Charge-breaking opportunities for the early Universe

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Based on: Aoki, Biermann, Borschensky, Ivanov, Mühlleitner, Shibuya, JHEP 02 (2024) 232 = arXiv:2308.04141

Yang, Ivanov, arXiv:2401.03264.

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Multi-Higgs models

Non-minimal Higgs sectors: a conservative framework for New Physics model building

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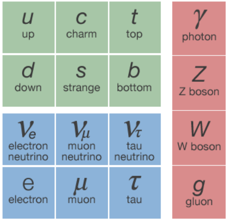

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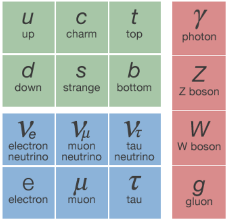
Multi-Higgs-doublet models



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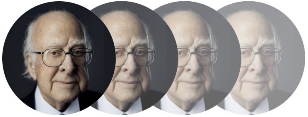

Several Higgs generations

Higgses can come in generations → N-Higgs-doublet models (NHDMs).

- T.D. Lee, 1973: 2HDM as a new source of CP-violation (CPV);
- Weinberg, 1976: 3HDM with natural flavor conservation and CPV;
- Intense activity in 70–80's: trying to reconstruct hierarchical quark and lepton masses and mixing patterns from symmetries and their breaking;
- 1990–2000's: MSSM requires two Higgs doublets;
- Cosmological consequences: scalar dark matter candidates protected by residual symmetries and strong first-order phase transitions → baryogenesis and GW signals.
- In total, $\mathcal{O}(10^4)$ papers over 40 years [Branco et al, 1106.0034; Ivanov, 1702.03776]

Charge-breaking vacuum in the early Universe

2HDM potential

2HDM with a softly broken $\mathbb{Z}_2$ symmetry (review Branco et al, 1106.0034):

$$V_{\rm tree} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + {\rm h.c.}\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+\lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + {\rm h.c.}\right].$$

- Vacuum stability: $\lambda_1, \lambda_2 > 0$, $\sqrt{\lambda_1\lambda_2} + \lambda_3 > 0$, $\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0$ [Deshpande, Ma, 1978]
- Perturbative unitarity: partial wave amplitudes $|a_\ell| < 1 \to$ eigenvalues of the $2 \to 2$ quartic coupling matrix are $< 16\pi$ [Lee, Quigg, Thacker, 1977; Kanemura, Kubota, Takasugi, 1993; Logan, 2207.01064], see also [Goodsell, Staub, 1805.07310].

Natural flavor conservation [Glashow, Weinberg; Paschos, 1977] each right-handed fermion sector (u_R, d_R, ℓ_R) couples only to one Higgs doublet. Let's choose Type I 2HDM: all RH fermions couple only to Φ_2.

Charge breaking vacuum

Minimization gives $\langle\Phi_1\rangle$, $\langle\Phi_2\rangle$, which can be written as

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}u \\ v_2 e^{i\zeta} \end{pmatrix},$$

- Neutral vacuum: $u = 0$, residual symmetry $SU(2)_L \times U(1)_Y \to U(1)_{EM}$; the world we live in.
- Charge-breaking (CB) vacuum: $u \neq 0$: no residual symmetry, $SU(2)_L \times U(1)_Y$ is broken completely, massive photon, no conserved electric charge.
- The usual procedure: disregard the CB vacuum, assume the neutral vacuum, choose v_1, v_2, ξ as input, compute m_{ij}^2, proceed with phenomenology.
- In general 2HDM, at tree level, the necessary and sufficient conditions for the CB minimum were established in [Ivanov, 2007].

