Probing SUSY at Gravitational Wave Observatories

based on: S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2307.04595
S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746
S.A, K. Hinze, S. Saad, arXiv:2406.xxxxx (in preparation)

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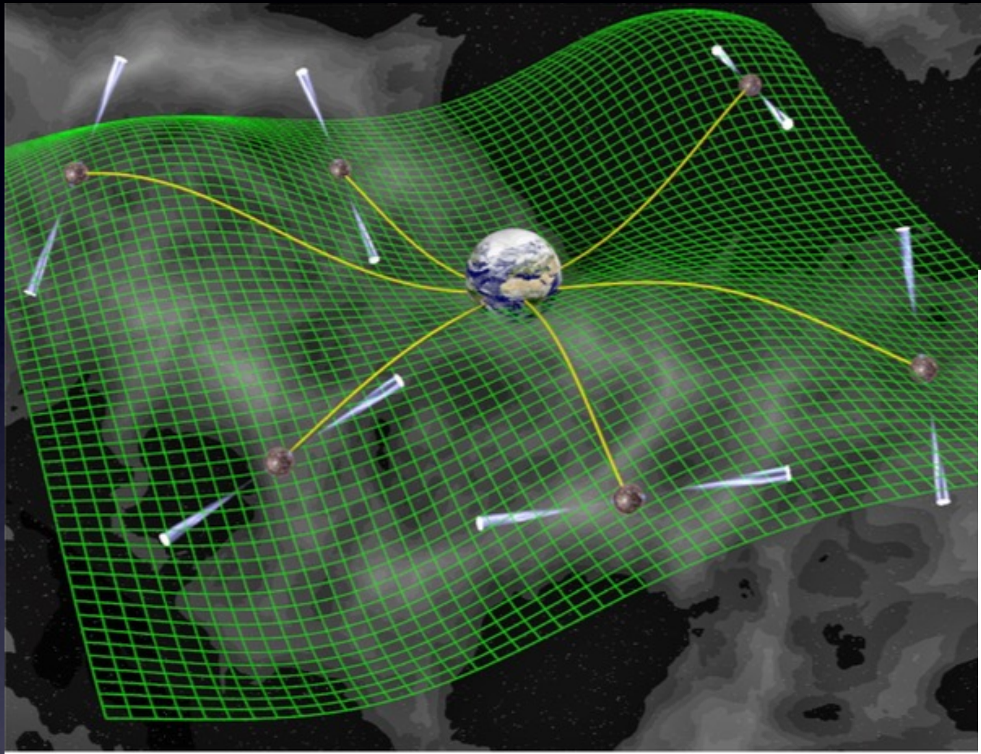
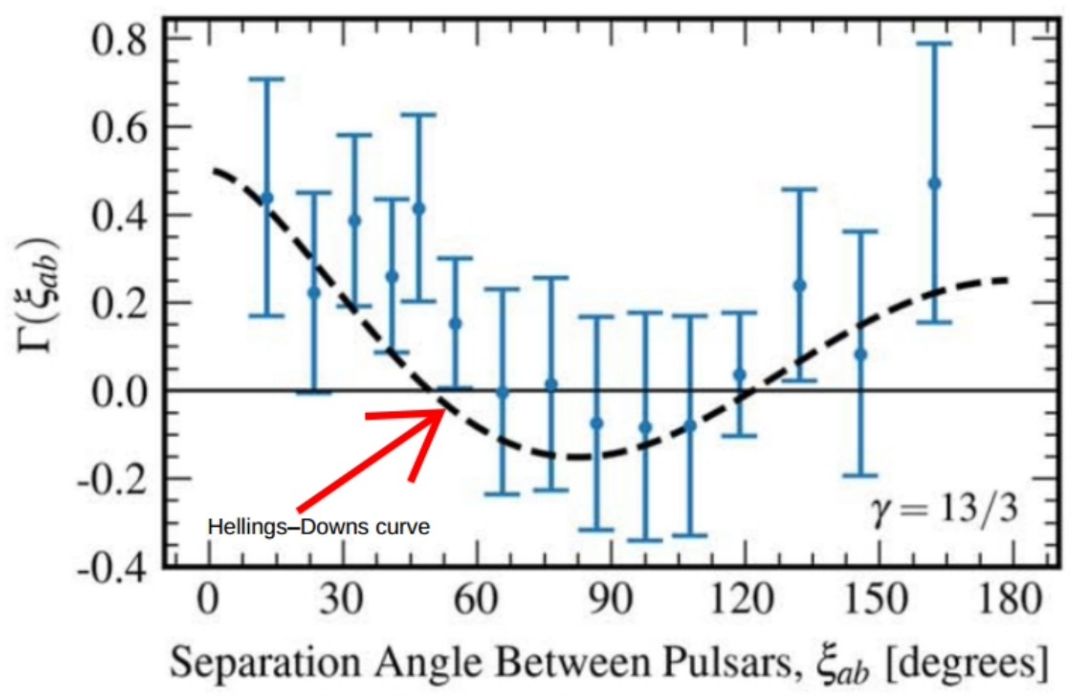
PTA results 2023 point at Stochastic Gravitational Wave Background (SGWB)

at nanohertz frequencies

NanoGrav, EPTA+InPTA, PPTA, CPTA

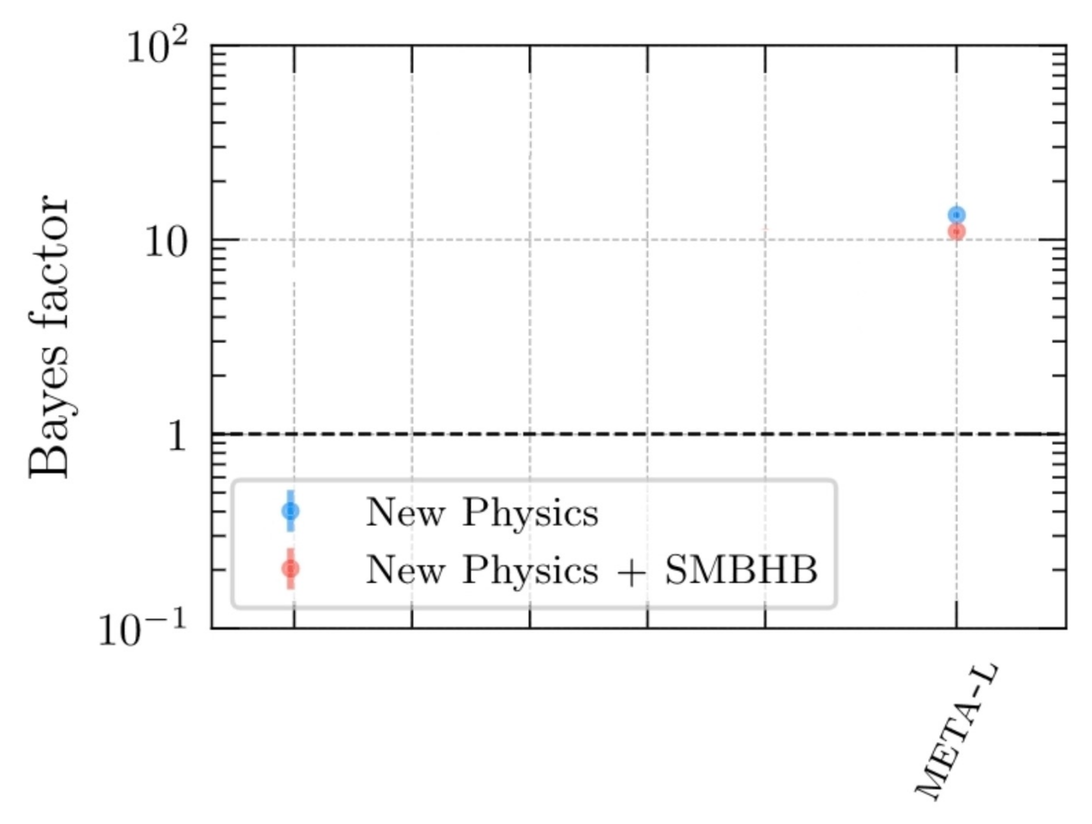




Gabriella Agazie et al 2023 ApJL 951 L8

- What is its physics origin? Supermassive BH binaries? Or BSM physics?

Among best fitting explanations: local Metastable cosmic strings



Adeela Afzal et al. (2023)

- MSCS provide a significantly better fit than SMBHB. Can arise in BSM scenarios with extended gauge symmetry (e.g. SO(10) GUTs)

Outline of my talk

When the 2023 PTA results for a SGWB at nanoherz frequencies are caused by metastable cosmic strings:

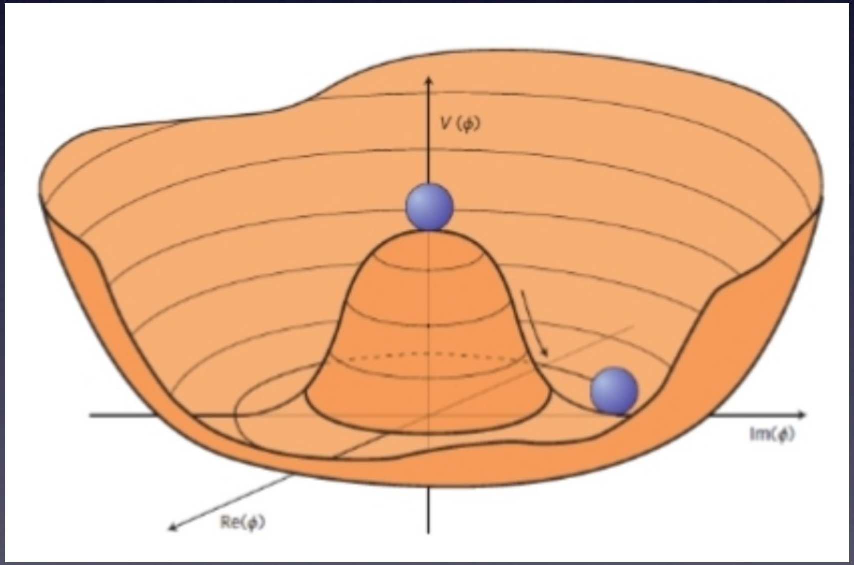
- Potential to discover signs of SUSY [or other NP with extra DOF] up to $m_{SUSY} \sim O(10^4\ \mathrm{TeV})$
- Might be a hint towards SO(10) GUTs, and would help to "single out" promising model classes

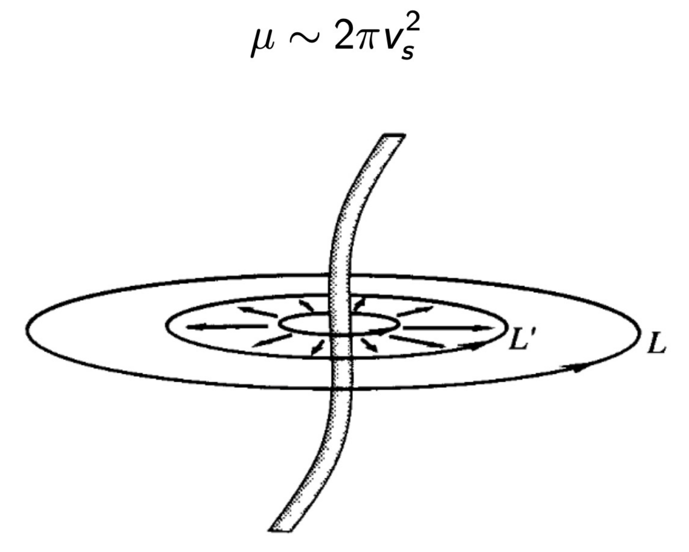
... but first: what are metastable cosmic strings?

Cosmic strings ...

