains the extrapolation

Deduced from $(^6\mathrm{Li}, d)$ data at $\sim 0.6A$ MeV

[Avila *et al.* PRC **91**, 048801 (2015)]

but new underground measurements predict a S(0) 21% smaller... and the discrepancy is traced back to $(C^{1/2+}_{^{13}\mathrm{C}-\alpha})^2$

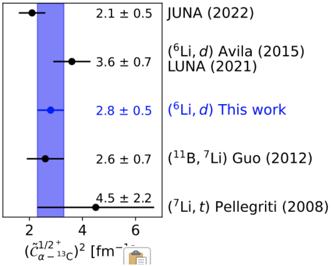




[Gao *et al.* (JUNA collaboration) PRL **129**, 132701 (2022)]

What can explain this discrepancy ? $\sigma_{^6\mathrm{Li},d} \approx C^2_{\alpha-d} C^2_{A-\alpha}\, \hat{\sigma}^{DWBA}_{^6\mathrm{Li},d}$

Using the ab initio $C_{\alpha-d}$ to reanalyze (^{6}Li, d) data, we reconcile both LUNA and JUNA analyses !



Previous $(C_{\alpha-d})^2$: [Blokhintsev *et al.* PRC **48**, 2390 (1993)]

→ unaccounted syst. uncertainties !

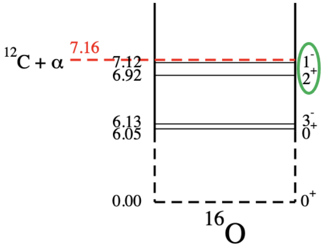
22% smaller than ab initio $(C_{\alpha-d})^2$

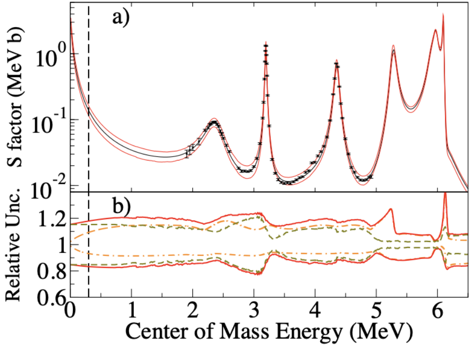
Our $(C_{\alpha-d})^2$ explains the discrepancy between JUNA and LUNA S(0), is more precise, & favors the JUNA evaluation of S(0) !

Another key astrophysical reaction $^{12}C(\alpha,\gamma)^{16}O$ have been constrained using $(^6Li, d)$ data and previous ANC !

$C_{\alpha-^{12}C}$ extracted from $(^6Li, d)$ data used in R-matrix fits

(large set of data : ANCs, S-factor, el. scattering, β-delayed α emission)



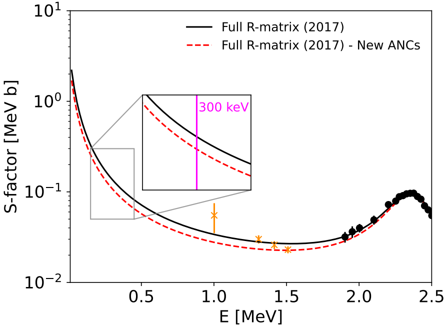


[deBoer *et al.* Rev. Mod. Phys. **89**, 035007 (2017)]

The ab initio $(C_{\alpha-d})^2$ leads to a reduction of 21% of the $(C_{\alpha-^{12}C})^2$ & S-factor at stellar energies !

J^π	E_{ex}	Probe	$(\mathcal{C}^{J^\pi}_{\alpha\text{-}^{12}C})^2$ Past Work		
0^+	6.05	$(^6\mathrm{Li}, d)$	2.43(30)	1.88(16)	$\times 10^6$
3^-	6.13	$(^6\mathrm{Li}, d)$	1.93(25)	1.49(14)	$\times 10^4$
2^+	6.92	$(^6\mathrm{Li}, d)$	1.24(24)	0.96(16)	$\times 10^{10}$
		$(^6\mathrm{Li}, d)$	1.48(16)	1.14(7)	
		$(^7\mathrm{Li}, t)$	1.33(29)		
		$(^7\mathrm{Li}, t)$	2.07(80)		
1^-	7.12	$(^6\mathrm{Li}, d)$	4.33(84)	3.34(58)	$\times 10^{28}$
		$(^6\mathrm{Li}, d)$	4.39(59)	3.39(34)	
		$(^7\mathrm{Li}, t)$	4.00(138)		

[Brune *et al.* PRL **83**, 4025 (1999)]
[Avila *et al.* PRL **114**, 071101 (2015)]
[Oulebsir *et al.* PRC **85**, 035804 (2012)]

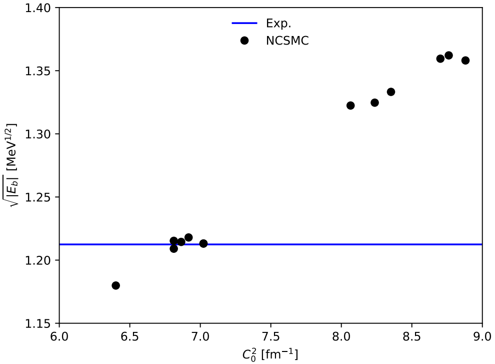


Data : [Schürmann *et al.* EPJA **26**, 301 (2005)]
Data : [Plag *et al.* PRC **86**, 015805 (2012)]

