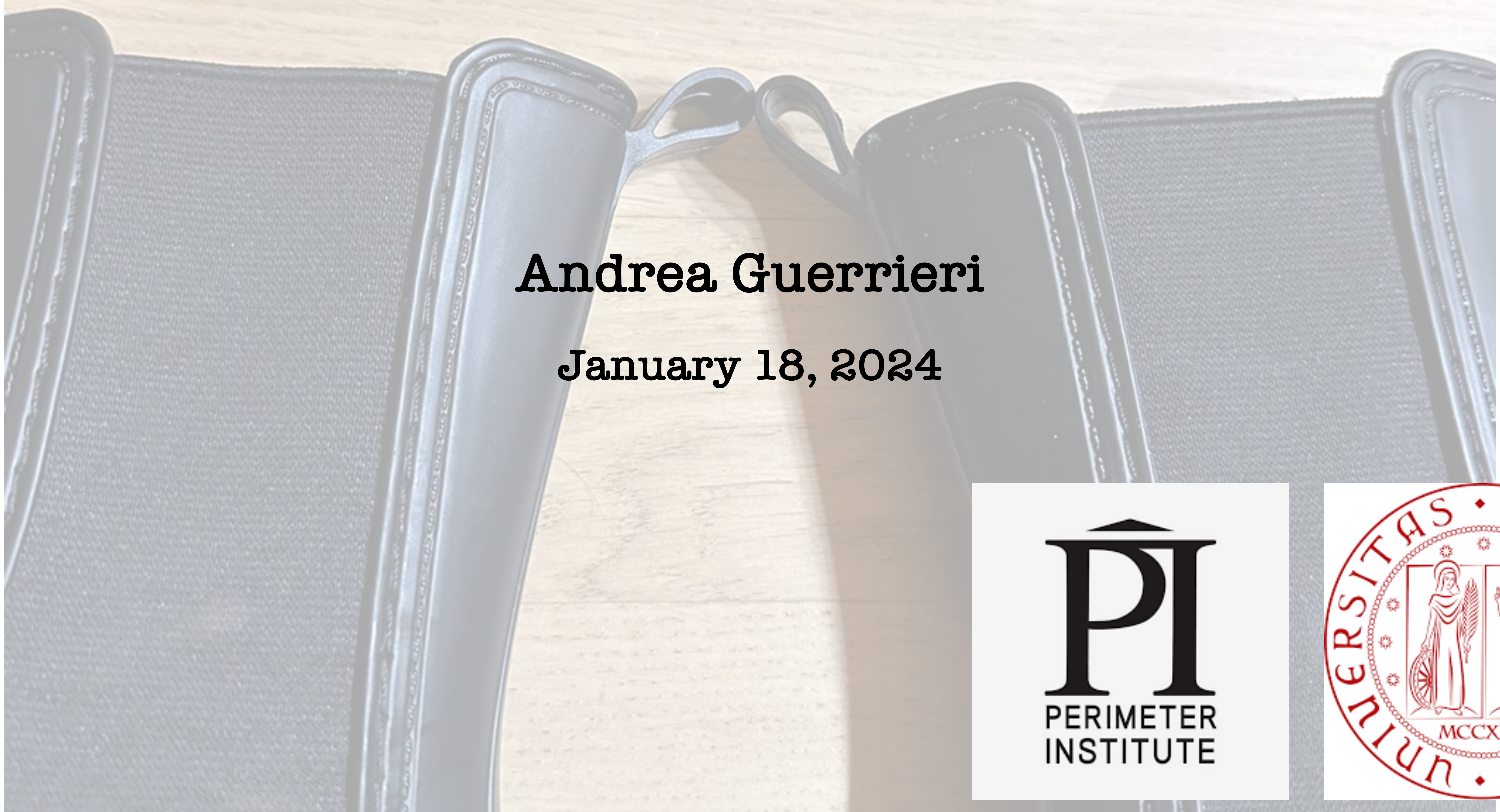


A Bootstrap Bridge between Gravity and QCD

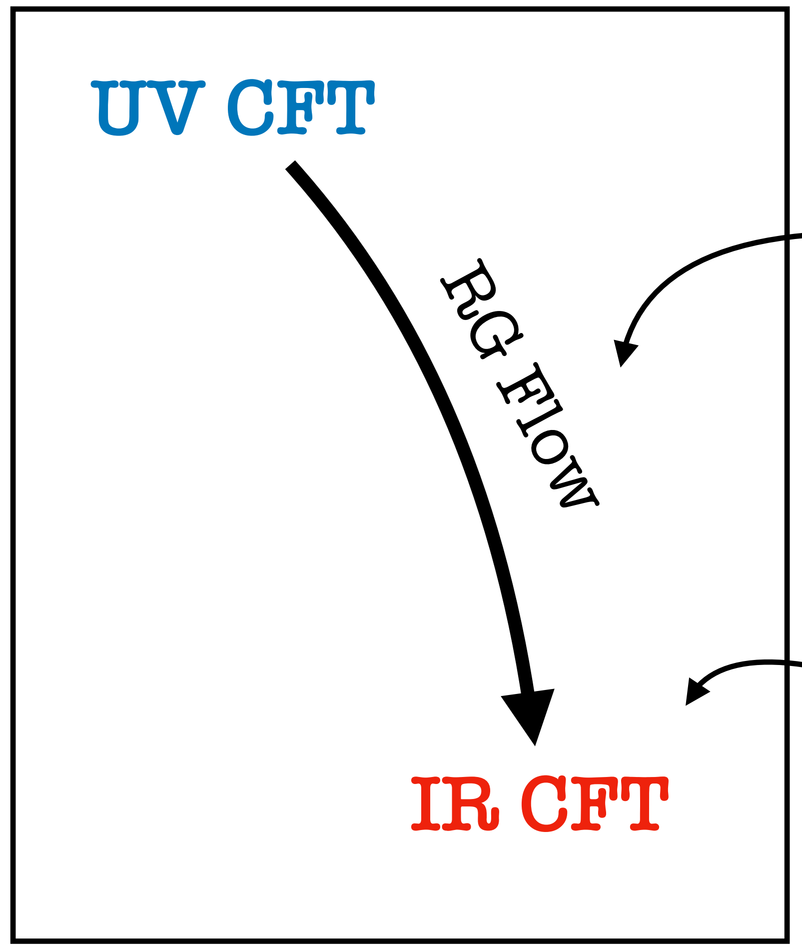
Andrea Guerrieri

January 18, 2024

PERIMETER INSTITUTE

UNIVERSITAS STUDII PADUANI MCCXXII

Bootstrap:
What is (im)possible in the Space of QFTs



Example of a QFT

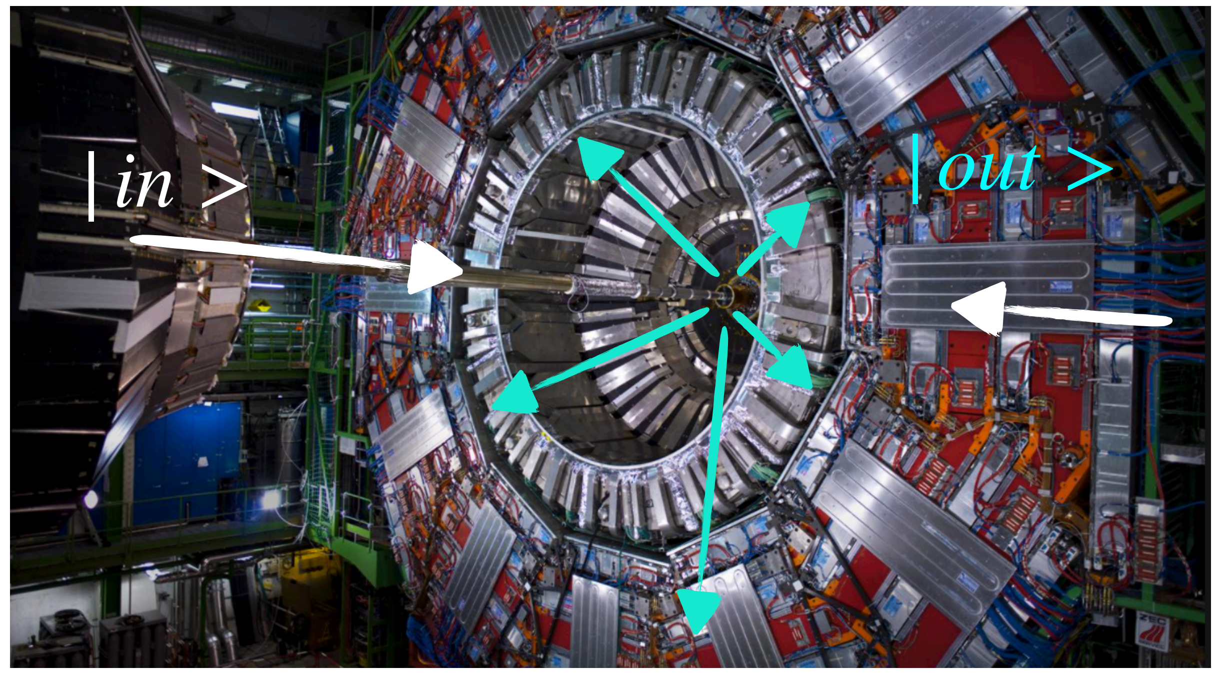
S-matrix Bootstrap: study the possible RG-flows

Conformal Bootstrap: study fixed points of the RG-flow

Bootstrap manifesto:
find the space of all possible physical observables assuming general principles

The S-matrix measures the probability of a scattering process

$$\mathcal{S}_{in \to out} \equiv < in | out >$$



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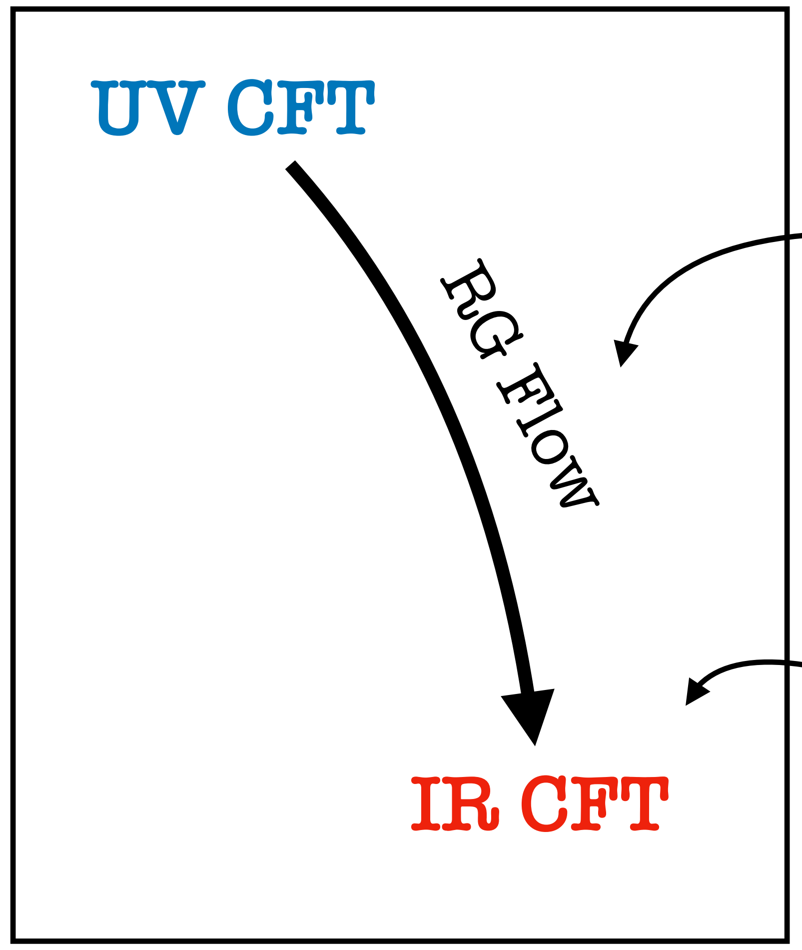

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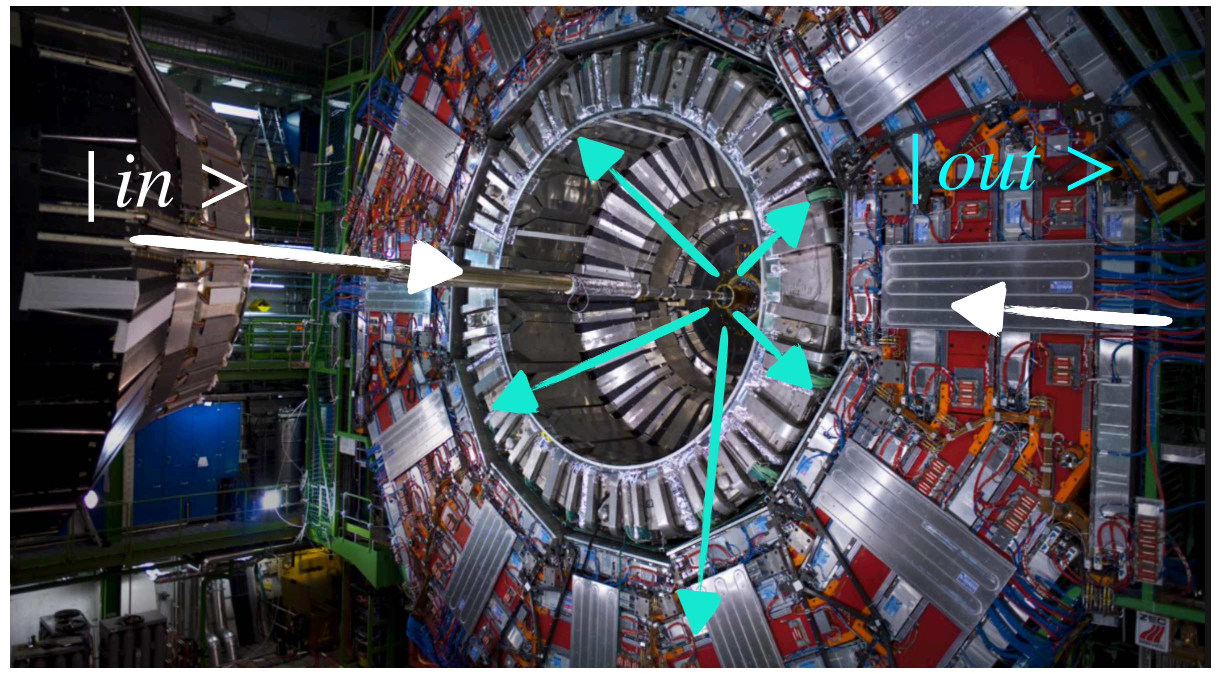
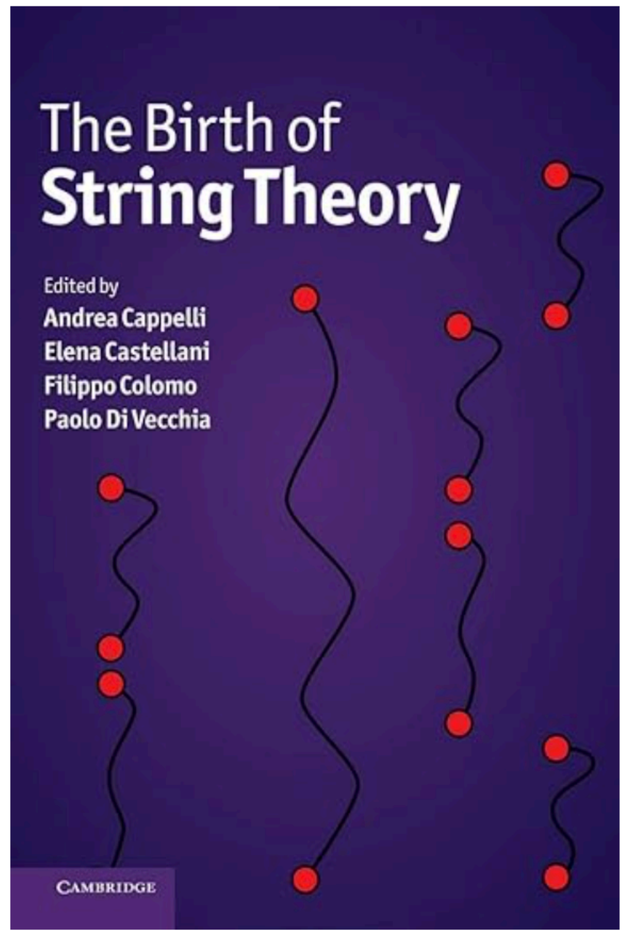


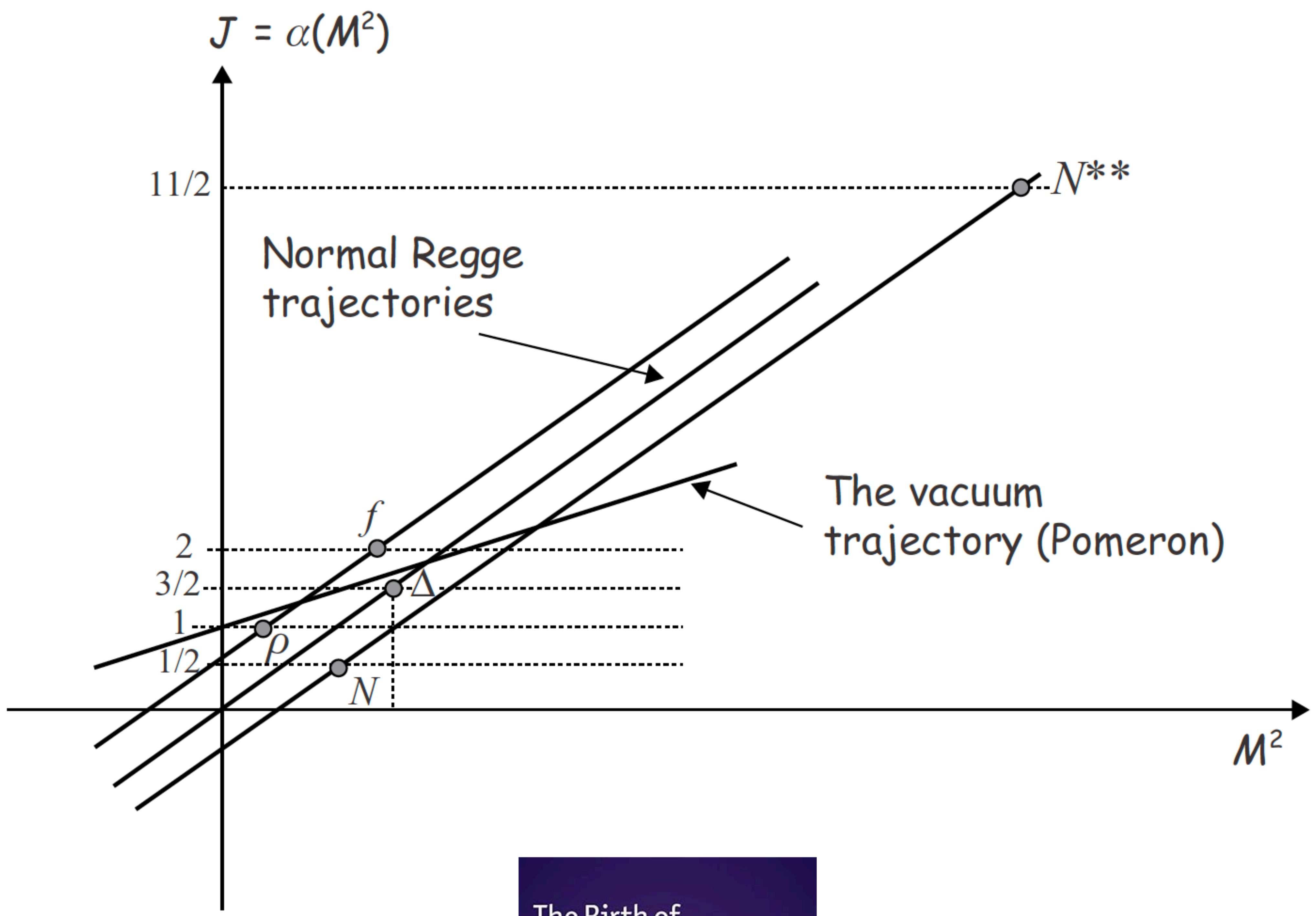
Point of the Talk: Strings populate the landscape of non-perturbative S-matrices

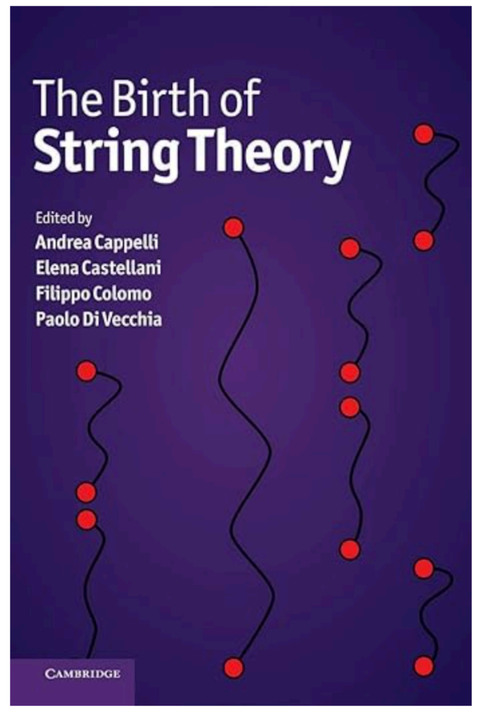


Strings and Regge trajectories

I will optimistically conjecture that behind Regge trajectories there is a dual string description

Veneziano's picture!





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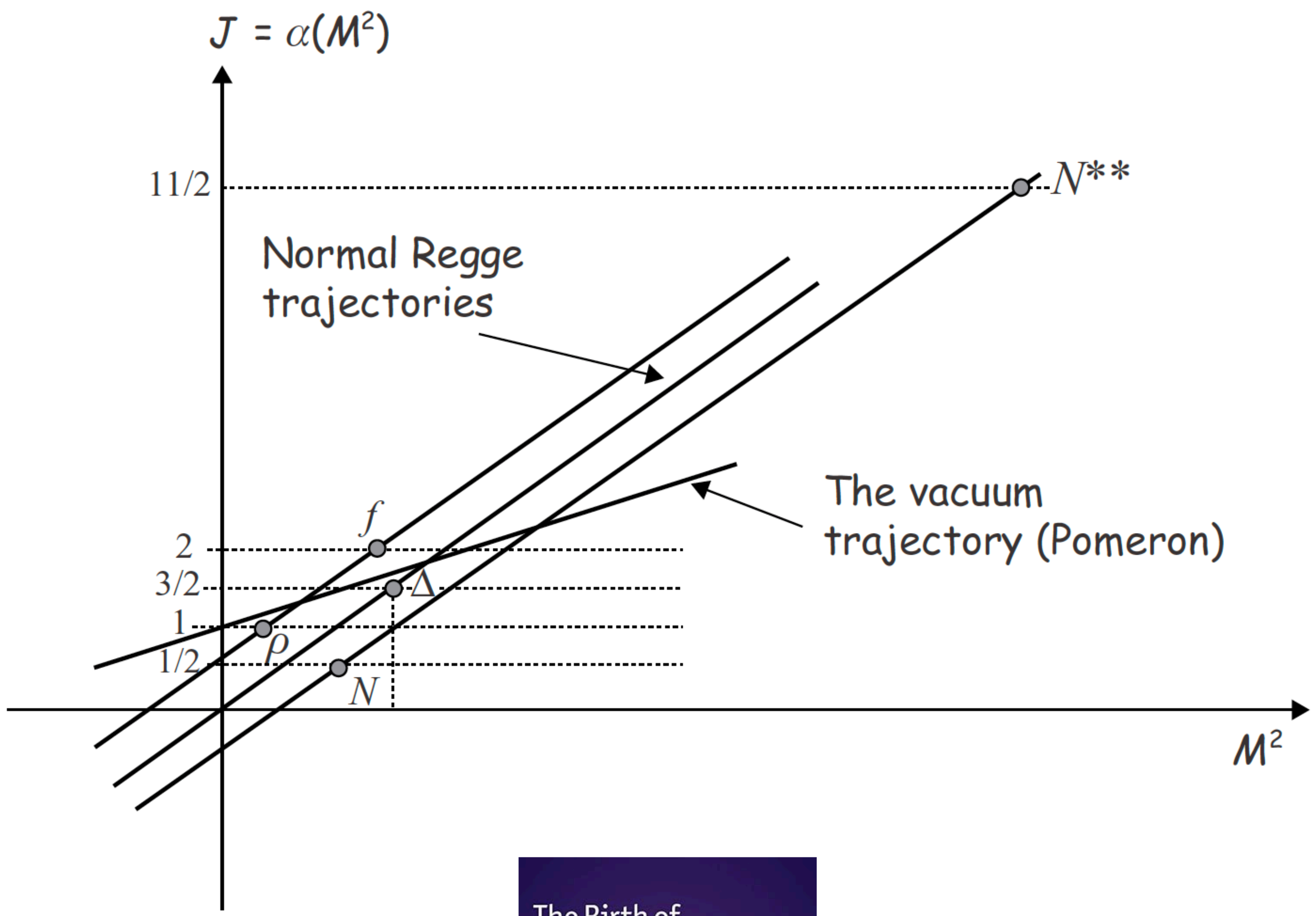
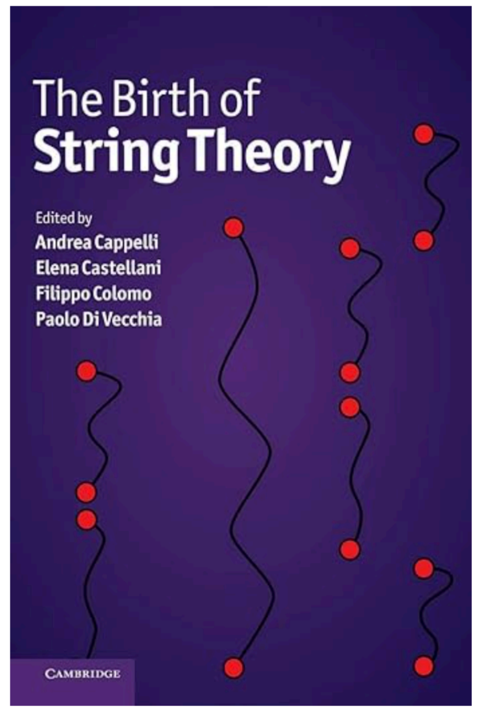
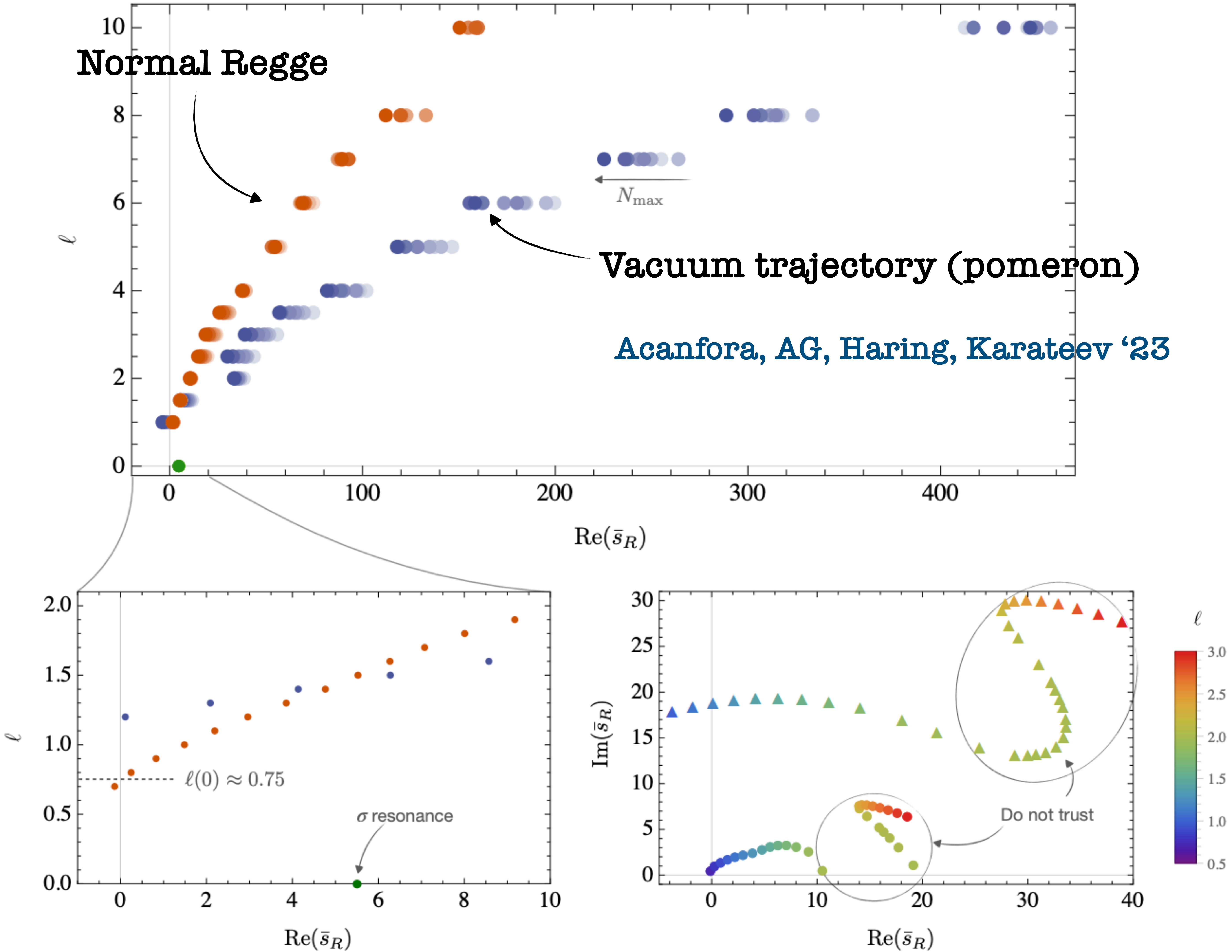
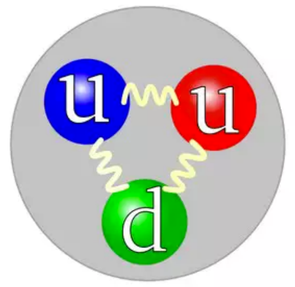




