# On-shell methods for SMEFT

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The LHC Precision Program – Benasque – 6 October 2023

# On-shell methods for SMEFT

- on-shell anomalous dimensions

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20], [Jiang et al. '20], [Elias Miró et al. '20, '21]
[Baratella et al. '20, '20, '21], [Accettulli Huber, De Angelis '21], [Delle Rose et al. '22], [Baratella '22]
[Machado, Renner, Sutherland '22], [Chala '23]

- on-shell operator enumeration

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Falkowski '19], [GD, Machado '19]
[Li, Ren, et al. '20, '20], [Harlander, Kempkens, Schaaf '23]

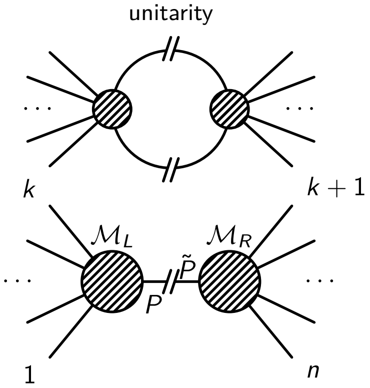
- on-shell massive amplitudes

[Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20]
[Balkin et al. '21], [Dong, Ma, Shu, Zheng '21, '22], [De Angelis '22]
[Bradshaw, Chang, Chen, Liu, Luty '22, '23], [Liu, Ma, Shadmi, Waterbury '23]

- on-shell matching

[Delle Rose, von Harling, Pomarol '22]
[De Angelis, GD '23]

# Recursive on-shell amplitude construction




- loops cut into trees
  - + rational terms
- trees cut into trees
  - + contact terms

bypass unphysical fields, operators, Lagrangians

avoid gauge and field redefinition redundancies

# Massless three points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1},2^{h_2},3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h>0 \\ \langle 12\rangle^{-h_1-h_2+h_3}\langle 23\rangle^{-h_2-h_3+h_1}\langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h<0 \end{cases}$$

up to a coefficient, encompassing all loop orders

$f^+f^+s$ $[12]$

$v^+v^+s$ $[12]^2$

$f^+f^-v^+$ $[13]^2/[12]$

$v^+v^+v^-$ $[12]^3/[23][31]$

$t^+t^+t^-$ $\left([12]^3/[23][31]\right)^2$

$$[g] = 1-|h| \qquad h \equiv \sum h_i$$

fully characterise massless renormalisable theories (except $\phi^4$)

# Massless helicity spinors

[Mangano, Parke '91]
[Dreiner, Haber, Martin '08]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for particle } i$$

Rewritting momenta

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \equiv p^i_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\ {}^\beta i\rangle[i_{\dot\alpha} \qquad \text{2-by-2 matrix of rank 1}$$

Trivializing $p_i^2 = \det(p^i_{\alpha\dot\alpha}) = \langle ii\rangle[ii]/2 = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\ i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\ i]_{\dot\alpha} i]_{\dot\beta} = 0$$

# Massive little-group-covariant spinors

[Arkani-Hamed, Huang, Huang '17]

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; q^i\rangle[q^i + k^i\rangle[k^i \;=\; i^J\rangle[i_J \qquad \text{with } k_i^2 = 0 = q_i^2,\;\; J = 1,2 \quad 2k^i \cdot q^i = m_i^2$$

Spin $s$ from $2s$ symmetrized spin $1/2$

Bolded spinors with implicit symmetrization

e.g. $\langle 1^I 3^J\rangle\langle 2^K 3^{J'}\rangle + (J \leftrightarrow J') \to \langle \mathbf{13}\rangle\langle \mathbf{23}\rangle$

Spin quantisation axis unspecified / little-group covariance

*ffs* $[\mathbf{12}],\ \langle \mathbf{12}\rangle$

*vvs* $\langle \mathbf{12}\rangle^2,\ \langle \mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

