Likelihood learning theory

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The LHC Precision Program — Benasque — 5/10/2023

mainly based on S. Chen, A. Glioti, G. P. and A. Wulzer
2007.10356 [hep-ph] and 2308.05704 [hep-ph]

Fundamental physics at colliders

The main goal of the collider program is to deepen our knowledge of fundamental physics

In practical terms, this means **testing the SM**

looking for its possible **failures** → evidence of **New Physics** (BSM)

Testing the SM

Complementarity

devising different strategies to test the SM predictions
and to cover different types of new physics

Optimality

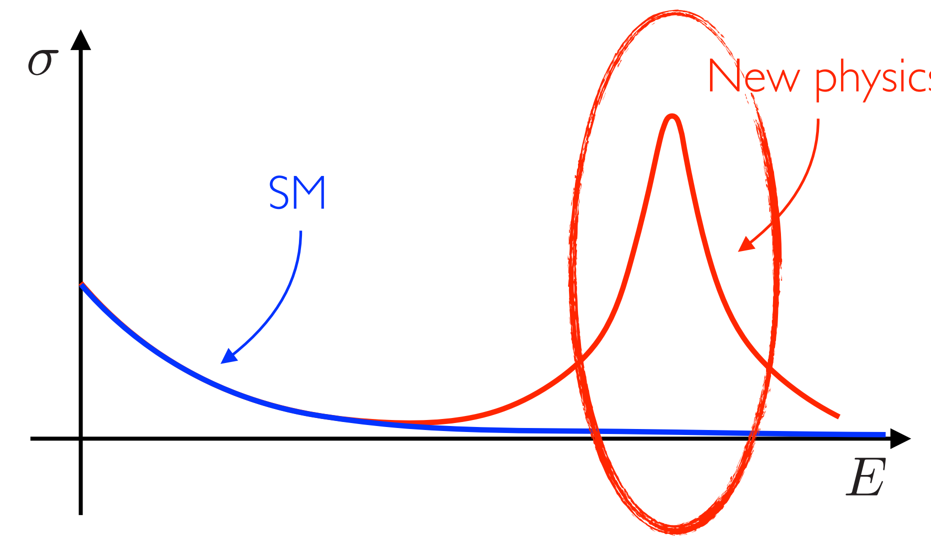
improve and optimize the new-physics probes to achieve better sensitivity

How to look for new physics

Direct searches:

look for signals of production
of new particles

- resonant effects in kinematic distributions
- "bump" on top of a smooth SM background
  (that can be often extracted from the data)



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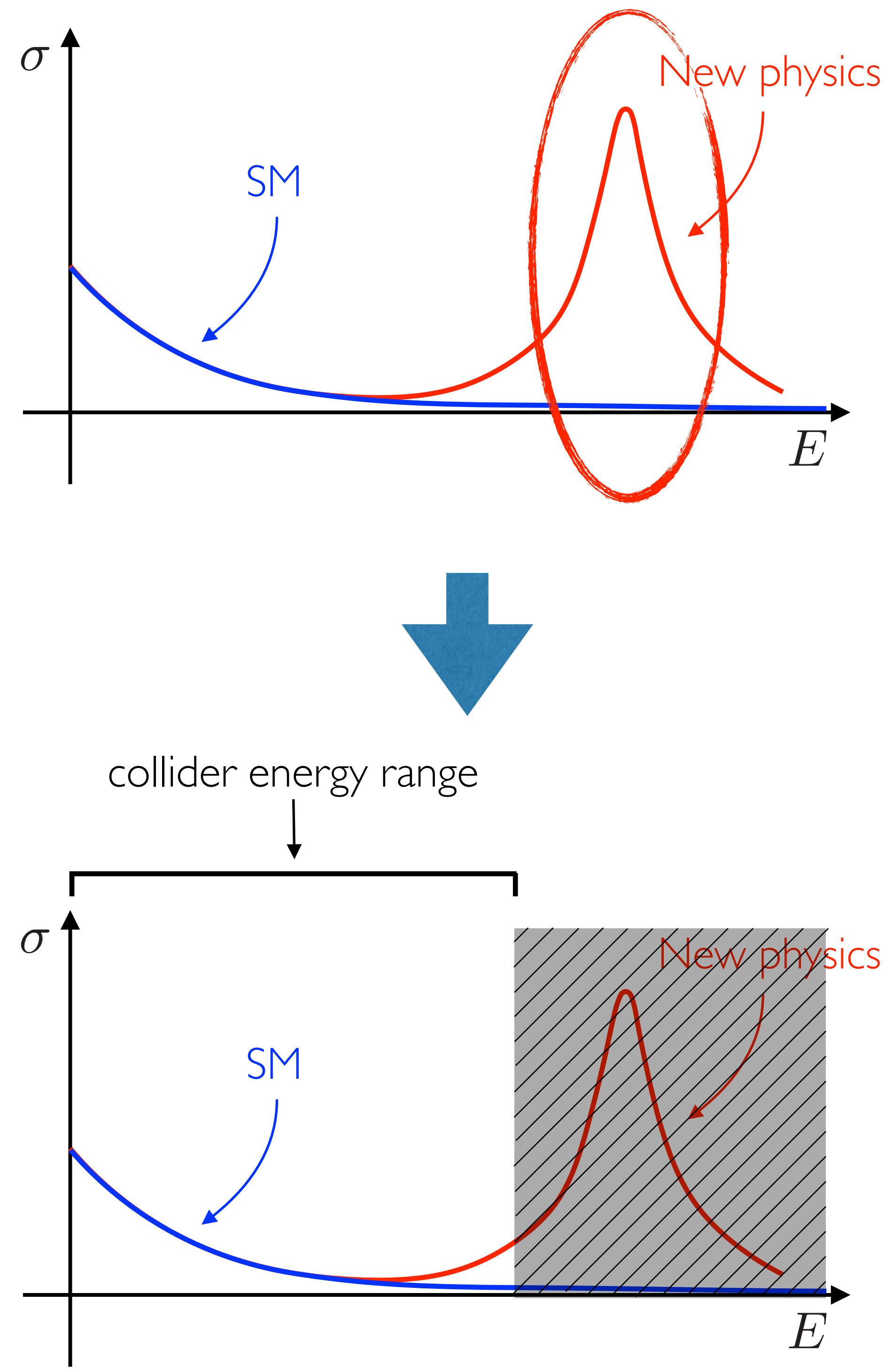

