

Entanglement in Few-Particle Scattering



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Motivation

- Entanglement as a measure of correlation in a system
- New perspective on nuclear systems
- Scattering of nucleons and deuterons
→ already previous work on it
- Extension to halo nuclei
 - $^{11}\text{Be} - n$
 - $^{15}\text{C} - n$
 - $^{19}\text{C} - n$

Examples - Entanglement Entropy

- Quantifying entanglement: entanglement entropies
- Von Neumann entropy $E_N(\hat{\rho}_1) = -\text{Tr}[\hat{\rho}_1 \ln(\hat{\rho}_1)]$ with reduced density matrix $\hat{\rho}_1 = \text{Tr}_2[|\psi\rangle\langle\psi|]$

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order expression

$$1 \quad E_1(\hat{\rho}) = 1 - \text{Tr}[\hat{\rho}^2]$$

$$2 \quad E_2(\hat{\rho}) = \frac{3}{2} - 2 \text{Tr}[\hat{\rho}^2] + \frac{1}{2} \text{Tr}[\hat{\rho}^3]$$

$$3 \quad E_3(\hat{\rho}) = \frac{11}{3} - 3 \text{Tr}[\hat{\rho}^2] + \frac{3}{2} \text{Tr}[\hat{\rho}^3] - \frac{1}{3} \text{Tr}[\hat{\rho}^4]$$

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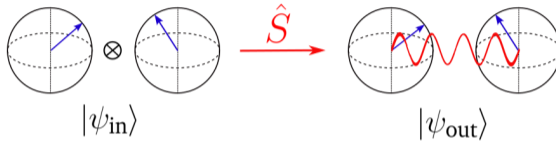
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- Rényi entropy $E_R(\alpha) = \frac{1}{1-\alpha} \ln\left(\text{Tr}[\hat{\rho}^\alpha]\right) = \frac{1}{1-\alpha} \ln(\sum_i \lambda_i^\alpha)$ for $\alpha > 0, \alpha \neq 1$



Formalism

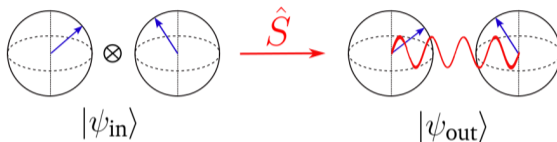
$S = \frac{1}{2}$:
Bloch sphere



■ $\hat{\rho} = |\psi_{out}\rangle \langle \psi_{out}|$ for $|\psi_{out}\rangle = \hat{S} |\psi_{in}\rangle$

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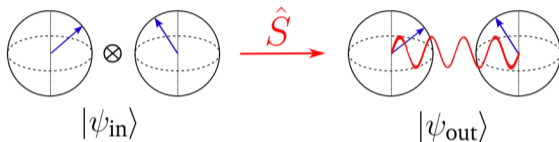
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- Scattering matrix $\hat{S} = \sum_{\text{spin channels } \sigma} \left(a_{\sigma} \mathbb{1} + b_{\sigma} \vec{S}_1 \cdot \vec{S}_2 + c_{\sigma} (\vec{S}_1 \cdot \vec{S}_2)^2 \right)$
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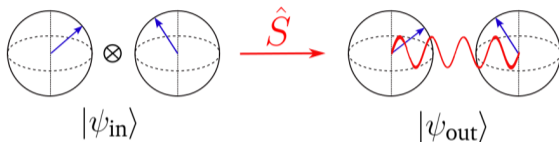
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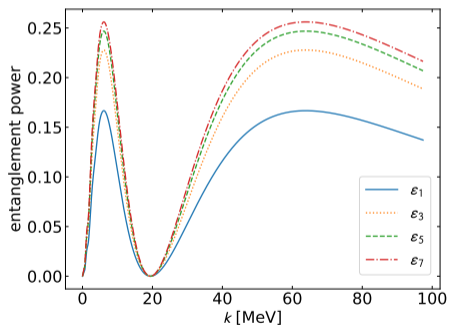
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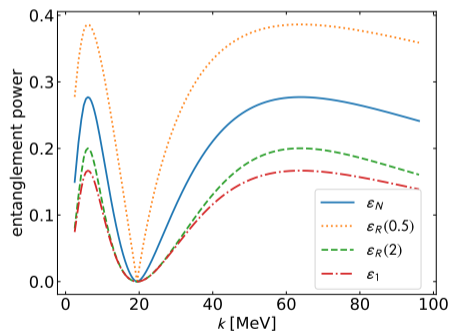
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- Spin $\frac{1}{2}$ and spin 1 - systems
 - np scattering
 - nd and pd scattering
 - dd scattering
 - spin- $\frac{1}{2}$ halo n scattering
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Proton-neutron scattering

