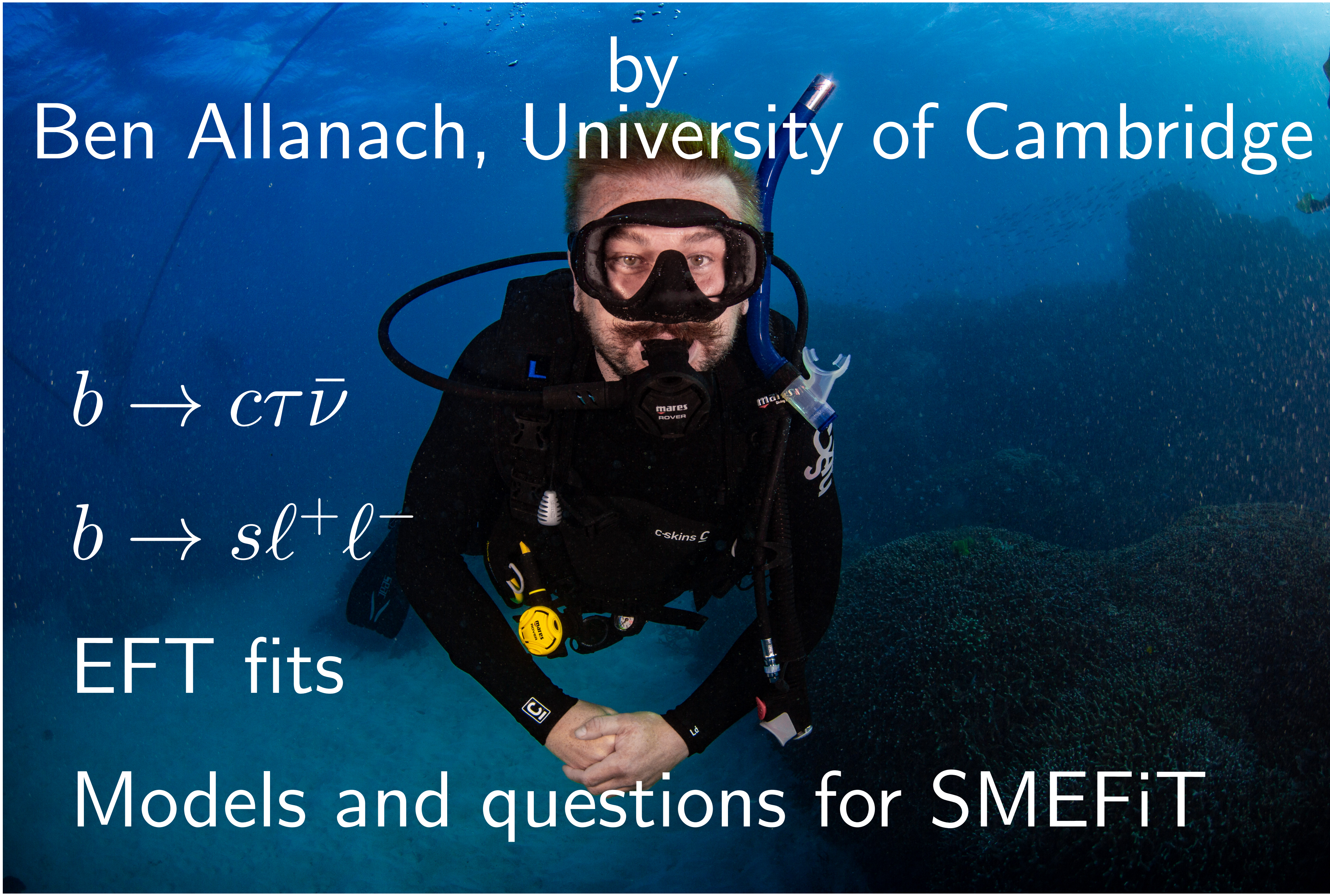
Flavour Physics



by

Ben Allanach, University of Cambridge

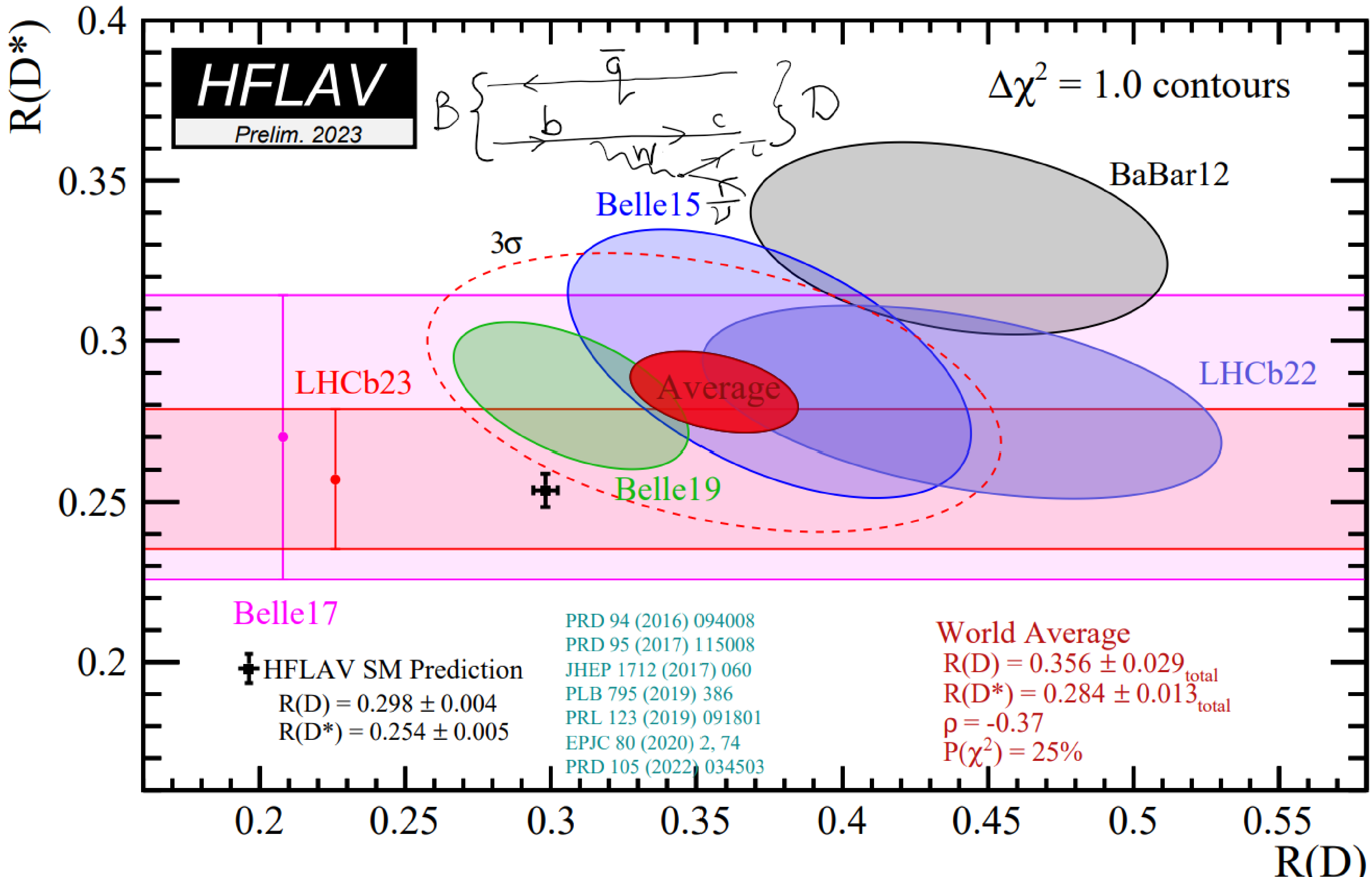
$b \to c \tau \bar{\nu}$

$b \to s \ell^+ \ell^-$

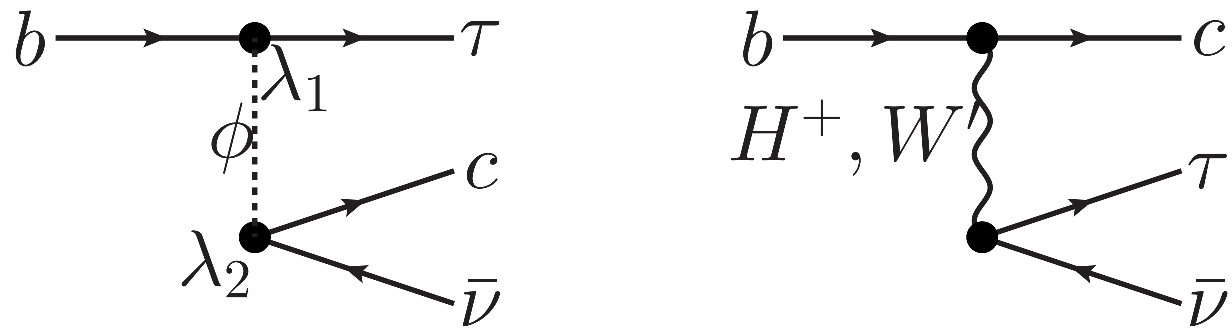
EFT fits

Models and questions for SMEFiT

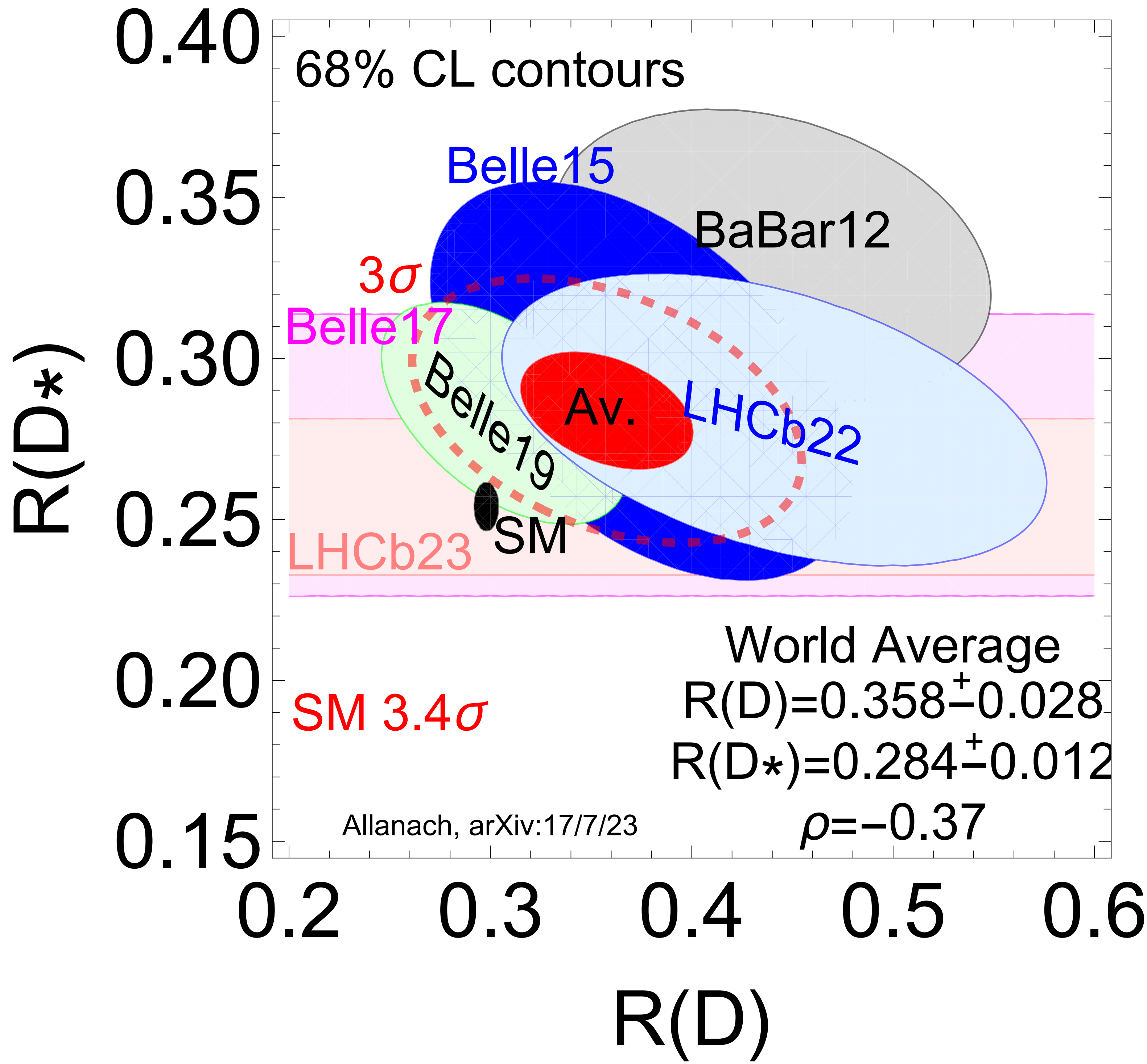
$$R_{D^{(*)}} = BR(B \to D^{(*)}\tau\nu)/BR(B \to D^{(*)}\ell\nu_\ell)$$



$R_{D^{(*)}}$: BSM Explanations



$$\mathcal{L}_{WET} = -\frac{2\lambda_1\lambda_2}{M^2}\left(\bar{c}\gamma^{\mu}P_L\nu\right)\left(\bar{\tau}\gamma_{\mu}P_L b\right) + H.c.$$


68% CL contours
Belle15
BaBar12
3σ
Belle17
Belle19
Av.
LHCb22
SM
LHCb23
World Average
R(D)=0.358±0.028
R(D*)=0.284±0.012
ρ=−0.37
SM 3.4σ
Allanach, arXiv:17/7/23
R(D*)
R(D)
0.40
0.35
0.30
0.25
0.20
0.15
0.2
0.3
0.4
0.5
0.6

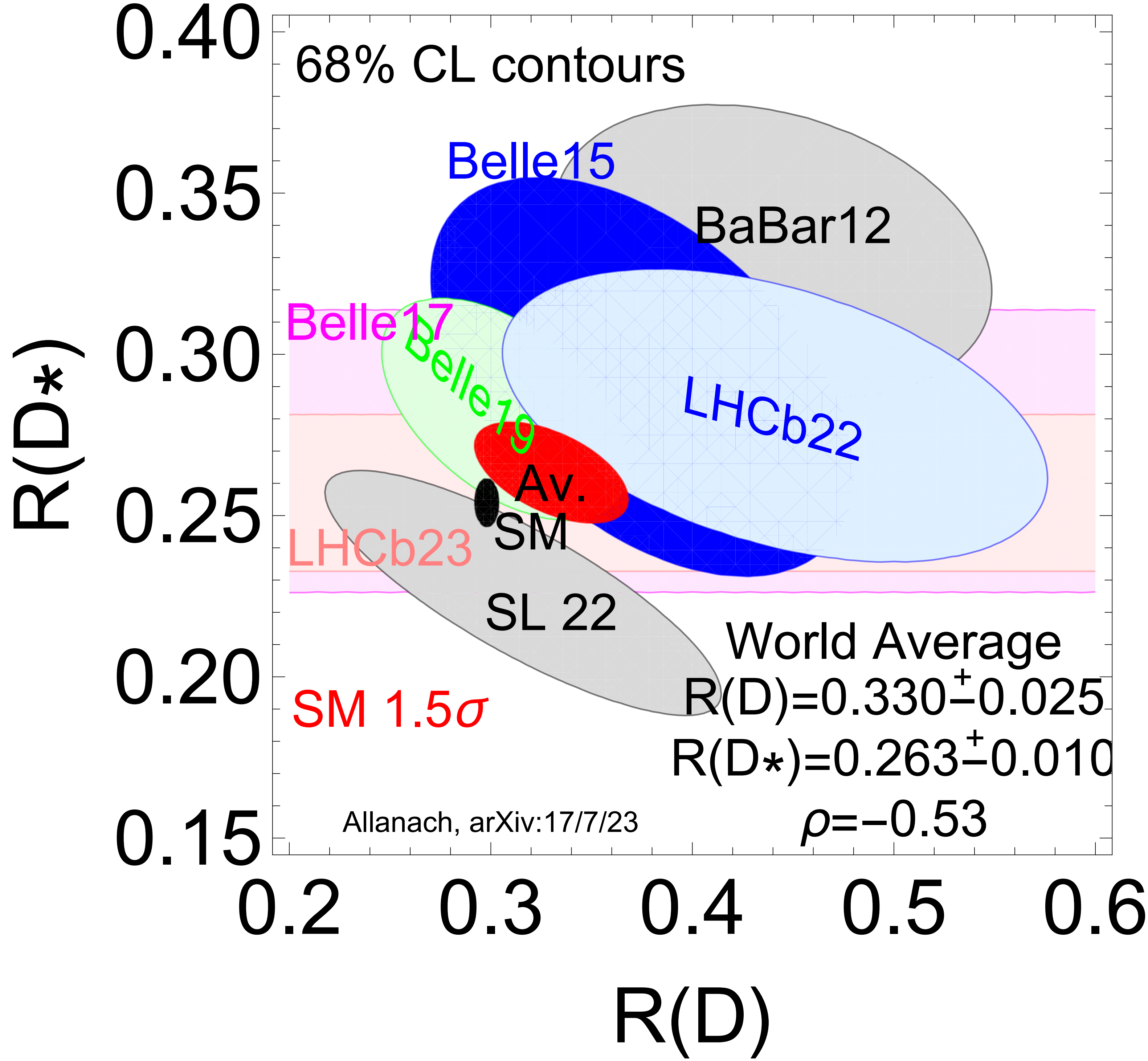
2022 Measurement

Using BaBar data (not official BaBar analysis) and *semi-leptonic* tag: (2012 used *hadronic*)

$$R(D) = 0.316 \pm 0.062 \pm 0.019$$
$$R(D*) = 0.226 \pm 0.022 \pm 0.012$$
$$\rho = -0.82$$

Yunxuan Li, *Search for Beyond Standard Model Physics at BaBar*, (2022), Caltech Ph.D. thesis

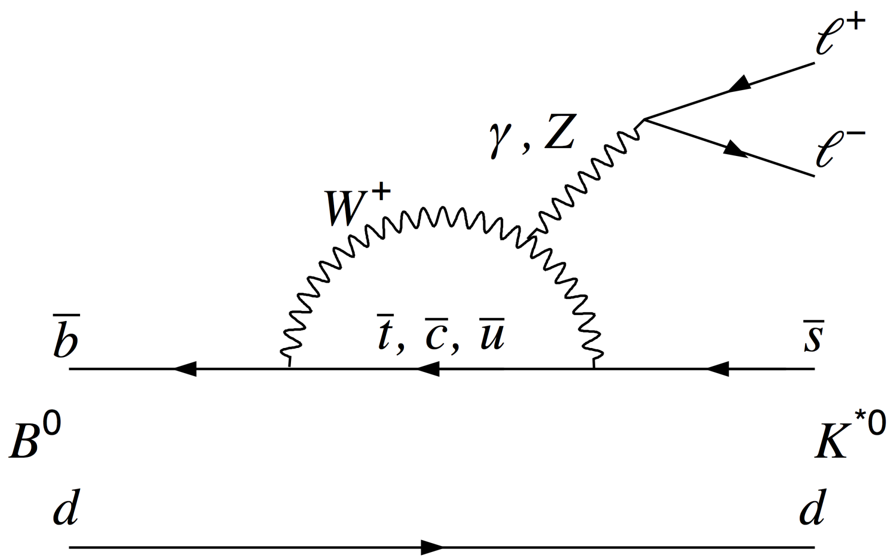
https://resolver.caltech.edu/CaltechTHESIS:05232022-144829107


68% CL contours
Belle15
BaBar12
Belle17
Belle19
LHCb22
Av.
SM
LHCb23
SL 22
World Average
R(D)=0.330±0.025
R(D*)=0.263±0.010
ρ=−0.53
SM 1.5σ
Allanach, arXiv:17/7/23
0.40
0.35
0.30
0.25
0.20
0.15
R(D*)
0.2
0.3
0.4
0.5
0.6
R(D)

