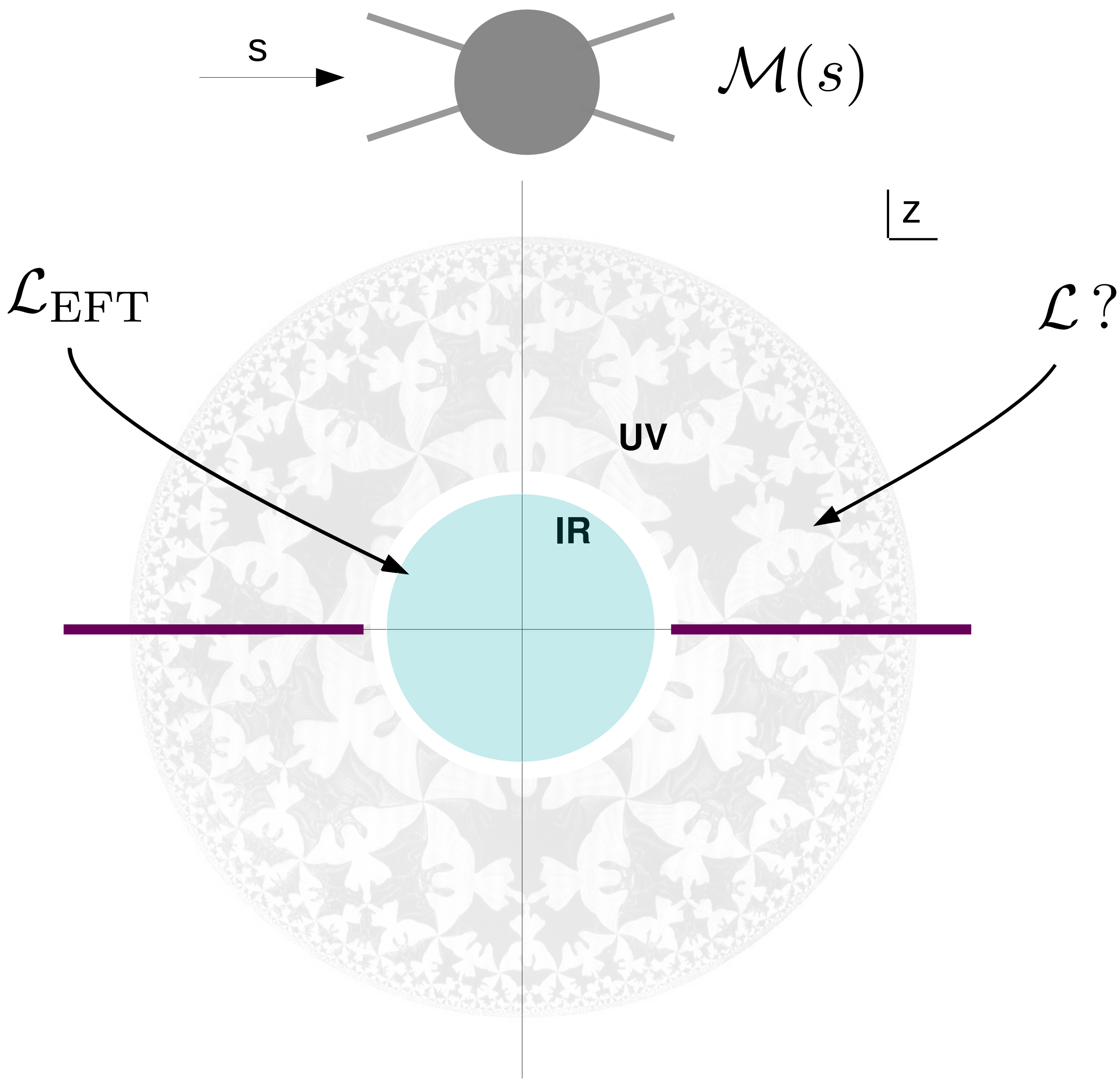
Quick Overview On Positivity

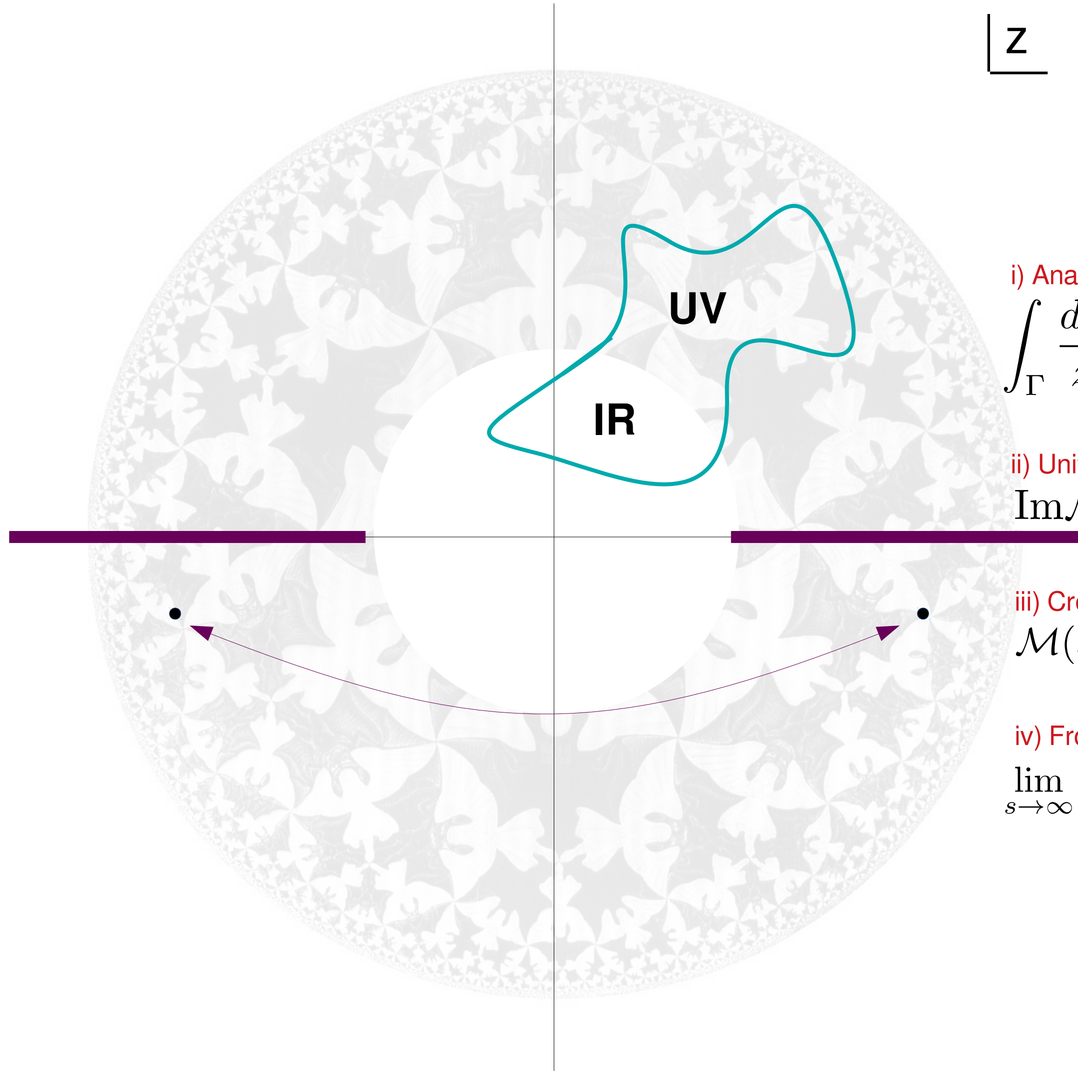
Marc Riembau

CERN

5th October 2023



s
$\mathcal{M}(s)$
z
$\mathcal{L}_{\rm EFT}$
$\mathcal{L}\,?$
UV
IR



i) Analyticity

$$\int_\Gamma \frac{dz}{z}\mathcal{M}(z) \;=\; 0$$

ii) Unitarity

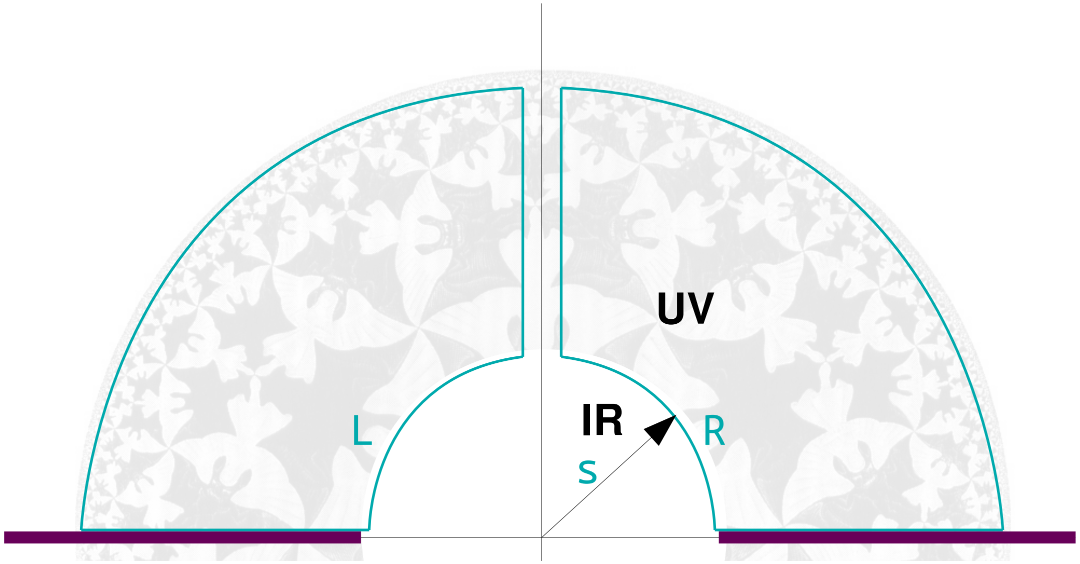
$$\mathrm{Im}\mathcal{M}(s+i\epsilon) \geq 0$$

iii) Crossing

$$\mathcal{M}(s) = \mathcal{M}^*(-s^*)$$

iv) Froissart bound

$$\lim_{s\to\infty} \mathcal{M}(s)/s^2 = 0$$