Limitations:

- new particle must be resonantly produced and must decay to reconstructable final state
- limited by collider energy range

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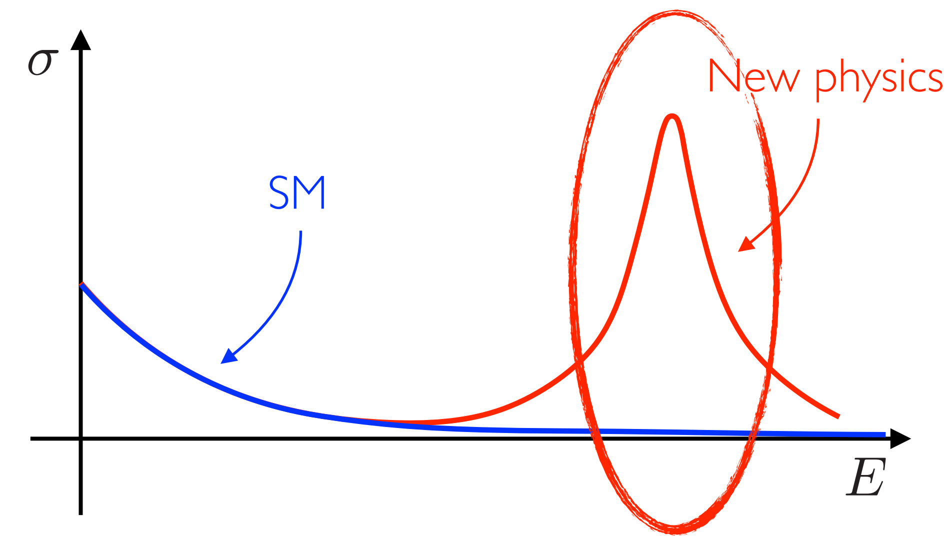
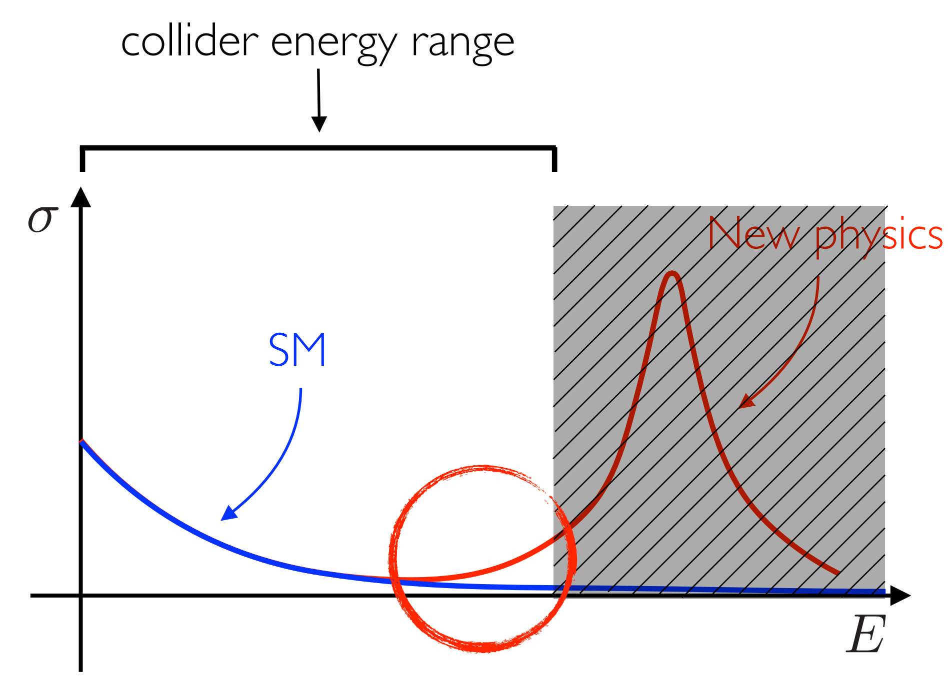


<u>Looking for the tail:</u> <u>Indirect searches</u>

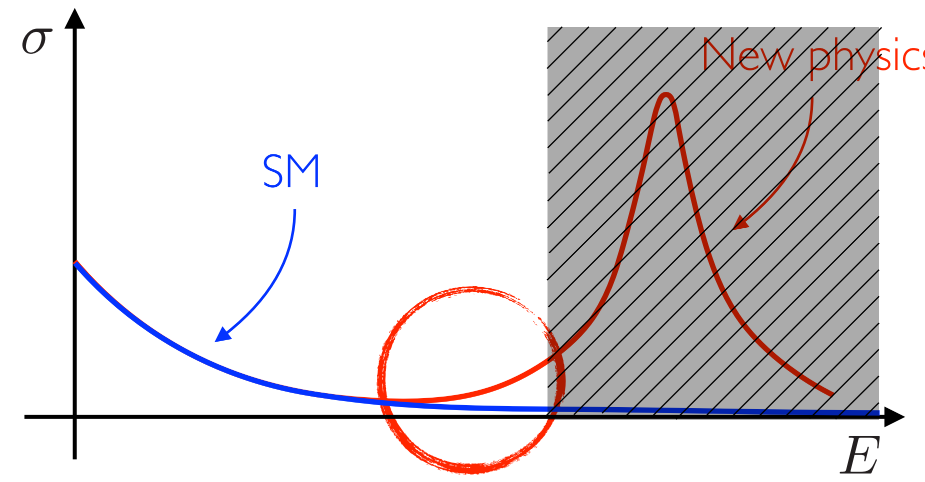
even if we can not directly produce
the new particles,
we can test their **indirect effects**

- LEP data at 200 GeV tested new particles with masses up to 3 TeV !



