

Minimal U(1) two-Higgs-doublet models for quark and lepton flavour

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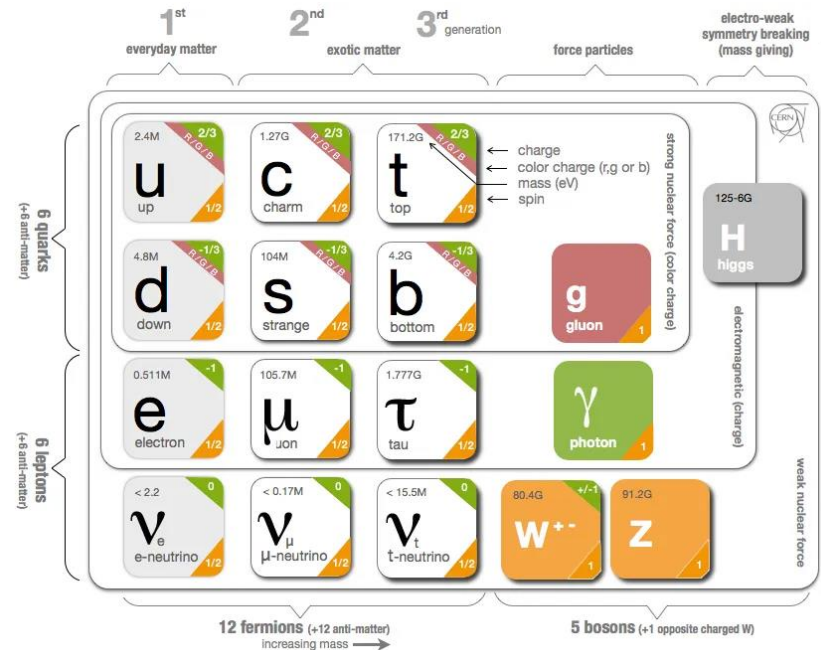
In collaboration with: H. B. Câmara, F. R. Joaquim and R. G. Felipe

arXiv: **2406.03331** [hep-ph]

Introduction

The **Standard Model** of Particle Physics:

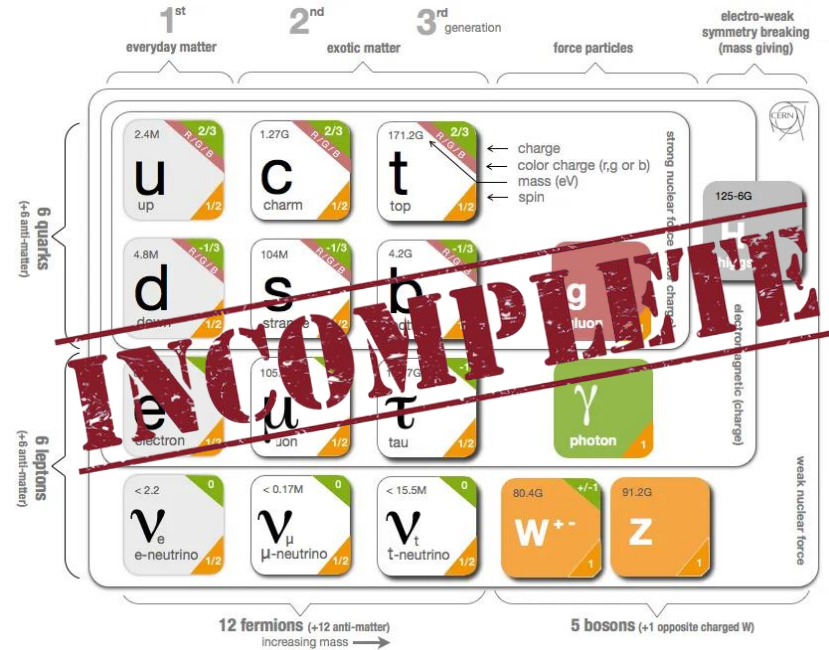
- ✓ Quark mixing is encoded in the CKM matrix;
- ✓ This flavour structure is the only known source of CP violation;
- ✓ The CKM parameters have been determined with extreme precision.



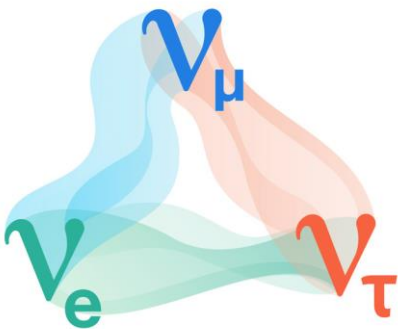
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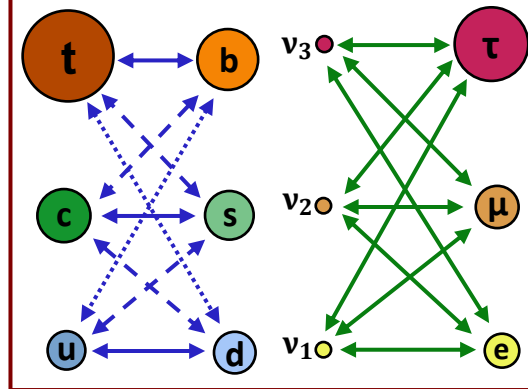
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Neutrino Oscillations



