北京大学物理学院
School of Physics, Peking University

HaloWeek'24, Chalmers University of Technology,
Gothenburg, June 10-14, 2024

Structure and decays of nuclei at and beyond driplines

Furong Xu

I. Gamow shell model based on realistic nuclear forces

The coupling to the continuum; 3NF

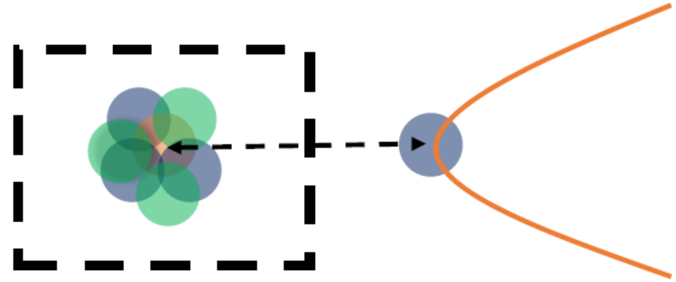
II. Calculations

Ground-state energy, spectrum, β decay, mirror symmetry breaking at driplines …

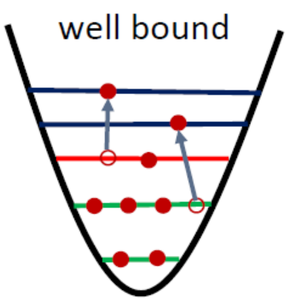
III. Conclusion

I. Gamow shell model with resonance and continuum

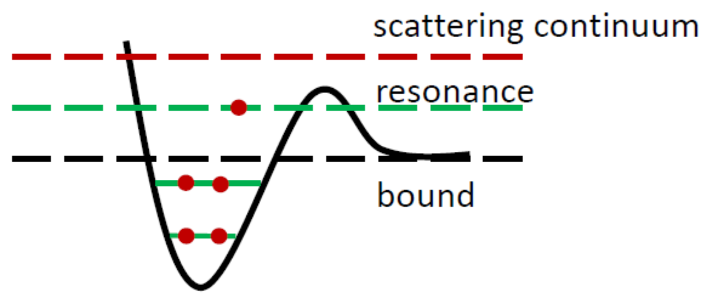
Closed quantum system



Open quantum system



HO basis



A time-dependent problem

One-body Berggren (Gamow) basis: T. Berggren, Nucl. Phys. A109 (1968) 265

A time-independent approach to the time-dependent problem

$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})$ **(Stationary)**

$[-\frac{\hbar^2}{2m}\nabla^2+\mathrm{V}(\mathbf{r})]\varphi_E(\boldsymbol{r})=\mathrm{E}\varphi_E(\boldsymbol{r})$

But E can be complex, and

the inner product: $\int\varphi_E(\boldsymbol{r})\varphi_E(\boldsymbol{r})=1$ (not the complex conjugate)

For a bound state, E is a negative real number

For continuum state, E is a positive real number

For a resonant state: $E=\frac{\hbar^2k^2}{2m}=E_R-i\frac{\Gamma}{2}$

$\psi(\boldsymbol{r},t)=e^{-iEt/\hbar}\varphi_E(\boldsymbol{r})=e^{-iE_nt/\hbar}\varphi_E(\boldsymbol{r})e^{-\Gamma\hbar t/2}$ $\quad T_{1/2}=\hbar\ln 2/\Gamma$

The Berggren ensemble provides a good basis for many-body calculations of nuclei as OQS's

The Berggren basis

Single-particle basis in the complex-*k* plane: bound, resonance and scattering on equal footing

The radial wave function $u(r)/r$

$$\frac{d^2 u(k,r)}{dr^2} = \left[\frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}U(r) - k^2\right]u(k,r)$$

boundary conditions

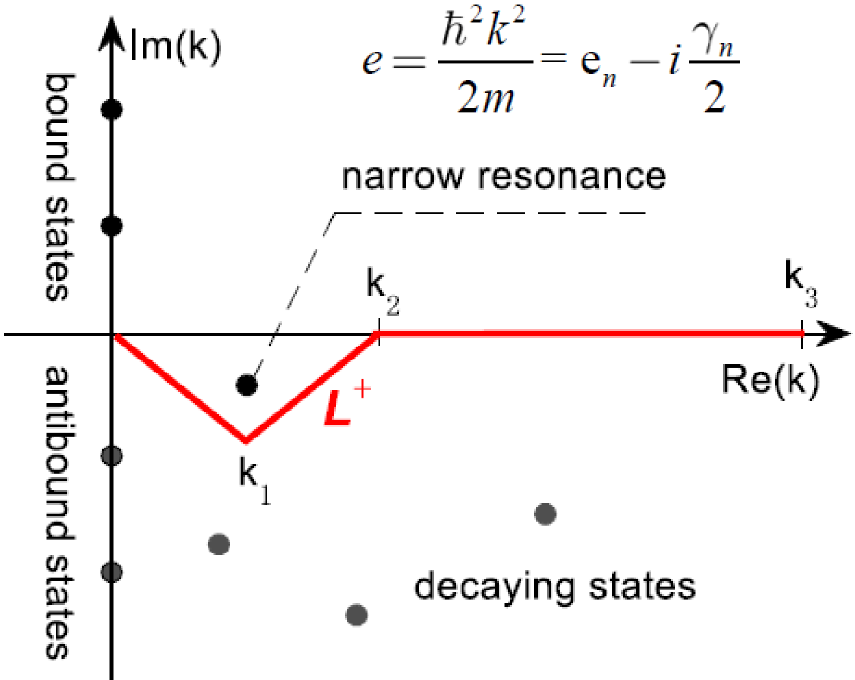
$$u(0) = 0,$$

$$u(a)O_l'(ka) - u'(a)O_l(ka) = 0$$

$$O_l(kr) \sim e^{i(kr - l\pi/2)}$$

Outgoing solution at large distance

$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$



$$u(k,r) \sim C^+ H^+_{l\eta}(kr) + C^- H^-_{l\eta}(kr), \qquad r \to +\infty$$

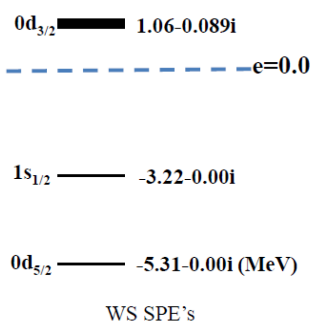
Asymptotically

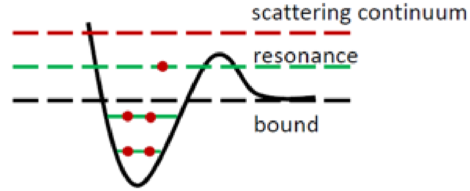
Orthogonality and Completeness

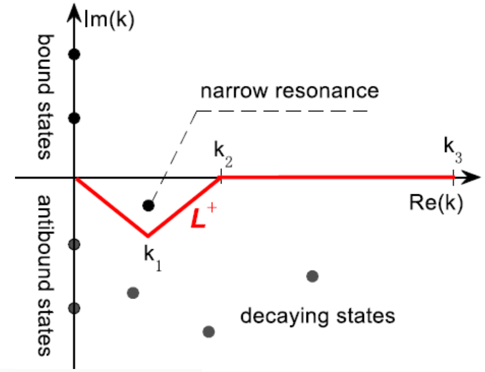
$$\delta(r - r') = \sum_n w_n(r, k_n) w_n(r', k_n) + \frac{1}{\pi}\int_{L^+} dq\, u(r,q) u(r',q)$$

