The beryllium isotopic chain: evolution of structure in neutron rich nuclei

Anna E. McCoy

HaloWeek'24
Gothenburg,Sweden
June 13, 2024

No-core shell model

Solve many-body Schrodinger equation

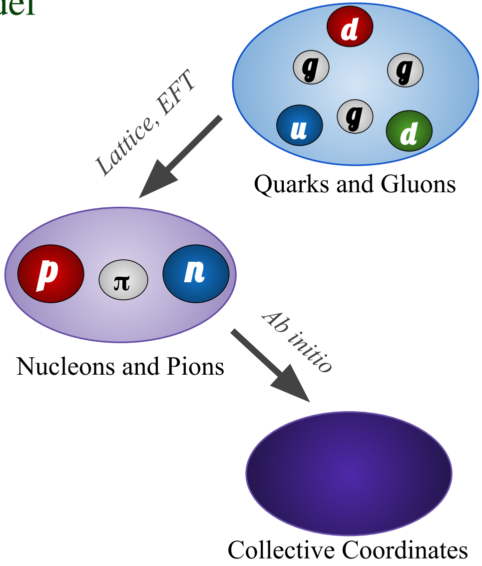
$$\sum_{i}^{A} -\frac{\hbar^2}{2m_i}\nabla_i^2 \Psi + \frac{1}{2}\sum_{i,j=1}^{A} V(|r_i - r_j|)\Psi = E\Psi$$

Expanding wavefunctions in a basis

$$\Psi = \sum_{k=1}^{\infty} a_k \phi_k$$

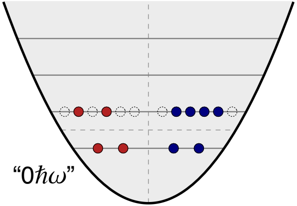
Reduces to matrix eigenproblem

$$\begin{pmatrix} H_{11} & H_{12} & \dots \\ H_{21} & H_{22} & \dots \\ \vdots & \vdots & \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} = E\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix}$$



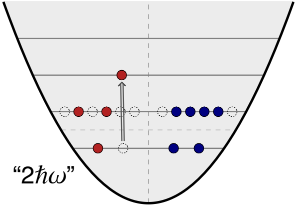
Harmonic oscillator basis

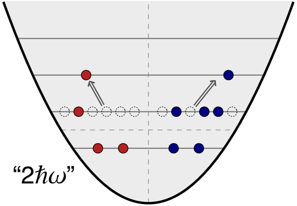
$N = 2n + l$



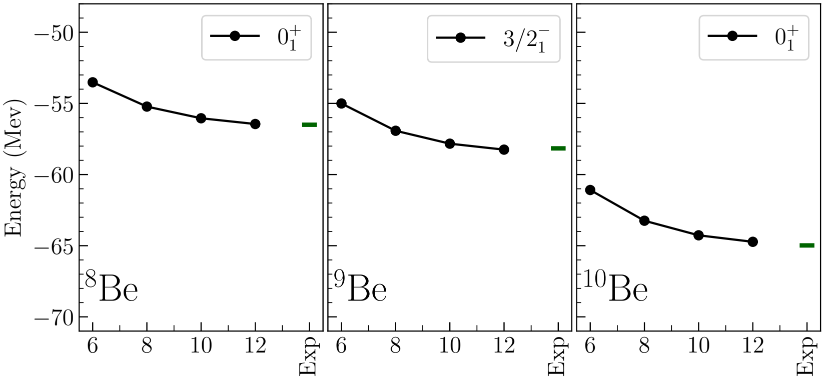
- Basis states are configurations, i.e., distributions of particles over harmonic oscillator shells (*nlj substates*)
- States are organized by total number of oscillator quanta above the lowest Pauli allowed number N_{ex}
- States with higher N_{ex} contribute less to the wavefunction
- Basis must be truncated:
  Restrict $N_{\mathrm{ex}} \leq N_{\mathrm{max}}$

Want results that are approximately independent of N_{max}





Binding energies



Beryllium isotopes

- Beryllium isotopes have well known 2α cluster structure
  See Dean Lee's talk
- Appearance of halo nuclei: ^{11}Be, ^{14}Be, others?
- Highly deformed states

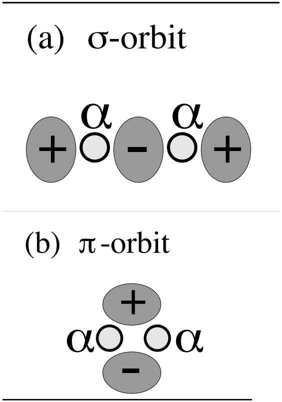
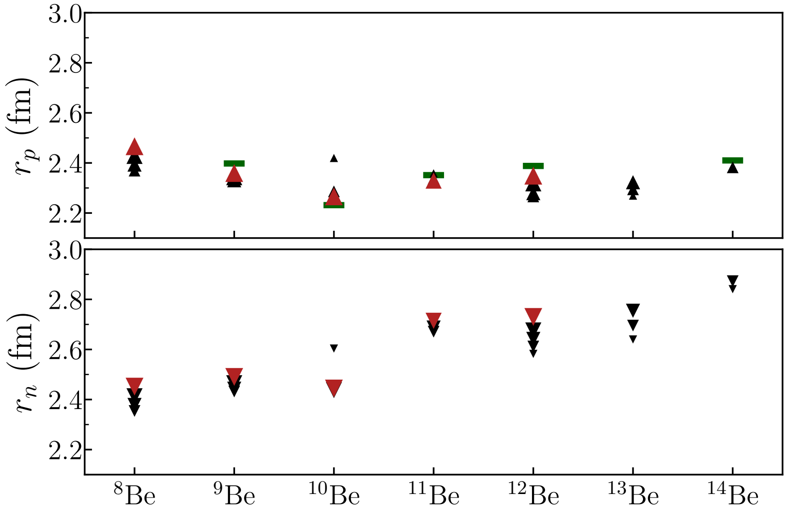