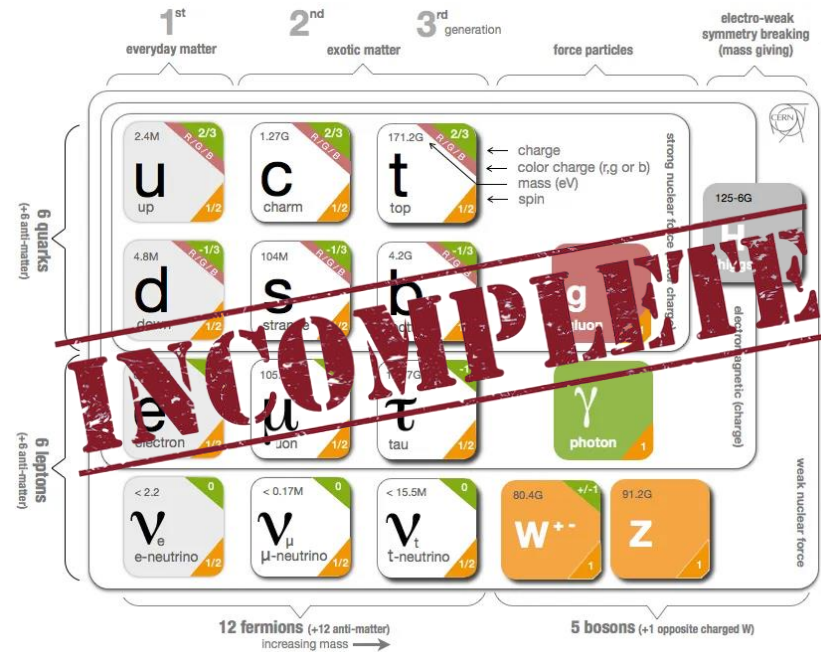
Flavour Puzzle



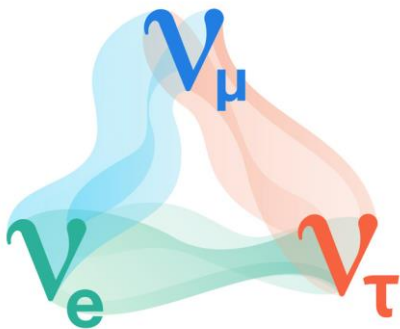
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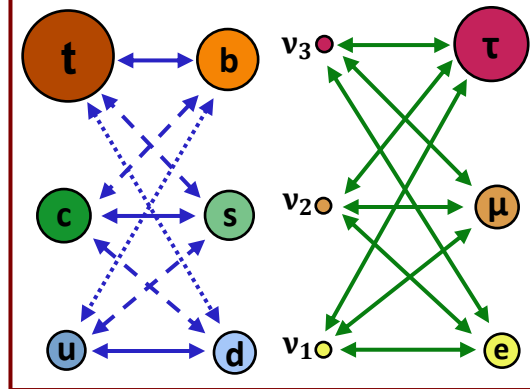
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Neutrino Oscillations



Flavour Puzzle



The SM must be extended!

Neutrino masses and mixing

EFFECTIVE THEORY with SM fields

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \dots, \quad \delta\mathcal{L}^{D=d} \equiv \sum_k \frac{\mathcal{O}_k^{(d)}}{\Lambda^{d-4}}$$

Neutrino masses and mixing

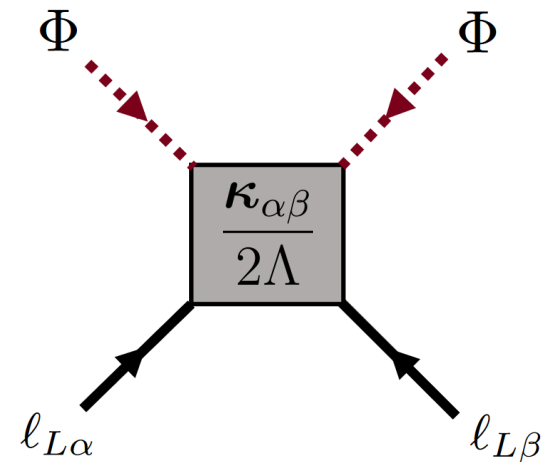
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The lowest $d > 4$ operator is **unique (Weinberg Operator)**

(Weinberg, 1979)

$$\delta\mathcal{L}^{d=5} = \frac{1}{2\Lambda} \kappa_{\alpha\beta} \left(\overline{\ell_{\alpha L}^C} \tilde{\Phi}^* \right) \left(\tilde{\Phi}^\dagger \ell_{\beta L} \right) + \text{H.c.}$$



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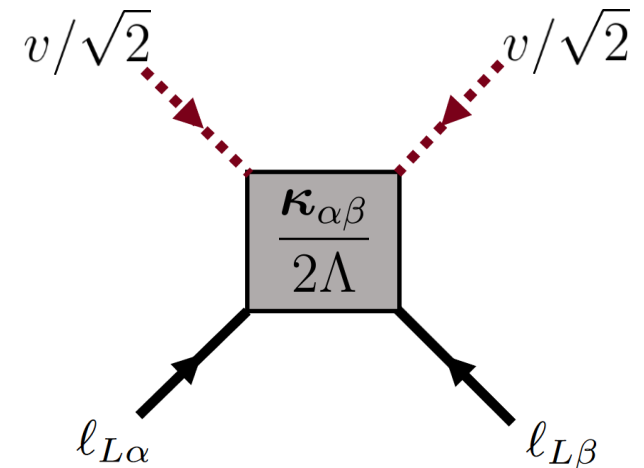
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EWSB



$$\mathcal{L}_m^{\text{Majorana}} = -\frac{1}{2} \mathbf{M}_{\nu\alpha\beta} \overline{\nu_{\alpha L}^C} \nu_{\beta L} + \text{H.c.}$$



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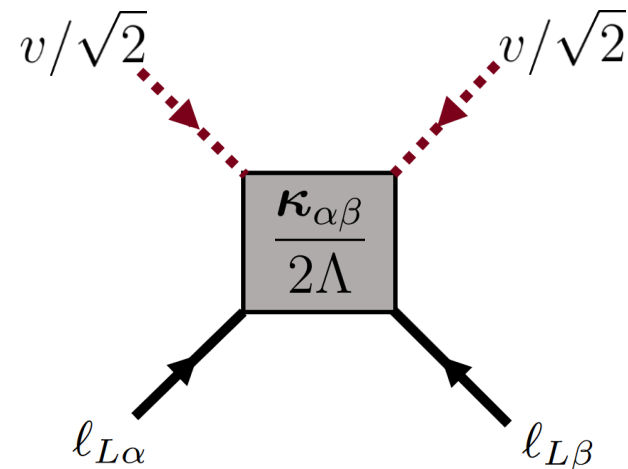
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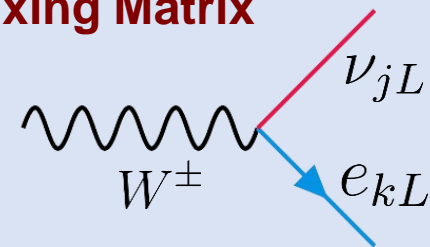
Majorana Mass Eigenstates

$$\nu_{\alpha L} \rightarrow (\mathbf{U}_L^\nu)_{\alpha j} \nu_{jL}$$

$$\mathbf{U}_L^{\nu T} \mathbf{M}_\nu \mathbf{U}_L^\nu = \text{diag}(m_1, m_2, m_3)$$

Lepton Mixing Matrix

$$\mathbf{U}_\ell = \mathbf{U}_L^{e\dagger} \mathbf{U}_L^\nu$$



Softly-broken U(1)-symmetric 2HDM

The **SM** does not allow for the implementation of **Abelian flavour symmetries**