Tails are "universal"

Indirect searches have important advantages



"universality"

- deviations from SM exhibit small number of behaviors dictated by symmetries
- simple parametrization in terms of EFT operators

"model independence"

- captures a huge class of new-physics models

"ubiquity"

- deviations are present also in channels with non-resonant new physics production
- can often be seen also in channels where the final state can not be fully reconstructed

The challenges of indirect searches

Performing indirect searches is a challenging task
that requires several key ingredients

- Accurate theoretical knowledge of the SM and BSM predictions
  (i.e. small theoretical systematic uncertainty)

  → needed to compare theoretical expectation with the experimental data

- Accurate experimental measurements
  (i.e. small experimental systematic and statistical uncertainty)

  → in many cases we expect small deviations with respect to the SM

- Use of effective search strategies and optimized statistical analysis

A simple example

Simplest approach: exploit partial kinematic information

- keep only few kinematic variables and ‘ignore’ the others
- reconstruct the distributions through binning

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Example: **di-lepton production**

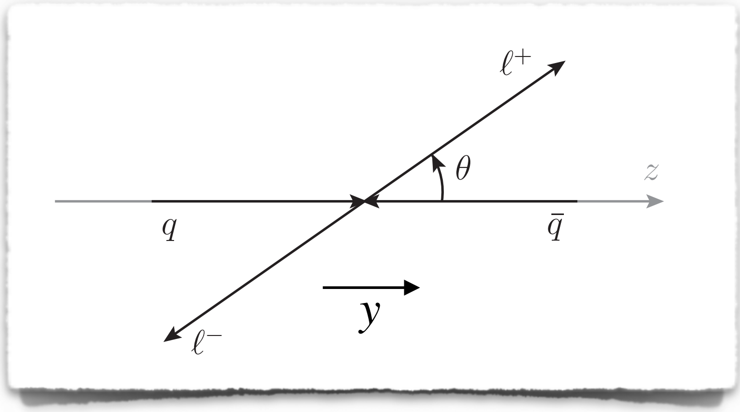
$$pp \to \ell^+\ell^-$$

three kinematic variables

invariant mass $m_{\ell^+\ell^-}$

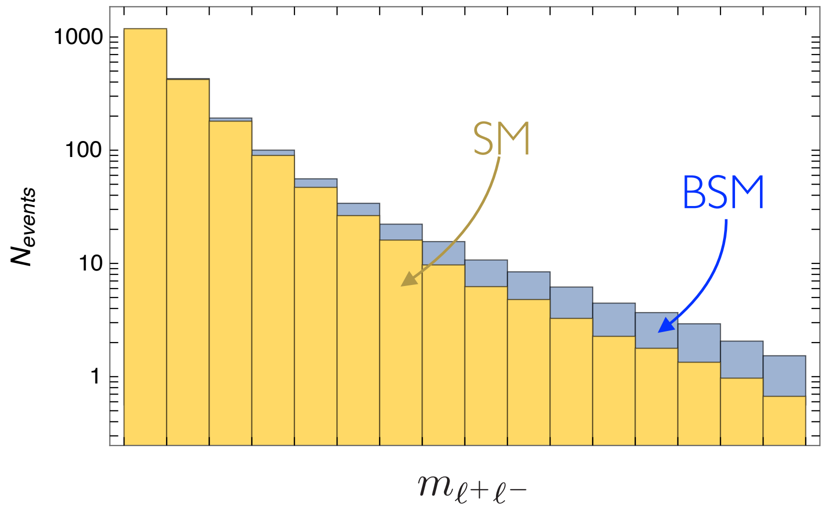
scattering angle θ

c.o.m. rapidity y



- can focus only on invariant mass
- distribution reconstructed with simple 1-dimensional binning

$$d\sigma(m_{\ell^+\ell^-}; C)$$



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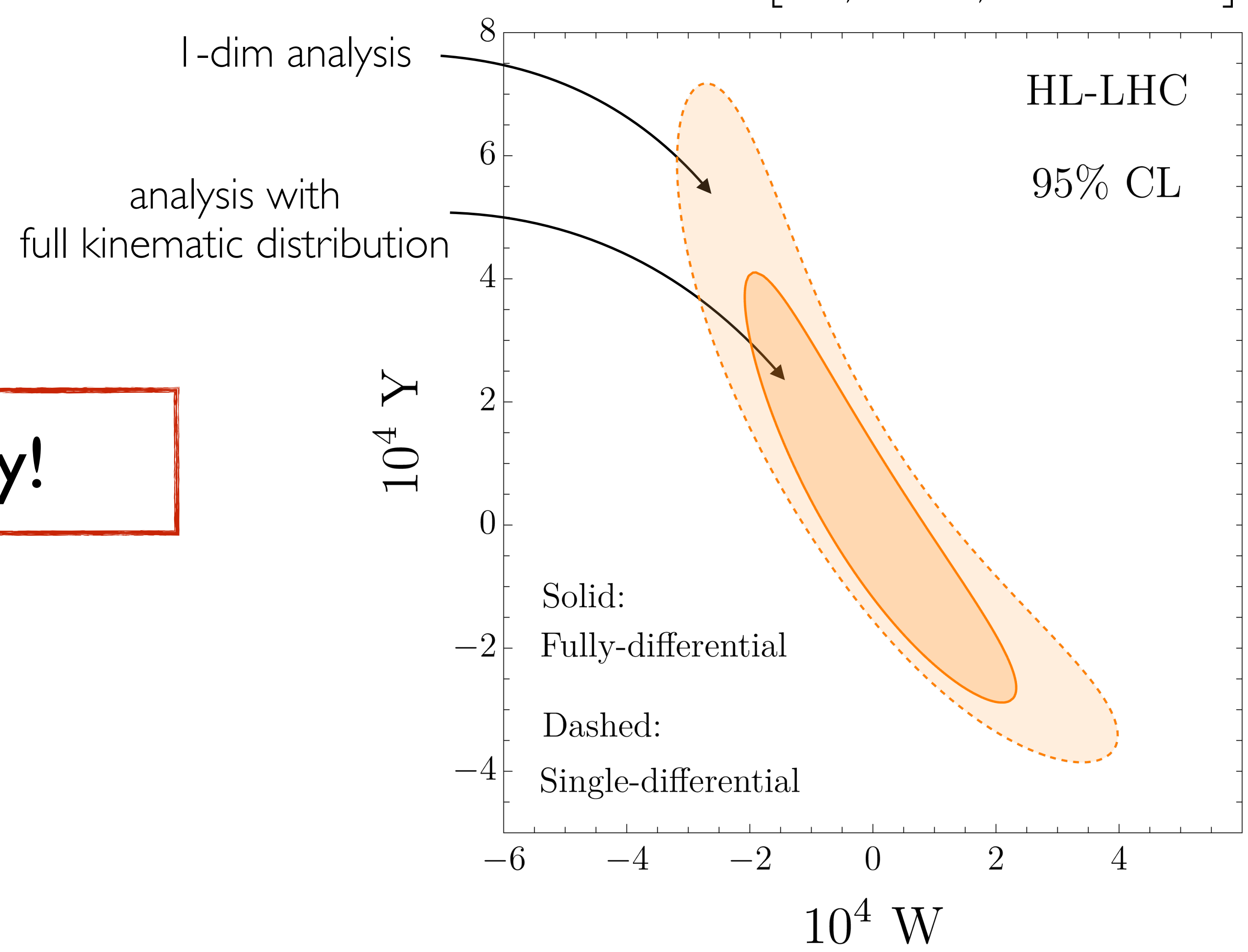
$$pp \to \ell^+\ell^-$$