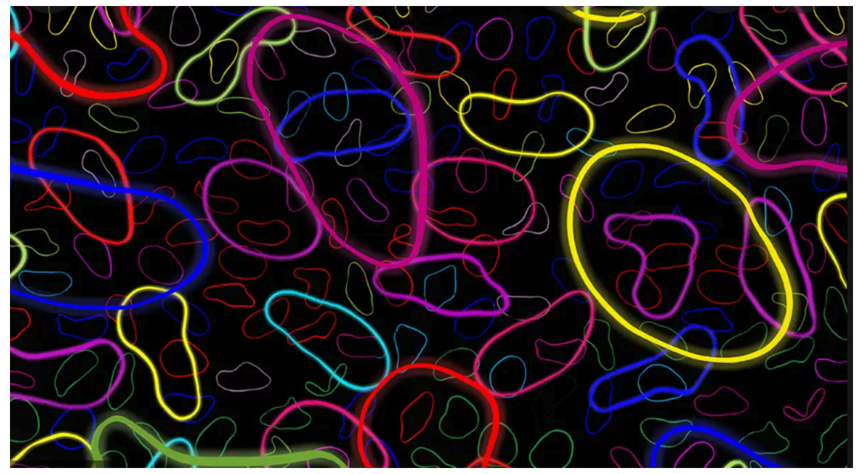


Plan of the Talk

1) The Prehistory of String Theory: the Analytic S-matrix



2) Strings from Wightman axioms: SU(3) YM Glueball Scattering

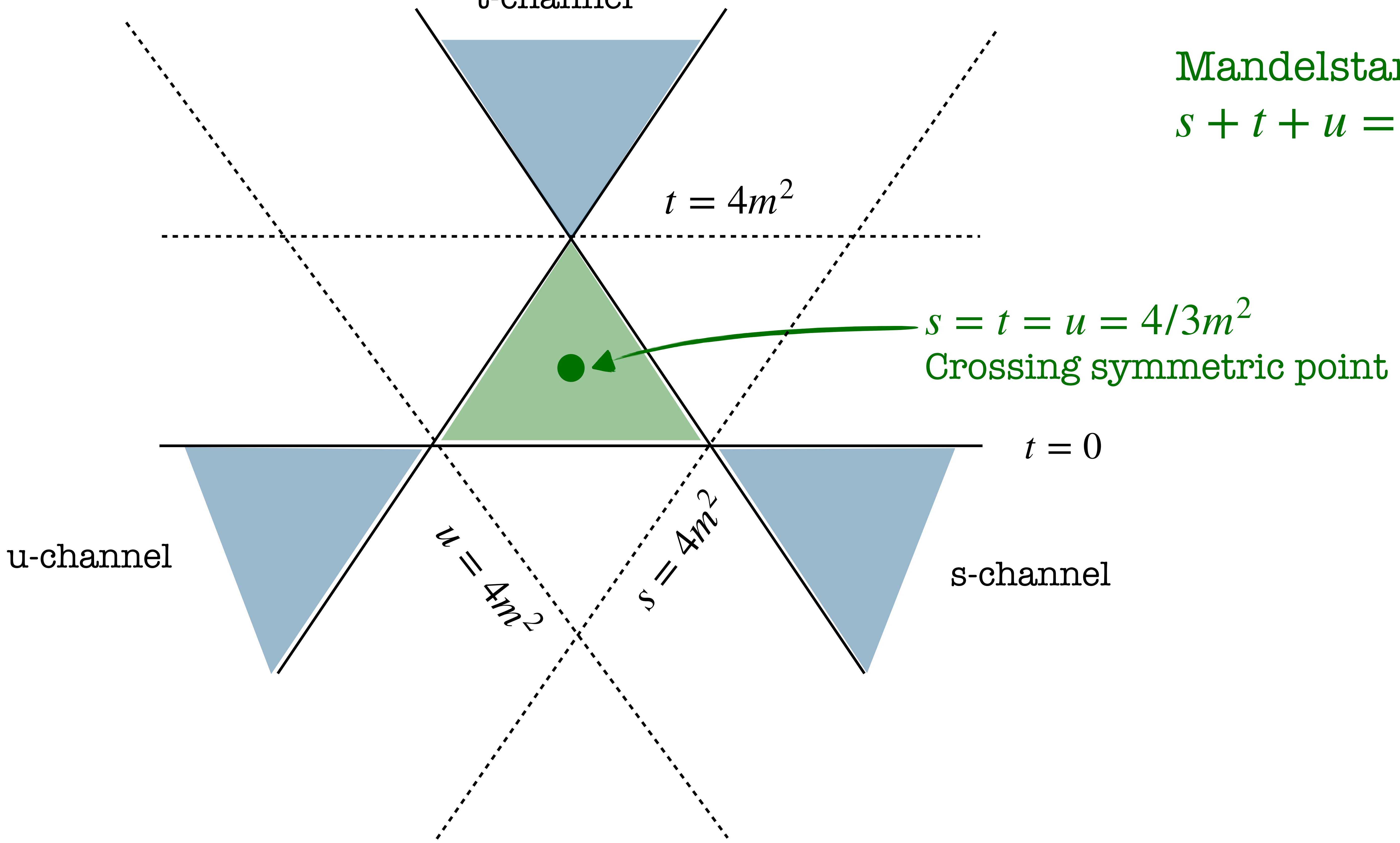


3) Strings from Gravity and no-go theorems

4) Bootstrap the world-sheet of confining Strings?

The Prehistory of String Theory: the analytic S-matrix 1

Example: 1 scalar field of mass m in the IR



Mandelstam plane
$s + t + u = 4m^2$

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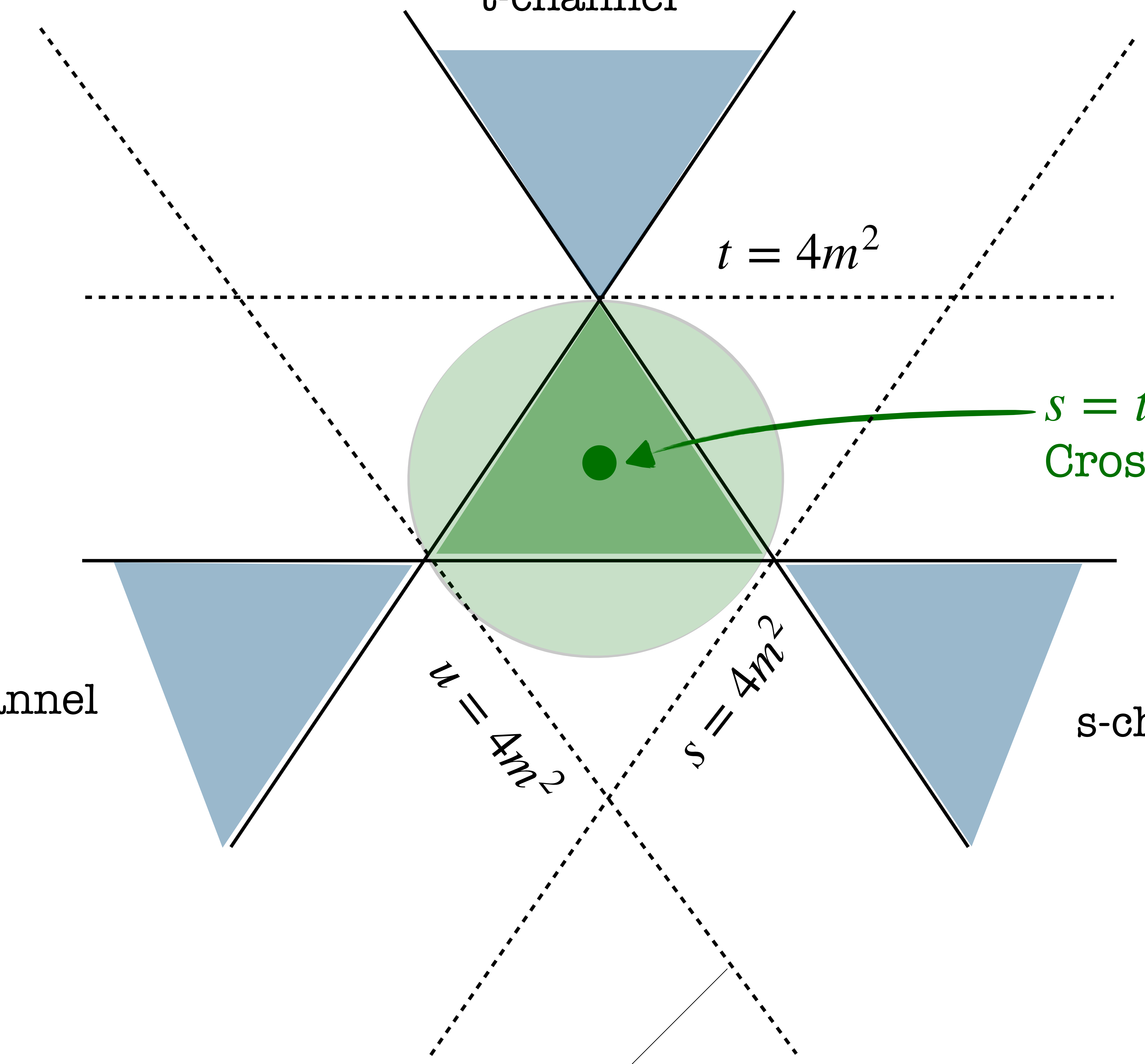


Mandelstam plane
$s + t + u = 4m^2$

$s = t = u = 4/3m^2$
Crossing symmetric point

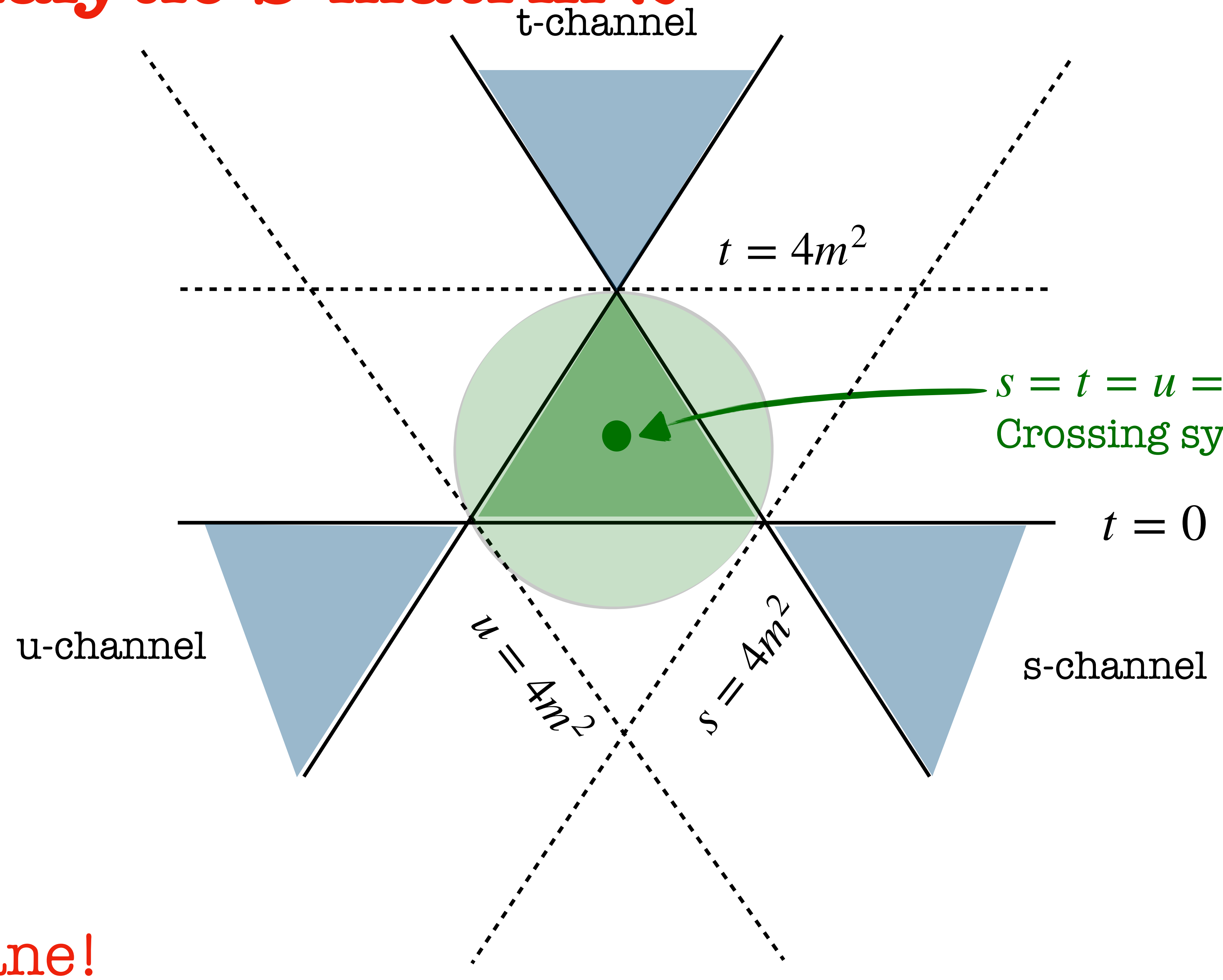
$$M(s, t, u) = -c_0 + c_2(\bar{s}^2 + \bar{t}^2 + \bar{u}^2) + c_3\bar{s}\bar{t}\bar{u} + \ldots \qquad \bar{x} = x - \frac{4}{3}m^2$$

The set $\{c_0, c_2, c_3, \ldots\}$ parametrizes the space of amplitudes

The Prehistory of String Theory: the analytic S-matrix 2

$$M(s,t,u) = -c_0 + c_2(\bar{s}^2 + \bar{t}^2 + \bar{u}^2) + c_3\bar{s}\bar{t}\bar{u} + \dots$$

Space of amplitudes parametrized by $\{c_0, c_2, c_3, \dots\}$



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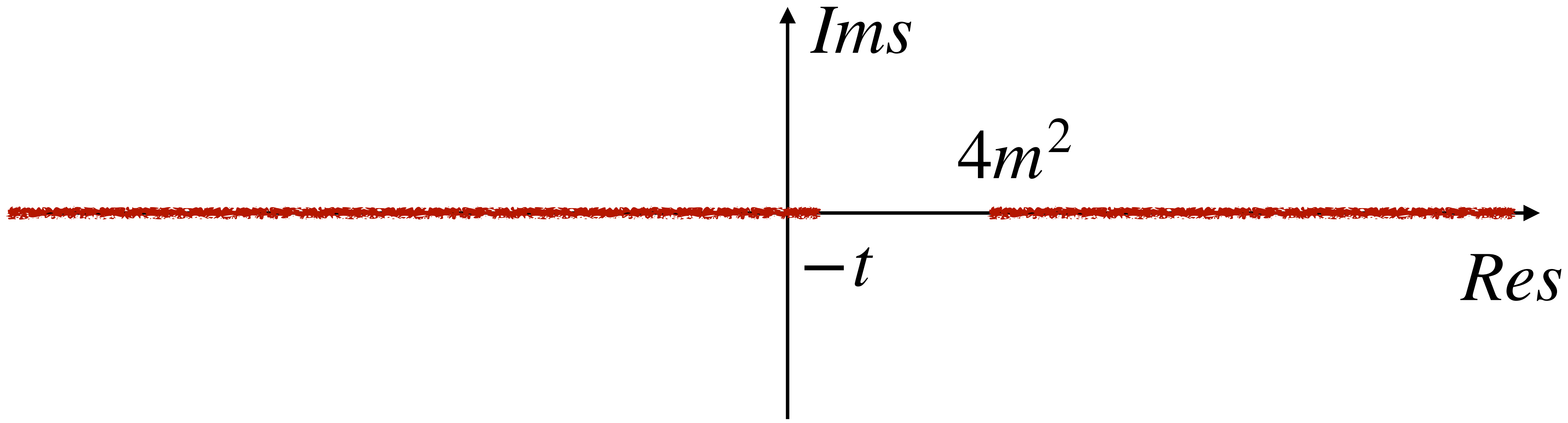
Analyticity means we can go into the complex plane!

Analytic in the s-plane away from the cuts for all $-28m^2 < t < 4m^2$

Martin, Jin, Froissart, Mandelstam, Lehmann, and many many others



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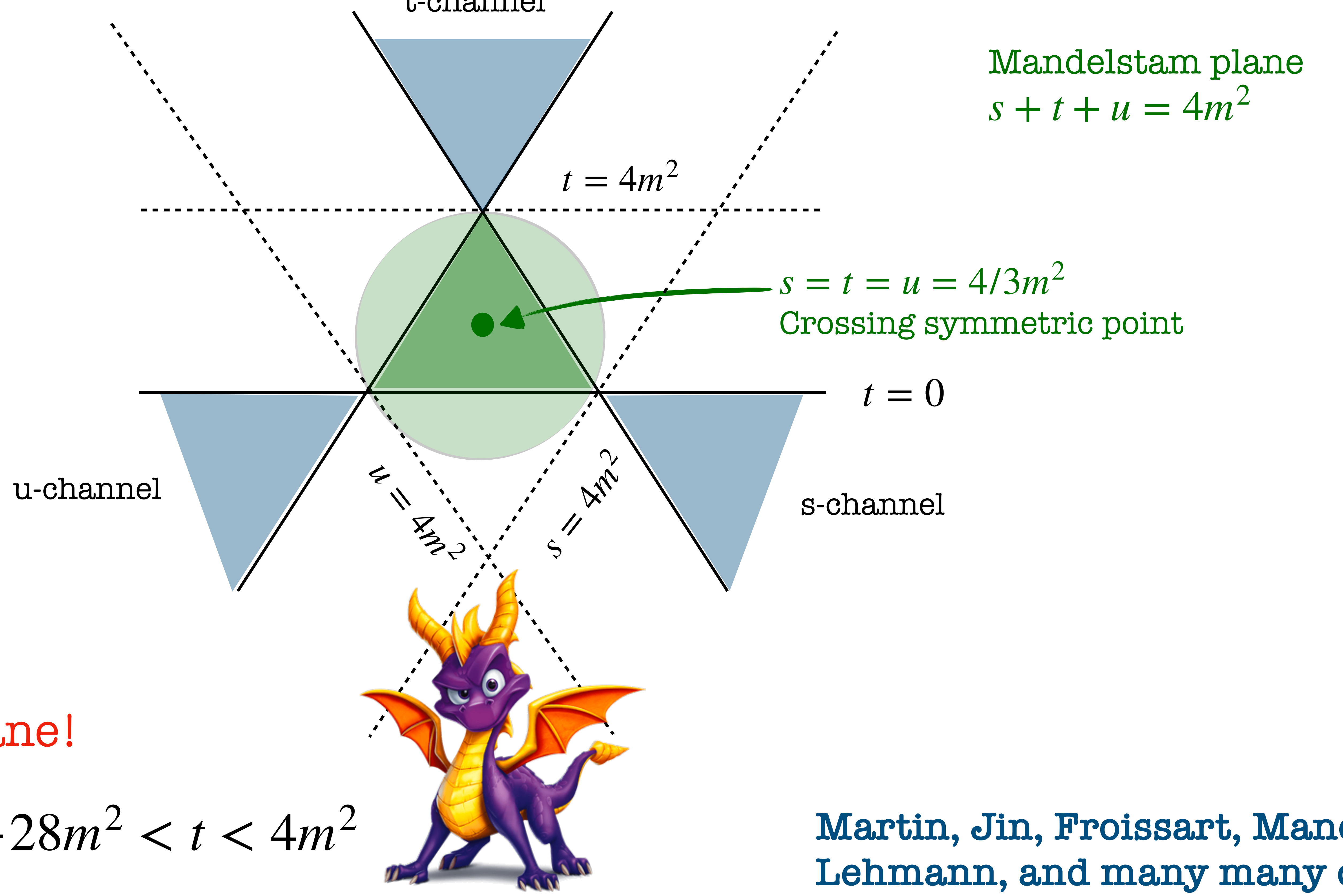

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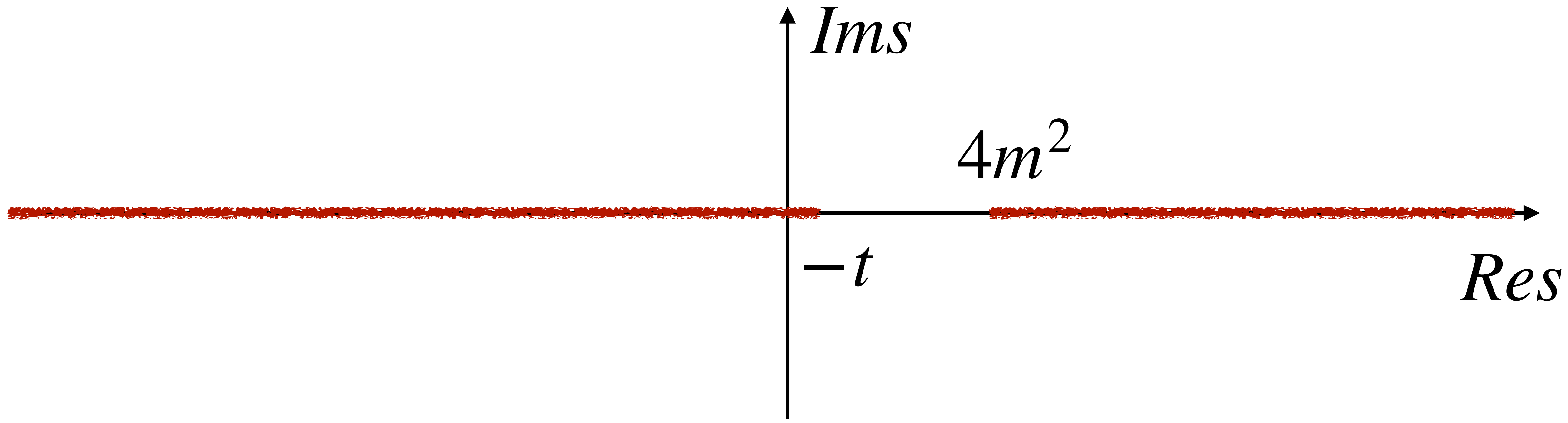

$t < -28m^2$: double discontinuity region!

Correia, Sever, Zhiboedov 2111.12100

Tourkine, Zhiboedov 2303.08839

The beauty of Unitarity

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Froissart bound

Dispersive parameters ≡ operators of dimension ≥ 8

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$$t_0 = s_0 = 4/3m^2$$

Subtraction point

$$T_v(v,t_0) \equiv 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) Im f_\ell(s) P_\ell(1 + 2t_0/(s-4)) \geq 0$$

Positivity

Legendre positivity

$$P_\ell(x) > 0, \quad x \geq 1$$

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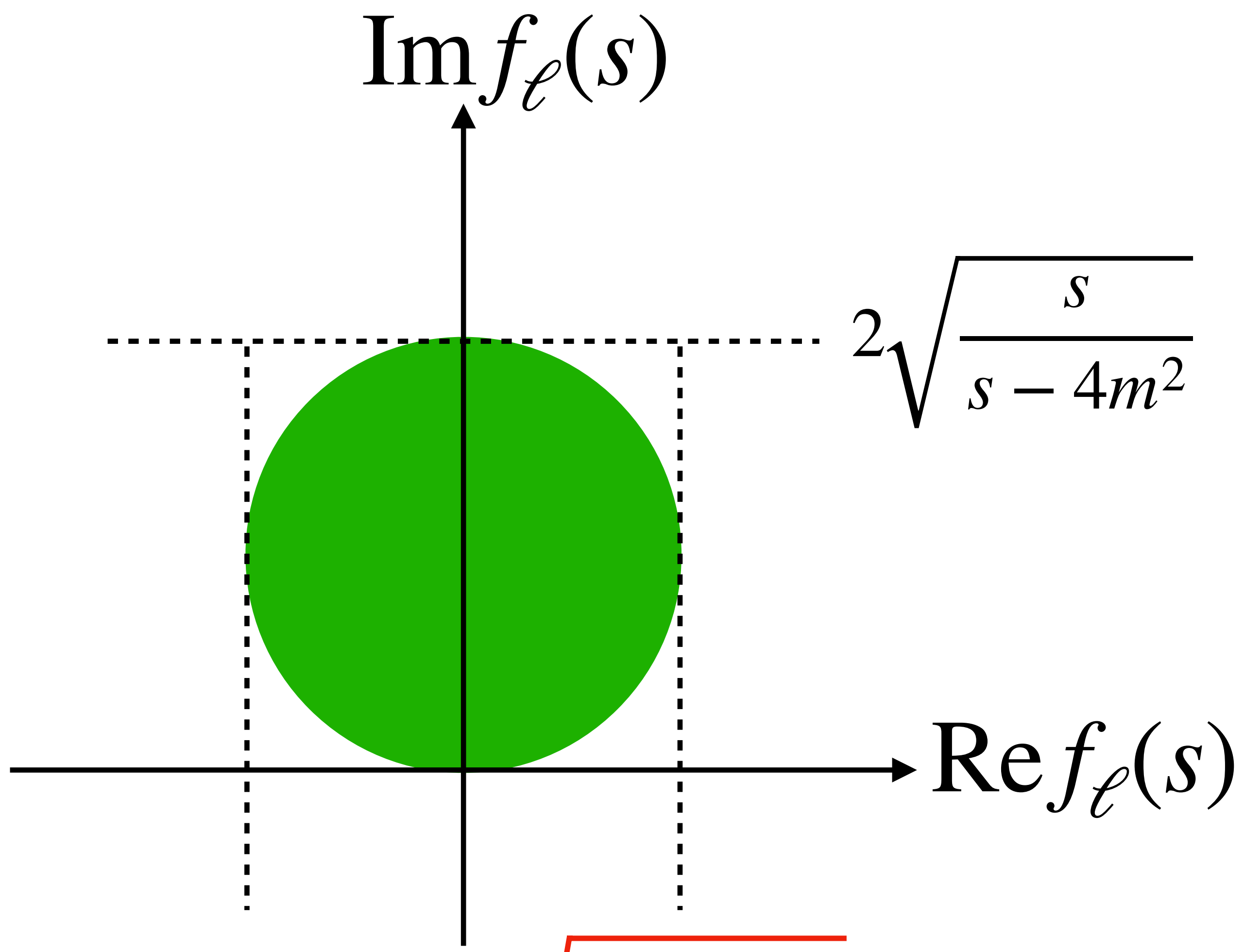

Unitarity saves the day!

Unitarity: $2 Im f_\ell \geq \sqrt{\frac{s-4m^2}{s}} |f_\ell|^2$

The Island of 4d scalar amplitudes

We bound c_0, c_2 using dispersion relations and unitarity!
It is an exercise in constrained optimization theory.