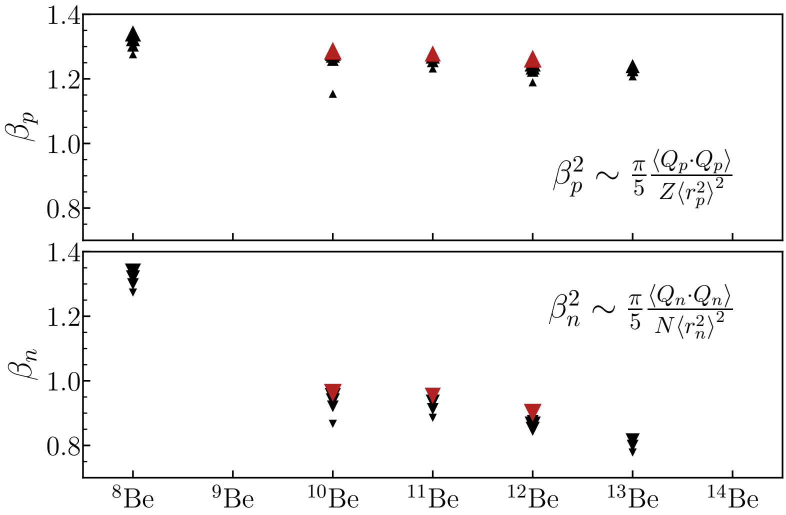
Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

Radii



Quadrupole deformation

D. J. Rowe. Rep. Prog. Phys. **48**(1985) 1419.



Beryllium isotopes

- Beryllium isotopes have well known 2α cluster structure
  See Dean Lee's talk
- Appearance of halo nuclei: ^{11}Be, ^{14}Be, others?
- Highly deformed states
- Exhibit rotational dynamics

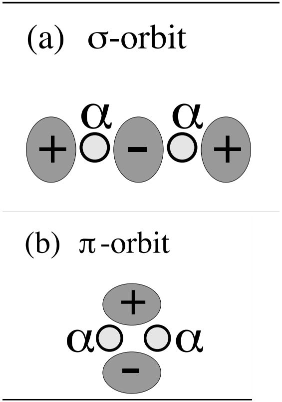


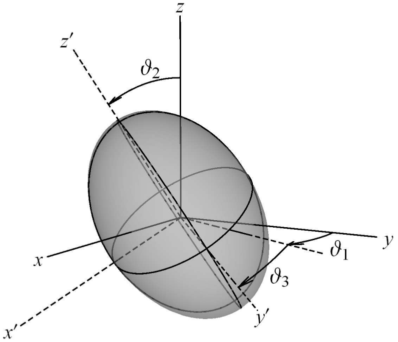
Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

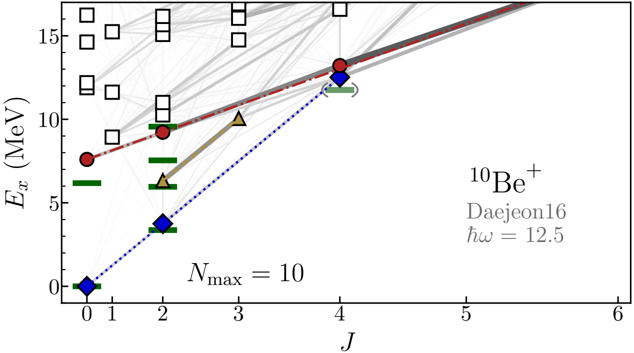
Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $(J = K, K+1, \ldots)$

$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy: $E(J) = E_0 + A[J(J+1)]$



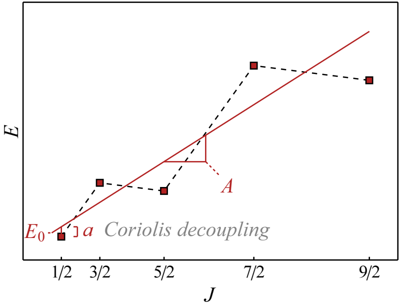


Nuclear rotations

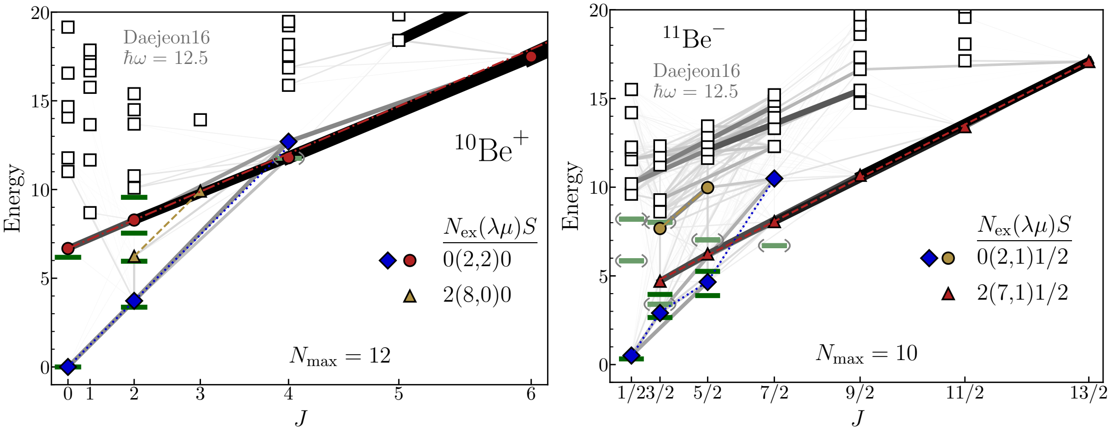
Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy: $E(J) = E_0 + A[J(J+1)] + \underbrace{a(-)^{J+1/2}(J+\frac{1}{2})}_{\text{Coriolis}\,(K=1/2)}$



$^{10}\mathrm{Be} + n$



Beryllium isotopes

- Beryllium isotopes have well known 2α cluster structure
  See Dean Lee's talk
- Appearance of halo nuclei: ^{11}Be, ^{14}Be, others?
- Highly deformed states
- Exhibit rotational dynamics
- Intruder states and and island of inversion around $N = 8$