[G.P., L. Ricci, A. Wulzer '21]

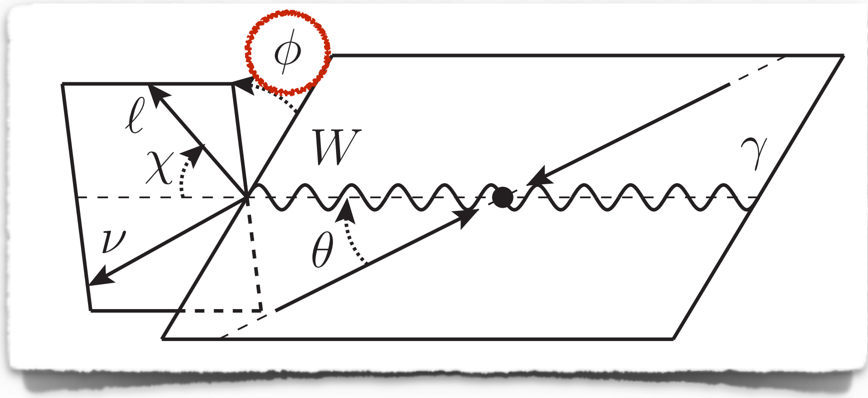
Big loss in sensitivity!



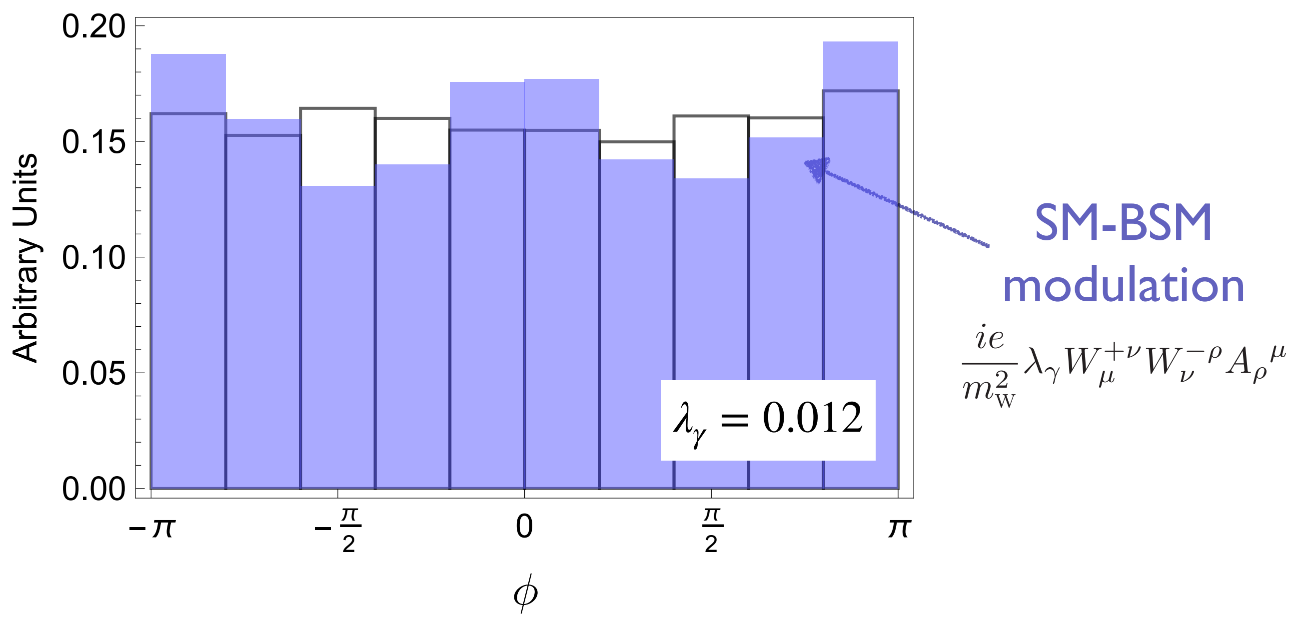
Interference resurrection

Interference resurrection can provide significant improvements
but requires *differential measurements* [GP, Riva, Wulzer '17]

example: trilinear gauge couplings in $W\gamma$ production



interference between
different polarization channels
makes BSM effects clearly visible