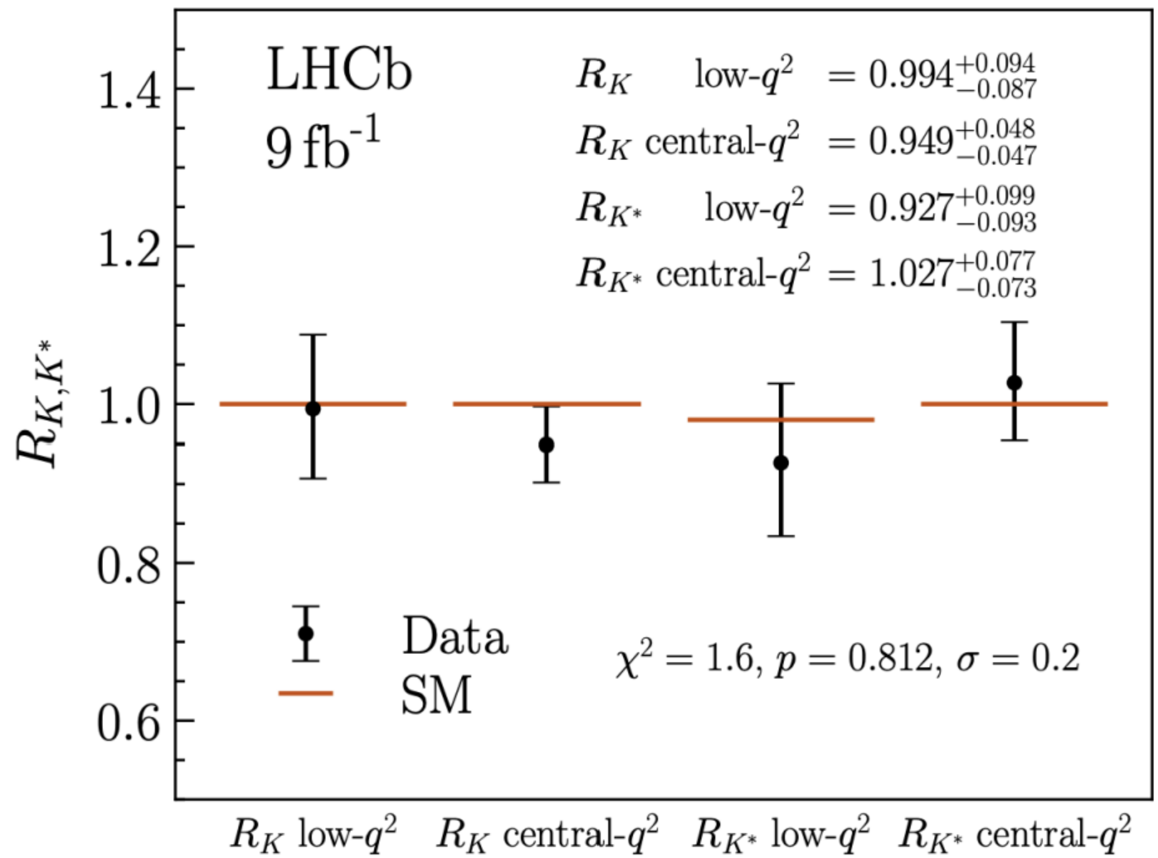
$b \to sl^+l^-$ in Standard Model

$$BR(B \to K\mu^+\mu^-) = BR(B \to Ke^+e^-)$$

BR$\sim \mathcal{O}(10^{-6})$: loop+EW+CKM



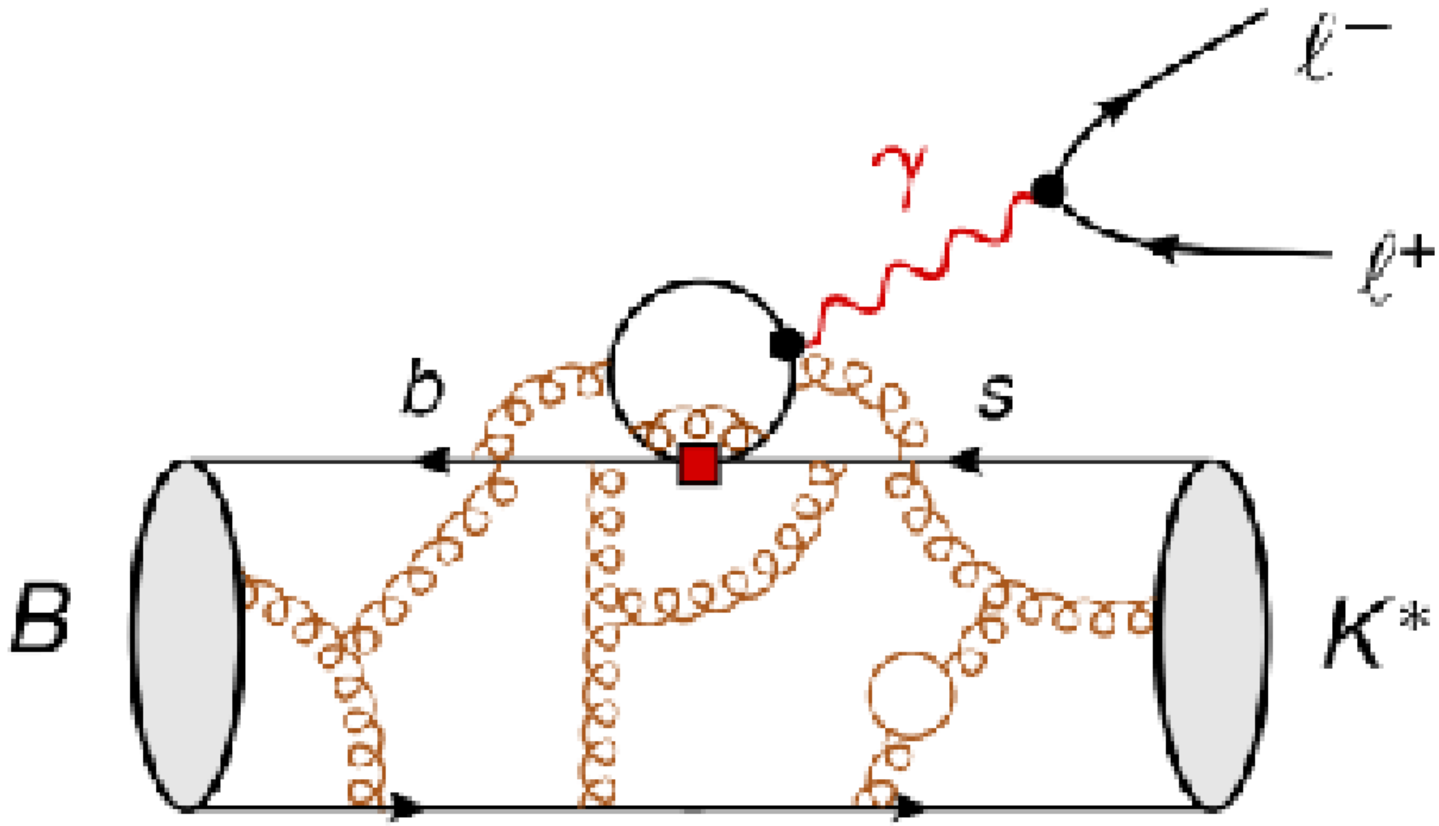
LHCb 2212.09152



$$R_X(q^2) = \frac{BR(B \to X\mu^+\mu^-)}{BR(B \to Xe^+e^-)}(q^2)$$


MUSTER STATION 1
FIRE HOSE
NITROX

Form Factors



Predicting $B \to M\ell^+\ell^-$: FFs

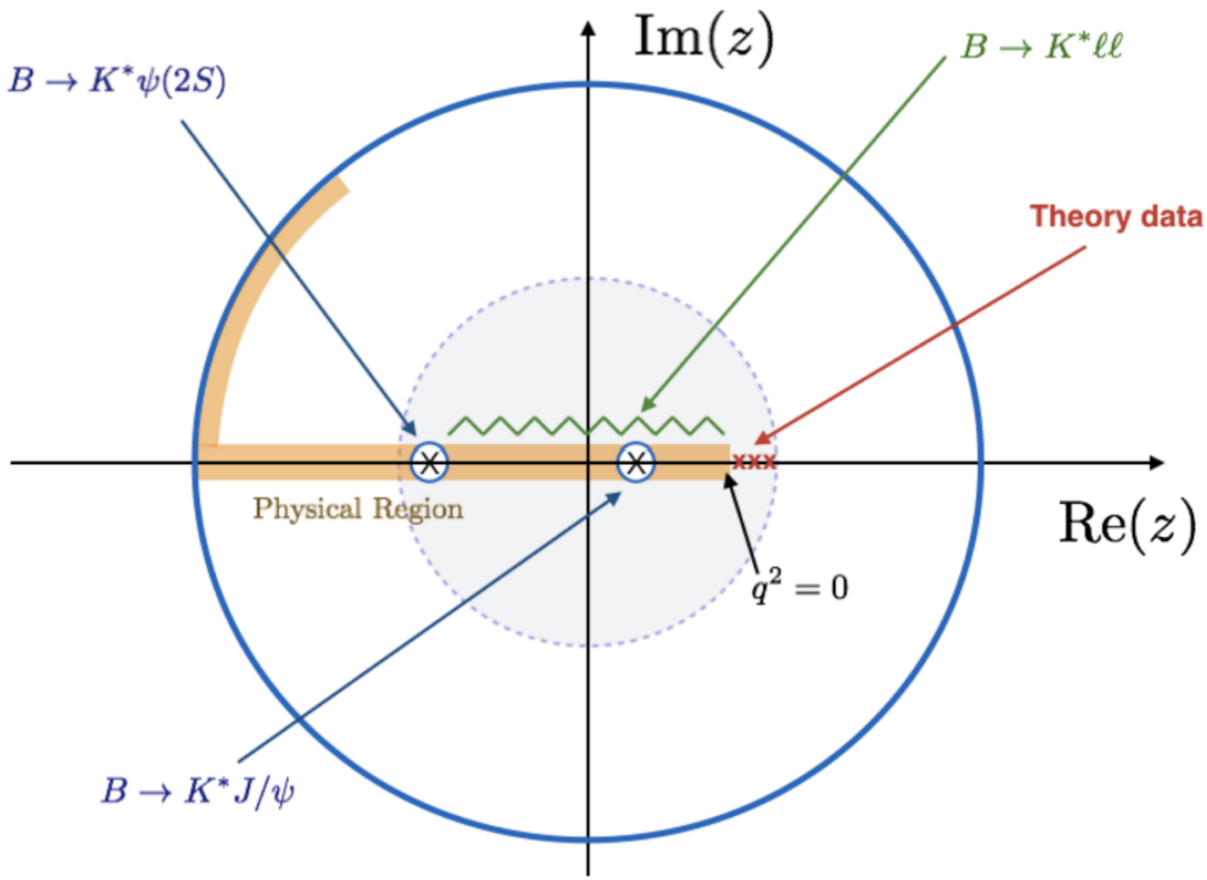
$$\mathcal{A} = \texttt{local} + \texttt{non-local}$$

`local`: interpolate lattice at high $q^2 = m_{ll}^2$ and LCSR at low q^2.

`non-local`: no lattice. Most use QCD factorisation: perturbative charm loop+ad-hoc

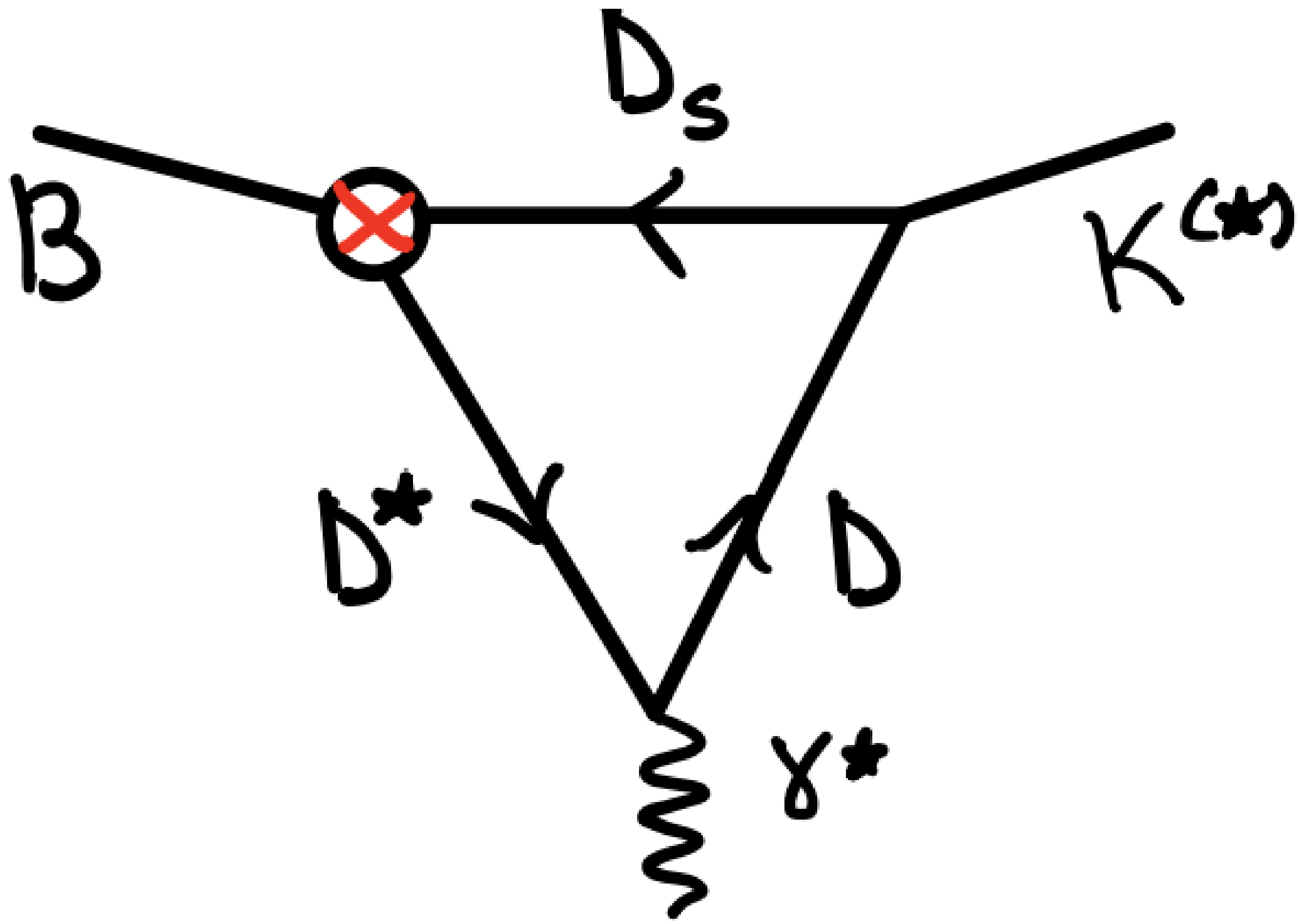
`EOS` approach: interpolate $q^2 < 0$ LCOPE and measurements of BRs/angular dists at $q^2 = M_{J/\psi}^2$.

$q^2 \to z(q^2),\ |z| < 1$



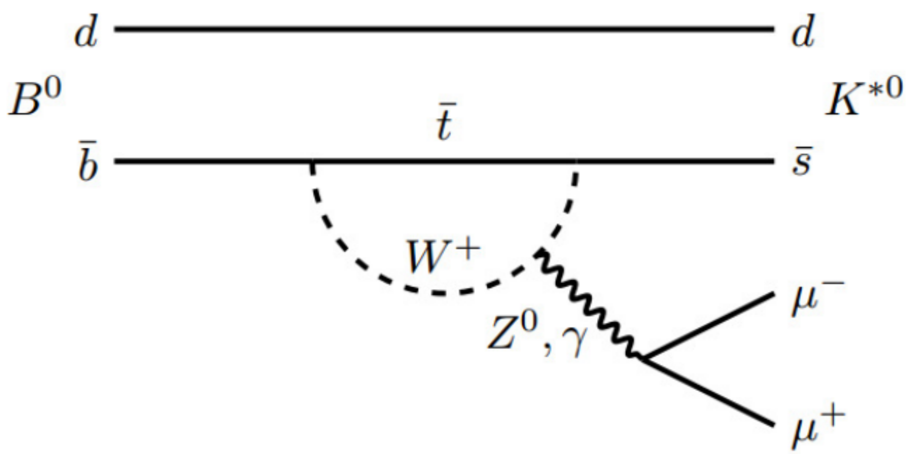
$C_9^{LD} \propto \sum_n a_n z^n$ 1707.07305 *truncation* 2205.03797

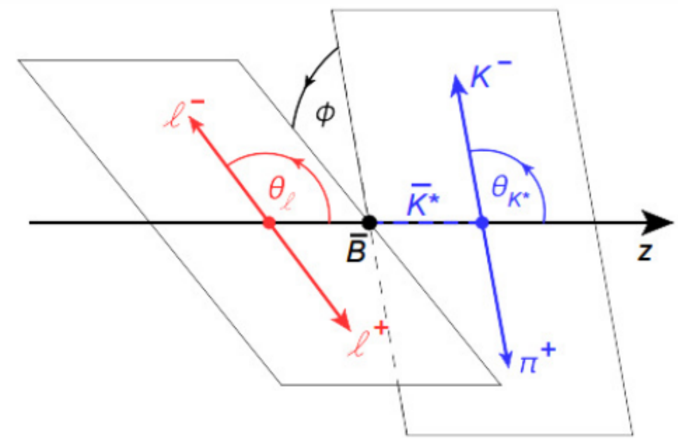
Caveat Emptor



2212.10516

$B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$

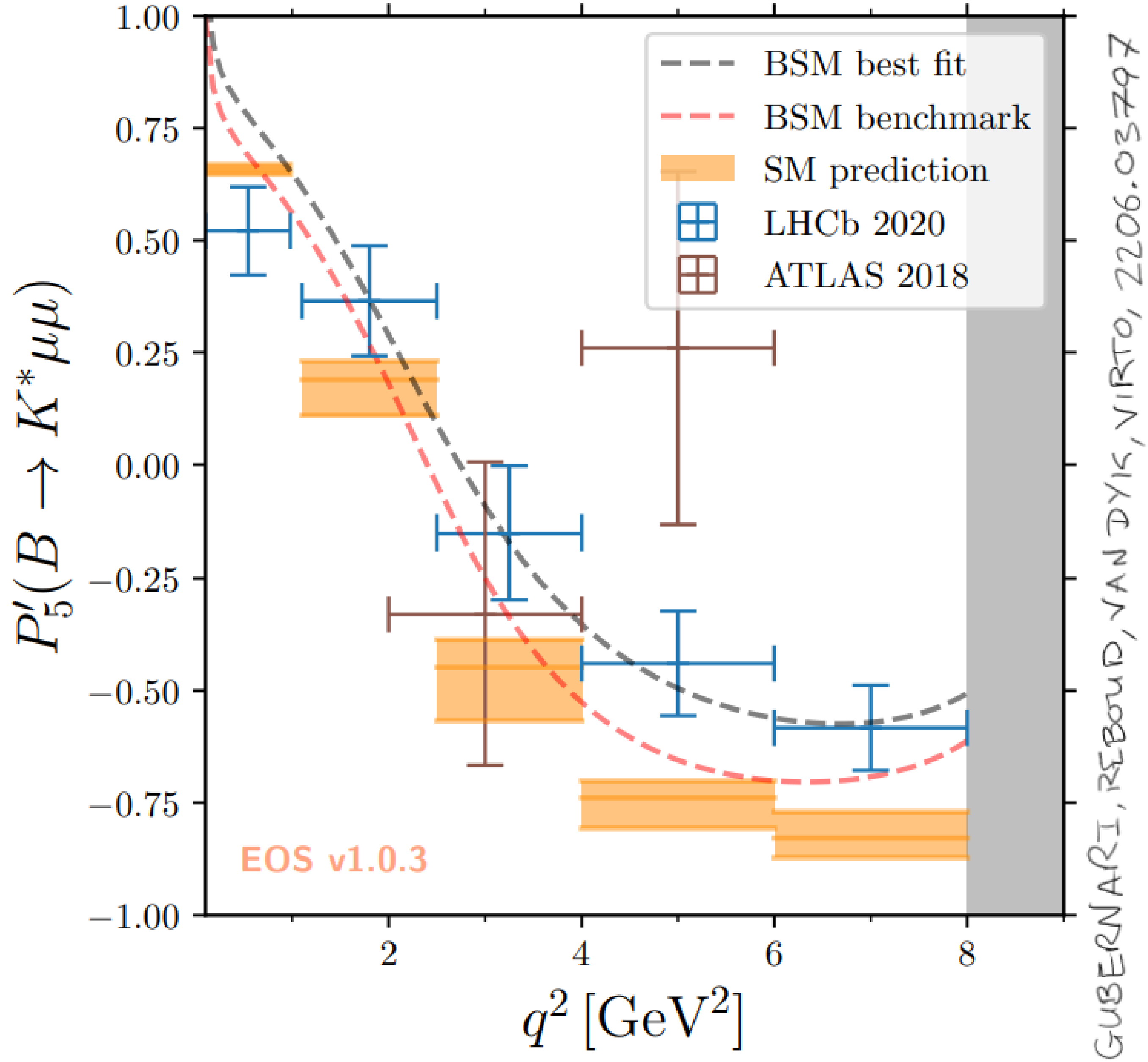




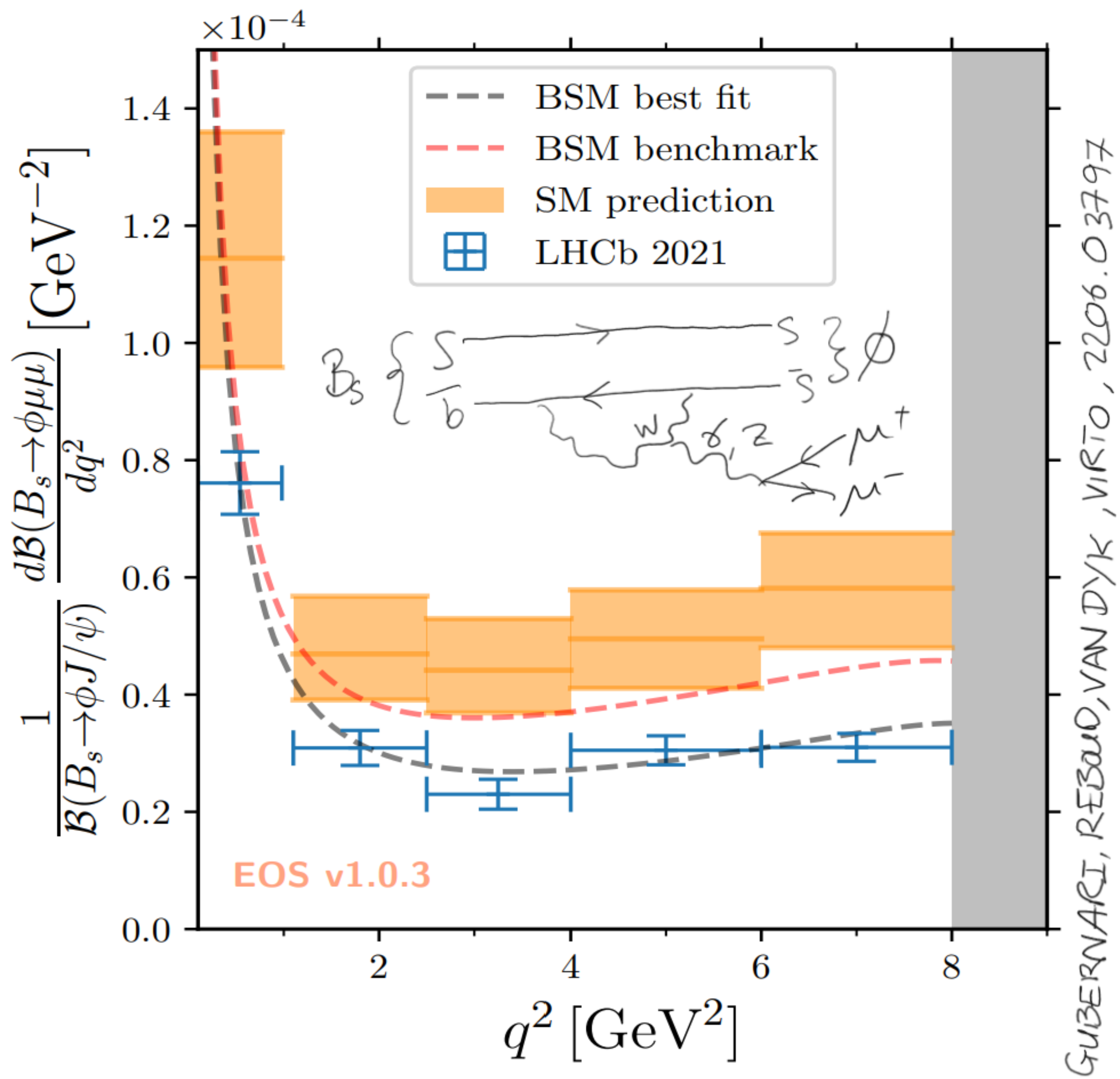
Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ \tfrac{4}{3}A_\mathrm{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\big]$$