- Cosmic string production: spontaneous symmetry breaking H → K with nontrivial homotopy group π_1 of H/K (vacuum manifold with "unshrinkable loops"), e.g.

$$U(1) \xrightarrow{v_s} \text{nothing}$$





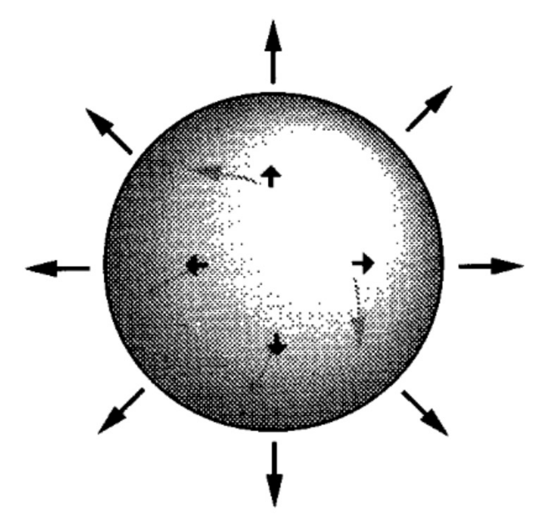
$$\mu \sim 2\pi v_s^2$$

$$\text{Dimensionless string tension} = G\mu \qquad (G = \text{Newton's constant})$$

... can be metastable when the theory allows for monopoles

- Monopole production: spontaneous symmetry breaking G → H with nontrivial homotopy group π_2 of G/H (vacuum manifold with "unshrinkable spheres"), e.g. when compact simple group G breaks into H that contains a U(1) factor

$$SU(2) \xrightarrow{v_m} U(1)$$



monopole mass:

$$m = \frac{4\pi v_m}{g}$$

- Important: After monopole production the monopoles have to be diluted by inflation in order not to overclose the universe (more later)

Metastable cosmic strings

P. Langacker, S.Y. Pi (1980)

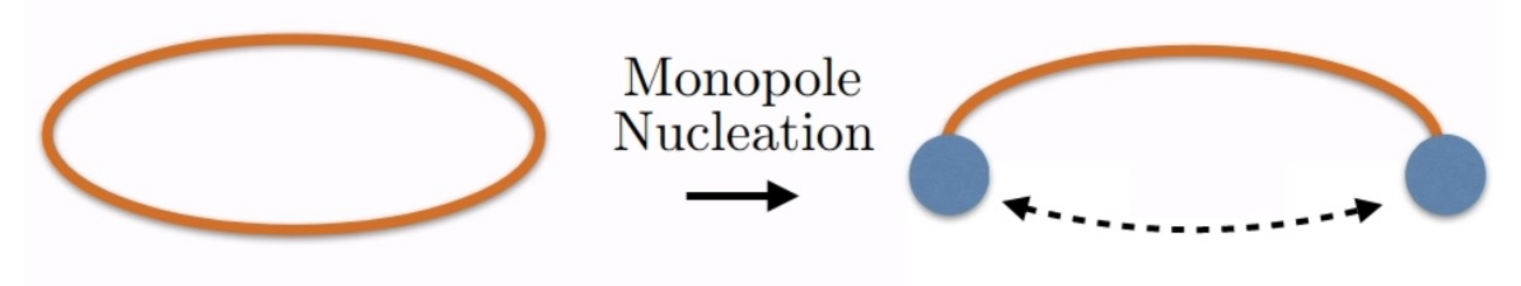
- Appear in multi-step SSB, when e.g. the U(1) from CS generation is involved in monopole formation: CS can decay via monopole-antimonopole production

monopole production

Inflation

cosmic string production

$$\text{Example: } SU(2) \xrightarrow{v_m} U(1) \xrightarrow{v_S} \text{broken}$$



- Lifetime depends on

$$t_s = \Gamma_d^{-1/2}, \qquad \Gamma_d = \frac{\mu}{2\pi} e^{-\pi\kappa_m}$$

Γ_d: decay rate per string unit length

$$\kappa_m = \frac{m^2}{\mu} \sim \frac{8\pi}{g^2}\left(\frac{v_m}{v_s}\right)^2$$

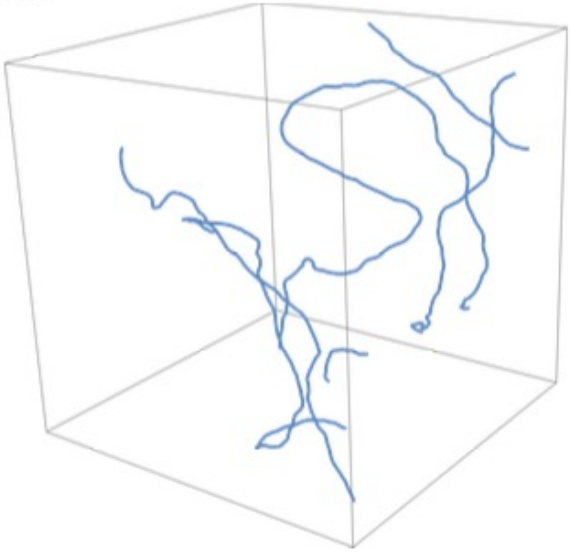
$$(\kappa_m^{1/2} < 9 \ \text{metastable})$$

Vilenkin ('82), Preskill, Vilenkin ('92), Monin, Voloshin ('08), Leblond, Shlaer, Siemens ('09), Chitose, Ibe, Nakayama, Shirai, Watanabe (arXiv:2312.15662)

Gravitational waves from metastable CS

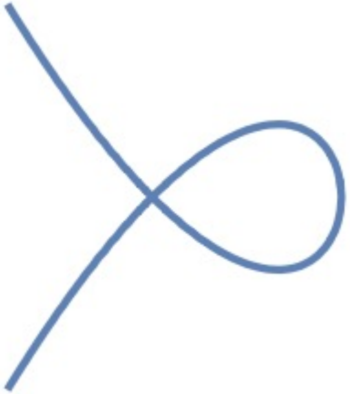
0. Monopoles diluted away by Inflation

1. Cosmic string network forms

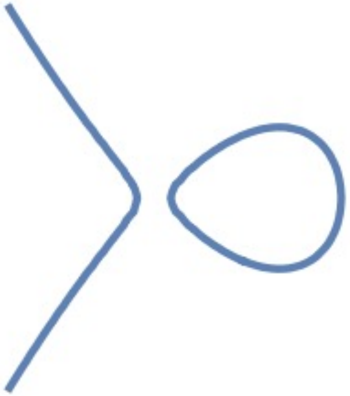


... and enters scaling regime

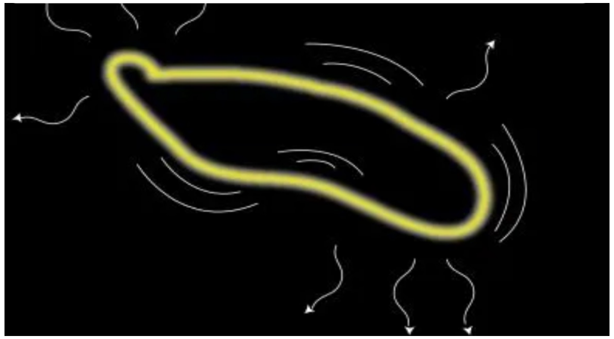
2.



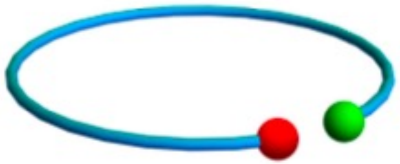
3.



4. CS loops wiggle and oscillate → GW



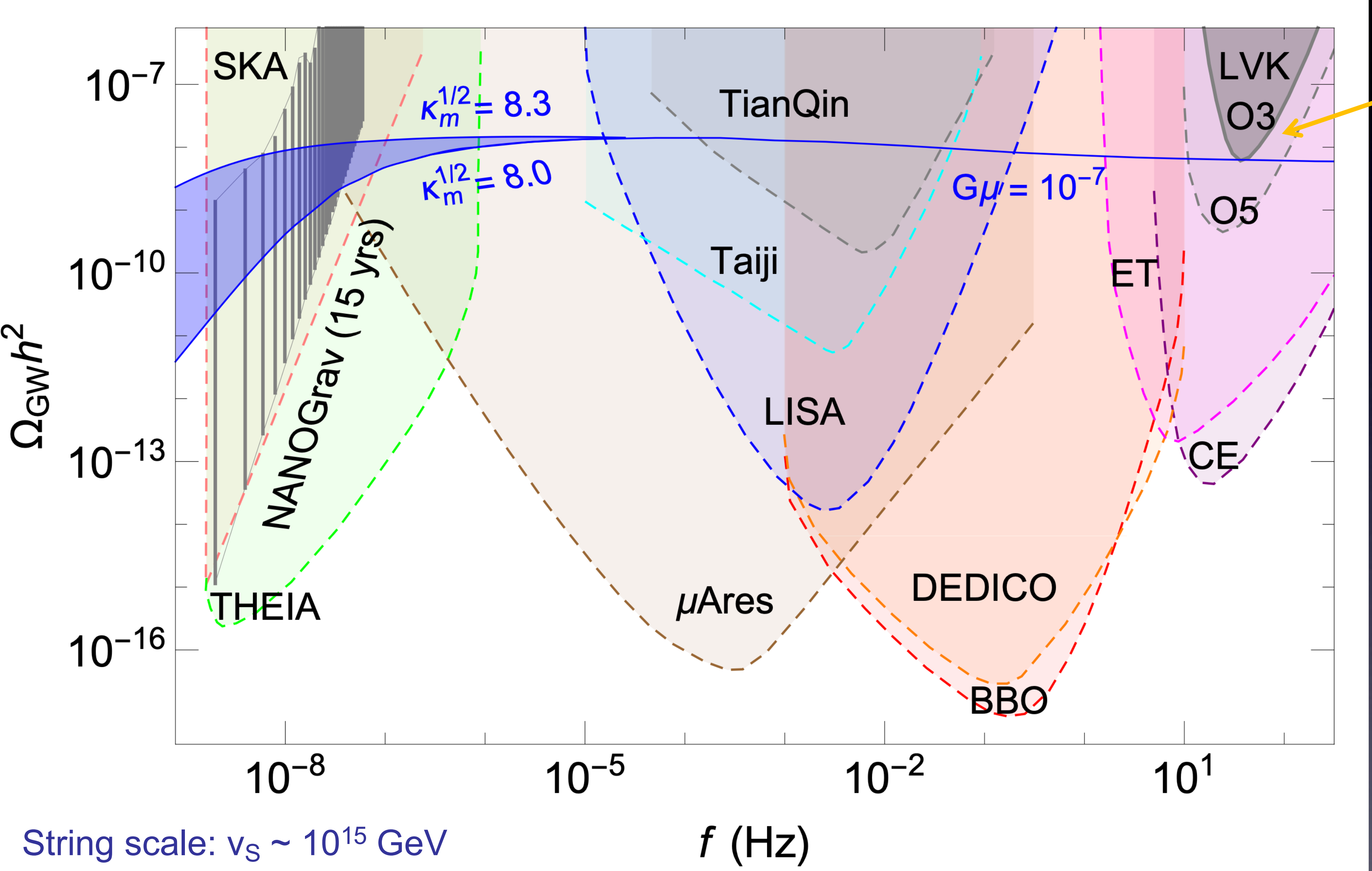
5. Cosmic strings decay



→ **Characteristic GW spectrum**

Characteristic SGWB spectrum

- Spans over large frequency range!