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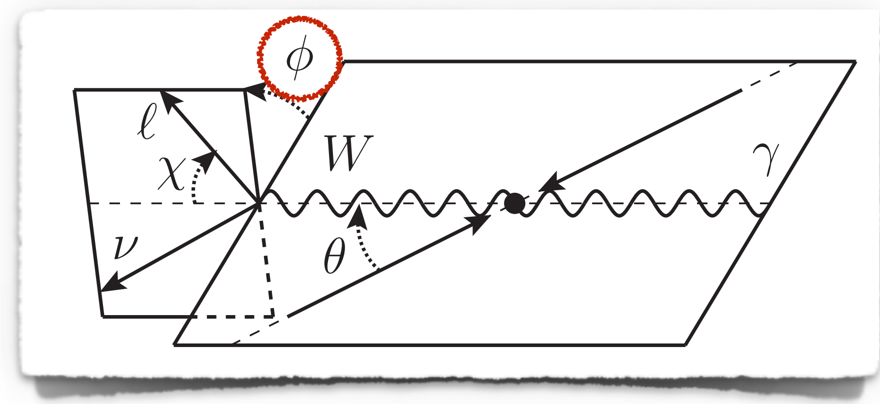
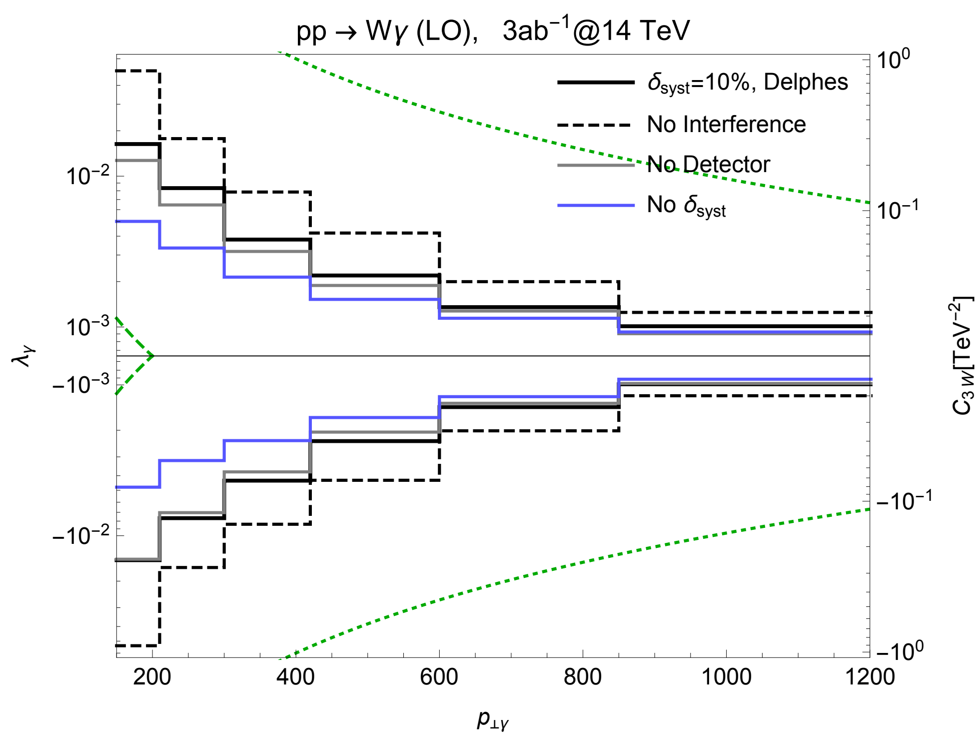
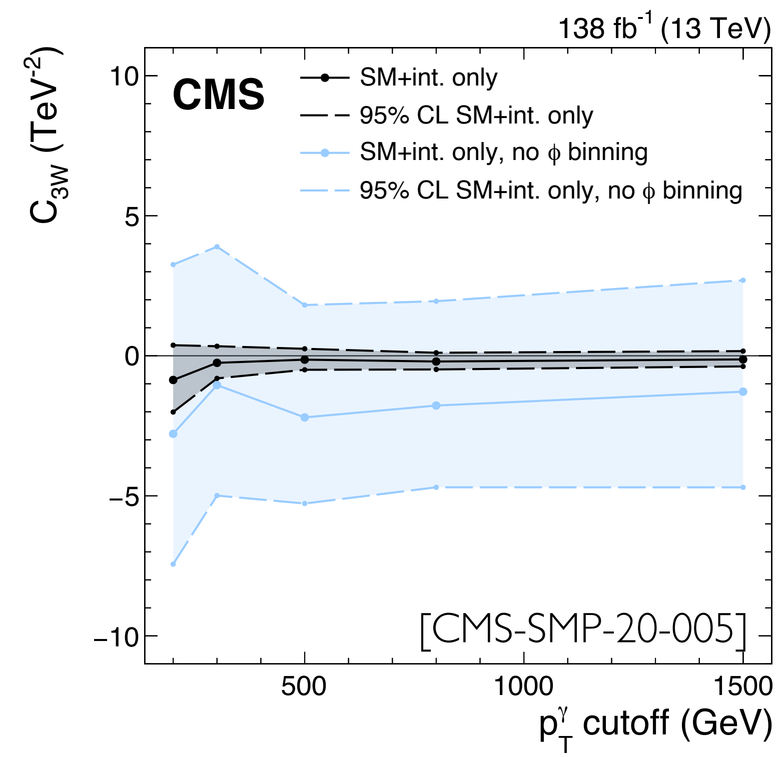


example: trilinear gauge couplings in $W\gamma$ production





- significant gain for operators that do not interfere with SM
- improved sensitivity to linear terms confirmed by experimental analysis
  (present sensitivity still dominated by quadratic terms)

Assessing optimality

Assessing **optimality** is crucial!

- Important to know if search strategies are close to optimal
- If not, help to design more efficient analyses
  (eg. identifying more sensitive observables)

Optimal tests of new physics

The **differential distribution** contain the maximal information about a process

$$d\sigma(x; C)$$

measurable kinematic quantities → x

new-physics parameters → C

- basis to perform optimal statistical tests (eg. likelihood ratio test)

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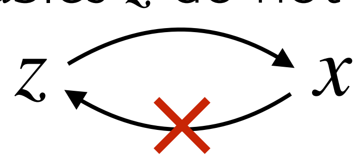
How to determine the theoretical kinematic distributions?

- not known in analytic form
- only available knowledge are Monte Carlos event samples following $d\sigma(x; C)$
  - "latent" Monte Carlo variables z do not coincide with measurable quantities x



  - higher-order effects generate "unphysical" events (negative weights)
  - showering, hadronization, detector effects not known "analytically"