$$a_n^R = \int_R \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}} = \int_s^\infty \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}} + \int_{i\infty}^{is} \frac{dz}{i\pi}\frac{\mathcal{M}(z)}{z^{3+n}}$$

n even:

$$a_n^R + a_n^L = \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Im}\mathcal{M}(z)}{z^{2+n}}$$

$$a_n^R - a_n^L = \frac{2}{i\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Re}\mathcal{M}(z)}{z^{2+n}} + 2I_n$$

Bellazzini, Elias-Miro, Rattazzi, MR, Riva ‘20

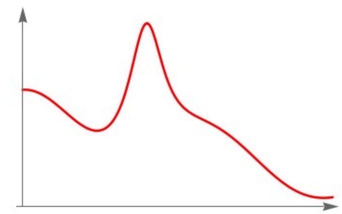
$$a_n^R + a_n^L = \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\frac{\mathrm{Im}\mathcal{M}(z)}{z^{2+n}}$$

$\{a_n\}$

IR–calculable

$$\int_s^\infty \frac{dz}{z}\frac{1}{z^{2+n}}\bullet$$

$\mathrm{Im}\mathcal{M}(z) \geq 0$



is this mapping complete? Yes.

$$m_n = \int_0^1 d\mu(x)x^n\,, \quad d\mu(x) \geq 0$$

Knowledge of the moments = knowledge of the measure

Knowledge of IR arcs = knowledge of UV spectrum

When is an EFT UV-completable? → **When a bunch of numbers can be identified with moments of a positive distribution?**

When can a bunch of numbers be identified with moments of a positive distribution?