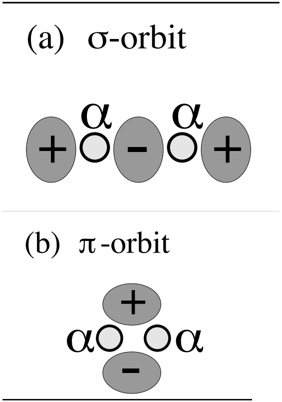
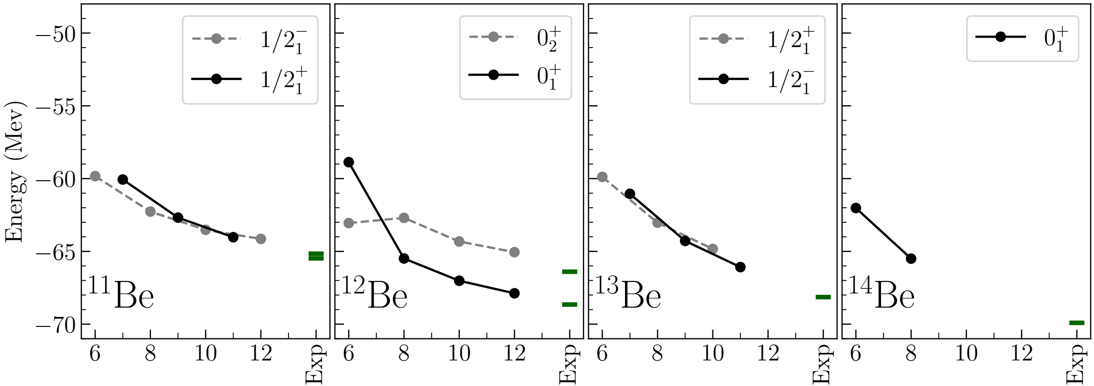
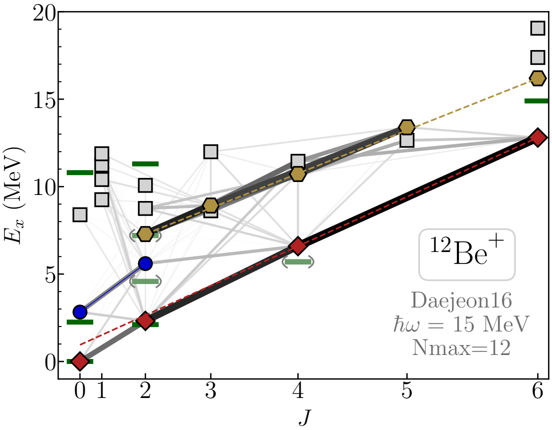


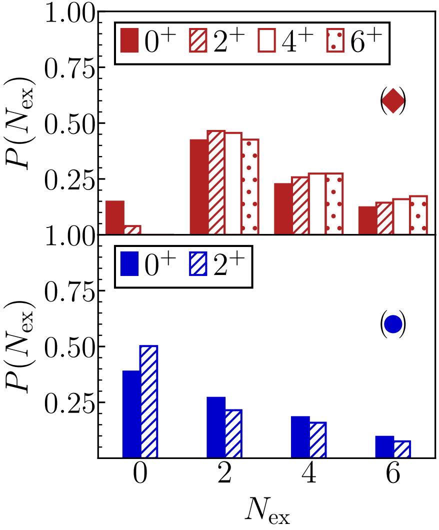
Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

Binding energies



Intruder ground state in ^{12}Be





A. E. McCoy, M. A. Caprio, P. Maris, P. J. Fasano arXiv:2402.12606.

Beryllium isotopes

- Beryllium isotopes have well known 2α cluster structure
  See Dean Lee's talk
- Appearance of halo nuclei: ^{11}Be, ^{14}Be, others?
- Highly deformed states
- Exhibit rotational dynamics
- Intruder states and and island of inversion around $N = 8$
- Shape coexistence
  Bands with very different moments of inertia
  Different deformation

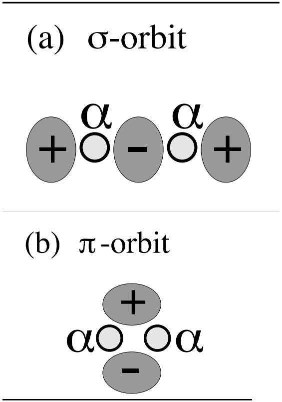
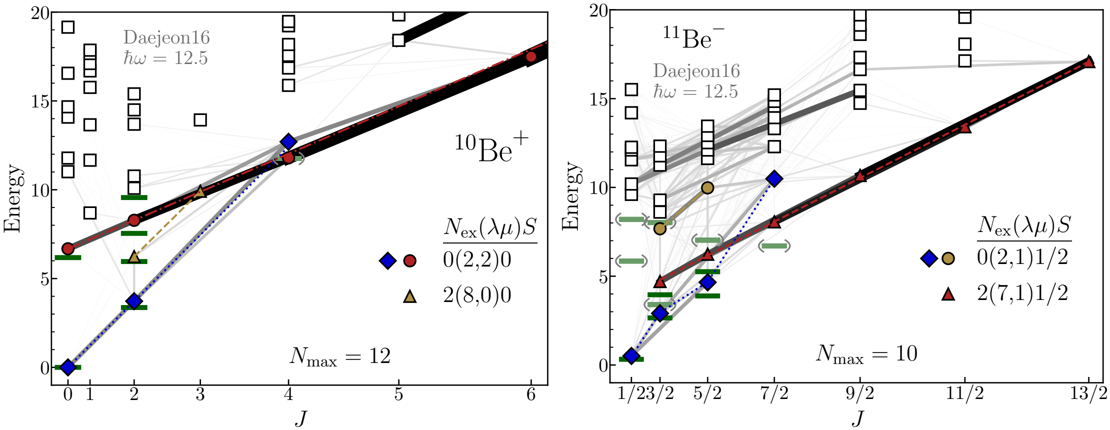
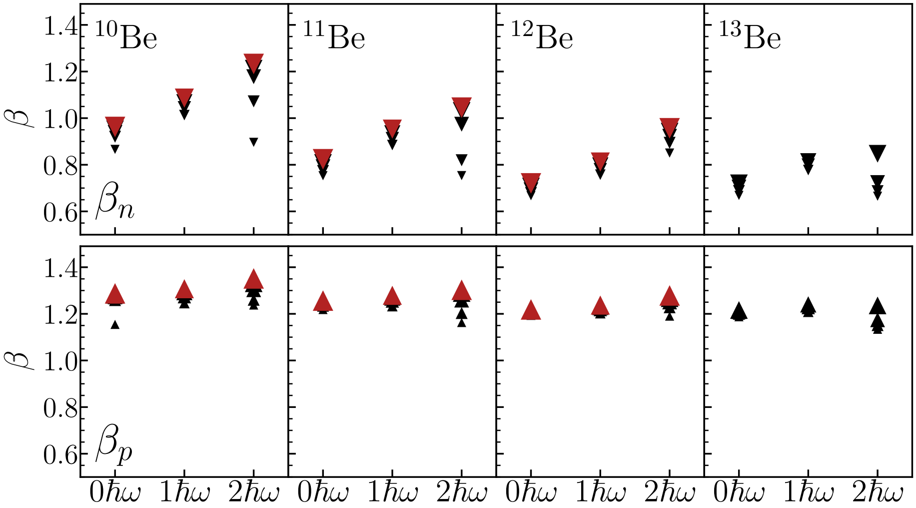


Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

$^{10}\mathrm{Be} + n$



Quadrupole deformation



Beryllium isotopes

- Beryllium isotopes have well known 2α cluster structure
  See Dean Lee's talk
- Appearance of halo nuclei: ^{11}Be, ^{14}Be, others?
- Highly deformed states
- Exhibit rotational dynamics
- Intruder states and and island of inversion around $N = 8$
- Shape coexistence
  Bands with very different moments of inertia
  Different deformation

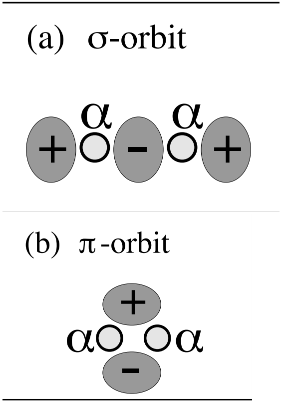


Figure: Y. Kanada-En'yo and H. Horiuchi. Phys. Rev. C **68** (2003) 014319.

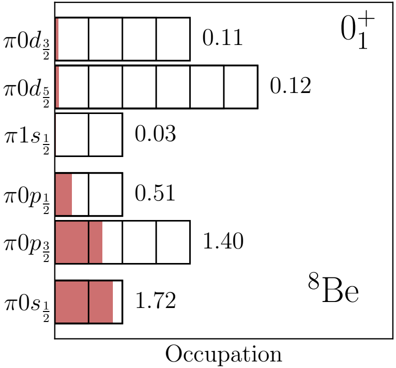
Effective single particle picture

– Many different ways to choose single particle basis

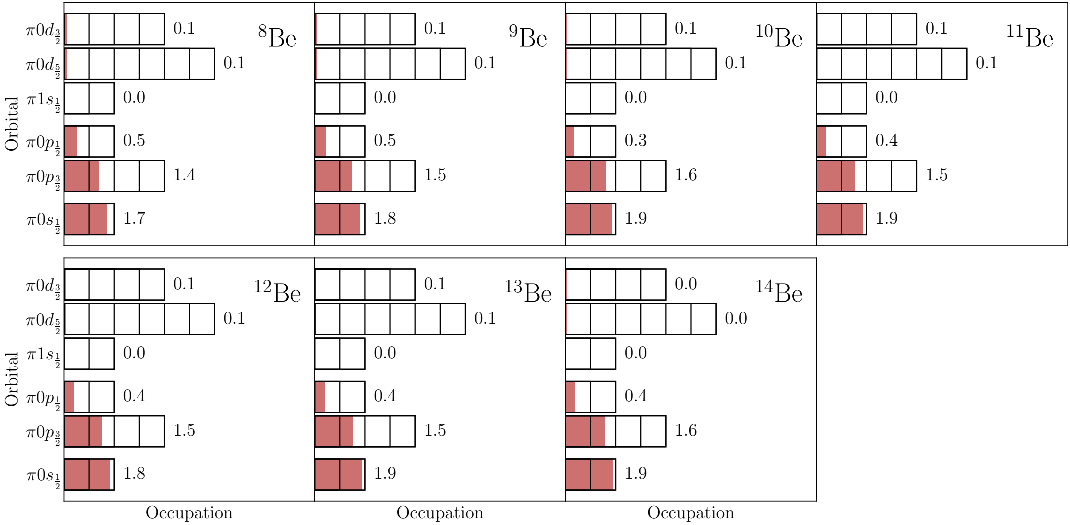
– Natural orbitals obtained by diagonalizing the density matrix

$$\hat{\rho} = \sum_{\alpha\beta} |\alpha\rangle \langle \Psi | a^\dagger_\alpha a_\beta | \Psi \rangle \langle \beta |$$

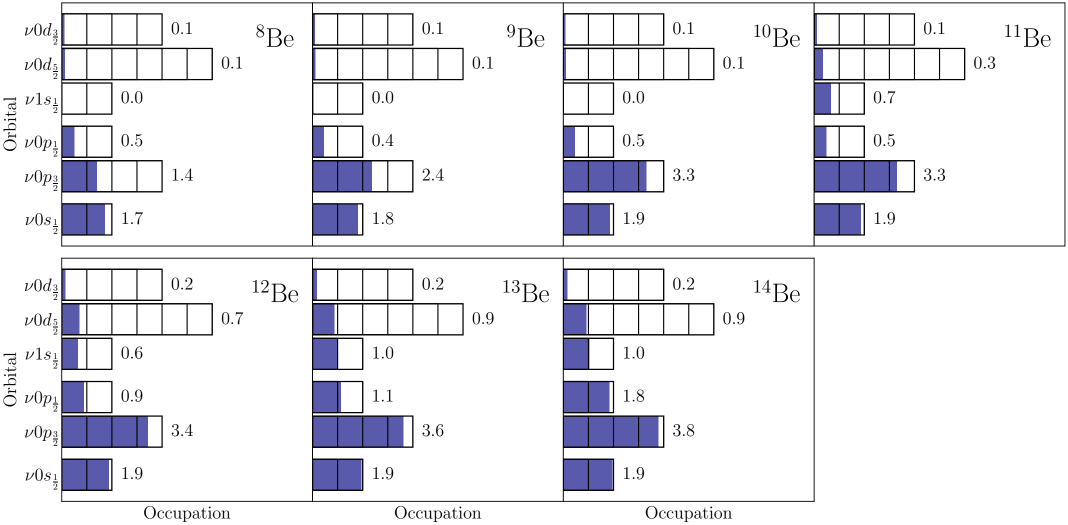
– Maximize occupation number of lowest orbitals