Bonnier, Lopez, Mennessier, '70s
AG, Sever 2106.10257

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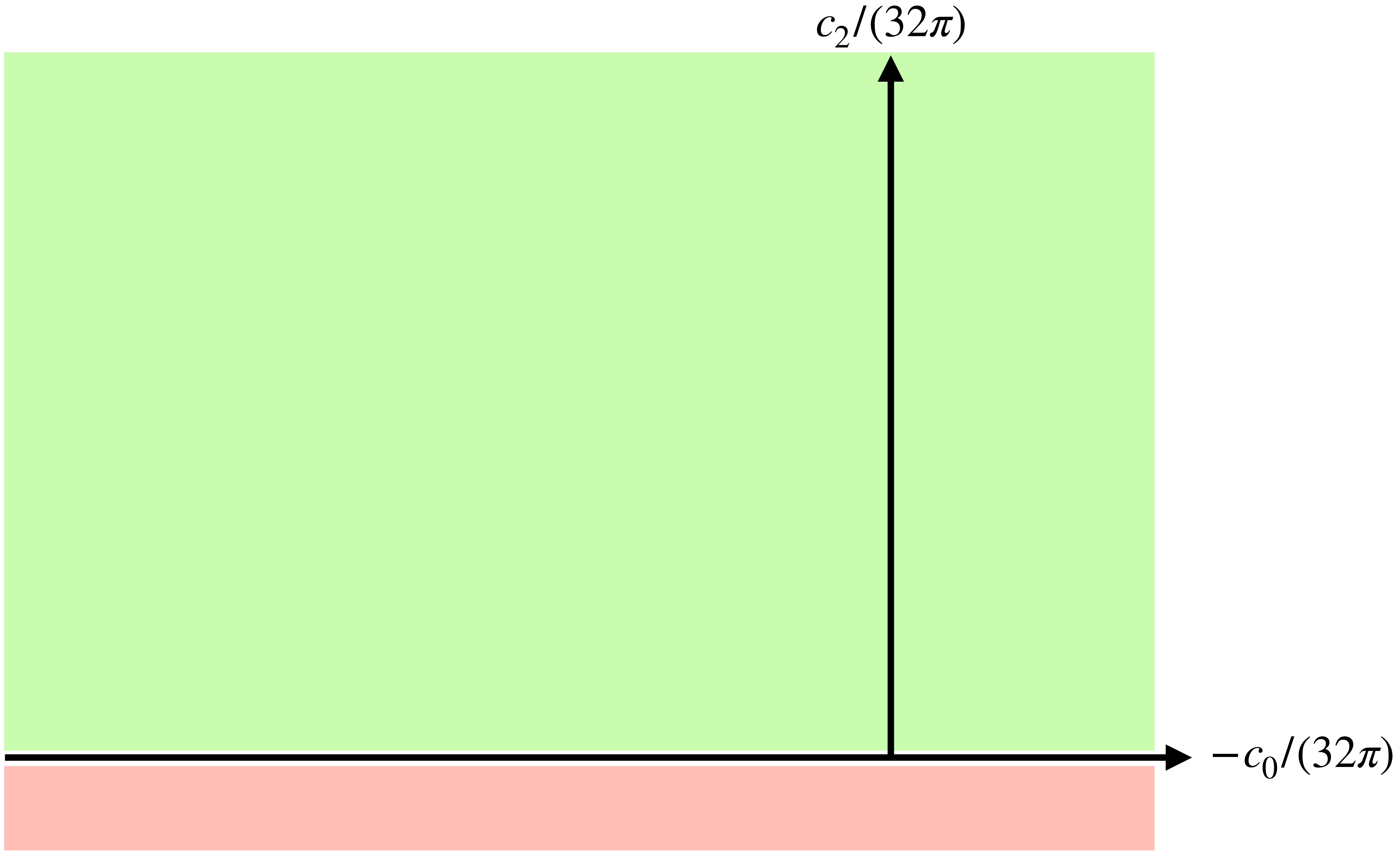

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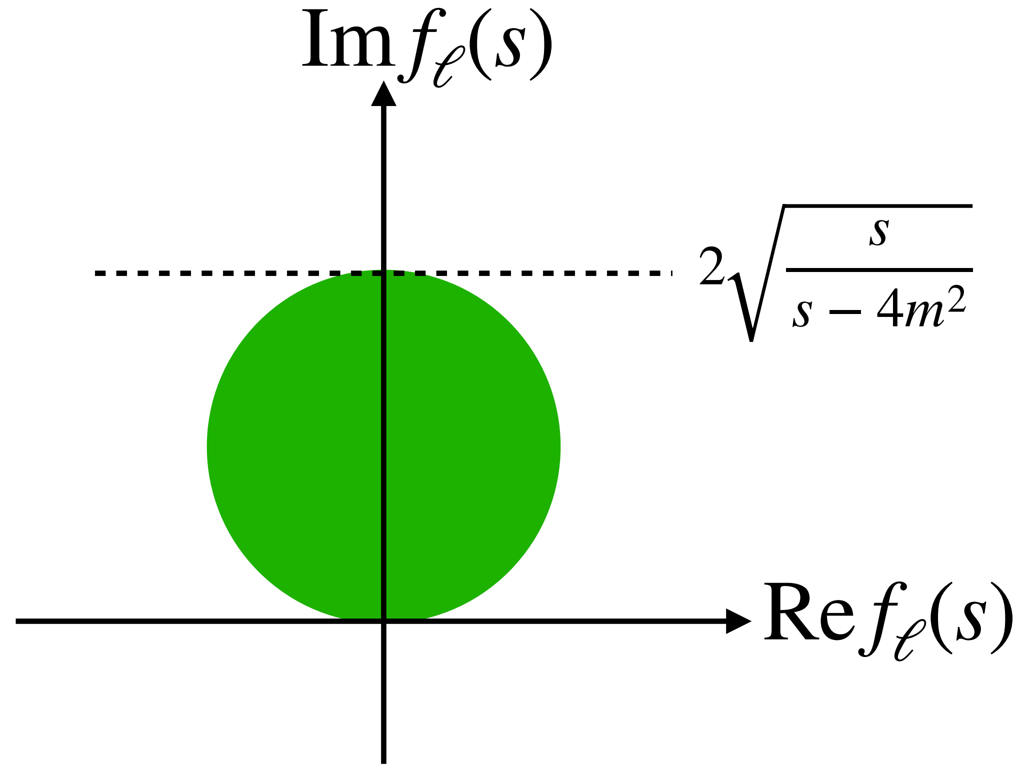
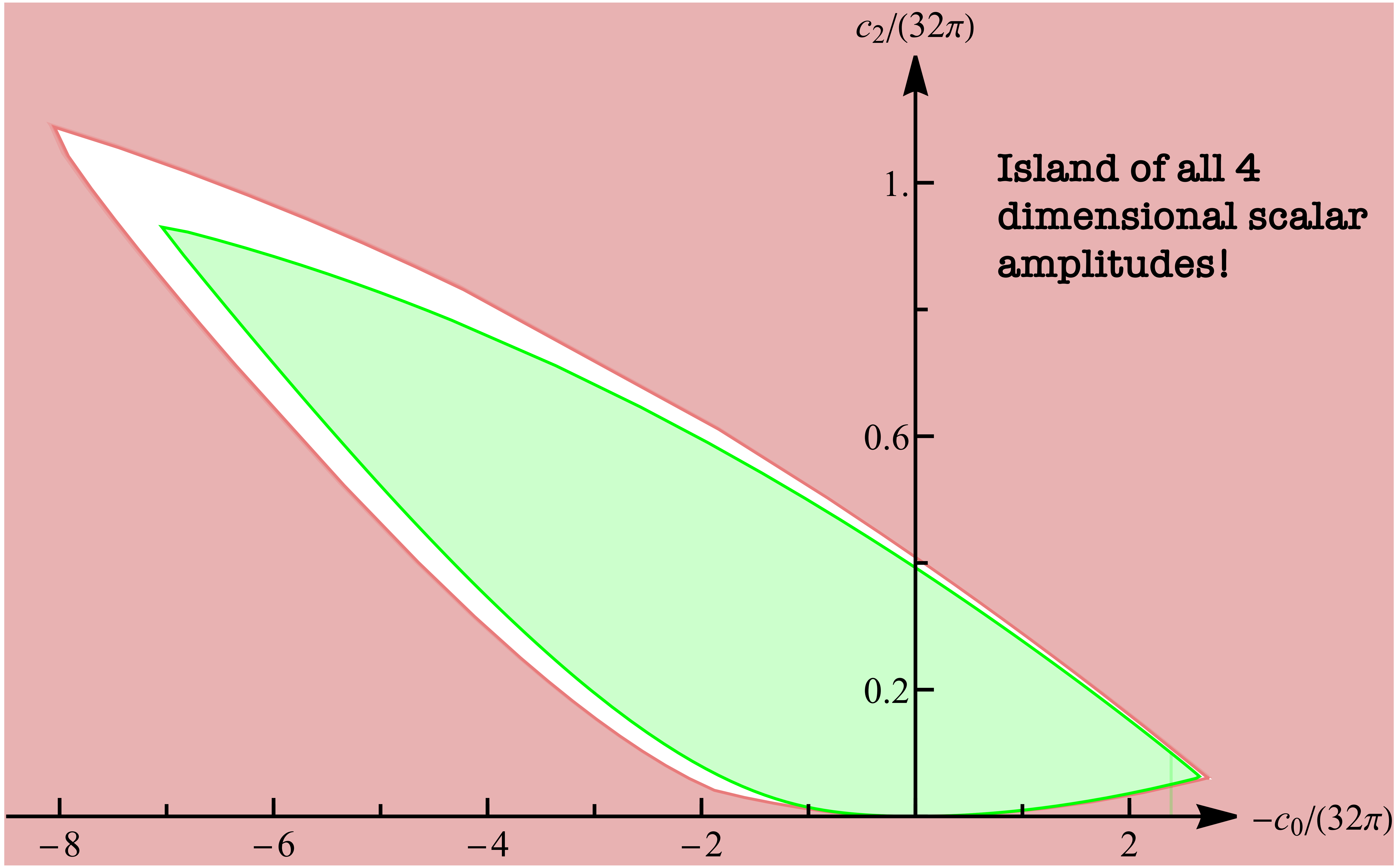


Elias-Miro, AG, Gumus 2210.01502
Elias-Miro, AG, Gumus (to appear)



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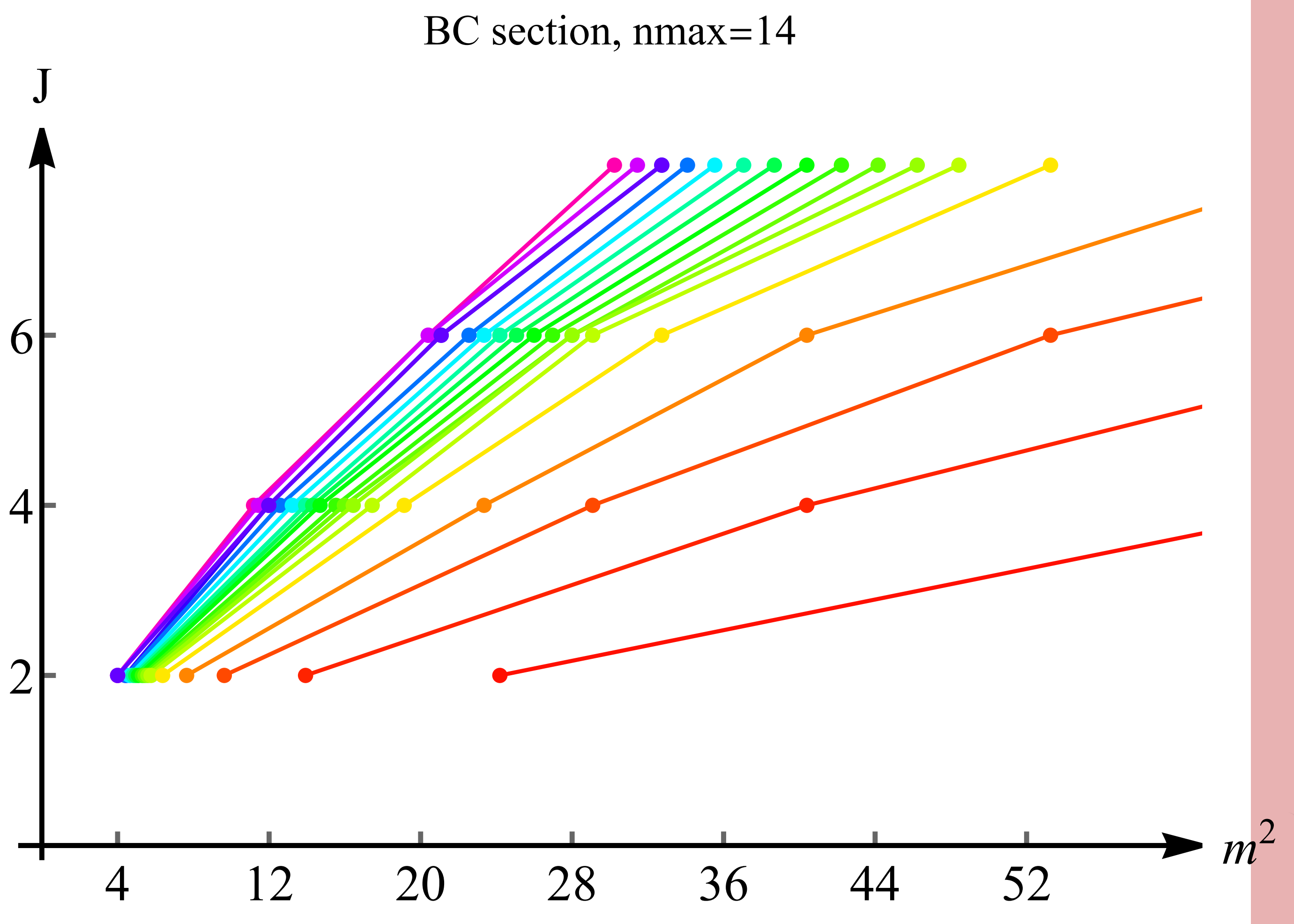

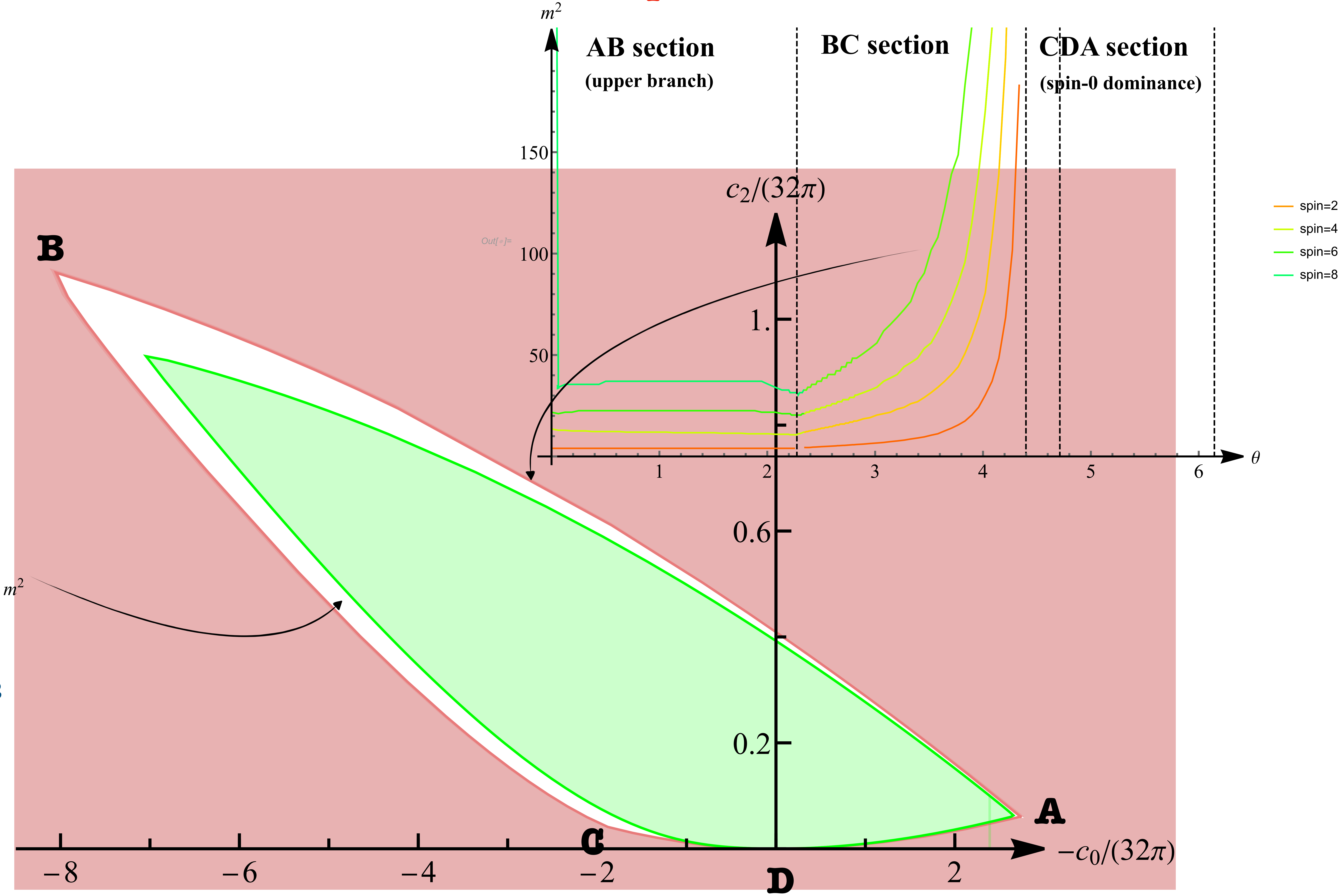

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Glueball Scattering in SU(3) pure YM

Regime in which the S-matrix Bootstrap shows its power: cutoff $\Lambda = 2m$, no small parameters.

(Still Hard after 50 years of QCD)

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Stable Glueballs spectrum

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H	2^{++}	1.437 ± 0.006
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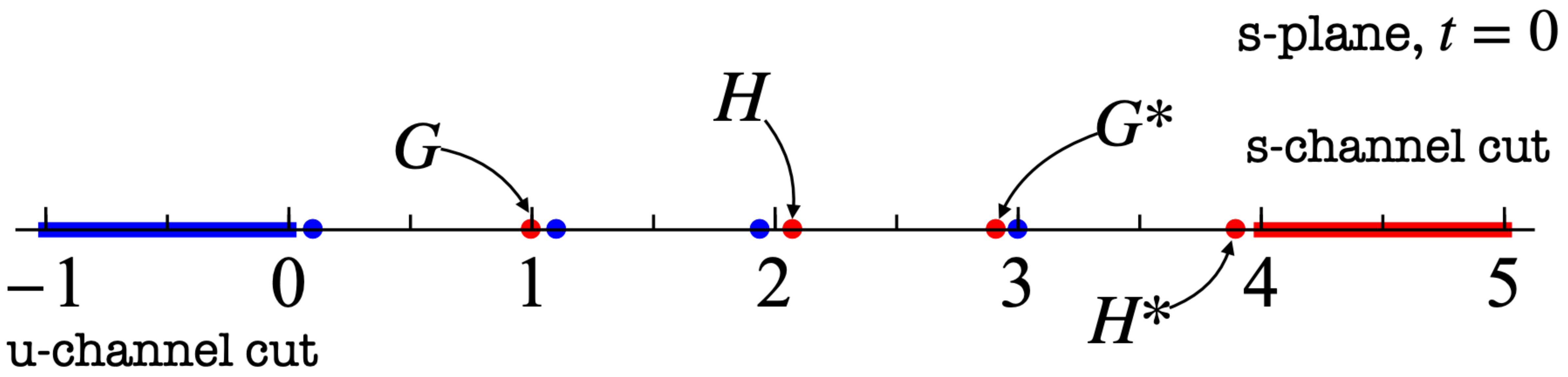
Athenodorou, Teper 2007.06422, 2106.00364

Pole Structure in GG->GG scattering



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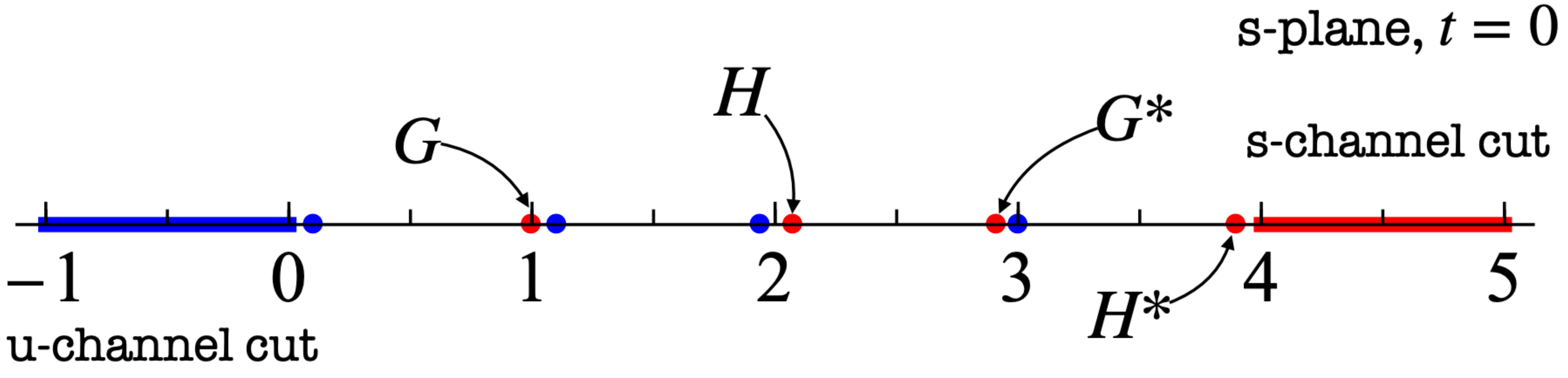
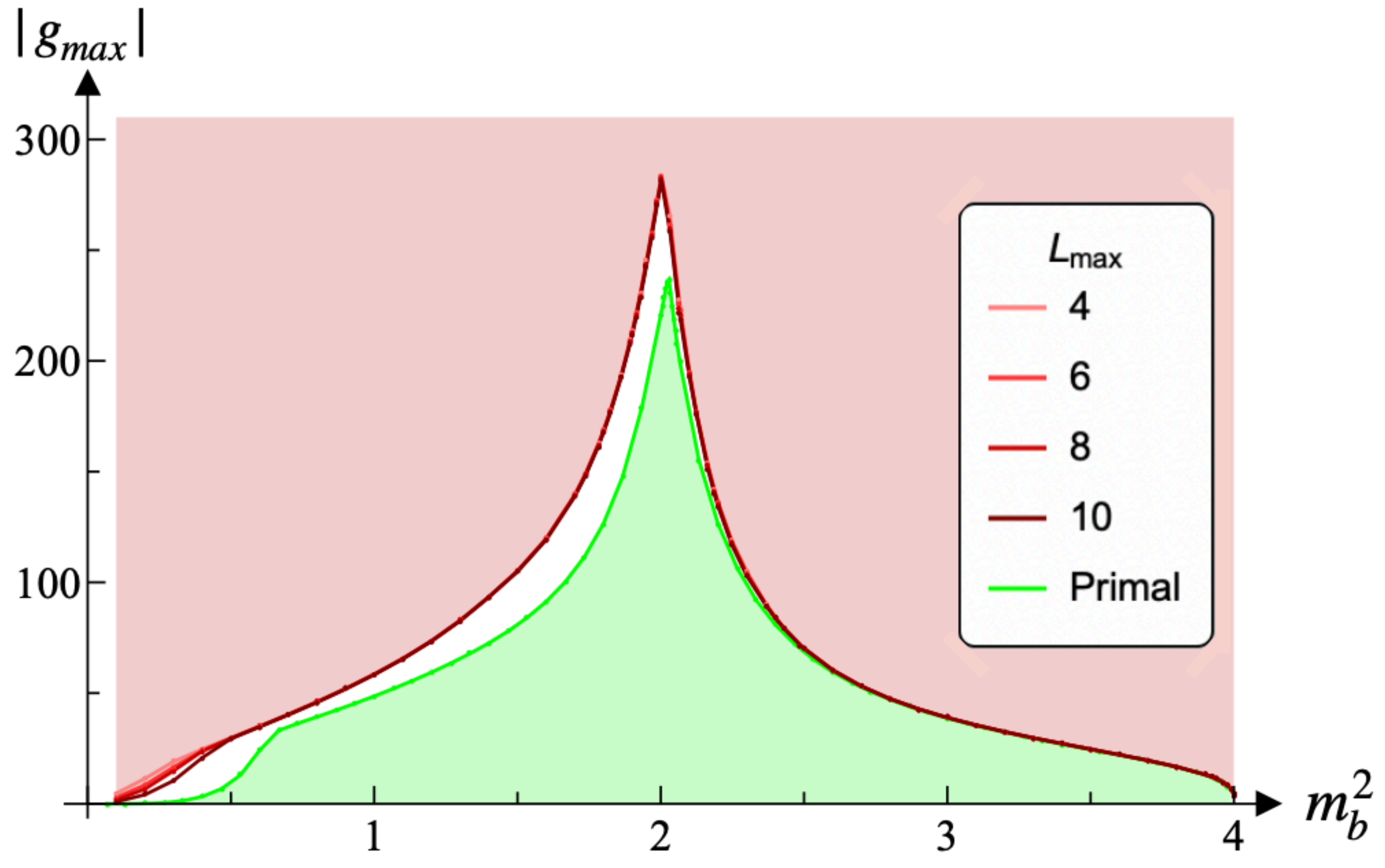


The maximum residue at the spin-0 pole is a simple problem

$$M \supset \frac{-g^2}{s - m_b^2} + \frac{-g^2}{t - m_b^2} + \frac{-g^2}{u - m_b^2} + \ldots$$

Paulos, Penedones, Toledo, van Rees, Vieira 1708.06765

AG, Hebbar, van Rees 2312.00127



Toy model: amplitudes maximizing the spin-2 coupling

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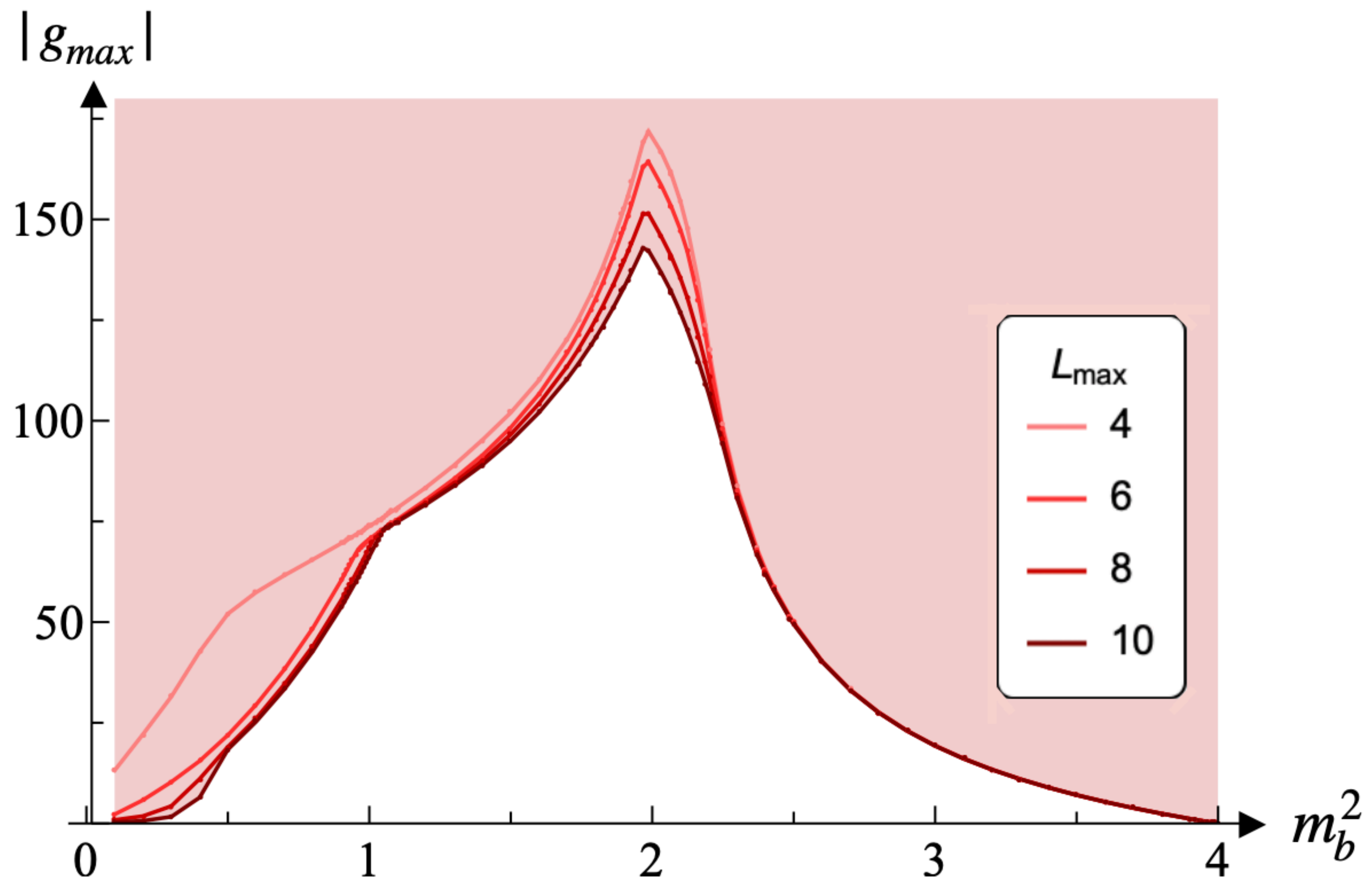

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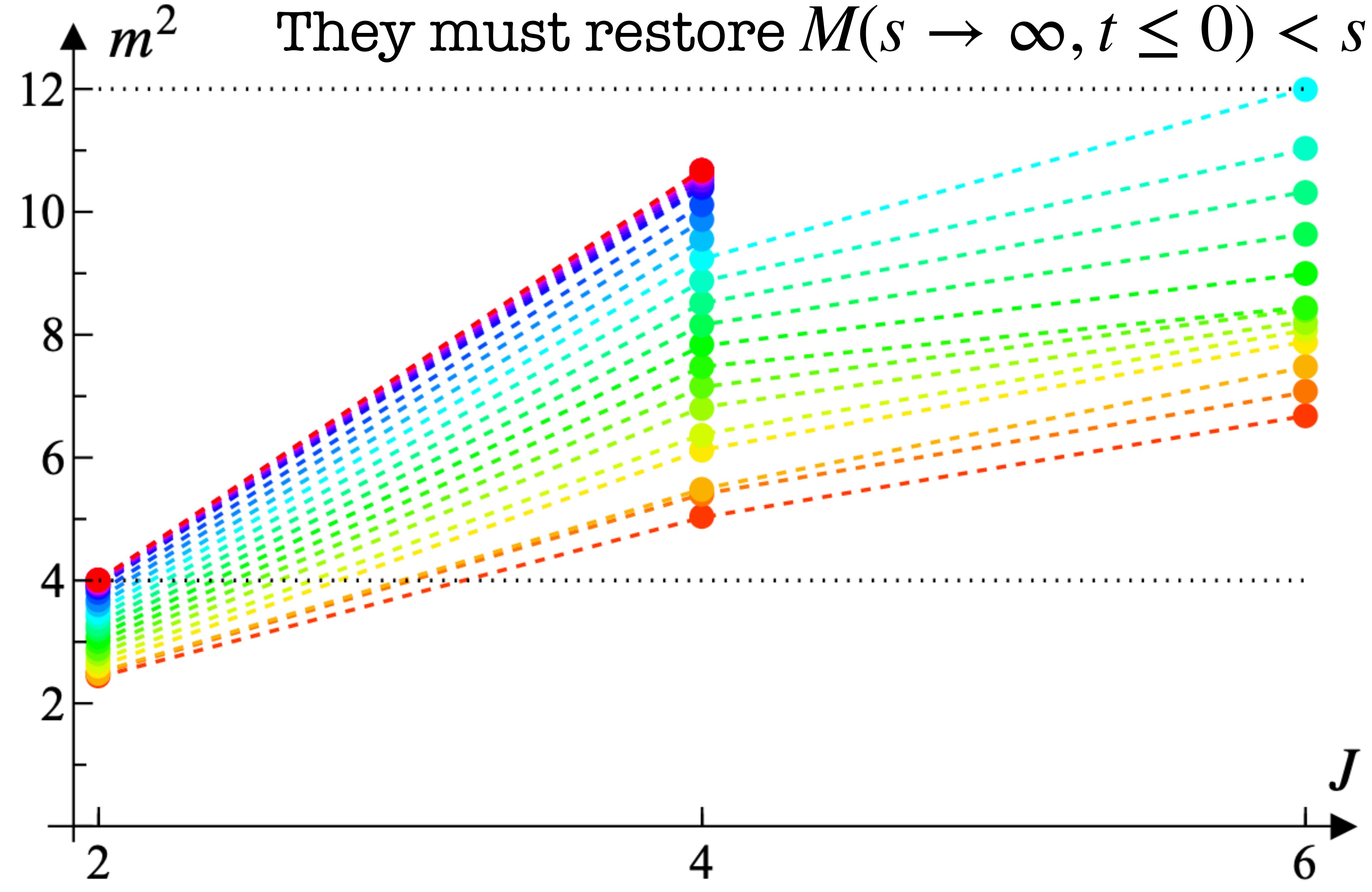