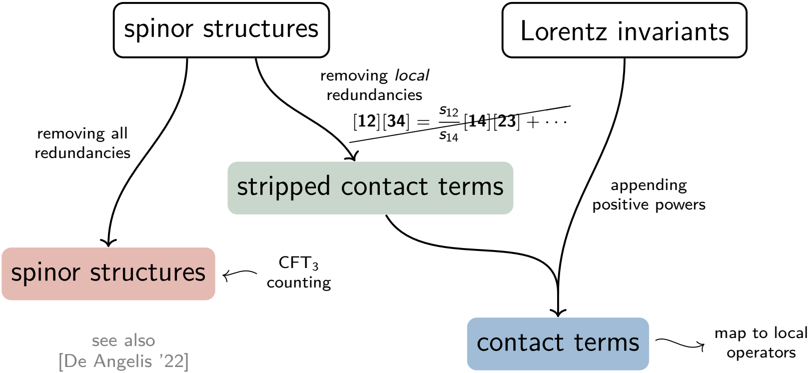
*ssv* $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

*ffv* $\langle \mathbf{13}\rangle\langle \mathbf{23}\rangle,\ \langle \mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle \mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

...

# Massive higher points

e.g. $W^+W^+W^-W^-$: $\dfrac{[\mathbf{13}][\mathbf{24}]\langle\mathbf{13}\rangle\langle\mathbf{24}\rangle - [\mathbf{14}][\mathbf{23}]\langle\mathbf{14}\rangle\langle\mathbf{23}\rangle}{m_1 m_2 m_3 m_4}$ $(\tilde{s}_{13} - \tilde{s}_{14} - \tilde{s}_{23} + \tilde{s}_{24})$




see also
[De Angelis '22]
[Dong, Ma, Shu, Zheng '22]

and stripped contact term enumeration
from Hilbert series numerators
[Bradshaw, Chang, Chen, Liu, Luty '22, '23]

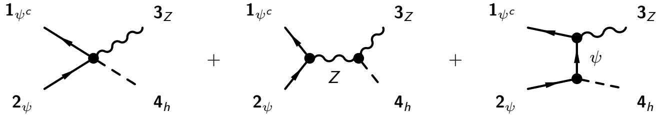
e.g. $\text{Hilbert}(\psi^c\psi Zh) = \dfrac{2d^5 + 6d^6 + 4d^7}{(1-d^2)^2}$

→ 4-points & spins≤1

# EW symmetry from perturbative unitarity

[GD, Kitahara, Shadmi, Weiss '19]

$\psi^c\psi\, Z\, h \sim$




contact terms — factorisable $s, t, u$ channels

$$\begin{matrix}[\mathbf{13}][\mathbf{23}]\\ [\mathbf{13}]\langle\mathbf{23}\rangle\\ [\mathbf{123}\rangle\langle\mathbf{23}\rangle\\ \cdots\end{matrix} \times f(s_{ij}) \qquad \begin{matrix}[\mathbf{1P}][\mathbf{2P}]\\ [\mathbf{1P}]\langle\mathbf{2P}\rangle\\ \langle\mathbf{1P}\rangle[\mathbf{2P}]\\ \langle\mathbf{1P}\rangle\langle\mathbf{2P}\rangle\end{matrix} \otimes \begin{matrix}[\mathbf{P3}]^2\\ [\mathbf{P3}]\langle\mathbf{P3}\rangle\\ \langle\mathbf{P3}\rangle^2\end{matrix} \qquad \begin{matrix}[\mathbf{13}][\mathbf{P3}]\\ [\mathbf{13}]\langle\mathbf{P3}\rangle\\ \langle\mathbf{13}\rangle[\mathbf{P3}]\\ \langle\mathbf{13}\rangle\langle\mathbf{P3}\rangle\end{matrix} \otimes \begin{matrix}[\mathbf{P2}]\\ \langle\mathbf{P2}\rangle\end{matrix}$$