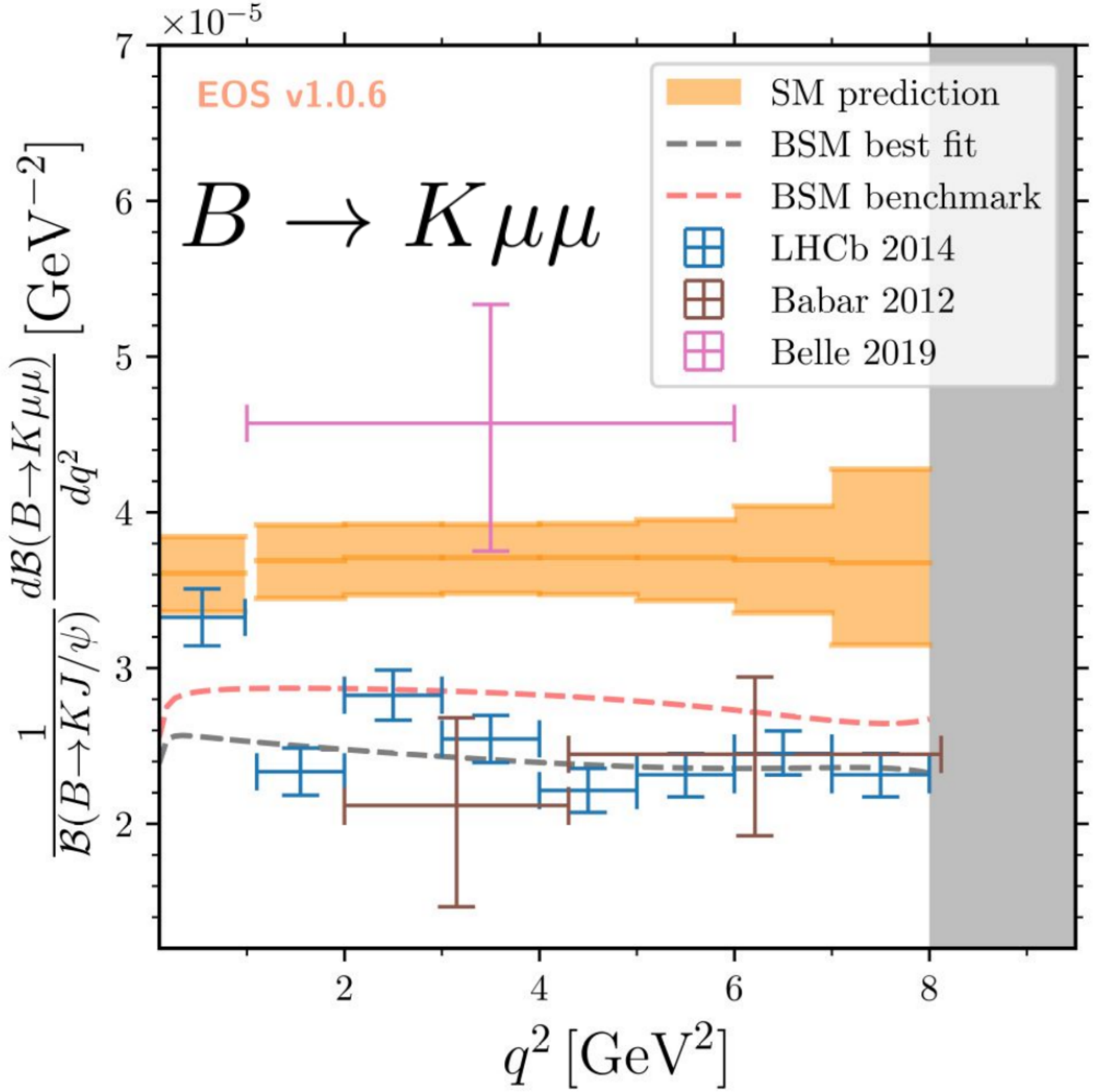
$$P_5' = S_5/\sqrt{F_L(1-F_L)}$$



$B_s \to \phi\mu^+\mu^-$: $\phi = (s\bar{s})$



$BR(B \to K\mu^+\mu^-)$



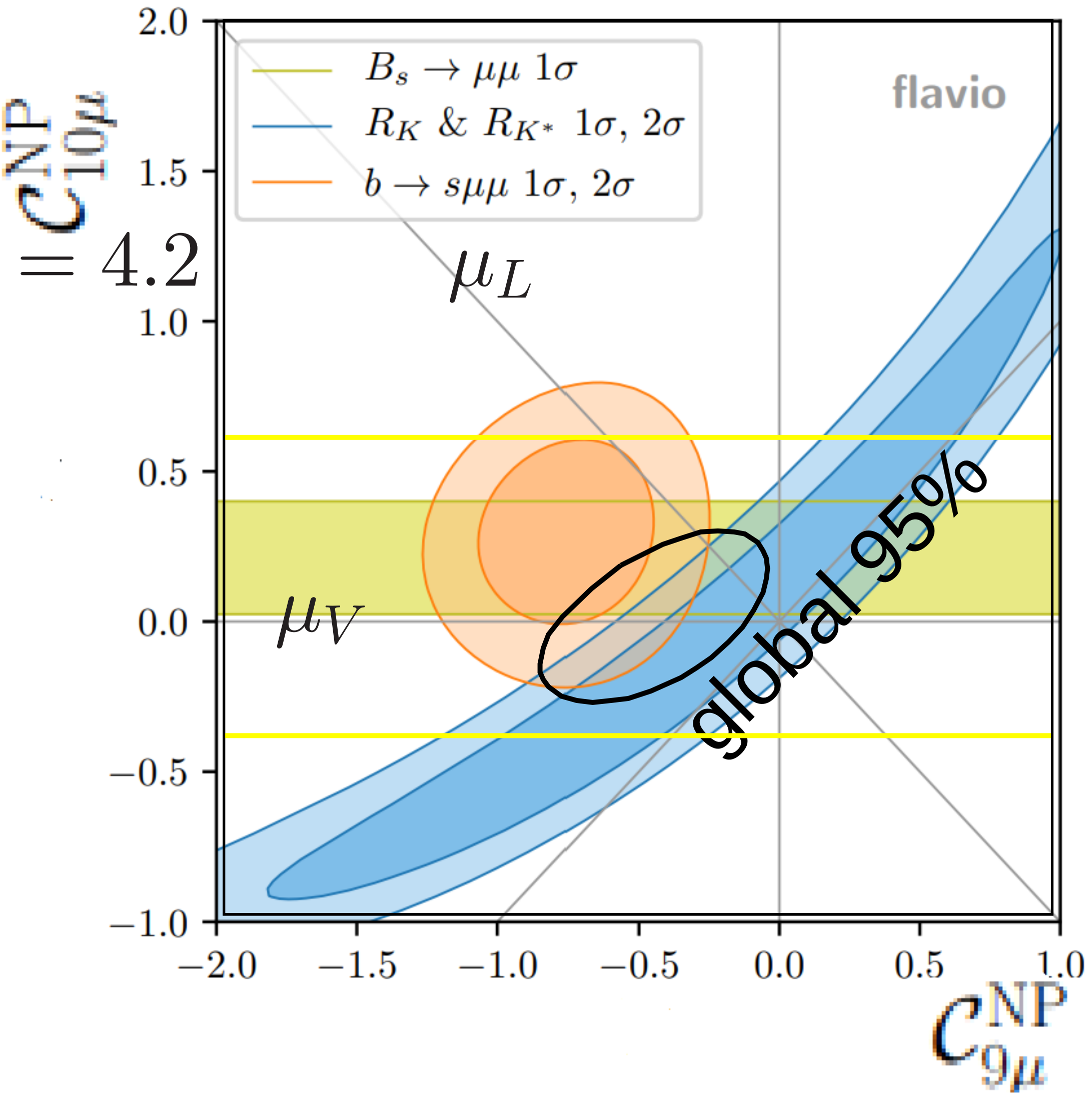
EOS v1.0.6
$B \to K\mu\mu$
SM prediction
BSM best fit
BSM benchmark
LHCb 2014
Babar 2012
Belle 2019
$\times 10^{-5}$
$\frac{1}{\mathcal{B}(B\to KJ/\psi)} \frac{d\mathcal{B}(B\to K\mu\mu)}{dq^2}$ [GeV^{-2}]
q^2 [GeV^2]

μ Neutral Current Fits

Greljo, Salko, Smolkovic, Stangl, 2212.10497

$$\mathcal{L} = N[C_{9\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\mu) + C_{10\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\gamma^5\mu)] + H.c.$$

$$C_9^{SM} = -C_{10}^{SM} = 4.2$$



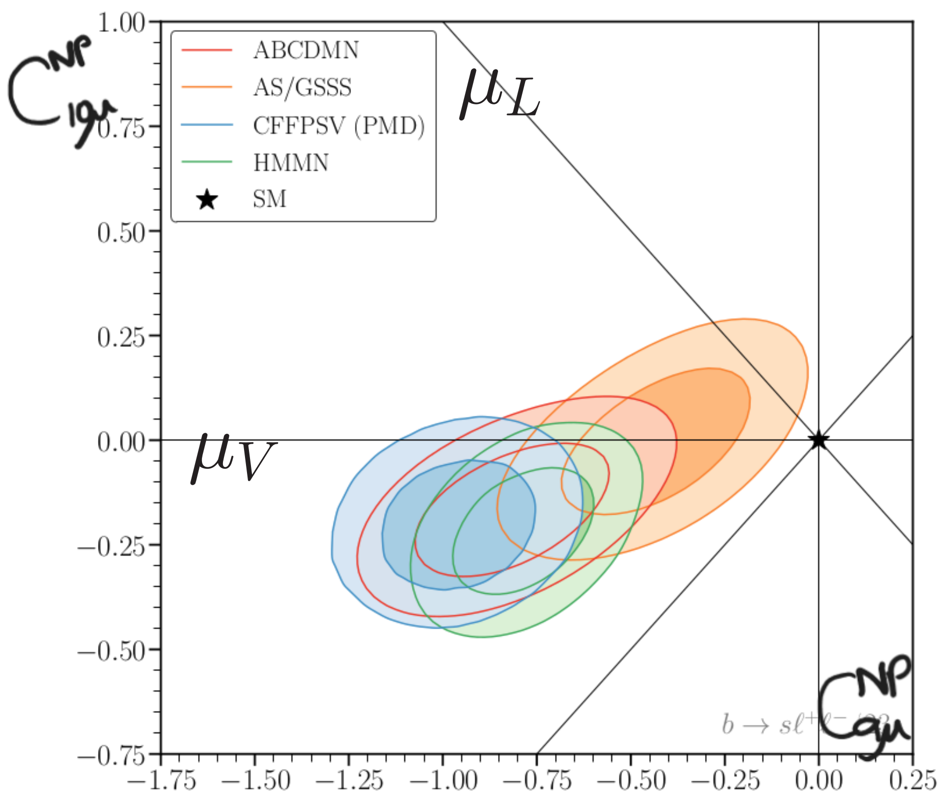
Neutral Current Fits Compare

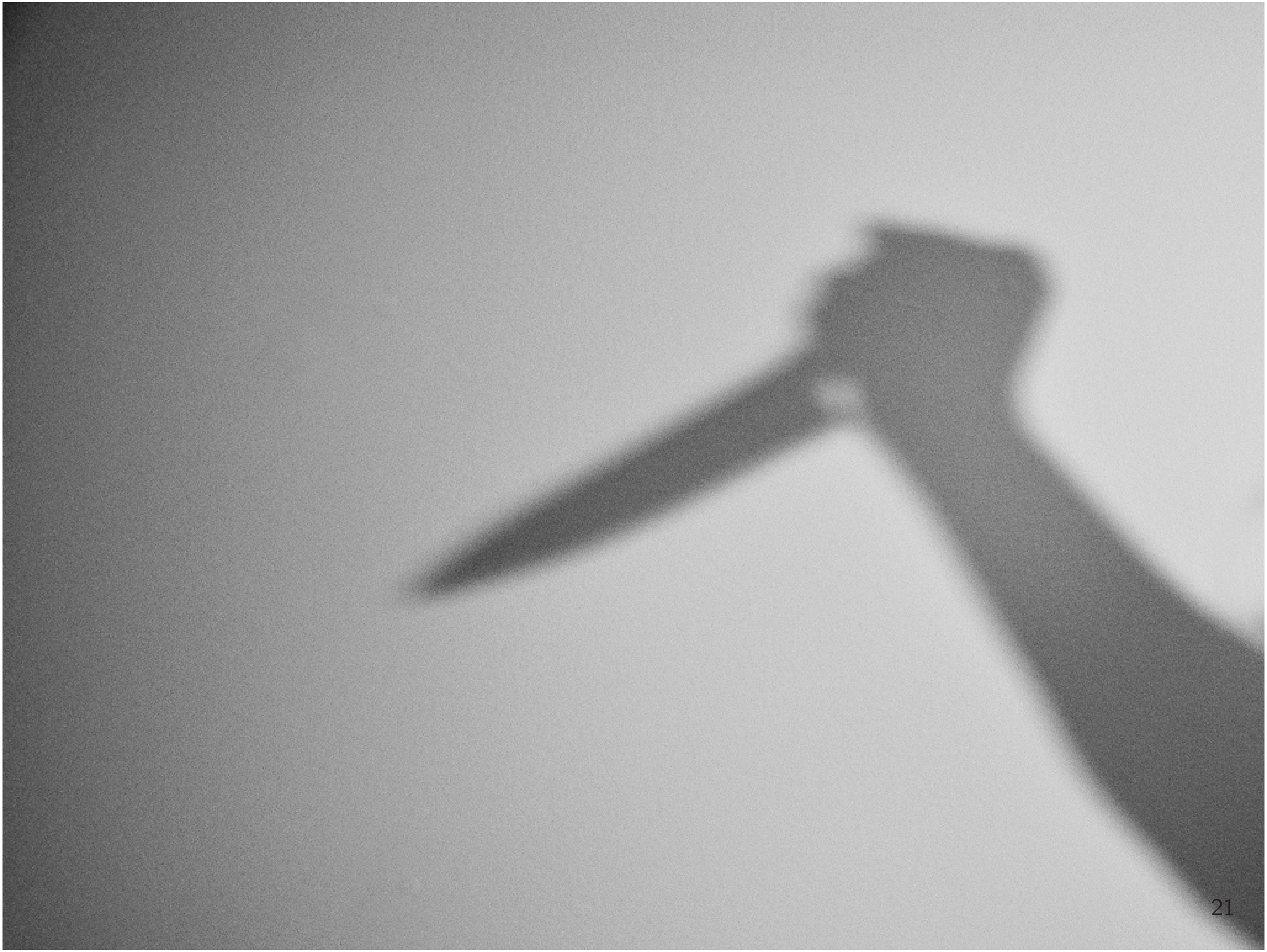
Alguero *et al*, 2304.07330; Altmannshofer, Stangl, flavio 2212.10497

Ciuchini *et al*, HEPfit 2212.10516; Hurth *et al*, superIso 23??.?????

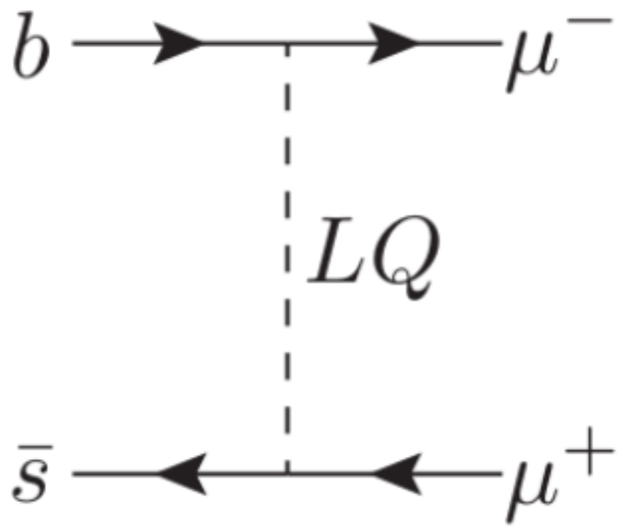
$$\mathcal{L} = N[C_{9\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\mu) + C_{10\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\gamma^5\mu)] + H.c.$$

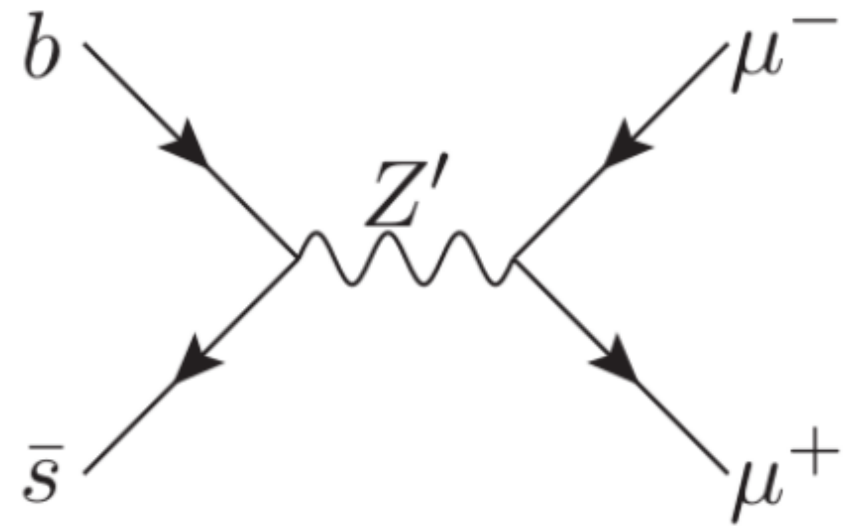
Plot from B Capdevila-Soler *Beyond Flavour Anomalies* workshop





Tree-level Explanations





Simple Z' Models

SM-singlet scalar ‘flavon’ θ

Additional $U(1)_X$ gauge symmetry broken by $\langle\theta\rangle$ ~TeV$\Rightarrow M_{Z'}$ ~TeV

SM$+3\nu_R$ fermion content

Zero charges for first two generations of quark

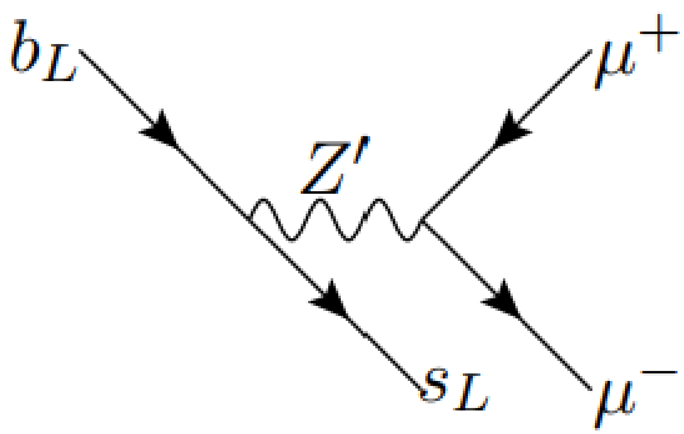
$X = 3B_3 - [L_1 L_e + L_2 L_\mu + L_3 L_\tau]$ postdicts some small CKM[1]. $L_3 = (3 - L_1 - L_2)$ anom free

[1] BCA, Mullin, 2306.08669

Important Z' Couplings

$$g_{Z'}\left[(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\begin{pmatrix}0 & 0 & 0\\ 0 & \sin^2\theta_{sb} & \frac{1}{2}\sin 2\theta_{sb}\\ 0 & \frac{1}{2}\sin 2\theta_{sb} & \cos^2\theta_{sb}\end{pmatrix}\not{Z}'\begin{pmatrix}d_L\\ s_L\\ b_L\end{pmatrix}\right.$$

$$\left.-(\overline{e}\ \overline{\mu}\ \overline{\tau})\begin{pmatrix}L_1 & 0 & 0\\ 0 & L_2 & 0\\ 0 & 0 & (3-L_1-L_2)\end{pmatrix}\not{Z}'\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix}\right]$$



– LFU Violating, $C_9 \neq 0$

SMEFT WCs$/(g_{Z'}^2/M_{Z'}^2)$

WC	value	WC	value
C_{ll}^{iiii}	$-\frac{1}{2}L_i^2$	C_{ll}^{iijj} $(i \neq j)$	$-L_i L_j$
$(C_{lq}^{(1)})^{iijk}$	$L_i(\Lambda_\Xi^{(d_L)})_{jk}$		
C_{ee}^{iijj} $(i \neq j)$	$-L_i L_j$	C_{uu}^{3333}	$-\frac{1}{2}$
C_{dd}^{3333}	$-\frac{1}{2}$	C_{ee}^{iiii}	$-\frac{1}{2}L_i^2$
C_{eu}^{ii33}	L_i	C_{ed}^{ii33}	L_i
$C_{ud}^{(1)\,3333}$	-1	C_{le}^{iijj}	$-L_i L_j$
C_{qe}^{ijkk}	$L_k(\Lambda_\Xi)_{ij}$	$C_{qu}^{(1)\,ij33}$	$-(\Lambda_\Xi)_{ij}$
$C_{qd}^{(1)\,ij33}$	$-(\Lambda_\Xi)_{ij}$	$C_{qq}^{(1)\,ijkl}$	$(\Lambda_\Xi)_{ij}(\Lambda_\Xi)_{kl}\frac{\delta_{ik}\delta_{jl}-2}{2}$
C_{lu}^{ii33}	L_i	C_{ld}^{ii33}	L_i

```
| wilson | flavio | smelli > output
```

Questions for SMEFiT

Flavour assumption wrong

Can you upgrade to 2499 parameters?

RGE done for you in `Wilson`

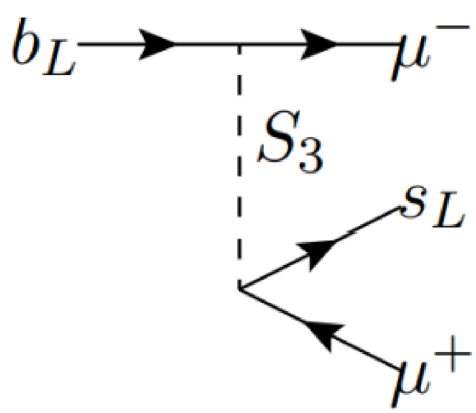
Matching also done in `Wilson`

Work is in observables: you'd be parallel to `flavio`

S_3 Leptoquark Model

TeV scale Scalar[2] $S_3 = (\bar{3},\ 3,\ 1/3)$:

$$\mathcal{L} = \ldots + \lambda Q_3' S_3 L_2 + \cancel{Y_{ij} Q_i Q_j S_3^\dagger} + \text{h.c.}$$
$$= \ldots + \lambda(\cos\theta_{23} Q_3 L_2 + \sin\theta_{23} Q_2 L_2) + \text{h.c.}$$



$$C_9 = -C_{10}$$

[2] Capdevila *et al* 1704.05340, Hiller and Hisandzic 1704.05444, D'Amico *et al* 1704.05438

Best fits `BCA, Davighi, 2211.11766`

2-parameter fits to 247 flavour observables:

`parameters | Wilson | flavio | smelli > output`

Model	$\sqrt{\chi^2_{SM} - \chi^2}$
$B_3 - L_2\ Z'$	3.6
S_3 LQ	3.6

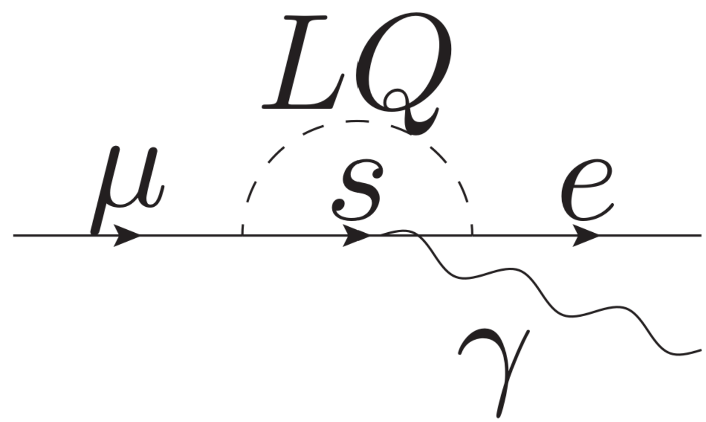
Coupling to electrons as well

Let's now switch on electron couplings in each model (eg $X_e \neq 0$ in Z' model).