The Glue-Hedron

$$M \supset -g_G^2 \frac{1}{s-m_G^2} - g_{G*}^2 \frac{1}{s-m_{G*}^2} - g_H^2 \frac{t^2+\dots}{s-m_H^2} - g_{H*}^2 \frac{t^2+\dots}{s-m_{H*}^2} + \dots$$

AG, Hebbar, van Rees 2312.00127

$\max \lvert g_G \rvert$	$\max \lvert g_H \rvert$	$\max \lvert g_{G^*} \rvert$	$\max \lvert g_{H^*} \rvert$
213	158	224	2.15
206	156	217	–

SU(3) YM Lattice $g_G \approx 50 \pm 7$

De Forcrand, Schierloz, Schneider, Teper '85

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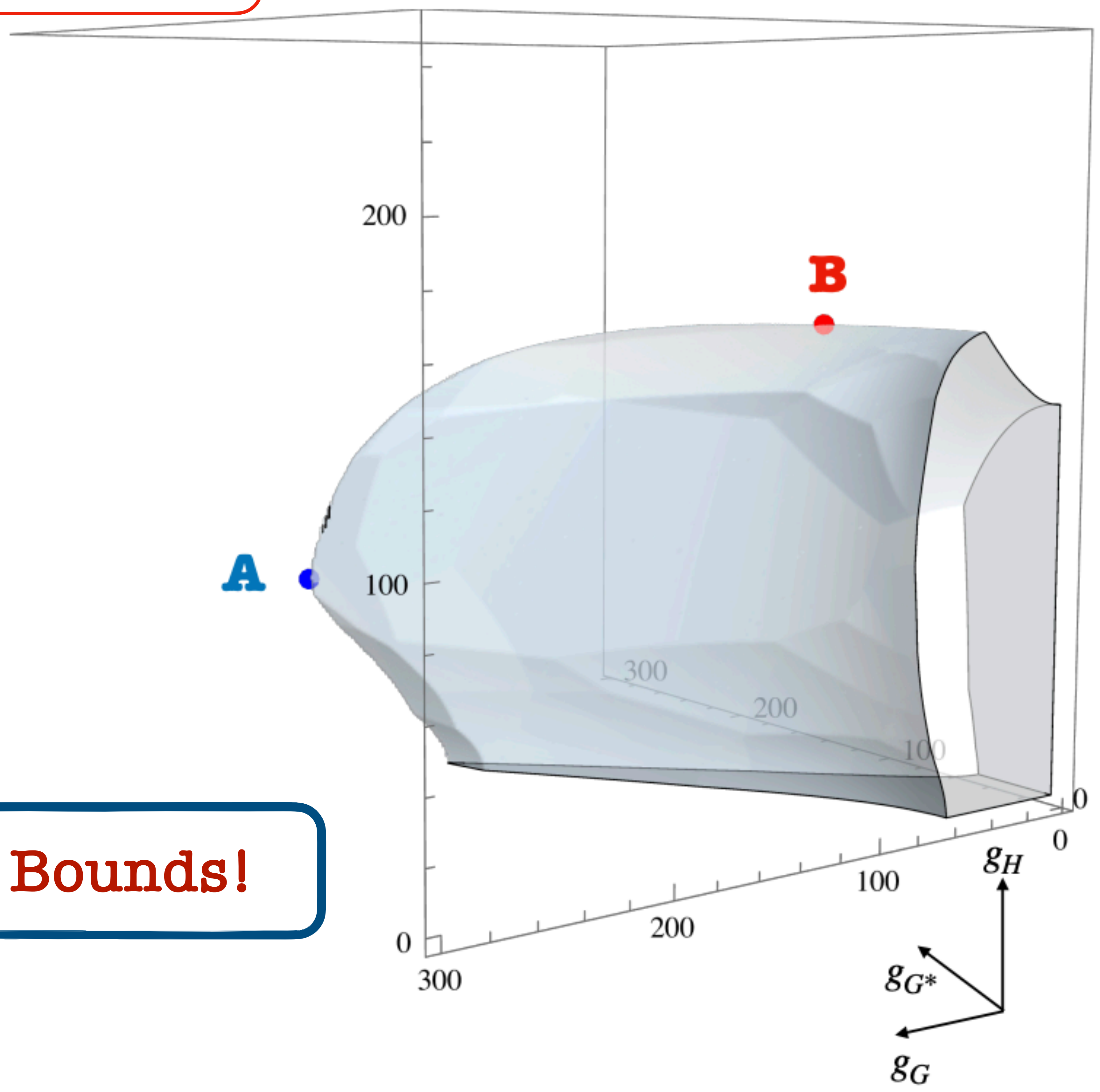
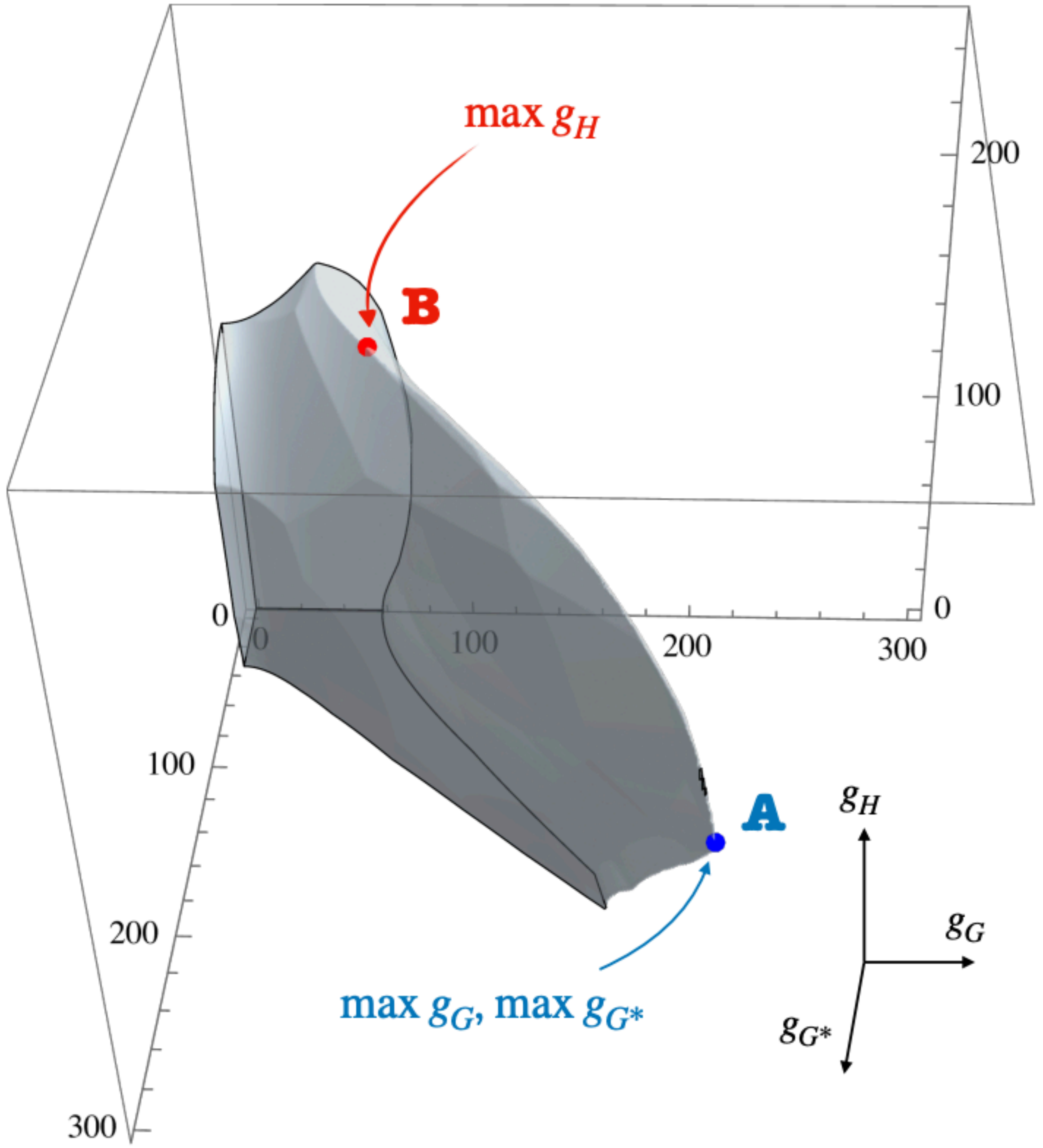
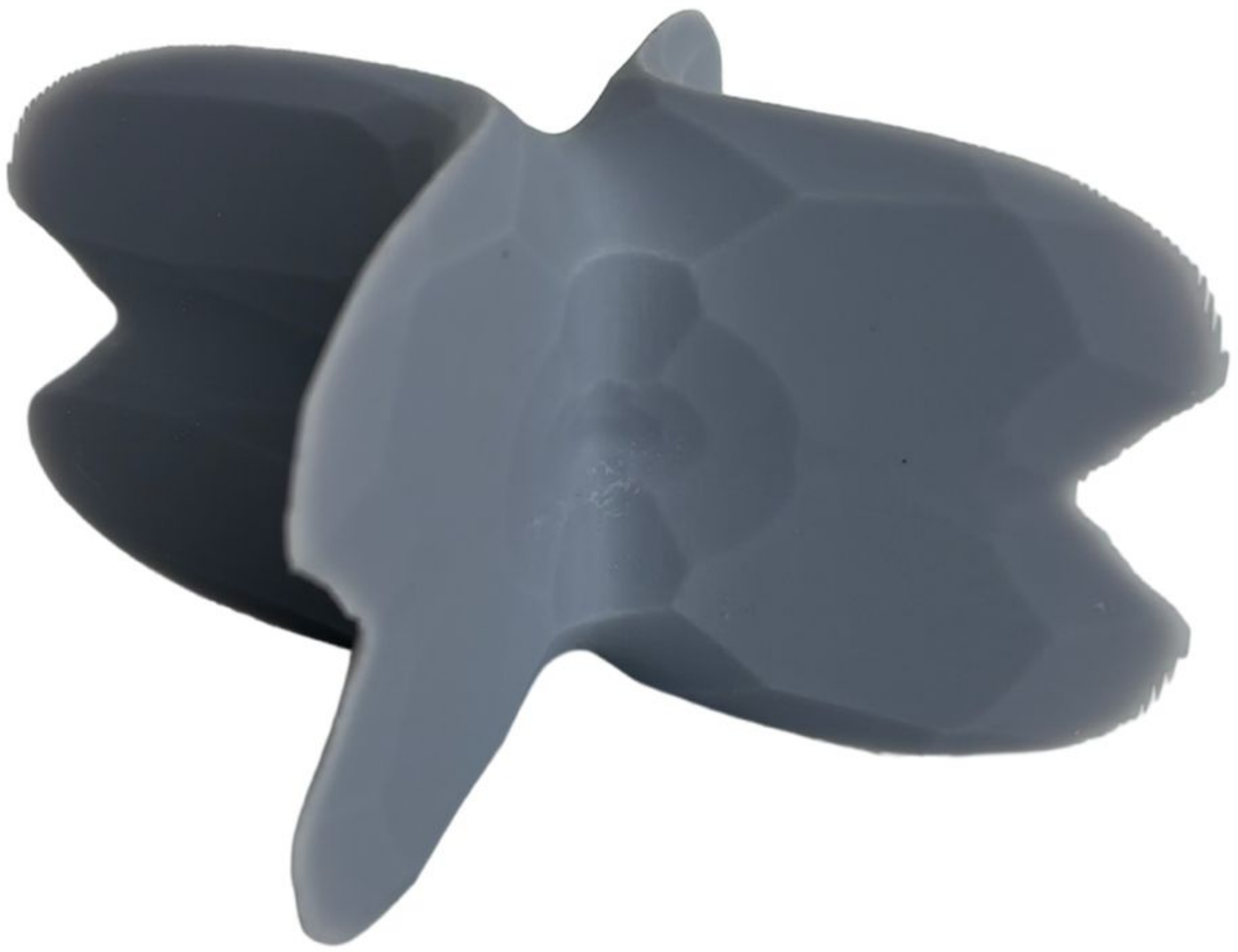




Rigorous Bounds!




QG Bootstrap and No Go Theorems

D=10, Maximal Susy, turn off all couplings except G_D

$$A_{QG} = \int \sqrt{-g}(R + 0 \times R^2 + 0 \times R^3 + \alpha_D R^4 + \ldots)$$

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Amplitude for graviton scattering in the EFT

$$A(s,t,u) = 8\pi G_D \left(\underbrace{\frac{1}{stu}}_{\text{Sugra}} + \alpha_D \ell_P^6 + \mathcal{O}(s \log s) \right)$$

First quantum correction $\alpha_D R^4$

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First quantum correction $\alpha_D R^4$

1) No-go Theorem: If gravity is quantum, it must be corrected at leading order!

Can be violated only if there are new physical principles!

- AG, J. Penedones and P. Vieira, 2102.02847
- AG, H. Murali, J. Penedones and P. Vieira, 2212.00151

E.g. D=10

$$\alpha(\text{Bootstrap}) \geq 0.126 \pm 0.006$$

$$\alpha(\text{String Theory}) \geq 3^{1/4}\zeta(\tfrac{3}{2})(\zeta(\tfrac{3}{2},\tfrac{1}{2}) - \zeta(\tfrac{3}{2},\tfrac{2}{3}))/\sqrt{2} \simeq 0.1389\ldots$$

The space of deformations compatible with String Theory!

Wilson coefficients as probes of gravity UV completions

Can α_D take any value?

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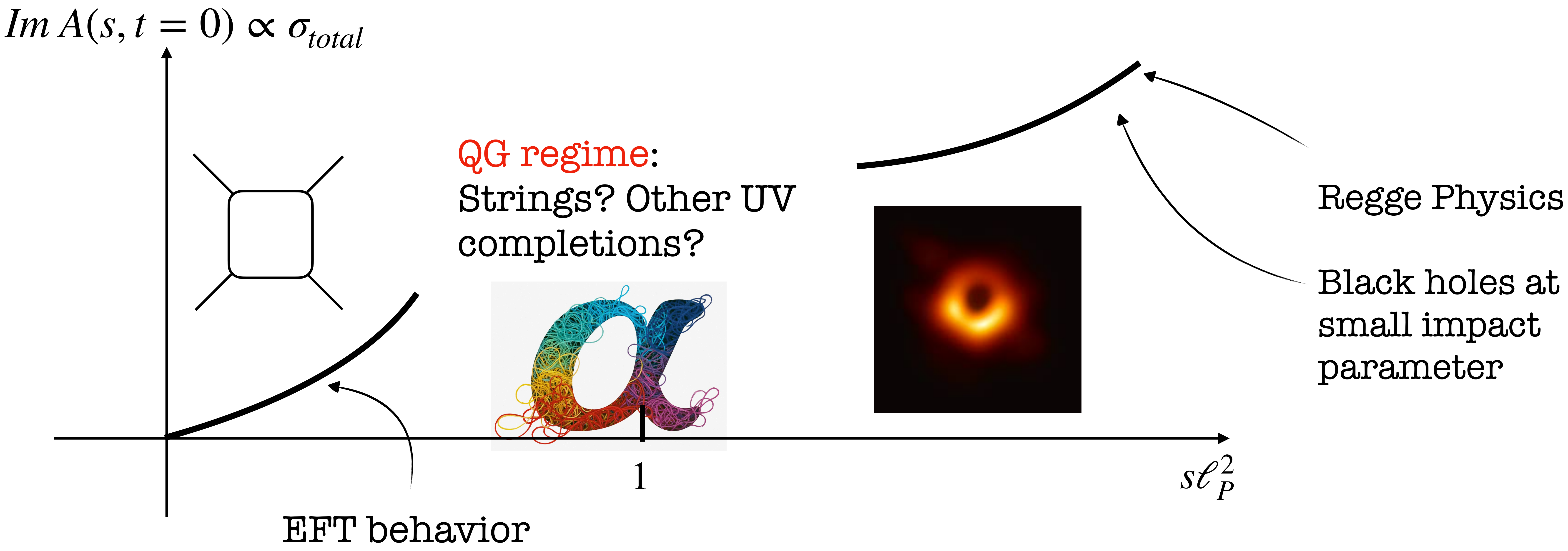
Can α_D take any value?

α_D knows about the theory at all scales! $\alpha_D \propto \int_0^{\infty} \frac{Im A(s, t=0)}{s} ds \geq 0$



QG Bootstrap: Methodology

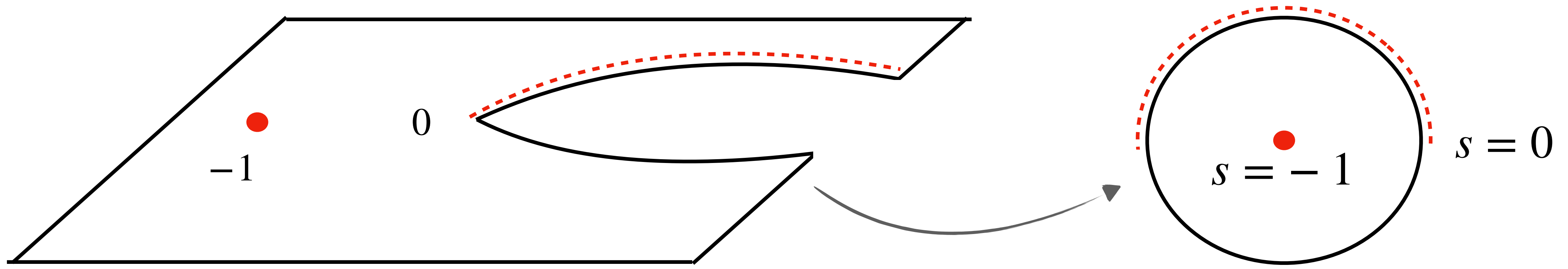
- There exist a parametrization of the non-perturbative amplitude manifestly **crossing symmetric** and **analytic**

$$A(s,t,u) = \underbrace{\frac{8\pi G_D}{stu}}_{\text{Sugra}} + \underbrace{\prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\leq N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c}_{\text{UV completion}}$$

Crossing

Analyticity

$$\rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$



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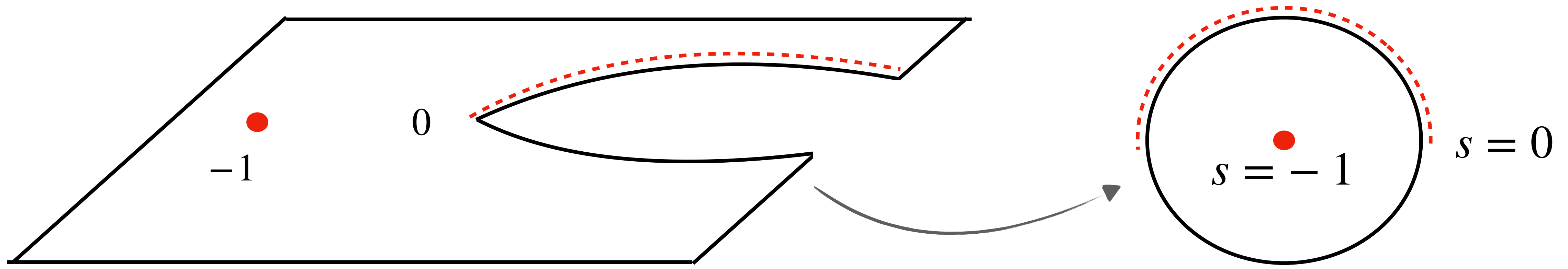
- There exist a parametrization of the non-perturbative amplitude manifestly **crossing symmetric** and **analytic**

$$A(s,t,u) = \underbrace{\frac{8\pi G_D}{stu}}_{\text{Sugra}} + \underbrace{\prod_{X=s,t,u} (\rho_X + 1)^2 \sum_{a+b+c\le N} \nu_{(a,b,c)} \rho_s^a \rho_t^b \rho_u^c}_{\text{UV completion}}$$

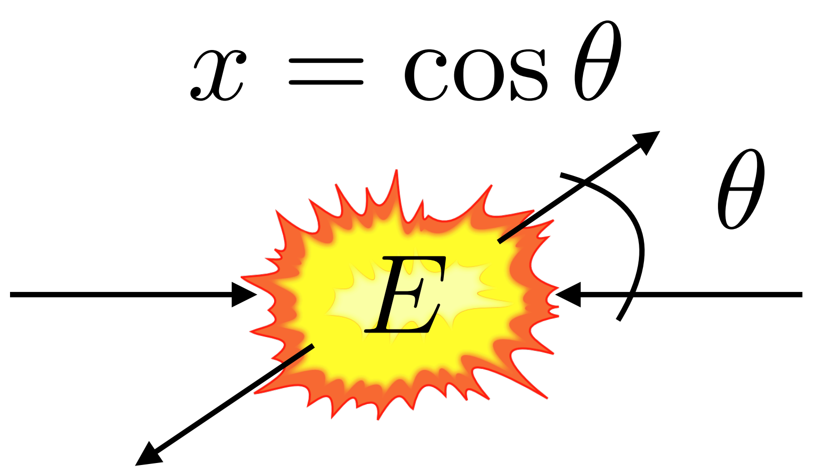
Crossing

Analyticity

$$\rho(s) = \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$



- Impose **unitarity** numerically (the $\nu_{(a,b,c)}$ cannot vary arbitrarily)



Linear operation: $T(s,x) \to S_\ell(s)$ $\quad |S_\ell(s)|^2 \le 1, \quad s>0, \quad \ell = 0,2,\ldots,\infty$ **(10D)**

$$S_\ell(s) = 1 + i\frac{s^3}{2^{18}3\pi^4}\int_{-1}^{1}(1-x^2)^3 \frac{C_\ell^{7/2}(x)}{C_\ell^{7/2}(1)} T(s,x)$$

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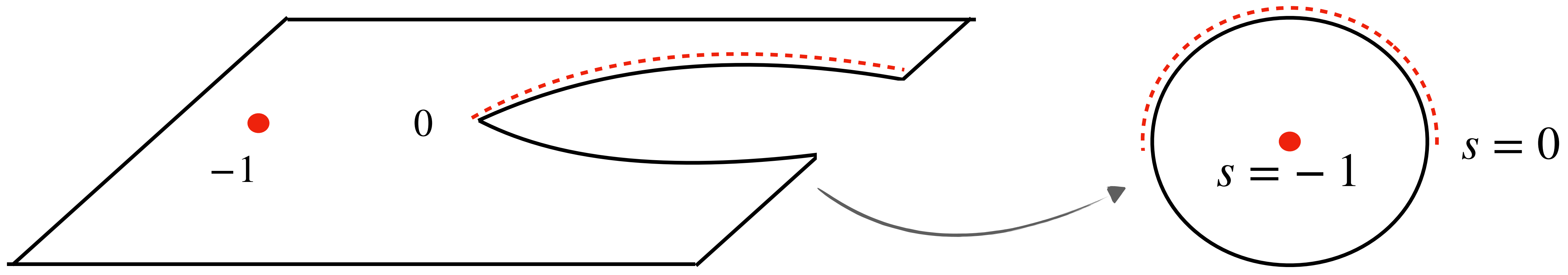

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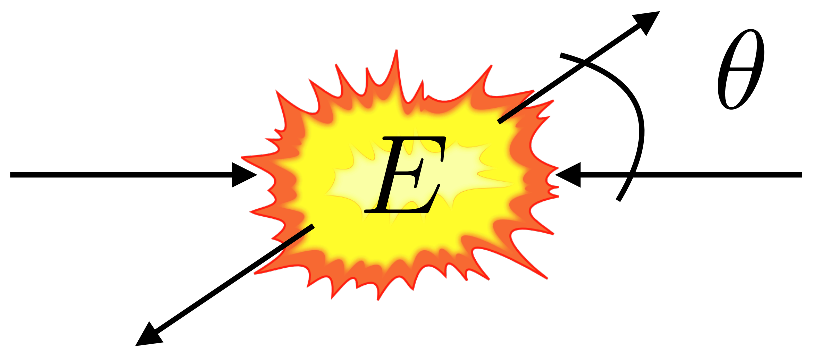

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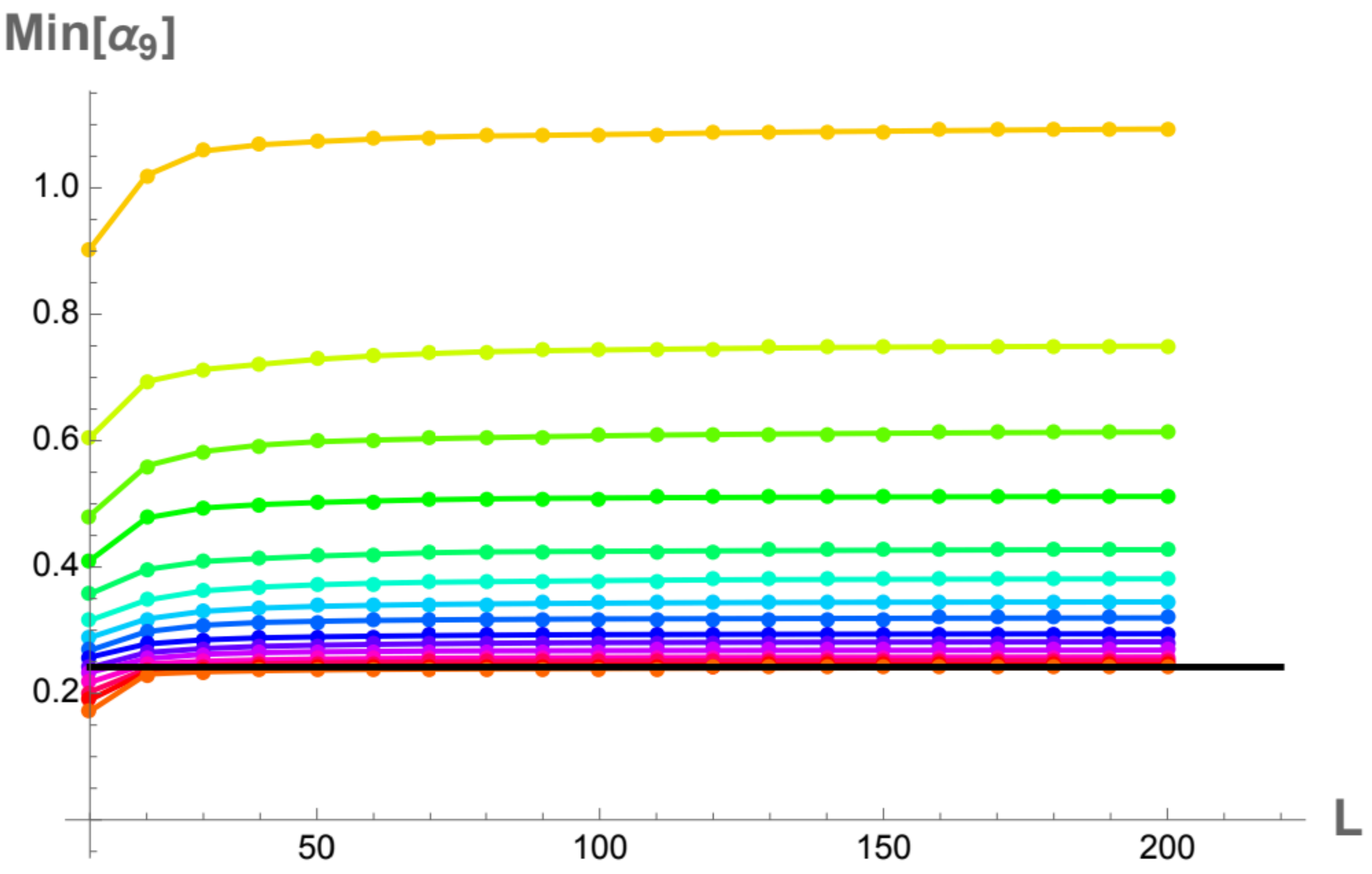
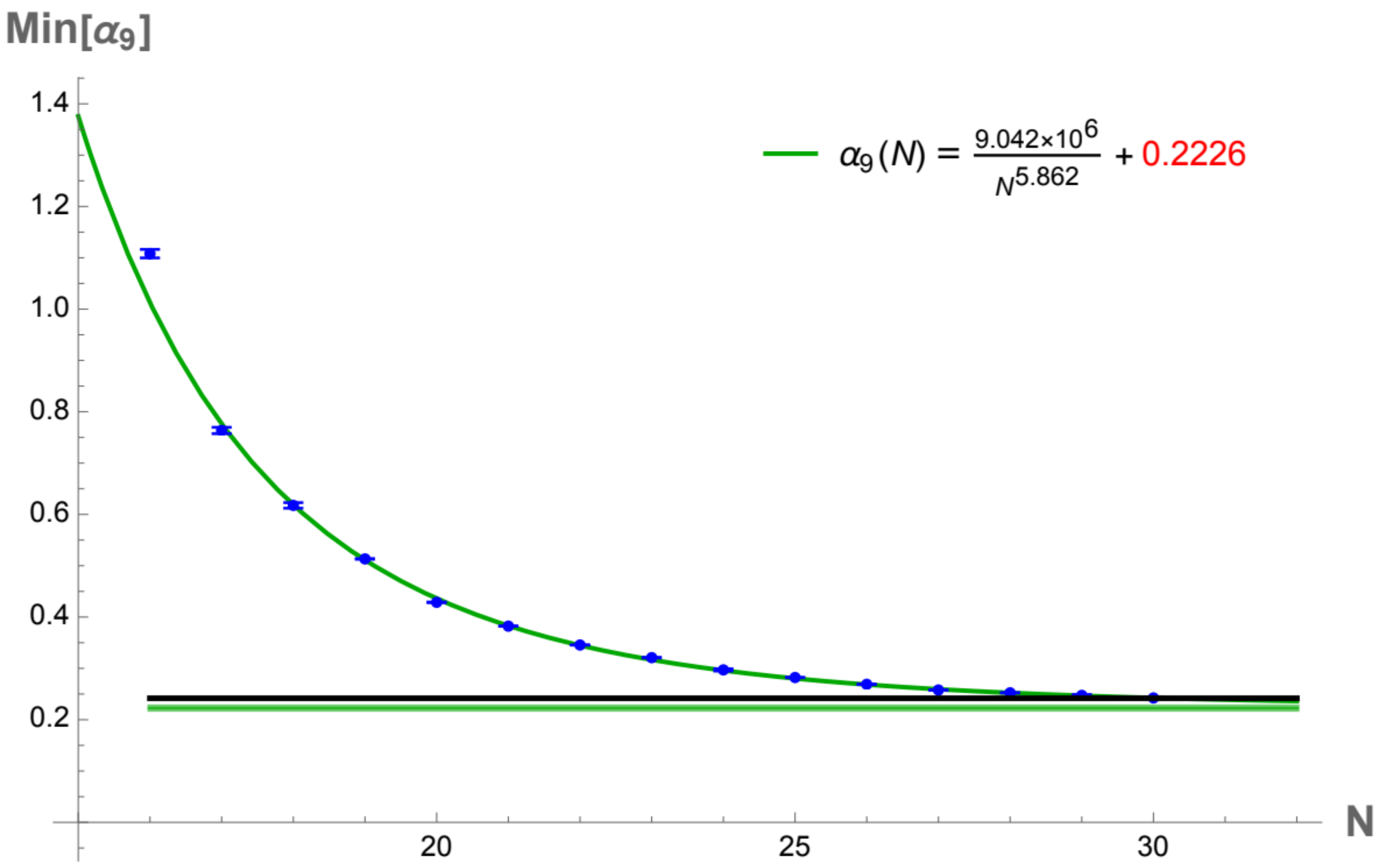
- **Bootstrap as a semi-definite optimization problem**