String scale: $v_S \sim 10^{15}$ GeV

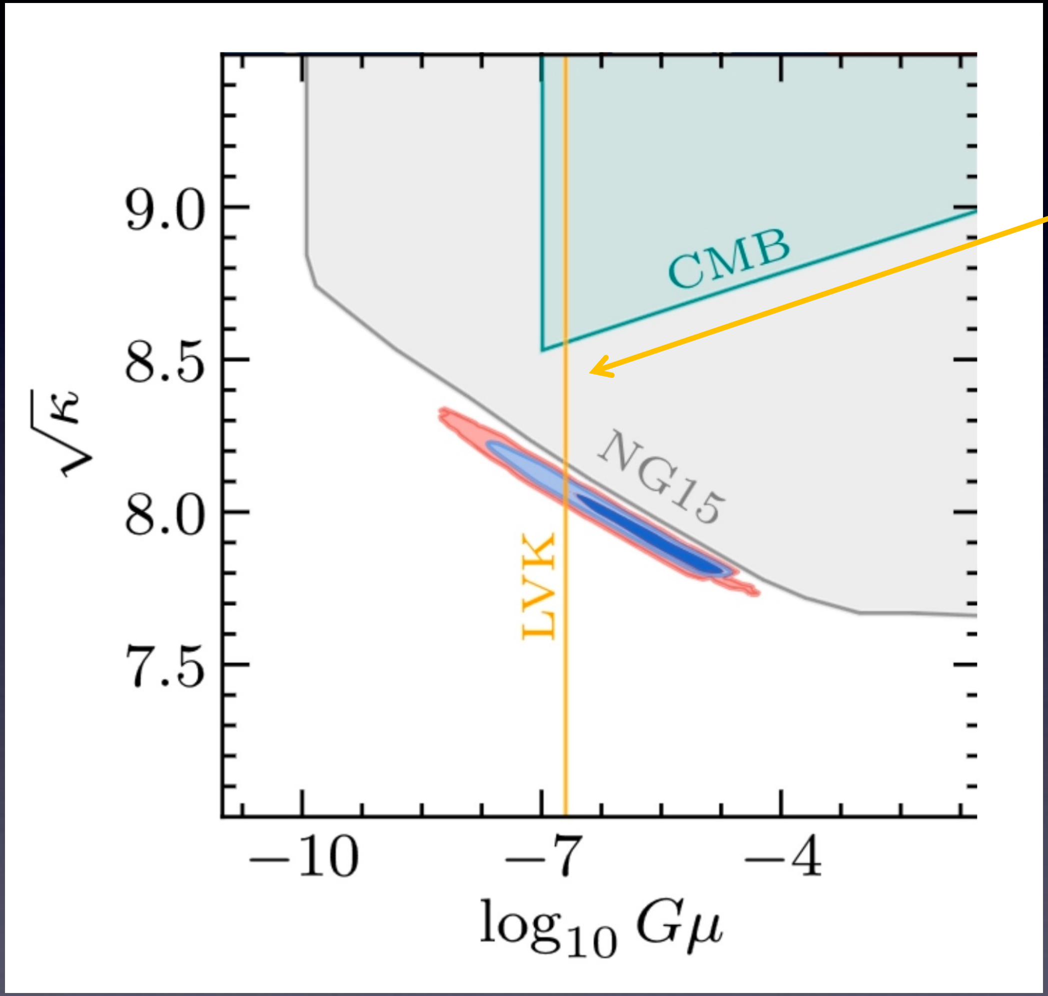
Constraints from LIGO-Virgo-Kagra (LVK)!

plot from: S.A, K. Hinze, S. Saad, J. Steiner (arXiv:2307.04595)

- MSCS explanation will be confirmed/dismissed at (future) GW ovservatories!

Preferred metastable CS parameters from PTA results

Adeela Afzal et al. (2023)



Constraints from LVK

Preferred range: (kappa)$^{1/2}$ ~ 8

Preferred range for $G\mu$: 10^{-8} – 10^{-4}
→ v_s: $5\text{x}10^{14}$ – $5\text{x}10^{16}$ GeV

- PTA data points at CS generation scale $v_s \sim v_m$ close to the (typical) GUT scale

If the metastable CS explanation of the PTA results will be confirmed:

Potential to discover signs of SUSY up to $m_{SUSY} \sim O(10^4\ \text{TeV})$

Note: Also applies to other new physics (NP) that predicts a signifficant increase of particle degrees of freedom (DOF)

Computation of the metastable cosmic string GW spectrum

for details, see Appendix of: S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746

- Step 1: Determine expansion history of the universe (→ Friedmann eq.)
- Step 2: Compute CS loop number density
- Step 3: Compute GW spectrum

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- Step 1: Determine expansion history of the universe (→ Friedmann eq.)

$$H(z) = H_0 \left(\Omega_\Lambda + (1+z)^3 \Omega_{\mathrm{mat}} + (1+z)^4 \boxed{\mathcal{G}(z)} \Omega_{\mathrm{rad}}\right)^{1/2}$$

$$\boxed{\mathcal{G}(z) = \frac{g_*(z) g_S^{4/3}(z_0)}{g_*(z_0) g_S^{4/3}(z)}}$$

→ # DOF modified by NP (SUSY)

- Step 2: Compute CS loop number density

- Step 3: Compute GW spectrum

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→ # DOF modified by NP (SUSY)

- Step 2: Compute CS loop number density

energy loss due to GW emission (Γ ~ 50) ↖

dilution from expansion ↖

decay due to monopole nucleation ↑

$$\left[-\Gamma\, G\mu\, \partial_\ell + \partial_t\right] n(\ell,t) = S(\ell,t) - \left(3H(t) + \Gamma_d \ell\right) n(\ell,t)$$

W. Buchmuller, V. Domcke, K. Schmitz (arXiv:2107.04578)

↘ loop production function S (simulation result) from: J. J. Blanco-Pillado, K. D. Olum, and B. Shlaer (arXiv:1309.6637)

- Step 3: Compute GW spectrum

z_c: creation of CS network

$$\Omega_{\text{GW}}(f,t) = \frac{8\pi (G\mu)^2}{3H^2(t)} \sum_{n=1}^{\infty} C_n P_n, \quad C_n = \frac{2n}{f^2}\int_{z(t)}^{z_c} \frac{dz}{H(z)(1+z)^6}\, n\!\left(\frac{2n}{f(1+z)}, t(z)\right)$$

P_n (power spectrum per mode, simulation result) from: J. J. Blanco-Pillado and K. D. Olum (arXiv:1709.02693)

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→ # DOF modified by NP (SUSY)

General, also beyond MSCS and # DOF: via H(z) a "loud" SGWB source with predicted shape allows to test for deviations from standard cosmic history

density

dilution from expansion ↖

decay due to monopole nucleation ↑

$$S(\ell,t) - \left(3H(t) + \Gamma_d \ell\right) n(\ell,t)$$

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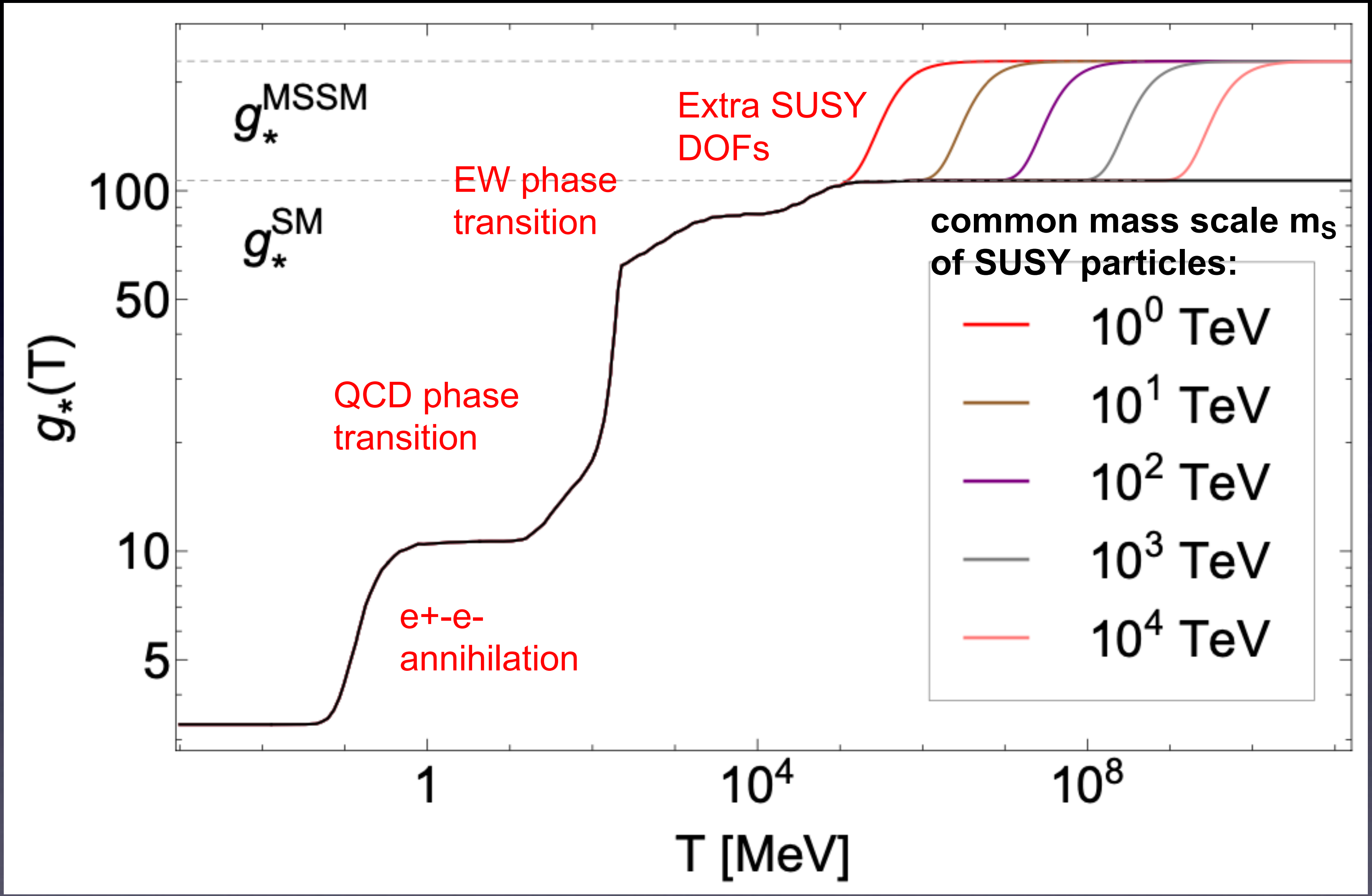
P_n (power spectrum per mode, simulation result) from: J. J. Blanco-Pillado and K. D. Olum (arXiv:1709.02693)

Main effect: SUSY particles modify g_*

Charaxteristic for SUSY:
DOF ~ 2x SM-DOF

Effective number of relativistic degrees of freedom

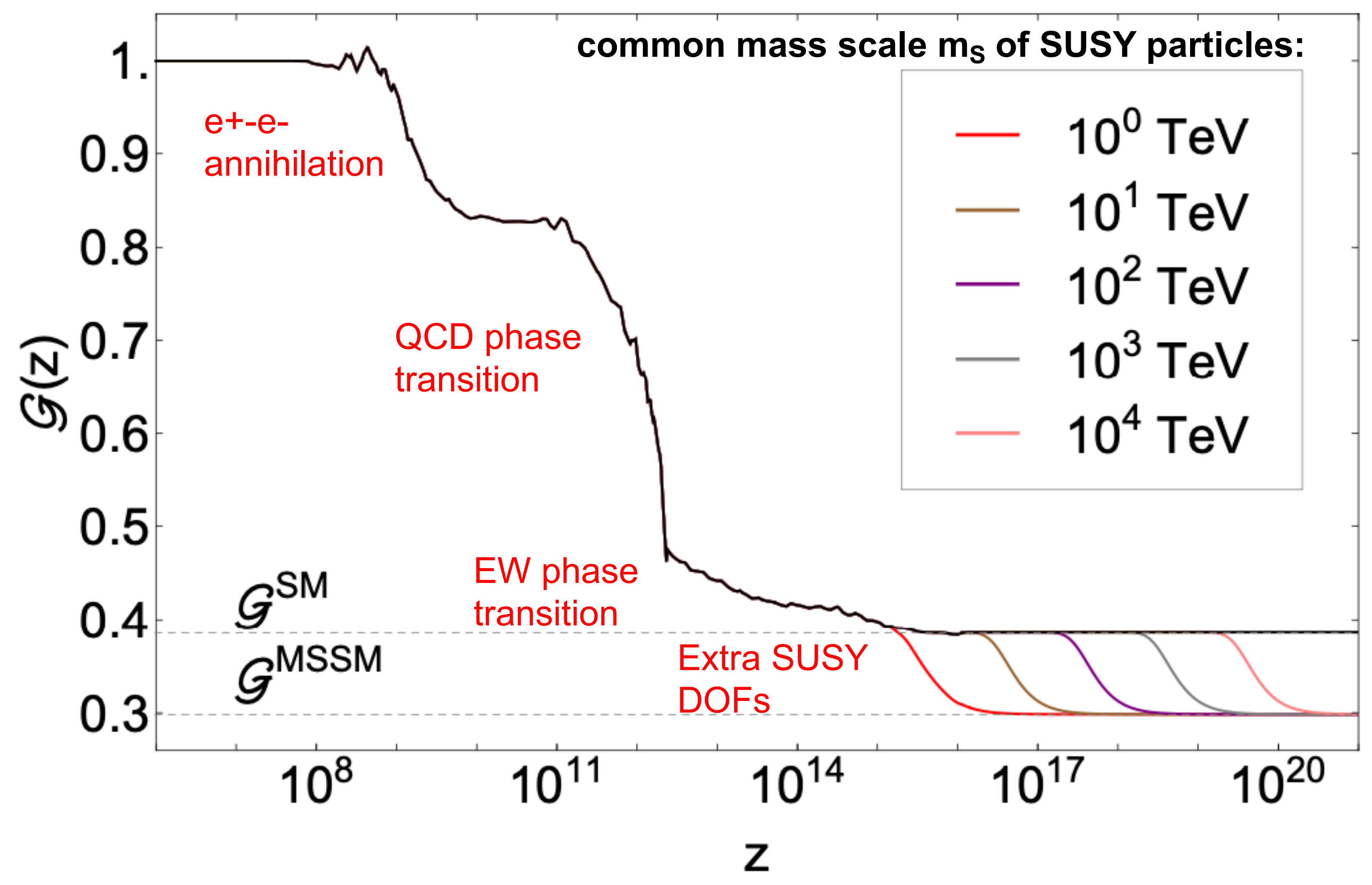


plot from: S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746

For discussions of effects of extra DOF on GW from CS, cf. also e.g.:
P. Auclair et al. (arXiv:1909.00819), Cui, Lewicki, Morrissey, Wells (arXiv:1808.08968), Battye, Caldwell, Shellard ('97)