Taylor expansions:



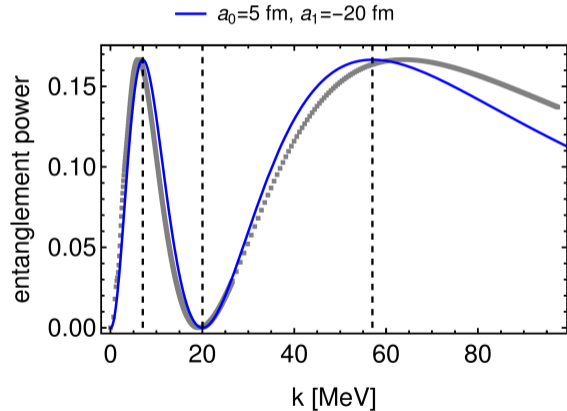
Entropies:



$$\varepsilon_1 = \frac{1}{6} \sin(2 \cdot (\delta_0 - \delta_1))^2 \quad [\text{Beane et al., PRL 122, 102001 (2019)}]$$

Proton-neutron scattering

- $\varepsilon_1 = \frac{1}{6} \sin(2 \cdot (\delta_0 - \delta_1))^2$
- $k \cot(\delta(k)) = -\frac{1}{a} + \frac{r}{2}k^2 + \dots$
- $a_0 \approx 5 \text{ fm}$ and $a_1 \approx -20 \text{ fm}$
- results in leading order:
 - ▣ $k_{\min,1} = 0$ and $k_{\min,2} = \frac{1}{\sqrt{-a_0 \cdot a_1}} \approx 20 \text{ MeV}$
 - ▣ $k_{\max,\pm} = \frac{\pm(a_0 - a_1) - \sqrt{a_0^2 - 6a_0a_1 + a_1^2}}{2a_0a_1}$
 - ▣ $k_{\max,+} \approx 7 \text{ MeV}$ and $k_{\max,-} \approx 57 \text{ MeV}$

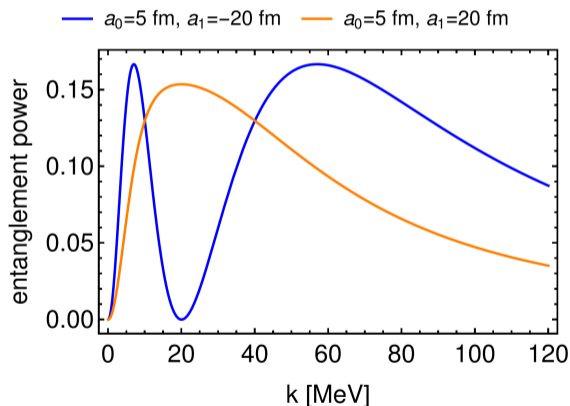


[Beane et al., 2021]

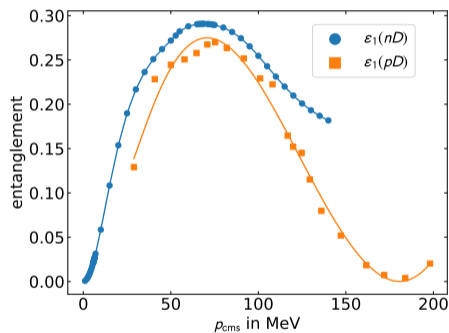
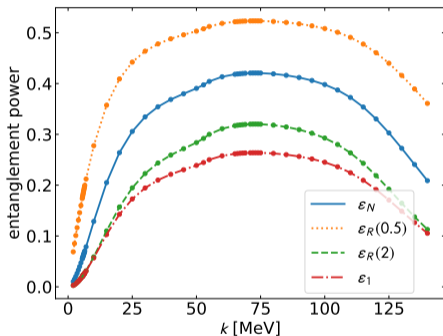
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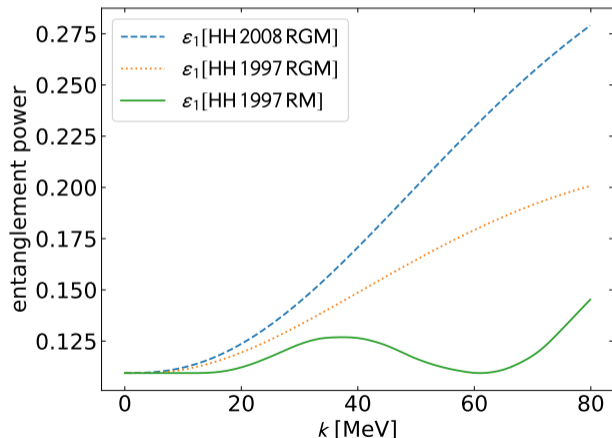
Neutron-deuteron and proton-deuteron scattering