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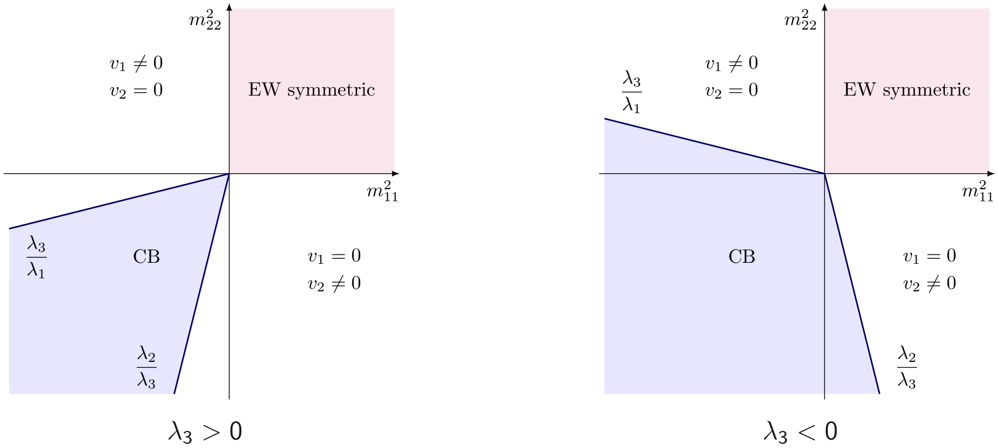
$\mathbb{Z}_2$ symmetric 2HDM: the phase diagram



Conditions for the CB minimum: $\sqrt{\lambda_1\lambda_2} > \lambda_3$, $\lambda_4 > |\lambda_5|$, the point (m_{11}^2, m_{22}^2) inside the CB wedge.

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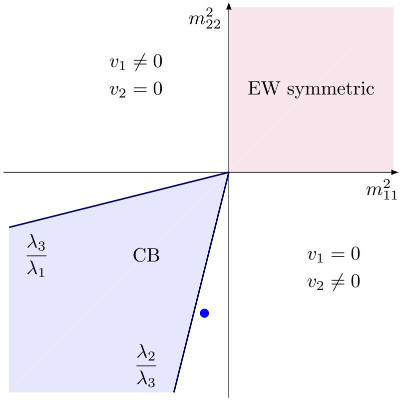

$\lambda_3 > 0$

The neutral/CB boundary is:

$$\frac{m_{22}^2}{m_{11}^2} = \frac{\lambda_2}{\lambda_3}\,.$$

The charged Higgs mass depends on the proximity to this boundary:

$$\frac{m_{H^\pm}^2}{v^2} = \frac{\lambda_3}{2}\left(1 - \frac{m_{11}^2}{m_{22}^2}\frac{\lambda_2}{\lambda_3}\right)\,.$$

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Charge breaking vacuum

In the hot early Universe, the Higgs potential and its minima evolve with temperature T $\to$ phase transitions are expected.

Electroweak phase transition (EWPT) ($v = 0 \Rightarrow v \neq 0$) is the most famous example. But other phase transitions could have taken place.

What if the charge-breaking vacuum existed in the hot early Universe in a range of T?

Ginzburg, Ivanov, Kanishev, 0911.2383: a simple tree-level study revealed benchmark 2HDMs with an intermediate CB vacuum at finite T.

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In Aoki et al, 2308.04141, we returned to this possibility with the finite-T loop-corrected effective potential and the code BSMPT v2 [Basler, Mühlleitner, Müller, 2007.01725].

- Is it possible at all to have a CB vacuum at intermediate T?
- Are such scenarios compatible with the LHC Higgs results?
- If they are, what are the characteristic features of such scenarios?

The formalism

Finite T one-loop corrected effective potential: $V = V_{\text{tree}} + V_{CW} + V_{CT} + V_T$, where

- V_{CW}: T-independent one-loop Coleman-Weinberg potential,
- V_{CT}: T-independent counterterms (keep v and m_h),
- V_T: one-loop thermal corrections at finite T:

$$V_T = \sum_k n_k \frac{T^4}{2\pi^2} J_{\pm}^{(k)}\left(\frac{m_k^2}{T^2}\right),$$

  with summation over all fields, n_k is the number of d.o.f., J's are the thermal integrals, m_k depend on the values of scalar fields; full expressions in [Basler et al, 1612.04086, 1803.02846].

- Thermal masses are consistently implemented at one loop using the Arnold-Espinosa resummation procedure [Arnold, Espinosa, hep-ph/9212235; Quiros, hep-ph/9901312].