... which enters G(z)

$$\mathcal{G}(z) = \frac{g_*(z) g_S^{4/3}(z_0)}{g_*(z_0) g_S^{4/3}(z)}$$

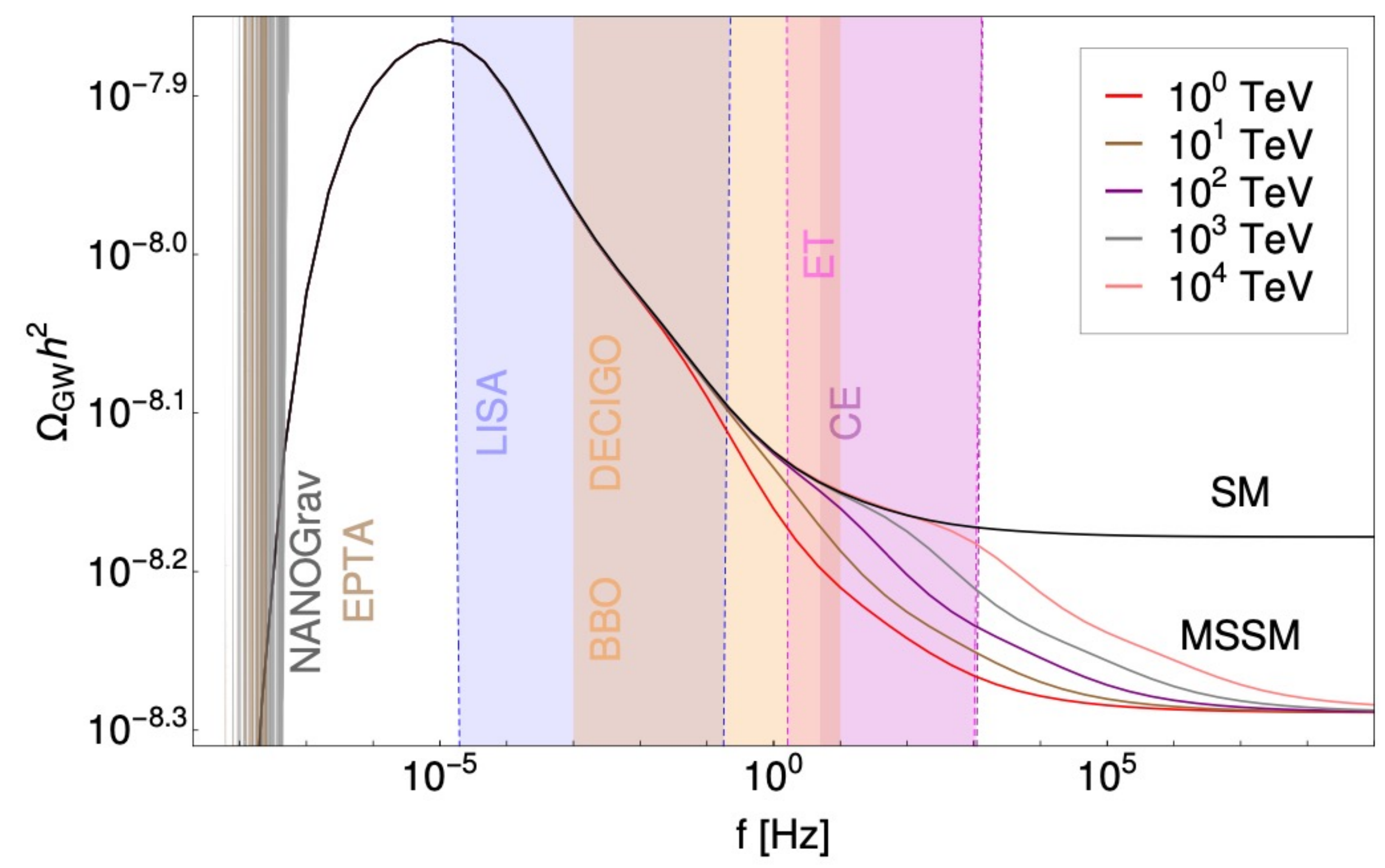


plot from: S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746

- Via H(z), extra DOF leave imprint on the GW spectrum produced by MSCSs

Imprint of SUSY on the GW spectrum

plot from: S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746



20% drop due to extra SUSY DOF

NP DOF lower the "plateau" at high frequencies:

$$\Omega_{\rm GW}^{\rm NP} \sim \Omega_{\rm GW}^{\rm SM} \left(\frac{g_*^{\rm SM}}{g_*^{\rm SM} + \Delta g_*^{\rm NP}} \right)^{1/3}$$