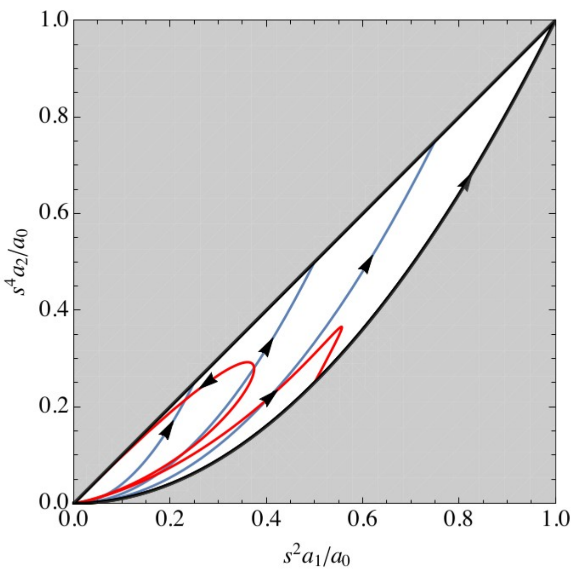
$\{a_0, a_1, \ldots\}$ moments of a positive distribution in [0,1] **iff**

$$\begin{pmatrix} a_0 & a_1 & \ldots & a_n \\ a_1 & a_2 & & \\ \ldots & & \ddots & \vdots \\ a_n & & \ldots & a_{2n} \end{pmatrix} \succ 0$$

$$\begin{pmatrix} a_0 - a_1 & a_1 - a_2 & \ldots & a_n - a_{n+1} \\ a_1 - a_2 & a_2 - a_3 & & \\ \ldots & & \ddots & \vdots \\ a_n - a_{n+1} & & \ldots & a_{2n} - a_{2n+1} \end{pmatrix} \succ 0$$

2d

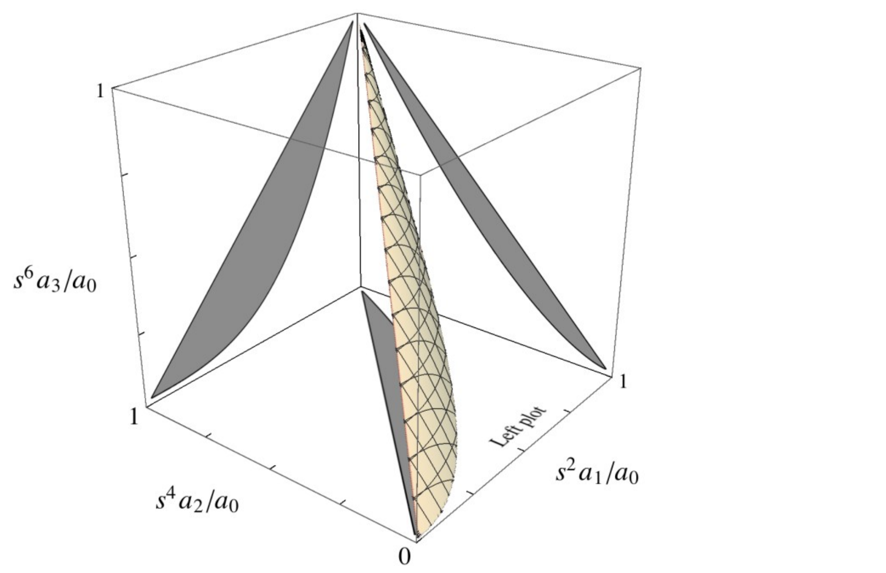
$$\begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \succ 0\,, \quad a_1 > 0\,, \quad a_0 > \hat{s}^2 a_1\,, \quad a_1 > \hat{s}^2 a_2$$