$$\varepsilon_1 = \frac{8}{243} (17 + 10 \cos[2(\delta_{1/2} + \delta_c - (\delta_{3/2} + \delta_c))]) \sin[\delta_{1/2} + \delta_c - (\delta_{3/2} + \delta_c)]^2$$

[TK et al., 2023]

Deuteron-deuteron scattering

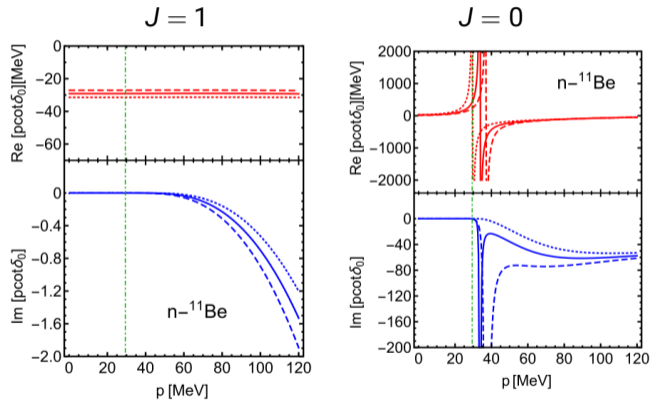


$$\varepsilon_1 = \frac{1}{576} (153 - 70 \cos[2(\delta_0 - \delta_2)] - 20 \cos[4(\delta_0 - \delta_2)])$$

- Datasets from [Hofmann et al., 1997] and [Hofmann et al., 2008]
- calculated with "R-matrix analysis" and "Resonating Group Model"

Why study halo nuclei

- use n-halo scattering to study entanglement
- not easy to measure - I use theoretically calculated phase shift data
- $a_{\text{Be}^{10}-\text{n}} = 6.741 \text{ fm}$

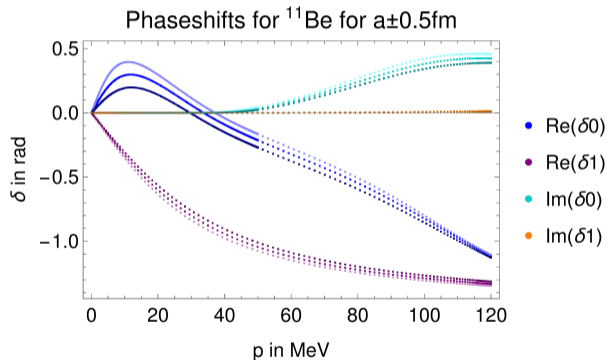


[Zhang et al., 2023]



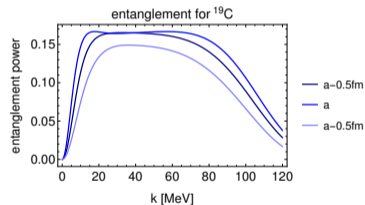
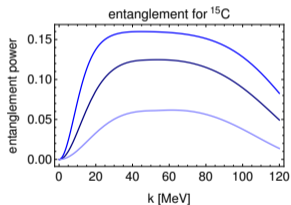
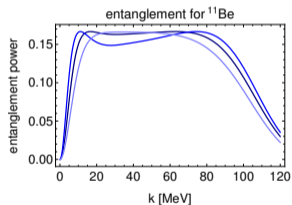
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Halo-neutron scattering

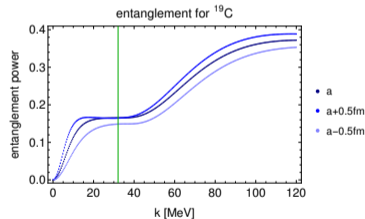
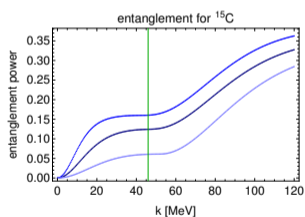
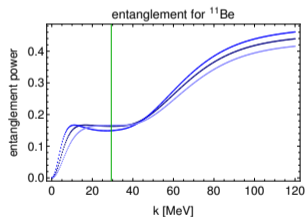
- $\varepsilon_1 = \frac{1}{6} \sin(2(\delta_0 - \delta_1))^2$ for non-imaginary phase shifts