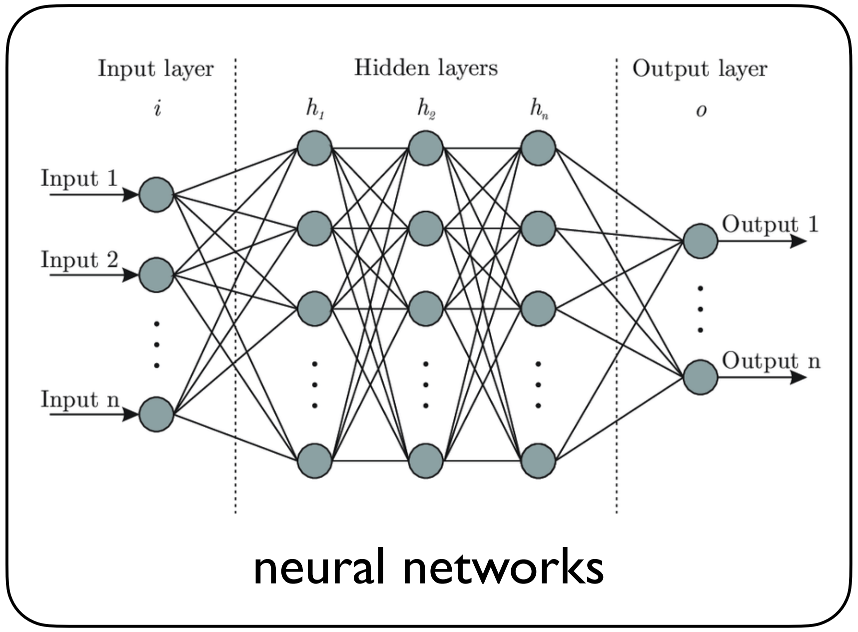
A Machine Learning approach

Full distributions through ML

[Baldi, Cranmer et al. '16;
Brehmer, Cranmer, Louppe, Pavez '18;
Cranmer, Pavez, Louppe '18;
Stoye, Brehmer at al. '18;
Brehmer, Louppe at al. '18;
Brehmer, Cranmer at al. '18;
Brehmer, Kling at al. "MadMiner" '19;
Chen, Glioti, G.P., Wulzer '20;
…]

Basic idea: reconstruct $d\sigma(x; C)$ with Neural Networks

kinematic variables x → neural networks → full distribution ratio $\frac{d\sigma(x; C)}{d\sigma(x; 0)}$ (reference hypothesis)



- fully differential (analytic) result in all measurable quantities
- obtained with a relatively small amount of Monte Carlo data
- systematically improvable
  - with more data, reacher NN structure, …
  - with more accurate Monte Carlo samples (eg. higher-order effects, backgrounds, …)

The Binary Classifier trick

A **binary classifier** can be used to reconstruct the distribution ratio from Monte Carlo data

- two samples, following new physics ($C = \overline{C}$) and reference ($C = 0$) distributions

$$\mathcal{S}_{\overline{C}} = \{x_i \sim d\sigma(x; \overline{C})\} \qquad \mathcal{S}_0 = \{x_i \sim d\sigma(x; 0)\}$$

notice that $\overline{C}$ is fixed

- binary classifier (eg. with quadratic loss)

$$L = \sum_{x_i \in \mathcal{S}_{\overline{C}}} [NN(x_i) - 1]^2 + \sum_{x_i \in \mathcal{S}_0} [NN(x_i)]^2$$

- in the infinite training sample limit

$$\frac{\delta L}{\delta NN} = 0 \longrightarrow NN(x) = \frac{d\sigma(x; \overline{C})}{d\sigma(x; \overline{C}) + d\sigma(x; 0)} \longleftrightarrow \frac{d\sigma(x; \overline{C})}{d\sigma(x; 0)} = \frac{NN(x)}{1 - NN(x)}$$

✦ weighted samples can be treated in an analogous way introducing weights in L

Application to WZ production

$$pp \to W^{\pm}Z \to (\ell^{\pm}\nu)(\ell^{+}\ell^{-})$$

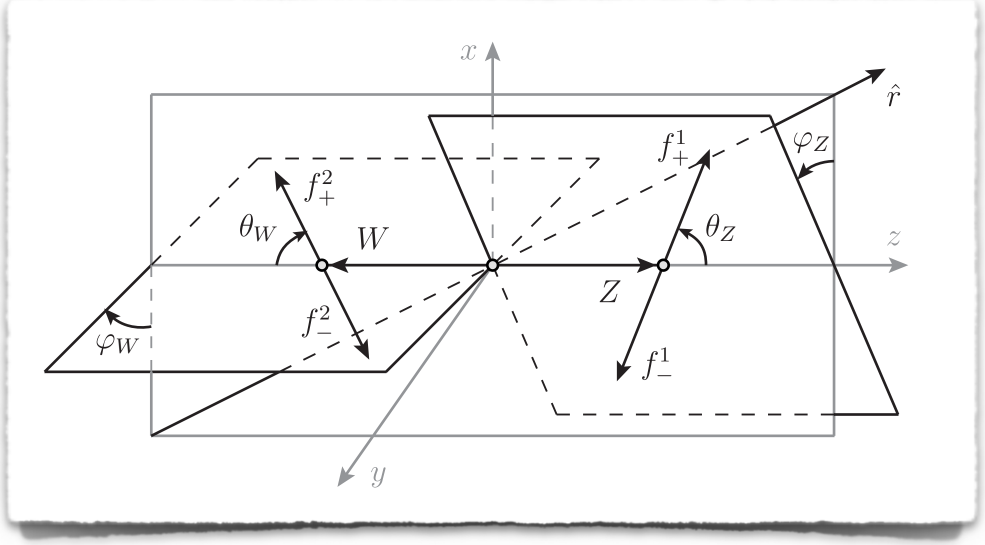
Final state described
by 6 kinematic variables!