Natural Orbital Occupations

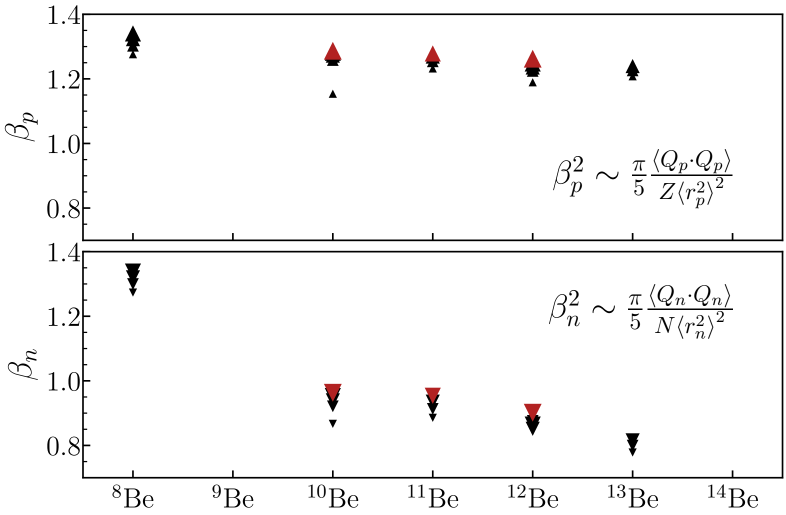


Natural Orbital Occupations



Quadrupole deformation

D. J. Rowe. Rep. Prog. Phys. **48**(1985) 1419.



Nilsson Model

Wood Saxon potential

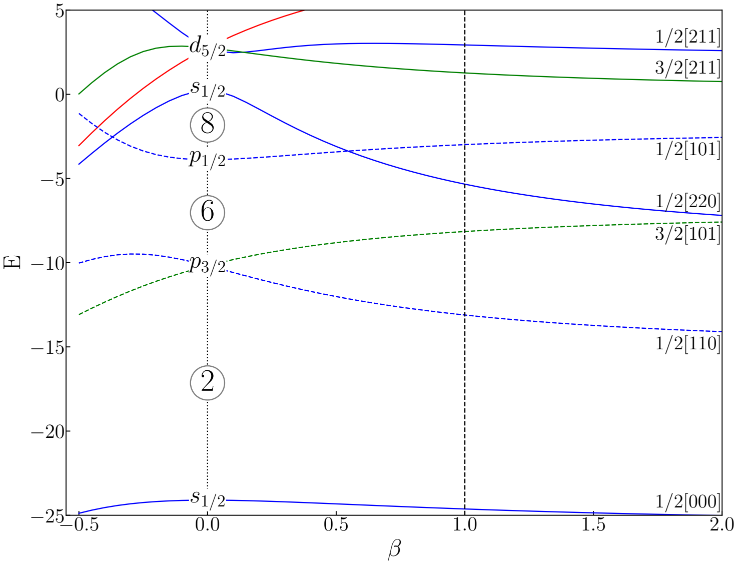
$$V_{WS} = \frac{-V_0}{1 + e^{(r-R)/a}},$$

$a = 0.67\,\text{fm},\ V_0 = 57\,\text{MeV},$
$R = 1.27(A = 8)^{1/3}\ \text{fm}.$

Nilsson Hamiltonian

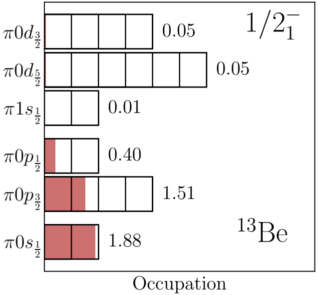
$$H = V_{WS} + \beta\hbar\omega r^2 Y_{20}$$

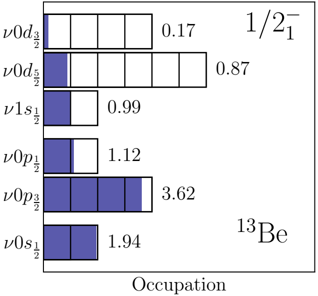
$\hbar\omega = 12.5, \beta = 1$

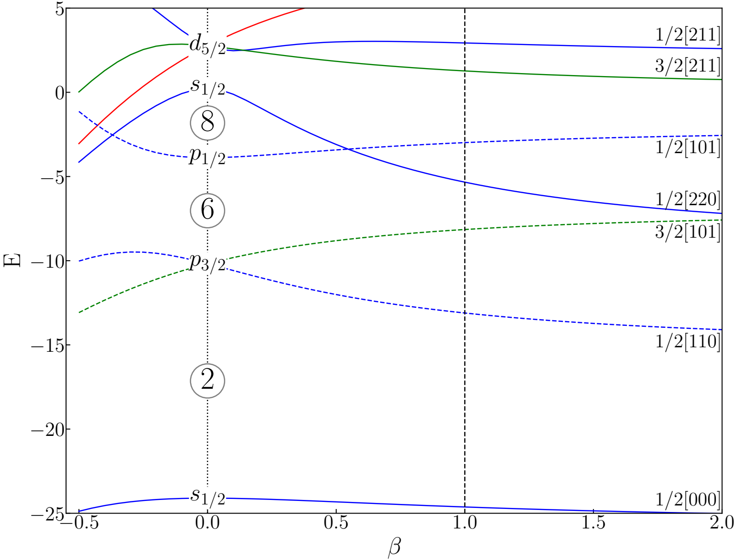


Wood Saxon parameters: J. Suhonen. From Nucleons to Nucl
Concepts of Microscopic Nuclear Theory, Chapter 3.