FindMinimum $\alpha_D[\nu_{(abc)}]$ subject to unitarity

QG Bootstrap: What we learn 1





Existence of Universal lower bound depending on low energy SUGRA, analyticity, crossing, and unitarity

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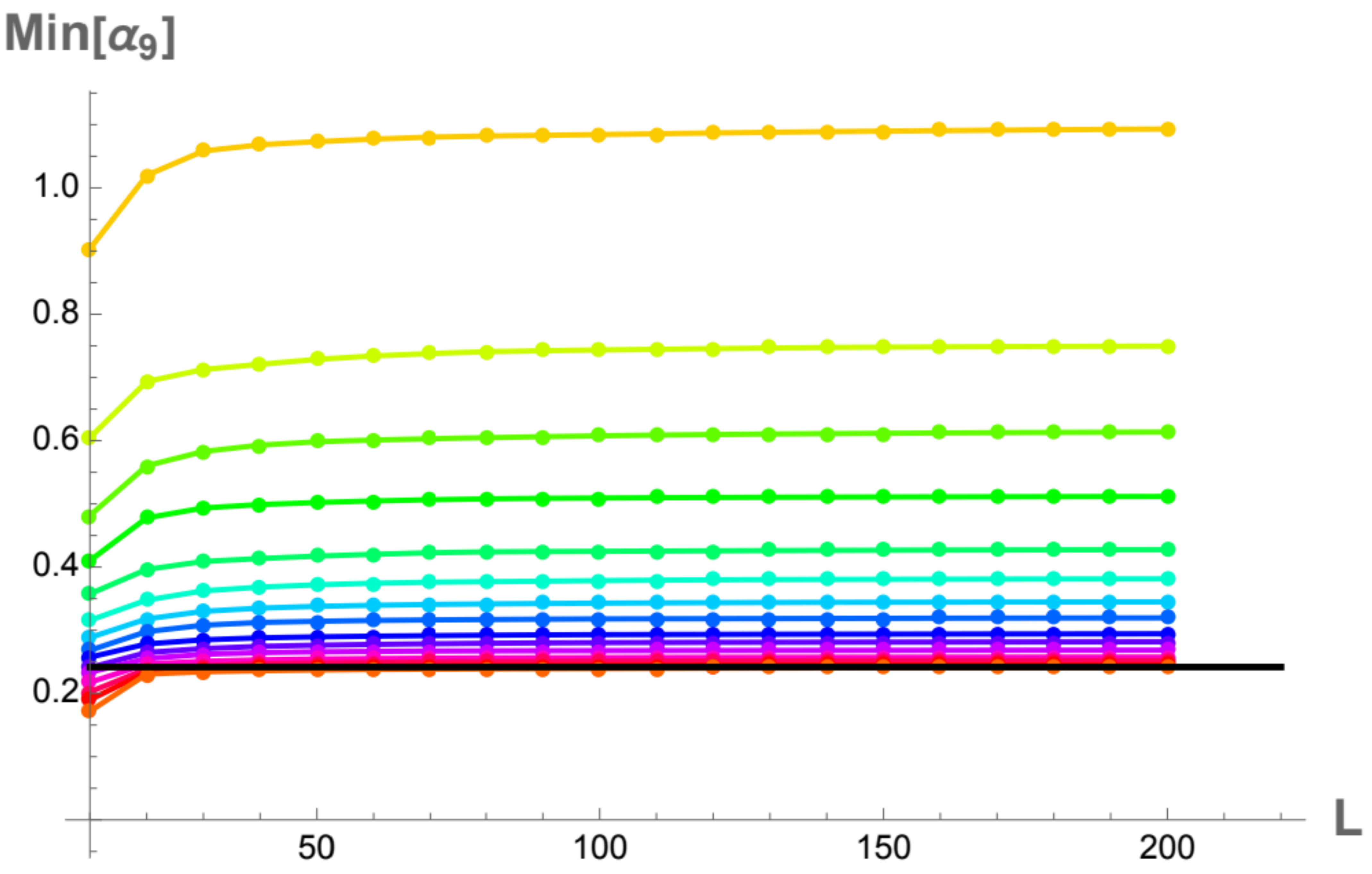
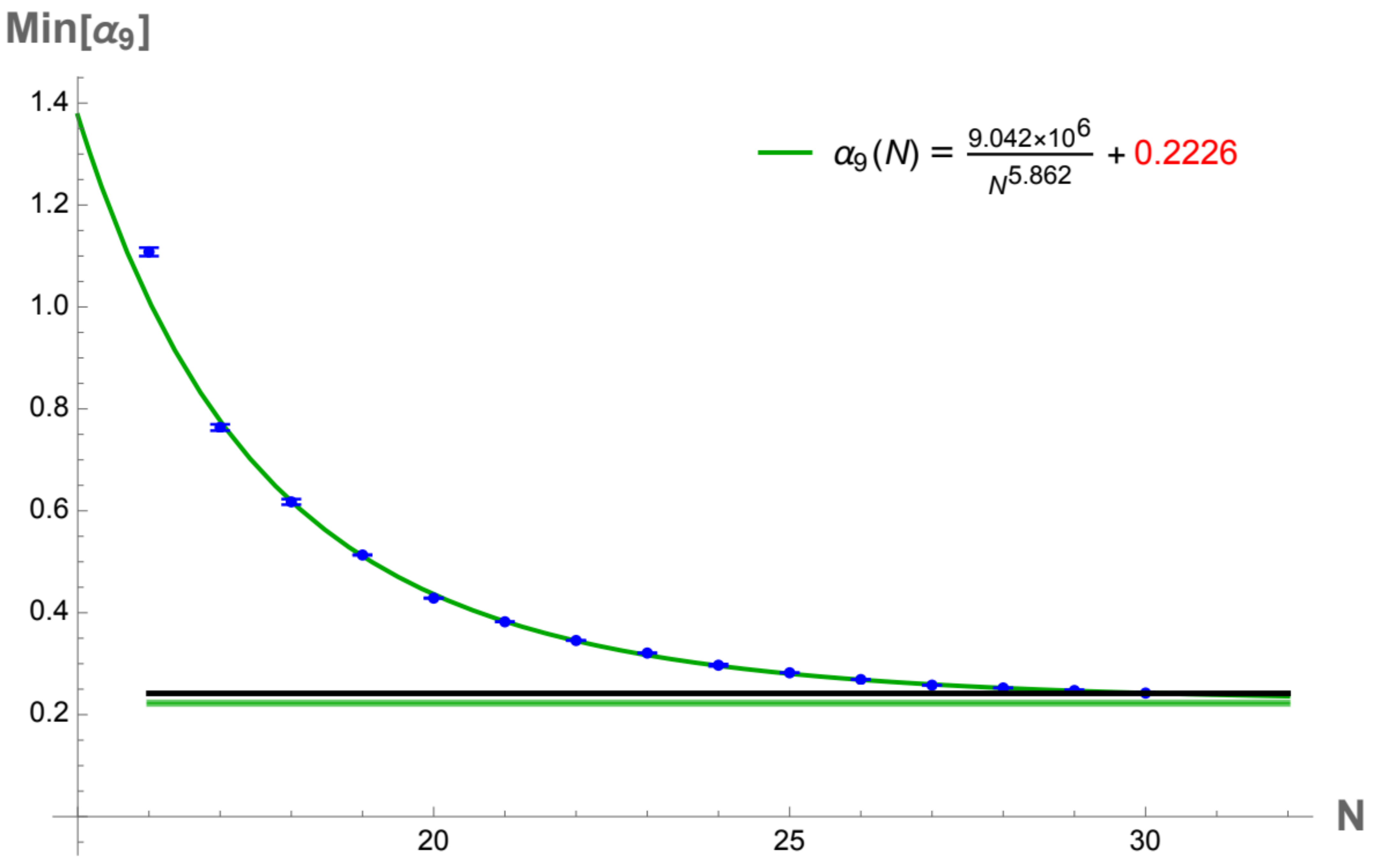




Existence of Universal lower bound depending on low energy SUGRA, analyticity, crossing, and unitarity

1. The Bound on α_D

$$\alpha_D^{\min} < \alpha_D < \infty$$

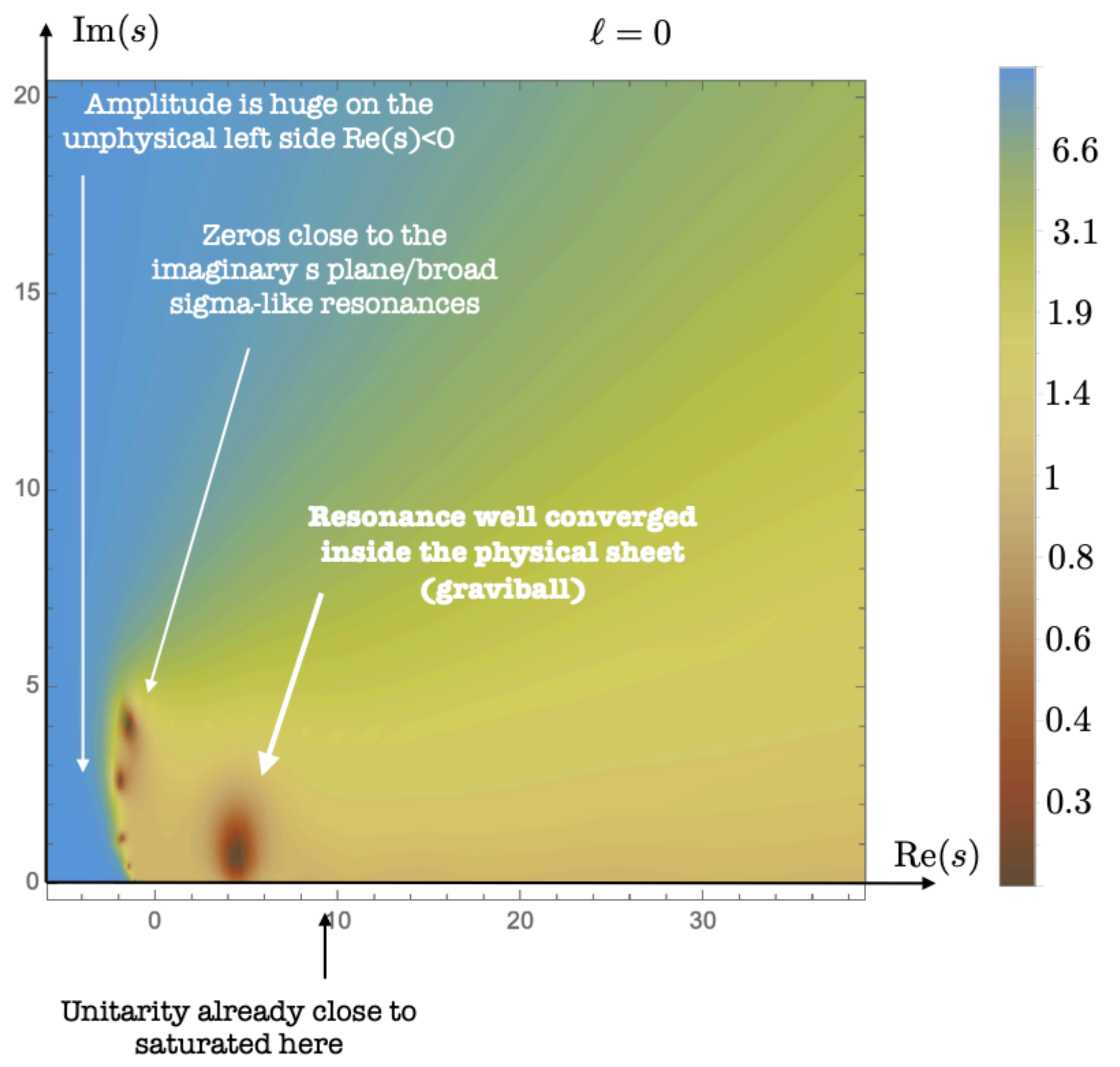
Dimension	String/M theory	Bootstrap $\alpha_D^{\min}$
9	≥ 0.2411	0.223 ± 0.002
10	≥ 0.1389	0.124 ± 0.003
11	0.1304	0.101 ± 0.005

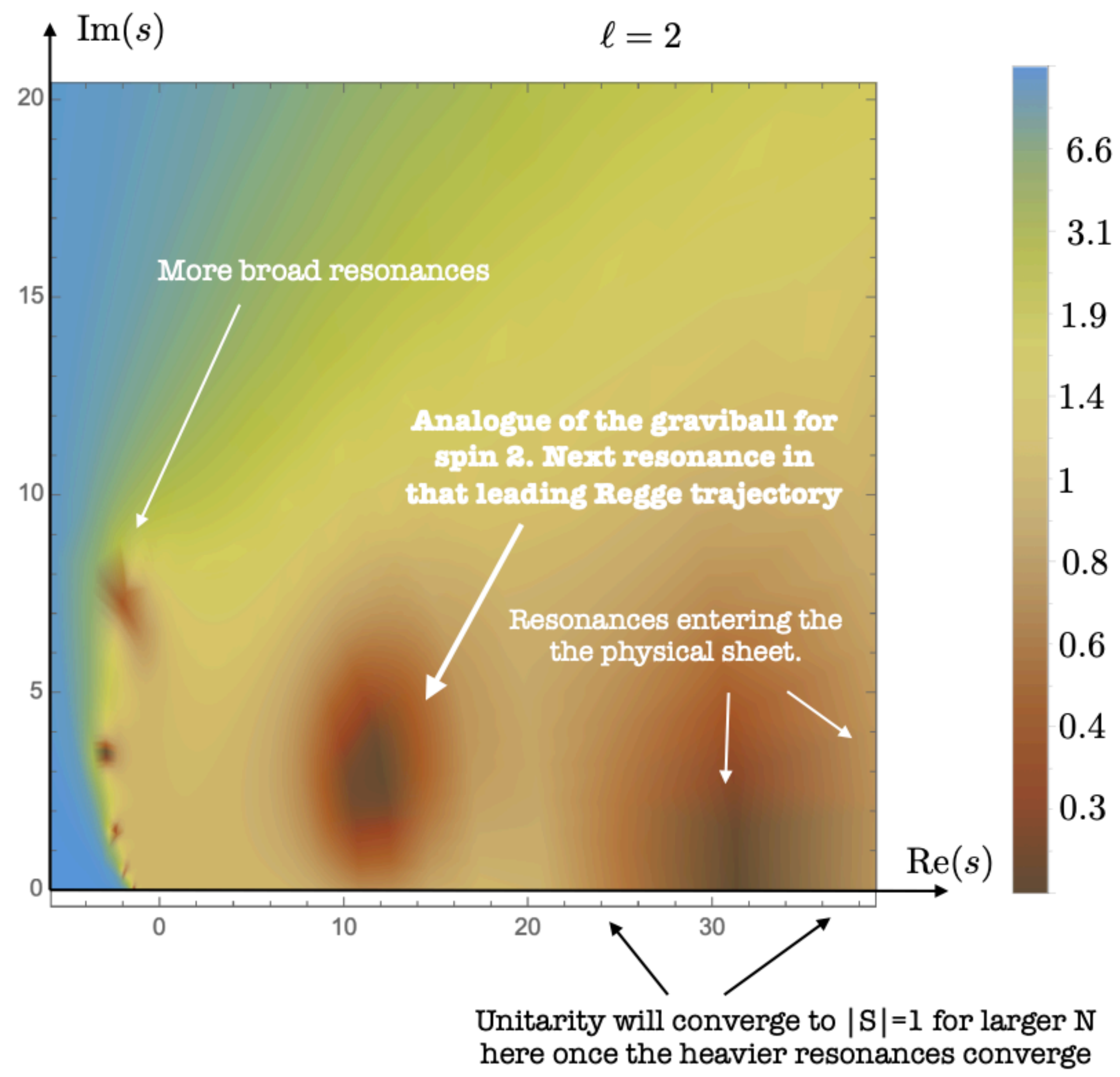
a) String Theory in 9, and 10 dimensions almost saturates the allowed region for α

b) α for M-theory is close to the boundary of the allowed region

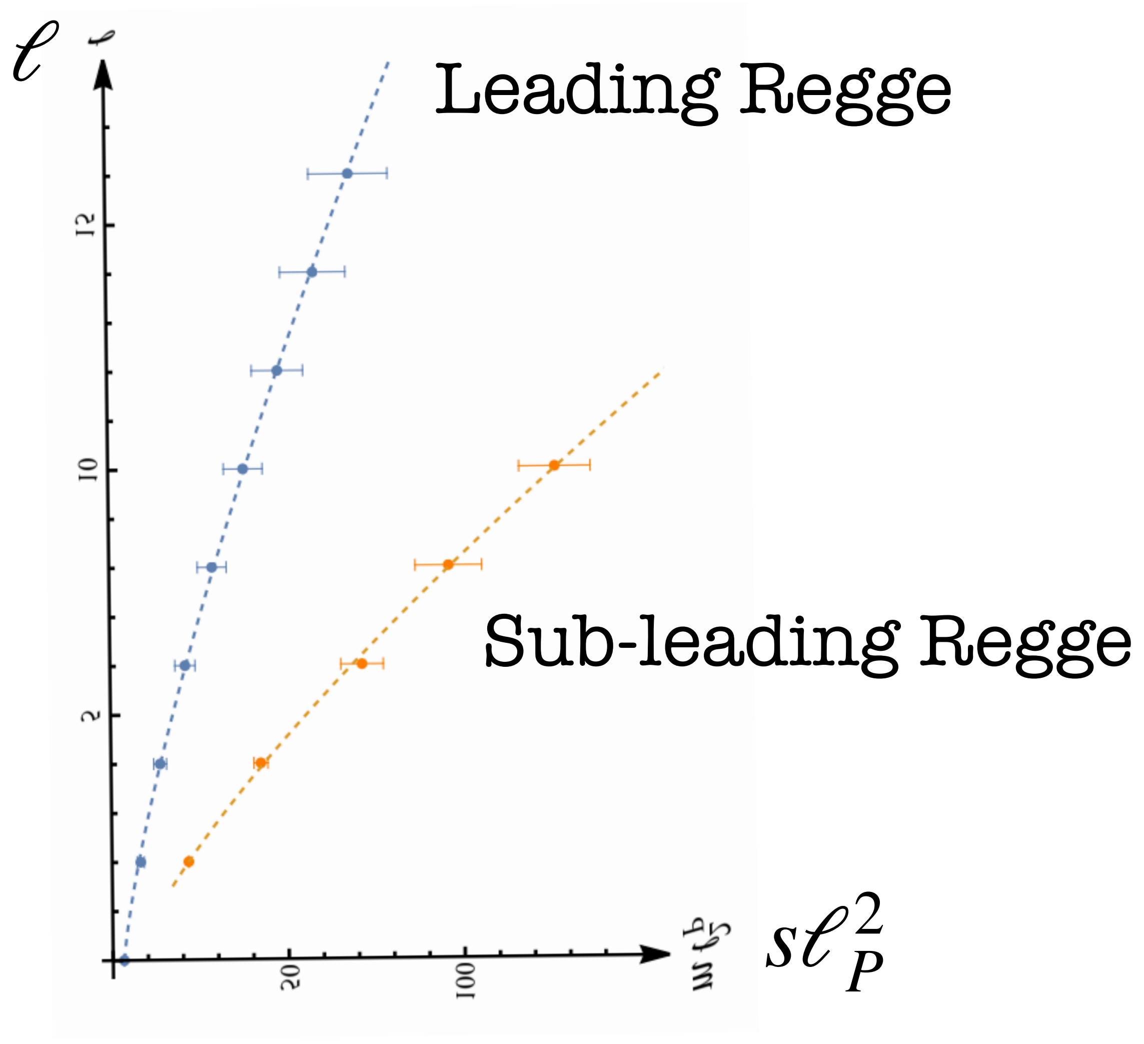
Green, Gutperle hep-th/9701093
Green, Vanhove hep-th/9704145
Green, Russo, Vanhove hep-th/0610299

QG Bootstrap: What we learn 2





We can reconstruct the solution that minimizes α_D and study this non-perturbative amplitude



Resonance spectrum organizes in (curved) Regge trajectories

Stringy Spectrum although there is no assumption about the UV completion

Can we directly Bootstrap the world-sheet of the Hadronic String?

String Theory from Gravity ✓

What about the Hadronic String Theory?

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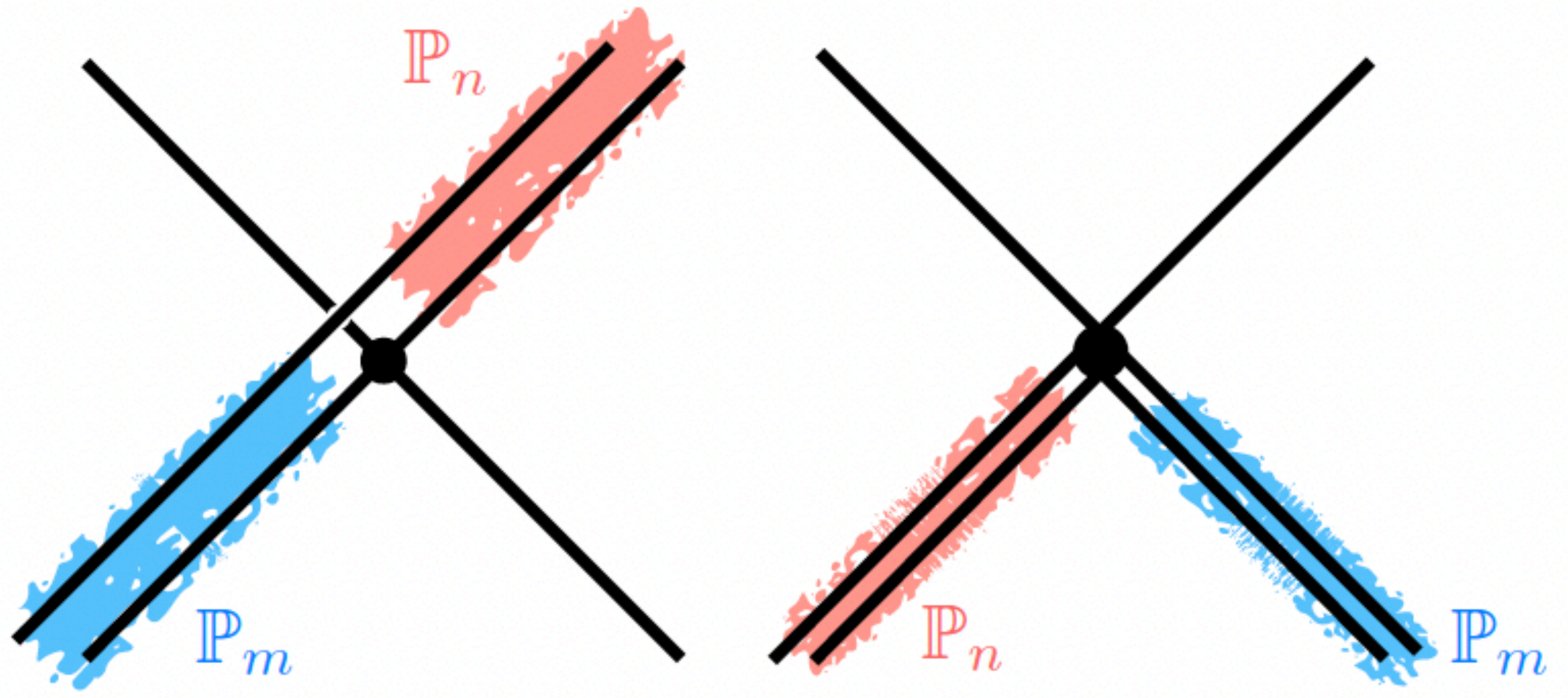
Simplest case: massless modes of long Strings in 3D

- AG, A. Homrich, J. Penedones and P. Vieira, to appear



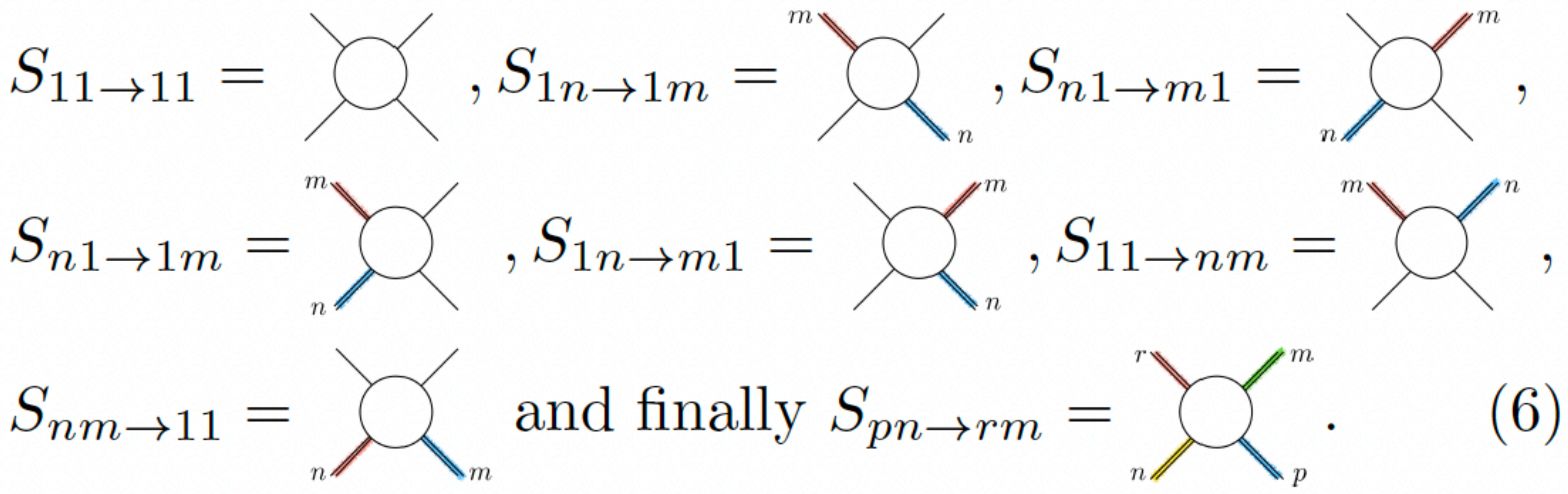
(a) $3 \to 3$ (b) $2 \to 4$

Idea: project multi-particle states into jet states

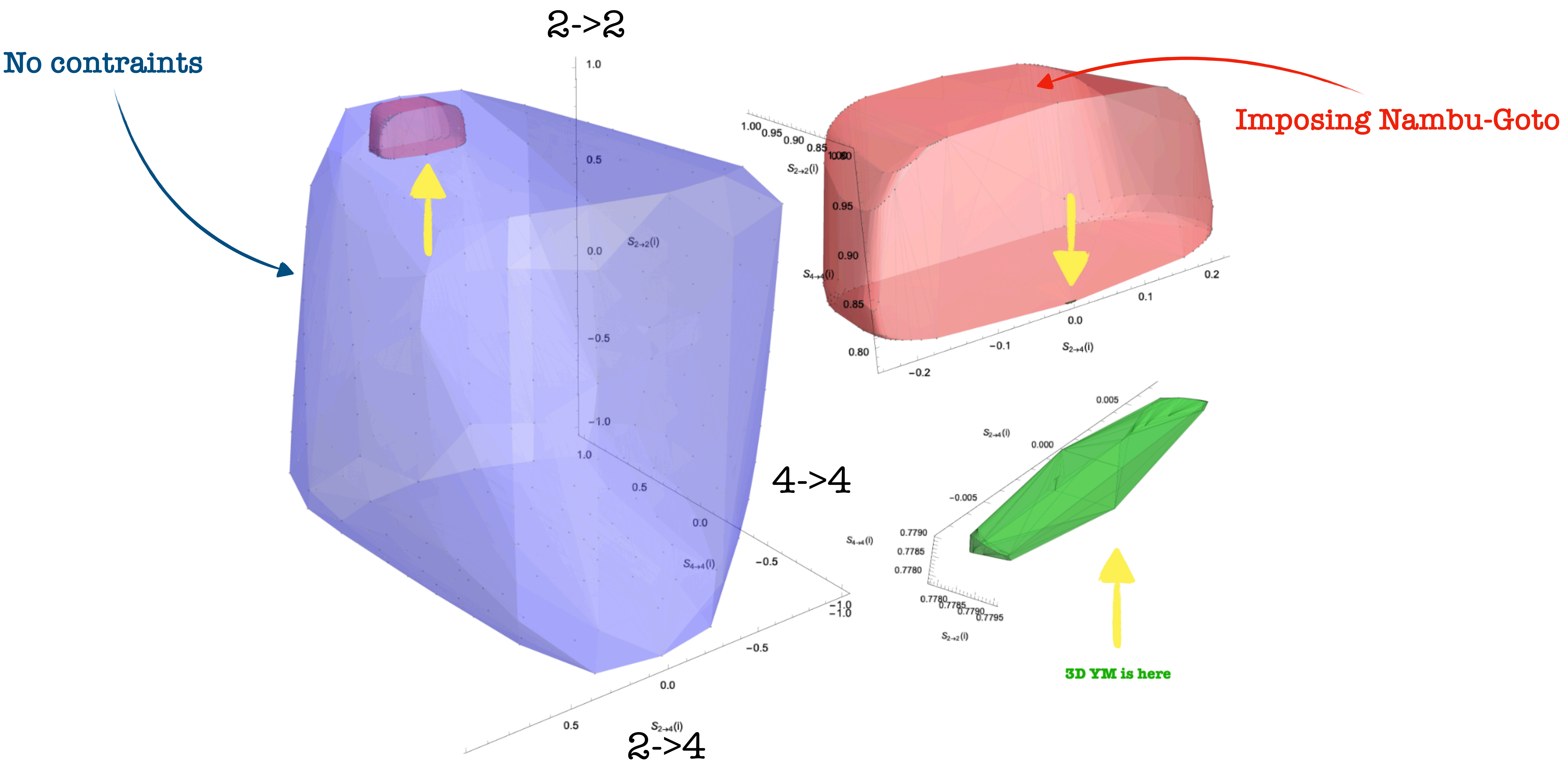
Problem decomposes into a bunch of 2->2 processes

2-particle Jet State

$$|n, P\rangle \equiv \sqrt{2n+1} \int_0^1 d\alpha \frac{P_n(2\alpha - 1)}{\sqrt{8\pi\alpha(1-\alpha)}} |\alpha, (1-\alpha), P\rangle_2$$



The Multi-Particle Matrioska coming soon…



What's next?