invariant mass m_{WZ}

scattering angle θ

W decay angles θ_W, ϕ_W

Z decay angles θ_Z, ϕ_Z



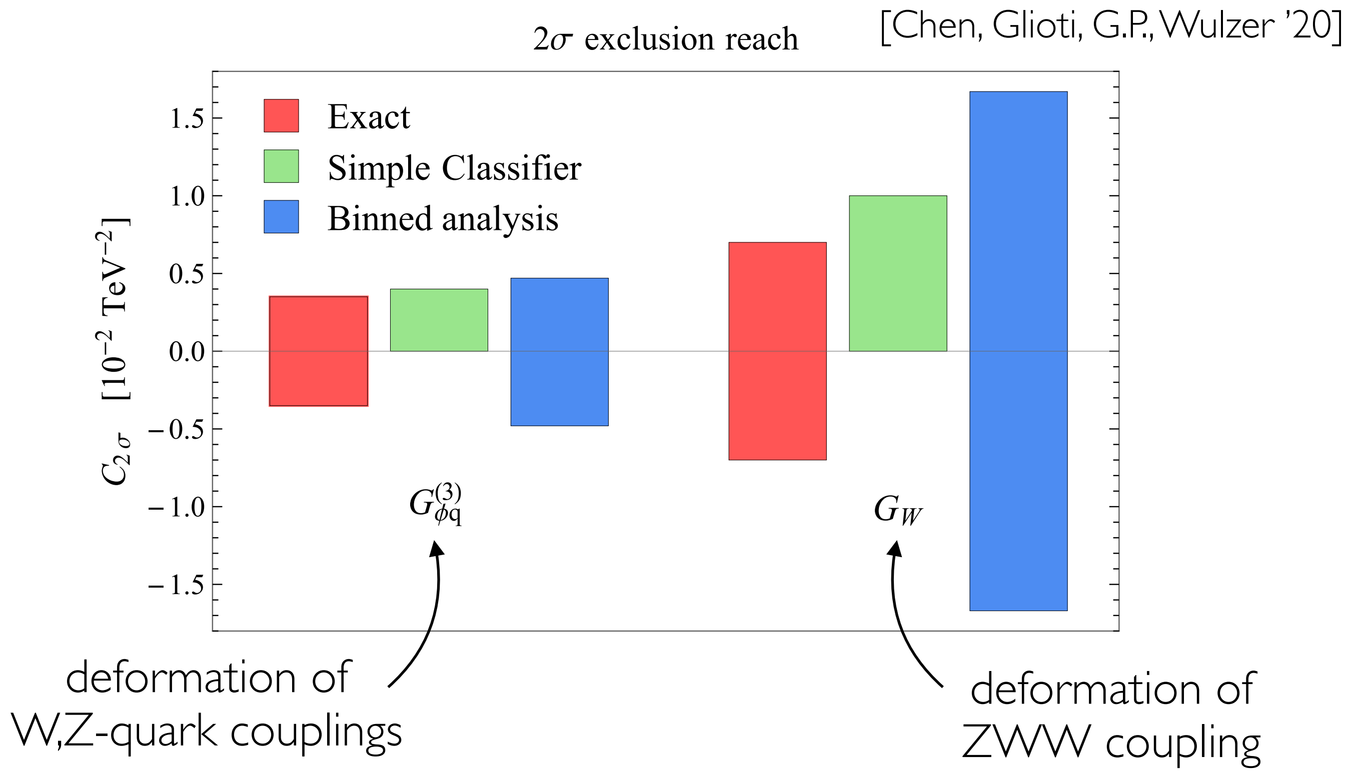
- standard binned analysis can not take into account all kinematic variables (at most two or three)

→ important features characterizing new-physics distributions are lost

→ huge loss in sensitivity

Simple Classifier performance

- The Simple Classifier approach works, but there is still some gap



<u>Drawbacks:</u>

- must be trained for 'every' value of $\overline{C}$ (with new Monte Carlo sample!)
- becomes inefficient for small values of $\overline{C}$
  (differential distribution very close to reference, large amount of training data are needed to reconstruct the ratio)

Joining Machine Learning with Theory:

Parametrized Neural Networks

The Quadratic Classifier

Theory fixes the structure of the differential distribution

$$d\sigma(x;C) = d\sigma(x;0)\left[(1 + C\,\alpha(x))^2 + C^2\,\beta^2(x)\right]$$

positive **quadratic polynomial** in C

We can exploit this information to solve the drawbacks of the Simple Classifier approach

The Quadratic Classifier

Theory fixes the structure of the differential distribution

$$d\sigma(x;C) = d\sigma(x;0)\underbrace{\Big[(1 + C\,\alpha(x))^2 + C^2\,\beta^2(x)\Big]}$$

positive **quadratic polynomial** in C

- we use a standard binary classifier loss

$$L = \sum_{\{C_i\}} \left\{ \sum_{x_i \in \mathcal{S}_0} [F(x_i;C_i) - 1]^2 + \sum_{x_i \in \mathcal{S}_{C_i}} [F(x_i;C_i)]^2 \right\}$$

- but the distribution ratio is parametrized in terms of **two neural networks**

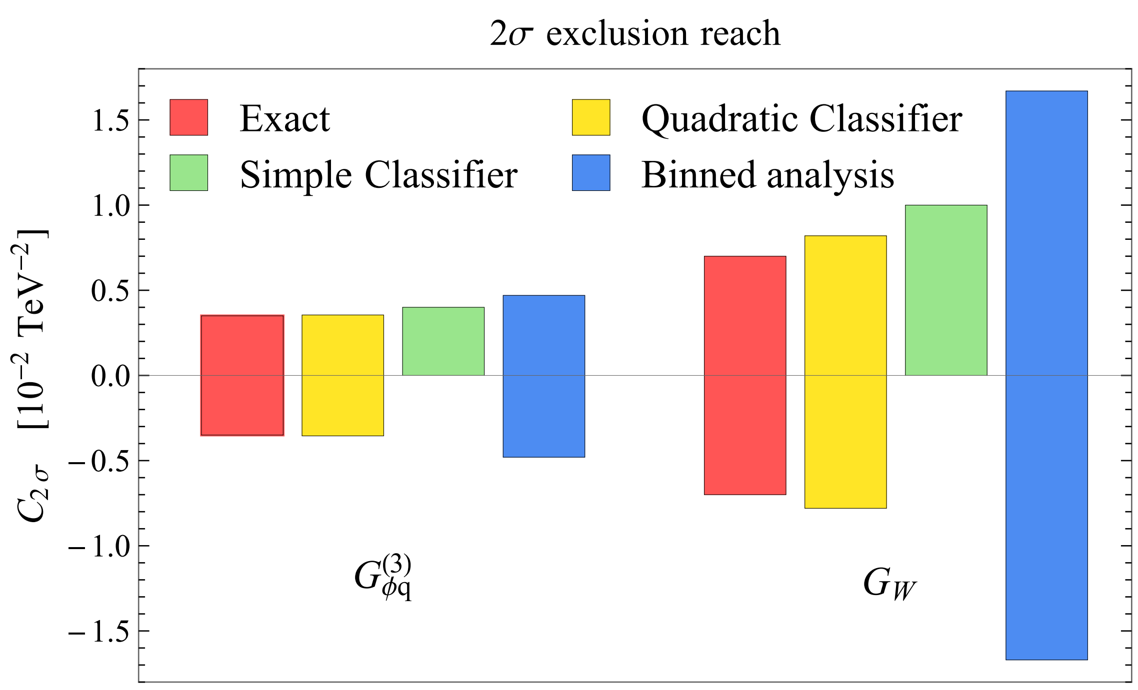
$$F(x;C) = \frac{1}{1 + (1 + C\,NN_\alpha(x))^2 + C^2\,NN_\beta^2(x)}$$