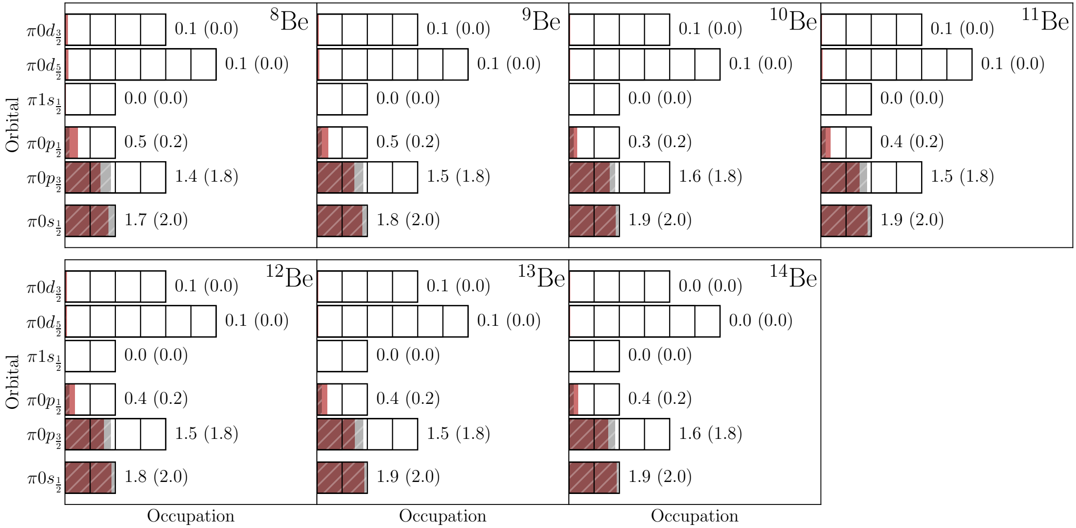
Nilsson Model



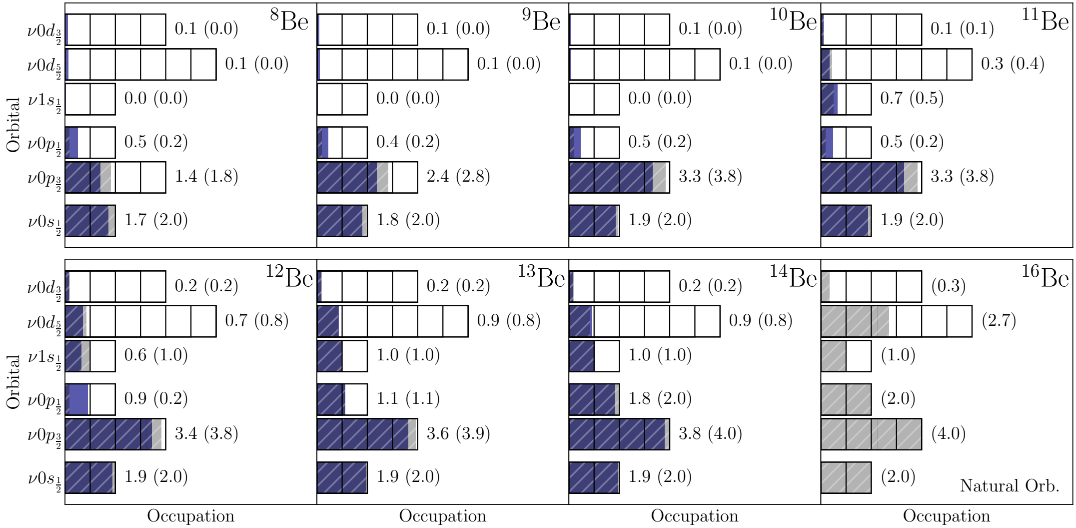




Natural Orbital Occupations



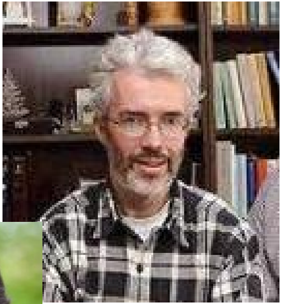
Natural Orbital Occupations



Acknowledgements

In collaboration with...

Patrick Fasano *ANL*
Mark Caprio *Univ. Notre Dame*
Pieter Maris *Iowa State Univ.*



Summary

– Calculated energies and proton radii for beryllium isotopes are in reasonable agreement with experiment.

– Proton deformation is similar across the beryllium isotopic chain. Neutron deformation is decreasing with increasing N, but is remarkably similar in $^{10-12}$Be.
Are similar patterns realized in other isotopic chains?

– We observe rotational dynamics, shape coexistence and an island of inversion.

– Occupations of single particle natural orbitals are qualitatively consistent with naive filling of Nilsson orbitals.
But we only considered one interaction here. How robust are these occupations?

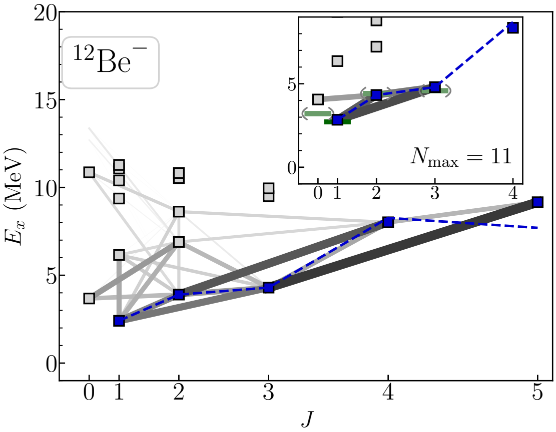
Nuclear rotations

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

$$|\psi_{JKM}\rangle \propto \int d\vartheta \Big[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \Big]$$