(Branco, et al., 2012)

2HDM

$$\Phi_{1,2} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\phi_{1,2}^+ \\ v_{1,2} + \rho_{1,2} + i\eta_{1,2} \end{pmatrix}$$

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$$\begin{aligned} V = & \mu_{11}^2 \left(\Phi_1^\dagger \Phi_1 \right) + \mu_{22}^2 \left(\Phi_2^\dagger \Phi_2 \right) + \mu_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\ & + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 \\ & + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \end{aligned}$$

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Mass Eigenstates

$$\begin{matrix} m_h & m_I \\ m_H & m_{H^\pm} \end{matrix}$$

Alignment Limit

$$\beta - \alpha = \pi/2$$

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Mass Eigenstates

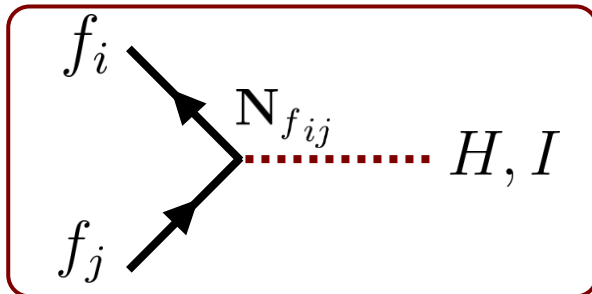
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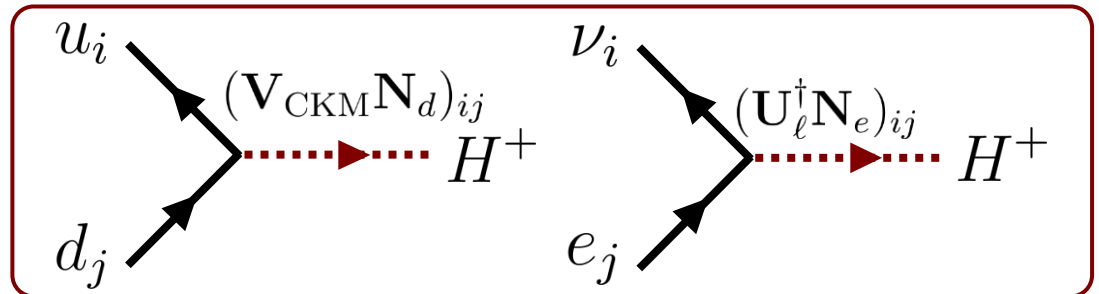
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Expanding the **Yukawa Lagrangian** in the **mass eigenstates**:

FCNC



FCCC

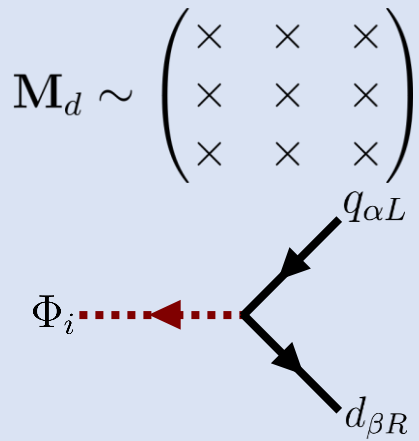


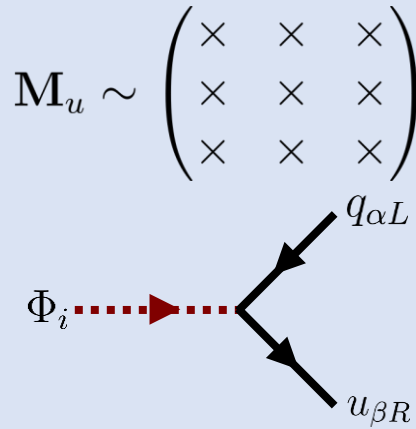
Abelian flavour symmetries

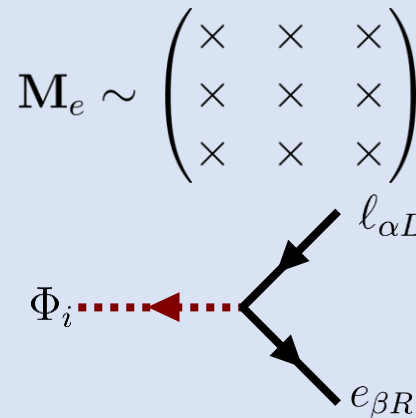
GOAL

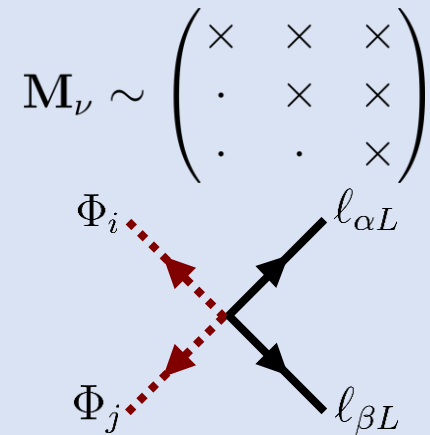
Reduce the number of free parameters in the mass matrices and make the theory more predictive

↓ Introduce flavour charges

$$\mathbf{M}_d \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$


$$\mathbf{M}_u \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$


$$\mathbf{M}_e \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$


$$\mathbf{M}_\nu \sim \begin{pmatrix} \times & \times & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}$$


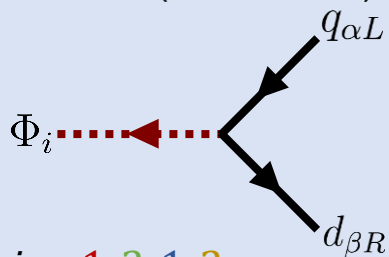
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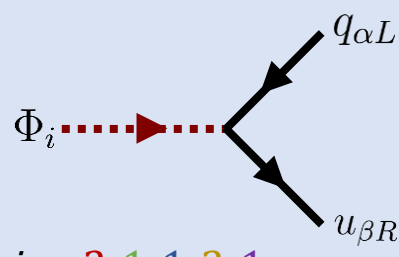
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$$M_d \sim \begin{pmatrix} 0 & 0 & \otimes \\ 0 & \otimes & 0 \\ \otimes & 0 & \otimes \end{pmatrix}$$