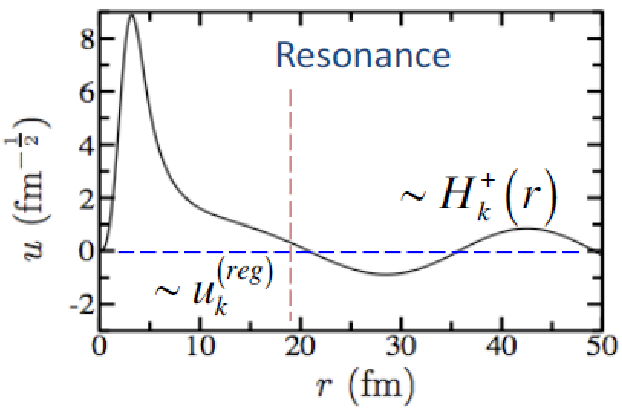
Discretized

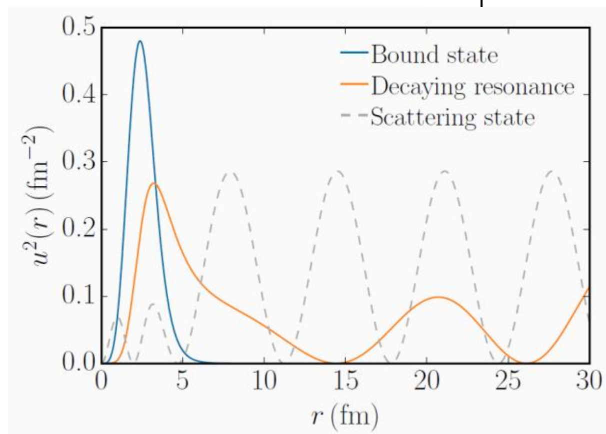
Complex-momentum space: **bound, resonance and continuum**











$$\int_0^\infty dr\, u^2_{\ell,\eta}(r) = 1$$

$$C^+(k)\,C^-(k) = \frac{1}{2\pi}$$

$$O_l(kr) \sim e^{\imath(kr - l\pi/2)}$$

In1968, Berggren used Zel'Dovich $e^{-\varepsilon r^2}$

In 1971, complex scaling:

$$U(\theta)\boldsymbol{r}_i U^{-1}(\theta) = \boldsymbol{r}_i\, e^{i\theta}, \qquad U(\theta)\boldsymbol{k}_i U^{-1}(\theta) = \boldsymbol{k}_i\, e^{-i\theta} \qquad U(\theta)\,U^{-1}(\theta) = 1$$

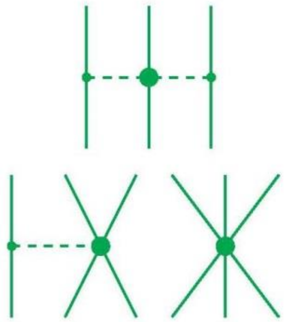
J. Aguilar and J. M. Combes, Comm. Math. Phys. 22, 269 (1971);

E. Balslev and J. M. Combes, Comm. Math. Phys. 22, 280 (1971)

Many-body problem: GSM within Berggren basis

$$H_{\rm int} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am} \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

$$\hat{H}_{int} = \sum_{i<j}^{A} \frac{(\vec{p}_i - \vec{p}_j)^2}{2mA} + \sum_{i<j}^{A} V_{NN,ij} + \sum_{i<j<k}^{A} V_{NNN,ijk}$$



The Gamow Shell Model

Two valence-particle systems, with phenomenological interactions:

1. R. Id Betan, R.J. Liotta, N. Sandulescu, T. Vertse, PRL 89, 042501 (2002) .
2. N. Michel, W. Nazarewicz, M. Płoszajczak, K. Bennaceur, PRL 89, 042502 (2002).

With realistic nuclear forces using the MBPT

1. G. Hagen, M. Hjorth-Jensen, N. Michel, PRC 73, 064307 (2006)
2. K. Tsukiyama, M. Hjorth-Jensen, G. Hagen, PRC 80, 051301(R) (2009)
3. G. Papadimitriou, J. Rotureau, N. Michel, M. Płoszajczak, B.R. Barrett, PRC 88, 044318 (2013): no-core Gamow shell model (very for light nuclei)
4. Z.H.Sun, Q.Wu, Z.H.Zhao, B.S.Hu, S.J.Dai, FRX, PLB 769, 227 (2017)

   Full Q-box folded diagrams

$$H=\sum_{i=1}^{A}\frac{p_i^2}{2m}+\sum_{i<j=1}^{A}v_{ij}^{NN}-\frac{\boldsymbol{P}^2}{2Am}=\sum_{i=1}^{A}\frac{p_i^2}{2m}+U+\sum_{i<j=1}^{A}\left(v_{ij}^{NN}-U-\frac{p_i^2}{2Am}-\frac{\boldsymbol{p}_i\boldsymbol{p}_j}{Am}\right)$$

$$=H_0+V. \qquad \boldsymbol{P}=\sum_{i=1}^{A}\boldsymbol{p}_i \qquad H_0=\sum_{i=1}^{A}\left(\frac{p_i^2}{2m}+U\right)$$

Complex valence-space effective Hamiltonian

Q-box

$$\hat{Q}(E)=PVP+PVQ\frac{1}{E-QHQ}QVP$$

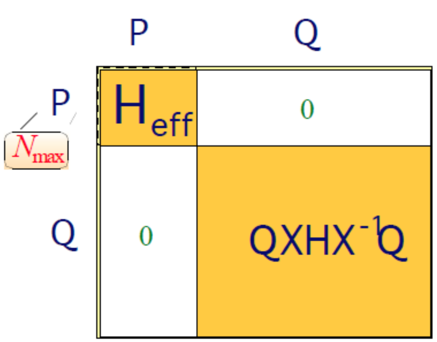
$$\hat{Q}(E)=PVP+PV\frac{Q}{E-QH_0Q}VP+PV\frac{Q}{E-QH_0Q}VP\frac{Q}{E-QH_0Q}VP+\ldots$$