3d

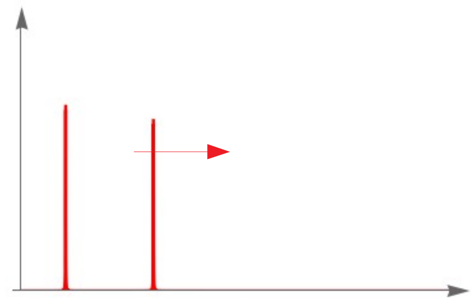
$$\begin{pmatrix} a_0 & a_1 \\ a_1 & a_2 \end{pmatrix} \succ 0\,, \qquad \begin{pmatrix} a_0 - a_1\hat{s}^2 & a_1 - a_2\hat{s}^2 \\ a_1 - a_2\hat{s}^2 & a_2 - a_3\hat{s}^2 \end{pmatrix} \succ 0\,,$$

$$\begin{pmatrix} a_1 & a_2 \\ a_2 & a_3 \end{pmatrix} \succ 0\,, \qquad a_1 > \hat{s}^2 a_2\,,$$



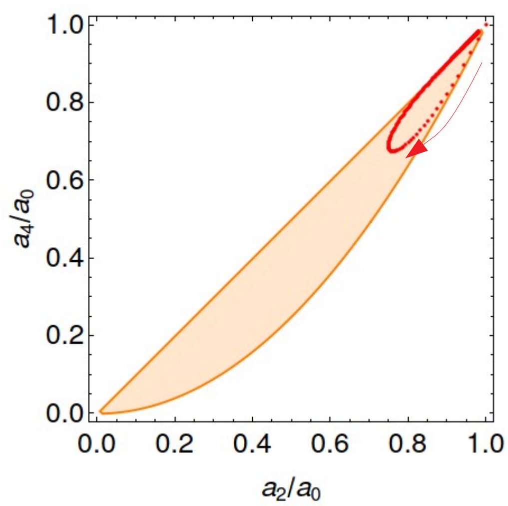
Convexity ➔ **Understanding of the boundary = Understanding of the entire space**

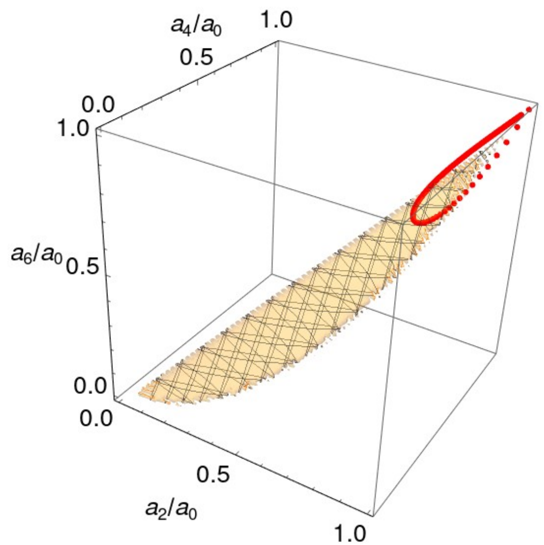
Boundary of n-dimensional space given by (n-1)-particles

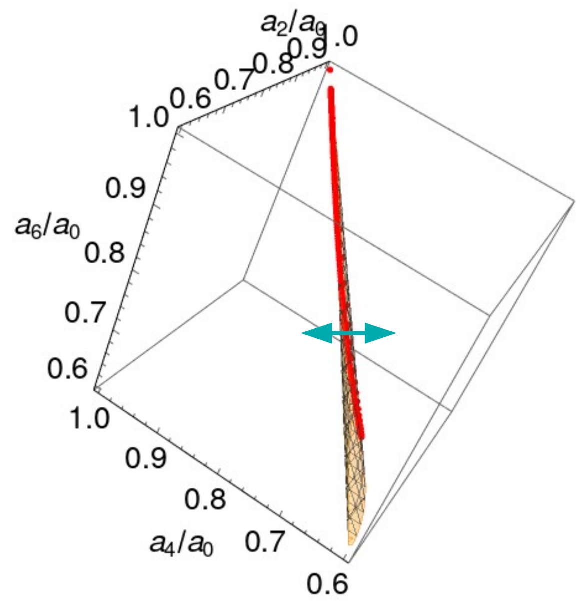


Arcs from two narrow resonances

Space of arcs shrinks
faster than exponentially with n!