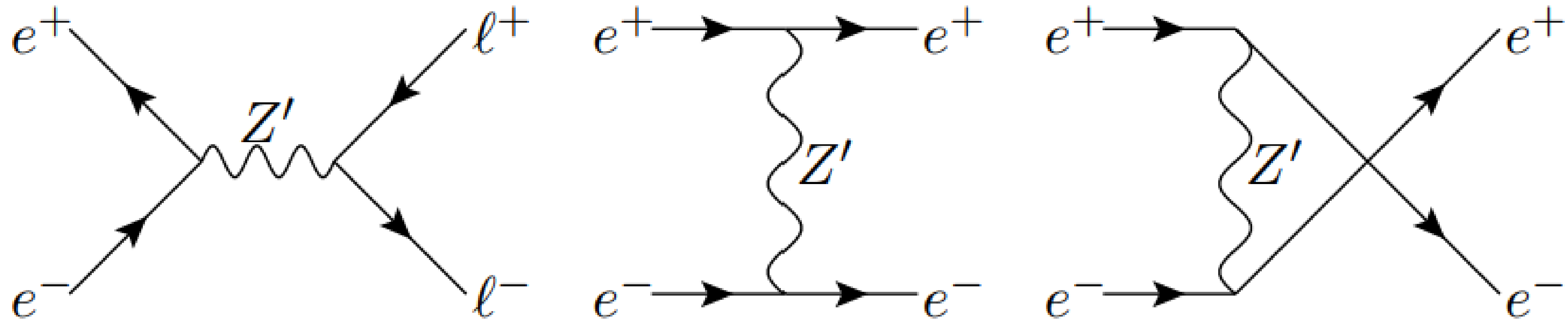
Leptoquark Explanation

Coupling a single LQ to electrons as well as muons will lead to trouble:

$BR(\mu \to e\gamma) < 4.2\ 10^{-13}$ (MEG 2016)



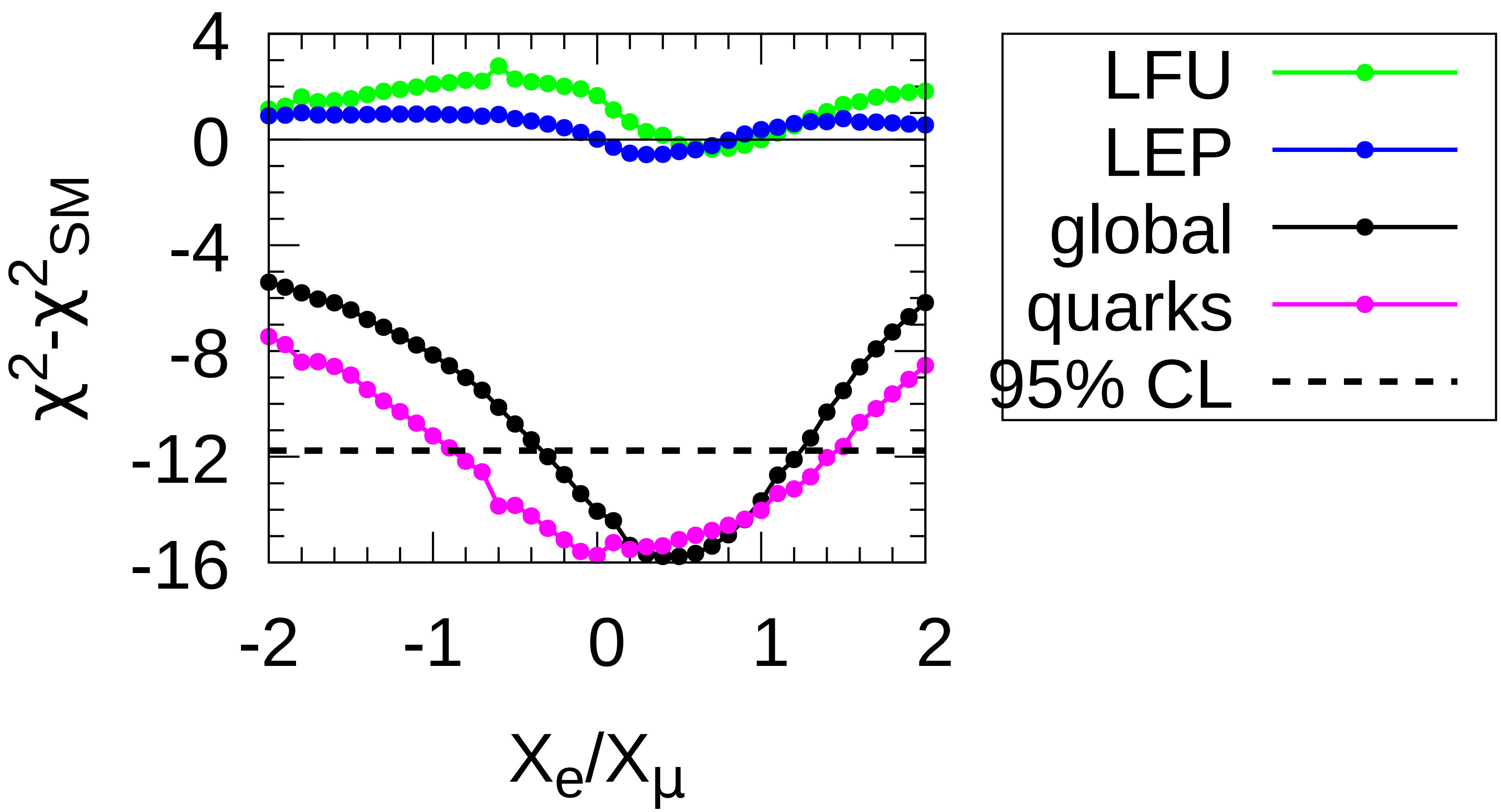
LEP constraints



Put into flavio (Falkowski, Mimouni 1511.07434)

Fit θ_{sb} and $g_{Z'}/M_{Z'}$

BCA, Mullin, 2306.08669

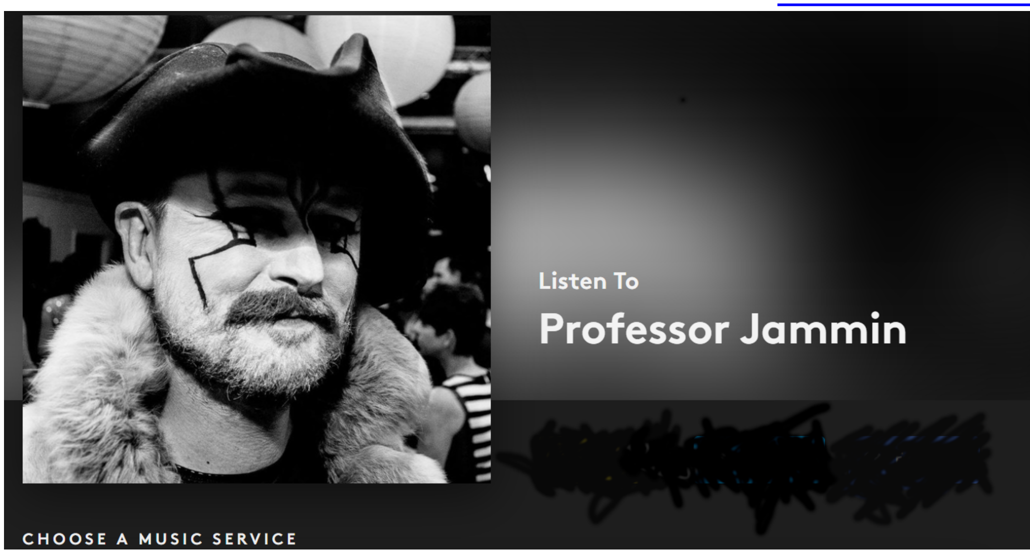


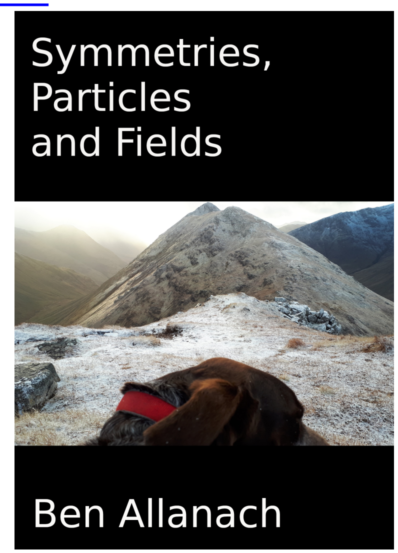
Epilogue

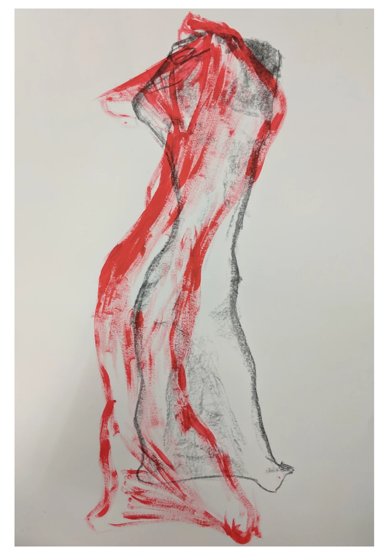
Need to upgrade to 2499 EFT parameters

Simple model with 2 parameters gives 100s of WCs

Links to my music, book (18€) and Quantum Selves art:







La Parilla for tonight

Benjamin A wrote a review
Yesterday

●●●●●

You go for the food and you stay for the art

"He was a nice friendly waiter and the restaurant had delicious food and wine. Try the lamb and mushroom ice cream. "

Date of visit: October 2023

Backup

Flavour problem

$$Y_u \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix},$$

Postdicts CKM angles $|V_{cb}|$, $|V_{ub}|$, $|V_{ts}|$, $|V_{td}|$ to be small

$$
\begin{aligned}
\mathcal{L}_{X\psi} &= g_X \Big(\overline{\mathbf{u_L}} \Lambda_{\xi}^{(u_L)} \not{Z}' \mathbf{u_L} + \overline{\mathbf{u_R}} \Lambda_{\xi}^{(u_R)} \not{Z}' \mathbf{u_R} \\
&\quad + \overline{\mathbf{d_L}} \Lambda_{\xi}^{(d_L)} \not{Z}' \mathbf{d_L} + \overline{\mathbf{d_R}} \Lambda_{\xi}^{(d_R)} \not{Z}' \mathbf{d_R} \\
&\quad - \overline{\mathbf{e_L}} \Lambda_{\Xi}^{(e_L)} \not{Z}' \mathbf{e_L} - \overline{\mathbf{e_R}} \Lambda_{\Xi}^{(e_R)} \not{Z}' \mathbf{e_R} \\
&\quad - \overline{\boldsymbol{\nu}_L} \Lambda_{\Xi}^{(\nu_L)} \not{Z}' \boldsymbol{\nu}_L - \overline{\boldsymbol{\nu}_R} \Lambda_{\Xi}^{(\nu_R)} \not{Z}' \boldsymbol{\nu}_R \Big) ,
\end{aligned}
$$

$$
\Lambda_{\substack{\xi \\ \Xi}}^{(I)} \equiv V_I^{\dagger} {}_{\Xi}^{\xi} V_I, \;\; \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \Xi = \begin{pmatrix} X_e & 0 & 0 \\ 0 & X_\mu & 0 \\ 0 & 0 & X_\tau \end{pmatrix}
$$

$$
X_\tau = 3 - X_e - X_\mu \; (\texttt{BCA, Mullin}, 2306.08669)
$$

Z' couplings, $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

A simple limiting case

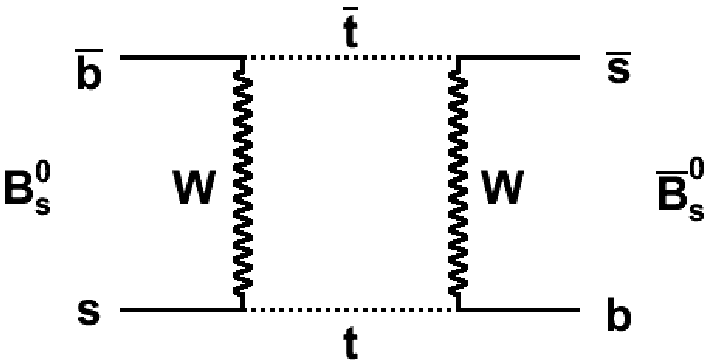
$$V_{u_R} = V_{d_R} = V_{e_L} = V_{e_R} = 1$$

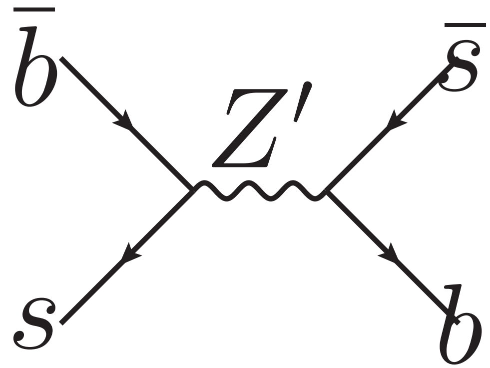
$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{sb} & -\sin\theta_{sb} \\ 0 & \sin\theta_{sb} & \cos\theta_{sb} \end{pmatrix}.$$

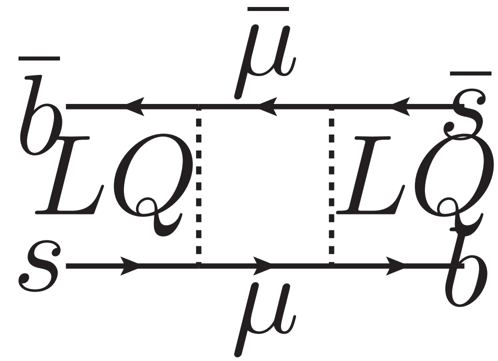
$\Rightarrow V_{u_L} = V_{d_L} V^{\dagger}_{CKM}$ and $V_{\nu_L} = V_{e_L} U^{\dagger}_{PMNS}$.

$B_s - \bar{B}_s$ Mixing

Measurement agrees with SM.







$$g_{sb} = \frac{g_X}{2} \sin 2\theta_{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}}$$ but uncertain

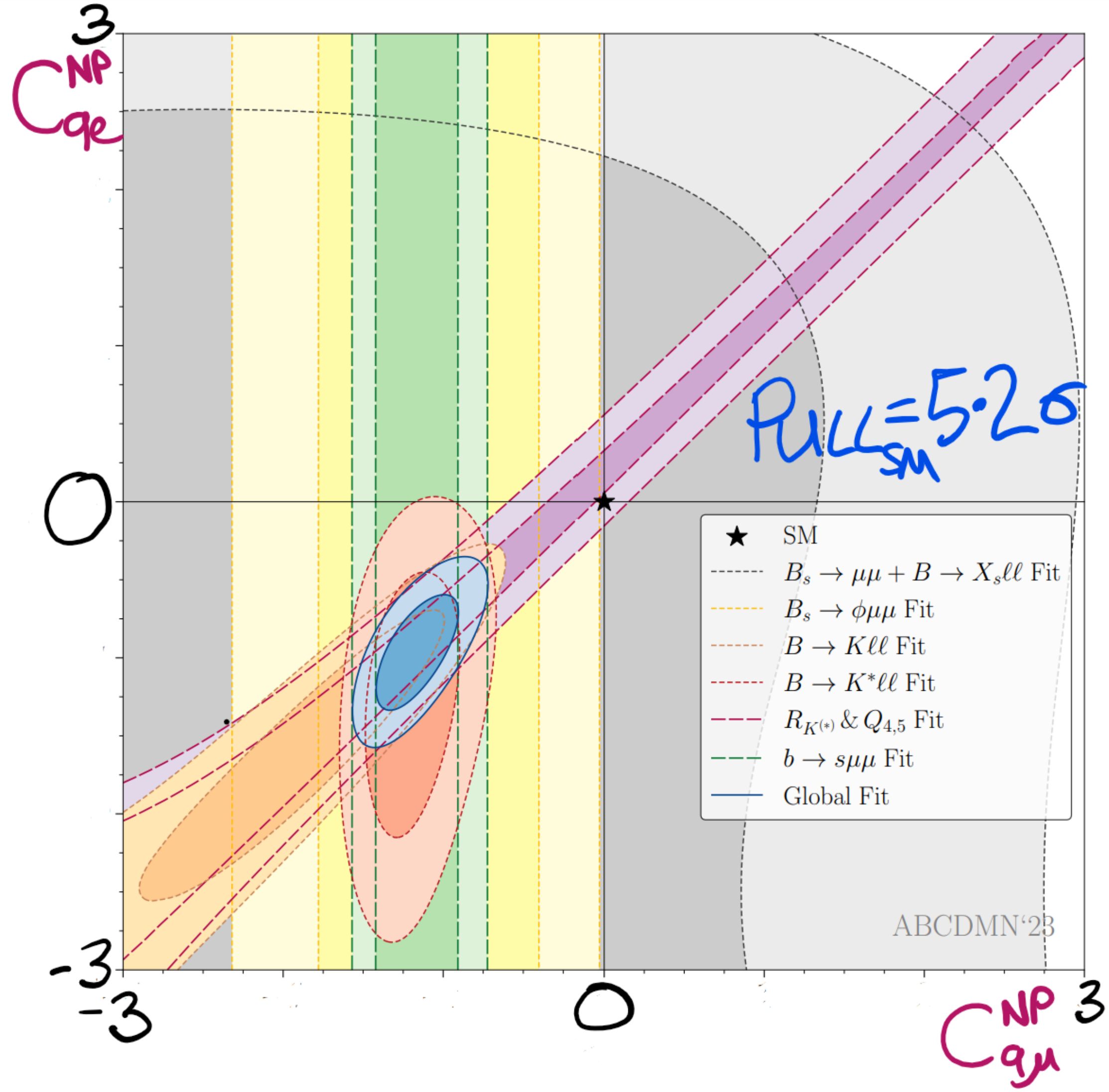
from QCD sum rules and lattice[3].

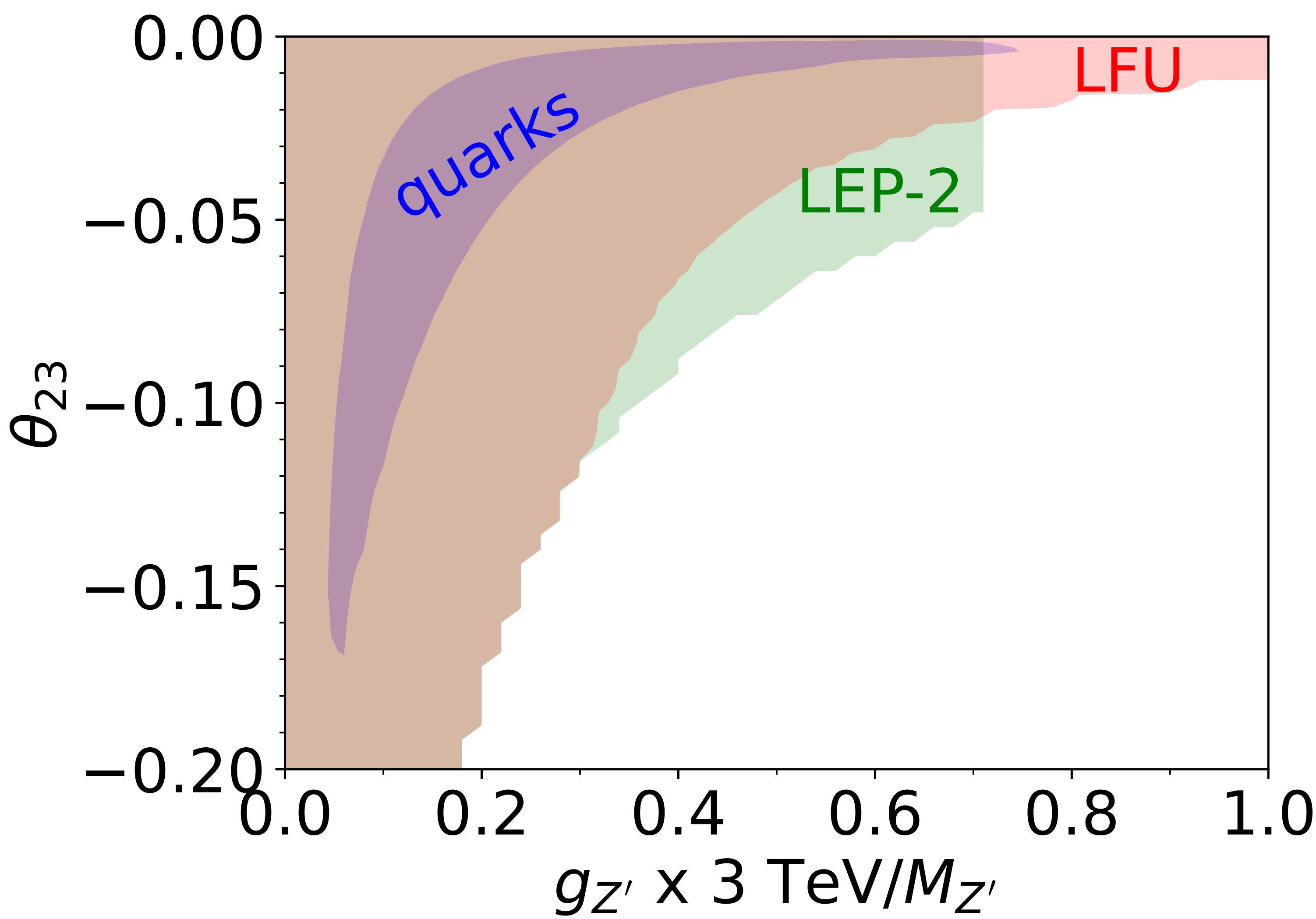
[3] King, Lenz, Rauh, arXiv:1904.00940

μ/e Neutral Current Fits

Alguero *et al*, 2304.07330

$$\mathcal{L} = N[C_{9\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\mu) + C_{9e}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{e}\gamma_\alpha e)] + H.c.$$