2nd order perturbation   3rd order perturbation

Q-box folded diagrams

$$V_{eff}=\hat{Q}(\varepsilon_0)-\hat{Q}'(\varepsilon_0)\int\hat{Q}(\varepsilon_0)+\hat{Q}'(\varepsilon_0)\int\hat{Q}(\varepsilon_0)\int\hat{Q}(\varepsilon_0)\ldots.$$

$$V_{eff}=\hat{Q}(\varepsilon_0)+\sum_{k=1}^{\infty}\hat{Q}_k(\varepsilon_0)[V_{eff}]^k$$



Q-box derivatives

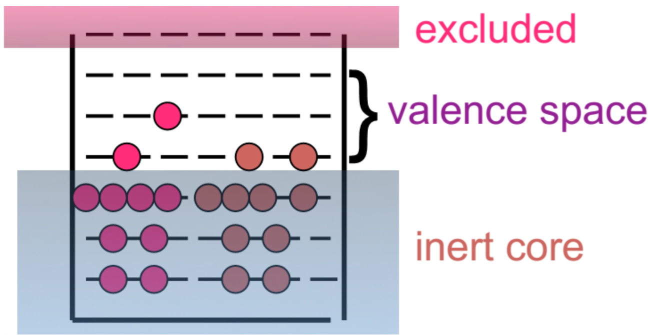
$$\hat{Q}_k(E)=\frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k}$$

$$=(-1)^kPVQ\frac{1}{(E-QHQ)^{k+1}}QVP$$

Iteration to include infinite-order folded diagrams

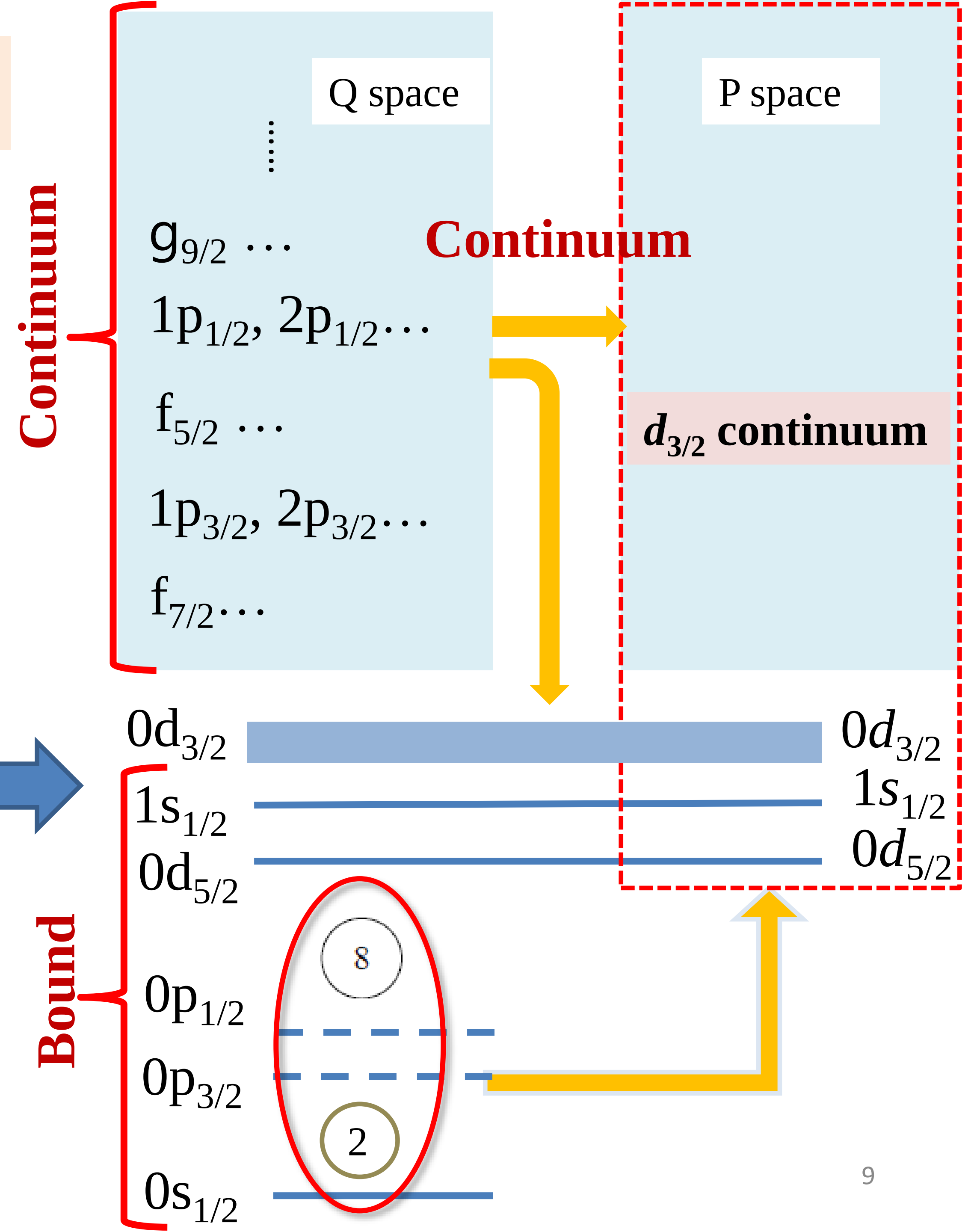
$$H_{\rm eff}^{(n)}=PH_0P+\hat{Q}(E)+\sum_{k=1}^{\infty}\frac{1}{k!}\frac{d^k\hat{Q}(E)}{dE^k}\{H_{\rm eff}^{(n-1)}-E\}^k$$