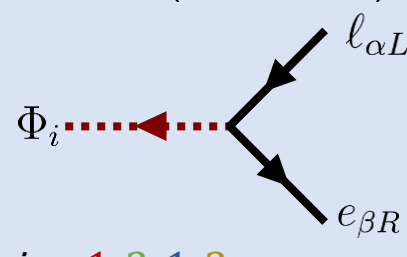
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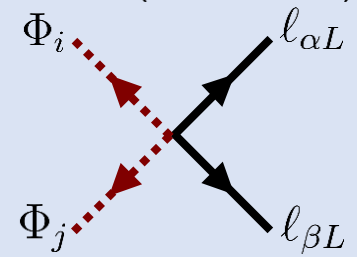
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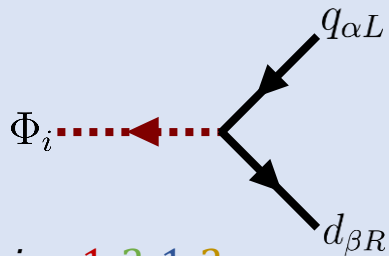
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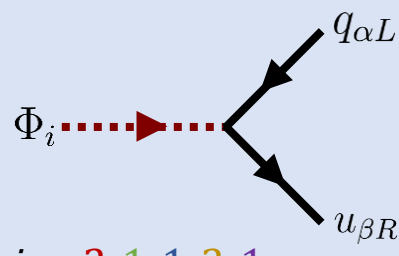
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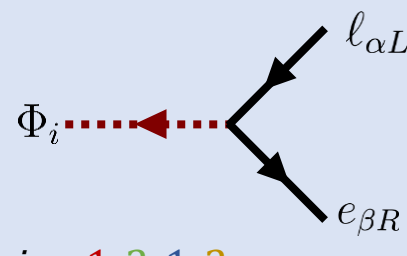
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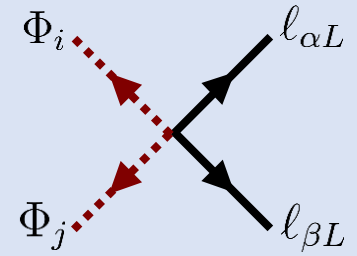
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Example:

$$\Phi_{1,2} \rightarrow q_{1L} + d_{2R}$$

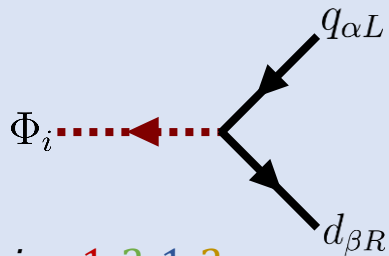
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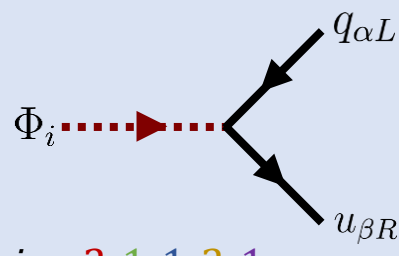
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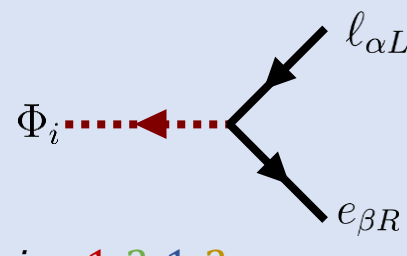
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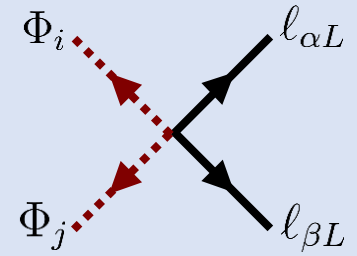
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Example:

~~$$\Phi_{1,2} \rightarrow q_{1L} + d_{2R}$$~~

Flavour charge is not conserved

$$Q_{\Phi_{1,2}} - Q_{q_{1L}} + Q_{d_{2R}} \neq 0$$

Maximally-restrictive textures from $U(1)$ symmetries

Procedure

Equivalence classes with
the maximum number of
zeros

Maximally-restrictive textures from $U(1)$ symmetries

Procedure

Equivalence classes with
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zeros



Solve system of equations
for the field charges

Maximally-restrictive textures from U(1) symmetries

Procedure

Equivalence classes with the maximum number of zeros



Solve system of equations for the field charges



Test compatibility at the 1σ CL for all observables

Experimental Data

Parameter	Best fit $\pm 1\sigma$
$m_d (\times \text{MeV})$	$4.67^{+0.48}_{-0.17}$
$m_s (\times \text{MeV})$	$93.4^{+8.6}_{-3.4}$
$m_b (\times \text{GeV})$	$4.18^{+0.03}_{-0.02}$
$m_u (\times \text{MeV})$	$2.16^{+0.49}_{-0.26}$
$m_c (\times \text{GeV})$	1.27 ± 0.02
$m_t (\times \text{GeV})$	172.69 ± 0.30
$\theta_{12}^q (^\circ)$	13.04 ± 0.05
$\theta_{23}^q (^\circ)$	2.38 ± 0.06
$\theta_{13}^q (^\circ)$	0.201 ± 0.011
$\delta^q (^\circ)$	68.75 ± 4.5

Quarks

Parameter	Best Fit $\pm 1\sigma$
$m_e (\times \text{keV})$	$510.99895000 \pm 0.00000015$
$m_\mu (\times \text{MeV})$	$105.6583755 \pm 0.0000023$
$m_\tau (\times \text{GeV})$	1.77686 ± 0.00012
$\Delta m_{21}^2 (\times 10^{-5} \text{ eV}^2)$	$7.50^{+0.22}_{-0.20}$
$ \Delta m_{31}^2 (\times 10^{-3} \text{ eV}^2) [\text{NO}]$	$2.55^{+0.02}_{-0.03}$
$ \Delta m_{31}^2 (\times 10^{-3} \text{ eV}^2) [\text{IO}]$	$2.45^{+0.02}_{-0.03}$
$\theta_{12}^\ell (^\circ)$	34.3 ± 1.0
$\theta_{23}^\ell (^\circ) [\text{NO}]$	49.26 ± 0.79
$\theta_{23}^\ell (^\circ) [\text{IO}]$	$49.46^{+0.60}_{-0.97}$
$\theta_{13}^\ell (^\circ) [\text{NO}]$	$8.53^{+0.13}_{-0.12}$
$\theta_{13}^\ell (^\circ) [\text{IO}]$	$8.58^{+0.12}_{-0.14}$
$\delta^\ell (^\circ) [\text{NO}]$	194^{+24}_{-22}
$\delta^\ell (^\circ) [\text{IO}]$	284^{+26}_{-28}