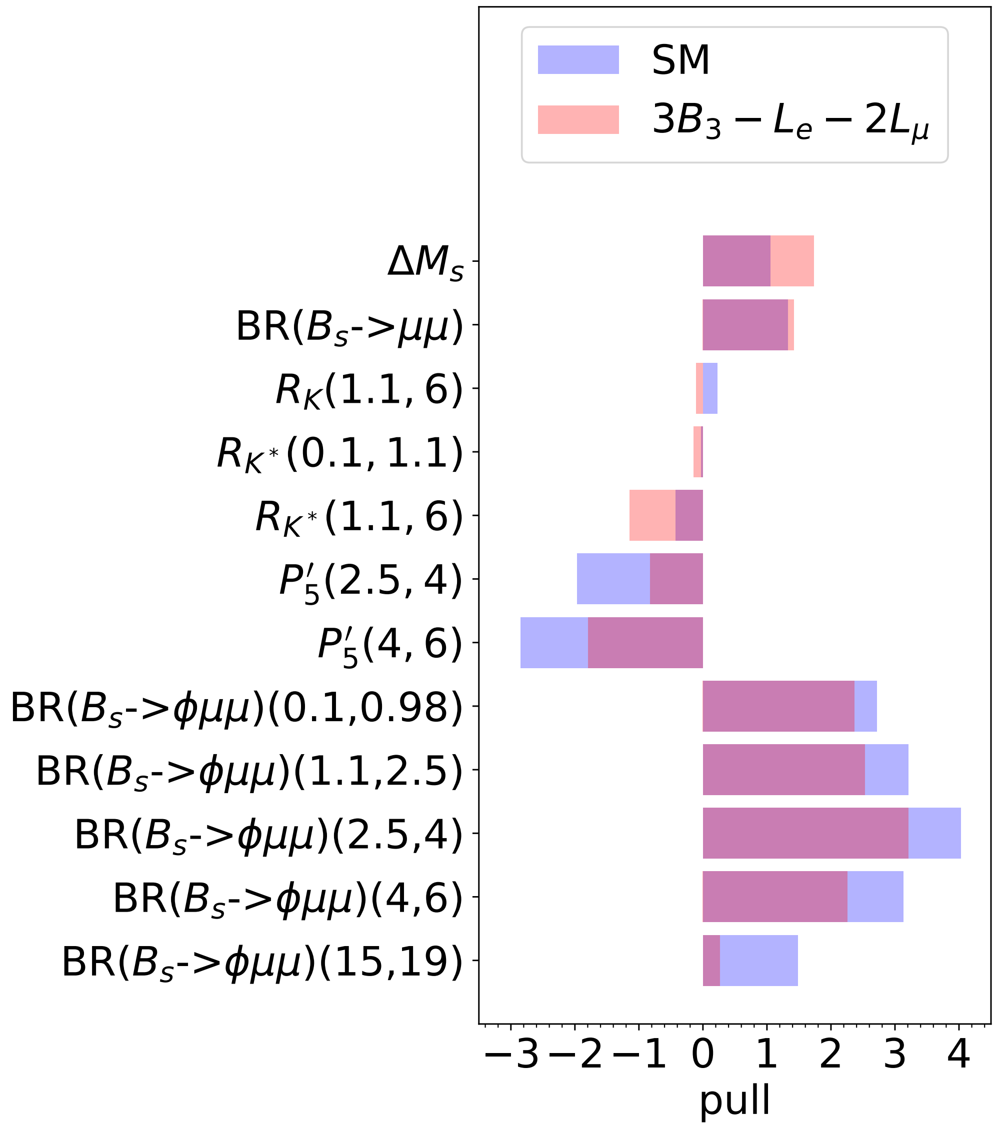
BCA, Mullin, 2306.08669

$3B_3 - L_e - 2L_\mu$ model

	$\chi^2 - \chi^2_{SM}$	p−value	measurement	pull
LFU	-0.2	.85	$R_{K^*}(0.1,\ 1.1)$	-0.1
LEP	-0.4	.58	$R_{K^*}(1.1,\ 6)$	-1.1
quarks	-14.7	.10	$R_K(0.1,\ 1.1)$	-0.3
global	-15.3	.28	$R_K(1.1,\ 6)$	-0.1

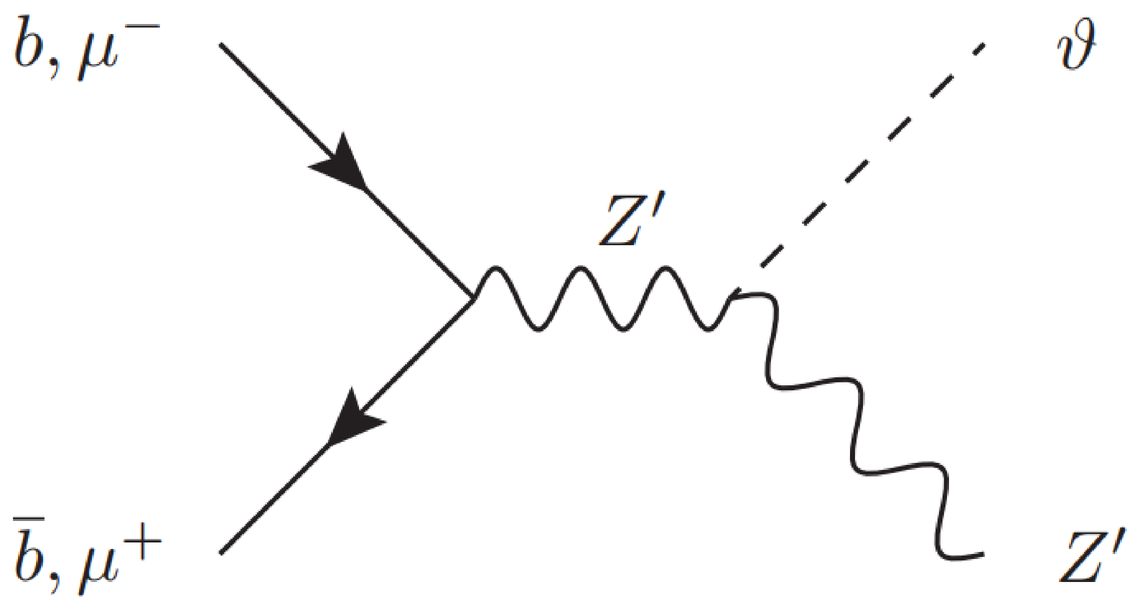
$g_{Z'} = 0.2,\ \theta_{sb} = -0.03$ best-fit

BCA, Mullin, 2306.08669


SM
$3B_3 - L_e - 2L_\mu$
ΔM_s
BR(B_s->$\mu\mu$)
$R_K(1.1, 6)$
$R_{K^*}(0.1, 1.1)$
$R_{K^*}(1.1, 6)$
$P_5'(2.5, 4)$
$P_5'(4, 6)$
BR(B_s->$\phi\mu\mu$)(0.1,0.98)
BR(B_s->$\phi\mu\mu$)(1.1,2.5)
BR(B_s->$\phi\mu\mu$)(2.5,4)
BR(B_s->$\phi\mu\mu$)(4,6)
BR(B_s->$\phi\mu\mu$)(15,19)
−3 −2 −1 0 1 2 3 4
pull

Flavonstrahlung

Models of Z' ilk possess $\mathcal{L} = \lambda H H^\dagger \theta \theta^\dagger \Rightarrow$ a *flavonstrahlung* signature:



BCA, 2009.02197; BCA, Loisa, 2212.07440

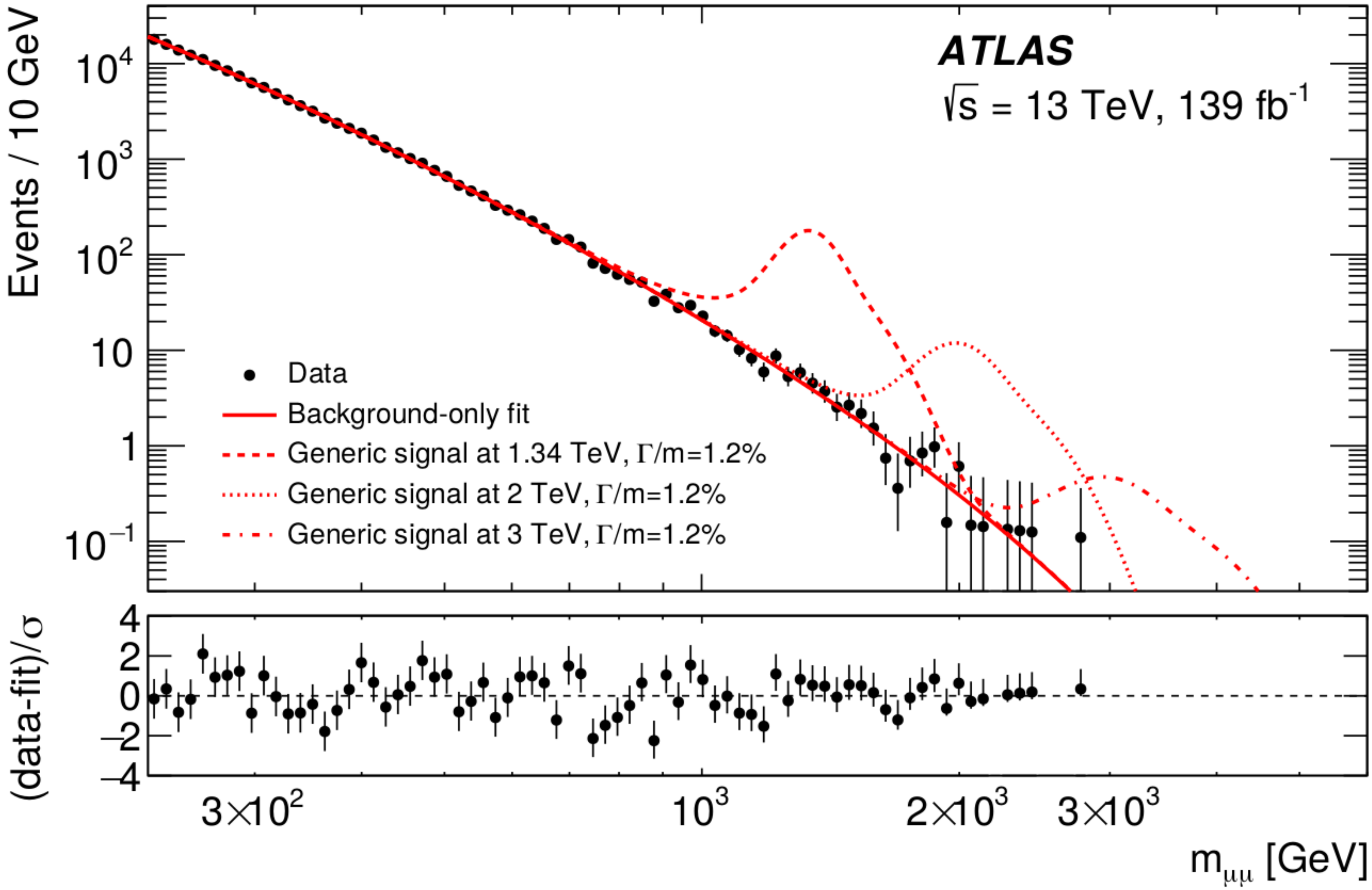
$\mu\mu$ ATLAS 13 TeV 139 fb^{-1}

2 track-based isolated μ, $p_T > 30$ GeV with reconstructed vertex.[4] Only keep pair with highest $(|p_{T_1}| + |p_{T_2}|)$.

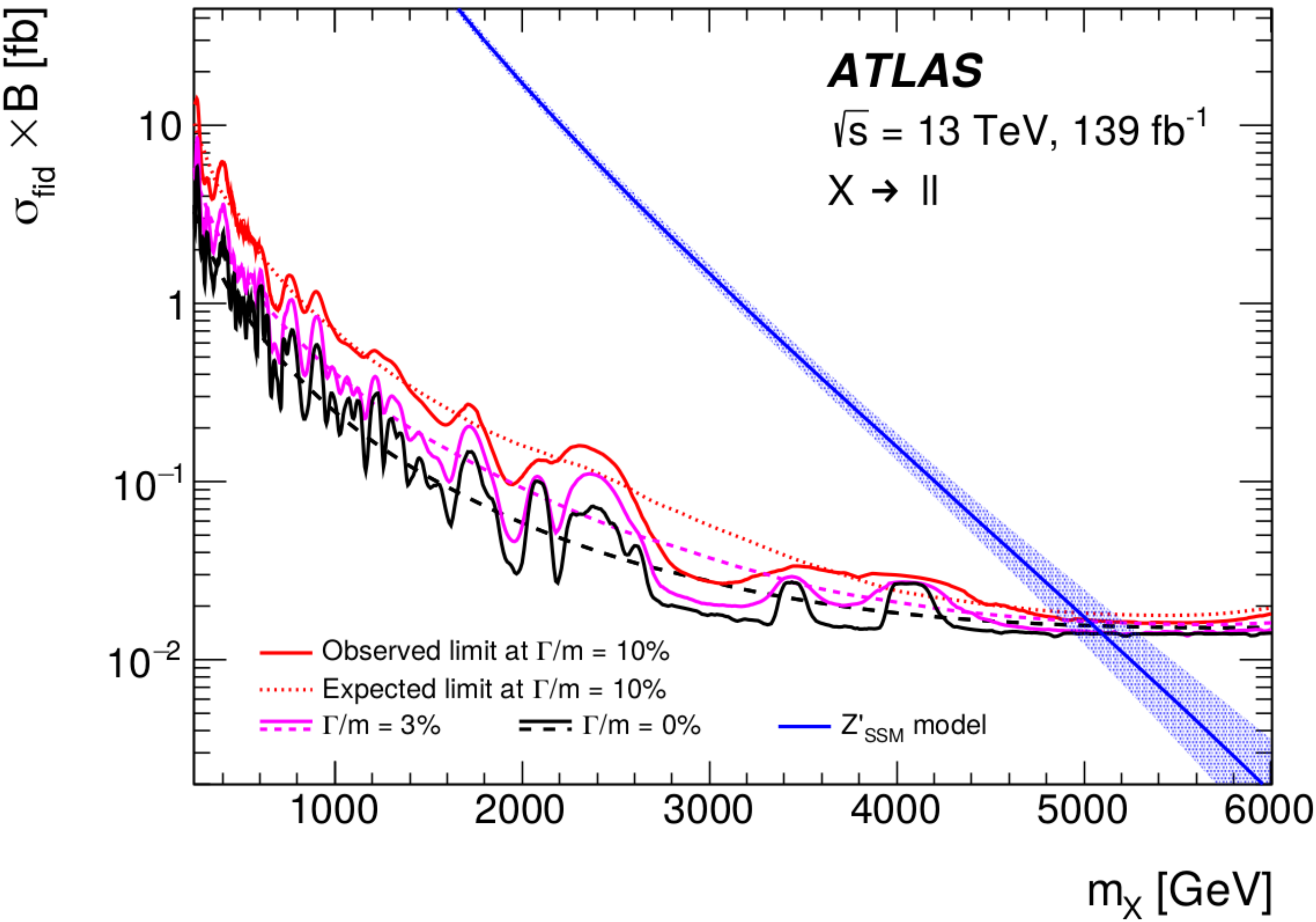
$$m^2_{\mu_1\mu_2} = (p_1^\mu + p_2^\mu)\left(p_{1_\mu} + p_{2_\mu}\right)$$

CMS also has a similar analysis[5]

[4] ATLAS, 1903.06248
[5] CMS, 2103.02708

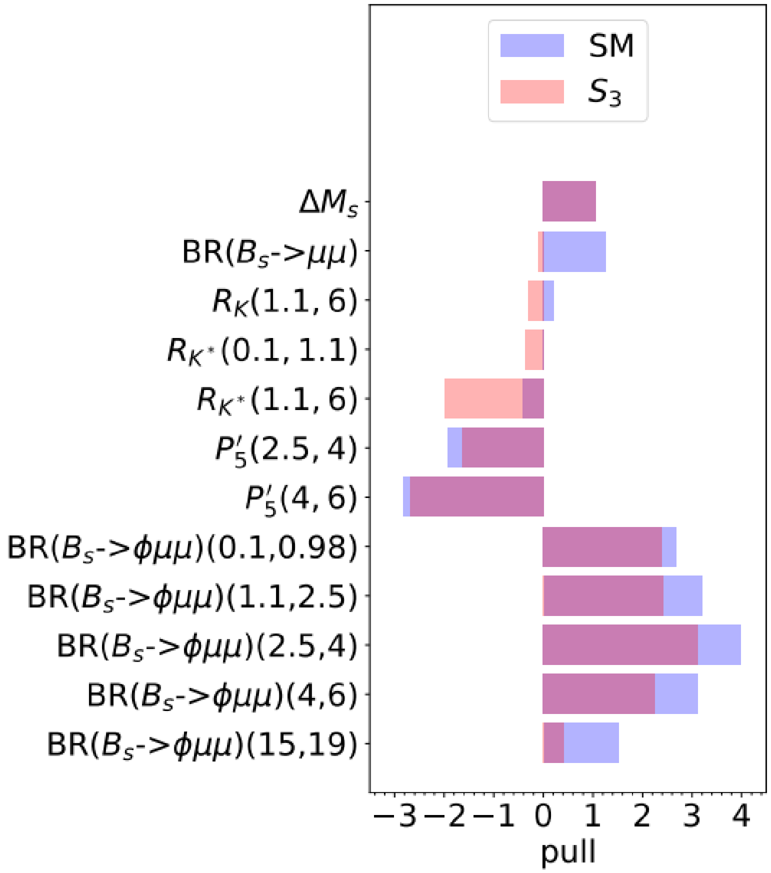

ATLAS
$\sqrt{s}$ = 13 TeV, 139 fb^{-1}
Events / 10 GeV
Data
Background-only fit
Generic signal at 1.34 TeV, Γ/m=1.2%
Generic signal at 2 TeV, Γ/m=1.2%
Generic signal at 3 TeV, Γ/m=1.2%
(data-fit)/σ
$m_{\mu\mu}$ [GeV]