Valence-space effective Hamiltonian

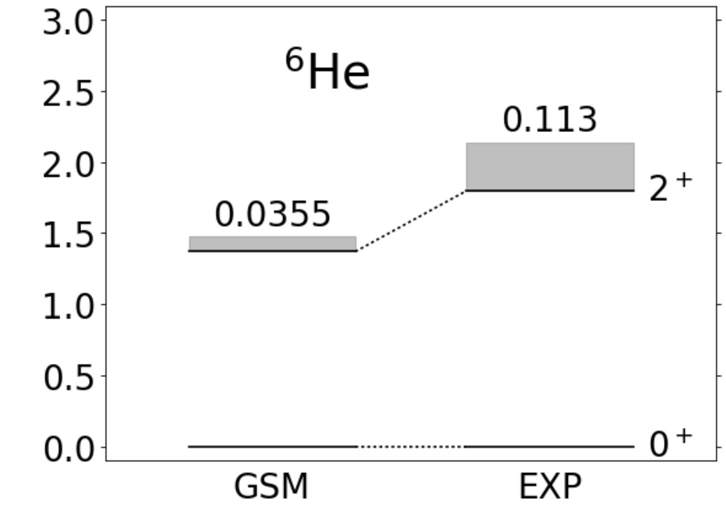


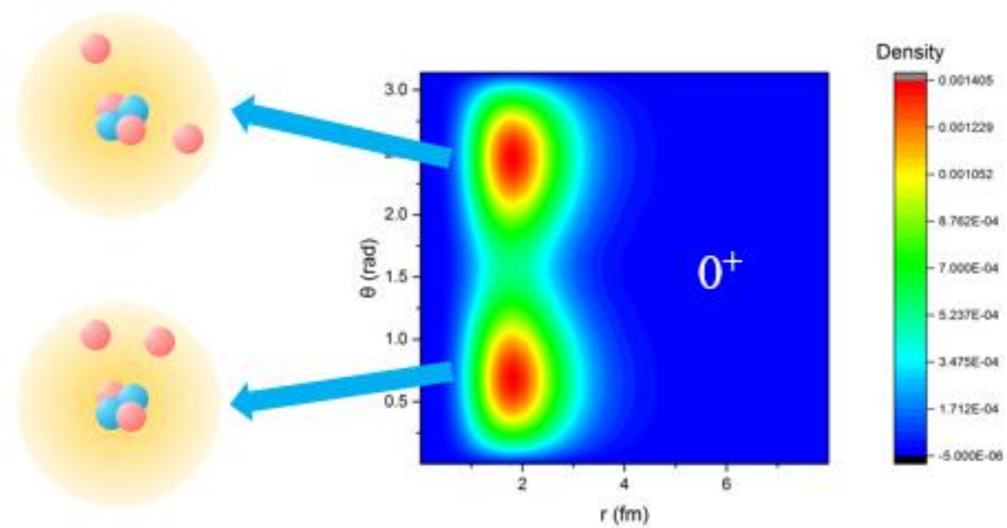
$$P + Q = 1$$

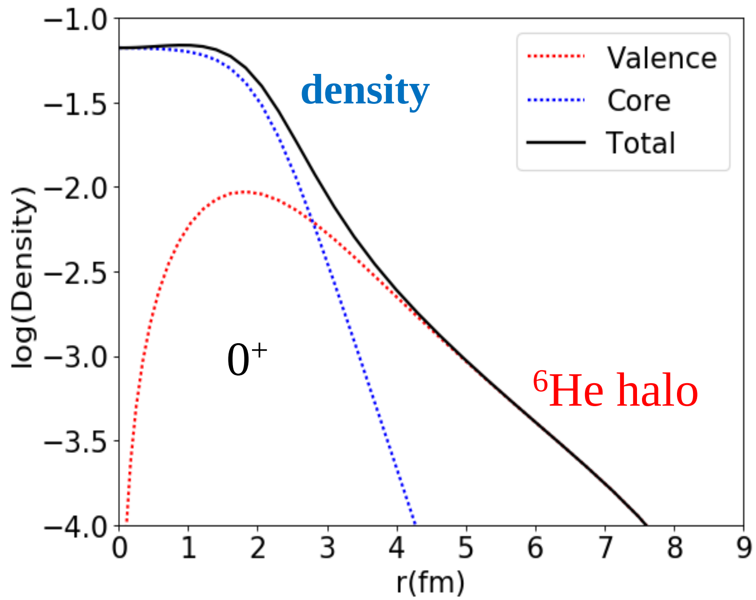
***S*-, *Q*-box folded diagrams in complex-*k* Berggren basis**



^{6}He halo





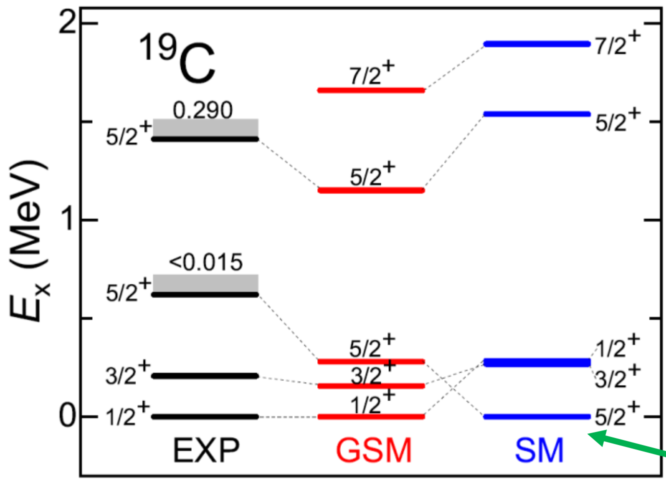


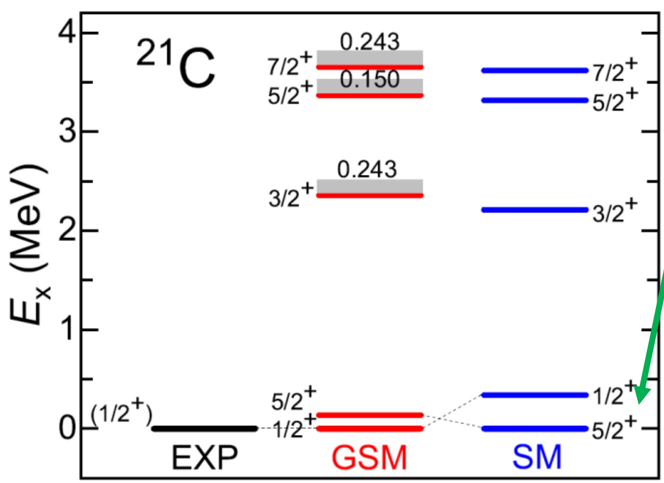
^{6}He correlated density distribution

$$\rho(r,\theta) = \langle \Psi | \delta(r_1 - r)\delta(r_2 - r)\delta(\theta_{12} - \theta) | \Psi \rangle$$

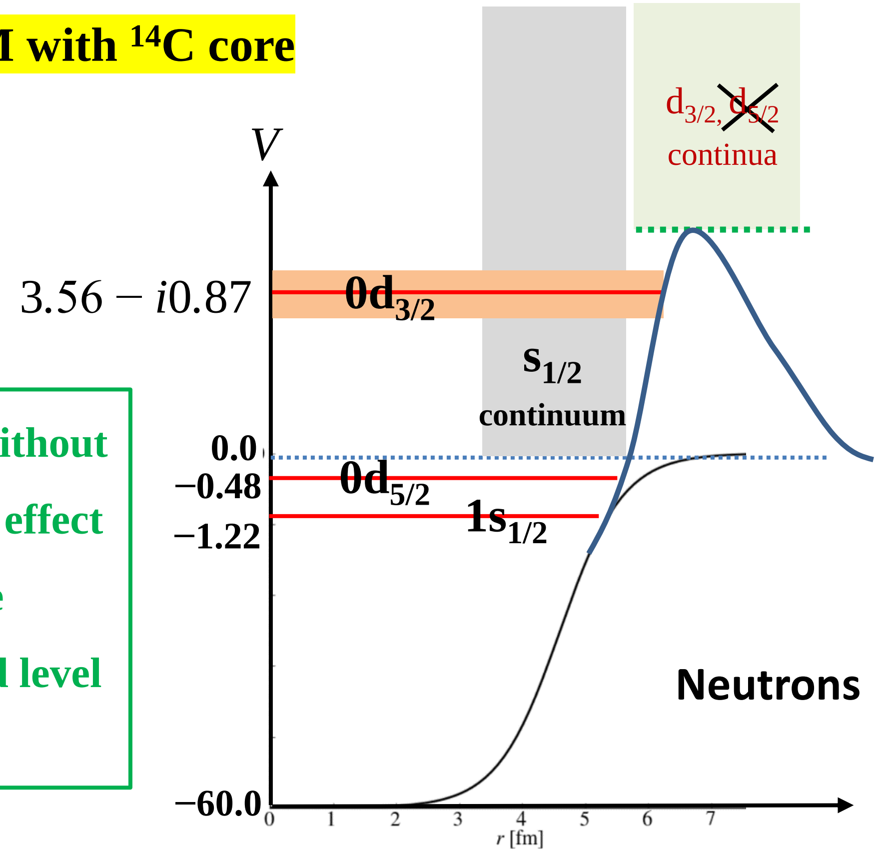
The continuum coupling is important in descriptions of carbon spectra, giving correct orders of levels