Data sets cannot constrained ANCs → renormalization factors

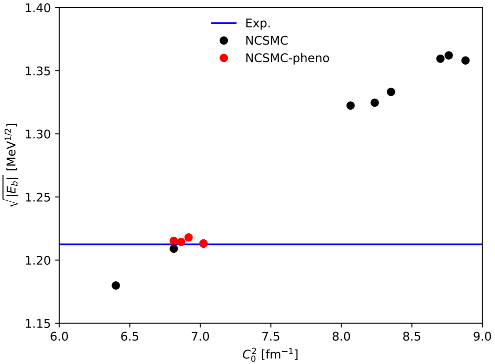
Ab initio $C_{\alpha-d}$ carries very small uncertainties, why ?

Few-body universality in the d-α system : the square root of the binding energy is correlated with the ANC^2



Calculations with various λ_{SRG}, with various 3N forces & model spaces
→ ANC is constrained with the binding energy... Is this universal behavior present in other non-halo nuclei ? How can we explain this ?

Few-body universality in the d-α system : the square root of the binding energy is correlated with the ANC^2



Calculations with various λ_{SRG}, with various 3N forces & model spaces

→ ANC is constrained with the binding energy... Is this universal behavior present in other non-halo nuclei ? How can we explain this ?

Can we explain this relationship with an analytic continuation of the effective range expansion ?

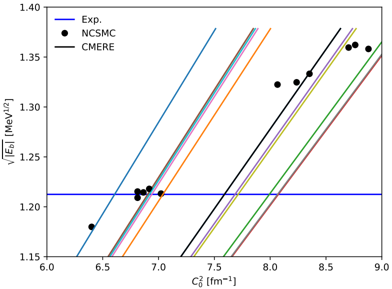
Coulomb effective range expansion (K_0^2 depends on $\cot\delta_0(k)$)

$$K_0(k^2) = -\frac{1}{a_0} + \frac{r_0}{2}k^2 - P_0 r_0^3 k^4 + Q_0 k^6 - R_0 k^8 + S_0 k^{10} + \mathcal{O}(k^{12})$$

→ Needs to impose the position of the bound state (pole of the S-matrix)

→ Convergence quite slow... up to k^{10}

→ ANC calculated from these coefficients [Sparenberg, Baye, Capel, PRC **81**, 011601(R) (2010)]



Universal features in ^{6}Li cannot be explained by ERE !

So what can explain it ?

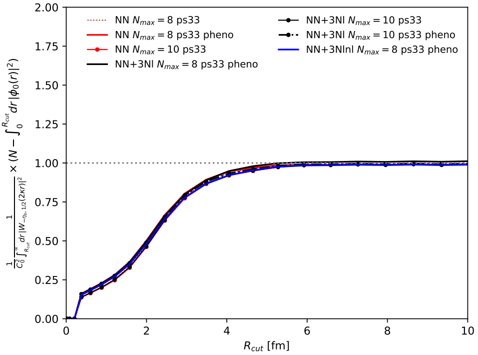
Can we explain the C_0^2-$\sqrt{E_b}$ relationship from bound state wavefunction ?

Normalization of the overlap wave function $N = \int dr\, r^2 \phi_0(r)$

$$\leftrightarrow\; N = \int_0^{R_{cut}} dr\, \phi(r) + C_0^2 \int_{R_{cut}}^{\infty} dr\, W(r)$$

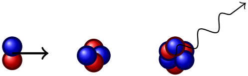
$$\leftrightarrow\; \left(N - \int_0^{R_{cut}} dr\, \phi(r)\right) \frac{1}{C_0^2 \int_{R_{cut}}^{\infty} dr\, W(r)} = 1$$

At which R_{cut} all wavefunctions look similar ?



Universal behavior of the overlap function

Summary and prospects

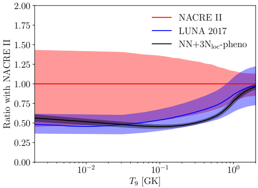


Ab initio methods are accurate for light systems

→ Start from a χ-EFT NN+3N Hamiltonian

& consistent treatment of structure & reaction

Ab initio prediction reduces the uncertainties on the $\alpha(d,\gamma)^6$Li rate by ~7 !



Use of ab initio input in the analysis of indirect measurements :

→ Reconciliation of LUNA & JUNA S-factors for ^{13}C$(\alpha, n)^{16}$O

→ ^{12}C$(\alpha,\gamma)^{16}$O S-factor at stellar energies reduced by 21% !

Small uncertainties due to universal behavior in ^{6}Li :

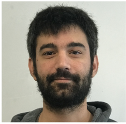
Is it present in other nuclei ? Up to which separation energies ?

How can we understand this universality ?

Thanks to my collaborators...

Lawrence Livermore National Laboratory


Sofia Quaglioni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TRIUMF


Petr Navratil


Peter Gysbers

IJCLab Irène Joliot-Curie


Guillaume Hupin

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Daniel Phillips


Carl Brune

And you for your attention ☺ !