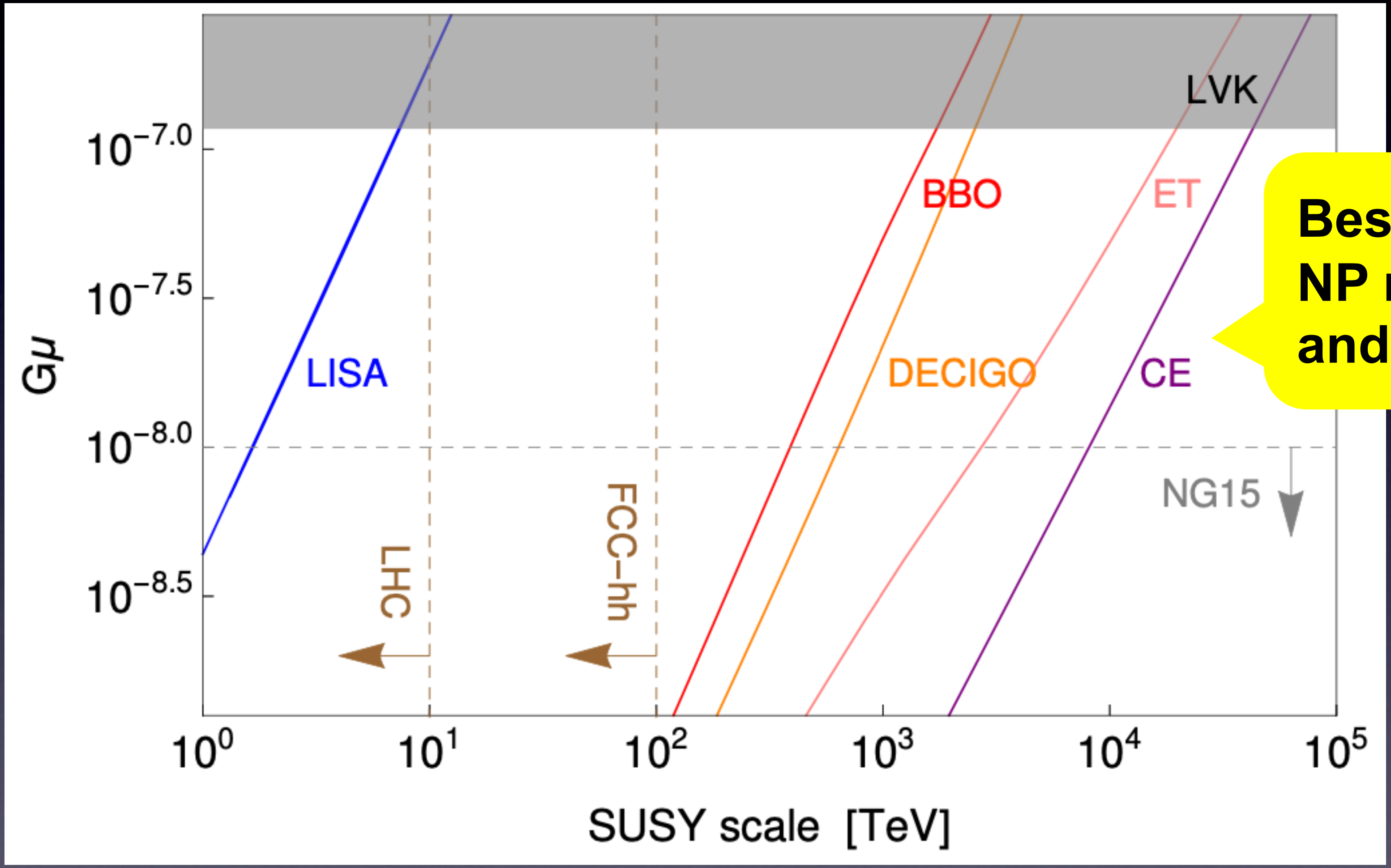
$$\Delta g_*^{\rm SUSY} = 122$$

$$\Omega_{\rm GW}^{\rm SUSY} / \Omega_{\rm GW}^{\rm SM} \approx 0.8$$

Discovery reach for signs of SUSY

S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746

- Signal-to-noise ratio analysis: Sensitivity to the SUSY scale

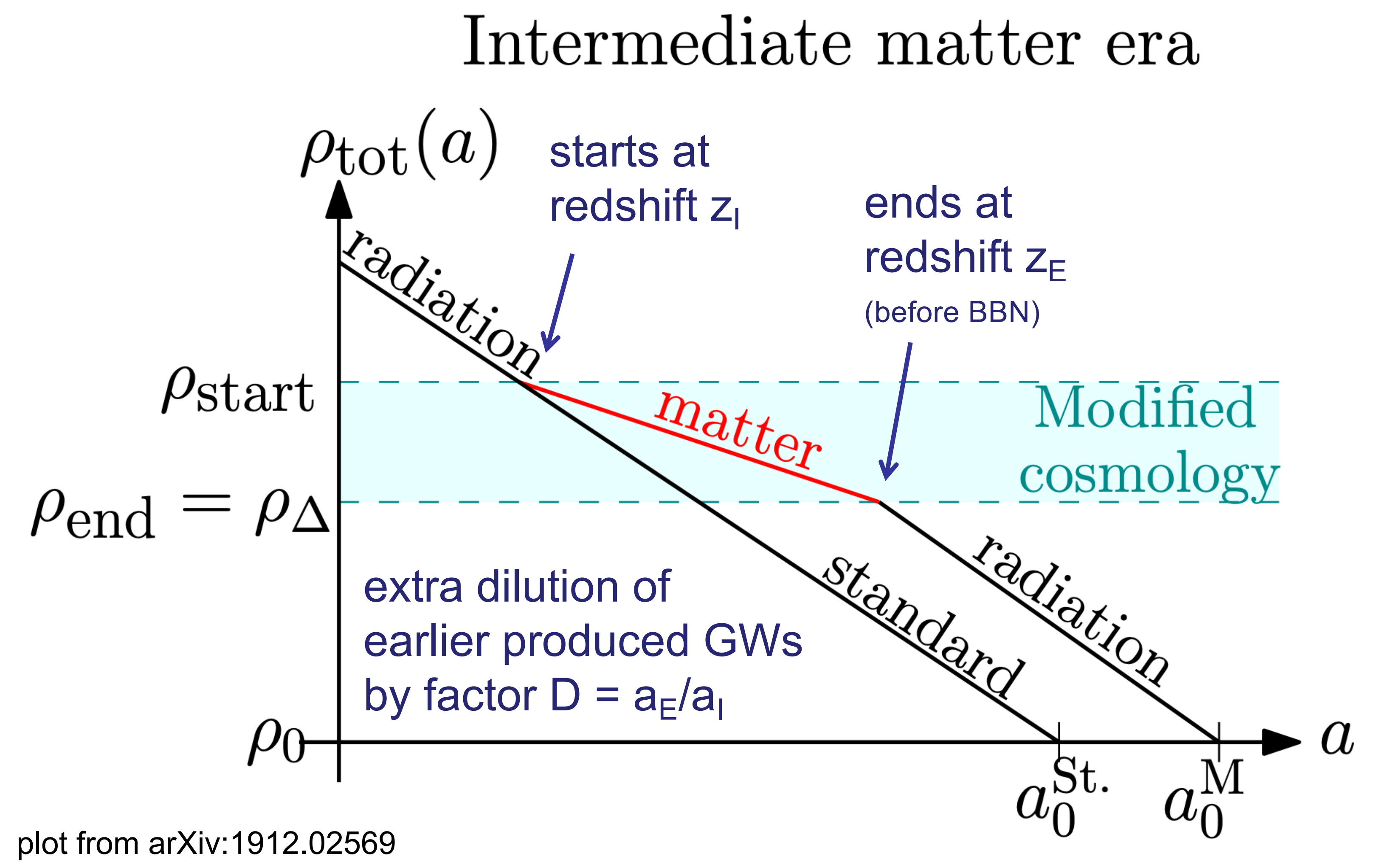


Best reach to high NP masses by ET and CE: O(10^4 TeV)

- From Fisher analysis: uncertainty of 10% for the # of DOF and 5% for the scale of NP (i.e. for m_{SUSY}) possible with ET and CE

Possible non-standard cosmology effect in SUSY models: Late-time entropy production

- Modeled by intermediate phase of matter domination (MD), changing H(z)



starts at redshift z_I

ends at redshift z_E (before BBN)

extra dilution of earlier produced GWs by factor $D = a_E/a_I$

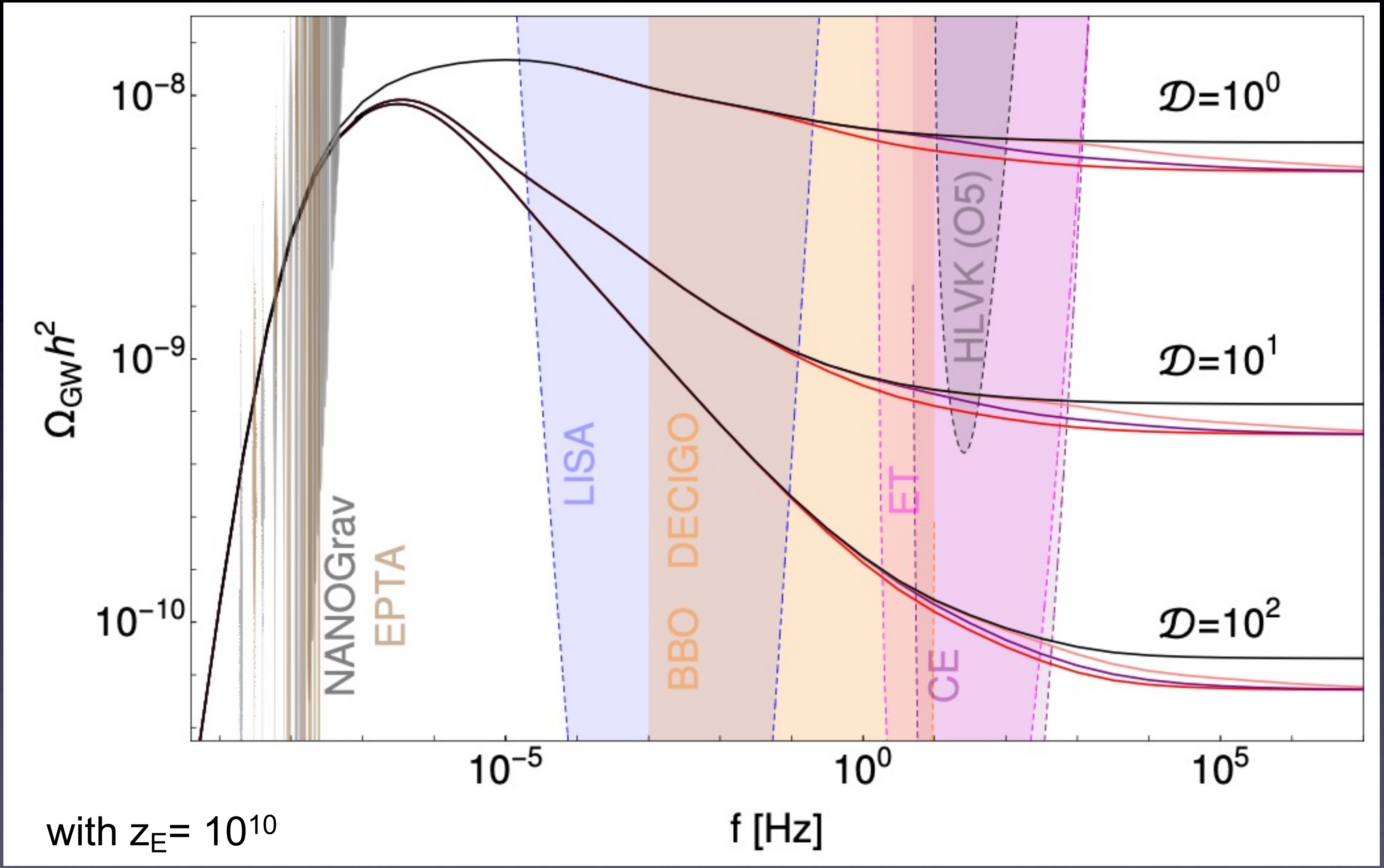
plot from arXiv:1912.02569

Caused e.g. by:
- sgoldstino (from SUSY breaking)
- gravitinos
- string moduli, ...

Affects CS GW spectrum, cf. e.g.: P. Auclair et al. (arXiv:1909.00819), Cui, Lewicki, Morrissey, Wells (arXiv:1711.03104, arXiv:1808.08968), Gouttenoire, Servant, Simakachorn (arXiv:1912.02569, arXiv:1912.03245) Blasi, Brdar, Schmitz (arXiv:2004.02889)

MSCS spectrum with SUSY DOF and extra dilution by factor D

S.A, K. Hinze, S. Saad, J. Steiner, arXiv:2405.03746



with $z_E = 10^{10}$

- LISA could measure $G\mu$ and dilution factor D; SGWB detection at LVK delayed, but still possible to see signs of SUSY at ET and CE

Assuming metastable CS explanation of PTA results will be confirmed:

"Singling out" promising SO(10) GUT scenarios

Which "promising SO(10) model routes" can explain the PTA results?

S.A, K. Hinze, S. Saad, J. Steiner arXiv:2307.04595
S.A, K. Hinze, S. Saad, arXiv:2406.xxxxx (in prep.)

- Superpotential:

$$W = W_{\mathrm{GUT-breaking}} + \underbrace{W_{\mathrm{Inflation}} + W_{\mathrm{Mixed}}}_{W_{\mathrm{Intermedite-breaking}}} + W_{\mathrm{DTS}} + W_{\mathrm{Yukawa}}$$

Our criteria:

Promising models:

- Gauge coupling unification
- Cosmic inflation
- Doublet-Triplet splitting

(without large tuning)

- Fermion mass
- Proton decay bounds

(without large tuning)

- ...

- Lower-dimensional reps.: 10, 16, 45

We use these criteria to "single out" "promising" classes of SO(10) GUT models ...

Which "promising SO(10) model routes" can explain the PTA results?

S.A, K. Hinze, S. Saad, J. Steiner (arXiv:2307.04595)

- SO(10) breaking by two 45-plets (in B-L & I_{3R} direction) + 16 $\overline{16}$:

$$\langle 45_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(a, a, a, 0, 0)\ ,\ \langle 45'_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(0, 0, 0, b, b)$$

(a) $\langle 45_H \rangle > \langle 45'_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

$$\begin{aligned} SO(10) &\xrightarrow[45_H]{M_{\mathrm{GUT}}} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\xrightarrow[45'_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_{II}} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

(b) $\langle 45'_H \rangle > \langle 45_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

$$\begin{aligned} SO(10) &\xrightarrow[45'_H]{M_{\mathrm{GUT}}} SU(4)_C \times SU(2)_L \times U(1)_R \\ &\xrightarrow[45_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_{II}} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

(c) $\langle 45_H \rangle = \langle 45'_H \rangle > \langle 16_H \rangle, \langle \overline{16}_H \rangle$:

$$\begin{aligned} SO(10) &\xrightarrow[45_H + 45'_H]{M_{\mathrm{GUT}}} SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L} \\ &\xrightarrow[16_H + \overline{16}_H]{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \end{aligned}$$

These are the only breaking chains that allow for "promising" (with our criteria) SO(10) models with metastable cosmic strings

See also works by K.S. Babu, S. M. Barr, Z. Berezhiani, R. N. Mohapatra, J. C. Pati, S. Raby, ...

Metastable CS from multi-step SO(10) breaking - with inflation before last breaking step

S.A, K. Hinze, S. Saad, J. Steiner (arXiv:2307.04595)

- Step 1: Adjoints (45-plets) get their vevs (example: case (a))

$\langle 45_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(a, a, a, 0, 0)$ $\langle 45'_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(0, 0, 0, b, b)$

$$SO(10) \xrightarrow{45_H} \overbrace{\underset{\text{LR symm. extension of SM}}{3_c 2_L 2_R 1_{B-L}} \xrightarrow[45'_H]{} 3_c 2_L 1_R 1_{B-L}}^{\text{Monopoles}} \xrightarrow{16_H+\overline{16}_H} 3_c 2_L 1_Y$$

- Step 2: **Inflation**

E.g.: via SUSY hybrid inflation: Linde ('91), Dvali, Shafi, Schaefer ('94)
or via Tribrid inflation: cf. S.A., Bastero-Gil, Baumann, Dutta, S.F. King ('10)
(where the **sneutrino can act as the inflaton**)

Inflation ends by the last step of SO(10) breaking: Dilutes away the monololes, production of CS after inflation

For example :

$W_{\text{Inflation}} \supset \kappa S(\overline{16}_H 16_H - m_{16}^2)$

- Step 3: 16-plets get their vevs

$$SO(10) \xrightarrow{45_H} \overbrace{3_c 2_L 2_R 1_{B-L} \xrightarrow[45'_H]{} \underbrace{3_c 2_L 1_R 1_{B-L} \xrightarrow{16_H+\overline{16}_H} 3_c 2_L 1_Y}_{\text{CS}}}^{\text{Monopoles}}$$

→ Metastable cosmic strings!

What is the preferred $G\mu \leftrightarrow v_s$ in the "singled out" class of SO(10) GUT models?

→ Gauge coupling unification analysis ...

Gauge coupling unification analysis

S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- Taking into account that some particles have intermediate scale masses (due to eff. operators → masses = $O(v^2/\Lambda)$, multi-step breaking → v_{45}, $v_{45'}$, v_{16}).