Leptons

Maximally-restrictive textures from U(1) symmetries

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Solve system of equations for the field charges



Test compatibility at the 1σ CL for all observables

Add nonzero entry

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$\delta^\ell (^\circ) [\text{NO}]$	194^{+24}_{-22}
$\delta^\ell (^\circ) [\text{IO}]$	284^{+26}_{-28}

Leptons

Maximally-restrictive textures from U(1) symmetries

Procedure

Equivalence classes with the maximum number of zeros



Solve system of equations for the field charges



Test compatibility at the 1σ CL for all observables

Add nonzero entry



Maximally-restrictive textures and U(1) charges

Experimental Data

Parameter	Best fit $\pm 1\sigma$
$m_d (\times \text{MeV})$	$4.67^{+0.48}_{-0.17}$
$m_s (\times \text{MeV})$	$93.4^{+8.6}_{-3.4}$
$m_b (\times \text{GeV})$	$4.18^{+0.03}_{-0.02}$
$m_u (\times \text{MeV})$	$2.16^{+0.49}_{-0.26}$
$m_c (\times \text{GeV})$	1.27 ± 0.02
$m_t (\times \text{GeV})$	172.69 ± 0.30
$\theta_{12}^q (^\circ)$	13.04 ± 0.05
$\theta_{23}^q (^\circ)$	2.38 ± 0.06
$\theta_{13}^q (^\circ)$	0.201 ± 0.011
$\delta^q (^\circ)$	68.75 ± 4.5

Quarks

Parameter	Best Fit $\pm 1\sigma$
$m_e (\times \text{keV})$	$510.99895000 \pm 0.00000015$
$m_\mu (\times \text{MeV})$	$105.6583755 \pm 0.0000023$
$m_\tau (\times \text{GeV})$	1.77686 ± 0.00012
$\Delta m_{21}^2 (\times 10^{-5} \text{ eV}^2)$	$7.50^{+0.22}_{-0.20}$
$ \Delta m_{31}^2 (\times 10^{-3} \text{ eV}^2) [\text{NO}]$	$2.55^{+0.02}_{-0.03}$
$ \Delta m_{31}^2 (\times 10^{-3} \text{ eV}^2) [\text{IO}]$	$2.45^{+0.02}_{-0.03}$
$\theta_{12}^\ell (^\circ)$	34.3 ± 1.0
$\theta_{23}^\ell [\text{NO}]$	49.26 ± 0.79
$\theta_{23}^\ell [\text{IO}]$	$49.46^{+0.60}_{-0.97}$
$\theta_{13}^\ell [\text{NO}]$	$8.53^{+0.13}_{-0.12}$
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$\delta^\ell (^\circ) [\text{NO}]$	194^{+24}_{-22}
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Leptons

Maximally-restrictive textures from U(1) symmetries

U(1) charges

\mathbb{Z}_5			
$(\mathbf{M}_e, \mathbf{M}_\nu)$	$(\delta_1, \delta_2, \delta_3)$	$(\epsilon_1, \epsilon_2, \epsilon_3)$	
$(5_1^e, 2_3^\nu)$	$(-1, -3, 1)$	$(1, -5, -1)$	
$(5_1^e, 2_7^\nu)$	$(-1, -2, 0)$	$(0, -3, -1)$	
$(5_1^e, 2_{10}^\nu)$	$(0, -1, 1)$	$(1, -2, 0)$	

\mathbb{Z}_4			
$(\mathbf{M}_d, \mathbf{M}_u)$	$(\alpha_1, \alpha_2, \alpha_3)$	$(\beta_1, \beta_2, \beta_3)$	$(\gamma_1, \gamma_2, \gamma_3)$
$(4_3^d, \mathbf{P}_{12} 5_1^u \mathbf{P}_{23})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 2, 0)$
$(4_3^d, \mathbf{P}_{123} 5_1^u \mathbf{P}_{12})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 0, 1)$
$(5_1^d, \mathbf{P}_{12} 4_3^u)$	$(0, -1, 1)$	$(1, -2, 0)$	$(2, 1, 0)$
$(5_1^d, \mathbf{P}_{321} 4_3^u \mathbf{P}_{23})$	$(0, -1, 1)$	$(1, -2, 0)$	$(-1, 1, 0)$

Maximally restrictive mass matrices

Quarks

$$4_3^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

$$5_1^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$$

$$\mathbf{P}_{12} 5_1^u \mathbf{P}_{23} \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \bullet & 0 \\ \times & \times & 0 \end{pmatrix}$$

$$\mathbf{P}_{123} 5_1^u \mathbf{P}_{12} \sim \begin{pmatrix} 0 & \times & \bullet \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}$$

$$\mathbf{P}_{12} 4_3^u \sim \begin{pmatrix} 0 & \bullet & \times \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}$$

$$\mathbf{P}_{321} 4_3^u \mathbf{P}_{23} \sim \begin{pmatrix} 0 & \bullet & \times \\ \times & 0 & \times \\ 0 & \times & 0 \end{pmatrix}$$

Leptons

$$5_1^e \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$$

$$2_3^\nu \sim \begin{pmatrix} \times & \times & \bullet \\ \cdot & 0 & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$$

$$2_7^\nu \sim \begin{pmatrix} \times & 0 & \bullet \\ \cdot & 0 & \times \\ \cdot & \cdot & \bullet \end{pmatrix}$$

$$2_{10}^\nu \sim \begin{pmatrix} \times & \bullet & 0 \\ \cdot & \times & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$$

Maximally-restrictive textures from U(1) symmetries

U(1) charges

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$(\mathbf{M}_e, \mathbf{M}_\nu)$	$(\delta_1, \delta_2, \delta_3)$	$(\epsilon_1, \epsilon_2, \epsilon_3)$	
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\mathbb{Z}_4			
$(\mathbf{M}_d, \mathbf{M}_u)$	$(\alpha_1, \alpha_2, \alpha_3)$	$(\beta_1, \beta_2, \beta_3)$	$(\gamma_1, \gamma_2, \gamma_3)$
$(4_3^d, \mathbf{P}_{12} 5_1^u \mathbf{P}_{23})$	$(0, 1, 2)$	$(2, 1, 0)$	$(3, 2, 0)$
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“Decoupled” entry in the matrices of type “5” lead to zeros in the N_k matrices