- training data must include different values of C

$$NN_\alpha(x) \to \alpha(x) \qquad\qquad NN_\beta(x) \to \beta(x)$$

The Quadratic Classifier

- The Quadratic Classifier provides a significantly better performance



[Chen, Glioti, G.P., Wulzer '20]

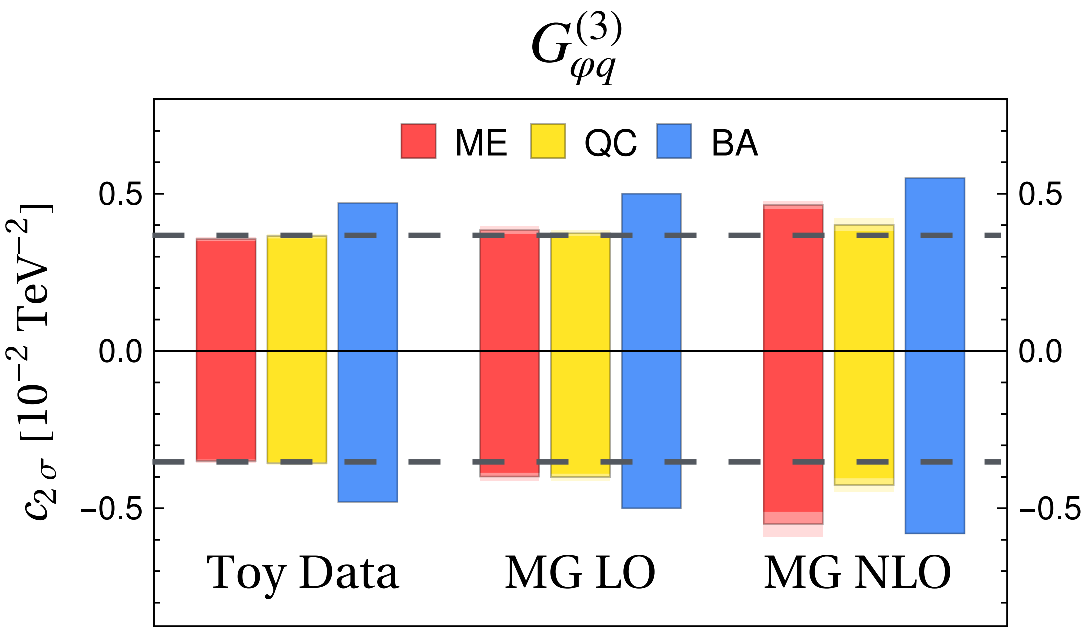
- a single training can reconstruct the distribution ratio for any C

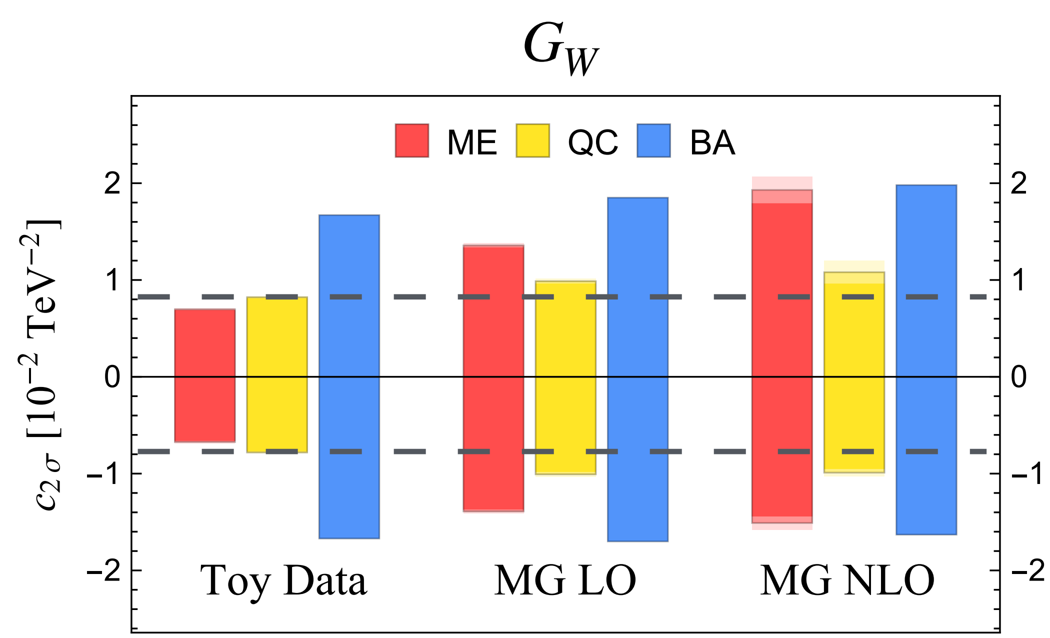
- training with "large" values of C avoids small differences from reference

  → limit $C \to 0$ properly reconstructed

The Quadratic Classifier

Additional effects can be included by changing the training data





- realistic Monte Carlo data can be used (eg. MadGraph @ LO or @ NLO)
- performance remains very stable

Conclusions and Outlook

Conclusions

Machine learning provides new tools to optimize the sensitivity in model-independent new-physics searches at colliders

Key ingredient: reconstruction of differential distributions from MC data

'Minimal' ML approach: **Simple binary Classifier**

- fair performance
- some drawbacks (lack of embedded theory knowledge)

Improved ML approach: **Quadratic Classifier**

- directly embeds theory knowledge (analytic dependence on parameters)
- only one training needed to test different new-physics parameters

Outlook

<u>Further developments:</u>

- ✦ Simultaneous treatment of **many new-physics deformations**

  [see talk by J. Ter Hoeve]

- ✦ Exploitation of **event 'reweighting'** to improve performance
  - faster generation of training data
  - better NN reconstruction (significantly smaller training sets needed)

  [see talk by A. Glioti]

- ✦ Inclusion of **systematic errors** (eg. pdf errors)