CD Bonn GSM with ^{14}C core





Calculations without the continuum effect cannot give the correct g.s. and level order.



GSM space:

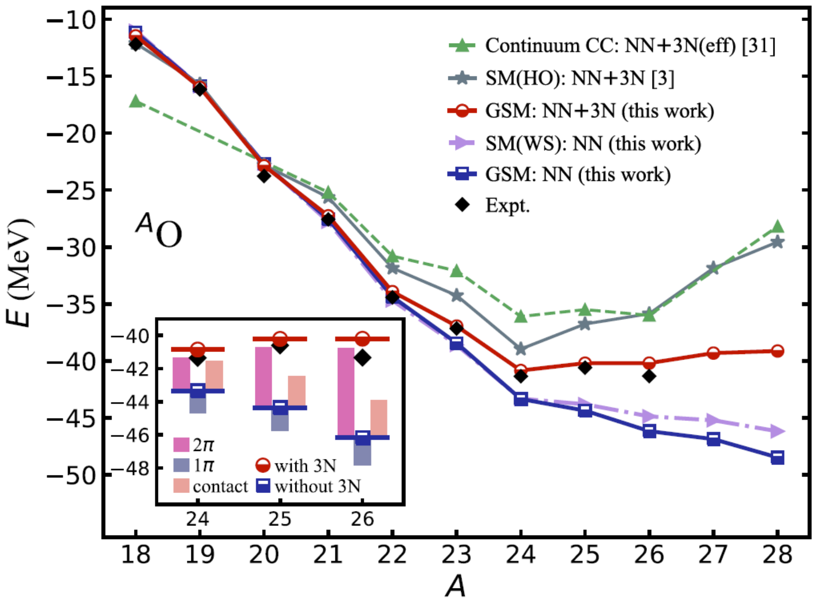
$0d_{5/2}$, $1s_{1/2}$+s-continuum, $0d_{3/2}$ (resonance)+$d_{3/2}$-continuum

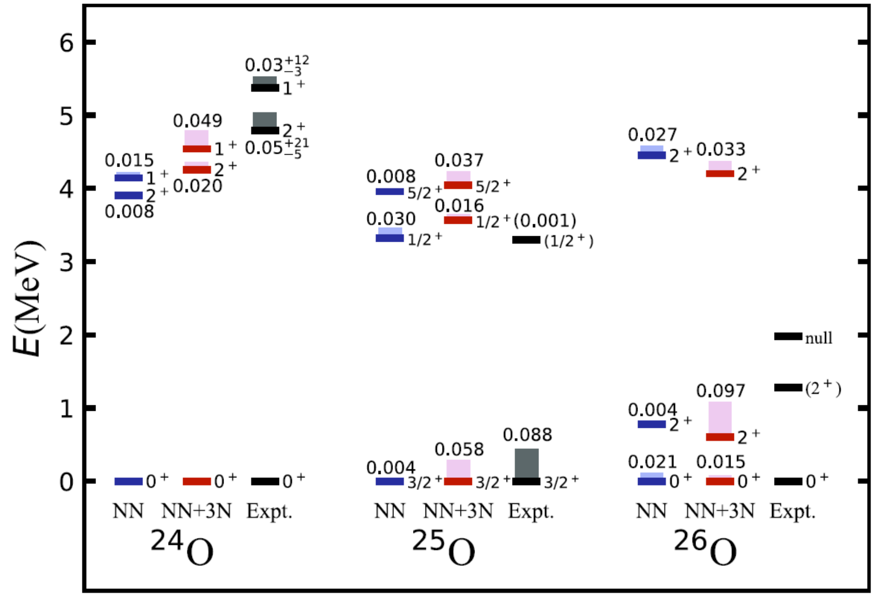
Y. F. Geng, J. G. Li, Y. Z. Ma, B. S. Hu, Q. Wu, Z. H. Sun, S. Zhang, and FRX, PRC 106, 024304 (2022)

GSM with ^{16}O core: N^{3}LO(NN) + N^{2}LO(NNN)

N^{3}LO(NN): Entem and Machleidt, PRC **66**, 014002 (2002)

N^{2}LO(NNN): c_D=-1, c_E=-0.34, P. Navrátil *et al.*, PRL 99, 042501 (2007)





[3] Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL 105 (2010) 032501

[31] Hagen, Hjorth-Jensen, Jansen, Machleidt, Papenbrock, PRL108 (2012) 242501

S_{2n}(MeV)	NN	NN+3N	Expt.
^{24}O	9.110	7.038	6.925
^{25}O	6.254	3.568	3.453
^{26}O	3.362	−0.150	−0.018

Y.Z. Ma, FRX *et al.*, PLB 802, 135257 (2020)

Gamow Hartree-Fock → complex S-box, Q-box → GSM with ^{16}O core

N3LO(NN) : D.R. Entem and R. Machleidt, Phys. Rev. C **66**, 014002(2002)

NNLO(NNN): c_D=-1, c_E=-0.34, Navrátil *et al*., PRL 99, 042501 (2007)