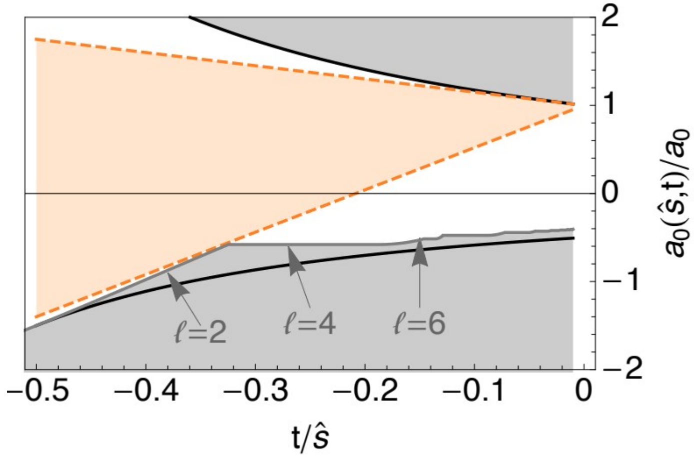
Finite t

Bellazzini, MR, Riva '21

Arcs at finite t get a t- and l- dependent kernel

$$a_n(s,t) = \frac{2}{\pi}\int_s^\infty \frac{dz}{z}\sum_\ell \frac{\mathrm{Im} f_\ell(z)}{z^{2+n}} I_{n,\ell}(t,z)$$

$$\min_{\ell,z} I_{n,\ell}(t,z) \leq \frac{a_n(s,t)}{a_n(s,0)} \leq \frac{s+t/2}{(s(s+t))^{n+2}}$$



Upper bound on the arc is t-dependent but lower bound is t- and l- dependent.

$$a_n(s,t) = c_2 - tc_3 + \ldots$$

Upper bound comes from t=0,

but lower bound from finite t,
at the intersection of l=2 and l=4 partial waves

Full unitarity? $2\text{Im} f_\ell(s) \geq |f_\ell(s)|^2$

MR '22

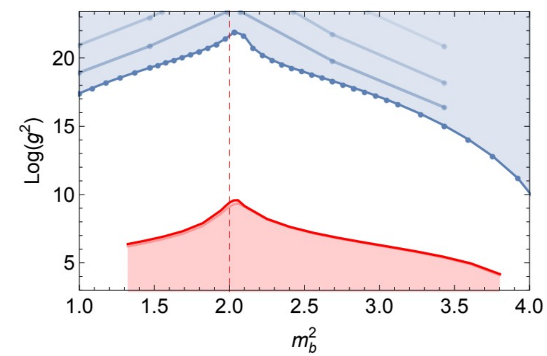
Pert. IR theory: $\mathcal{M}(s) = c_2 s^2 + s^4(c_4 + \beta_4 \log(-is)) + s^6(c_6 + \beta_6 \log(-is)) + \ldots$

$$\frac{\text{relevance of Unitarity bounds}}{\text{relevance of Convexity bounds}} \sim \frac{\beta_4}{c_4}\frac{c_2}{c_4 s^2} \sim \frac{\text{loop expansion}}{\text{derivative expansion}}$$

Bounds from full unit. become more relevant in theories and regimes where the loop expansion is more relevant than the derivative expansion