Q0: Can we construct non-perturbative scattering amplitudes and understand their properties?

Q1: Is it String Theory the unique UV completion of Sugra?

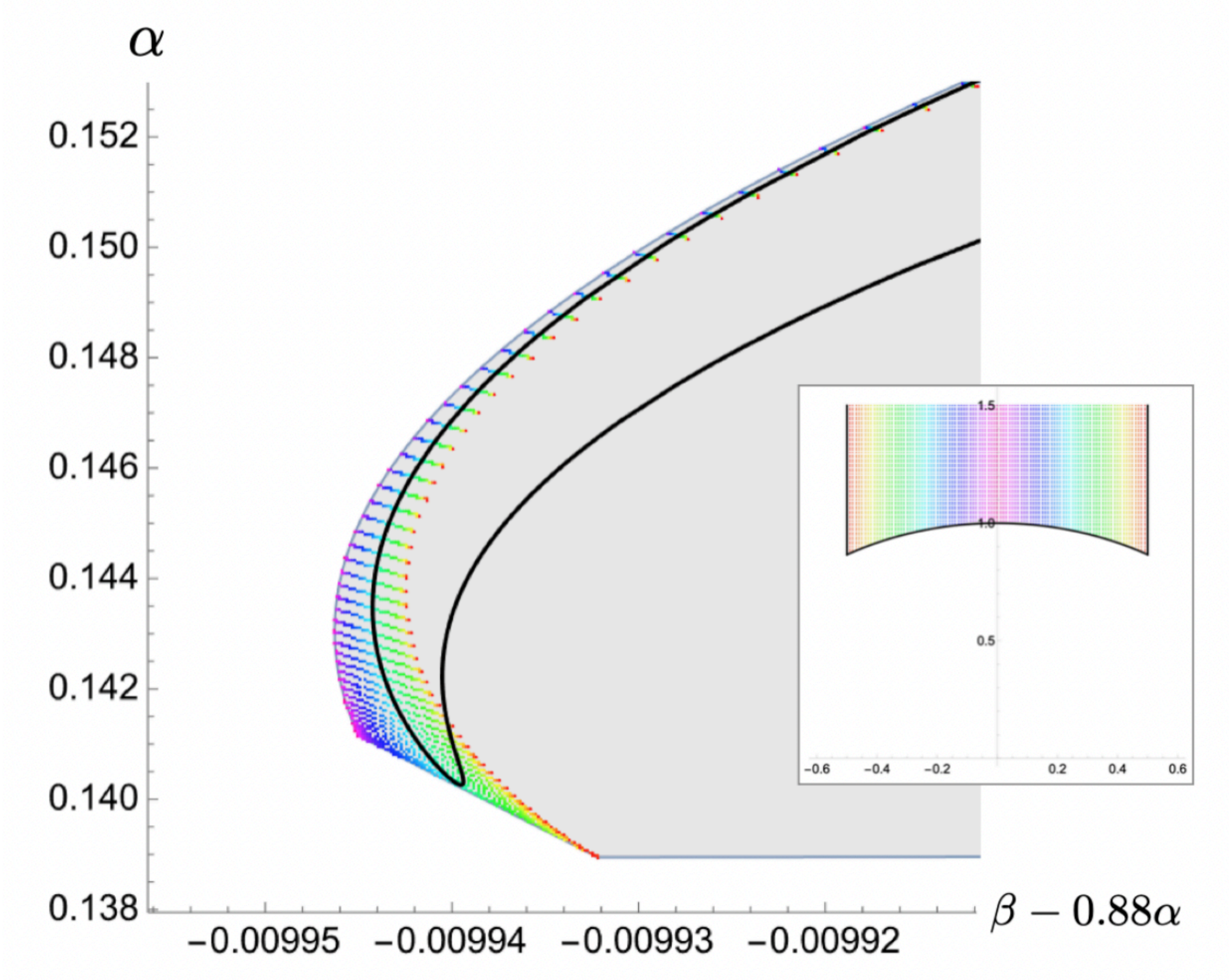
Q1.1: What bounds on D^4R^4, D^6R^4, ... operators?
What happens in lower dimensions?

Q1.2: Is the discrepancy compatible with black hole production?

$$\alpha \geq \frac{16}{3\pi^4 \ell_P^{14}} \sum_l (l+1)_6(2l+7) \int_0^\infty ds \frac{\eta_l(s)}{s^8}$$

The value of α increases as soon as we have inelasticity

Rotating black hole in 10D



We need a model: $Prob_{2\to 2} \sim Exp(-S_{BH}(Area))$

Q2: How the UV completion of pure Einstein Gravity in $D \geq 5$ looks like?

$$A^{QG} = 8\pi G_N^d \int d^d x \sqrt{-g}\left(R + a_2 R^2 + a_4 R^3 + \alpha R^4 + \ldots\right).$$

Q3: Can we perform a non-perturbative gravity Bootstrap in 4D?

Q4: Can we put QCD phenomenology on a more rigorous footing?

After dinner suggestion

Cachaça Degustation at **Chachaçaria.bar**

This is a great caipirinha

See you there, or look for me or these gentlemen after dinner

Backup Slides

An analytic bound on scattering

Goal: we bound $c_4 \Longleftrightarrow$ we bound Δ_3

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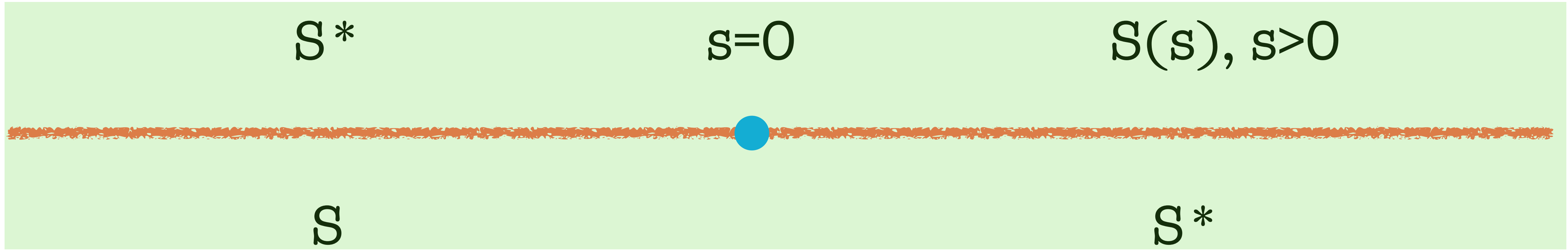

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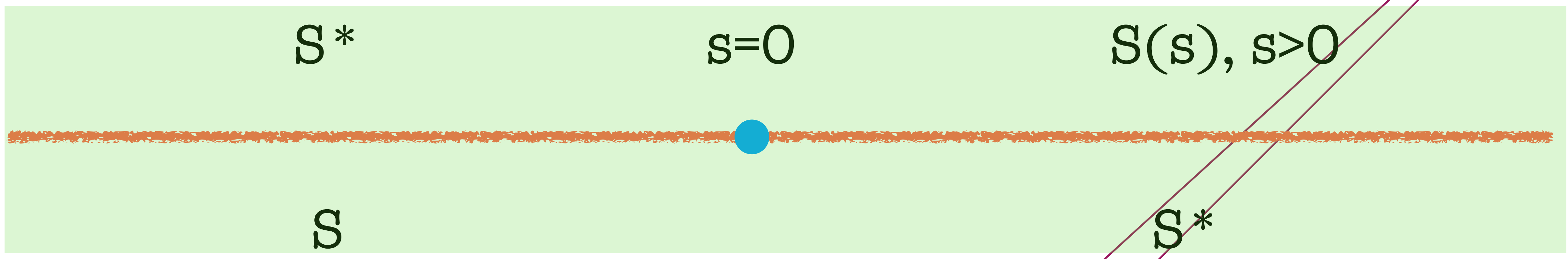

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Lattice for SU(2) YM $\gamma_3 = -0.00034(6)$

Caristo, Caselle, Magnoli, Nada, Panero '21

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Baffigo, Caselle '23

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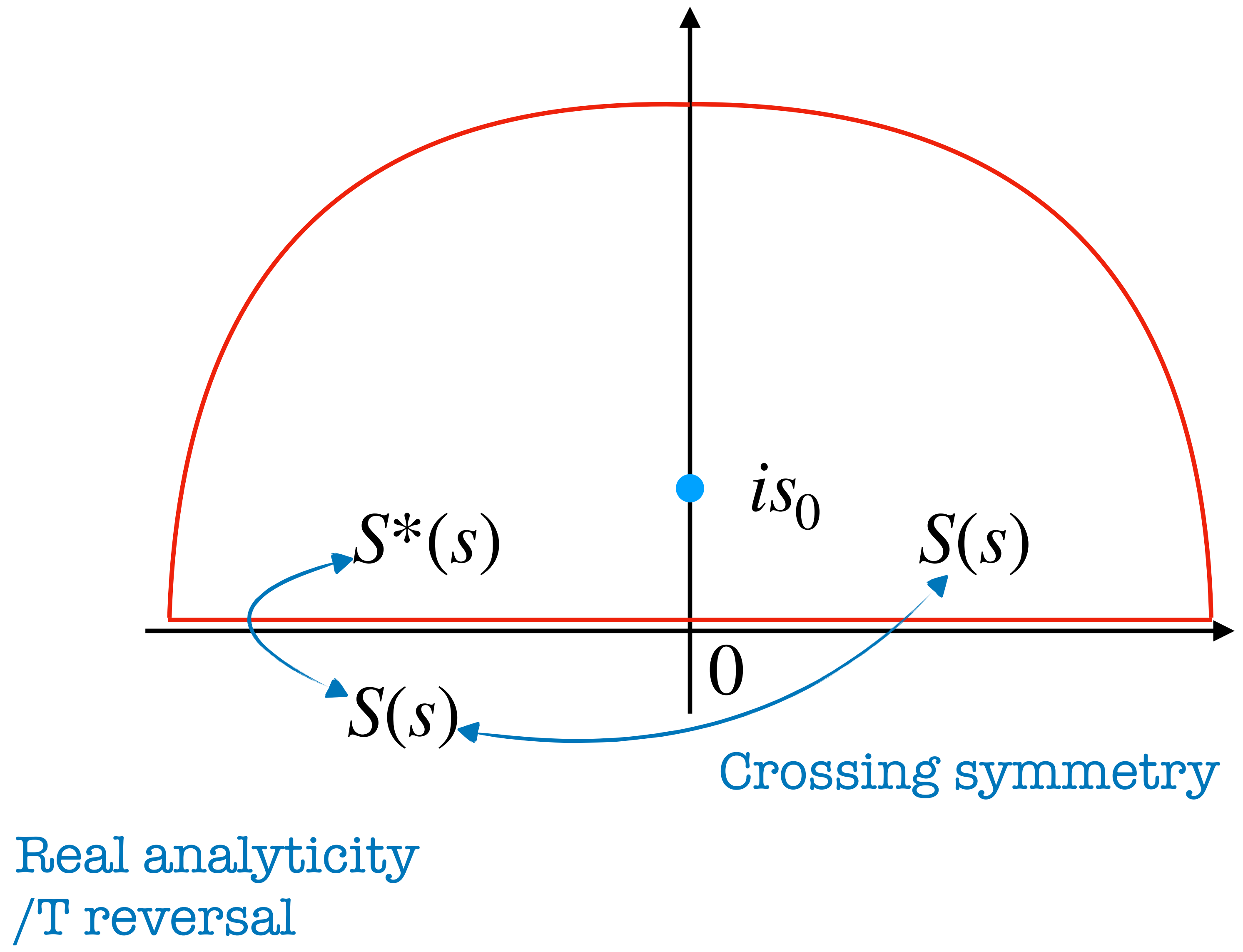

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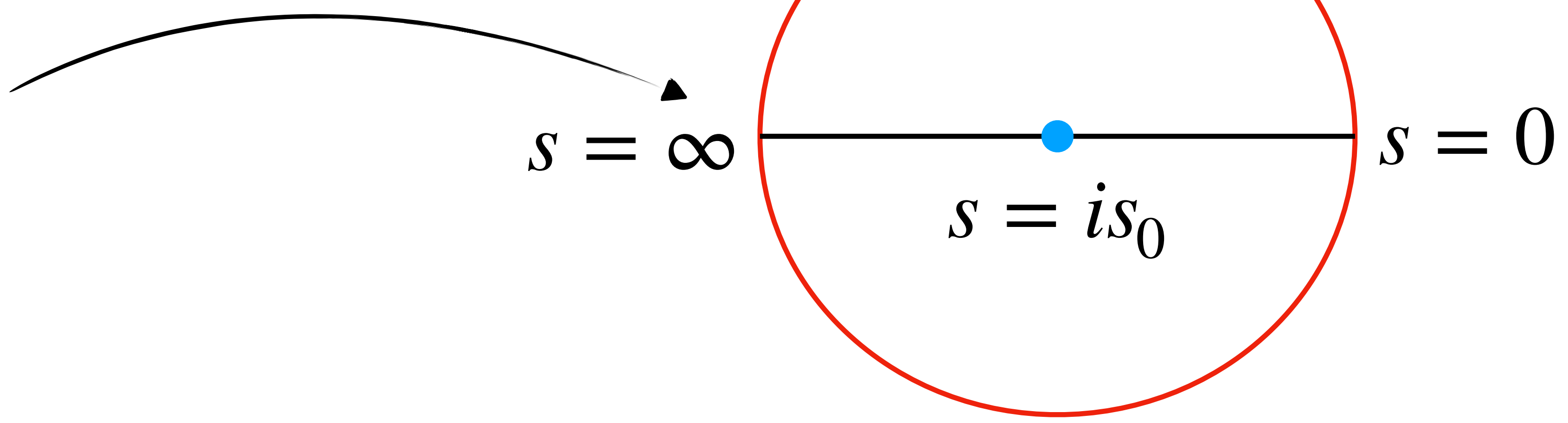

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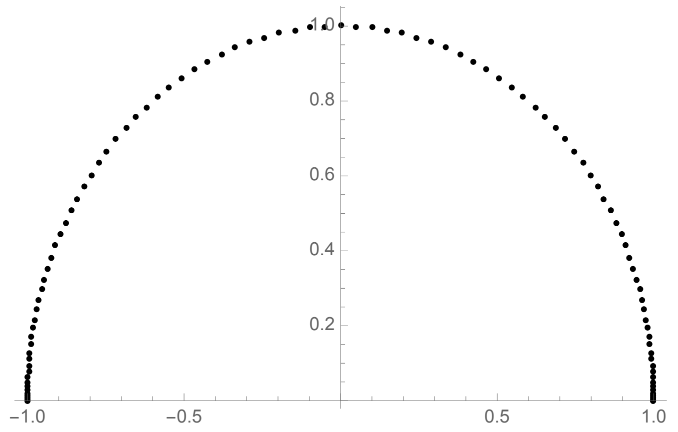

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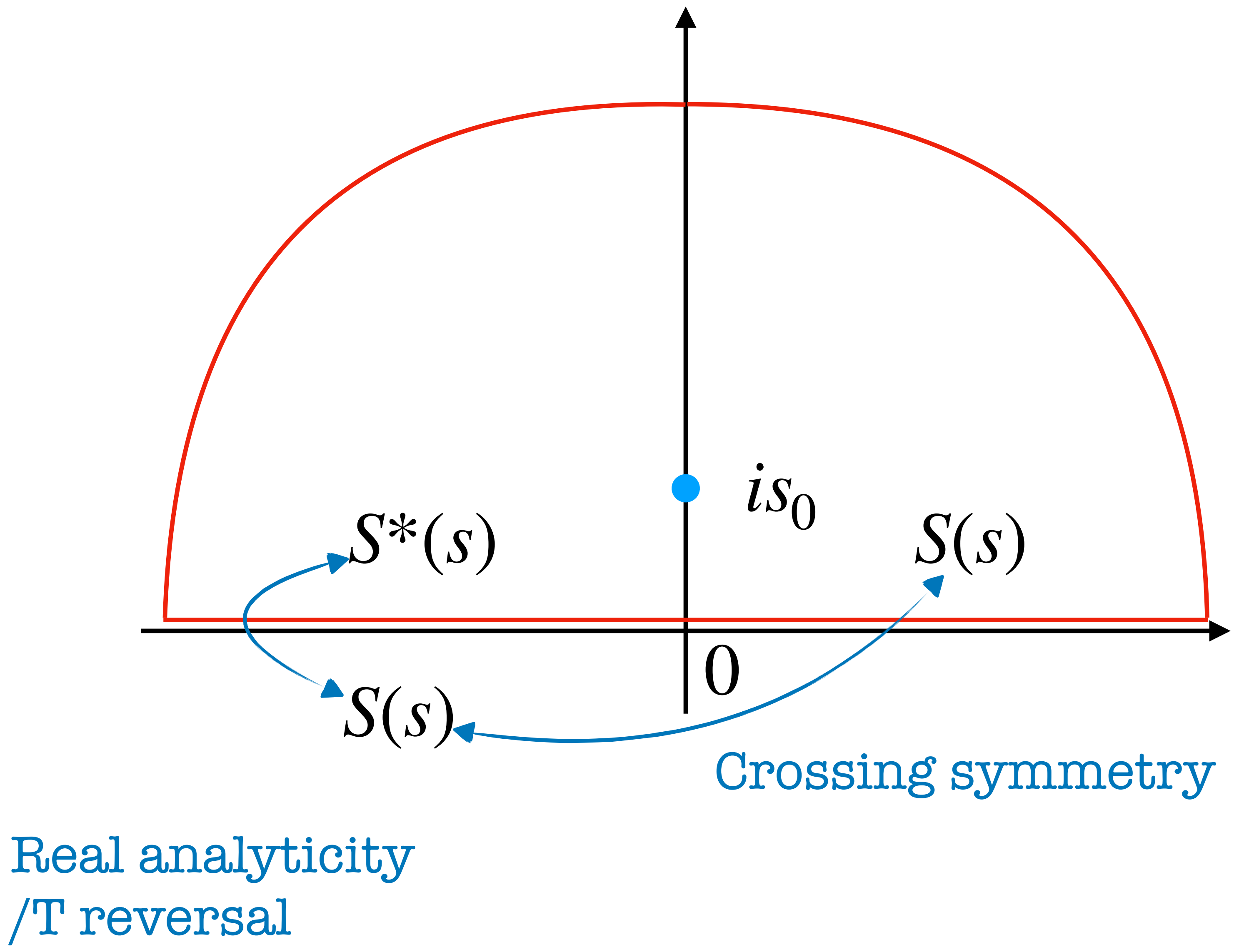

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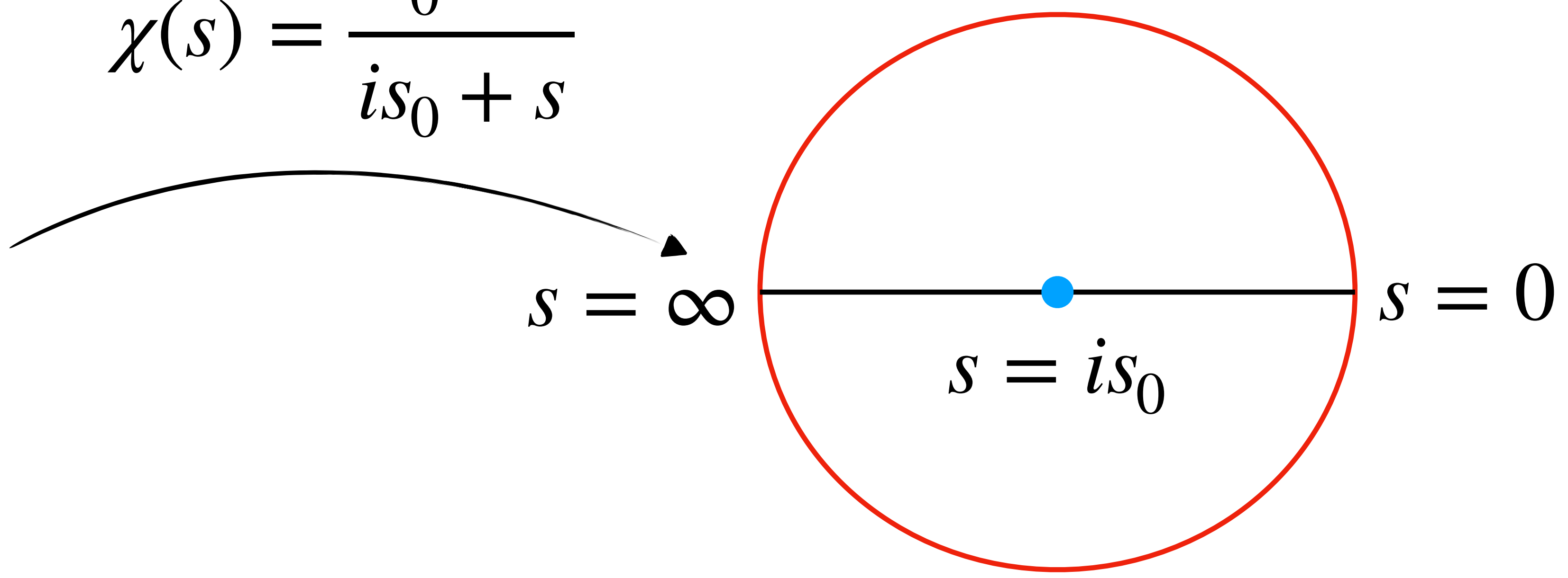

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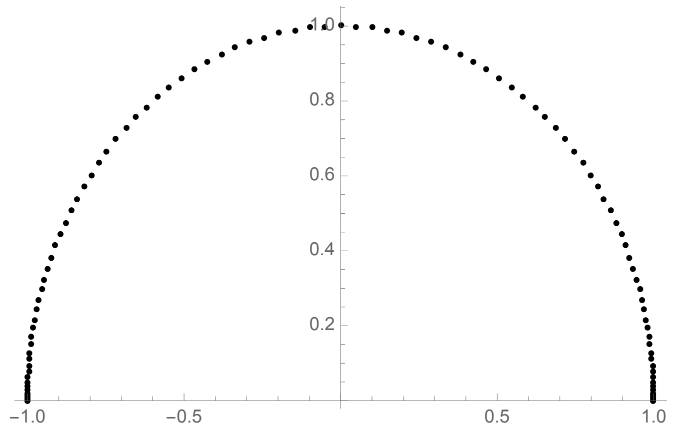

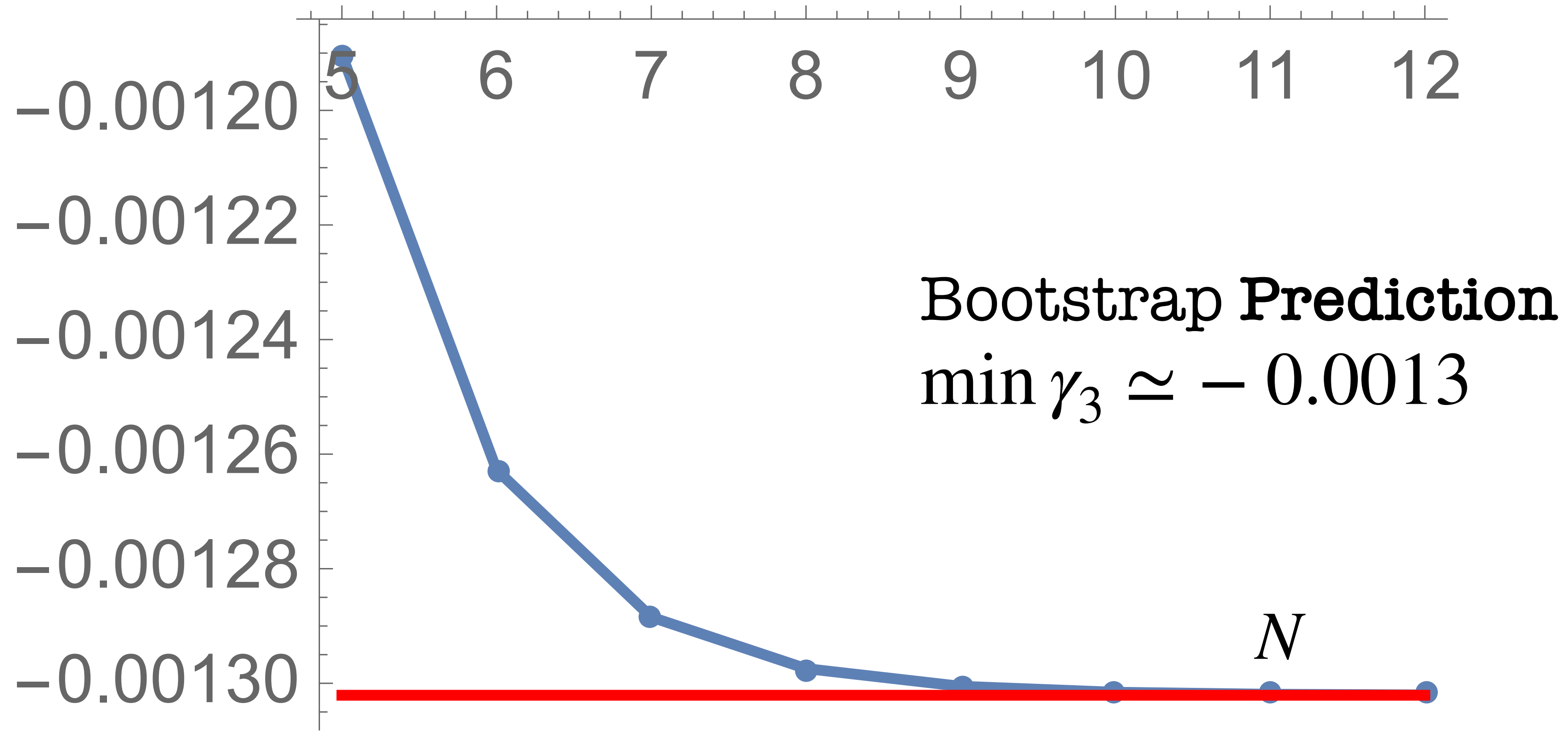

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$$\sum_n P_{2\to n} = 1 \implies P_{2\to 2} + P_{2\to 4} + \ldots \leq 1$$