Recently extended to the general 2HDM using the bilinear formalism [Cao, Cheng, Xu, 2305.12764]

Qualitative analysis

To gain qualitative insights, let's consider a toy model:

- stay with the tree-level potential,
- assume that the main thermal effect is in the quadratic coefficients:

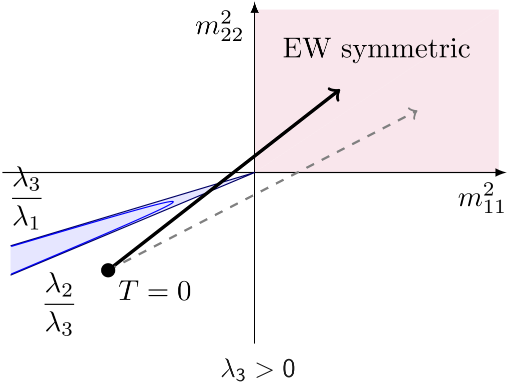
$$m_{11}^2(T) = m_{11}^2 + c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2 + c_2 T^2\,, \quad m_{12}^2(T) = m_{12}^2\,,$$

  where for Type I 2HDM we have

$$\begin{aligned} c_1 &= \frac{1}{12}(3\lambda_1 + 2\lambda_3 + \lambda_4) + \frac{1}{16}(3g^2 + g'^2)\,,\\ c_2 &= \frac{1}{12}(3\lambda_2 + 2\lambda_3 + \lambda_4) + \frac{1}{16}(3g^2 + g'^2) + \frac{1}{12}(y_\tau^2 + 3y_b^2 + 3y_t^2)\,. \end{aligned}$$

Then one can describe thermal evolution as a straight trajectory on the phase diagram.

Qualitative analysis



The ray passes through the CB wedge if

$$\frac{c_2}{c_1} > \frac{|m_{22}^2|}{|m_{11}^2|} > \frac{\lambda_2}{\lambda_3}.$$

- Not easy to satisfy!
- Placing $T = 0$ point close to the wedge will lead to a dangerously light charged Higgs!
- The plot is for

$$\lambda_1 = 2,\ \lambda_2 = 0.25,\ \lambda_3 = 0.6,\ \lambda_4 = 2.8,$$

  which leads to $m_{H^\pm} = 82$ GeV.
- Adding m_{12}^2 plays against the CB phase.

Scan of the 2HDM parameter space

The procedure adopted in Aoki et al, 2308.04141:

- Scan over parameter space of the tree-level potential to generate seed points:
  - At $T = 0$: neutral vacuum, $v = 246.22$ GeV, $m_h = 125.09$ GeV
  - At $T \neq 0$: intermediate CB phase.
- For each seed point, analyze the full finite-T one-loop corrected effective potential using BSMPT v2.
- Select points for which the intermediate CB phase survives for the effective potential.
- Use ScannerS [Coimbra et al, 1301.2599] to apply scalar sector constraints (unitarity, STU, flavor physics, HiggsSignals/HiggsBounds).
- Unfortunately, all such seed points are excluded by the LHC data, mainly by $\mu_{\gamma\gamma}$, due to the presence of a light $H^{\pm}$.
- So, one more tweak: we explore the parameter space patches in the vicinity of seed points: the CB phase must be present in the full effective potential, but no need to require it in V_{tree}.

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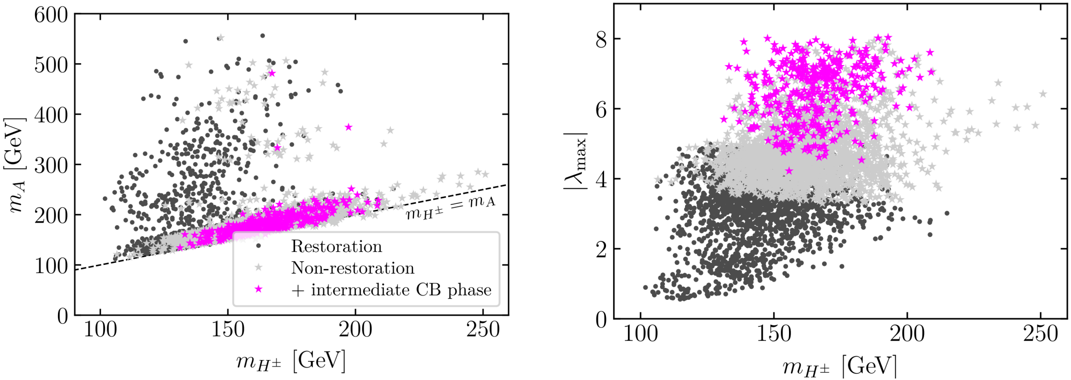
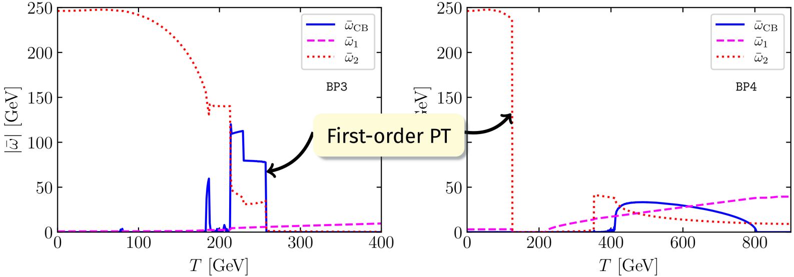