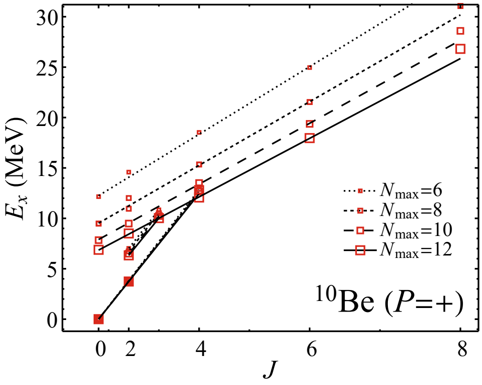
Rotational energy:

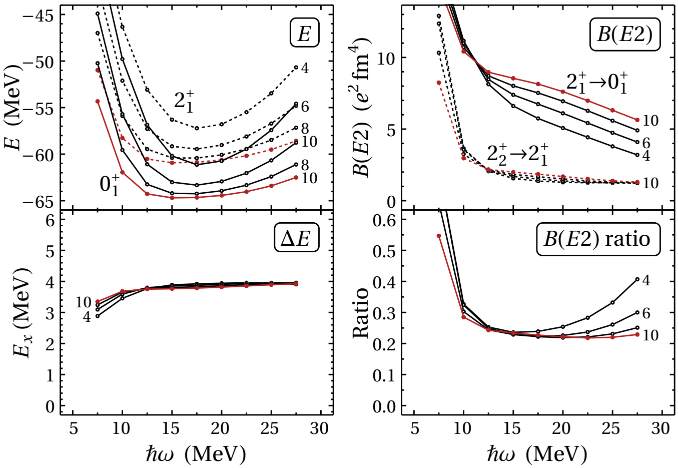
$$E(J) = E_0 + A[J(J+1)] + \underbrace{b(-)^{J+1}J(J+1)}_{\text{Coriolis}\,(K=1)}$$



Bohr and Mottelson Vol 2

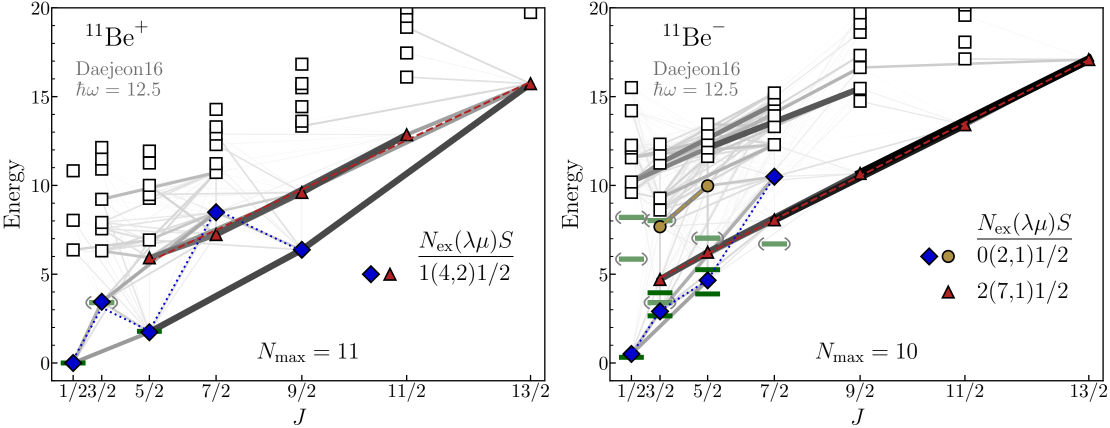
Robustness of band properties



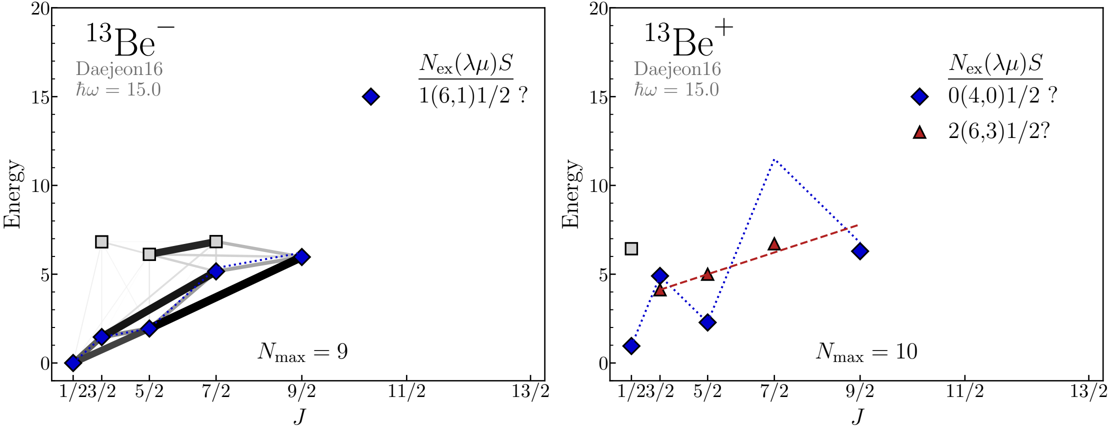


M. A. Caprio, P. J. Fasano, A. E. McCoy, P. Maris, J. P. Vary, Bulg. J. Phys. **46**, 455 (2019).

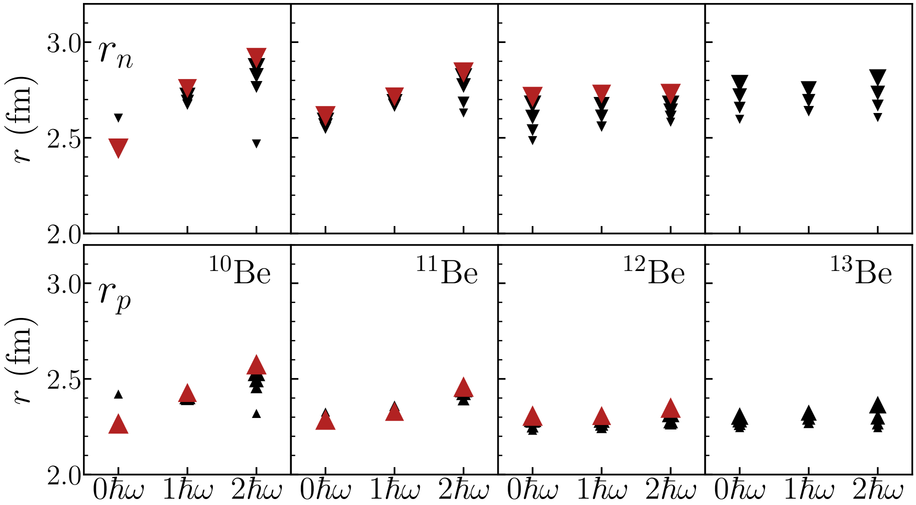
Parity inversion in ^{11}Be



Parity inversion ^{13}Be?