String Theory and M-theory Expectations

α_D is 1-loop exact up to non-perturbative corrections

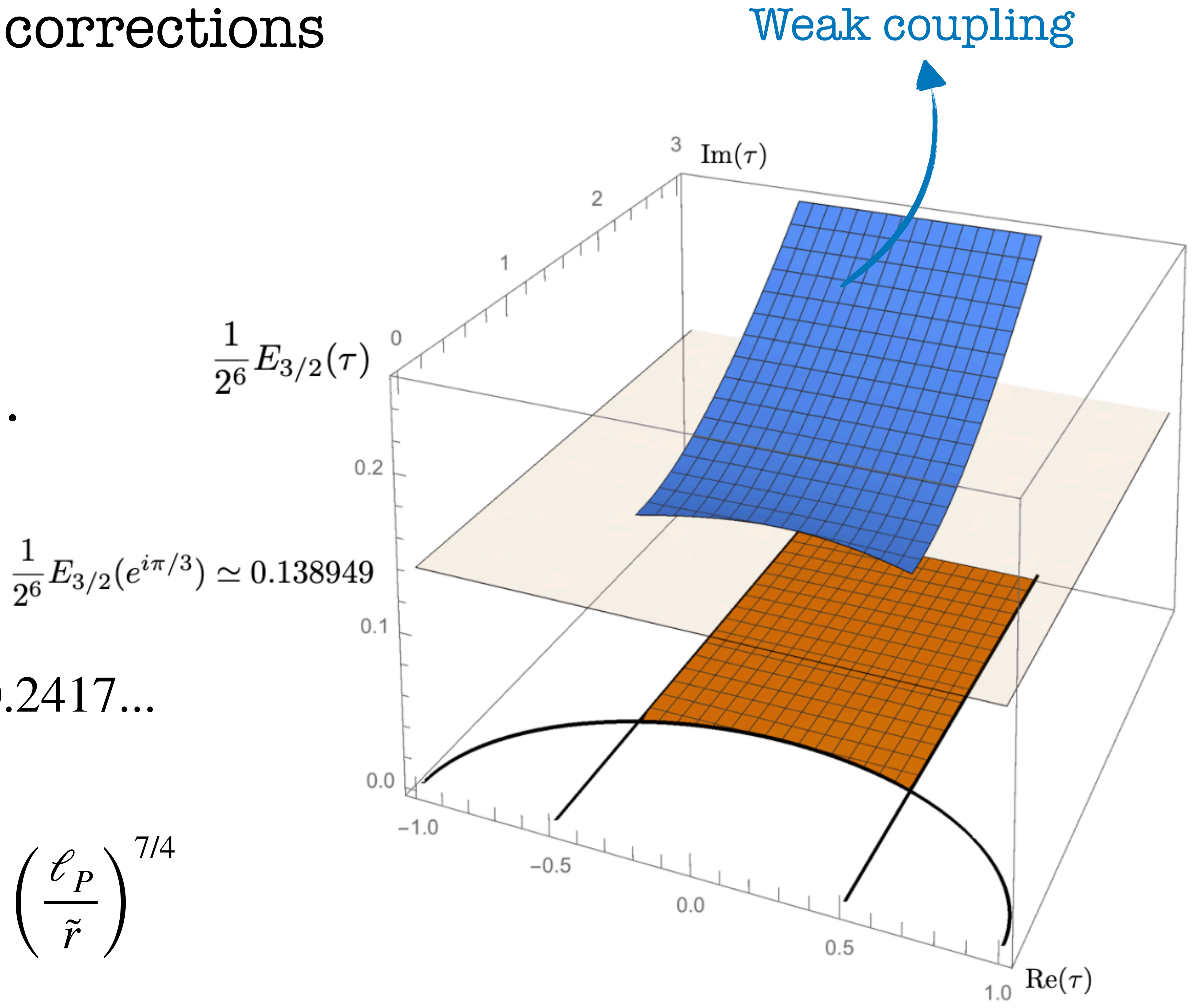
Min α_{10} realized in type IIB

D=10 Type IIB: $\alpha_{10}^{IIB} = \frac{1}{2^6} E_{3/2}(\tau,\bar{\tau}) \geq 0.139...$

D=9: $\alpha_9(\tau,\nu) = \frac{1}{2^6}\left(\nu^{-3/7}E_{3/2}(\tau,\bar{\tau}) + \frac{2\pi^2}{3}\nu^{4/7}\right) \geq 0.2417...$

$$\nu = \left(\frac{r}{\ell_s}\right)^{7/4}\sqrt{g_9} = \left(\frac{\ell_P}{\tilde{r}}\right)^{7/4}$$

D=11: $\alpha_{11} = \frac{(2\pi)^2}{3\times 2^7} = 0.1028...$



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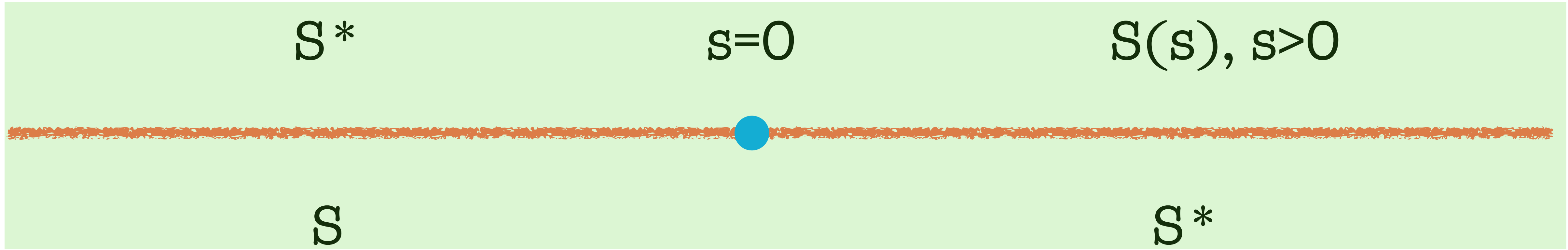

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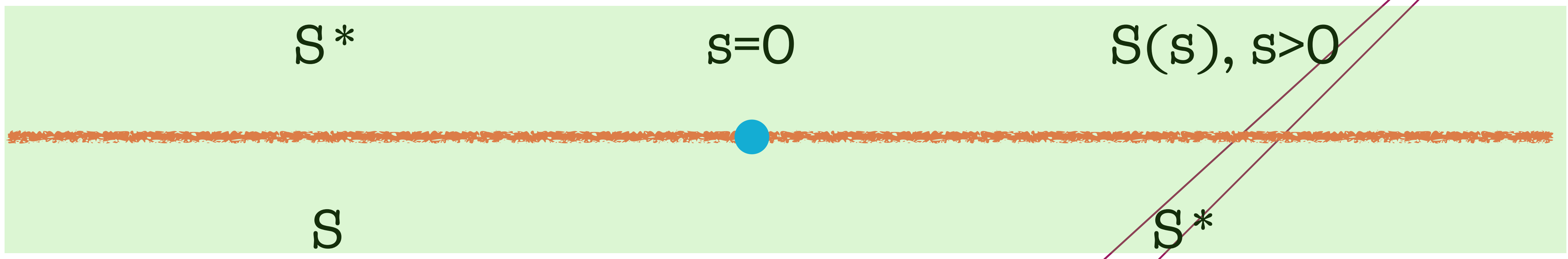

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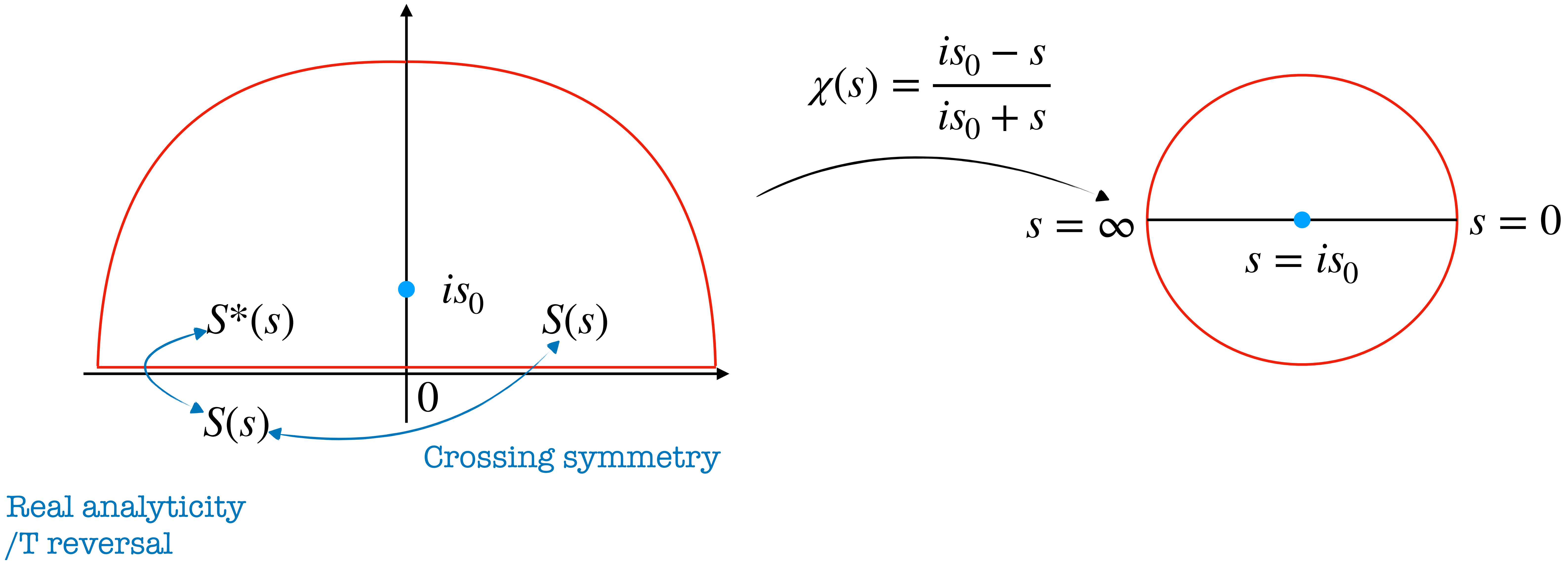

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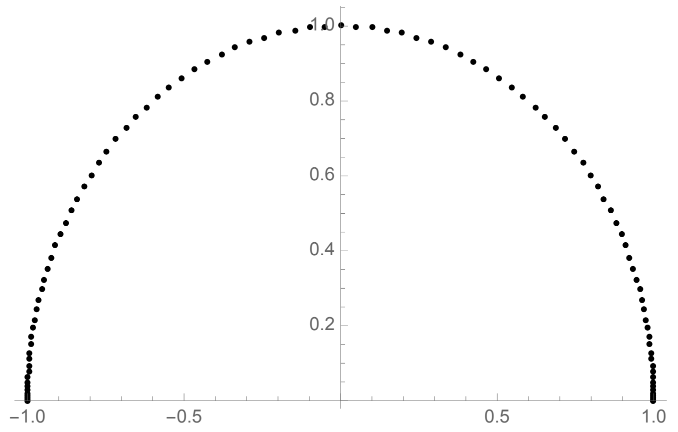

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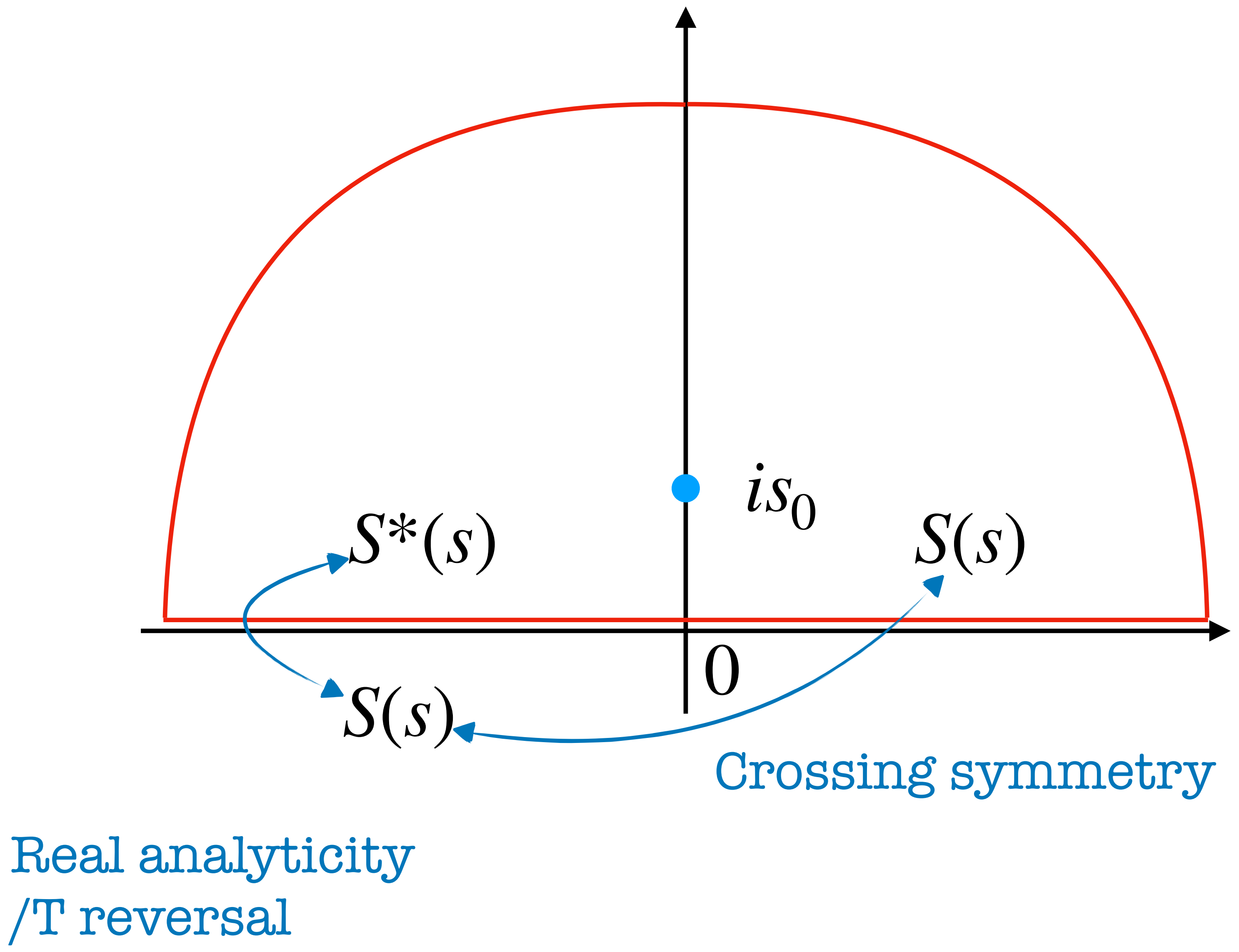

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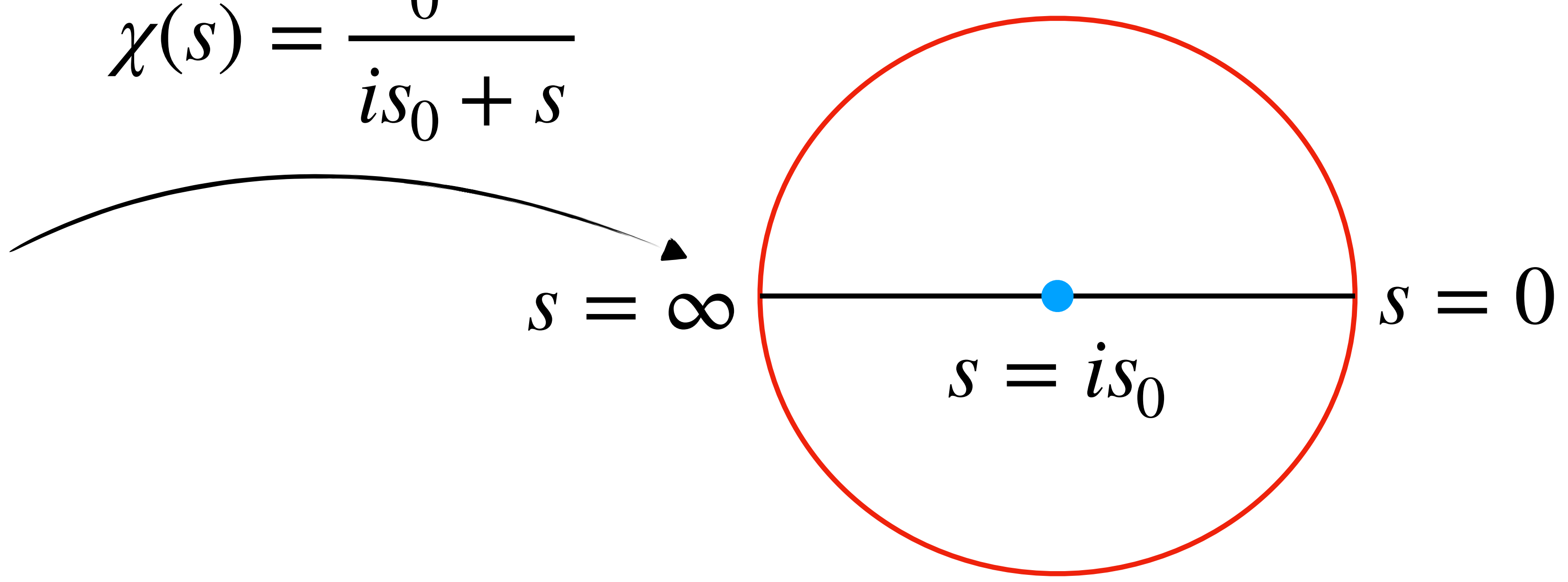

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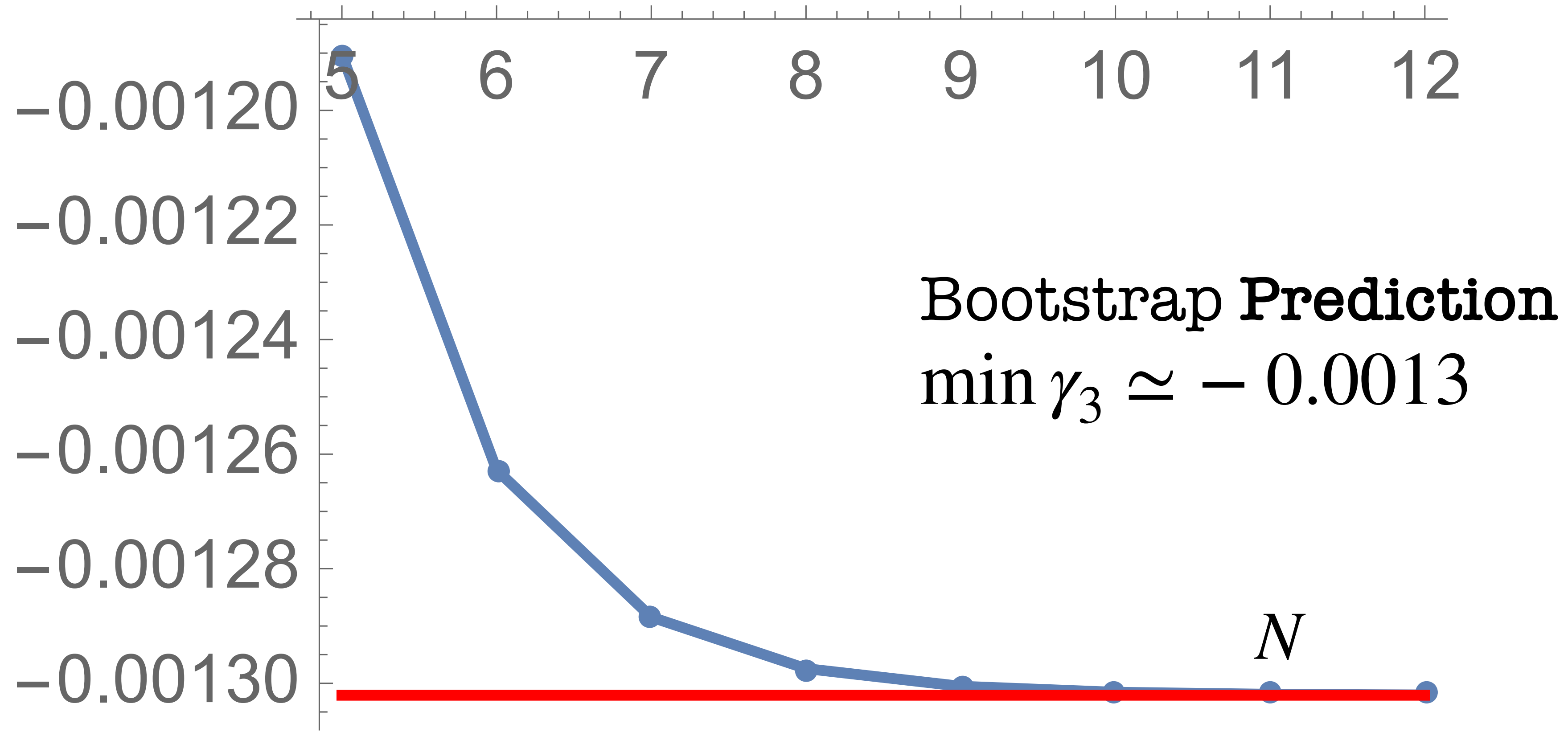

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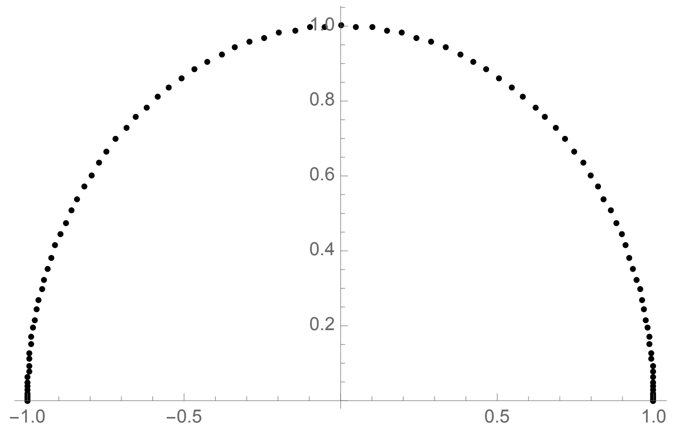

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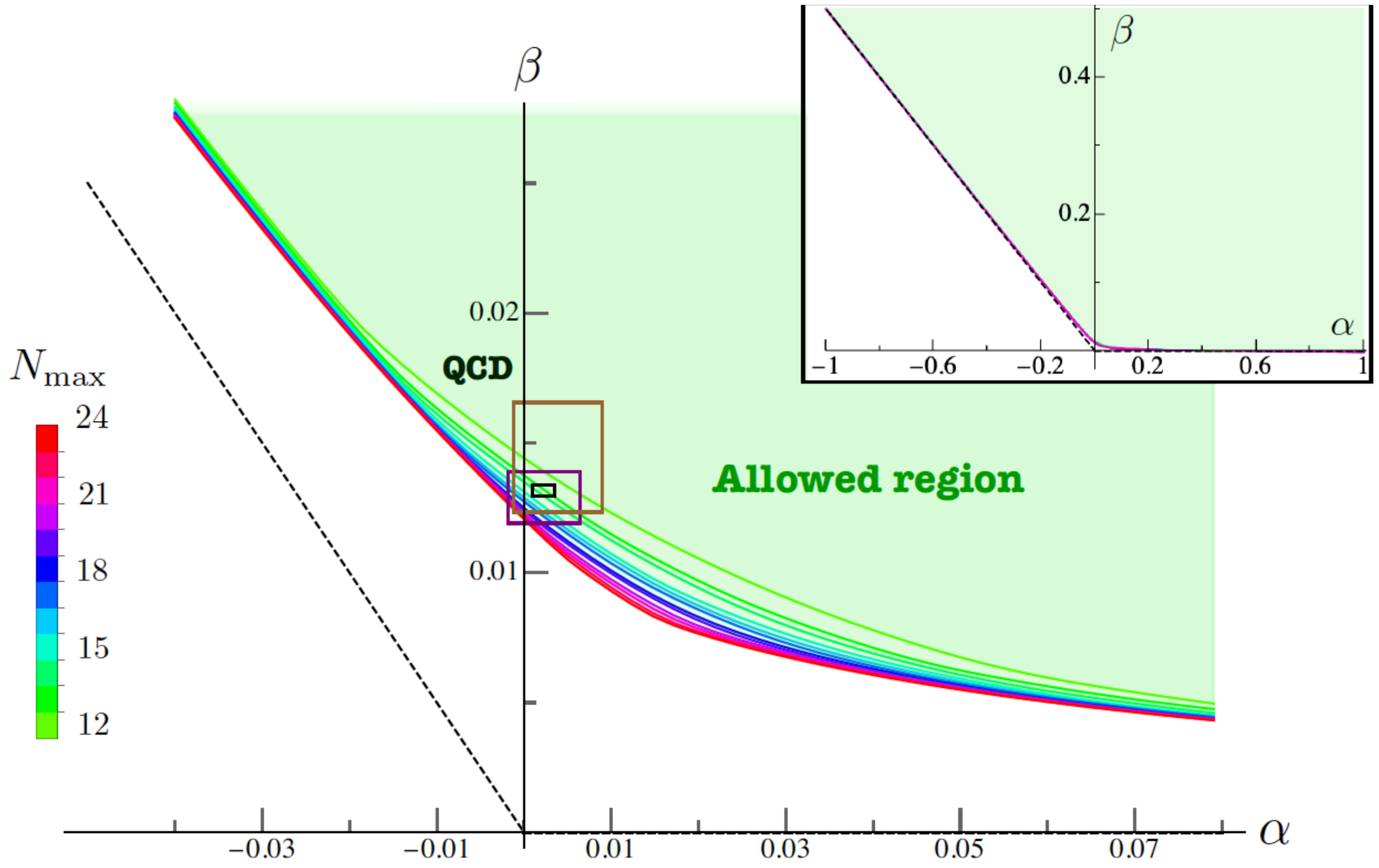
Low energy QCD

In QCD dynamical mass generation, non-perturbative RG flow

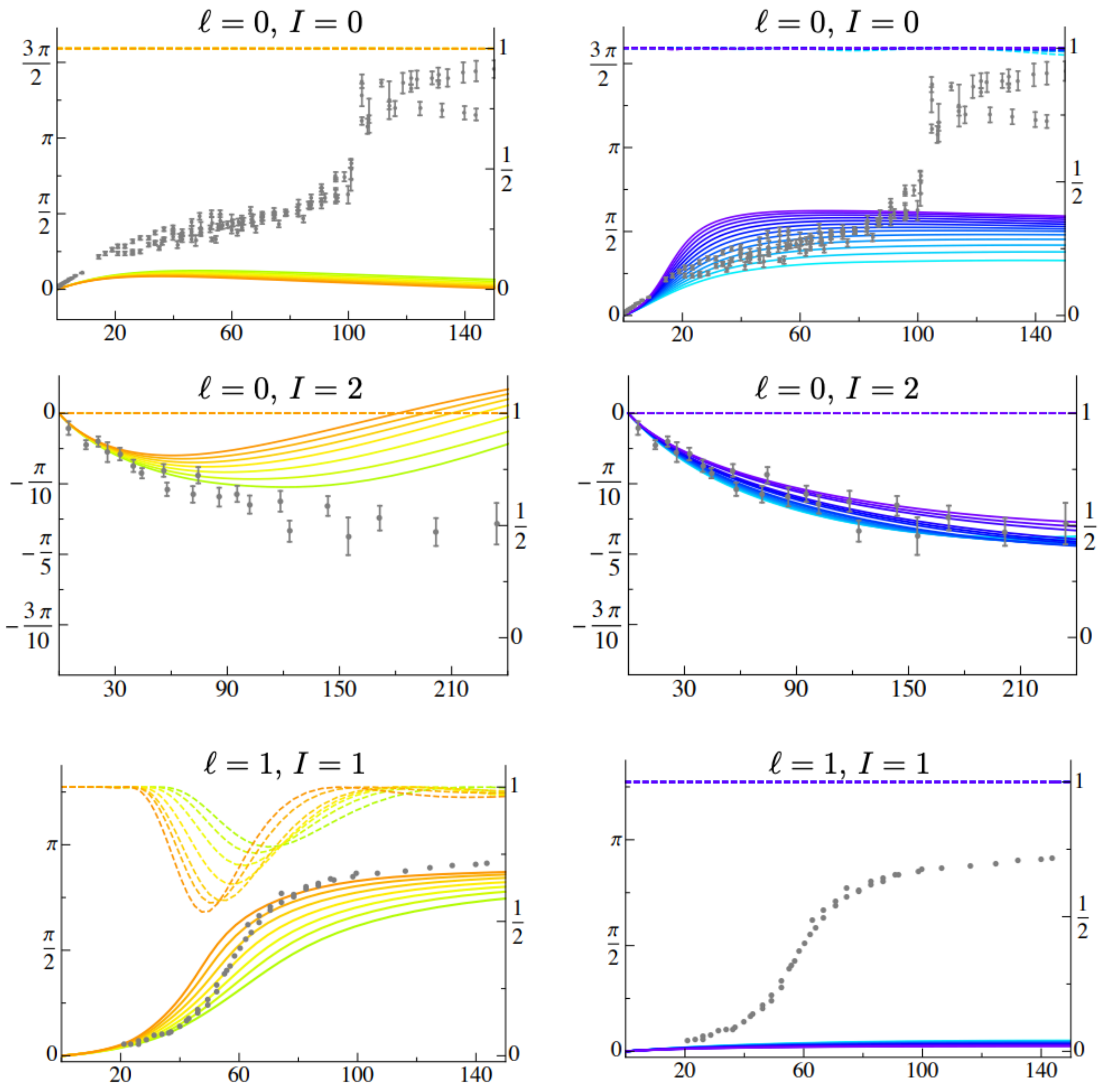
$$\text{Amplitude} = \frac{s}{f^2} + \alpha\frac{s^2}{f^4} + \beta\frac{t^2+u^2}{f^4} + \text{logs} + \text{UV completion}$$



ALG, Penedones, Vieira 1810.12849 (gapped), 2011.02802 (gapless)

α, β can be only computed using lattice QCD today or extracted from data!!!

Non perturbative S-matrices from Bootstrap



Left side of the boundary

Right side of the boundary

What can we add to nail down QCD?