$$\xrightarrow[\text{energy}]{\text{high}} \begin{cases} \dfrac{[12]}{m_Z}\left(c^{\text{left}}_{\psi^c\psi Z} - c^{\text{right}}_{\psi^c\psi Z}\right)\left(c^{\text{right}}_{\psi^c\psi h} - c_{ZZh}\dfrac{m_\psi}{2m_Z}\right) \\ \dfrac{\langle 12\rangle}{m_Z}\left(c^{\text{left}}_{\psi\psi Z} - c^{\text{right}}_{\psi^c\psi Z}\right)\left(c^{\text{left}}_{\psi^c\psi h} - c_{ZZh}\dfrac{m_\psi}{2m_Z}\right) \end{cases}$$

as for the SM in the '70

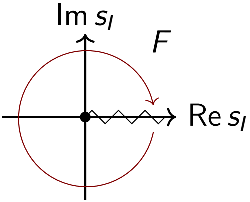
[Llewellyn-Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

→ parameterisation of measurable quantities

→ high operator dimensions easily accessible

# Anomalous dimensions from cuts

Relate dilation operator $D \equiv \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$
to $S$-matrix phase:




$e^{-i\pi D} F^* = S{\cdot}F^*$ on form-factor $F \equiv {}_{\text{out}}\langle p_1, \ldots p_n | \mathcal{O}(q) | 0 \rangle_{\text{in}}$

momentum influx

since $e^{-i\pi D} F^* = F$

$F - F^* = iT \cdot F^*$ or $F = S \cdot F^*$

With $D \sim -\mu \frac{\partial}{\partial \mu}$, at one-loop,

$$(\gamma_{ij} - \gamma_{ij}^{\text{IR}} + \beta_g \partial_g)^{(1)} \langle p1, \ldots p_n | \mathcal{O}_i | 0 \rangle^{(0)} = -\frac{1}{\pi} \langle p_1, \ldots p_n | \mathcal{A}{\cdot}\mathcal{O}_j | 0 \rangle^{(0)}$$

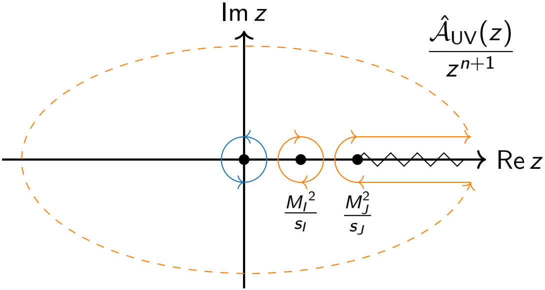
two-particle phase-space integral

absent for 'minimal' form factors

absent for off-diagonal elements

# Matching from non-analyticities

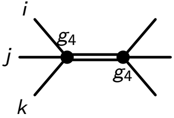
- equate $\mathcal{A}_{\text{EFT}}$ and $\mathcal{A}_{\text{UV}}$ order by order in $p_i \to 0$ (massless EFT assumed)
- dilate $p_i{\cdot}p_j \to z\, p_i{\cdot}p_j$ (i.e. act with $z^{D/2}$) and get $n$th order with $\operatorname*{Res}_{z=0} \frac{\hat{\mathcal{A}}(z)}{z^{n+1}}$
- use $\operatorname*{Res}_{z=0} \frac{\hat{\mathcal{A}}(z)}{z^{n+1}} = \oint dz \frac{\hat{\mathcal{A}}(z)}{z^{n+1}}$ around $z = 0$ and deforme it




$$\mathcal{A}^{\text{tree}}_{\text{EFT@}n} = \left[ \sum \text{residues} + \int \text{cuts} \quad \left( + \int \text{tree boundary} \right) \right] \frac{\hat{\mathcal{A}}_{\text{UV}}(z)}{z^{n+1}}$$

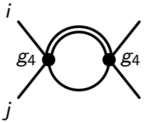
residues → fewer legs; cuts → fewer loops