Intermediate CB vacuum is possible in the 2HDM — but only at the expense of a large λ_1 and EW symmetry non-restoration at high T! Typical predictions: large $\tan\beta \sim 10 - 100$ and rather small $m_{H^+} \sim 150 - 200$ GeV.

Benchmark models



	m_H (GeV)	m_A (GeV)	$m_{H^\pm}$ (GeV)	$\tan\beta$	$\cos(\beta-\alpha)$	m_{12}^2 (GeV^2)
BP3	342.52	230.02	183.72	286.00	0.009	410.17
BP4	558.56	194.52	168.43	80.84	0.026	3857.90

Christoph Borschensky – Intermediate CB phases in the 2HDM – 16/09/23 10

Karlsruhe Institute of Technology

[Slide borrowed from the talk by Christoph Borschensky at Scalars 2023]

BSMPT v3

Recent update: BSMPT v3, Basler et al, 2404.19037:

- tracks T-dependent coexisting minima and computes the bounce solution;
- $\Rightarrow$ critical, nucleation, and percolation temperatures; checks EW restoration at high T;
- determines the properties of the GW signal.

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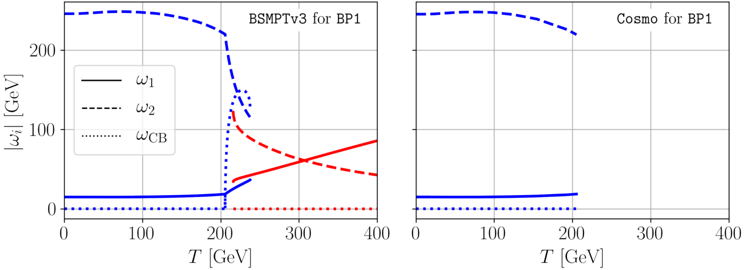
CB phase transitions confirmed with BSMPT v3.



BSMPT v3 vs CosmoTransitions

Benchmark point: $M_H = 563$ GeV, $M_A = 169$ GeV, $M_{H^\pm} = 164.5$ GeV $\tan\beta = 16.5$, $\cos(\beta-\alpha) = 0.128$ $m_{12}^2 = 18933\,\text{GeV}^2$

Charge-breaking bubbles walls in multi-Higgs-doublet models

Symmetries in 3HDM

A powerful feature of 3HDM: a lot of new symmetry options available!

Full classification of symmetries in the 3HDM scalar sector:

- abelian groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1)\,.$$

- discrete non-abelian groups: [Ivanov, Vdovin, 1210.6553; Darvishi, Pilaftsis, 1912.00887]:

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- The classification is exhaustive: imposing any other discrete group in the 3HDM scalar sector will produce an accidental continuous symmetry. Accidental symmetries beyond $SU(3)$ were classified in [Darvishi, Pilaftsis, 1912.00887].

$\Sigma(36)$ 3HDM

Large finite groups come up with many minima and saddle points

$\Rightarrow$ consequences for phase transitions!

The largest discrete group in 3HDM scalar sector is

$$\Sigma(36) \simeq (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4$$

generated by:

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad d = \frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix},$$

where $\omega = \exp(2\pi i/3)$. Orders of generators:

$$a^3 = \mathbf{1}, \quad b^3 = \mathbf{1}, \quad d^4 = \mathbf{1}.$$

$\Sigma(36)$ 3HDM

The scalar potential

$$
\begin{aligned}
V \quad = \quad & -m^2 \left[\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right] + \lambda_1 \left[\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right]^2 \\
& -\lambda_2 \left[|\phi_1^\dagger\phi_2|^2 + |\phi_2^\dagger\phi_3|^2 + |\phi_3^\dagger\phi_1|^2 - (\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) - (\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\
& +\lambda_3 \left(|\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_3|^2 + |\phi_2^\dagger\phi_3 - \phi_3^\dagger\phi_1|^2 + |\phi_3^\dagger\phi_1 - \phi_1^\dagger\phi_2|^2\right) ,
\end{aligned}
$$

where terms in blue are $U(3)$-invariant and λ_3 term selects out $\Sigma(36)$ subgroup.