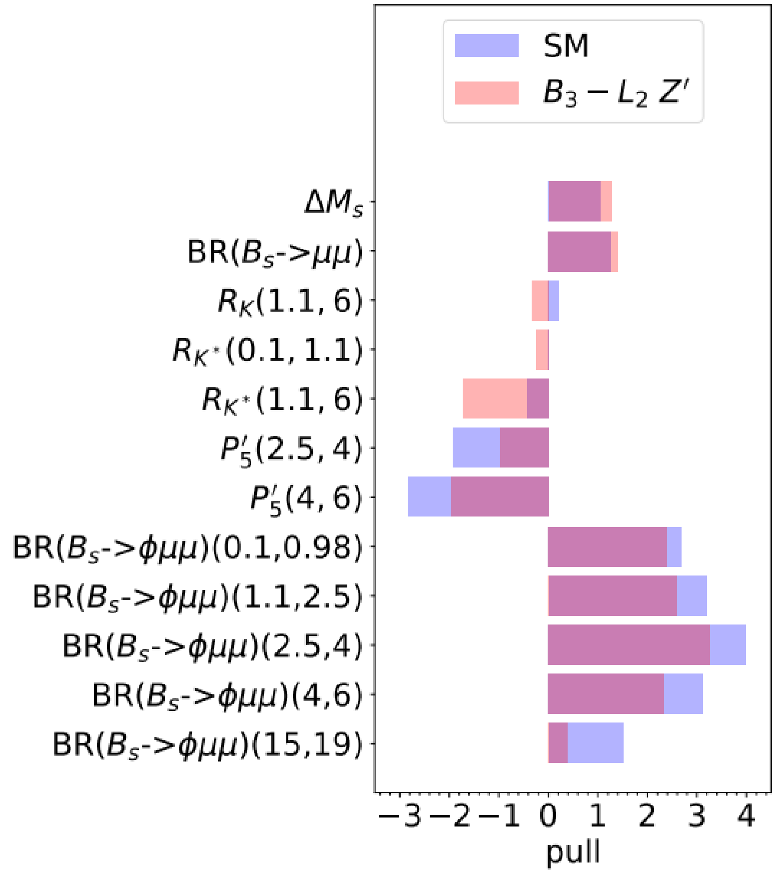
ATLAS l^+l^- limits



ATLAS, 1903.06248

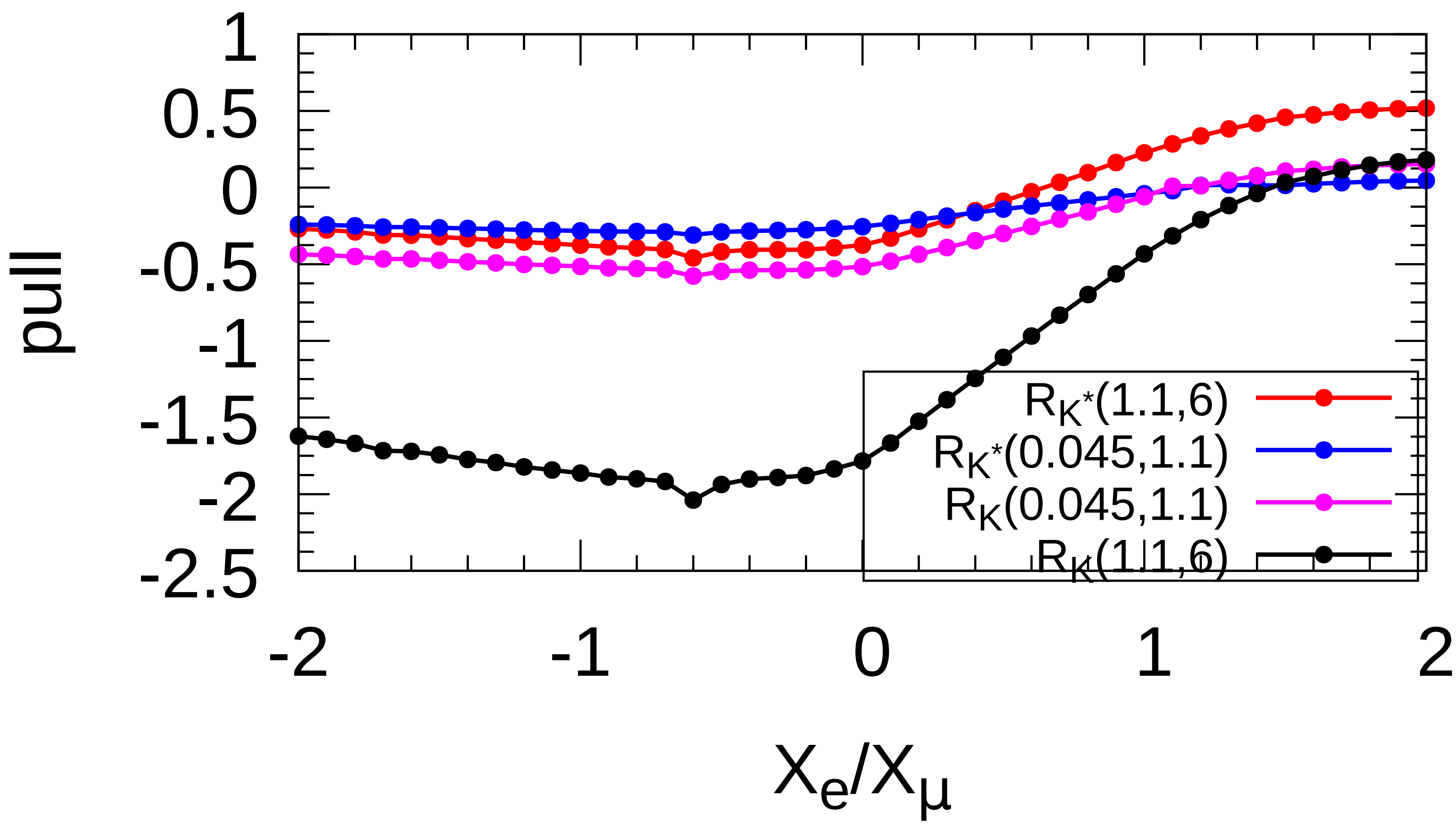
Pull=(theory-exp)/error

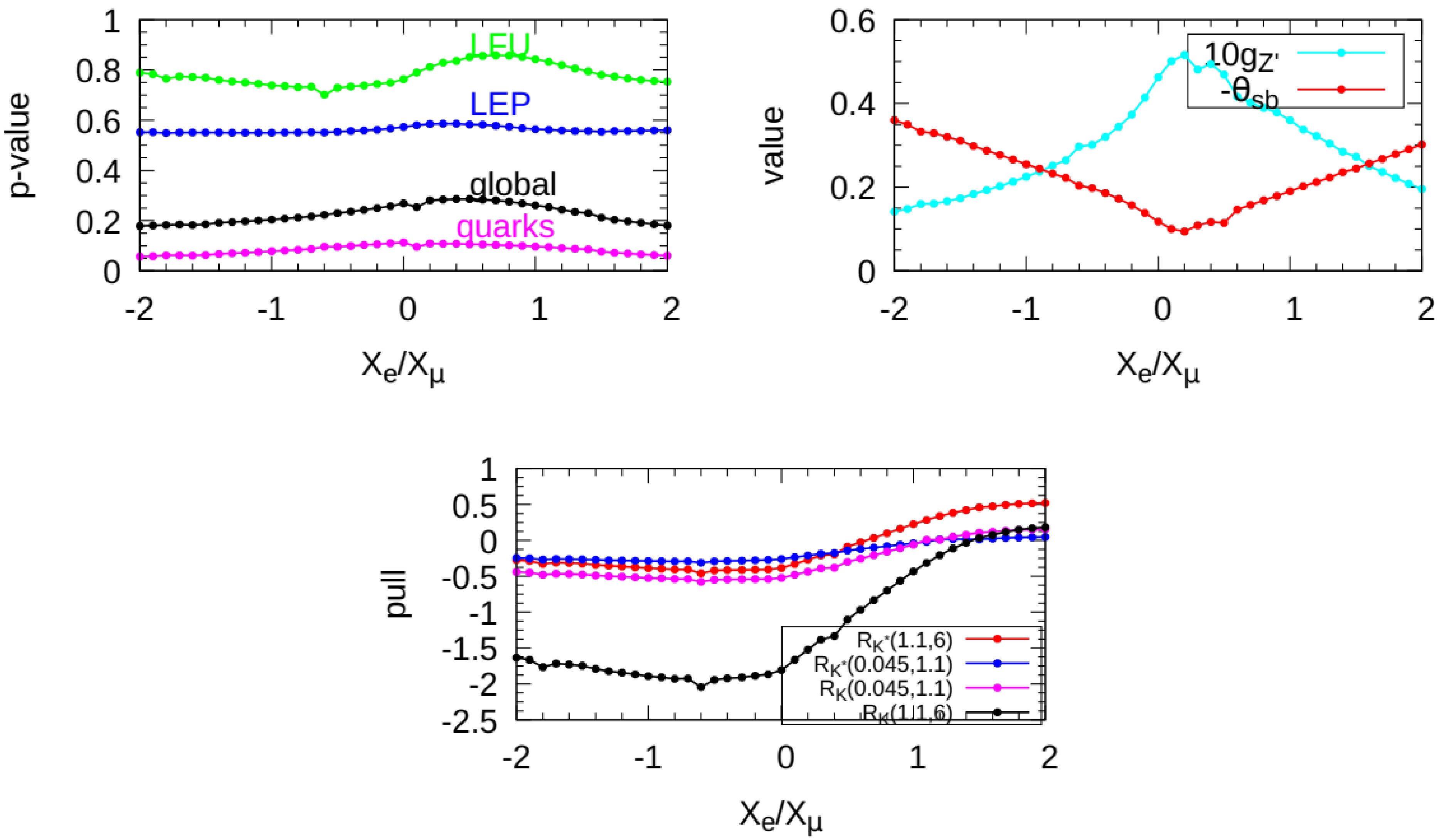




BCA, Davighi, 2211.11766

BCA, Mullin, 2306.08669



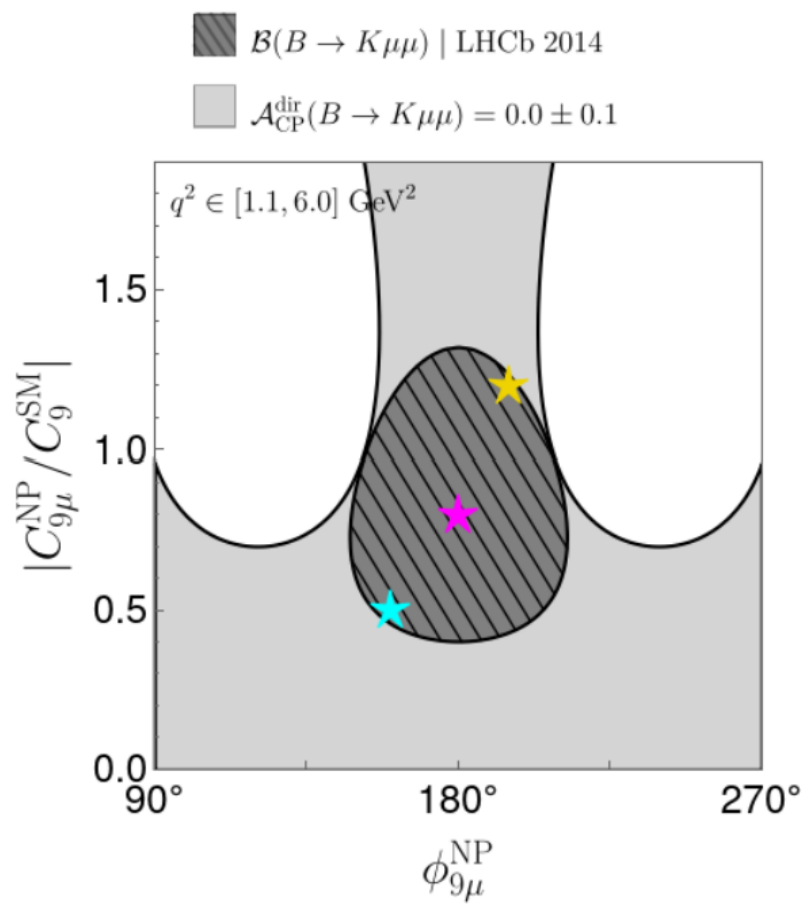


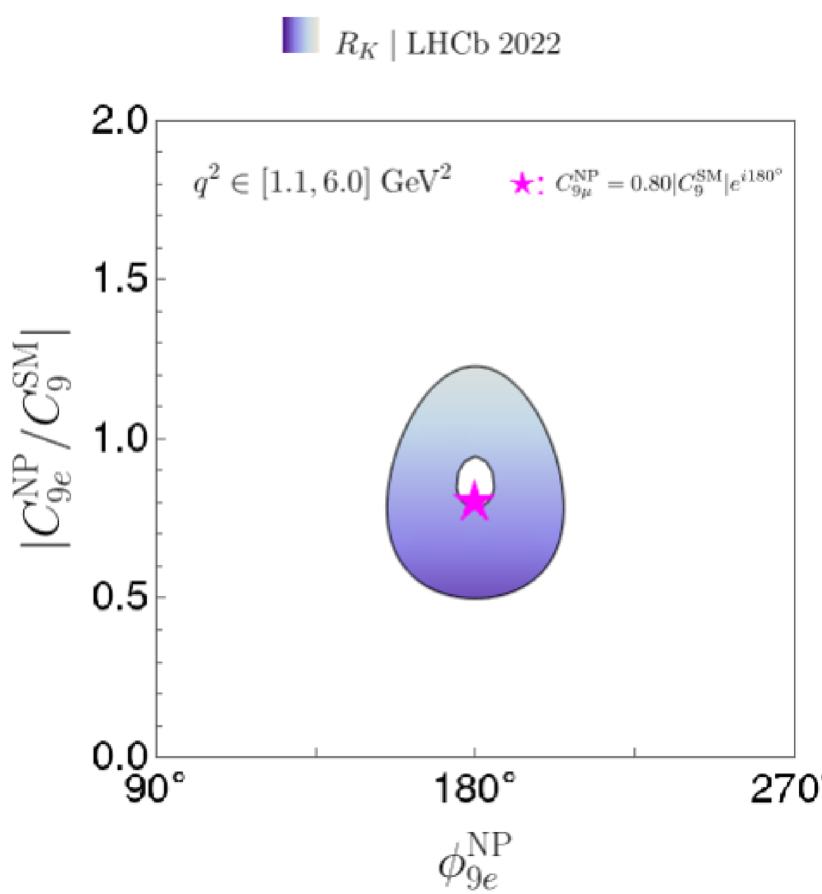
p-value
LFU
LEP
global
quarks
X_e/X_μ
value
$10g_{Z'}$
$-\theta_{sb}$
X_e/X_μ
pull
$R_{K^*}(1.1,6)$
$R_{K^*}(0.045,1.1)$
$R_K(0.045,1.1)$
$R_K(1.1,6)$
X_e/X_μ

$e \neq \mu$ allowed

Fleischer, Malami, Rehult, Keri Vos, 2303.08764; $C_{9\ell}^{NP} = |C_{9\ell}^{NP}| e^{i\phi_{9\ell}^{NP}}$

$$\mathcal{L} = N(\bar{b}_L \gamma^\alpha s_L)[C_{9\mu}^{NP}(\bar{\mu}\gamma_\alpha \mu) + C_{9e}^{NP}(\bar{e}\gamma_\alpha e)] + H.c.$$





Anomaly cancellation

Need to pick X charges for fermions consistent with QFT anomaly cancellation.

$$X = 3B_3 - (X_e L_e + X_\mu L_\mu + [3 - X_e - X_\mu] L_\tau)$$

works (proof in 2306.08669).

Trident Neutrino Process

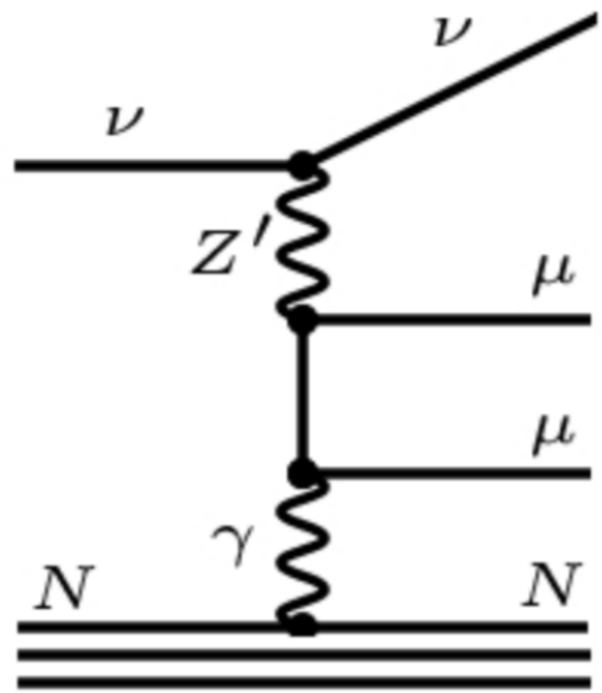
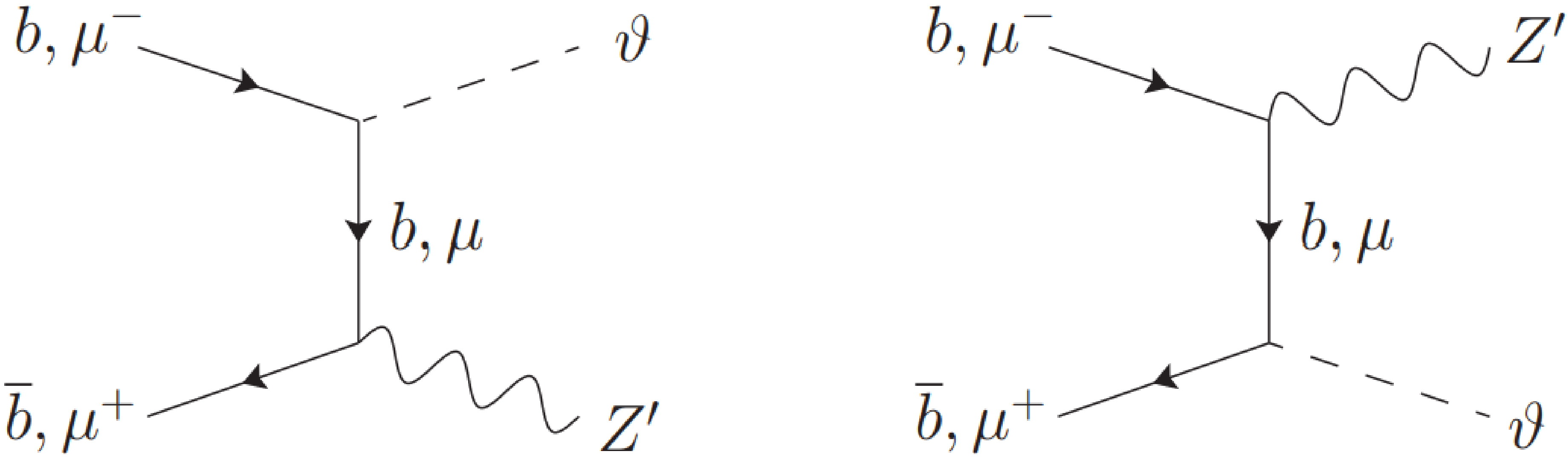
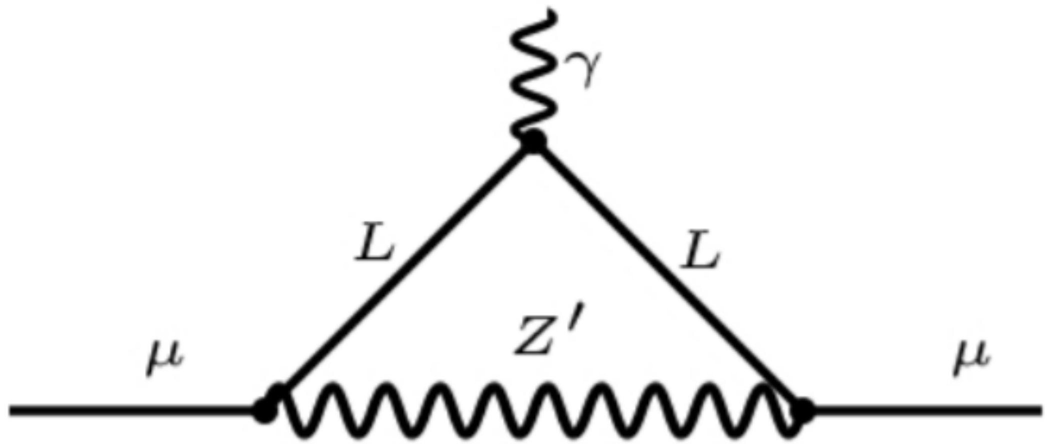


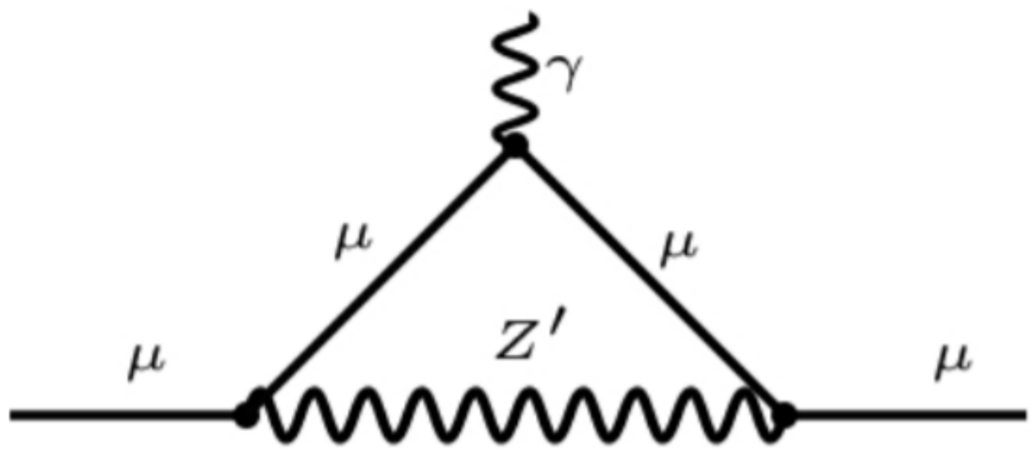
FIG. 10. Neutrino trident process that leads to constraints on the Z^{μ} coupling strength to neutrinos-muons, namely $M_{Z'}/g_{v\mu} \gtrsim 750$ GeV.

$t-$channel



$(g-2)_\mu$





$H\vartheta$ potential

$$
\begin{aligned}
V &= -\mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_\theta^2 \theta^* \theta + \\
&\quad \lambda_\theta (\theta^* \theta)^2 + \lambda_{\theta H} \theta^* \theta H^\dagger H \\
&= -\frac{1}{2} \begin{pmatrix} h' & \vartheta' \end{pmatrix} M^2 \begin{pmatrix} h' \\ \vartheta' \end{pmatrix} + \ldots \\
M^2 &= \begin{pmatrix} 2\lambda_H \upsilon_H^2 & \lambda_{\theta H} \upsilon_H \upsilon_\theta \\ \lambda_{\theta H} \upsilon_H \upsilon_\theta & 2\lambda_\theta \upsilon_\theta^2 \end{pmatrix}
\end{aligned}
$$

$H\vartheta$ **mixing**

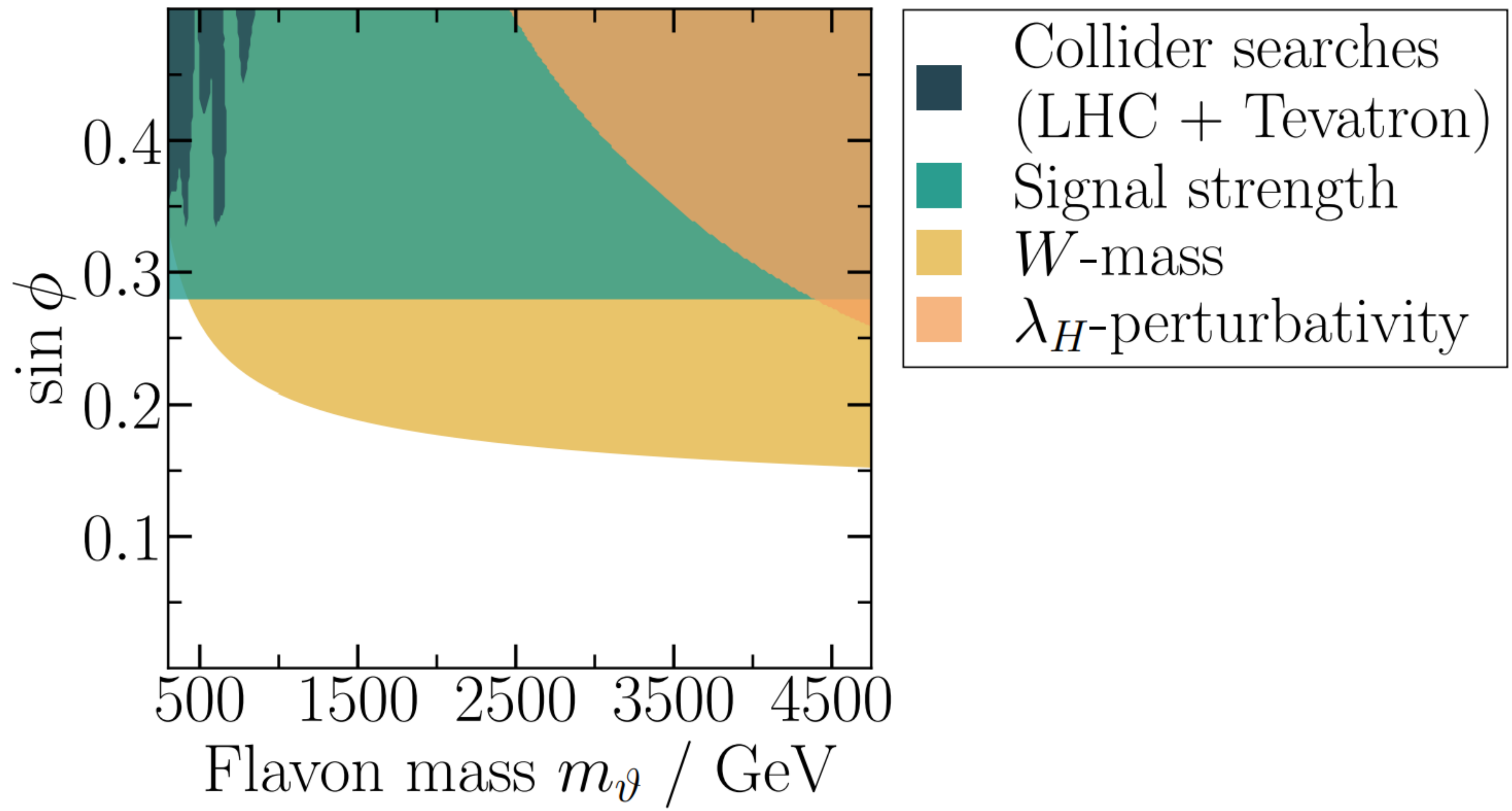
$$\begin{pmatrix} h \\ \vartheta \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} h' \\ \vartheta' \end{pmatrix}$$

$$\sin 2\phi = \frac{2\lambda_{\theta H} v_h v_\theta}{m_\vartheta^2 - m_h^2}. \qquad (1)$$