$h\omega$ = 14 MeV

$e = 2n + l \le e_{max} = 12$

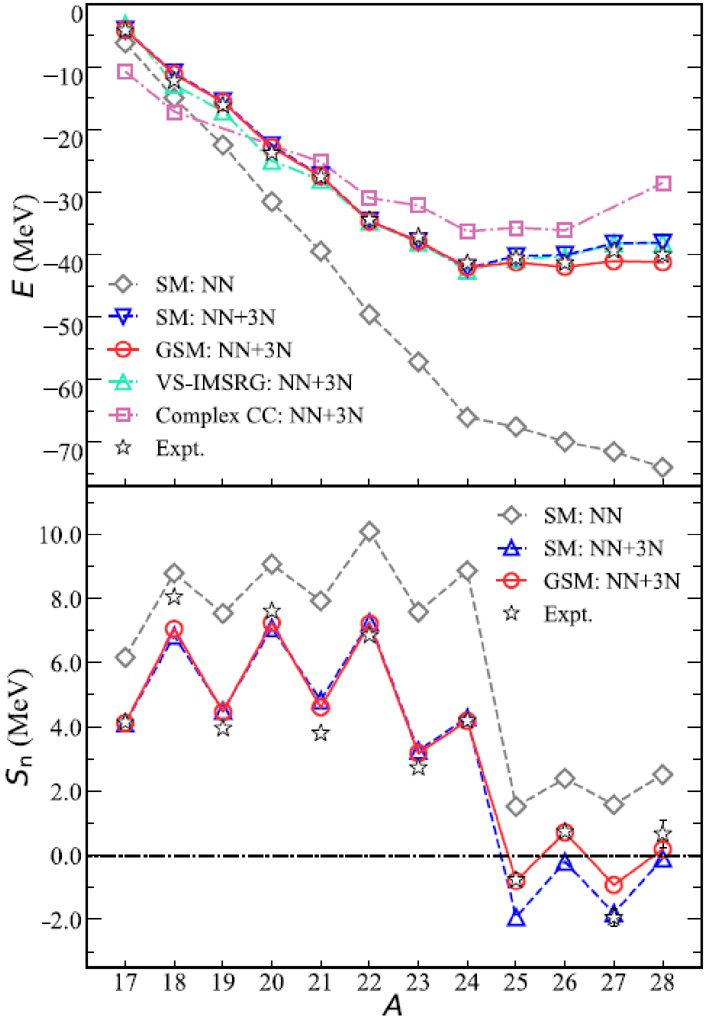
$\lambda_{SRG} = 2.3\ fm^{-1}$

For 3NF, $e_{3max} = 6$

$sdpf + \{d_{3/2}\ p_{3/2}\}$continua

AME2020: Wang *et al*., Chin. Phys. C 45 (3) (2021) 030003.

S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, FRX, PLB 827, 136958 (2022)

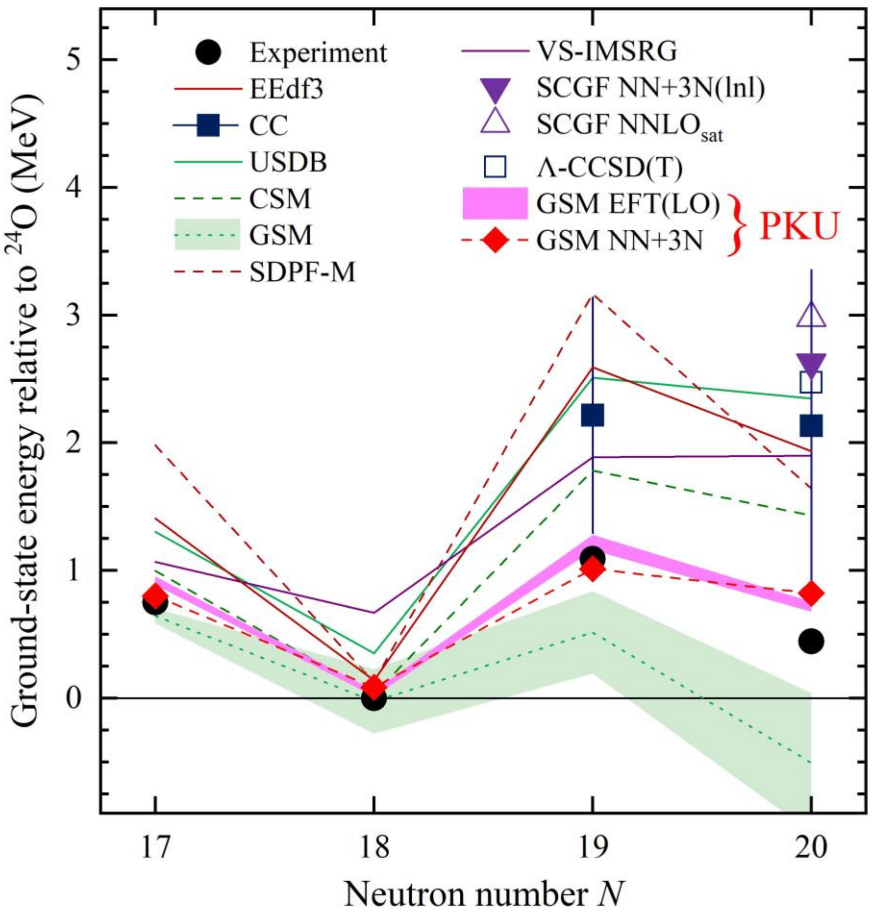


Relative to ^{24}O

	SM(2NF+3NF)	GSM(2NF+3NF)		EXP	
	$E_{g.s.}$(MeV)	$E_{g.s.}$(MeV)	Γ(keV)	$E_{g.s.}$(MeV)	Γ(keV)
^{25}O	1.924	0.801	84	0.749	88
^{26}O	2.124	0.086	327	0.018	~ 0
^{27}O	3.93	1.01	463	1.09	≤ 180
^{28}O	4.03	0.82	357	0.46	≤ 700

Y. Kondo *et al*., Nature 620, 965 (2023)

S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, FRX, PLB 827, 136958 (2022)

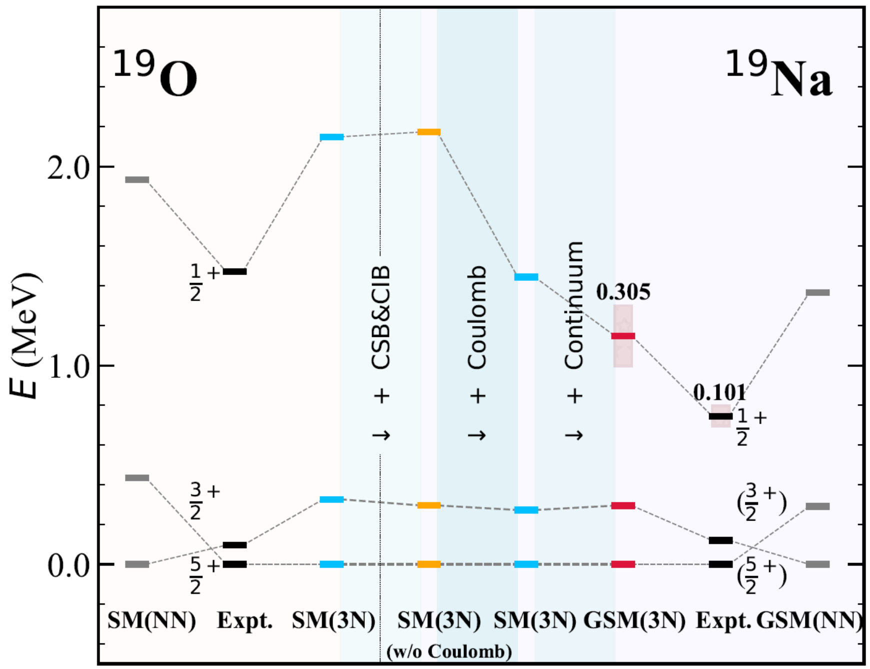


Y. Kondo *et al*., Nature 620, 965 (2023)

S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, FRX, PLB 827, 136958 (2022)

J.G. Li, N. Michel, W. Zuo, FRX, PRC 103, 034305 (2021)

Thomas-Ehrman shift



CIB: charge independence breaking, a violation of rotation invariance in isospin space.