- The model is extremely constrained $\rightarrow$ numerous relations among scalar masses and couplings.
- Many features remain even if $\Sigma(36)$ is softly broken [Varzielas, Ivanov, Levy, 2107.08227].

$\Sigma(36)$ 3HDM

[Ivanov, Nishi, 1410.6139]: up to cyclic permutations, the global minimum can only be at

$$A: \ (\omega,\, 1,\, 1)\,, \quad A': \ (\omega^2,\, 1,\, 1)\,, \quad B: \ (1,\, 0,\, 0)\,,$$
$$C: \ (1,\, 1,\, 1)\,, \quad (1,\, \omega,\, \omega^2)\,, \quad (1,\, \omega^2,\, \omega)\,.$$

Notation: for example, $(1,1,1)$ denotes the case $v_1 = v_2 = v_3$, that is

$$(\langle\phi_1^0\rangle,\, \langle\phi_2^0\rangle,\, \langle\phi_3^0\rangle) = \frac{v}{\sqrt{6}}\,(1,\, 1,\, 1)\,.$$

- In each case, there are 6 degenerate global minima: $A + A'$ or $B + C$.
- But if we study phase transitions, we want to know:
  - Can we have local minima? Can we have CB minima?
  - Can we have CB saddle points which would separate neutral minima?
  - In 2HDM, CB domain walls were recently studied in [Sassi, Moortgat-Pick, 2309.12398] and [Battye et al, 2006.13273] → see talk by Mohamed Sassi on Thursday, Parallel Session PII.5.

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- Large symmetry group $\to$ many competing minima and many competing saddle points.
- We can expect intriguing multi-step phase transitions in the early Universe, perhaps with exotic features which are not available even in the 2HDM.
- In Yang, Ivanov, arXiv:2401.03264, we undertook an exploratory study of the phase diagram and possible phase transitions on the 3HDM with exact or softly broken $\Sigma(36)$.
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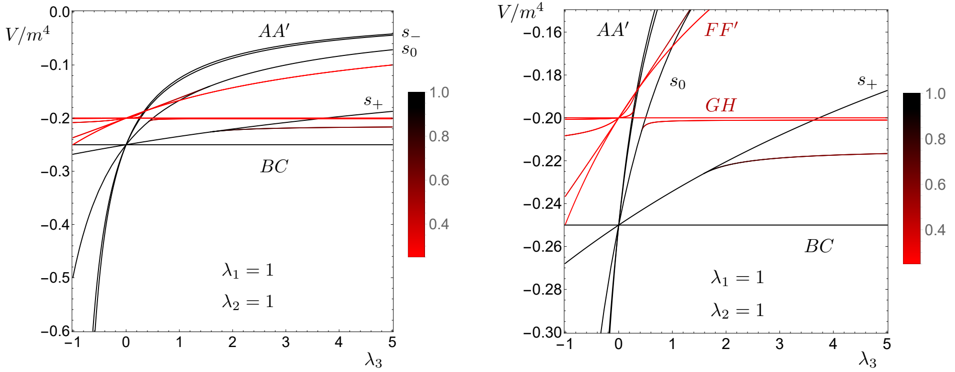
Exact Σ(36)

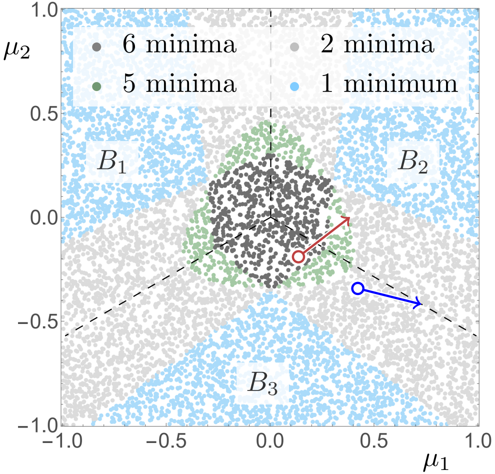


We find an extremely rich picture, with 69 or 78 extrema in total.