# Matching from non-analyticities: $\Phi\phi^3$ example




$$: \sum_{ijk\ \text{channels}} \underset{z=M^2/s_{ijk}}{\text{Res}} \frac{|\mathcal{A}(\phi\phi\phi\to\Phi)|^2}{z s_{ijk} - M^2}\frac{1}{z^{n+1}}$$

$$= \frac{g_4^2}{M^2}\sum_{\text{channels}}\left(\frac{s_{ijk}}{M^2}\right)^n$$



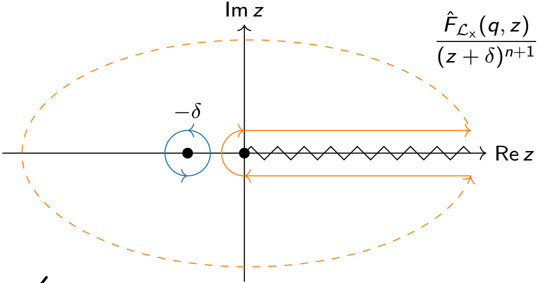

$$: \frac{1}{2\pi}\sum_{\text{channels}}\int_{M^2/s_{ij}}^{\infty}\frac{\text{d}z}{z^{n+1}}\int \text{dLIPS}\ |\mathcal{A}(\phi\phi\to\phi\Phi)|^2$$

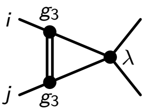
$$=\frac{1}{2\pi}\sum_{\text{channels}}\int_{M^2/s_{ij}}^{\infty}\frac{\text{d}z}{z^{n+1}}\frac{1}{8\pi}\left(1-\frac{M^2}{z s_{ij}}\right)g_4^2$$

$$=\frac{g_4^2}{16\pi^2 n(n+1)}\sum_{\text{channels}}\left(\frac{s_{ij}}{M^2}\right)^n \qquad \text{for } n>0$$

- all EFT orders obtained at once
- nothing to compute in the EFT (i.e. hard expansion extracted)
- no Green's basis, field redef./EOMs, evanescents?, massive vectors?
- fewer legs/loops: new computations and insight? [Delle Rose, von Harling, Pomarol '22]

# Matching from non-analyticities: massless cuts




$$: \int_0^\infty \frac{dz}{z^{n+1}} \int \mathrm{dLIPS}_d \frac{\lambda g_3^2}{(l-p_i)^2 - M^2}$$

$$= \frac{\lambda g_3^2 M^{d-6}}{8(4\pi)^{\frac{d}{2}-2}} \left(\frac{s_{ij}}{M^2}\right)^n \frac{(-1)^{n+1} n! \csc\frac{\pi d}{2}}{\Gamma[\frac{d}{2}+n]}$$

$$= \frac{\lambda g_3^2}{16\pi^2 M^2} \frac{(-1)^n}{n+1} \left(\frac{s_{ij}}{M^2}\right)^n \left(\frac{1}{\bar{\epsilon}} + H_{n+1} + \log\frac{\mu^2}{M^2} + \mathcal{O}(\epsilon)\right)$$

- phase-space integral can be complicated and require an IR regulator
- dimreg renders tree-level exact the (scaleless) EFT and UV boundary

# Matching from non-analyticities: boundary

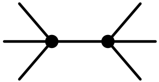
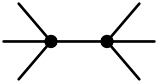
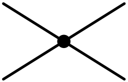
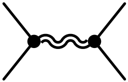
- no matching information unless $n \geq \min(4 - m - [c_{\mathrm{EFT}}])/2$
- no boundary term unless $n \leq \min(4 - m - [c_{\mathrm{UV}}])/2$

  $\rightarrow \max[c_{\mathrm{EFT}}] \geq 4 - m - 2n \geq \max[c_{\mathrm{UV}}]$

- but $\max[c_{\mathrm{EFT}}] \leq \max[c_{\mathrm{UV}}]$ by definition

  $\rightarrow$ boundary needed only for $\max[c_{\mathrm{EFT}}] = \max[c_{\mathrm{UV}}]$ $(= 4 - m - 2n)$

e.g.

| n | EFT | renorm. UV |
|---|---|---|
| $-1$ |  |  |
| 0 |  |  |

# On-shell methods for SMEFT

provide an alternative approach

lead to new insight

facilitate computations