T=1 channel of the interaction: Tz=+1 (pp), 0 (np) and -1 (nn)

The main reasons: $m_p \neq m_n$, π^0, $\pi^{\pm}$ **mass splitting**

CSB: charge symmetry breaking, a violation of rotation invariance by 180° (only for *pp* and *nn)*

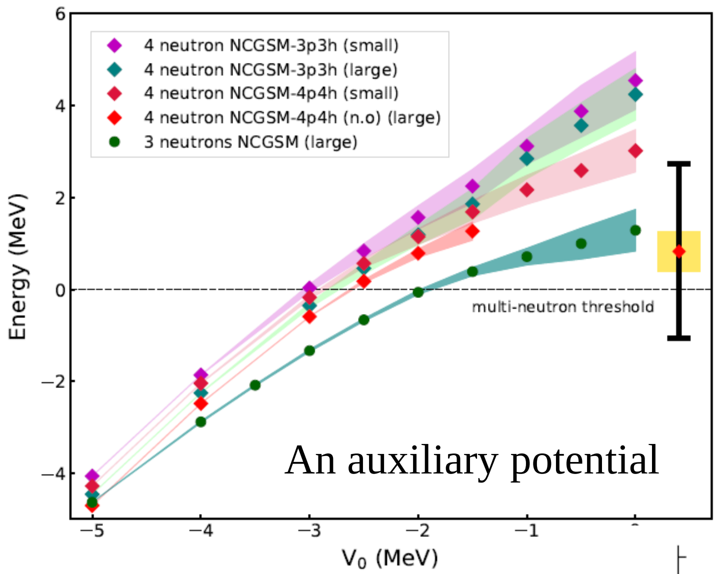
CIB is more significant than CSB

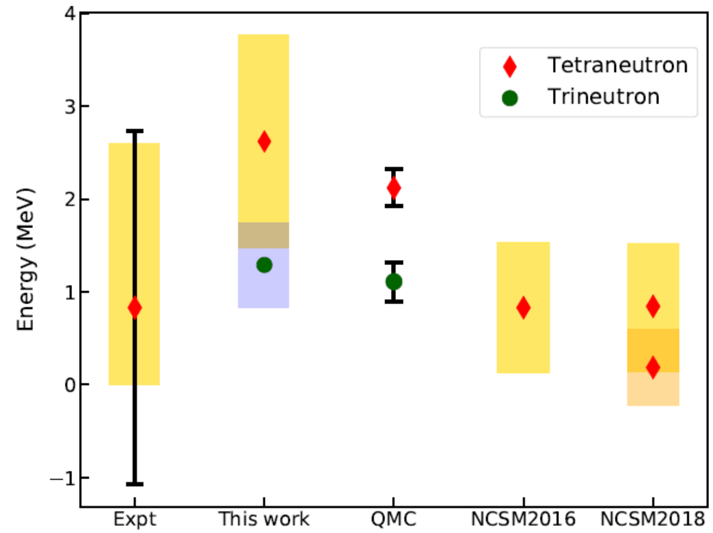
S. Zhang, Y.Z. Ma, J.G. Li, B.S. Hu, Q. Yuan, Z.H. Cheng, FRX, PLB 827, 136958 (2022)

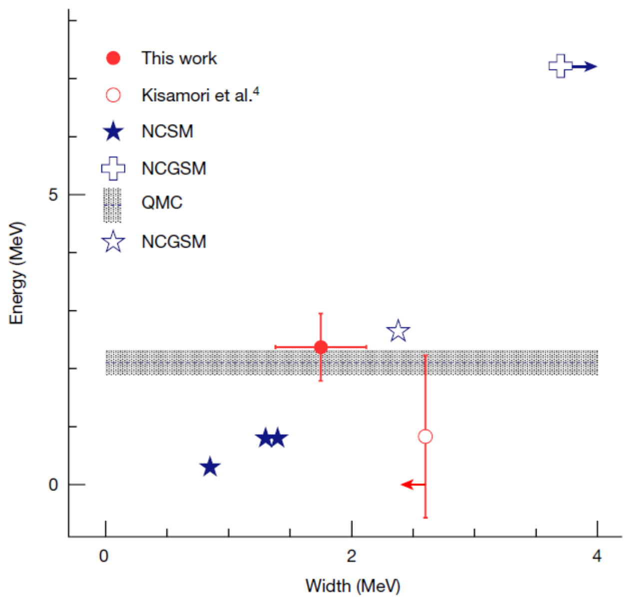
Tetraneutron (4n) and Trineutron (3n)

NCGSM, N3LO ($V_{low\text{-}k}$ Λ=2.1 fm^{-1}), N_{max}=2n+l=20, natural orbitals (n.o.)

J. G. Li, N. Michel, B. S. Hu, W. Zuo, F. R. Xu, PRC 100, 054313 (2019)







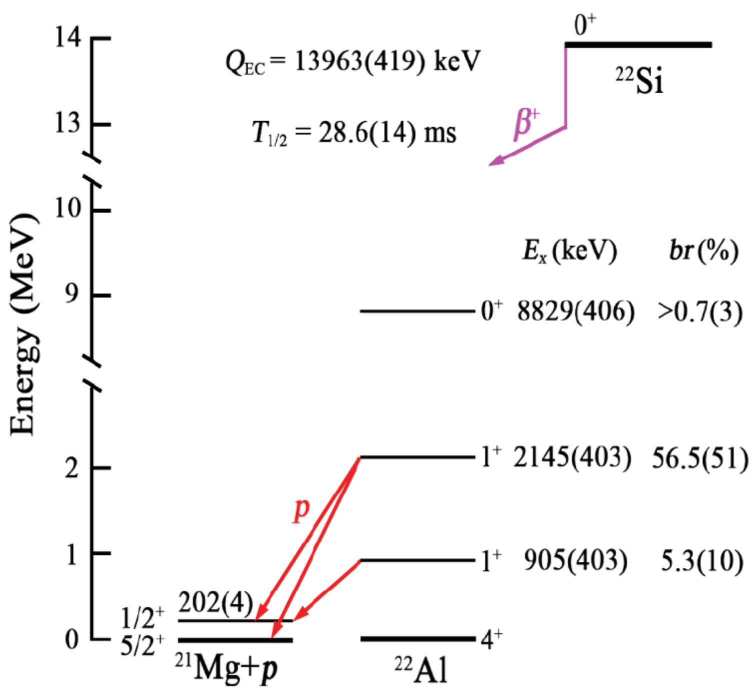
M. Duer *et al.*, Nature 606, 678 (2022)