Work in progress with H. Murali

D=4 Strings and the Axion

In D=4 two leading deviations from Nambu-Goto α_3, β_3

$$\mathscr{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \ldots + \alpha_3\ell_s^6K^4 + \beta_3\ell_s^6R^2 + \ldots\right)$$

$$\gamma_3 = \alpha_3 - \beta_3$$

New Effect in the amplitude: **Polchinski-Strominger** term $\propto \alpha_2 = \frac{D-26}{384\pi}$

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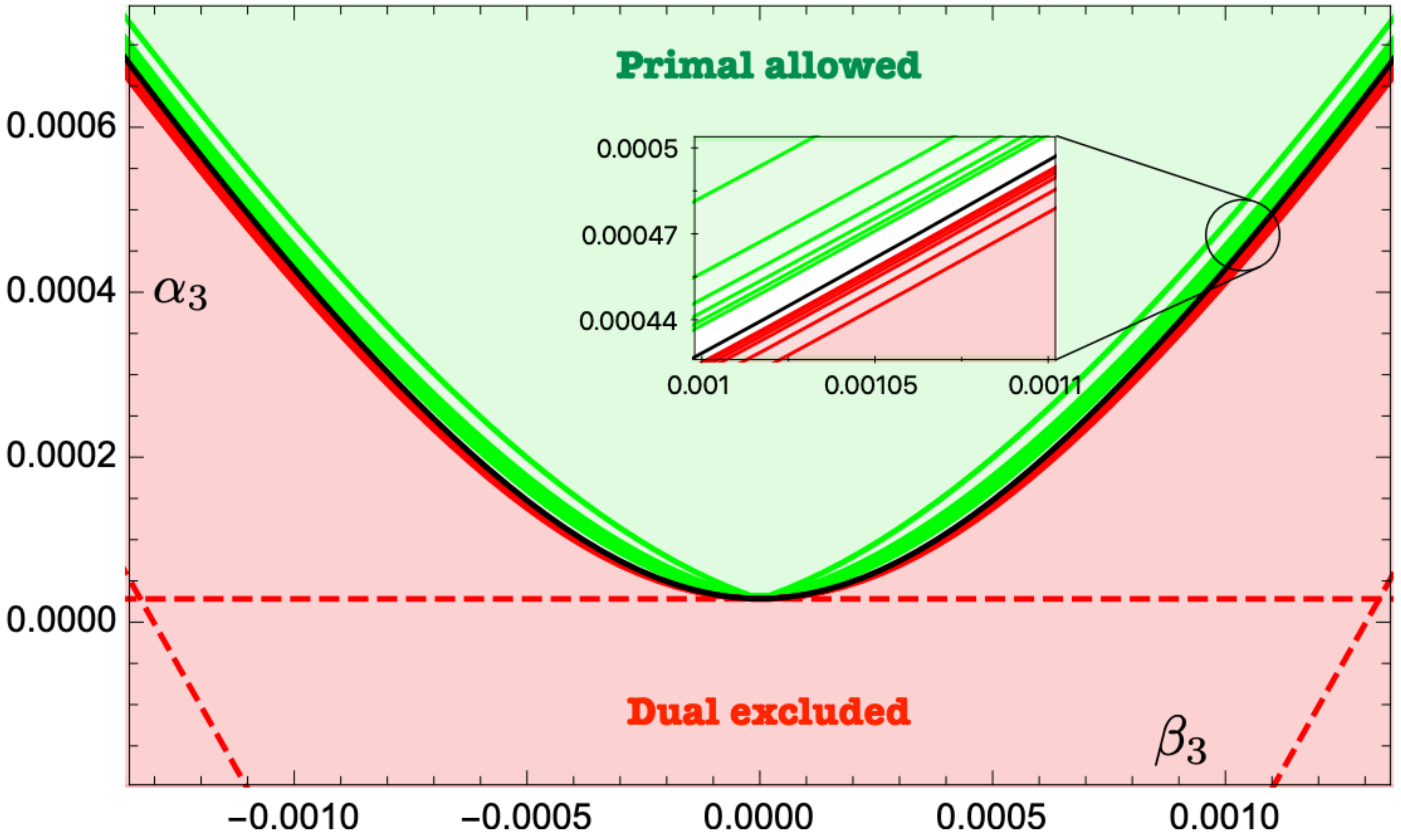
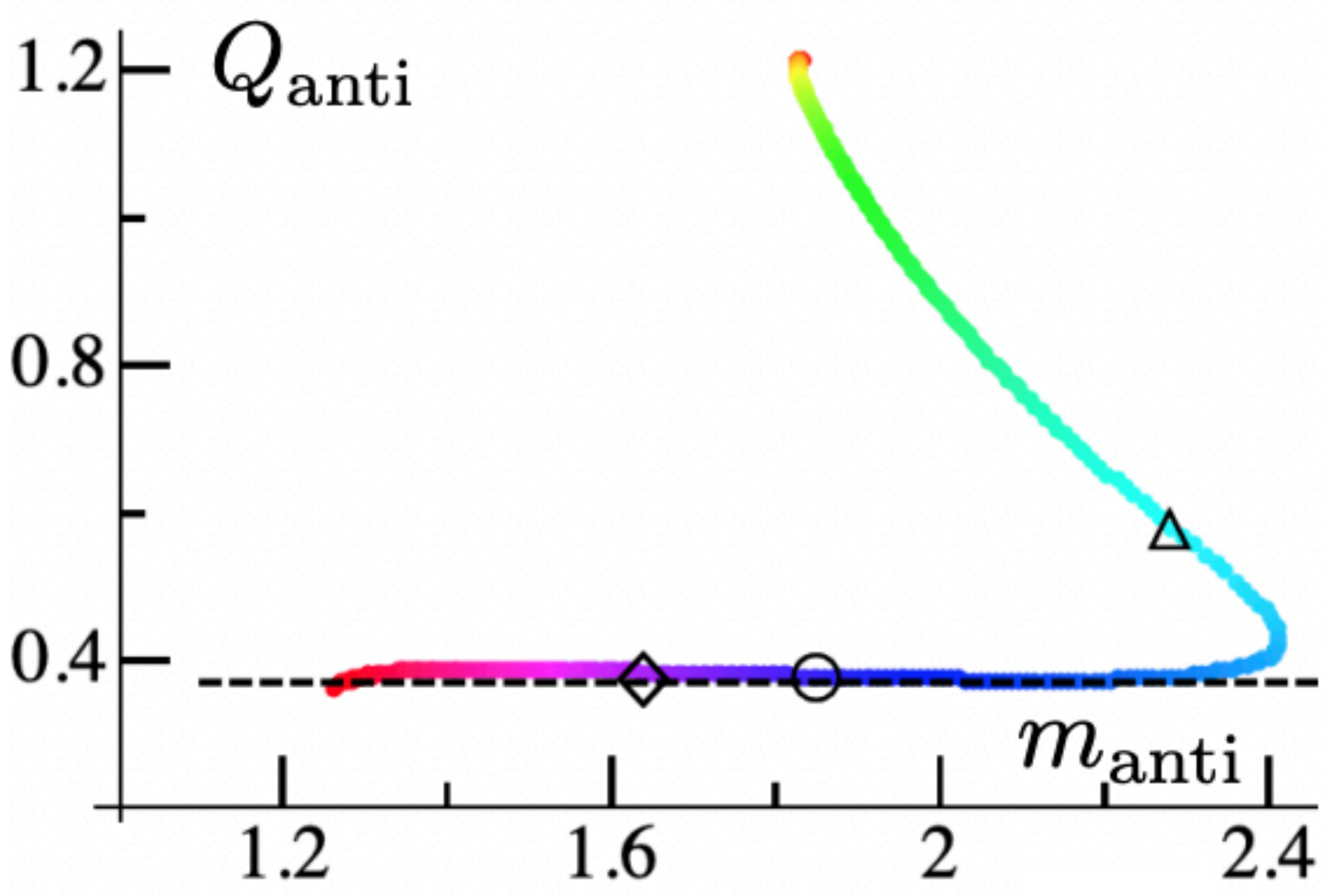
The two dimensional bound



Elias-Miró, ALG, Hebbar, Penedones, Vieira 1906.08098
Elias-Miró, ALG 2106.07957

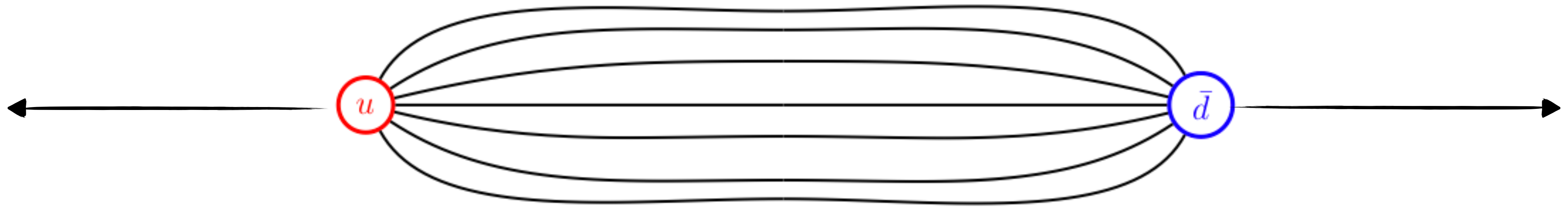
The extremal amplitude has an axion resonance



Bounds on the static $q\bar{q}$ potential
(toy for quantum gravity)

Application of the S-matrix Bootstrap to fundamental questions about confinement

Distance between quarks $R/\ell_s \to \infty$



Theory-dependent,
but bounded from first principles

Universal, consequence of non-linearly realized Lorentz

D=3 Target Space

$$E_0(R) = \frac{R}{\ell_s^2} - \frac{\pi}{6R} - \frac{\pi^2 \ell_s^2}{72R^3} - \frac{\pi^3 \ell_s^4}{432R^5} + \frac{\Delta_3 \ell_s^6}{R^7} + \mathcal{O}\left(\frac{1}{R^9}\right)$$

$$\Delta_3 \leq \frac{\pi^6}{5400} - \frac{5\pi^4}{10368}$$

String tension
$\sigma = \ell_s^{-2}$

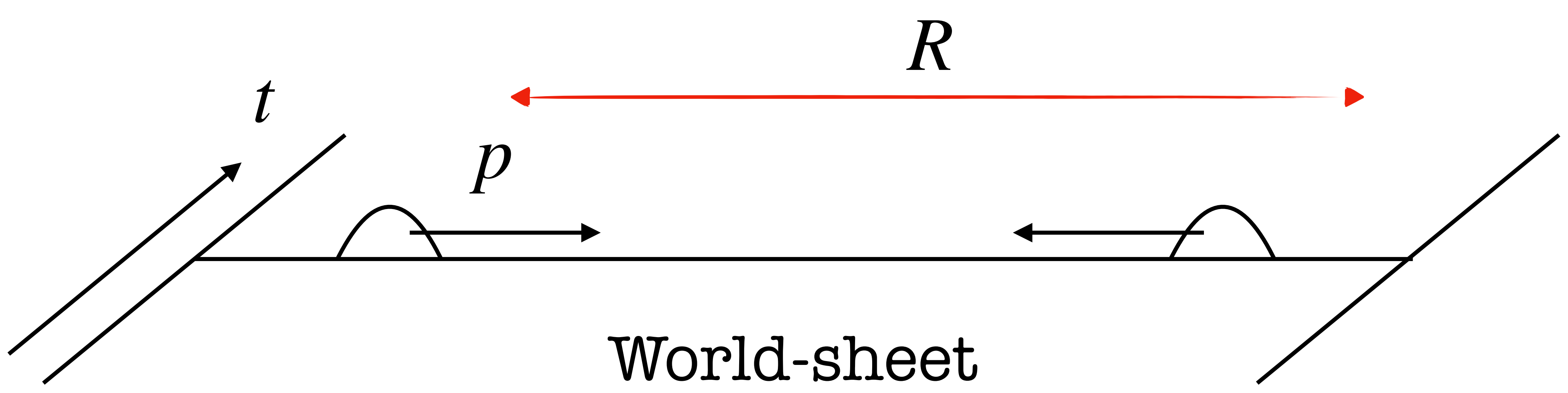
Lüscher
term '80

Lüscher, Weisz,
Drummond '04

Aharony,
Komargodsky,
Dubovsky, Flauger,
Gorbenko,...

Elias-Miró, ALG,
Hebbar, Penedones, and
Vieira, PRL 123, no.22
(2019)

Effective String Theory



2d gravity theory

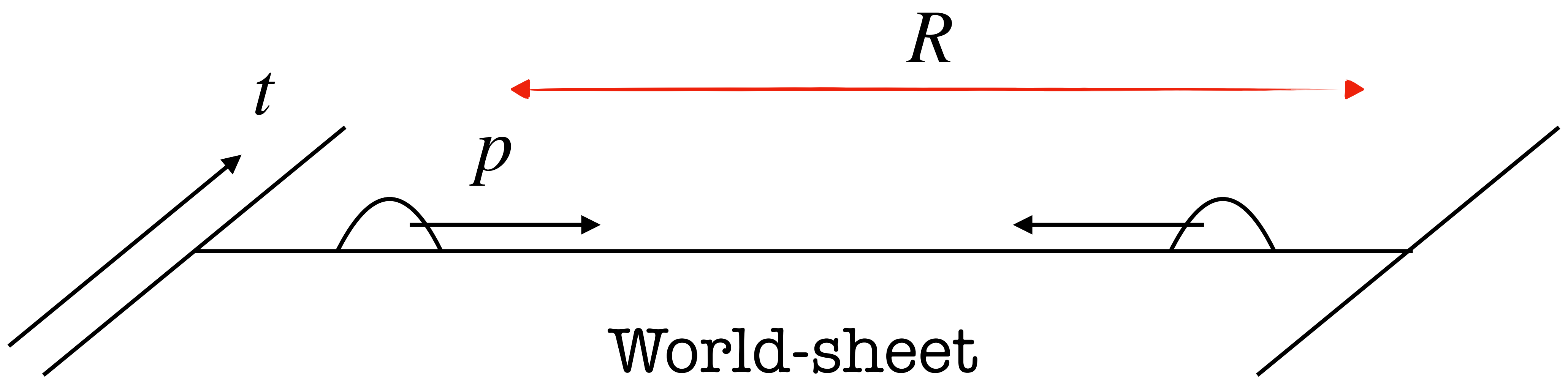
$$\mathcal{A}_{EFF} = \int d^2\sigma\sqrt{-h}\left(\frac{1}{\ell_s^2} + \gamma_2 \ell_s^2 R + \gamma_3 \ell_s^4 R^2 + \dots\right)$$

In $D \neq 26$, infinite corrections

Physical Degrees of freedom: X^i with i=1,...,D-2, massless Goldstones

We have an action, we can compute the S-matrix, but is this useful?

Effective String Theory



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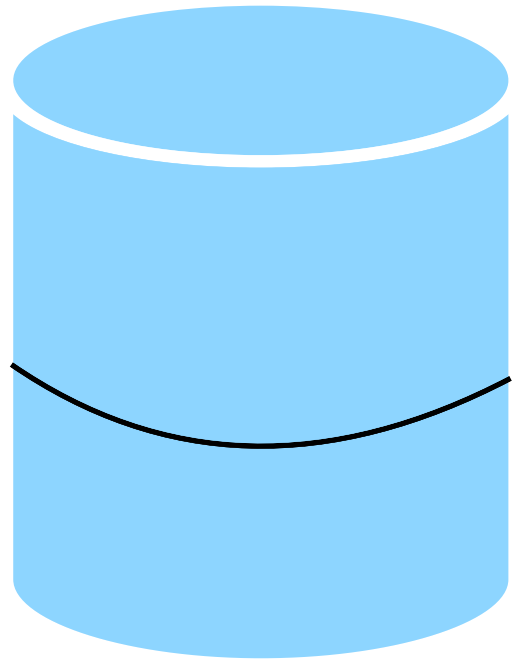
Thermodynamic Bethe Ansatz

$$T_{2\to 2}(s) = \ell_s^2 \frac{s^2}{2} + i\ell_s^4 \frac{s^3}{16} + \ell_s^6 (2\gamma_3 - \frac{1}{192}) s^4 + \dots$$

$$E_0(R) = R + \frac{1}{\pi R} \int_0^\infty dq \log(1 - e^{-\epsilon(q)})$$

$$\epsilon(p) = pR + \frac{1}{2\pi} \int_0^\infty dq \frac{\partial}{\partial q} \delta(4pq) \log(1 - e^{-\epsilon(q)})$$



Finite Volume Energy Levels from Infinite Space Scattering

$$\Delta_3 = -\frac{32\gamma_3 \pi^6}{225} - \frac{5\pi^4}{10368}$$

We solve them in the 1/R expansion

An analytic bound on scattering

Goal: we bound $c_4 \iff$ we bound Δ_3

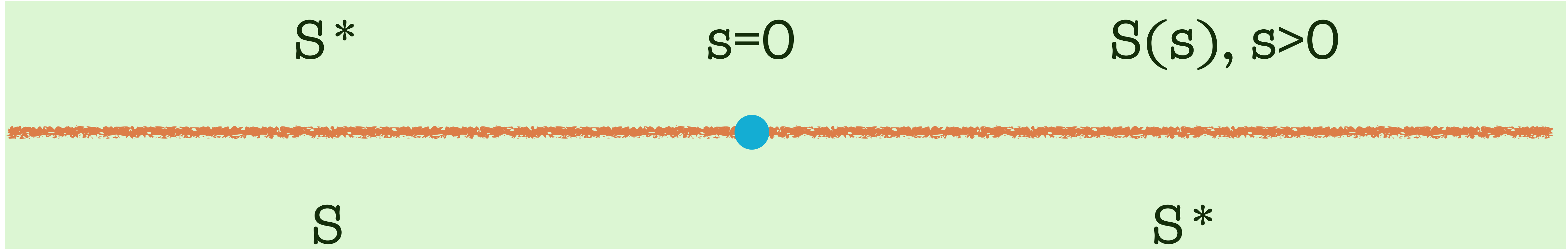
What are the non-perturbative properties of the branons scattering amplitude?

Unitarity: define $S(s) = 1 + \frac{i}{2s} T_{2\to 2}(s)$ then $|S(s)|^2 \leq 1$ for s>0

Crossing: $S(s) = S(u)$ where t=0, u=-s

Analyticity

Analytic away from the real axis



Low Energy Constraints: $S(s) = 1 + i\frac{s}{4} - \frac{s^2}{32} + i(\gamma_3 - \frac{1}{384})s^3 + \dots$

Solution (Schwarz-Pick Theorem)

$$\gamma_3 \geq -\frac{1}{768}$$

$$S(s) = \frac{8i - s}{8i + s}$$

gauge group	$\mathbb{Z}_2$	$SU(2)$	$SU(6)$	$SU(\infty)$
$\gamma_3 \times 768$	-0.4 [4]	-0.3 [5]	0.2 [1, 6]	0.3

[4] Baffigo, Caselle '23

[5] Caristo, Caselle, Magnoli, Nada, Panero '21

[1,6] Dubovsky, Gorbenko, et al

The S-matrix Bootstrap represent a novel framework connecting different fields in physics and mathematics

Optimization theory

Development of new algorithms and strategies

Analytic solutions and Geometric Function Theory

1+1 QFTs and Integrability

Integrable Models as application and testing ground for the Bootstrap

Approximate integrability to study the QCD String

String and M-theory

Non-perturbative Dualities

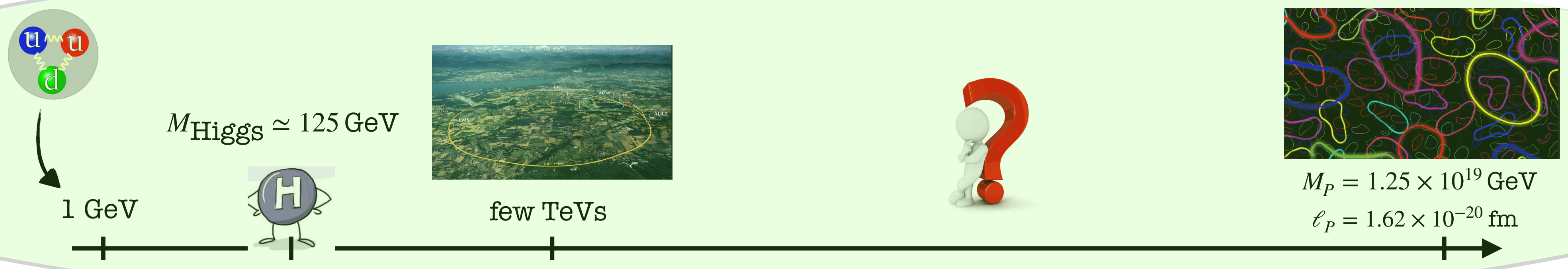
Nonperturbative properties of String scattering Amplitudes

Perturbative regimes

Large Spin expansion, and Spin analyticity

Loop expansions and match with QFT

S-matrix Bootstrap



CFT Bootstrap

Celestial Holography

Matrix Models Bootstrap

Q: Can we get rid of Lagrangians and reformulate QFTs and Strings
using more general physical principles?

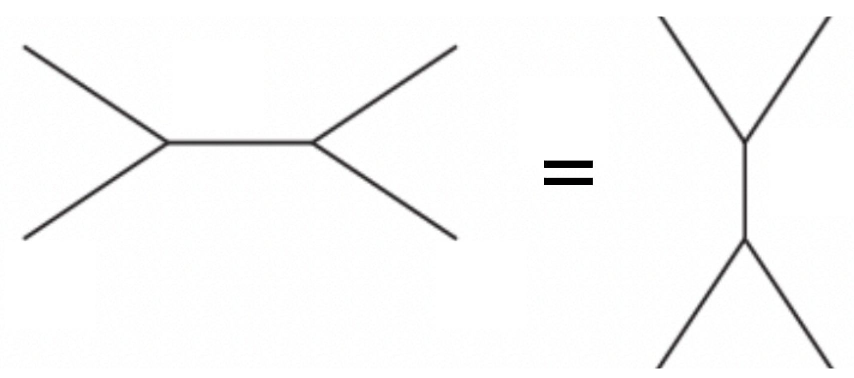
Bootstrap as an Optimization Problem

Goal: Find the optimal value of a physical observables constrained by the laws of nature

Math + Physics

Causality, crossing, and global symmetries

Linear Constraints: $M_{2\to 2}(s\,|\,t) = c + \int_{4m^2}^{\infty} \mathrm{Disc}_z M_{2\to 2}(z\,|\,t) K(z,s\,|\,t) dz$



$$M_{2\to 2}(s\,|\,t) = M_{2\to 2}(s\,|\,4m^2 - s - t)$$

Quantum mechanics

Unitarity Constraints: $\mathbb{I} - \mathbb{S}\mathbb{S}^\dagger \succeq 0$

$\mathbb{S} \supset M_{n\to m}$ all possible processes

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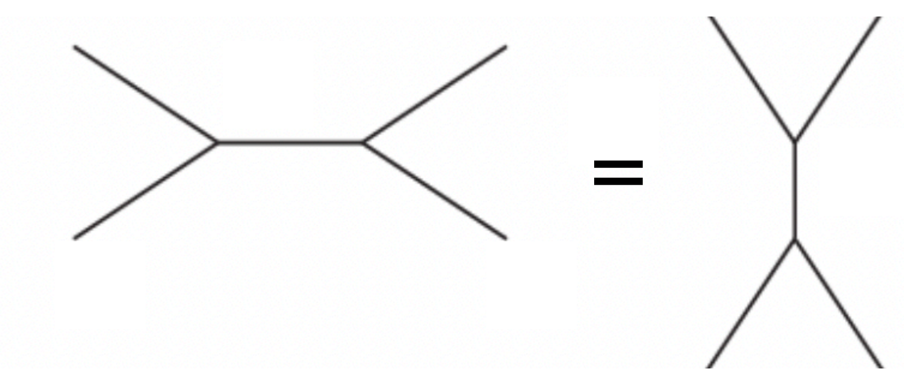

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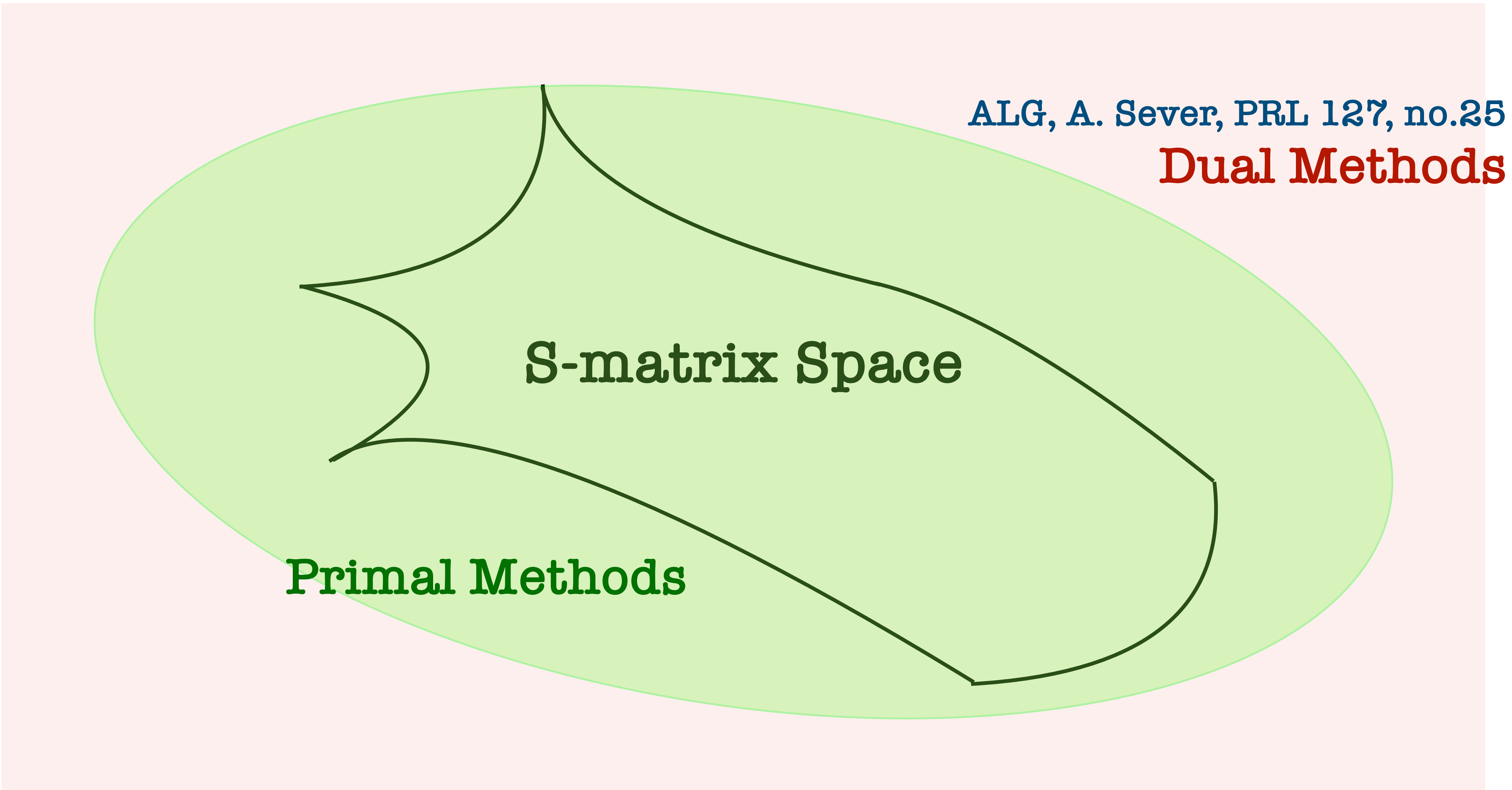
Computer Science

With Standard Optimization Tools (**SDPB**), we only explored the Convex Hull

ALG, A. Sever, PRL 127, no.25 (2021)



We relax Unitarity

$$\sum_n P_{2\to n} = 1 \implies P_{2\to2} \le 1$$

Use AI and Deep Learning Algorithms

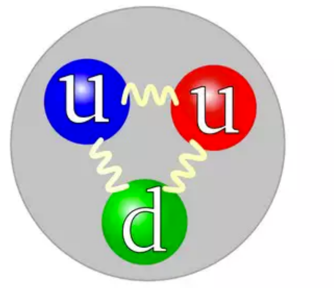
Unsupervised Reinforcement Learning to explore the S-matrix Space

Standard Search Algorithms

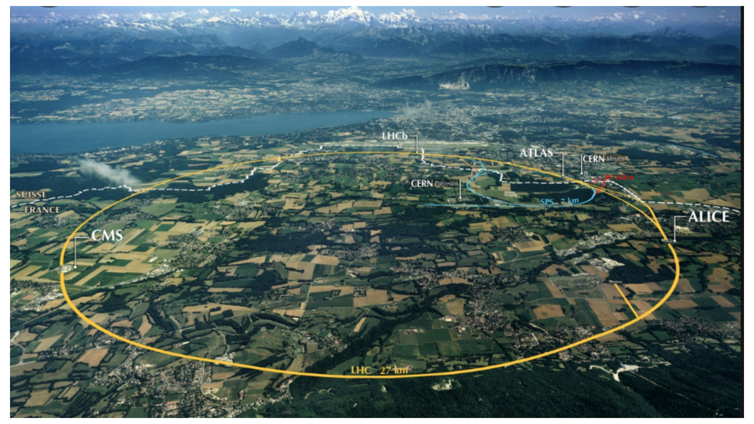
Combine Gradient Methods with SDPB (software created by the Bootstrap Collaboration)

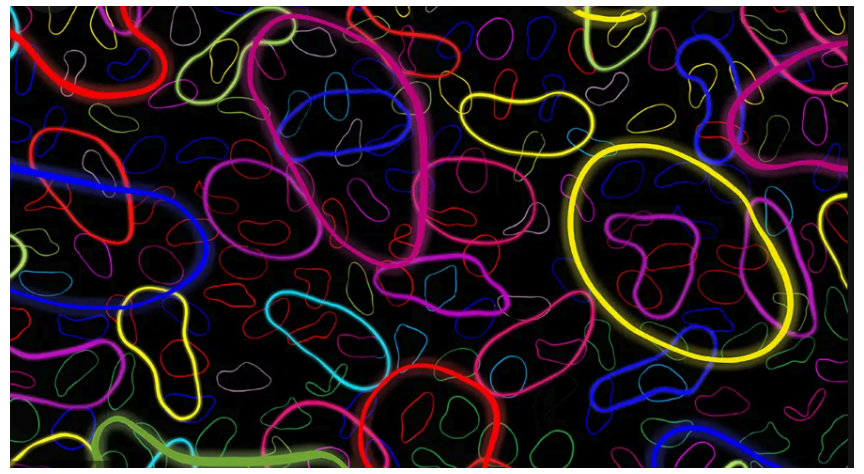
Why we need for a theoretical collider

Why we need for a theoretical collider



$M_{\text{Higgs}} \simeq 125\,\text{GeV}$





$M_P = 1.25 \times 10^{19}\,\text{GeV}$

$\ell_P = 1.62 \times 10^{-20}\,\text{fm}$

1 GeV — few TeVs

Particle physics — BSM — Quantum Gravity

Theoretical physicists are impatient!

The New Bootstrap Manifesto:
find the space of all possible physical observables assuming general principles

$$\mathcal{S}_{in\to out} \equiv < in\,|\,out >$$