Maximally restrictive mass matrices

Quarks	Leptons
$4_3^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$5_1^e \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$
$5_1^d \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}$	$2_3^\nu \sim \begin{pmatrix} \times & \times & \bullet \\ \cdot & 0 & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$
$\mathbf{P}_{12} 5_1^u \mathbf{P}_{23} \sim \begin{pmatrix} 0 & 0 & \times \\ 0 & \bullet & 0 \\ \times & \times & 0 \end{pmatrix}$	$2_7^\nu \sim \begin{pmatrix} \times & 0 & \bullet \\ \cdot & 0 & \times \\ \cdot & \cdot & \bullet \end{pmatrix}$
$\mathbf{P}_{123} 5_1^u \mathbf{P}_{12} \sim \begin{pmatrix} 0 & \times & \bullet \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}$	$2_{10}^\nu \sim \begin{pmatrix} \times & \bullet & 0 \\ \cdot & \times & \bullet \\ \cdot & \cdot & 0 \end{pmatrix}$
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Maximally-restrictive textures from $U(1)$ symmetries

Minimal flavour patterns for **quarks**:

- ✓ Four different models;
- ✓ There is a total of ten independent parameters, **matching** the number of observables;

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Predictions



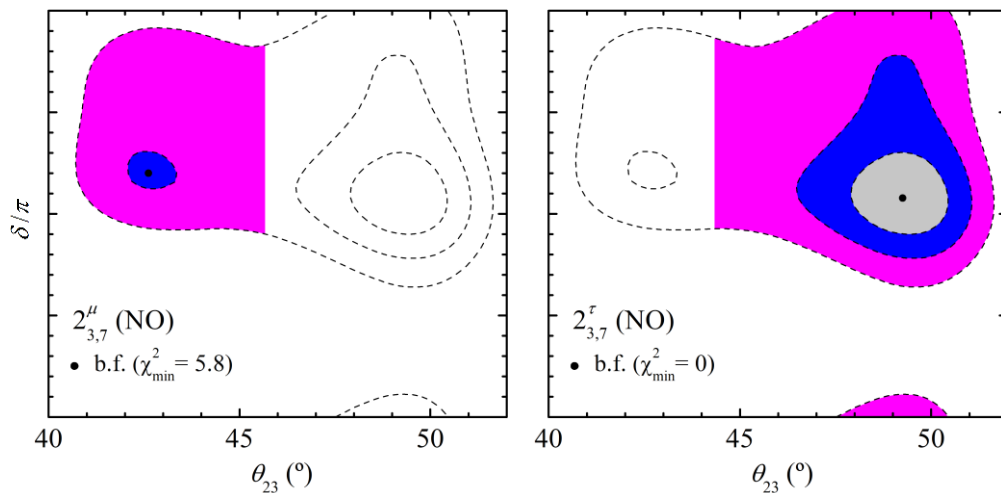
$$\mathbf{NO:} \quad m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$$

$$\mathbf{IO:} \quad m_1 = \sqrt{m_3^2 + |\Delta m_{31}^2|}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{21}^2 + |\Delta m_{31}^2|}$$

$$m_{\beta\beta} = \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 e^{-i\alpha_{21}} + s_{13}^2 m_3 e^{-i\alpha_{31}} \right|$$

Lepton sector predictions - NO

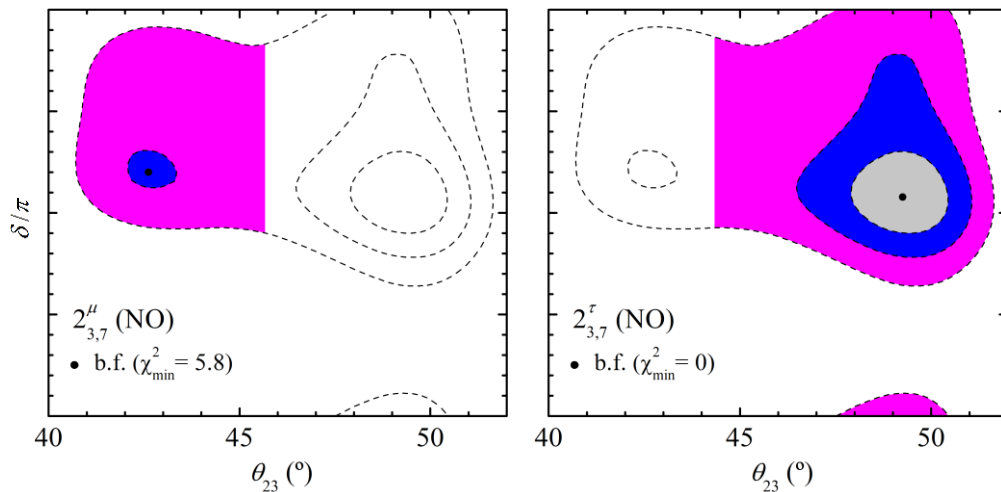
The symmetry-constrained lepton models provide **predictions** for the **neutrino sector**, for example:



For NO, $2_{3,7}^{\mu}$ and $2_{3,7}^{\tau}$ select the **first** and **second octant** for the atmospheric mixing angle θ_{23} , respectively