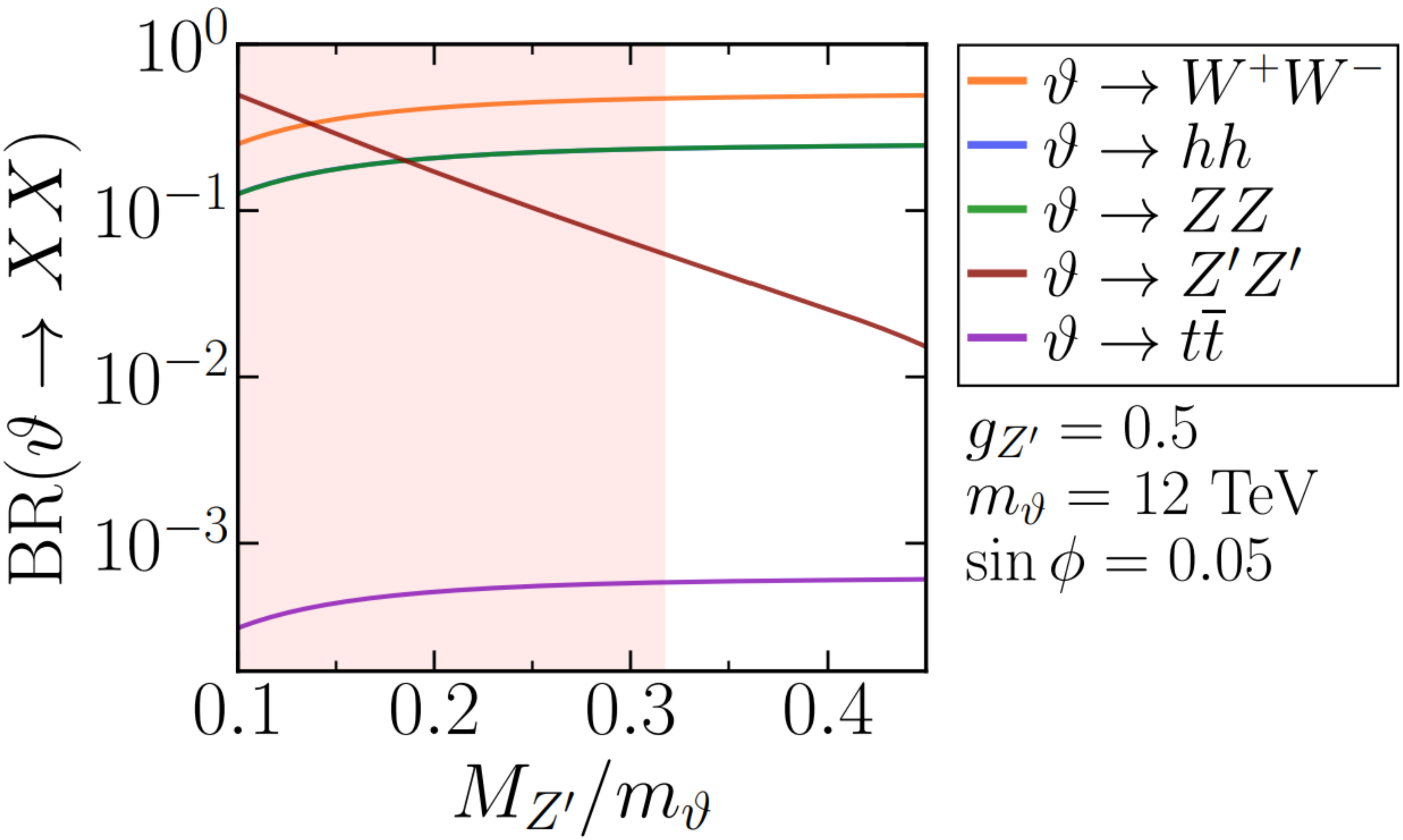
Three parameters: $v_\theta = M_{Z'}/g_{Z'}$, m_ϑ and ϕ.

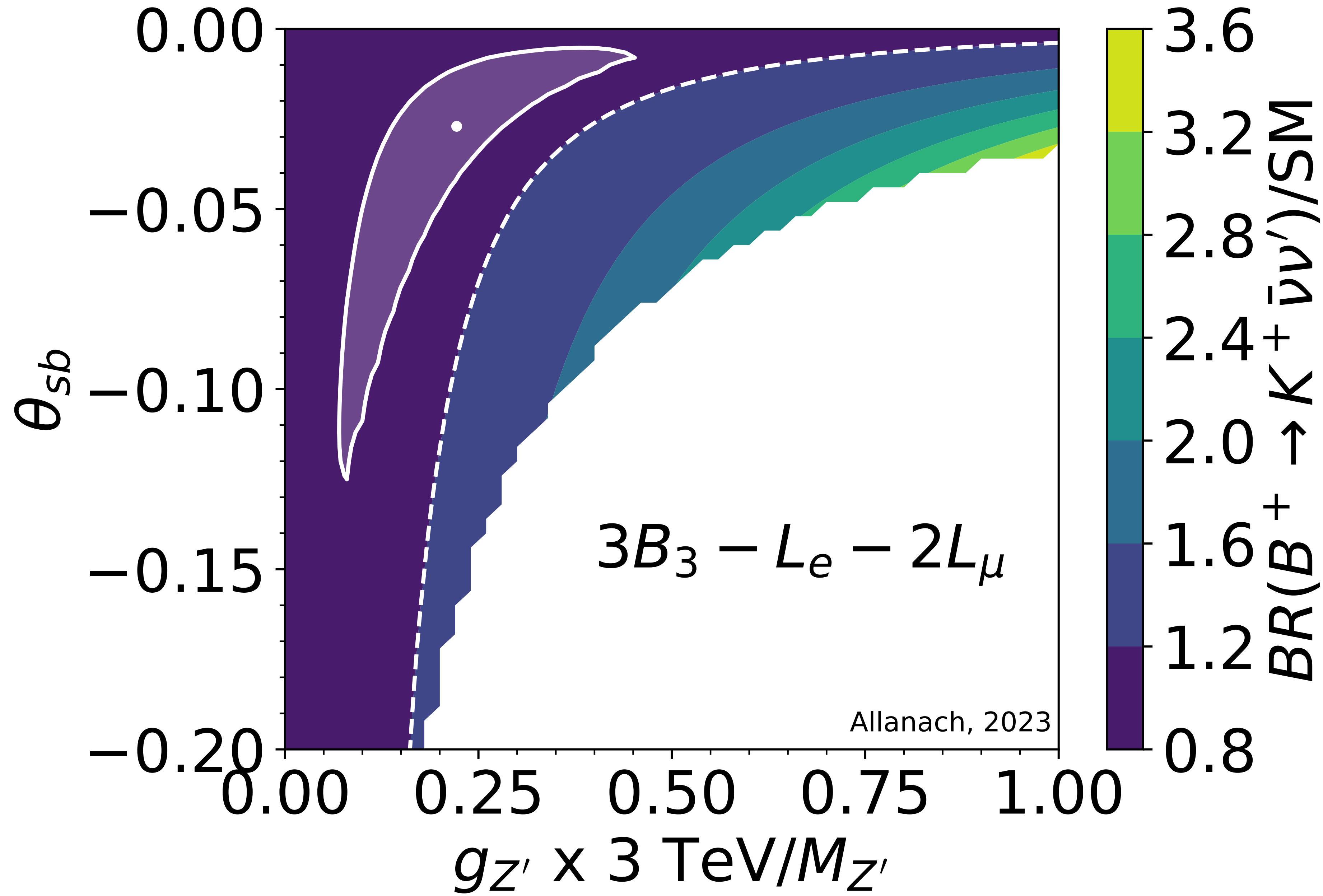
Higgs Signal Strength

BCA, Loisa, 2212.07440



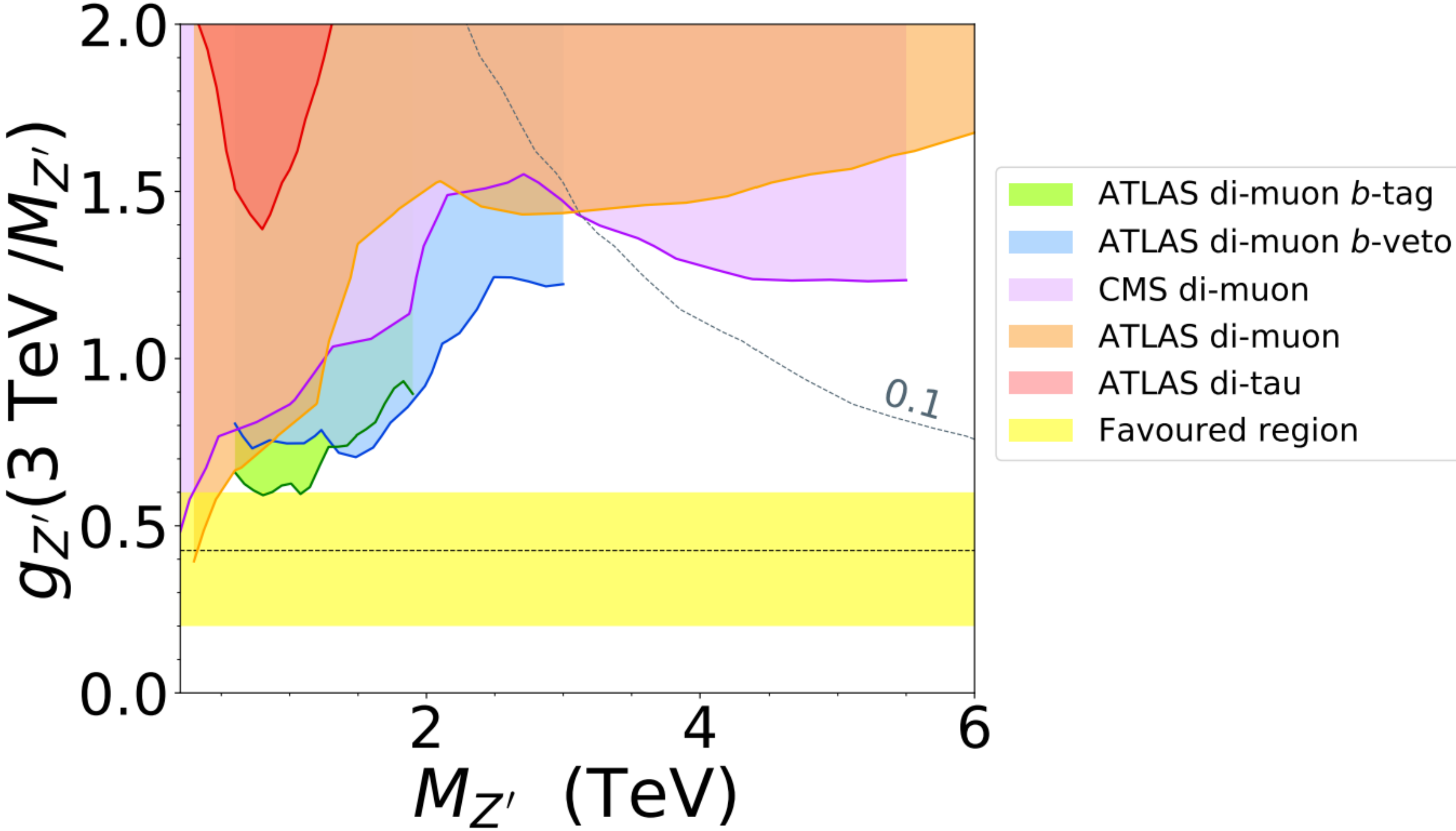
ϑ BRs



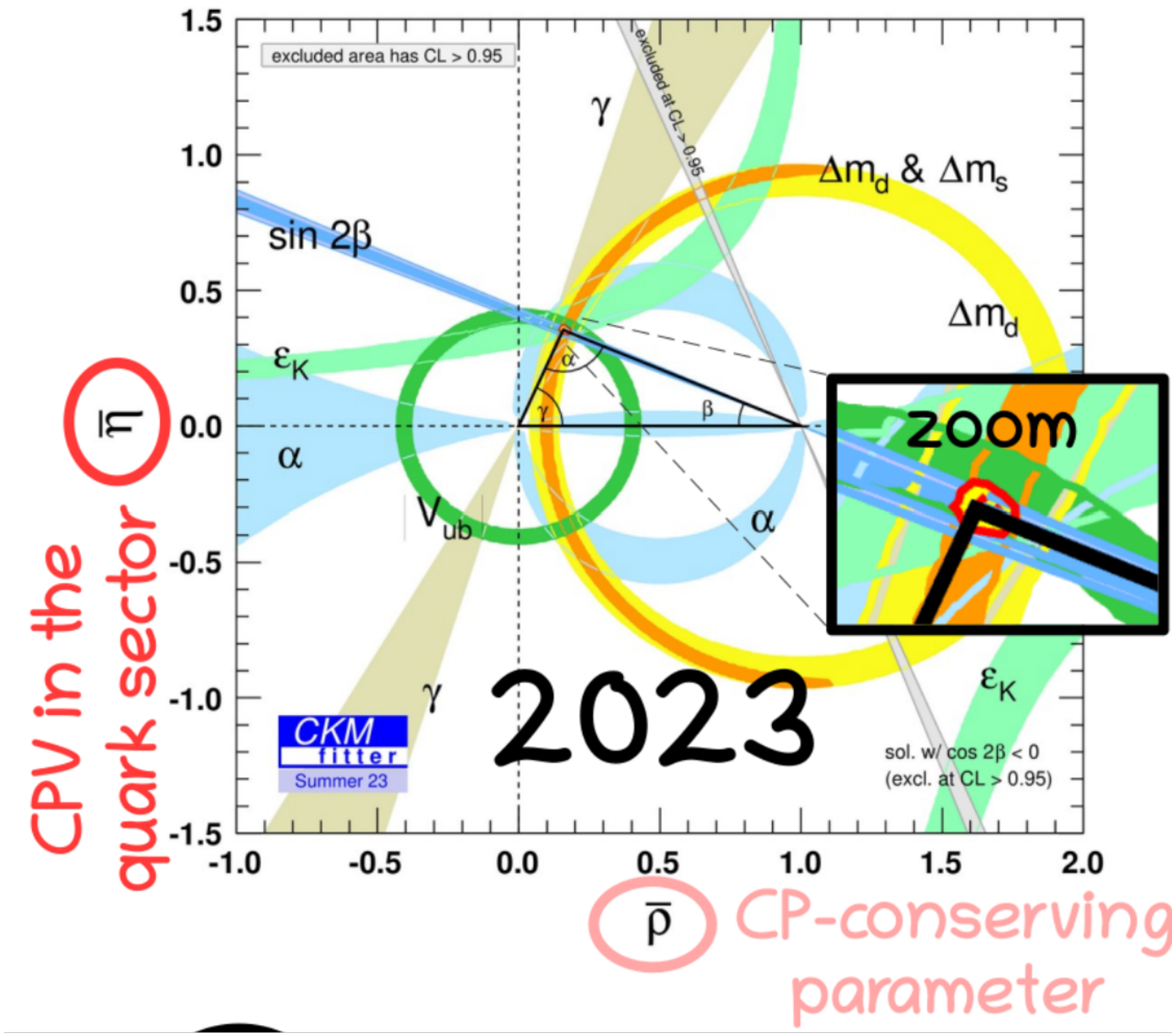


0.00
−0.05
−0.10
−0.15
−0.20
θ_{sb}
0.00
0.25
0.50
0.75
1.00
$g_{Z'}$ x 3 TeV/$M_{Z'}$
$3B_3 - L_e - 2L_\mu$
Allanach, 2023
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
$BR(B^+ \to K^+ \bar{\nu}\nu')$/SM

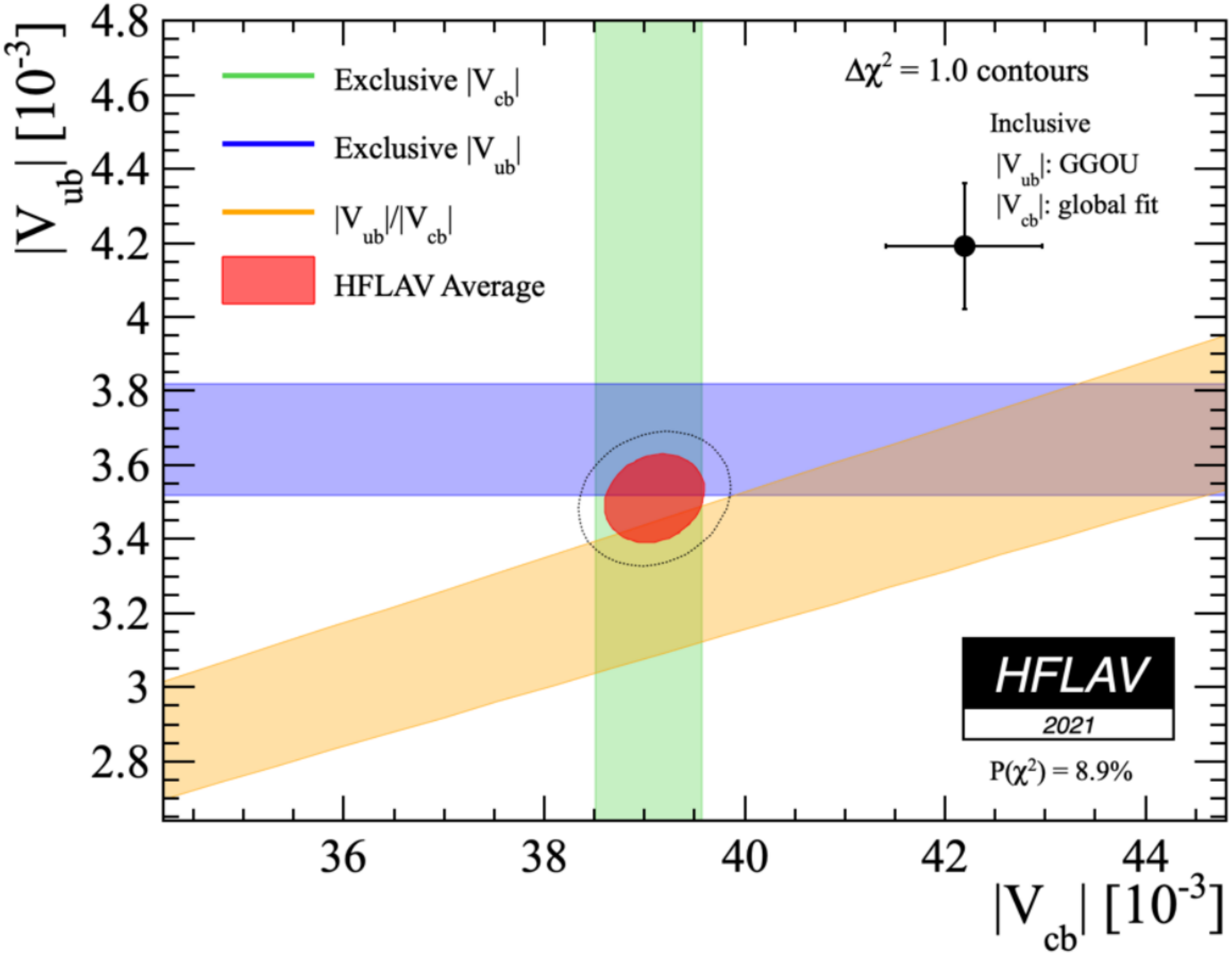
Z' Searches[6]



[6] BCA, Banks, 2111.06691



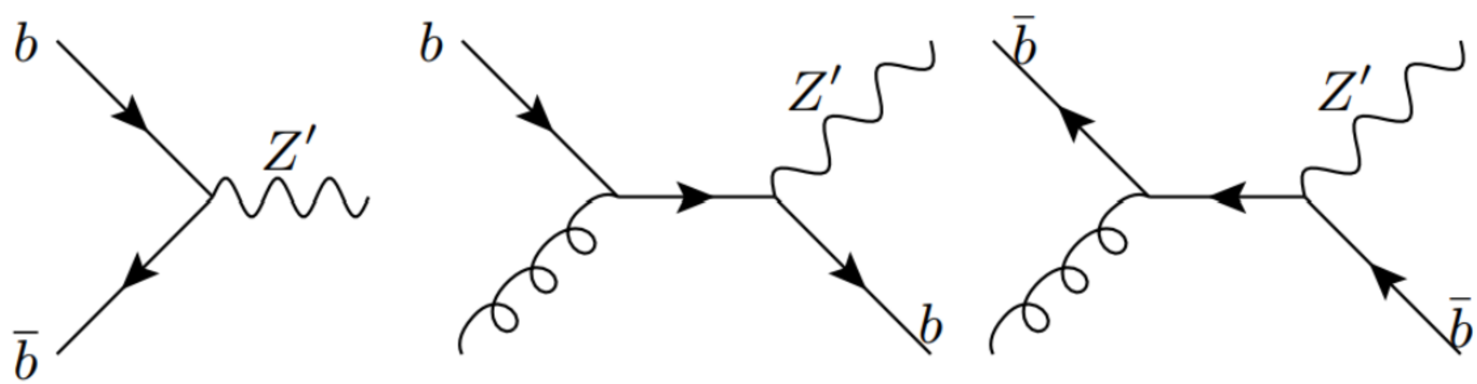
excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
γ
zoom
α
V_ub
α
ε_K
γ
2023
CKM fitter
Summer 23
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
η̄
CPV in the quark sector
ρ̄
CP-conserving parameter


Exclusive |V_cb|
Exclusive |V_ub|
|V_ub|/|V_cb|
HFLAV Average
Δχ² = 1.0 contours
Inclusive
|V_ub|: GGOU
|V_cb|: global fit
HFLAV
2021
P(χ²) = 8.9%
|V_ub| [10^-3]
|V_cb| [10^-3]
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
36
38
40
42
44