Paulos, Penedones, Toledo, v Rees, Vieira '17

Generic ansatz: $\mathcal{M}(s) = \sum_{a,b,c} \rho^a(s)\rho^b(t)\rho^c(u)$

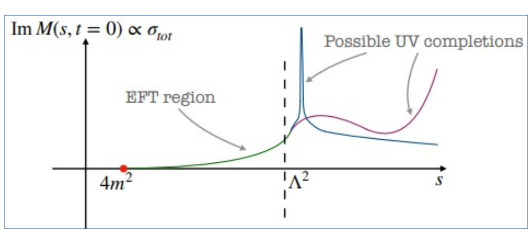


Impose unit. Numerically & maximise e.g. amplitude at a point

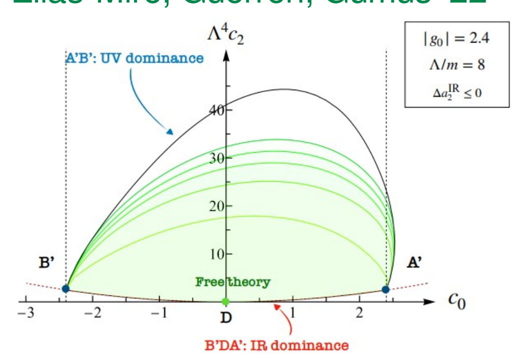
Generic ansatz + perturbativity:

Elias-Miró, Guerreri, Gumus '22

$$\mathcal{M}(s) = \sum_{a,b,c} \rho^a(s)\rho^b(t)\rho^c(u) \; +$$



=



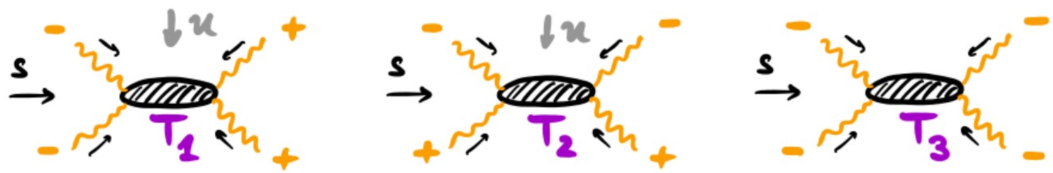
Multichannel EFTs

Remmen, Rodd '19
Li, Xu, Yang, Zhang, Zhou '21
Haring, Hebbar, Karateev, Meineri, Penedones '22
Durieux, Remmen, MR, Rodd 'WIP

Example, a U(1) vector:

$$\mathcal{L} = -(FF) + c_1(FF)^2 + c_2(F\widetilde{F})^2 + c_3(FF)(F\widetilde{F}) + \dots \qquad (FF) \equiv \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \qquad (F\widetilde{F}) = \frac{1}{4}F_{\mu\nu}\widetilde{F}_{\mu\nu}$$

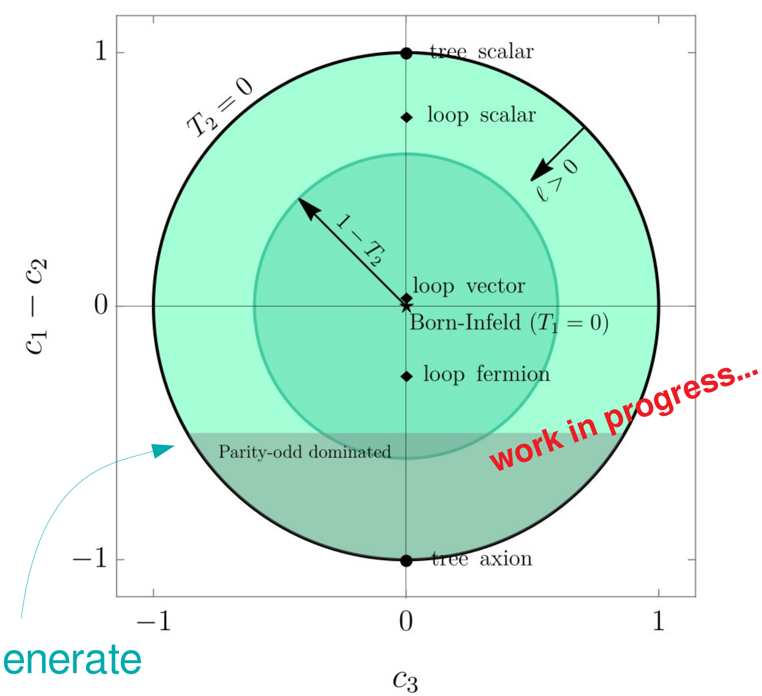
Three distinct scattering channels in the forward limit, and Wilson coeff. written as integrals of them:



$$\begin{aligned} c_1 &= T^1 + T^2 + \mathrm{Re}T^3 \\ c_2 &= T^1 + T^2 - \mathrm{Re}T^3 \\ c_3 &= \mathrm{Im}T^3 \end{aligned}$$

$$\begin{pmatrix} T^1 & 0 & 0 & T^3_r - iT^3_i \\ 0 & T^2 & 0 & 0 \\ 0 & 0 & T^2 & 0 \\ T^3_r + iT^3_i & 0 & 0 & T^1 \end{pmatrix} \succ 0$$

Now the measure is a positive matrix, relating arcs with same subtractions but different channels.



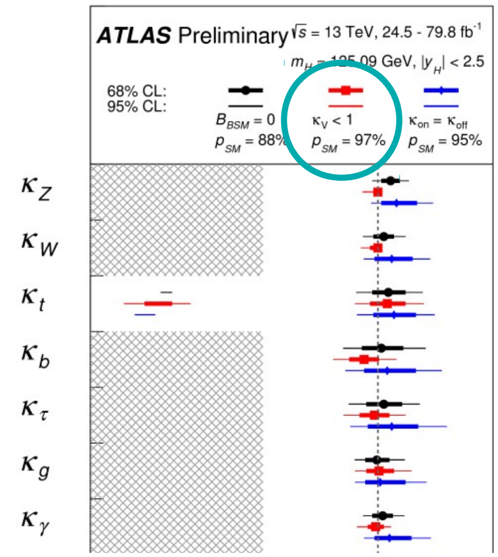
Boundary made of degenerate scattering channels

Dimension 6

Further assumptions on UV physics allow to set positivity bounds on dimension 6 effects

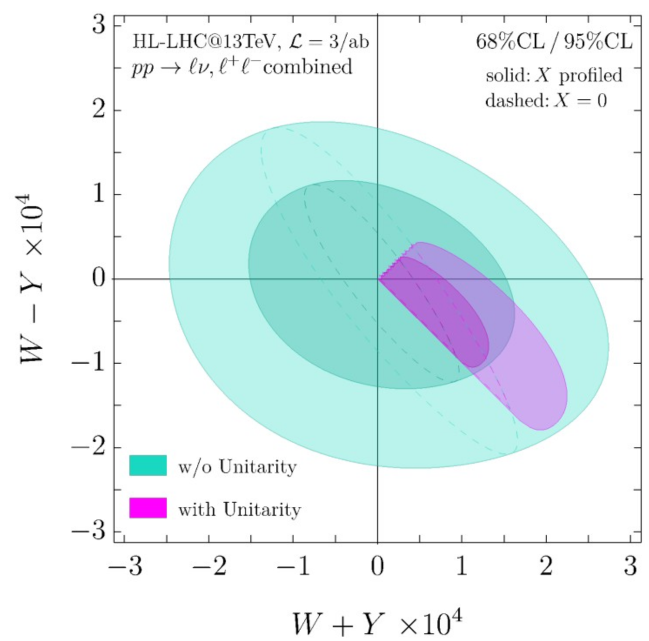
Low, Rattazzi, Vichi '09

No doubly-charged state → $c_H > 0$ →



WIP w/ McCullough, Ricci

Universal UV completion → $W > 0,\, Y > 0$ →



Conclusions

Generic UV properties

Moment structure in the forward limit

Relations between different scattering channels

Bounds on arc hierarchies from full unitarity

Bounds on finite angle scattering

Dispersion relations for scattering amplitudes allow to constrain the type of EFTs

and

map regions of parameter space to generic features of the UV completion