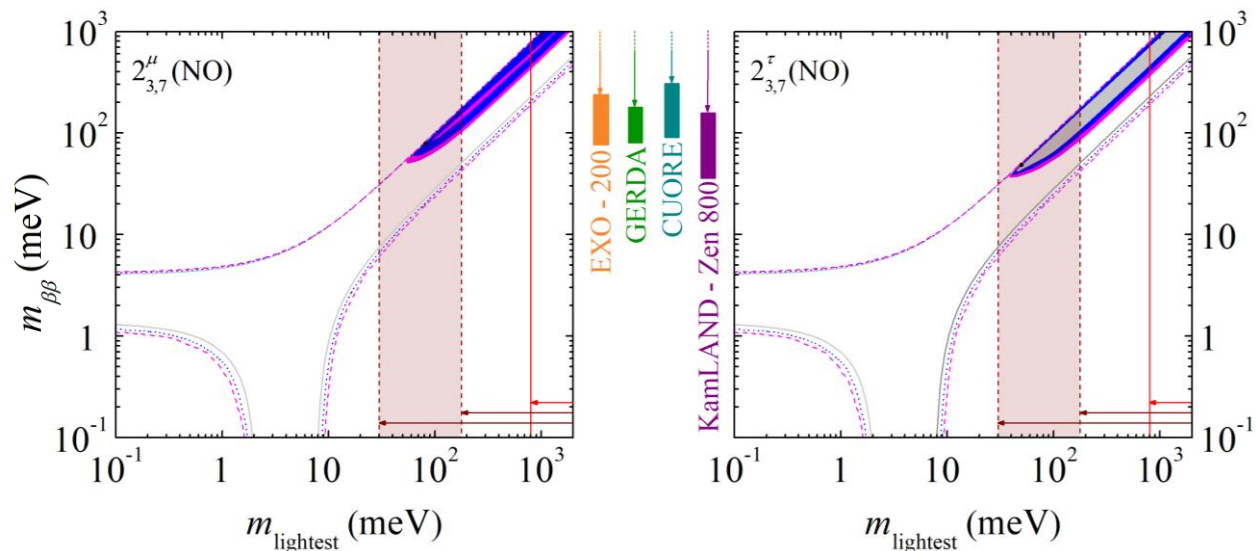
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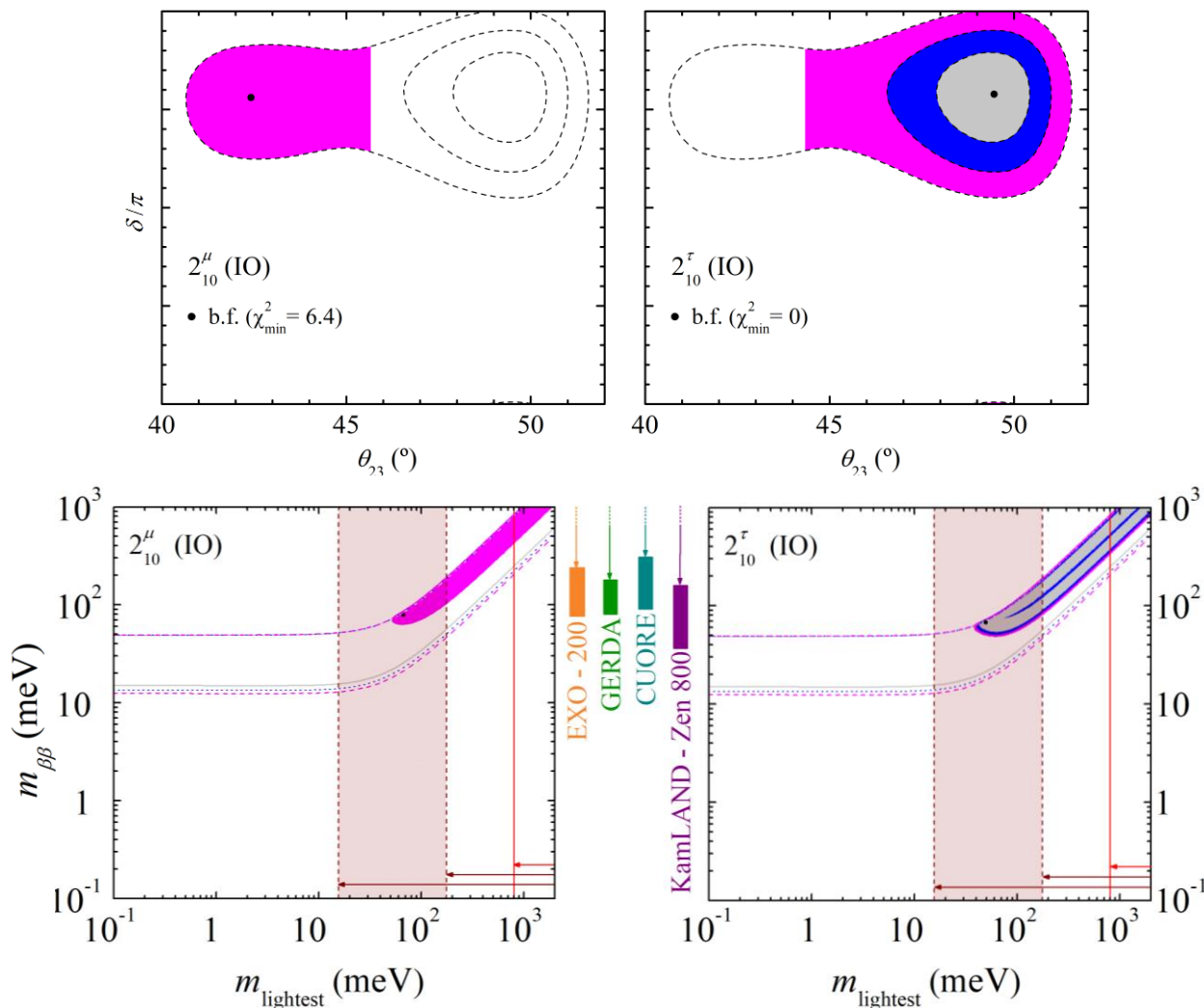
For NO, $2_{3,7}^\mu$ and $2_{3,7}^\tau$ select the **first** and **second octant** for the atmospheric mixing angle θ_{23} , respectively

The lower bounds on $m_{\beta\beta}$ are **within the sensitivity** of $0\nu\beta\beta$ decay experiments, while being simultaneously in **tension with cosmological** constraints on $m_{lightest}$



Lepton sector predictions - IO

There are models that behave similarly for **inverted ordering** (IO), namely 2_{10}^{μ} and 2_{10}^{τ}



Numerical procedure and phenomenological analysis

For the numerical analysis of the phenomenology of maximally-restrictive matrices, a private *Python* code was developed, which works as follows:

Random values for $\tan \beta$, m_I , m_H , m_{H^\pm}

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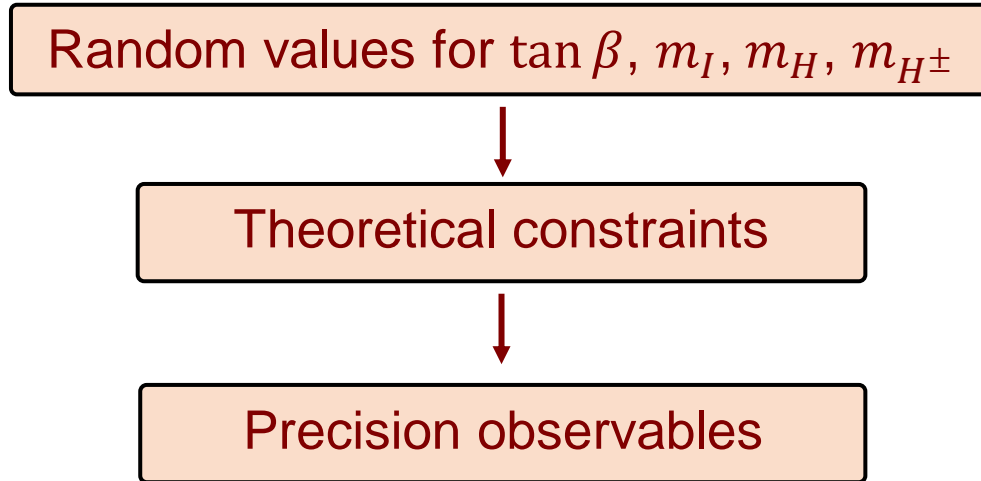
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Theoretical constraints