Halo-neutron scattering

- breakup of core and neutron gives new channel
→ imaginary phase shifts
- entanglement power:

$$\epsilon_1^{nh} = 1 - \frac{1}{24} e^{-8\delta_0^l} - \frac{13}{24} e^{-8\delta_1^l} - \frac{1}{3} e^{-4(\delta_0^l + \delta_1^l)} - \frac{1}{24} e^{-4(\delta_0^l + i\delta_0^R + \delta_1^l - i\delta_1^R)} - \frac{1}{24} e^{-4(\delta_0^l - i\delta_0^R + \delta_1^l + i\delta_1^R)}$$

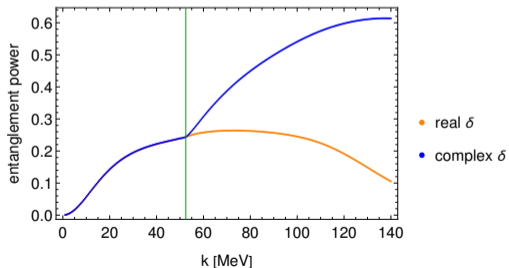


- green lines show breakup threshold

Halo-neutron scattering

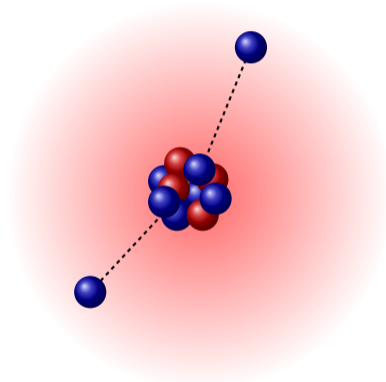
- breakup of core and neutron gives new channel
→ imaginary phase shifts

- nd scattering with deuteron breakup threshold

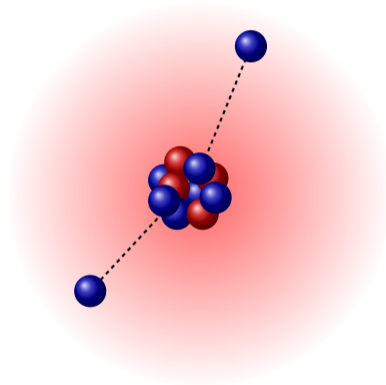


Outlook

- Study of two particle scattering with spin $\frac{1}{2}$ and spin 1
 - First Taylor expansion valid quantity
- Deuteron-deuteron scattering requires further study
- Compare with other halo nuclei
 - Rise in entanglement after breakup threshold
- Only spin entanglement so far
 - Interesting approach: investigate spatial entanglement for halo nuclei



Thank you for your
attention!



Coulomb influence

- $\delta_{\text{tot}}(k) = -\eta_k \log(2kr) + \sigma(k) + \delta_N(k)$
- $\sigma(k) = \arg \Gamma(1 + i\eta_k)$ pure s-wave Coulomb phase shift
 - $\eta_k = \alpha e Z_1 Z_2 \mu / k$ Sommerfeld Parameter
 - $\alpha = e^2 / (4\pi)$ electromagnetic fine structure constant in Heaviside-Lorentz units
 - μ reduced mass, Z_i charge numbers of both particles
- δ_N Coulomb-modified nuclear part (Coulomb-subtracted phase shift)
- influence cancels out as long as entanglement depends on phase shift differences
→ in all our systems the case

Deuteron Deuteron scattering

- spin channel $S = 1$ forbidden due to Bose symmetry

- $\langle S = 1 | \hat{S}_{dd} | S = 1 \rangle = 0$

- $\hat{S}_{dd} = -\frac{1}{3}(e^{2i\delta_0} - e^{2i\delta_2})\mathbb{1} + \frac{1}{2}e^{2i\delta_2}\vec{S}_1 \cdot \vec{S}_2 + \frac{1}{6}(2e^{2i\delta_0} + e^{2i\delta_2})(\vec{S}_1 \cdot \vec{S}_2)^2$
→ forbids the free propagation in the $S = 1$ channel

- $\varepsilon_1 = \frac{1}{576}(153 - 70 \cos[2(\delta_0 - \delta_2)] - 20 \cos[4(\delta_0 - \delta_2)])$ depends only on phase shift differences