Approximate analysis with the 3 scales

$$v_{\rm GUT} = \max\{v_{45}, v_{45'}\}$$

$$v_{16} \approx \min\{v_{45}, v_{45'}\}$$

(since $v_s \sim v_m$, i.e. lowest monopole scale close to cosmic string scale)

$$\Lambda \quad \text{(cutoff scale of eff. operators)}$$

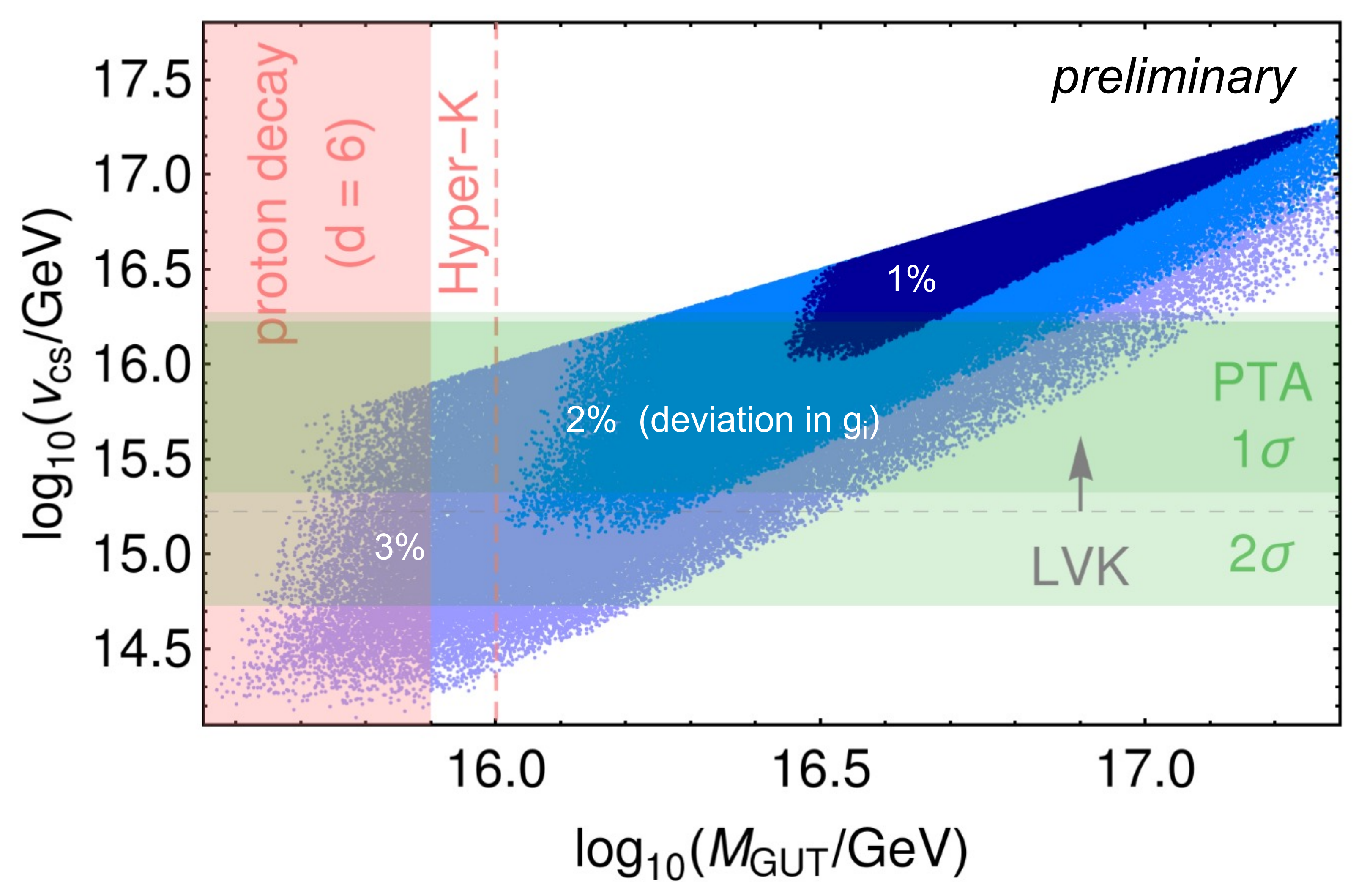
→ varied between 10 x $v_{\rm GUT}$ and $m_{\rm Pl}$

- Furthermore: in the analysis couplings are "set to 1":

Note: Ignoring them can easily introduce variations of ~ O(3%) for the gauge couplings. For example, when color octet mass is shifted by factor "e", then $g_3(m_{SUSY})$ changes by 3%. We therefore allow for uncertainties of up to O(3%) in the gauge couplings.

- *In addition: in the analysis, m_{SUSY} is set to 3 TeV*

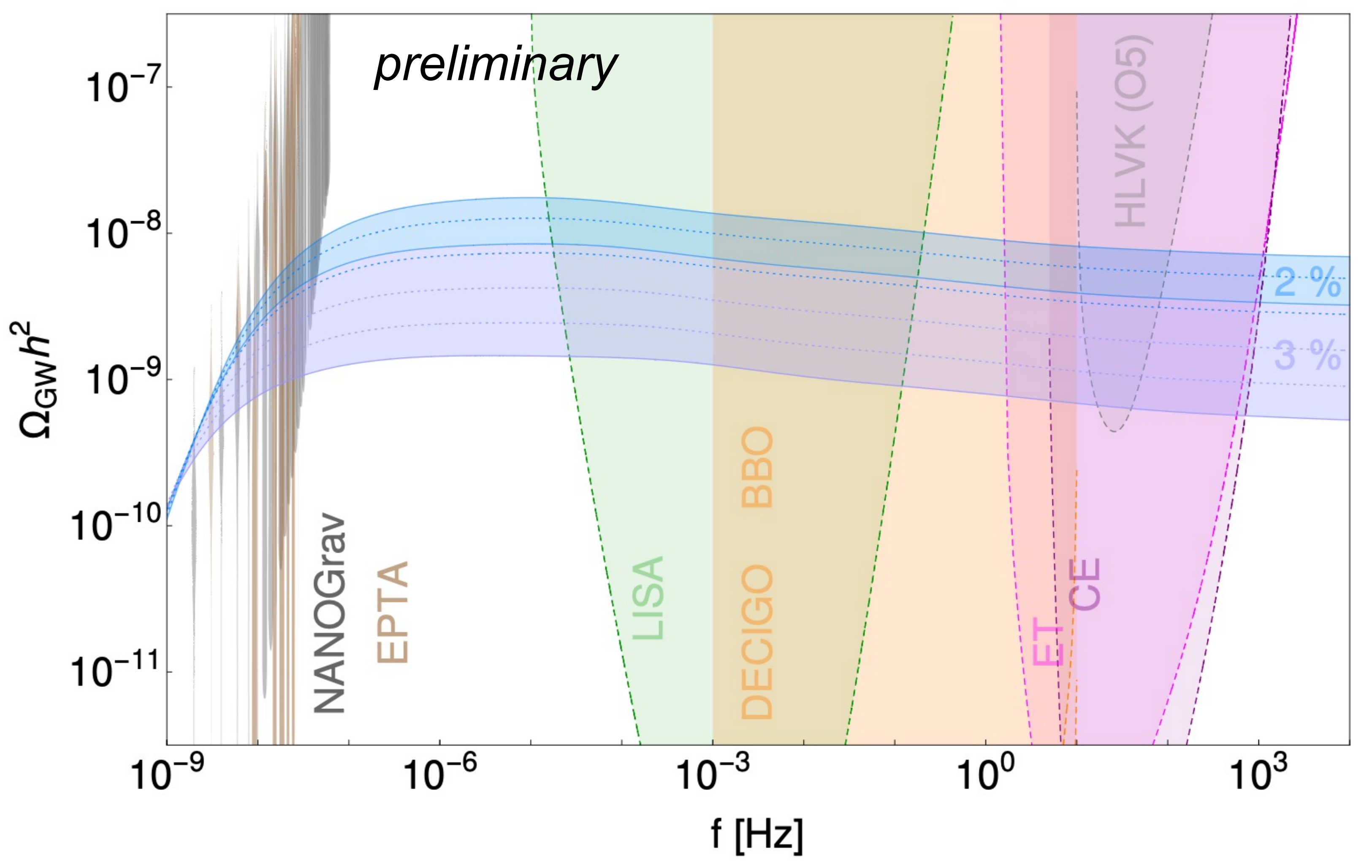
Gauge coupling unification (model case a)



S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- Overlap with PTA preferred region, constrained by LVK (standard cosmology)

GW spectra for MSCS parameters preferred by GCU (model case a)

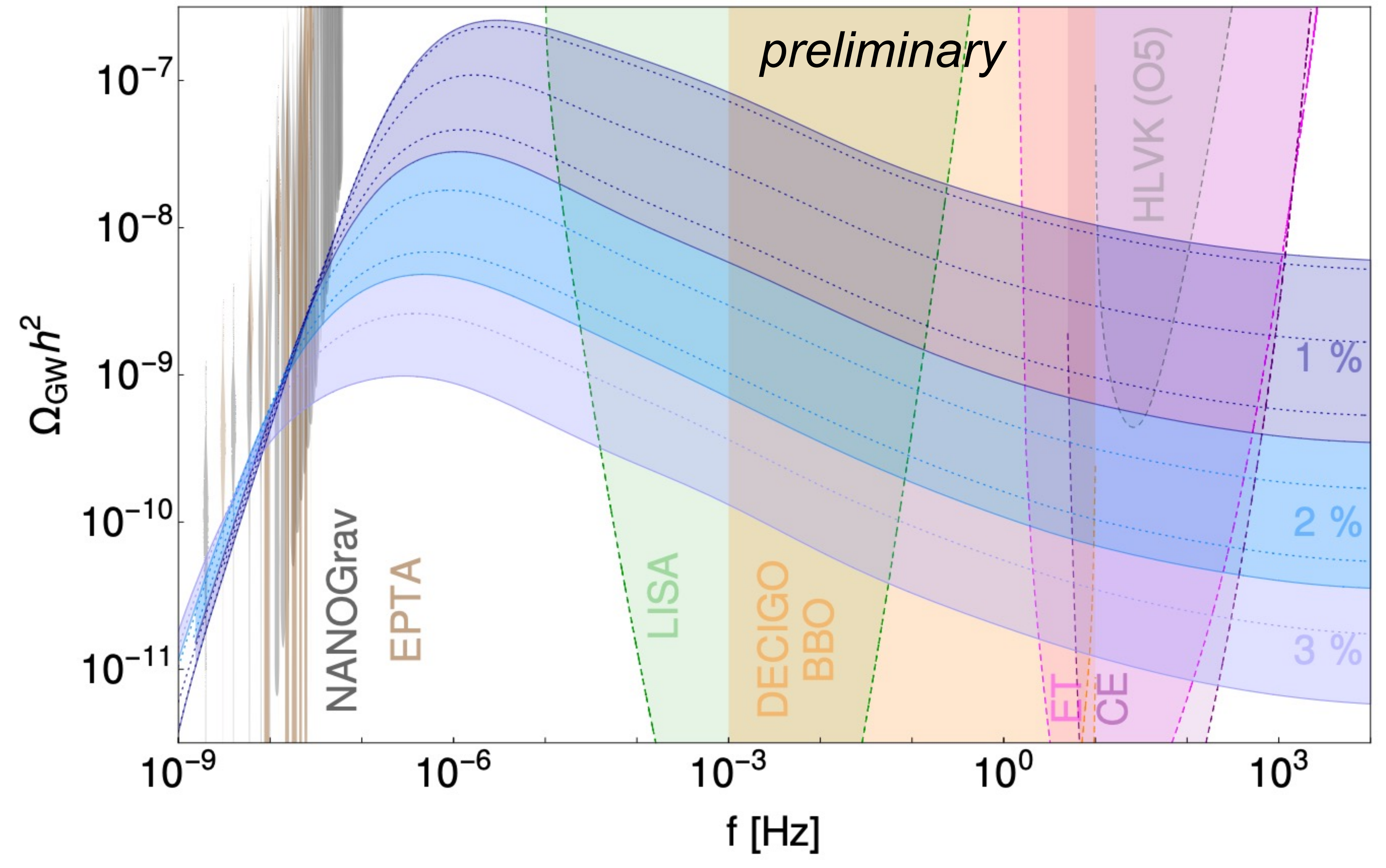


S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- SGWB from MSCS within sensitivity of HLVK (05), excellent prospects for detecting signs of SUSY with ET and CE

GW spectra for MSCS parameters preferred by GCU (case a, with dilution D = 100)

Example:



S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- Dilution from late-time entropy production can (i) delay SGWB discovery at LVK, (ii) avoid present LVK bound and thus optimise PTA 2023 data fit

Summary

When the MSCS explanation of PTA result gets confirmed ...