Z' Decay Modes

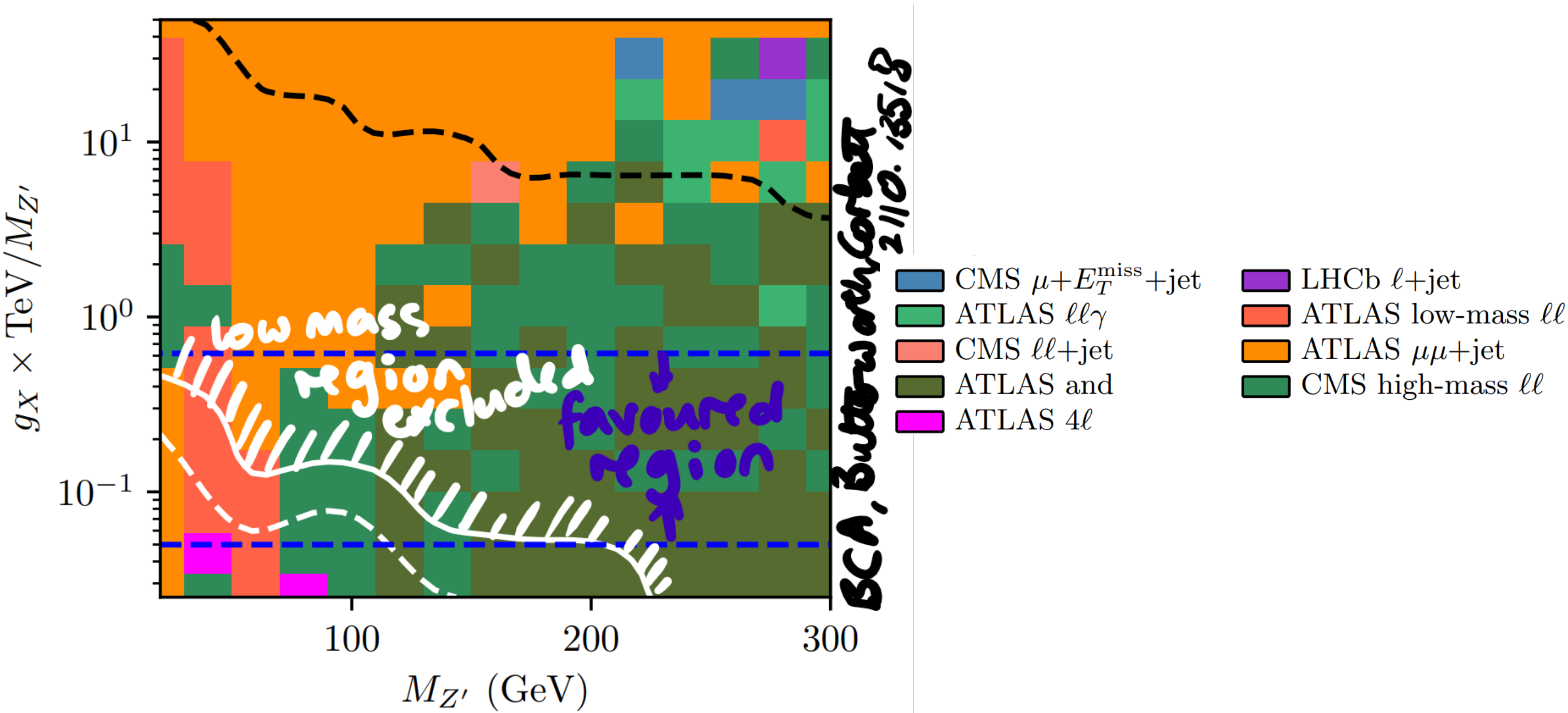
Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.15	$b\bar{b}$	0.15	$\nu\bar{\nu}'$	0.23
$\mu^+\mu^-$	0.46				

pp Z' Production:



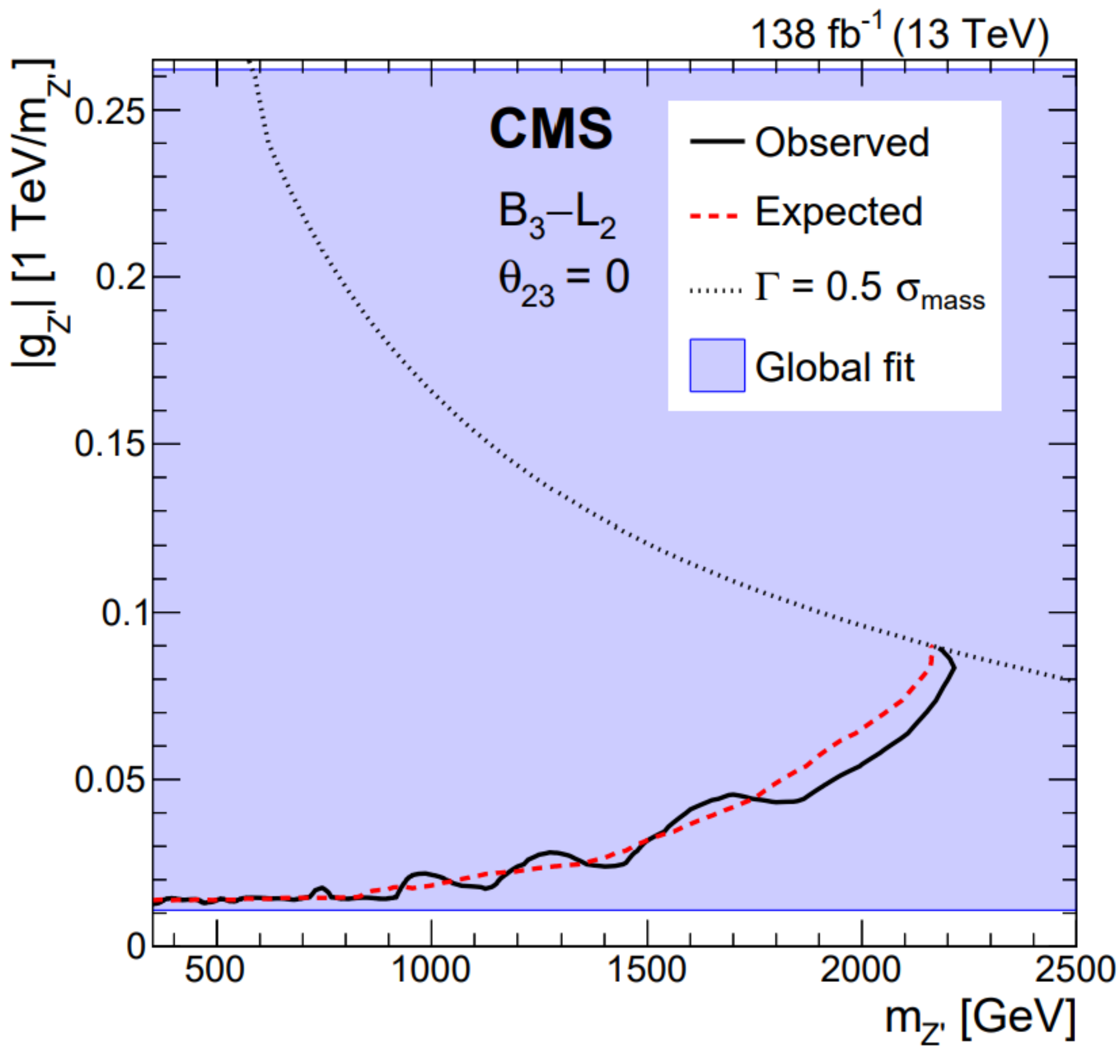
$$\sigma_{prod} \propto g_X^2 \cos^4\theta_{sb} = g_X^2 \left(1 - 2\theta_{sb}^2 + \mathcal{O}(\theta_{sb}^4)\right)$$

$B_3 - L_2$ model's[7] Z'

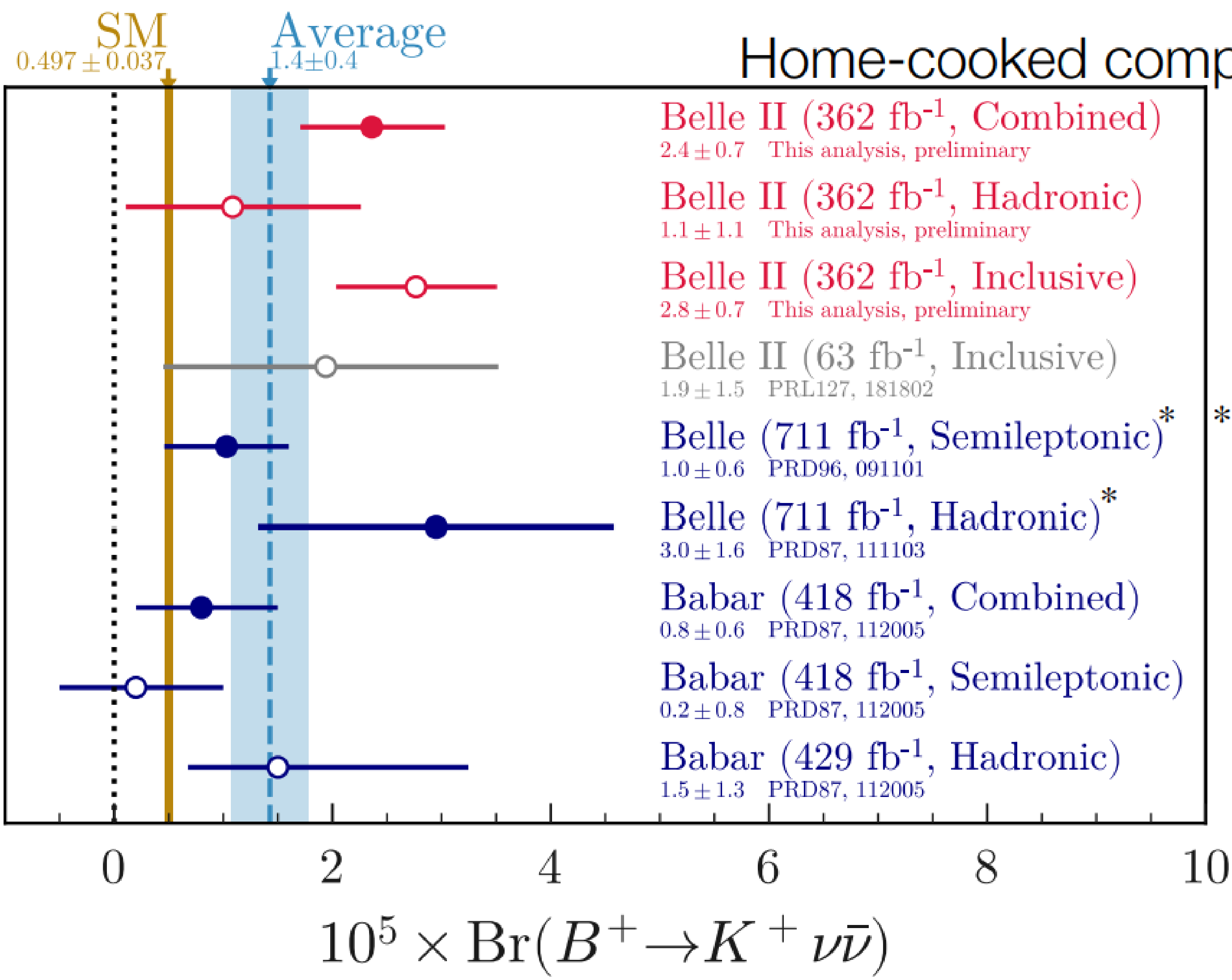


[7] Bonilla, Modak, Srivastava, Valle, 1705.00915; Alonso, Cox, Han, Yanagida 1705.03858

CMS $\mu^+\mu^- b$ 2307.08708

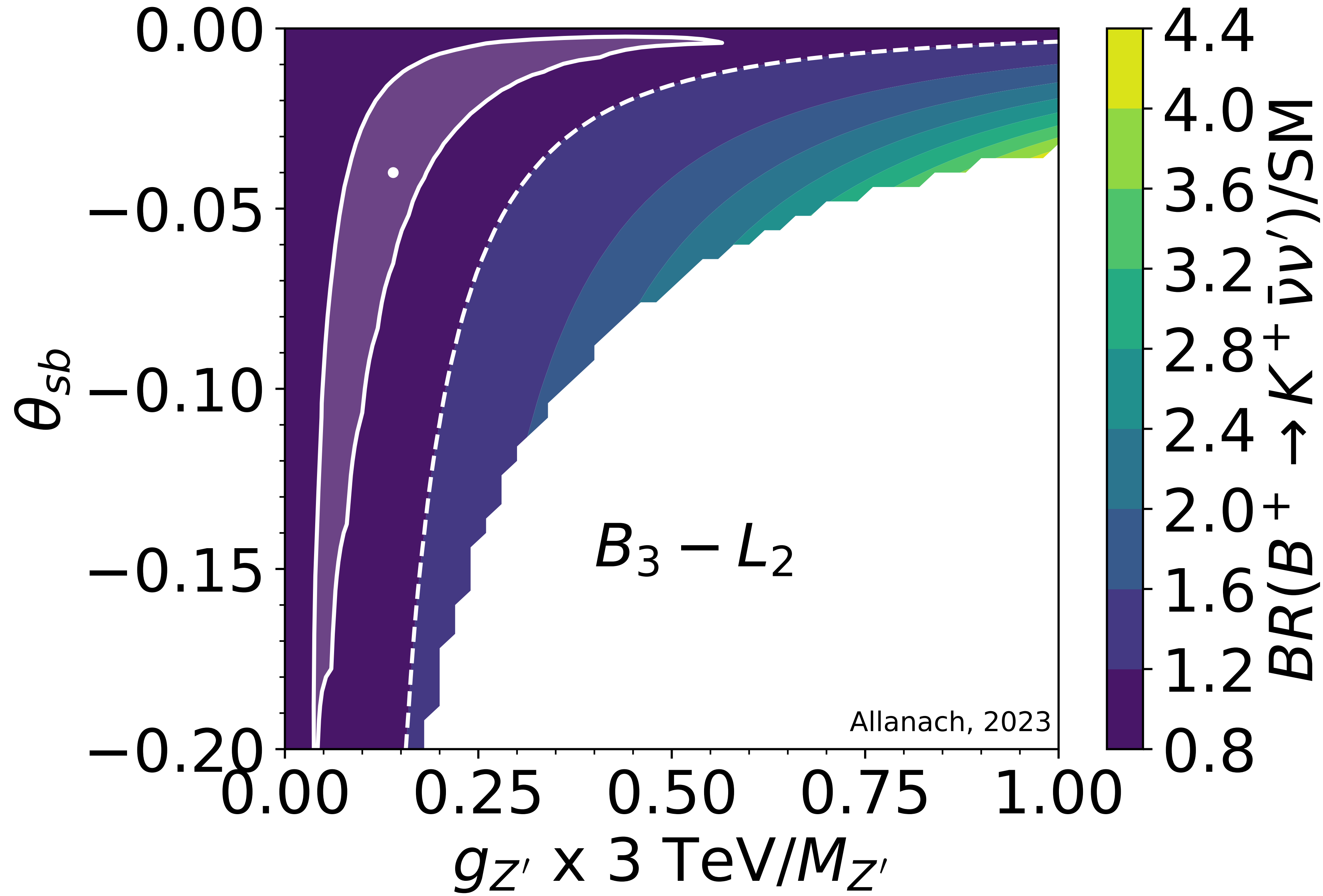


Home-cooked comparison



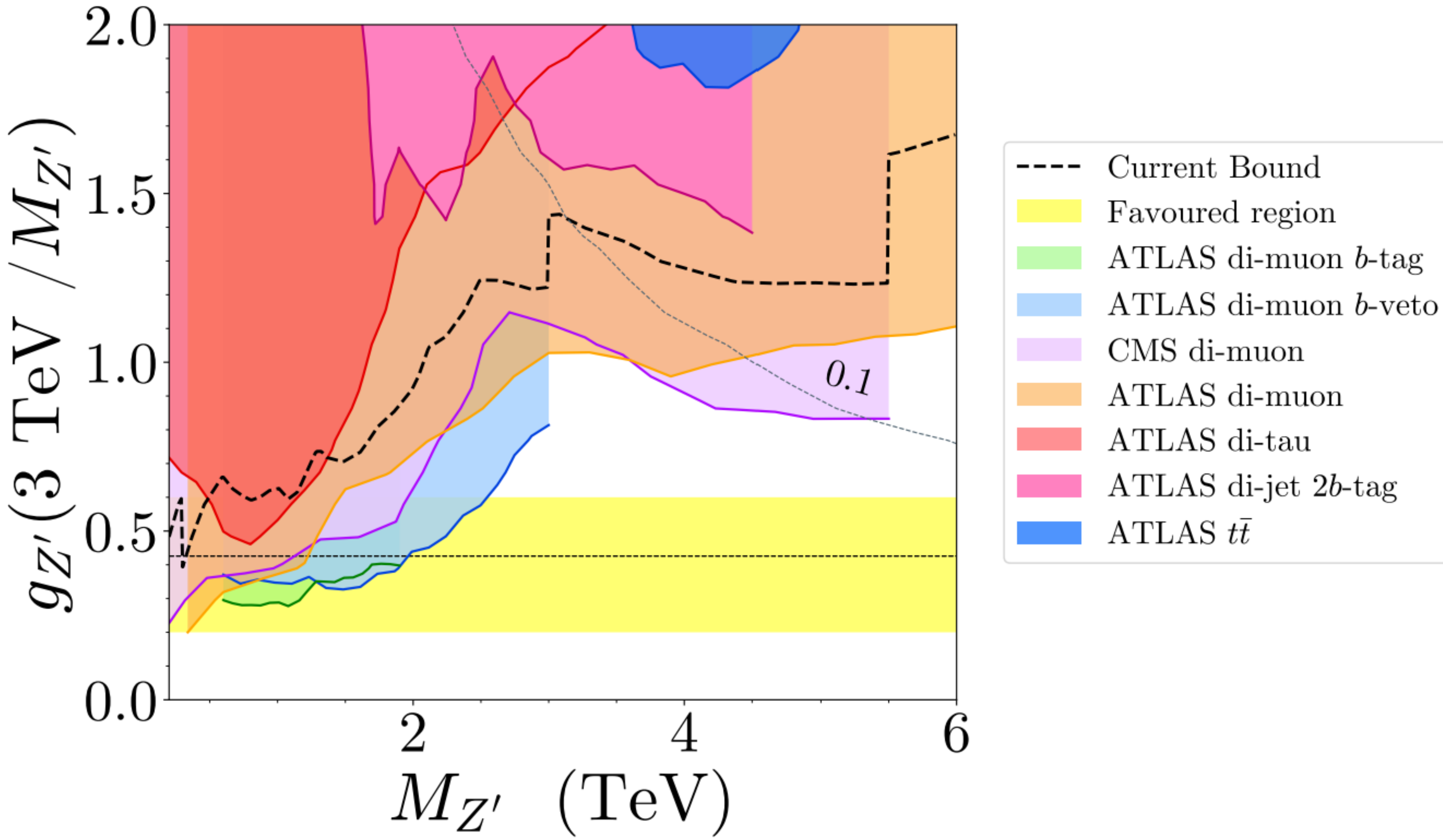
*Belle reports only upper limits. We calculate BF ourselves

Overall compatibility is good χ^2/ndf = 4.3/4



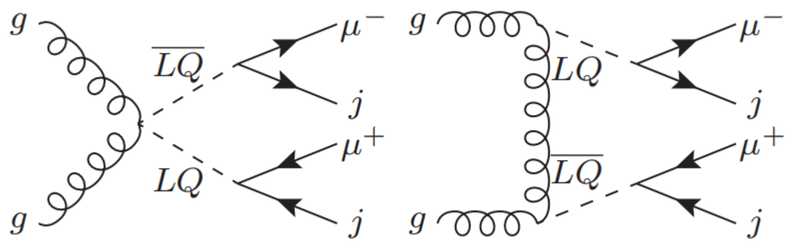
0.00
−0.05
−0.10
−0.15
−0.20
0.00
0.25
0.50
0.75
1.00
θ_{sb}
$g_{Z'}$ x 3 TeV/$M_{Z'}$
$B_3 - L_2$
Allanach, 2023
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
$BR(B^+ \to K^+ \bar{\nu}\nu')$/SM

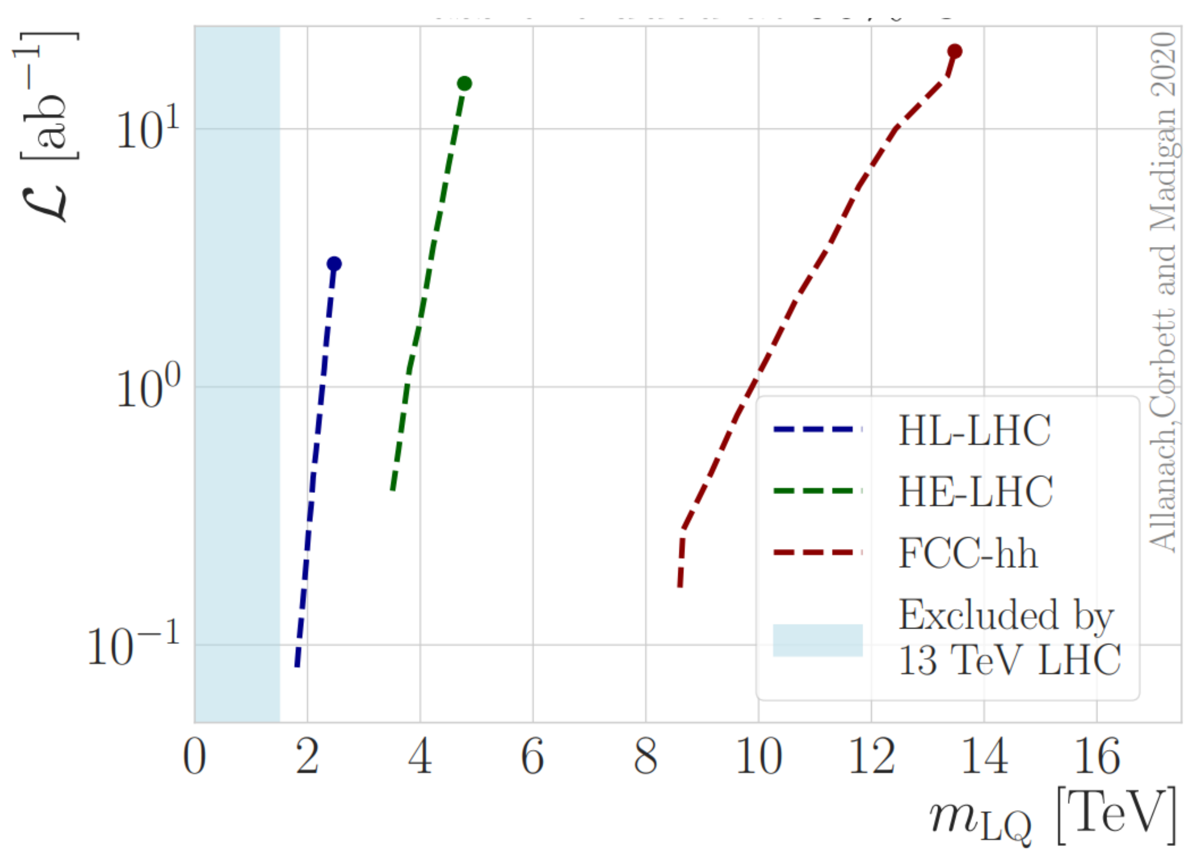
Y_3 HL-LHC sensitivity[8]



[8] BCA, Banks, 2111.06691

Scalar LQ[9]: eg $S_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$





[9]BCA, Corbett, Madigan, 1911.0445