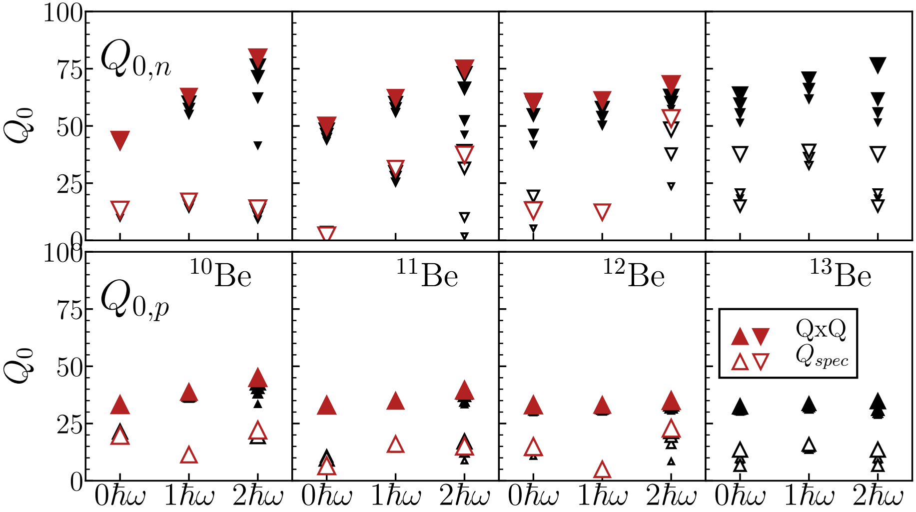


Radii



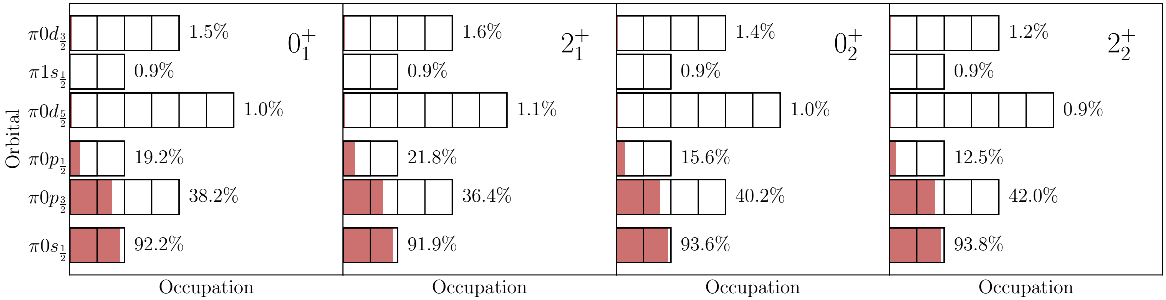
r_n
r_p
r (fm)
3.0
2.5
2.0
^{10}Be
^{11}Be
^{12}Be
^{13}Be
$0\hbar\omega$
$1\hbar\omega$
$2\hbar\omega$

Intrinsic quadrupole moment



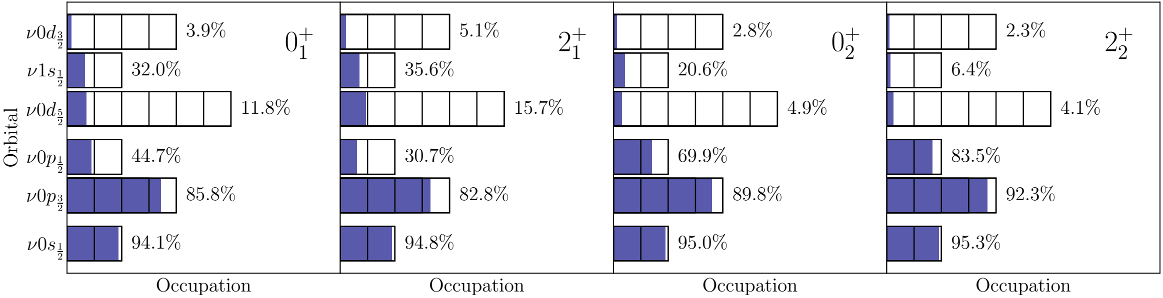
Natural orbital occupations

Protons



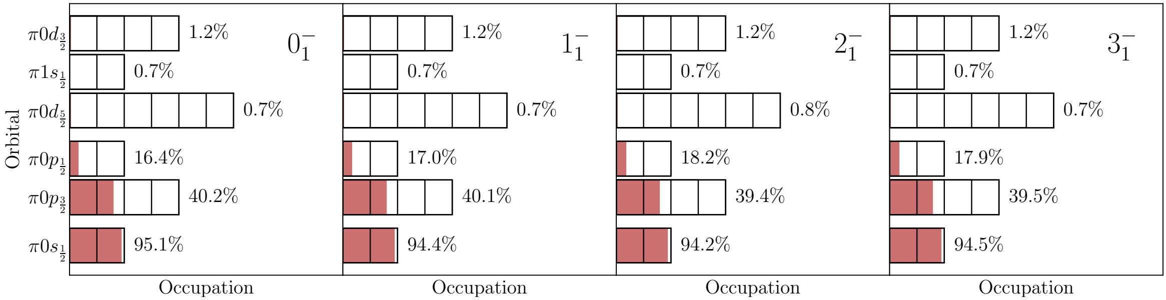
Natural orbital occupations

Neutrons



Natural orbital occupations

Protons



Natural orbital occupations

Neutrons