- Fantastic reach for NP with signifficant extra DOF (such as SUSY)
  - Best reach by Einstein Telescope (ET) and Cosmic Explorer (CE): Can detecxt signs of extra DOF up to O(10^4 TeV), with uncertainty of 10% for the # of DOF and 5% for the scale of NP
- PTA signal can be explained by "promising" SO(10) GUT scenarios
  - Preferred values of cosmic string scale $c_s = v_{16}$ from gauge coupling unification (example: case a) matches well with preferred string tension from PTA results ... which also works nicely with neutrino masses via the seesaw mechanism
- We also discussed: potential of additional non-standard cosmic history from late time entropy production
  - Discovery at LVK might be delayed, but signs of extra patricle DOF from NP (e.g. from SUSY) could nevertheless be discovered

Thanks for your attention!

Extra Slides

Doublet triplet splitting solved

For details see S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)
and references therein

- DT splitting solved by the "missing vev" (DW) mechanism

Only 10 couples to SM fermions

$$W_{\mathrm{DTS}} \supset \gamma_1 10_H 45_H 10'_H + \frac{\gamma_2}{\Lambda} 10'_H 45'^2_H 10'_H$$

$$\langle 45_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(a, a, a, 0, 0)$$

$$10_H \langle 45_H \rangle 10'_H \supset \cancelto{0}{\bar{2}_H 2'_H} + \cancelto{0}{\bar{2}'_H 2_H} + \bar{3}_H 3'_H + \bar{3}'_H 3_H$$

→ all triplets heavy

$$\langle 45'_H \rangle \propto i\tau_2 \otimes \mathrm{diag}(0, 0, 0, b, b)$$

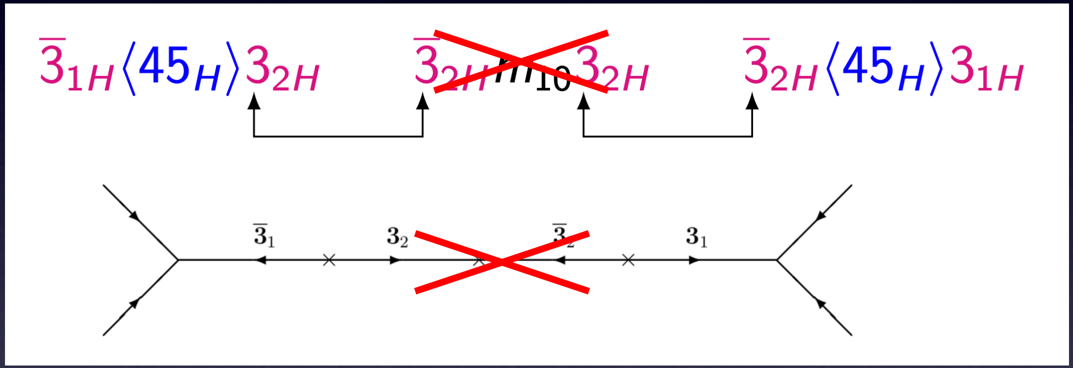
$$10'_H 45'^2_H 10'_H \supset \bar{2}'_H 2'_H + \cancelto{0}{\bar{3}'_H 3'_H}.$$

→ only one doublet (from 10 rep.) remains light

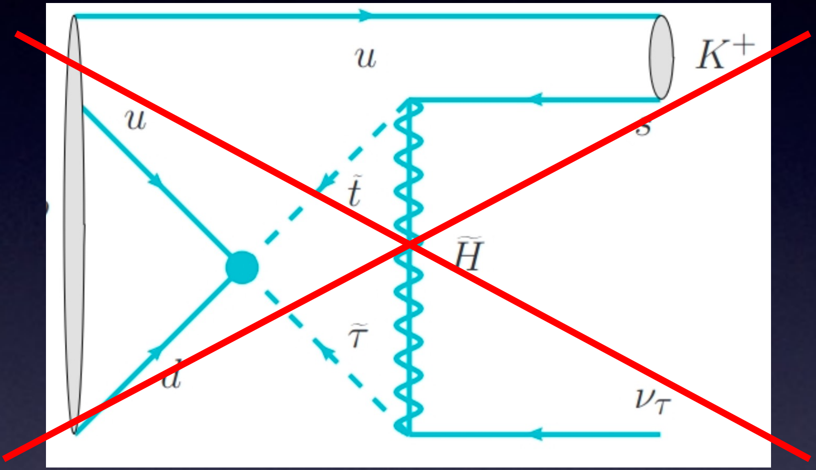
Proton decay not too fast ...

S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- DT splitting solution eliminates d=5 proton decay
  - Absence of direct mass term $m_{10'}$ 10' 10' forbids direct as wee as (suppressed) effective triplet mass:

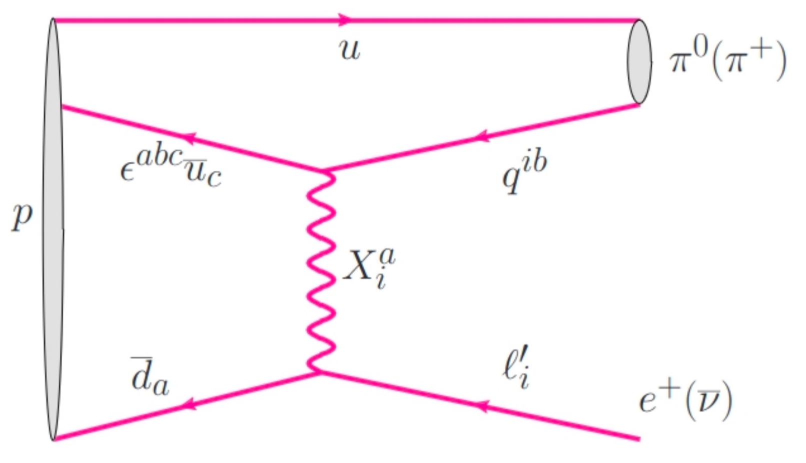


Note: only 10 couples to SM fermions (not 10')



- Proton decay dominated by d=6 contribution

(no conflict with current bounds for the typical GUT scale of 2 x 10^{16} GeV)

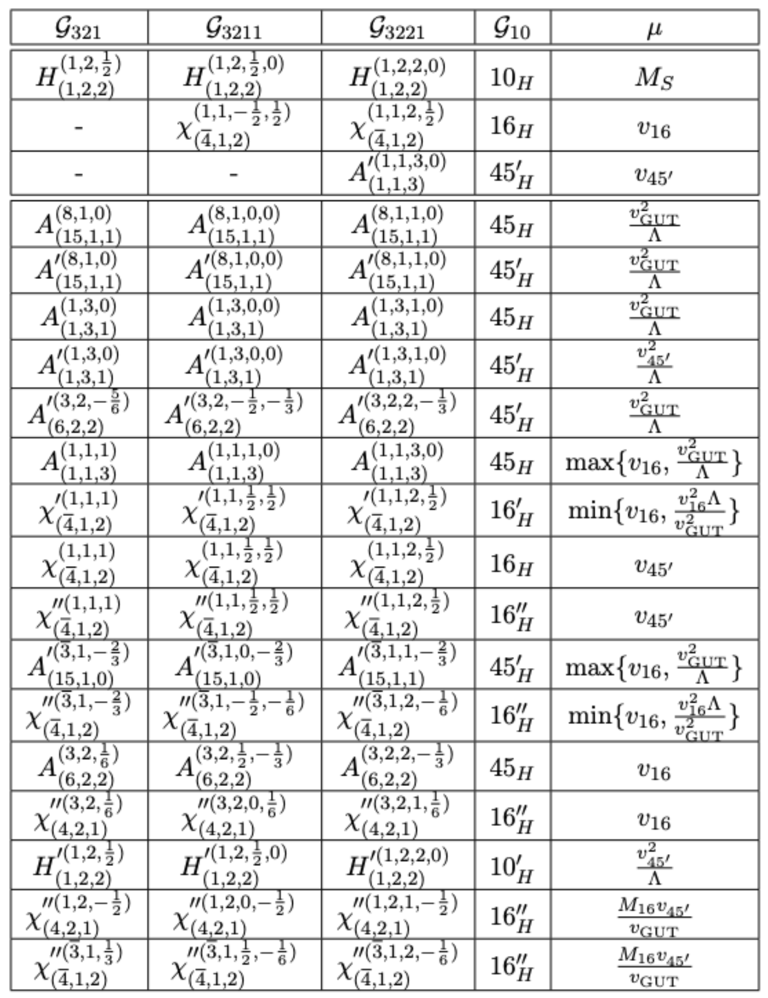


Gauge coupling unification analysis

S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

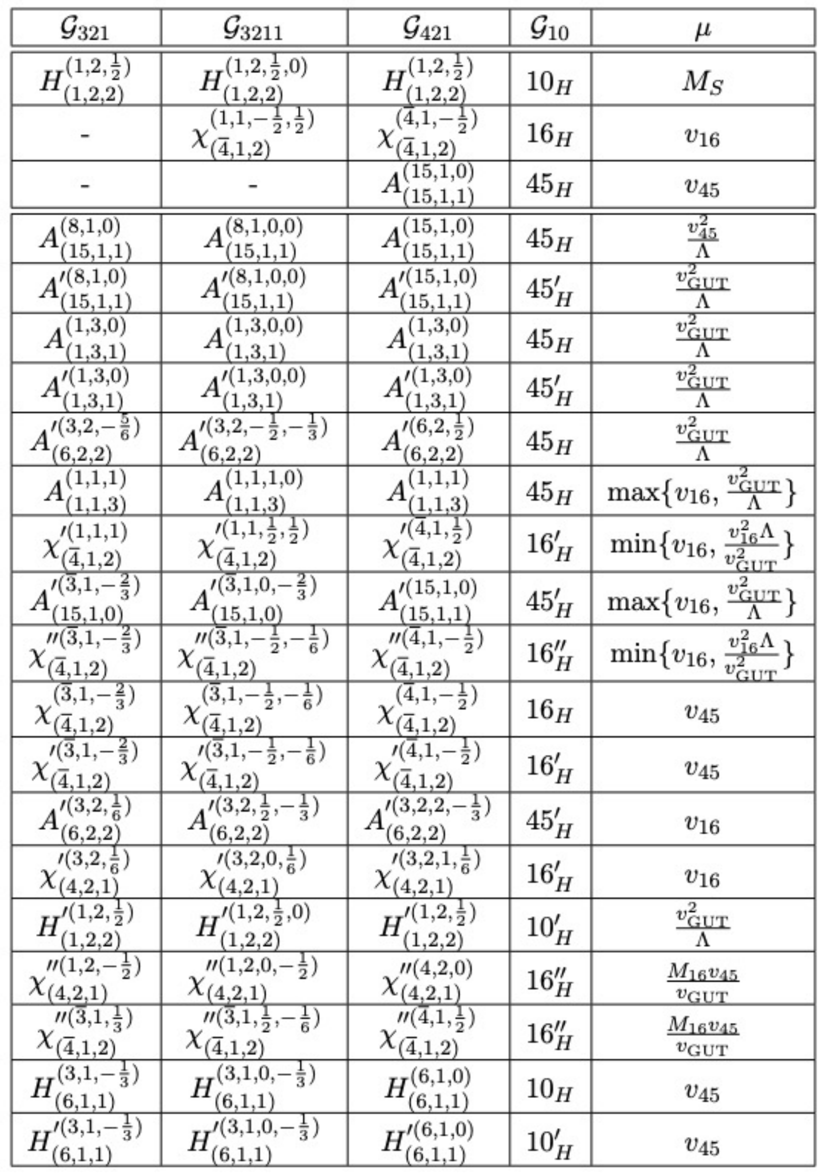
Approximate masses and scales:

Case (a)