Color encodes neutral (black) and charge-breaking extrema (shades of red).

Note: for $\lambda_3 > 2$, the deepest saddle point is charge-breaking.

Softly broken $\Sigma(36)$



Adding $\mathbb{Z}_3$-preserving soft breaking terms: $m_{ii}^2(\phi_i^\dagger\phi_i)$ with $m_{11}^2 + m_{22}^2 + m_{33}^2 = 0$.

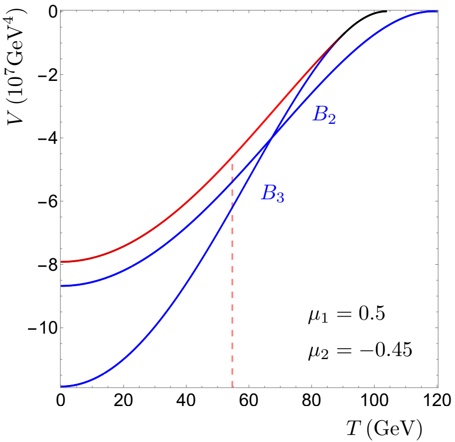
Parametrizing them via

$$\mu_1 = \frac{1}{\sqrt{2}}\frac{m_{11}^2 - m_{22}^2}{m^2}\,, \quad \mu_2 = \frac{\sqrt{6}}{2}\frac{m_{33}^2}{m^2}\,.$$

Coexistence of local and global minima on the plane of soft breaking parameters (μ_1, μ_2).

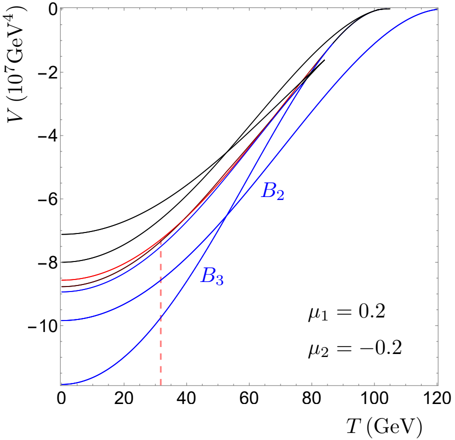
Also shown: T evolution in benchmark model 1 and benchmark model 2.

Softly broken Σ(36) 3HDM: benchmark 1



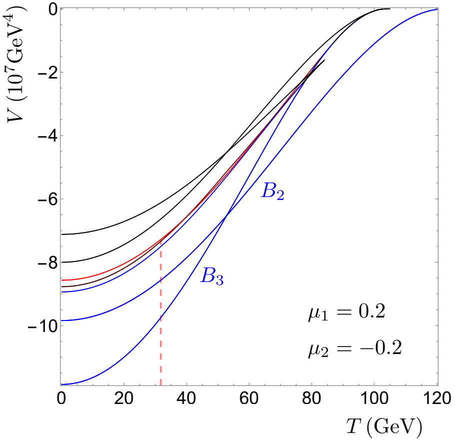
- Used the same simple tree-level thermal evolution with $m_{ii}^2(T) = m_{ii}^2 + c_i T^2$.
- A clear example of a deepest CB saddle point.
- The red dashed line indicates an approximate nucleation temperature (criterion: equal depth differences).
- These features should survive in an accurate numerical study.

Softly broken $\Sigma(36)$ 3HDM: benchmark 2



- Here, we have several saddle points, either neutral or charge-breaking, which closely follow each other.
- Which bounce trajectory corresponds to the most probable bubble nucleation? Impossible to answer with this simplistic analysis!
- If several saddle points compete, it may happen than bubbles of the same true and the same false vacua but completely different bubble wall profiles emerge in the Universe. How do they merge? What GW signatures are expected?
- A dedicated numerical study is required!

Softly broken $\Sigma(36)$ 3HDM: benchmark 2



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Conclusions

- Multi-Higgs models can accommodate rich phase transition dynamics around the EW scale!
- Intermediate charge-breaking phases at finite T or charge-breaking bubble walls between neutral vacua are possible within 2HDM and become more intriguing in the 3HDM.
- Within the 3HDM, competing minima and saddle points are ubiquitous and may lead to highly non-trivial bubble nucleation and coalescence dynamics.
- What happens to fermions during evolution through a CB phase or upon the passage of a CB bubble wall? Any consequences for baryogenesis?