GT transitions at driplines

PHYSICAL REVIEW LETTERS **125**, 192503 (2020)

Large Isospin Asymmetry in $^{22}Si/^{22}O$ Mirror Gamow-Teller Transitions Reveals the Halo Structure of ^{22}Al

J. Lee (李晓菁),[1,*] X. X. Xu (徐新星),[1,2,3,4,5,†] K. Kaneko (金子和也),[6] Y. Sun (孙扬),[7,2,3,‡] C. J. Lin (林承键),[3,8,§]



GT β-decay calculation

Z.C. Xu *et al.*, PRC **108**, L031301 (2023)

GSM with 3NF included, using MBPT (S-box, Q-box, Θ-box)

Free-space bare transition operator of GT: $\mathcal{O}(\mathrm{GT}_\pm) = \sum_j \sigma^j \tau_\pm^j$ $\quad M_{fi}^{\mathrm{GT}} = \langle f | \mathcal{O}(\mathrm{GT}_\pm) | i \rangle$

Θ-box perturbation (up to the third order) to obtain valence-space effective GT matrix elements M_{GT}^{pq}

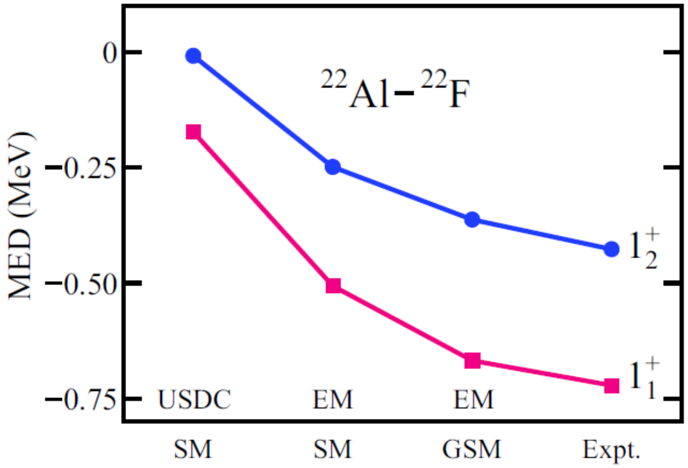
$$M_{\mathrm{GT}} = \sum_{p,q \in \text{valence space}} M_{\mathrm{GT}}^{pq} \langle \psi_f \| [\hat{a}_p^\dagger \hat{a}_q] \| \psi_i \rangle \qquad ft = \frac{D}{\left(\frac{g_A}{g_V}\right)_{\mathrm{eff}}^2 |M_{\mathrm{GT}}|^2}$$

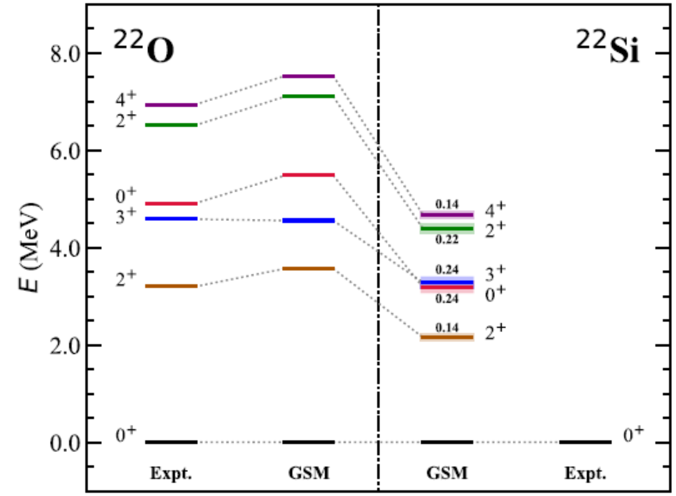
Quenching factor by two-body currents

$$q = 1 - \frac{\rho}{F_\pi^2}\left[\frac{c_4}{3}\left[3I_2^\sigma(\rho, |\mathbf{q}|) - I_1^\sigma(\rho, |\mathbf{q}|)\right] - \frac{1}{3}\left(c_3 - \frac{1}{4m_N}\right)I_1^\sigma(\rho, |\mathbf{q}|) - \frac{c_6}{12}I_{c6}(\rho, |\mathbf{q}|) - \frac{c_D}{4g_A\Lambda_\chi}\right]$$

$\rho = 0.10\ \mathrm{fm}^{-3}$ $\qquad q = 0.78$

EM1.8/2.0





GT transition matrix elements $|M_{GT}|$

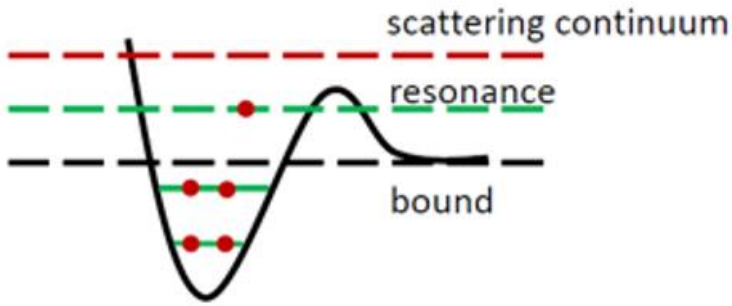
		SM		GSM	Ref. [54]	
		USDC	EM	EM	Expt.	Cal.
$^{22}Si \rightarrow {}^{22}Al$	1_1^+	0.236	0.343	0.257	0.176(16)	0.242
	1_2^+	0.721	1.042	1.012	0.750(41)	0.863
$^{22}O \rightarrow {}^{22}F$	1_1^+	0.198	0.569	0.497	0.310(32)	0.428
	1_2^+	0.719	1.092	1.068	0.775(77)	0.848

[54] J. Lee *et al.*, PRL 125, 192503 (2020)

Z.C. Xu, S. Zhang, J.G. Li, S.L.Jin, Q. Yuan, Z.H. Cheng, N. Michel, and FRX, PRC 108, **L**031301 (2023)

III. Summary

Berggren complex space



***Ab initio* nuclear structure calculations with resonance and continuum**

1. **Complex-space 2NF + 3NF**

2. **Gamow shell model**

Complex-space: *S*-box, *Q*-box, Θ-box folded diagrams

3. Both continuum coupling and 3NF are important for nuclei around driplines

In progress, computing the radii of nuclei: need “better” EFT forces?

Thank you for your attention

HaloWeek'24

Chalmers University of Technology

Gothenburg, June 10-14, 2024

J.G. Li, S. Zhang, Z.C. Xu, Y.Z. Ma, Y.F. Geng, Z.H. Cheng, B.S. Hu, Z.H. Sun, Q. Wu, N. Michel, L. Coraggio, T. Fukui, L. DeAngelis, N. Itaco, A. Gargano