$\mathcal{G}_{321}$	$\mathcal{G}_{3211}$	$\mathcal{G}_{3221}$	$\mathcal{G}_{10}$	μ
$H^{(1,2,\frac{1}{2})}_{(1,2,2)}$	$H^{(1,2,\frac{1}{2},0)}_{(1,2,2)}$	$H^{(1,2,2,0)}_{(1,2,2)}$	10_H	M_S
-	$\chi^{(1,1,-\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi^{(1,1,2,\frac{1}{2})}_{(\bar{4},1,2)}$	16_H	v_{16}
-	-	$A'^{(1,1,3,0)}_{(1,1,3)}$	$45'_H$	$v_{45'}$
$A^{(8,1,0)}_{(15,1,1)}$	$A^{(8,1,0,0)}_{(15,1,1)}$	$A^{(8,1,1,0)}_{(15,1,1)}$	45_H	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A'^{(8,1,0)}_{(15,1,1)}$	$A'^{(8,1,0,0)}_{(15,1,1)}$	$A'^{(8,1,1,0)}_{(15,1,1)}$	$45'_H$	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A^{(1,3,0)}_{(1,3,1)}$	$A^{(1,3,0,0)}_{(1,3,1)}$	$A^{(1,3,1,0)}_{(1,3,1)}$	45_H	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A'^{(1,3,0)}_{(1,3,1)}$	$A'^{(1,3,0,0)}_{(1,3,1)}$	$A'^{(1,3,1,0)}_{(1,3,1)}$	$45'_H$	$\frac{v^2_{45'}}{\Lambda}$
$A'^{(3,2,-\frac{5}{6})}_{(6,2,2)}$	$A'^{(3,2,-\frac{1}{2},-\frac{1}{3})}_{(6,2,2)}$	$A'^{(3,2,2,-\frac{1}{3})}_{(6,2,2)}$	$45'_H$	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A^{(1,1,1)}_{(1,1,3)}$	$A^{(1,1,1,0)}_{(1,1,3)}$	$A^{(1,1,3,0)}_{(1,1,3)}$	45_H	$\max\{v_{16}, \frac{v^2_{\rm GUT}}{\Lambda}\}$
$\chi'^{(1,1,1)}_{(\bar{4},1,2)}$	$\chi'^{(1,1,\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi'^{(1,1,2,\frac{1}{2})}_{(\bar{4},1,2)}$	$16'_H$	$\min\{v_{16}, \frac{v^2_{16}\Lambda}{v^2_{\rm GUT}}\}$
$\chi^{(1,1,1)}_{(\bar{4},1,2)}$	$\chi^{(1,1,\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi^{(1,1,2,\frac{1}{2})}_{(\bar{4},1,2)}$	16_H	$v_{45'}$
$\chi''^{(1,1,1)}_{(\bar{4},1,2)}$	$\chi''^{(1,1,\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi''^{(1,1,2,\frac{1}{2})}_{(\bar{4},1,2)}$	$16''_H$	$v_{45'}$
$A'^{(\bar{3},1,-\frac{2}{3})}_{(15,1,0)}$	$A'^{(\bar{3},1,0,-\frac{2}{3})}_{(15,1,0)}$	$A'^{(\bar{3},1,1,-\frac{2}{3})}_{(15,1,1)}$	$45'_H$	$\max\{v_{16}, \frac{v^2_{\rm GUT}}{\Lambda}\}$
$\chi''^{(\bar{3},1,-\frac{2}{3})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,-\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,2,-\frac{1}{6})}_{(\bar{4},1,2)}$	$16''_H$	$\min\{v_{16}, \frac{v^2_{16}\Lambda}{v^2_{\rm GUT}}\}$
$A^{(3,2,\frac{1}{6})}_{(6,2,2)}$	$A^{(3,2,\frac{1}{2},-\frac{1}{3})}_{(6,2,2)}$	$A^{(3,2,2,-\frac{1}{3})}_{(6,2,2)}$	45_H	v_{16}
$\chi''^{(3,2,\frac{1}{6})}_{(4,2,1)}$	$\chi''^{(3,2,0,\frac{1}{6})}_{(4,2,1)}$	$\chi''^{(3,2,1,\frac{1}{6})}_{(4,2,1)}$	$16''_H$	v_{16}
$H'^{(1,2,\frac{1}{2})}_{(1,2,2)}$	$H'^{(1,2,\frac{1}{2},0)}_{(1,2,2)}$	$H'^{(1,2,2,0)}_{(1,2,2)}$	$10'_H$	$\frac{v^2_{45'}}{\Lambda}$
$\chi''^{(1,2,-\frac{1}{2})}_{(4,2,1)}$	$\chi''^{(1,2,0,-\frac{1}{2})}_{(4,2,1)}$	$\chi''^{(1,2,1,-\frac{1}{2})}_{(4,2,1)}$	$16''_H$	$\frac{M_{16}v_{45'}}{v_{\rm GUT}}$
$\chi''^{(\bar{3},1,\frac{1}{3})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,2,-\frac{1}{6})}_{(\bar{4},1,2)}$	$16''_H$	$\frac{M_{16}v_{45'}}{v_{\rm GUT}}$

Case (b)



$\mathcal{G}_{321}$	$\mathcal{G}_{3211}$	$\mathcal{G}_{421}$	$\mathcal{G}_{10}$	μ
$H^{(1,2,\frac{1}{2})}_{(1,2,2)}$	$H^{(1,2,\frac{1}{2},0)}_{(1,2,2)}$	$H^{(1,2,\frac{1}{2})}_{(1,2,2)}$	10_H	M_S
-	$\chi^{(1,1,-\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi^{(\bar{4},1,-\frac{1}{2})}_{(\bar{4},1,2)}$	16_H	v_{16}
-	-	$A^{(15,1,0)}_{(15,1,1)}$	45_H	v_{45}
$A^{(8,1,0)}_{(15,1,1)}$	$A^{(8,1,0,0)}_{(15,1,1)}$	$A^{(15,1,0)}_{(15,1,1)}$	45_H	$\frac{v^2_{45}}{\Lambda}$
$A'^{(8,1,0)}_{(15,1,1)}$	$A'^{(8,1,0,0)}_{(15,1,1)}$	$A'^{(15,1,0)}_{(15,1,1)}$	$45'_H$	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A^{(1,3,0)}_{(1,3,1)}$	$A^{(1,3,0,0)}_{(1,3,1)}$	$A^{(1,3,0)}_{(1,3,1)}$	45_H	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A'^{(1,3,0)}_{(1,3,1)}$	$A'^{(1,3,0,0)}_{(1,3,1)}$	$A'^{(1,3,0)}_{(1,3,1)}$	$45'_H$	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A'^{(3,2,-\frac{5}{6})}_{(6,2,2)}$	$A'^{(3,2,-\frac{1}{2},-\frac{1}{3})}_{(6,2,2)}$	$A'^{(6,2,\frac{1}{2})}_{(6,2,2)}$	45_H	$\frac{v^2_{\rm GUT}}{\Lambda}$
$A^{(1,1,1)}_{(1,1,3)}$	$A^{(1,1,1,0)}_{(1,1,3)}$	$A^{(1,1,1)}_{(1,1,3)}$	45_H	$\max\{v_{16}, \frac{v^2_{\rm GUT}}{\Lambda}\}$
$\chi'^{(1,1,1)}_{(\bar{4},1,2)}$	$\chi'^{(1,1,\frac{1}{2},\frac{1}{2})}_{(\bar{4},1,2)}$	$\chi'^{(\bar{4},1,\frac{1}{2})}_{(\bar{4},1,2)}$	$16'_H$	$\min\{v_{16}, \frac{v^2_{16}\Lambda}{v^2_{\rm GUT}}\}$
$A'^{(\bar{3},1,-\frac{2}{3})}_{(15,1,0)}$	$A'^{(\bar{3},1,0,-\frac{2}{3})}_{(15,1,0)}$	$A'^{(15,1,0)}_{(15,1,1)}$	$45'_H$	$\max\{v_{16}, \frac{v^2_{\rm GUT}}{\Lambda}\}$
$\chi''^{(\bar{3},1,-\frac{2}{3})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,-\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{4},1,-\frac{1}{2})}_{(\bar{4},1,2)}$	$16''_H$	$\min\{v_{16}, \frac{v^2_{16}\Lambda}{v^2_{\rm GUT}}\}$
$\chi^{(\bar{3},1,-\frac{2}{3})}_{(\bar{4},1,2)}$	$\chi^{(\bar{3},1,-\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi^{(\bar{4},1,-\frac{1}{2})}_{(\bar{4},1,2)}$	16_H	v_{45}
$\chi'^{(\bar{3},1,-\frac{2}{3})}_{(\bar{4},1,2)}$	$\chi'^{(\bar{3},1,-\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi'^{(\bar{4},1,-\frac{1}{2})}_{(\bar{4},1,2)}$	$16'_H$	v_{45}
$A'^{(3,2,\frac{1}{6})}_{(6,2,2)}$	$A'^{(3,2,\frac{1}{2},-\frac{1}{3})}_{(6,2,2)}$	$A'^{(3,2,2,-\frac{1}{3})}_{(6,2,2)}$	$45'_H$	v_{16}
$\chi'^{(3,2,\frac{1}{6})}_{(4,2,1)}$	$\chi'^{(3,2,0,\frac{1}{6})}_{(4,2,1)}$	$\chi'^{(3,2,1,\frac{1}{6})}_{(4,2,1)}$	$16'_H$	v_{16}
$H'^{(1,2,\frac{1}{2})}_{(1,2,2)}$	$H'^{(1,2,\frac{1}{2},0)}_{(1,2,2)}$	$H'^{(1,2,\frac{1}{2})}_{(1,2,2)}$	$10'_H$	$\frac{v^2_{\rm GUT}}{\Lambda}$
$\chi''^{(1,2,-\frac{1}{2})}_{(4,2,1)}$	$\chi''^{(1,2,0,-\frac{1}{2})}_{(4,2,1)}$	$\chi''^{(4,2,0)}_{(4,2,1)}$	$16''_H$	$\frac{M_{16}v_{45}}{v_{\rm GUT}}$
$\chi''^{(\bar{3},1,\frac{1}{3})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{3},1,\frac{1}{2},-\frac{1}{6})}_{(\bar{4},1,2)}$	$\chi''^{(\bar{4},1,\frac{1}{2})}_{(\bar{4},1,2)}$	$16''_H$	$\frac{M_{16}v_{45}}{v_{\rm GUT}}$
$H^{(3,1,-\frac{1}{3})}_{(6,1,1)}$	$H^{(3,1,0,-\frac{1}{3})}_{(6,1,1)}$	$H^{(6,1,0)}_{(6,1,1)}$	10_H	v_{45}
$H'^{(3,1,-\frac{1}{3})}_{(6,1,1)}$	$H'^{(3,1,0,-\frac{1}{3})}_{(6,1,1)}$	$H'^{(6,1,0)}_{(6,1,1)}$	$10'_H$	v_{45}

Fermion masses

S.A, K. Hinze, S. Saad (arXiv:2406.xxxxx)

- Charged fermions from tree-level + effective operators:

→ allows to fit all charged fermion masses

$$W_{\text{Yukawa}} = Y_{10} 16_F 16_F 10_H + \frac{Y_a}{\Lambda}(16_F 45_H)_{16}(10_H 16_F)_{\overline{16}} + \frac{Y_b}{\Lambda}(16_F 45'_H)_{16}(10_H 16_F)_{\overline{16}} + \frac{Y_{\nu^c}}{\Lambda}(\overline{16}_H 16_F)_1(\overline{16}_H 16_F)_1 \, .$$

- Light neutrino masses via type I seesaw (with RH neutrino masses from effective operator):

$$M_\nu^D = Y_{10} v_{10}^u + \sqrt{6}\frac{\langle 45_H \rangle}{\Lambda} Y_a v_{10}^u + 2\frac{\langle 45'_H \rangle}{\Lambda} Y_b v_{10}^u$$

$$M_{\nu^c} = \frac{\langle \overline{16}_H \rangle^2}{\Lambda} Y_{\nu^c}$$

$$\mathcal{M}_\nu = -\left(M_\nu^D\right)^T M_{\nu^c}^{-1} M_\nu^D \quad \rightarrow \quad m_\nu \sim \frac{\Lambda v_{\text{ew}}^2}{v_{\text{s}}^2}$$

→ works well with cosmic string scale v_s around 10^{15} – 10^{16} GeV (as preferred by PTA results)