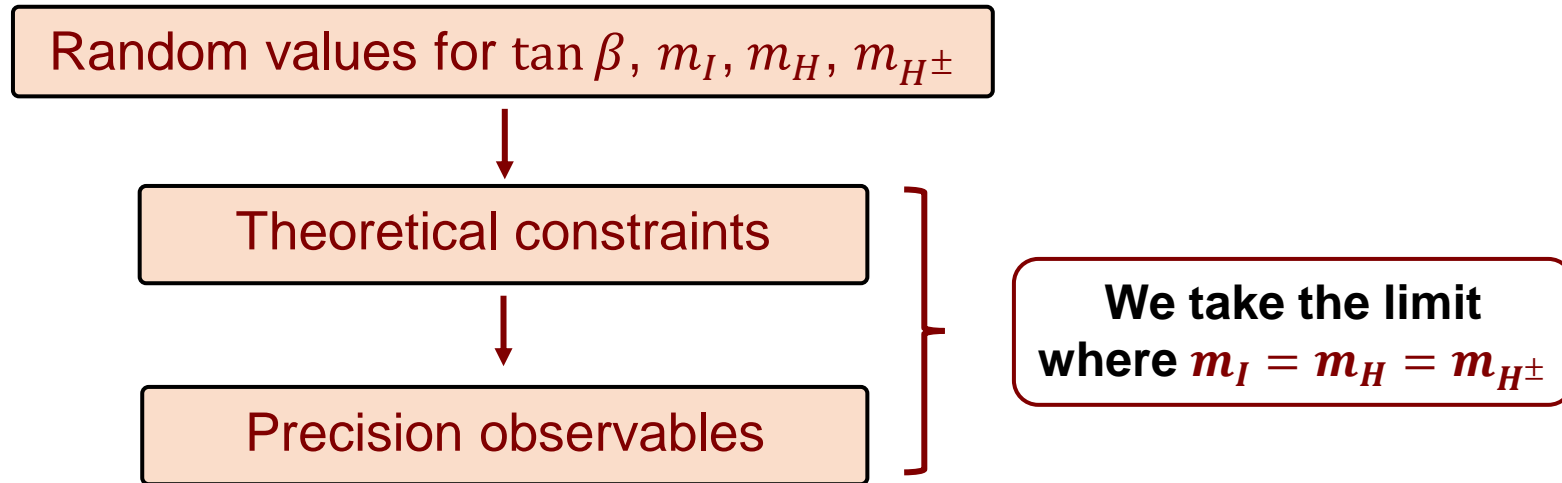
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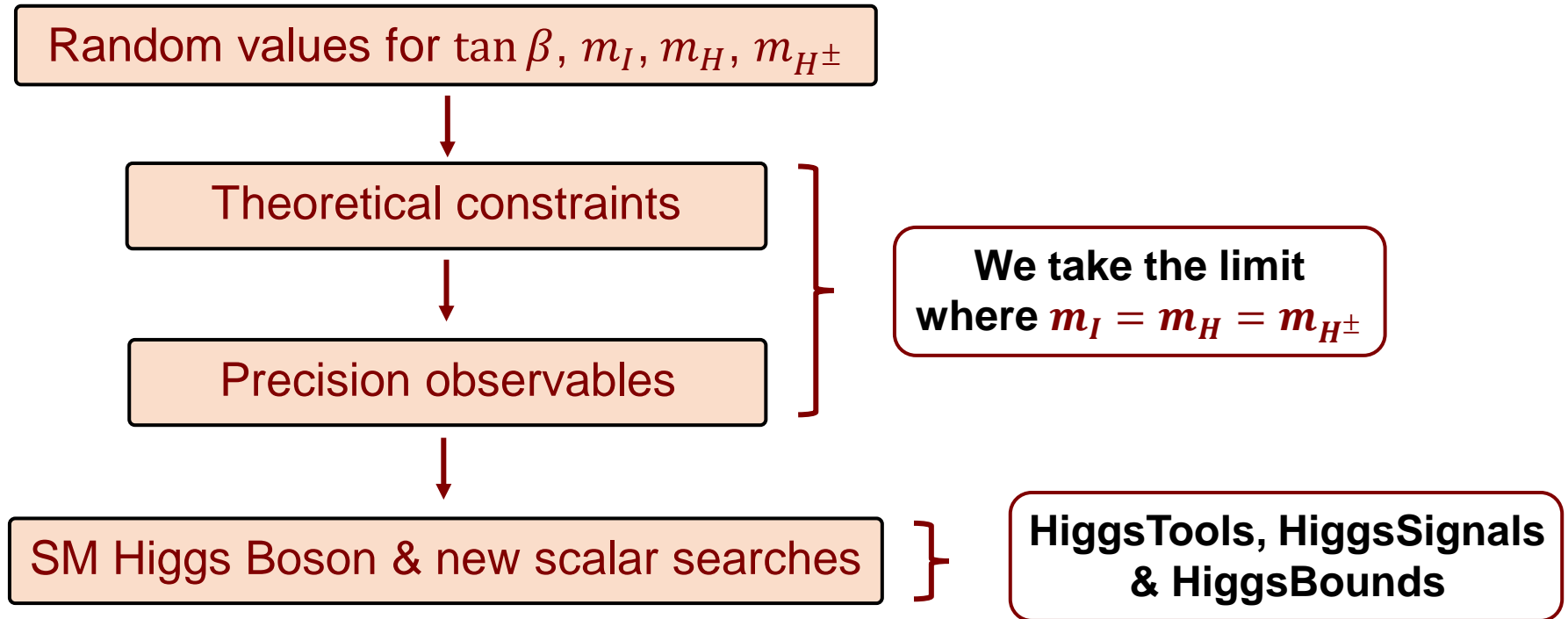
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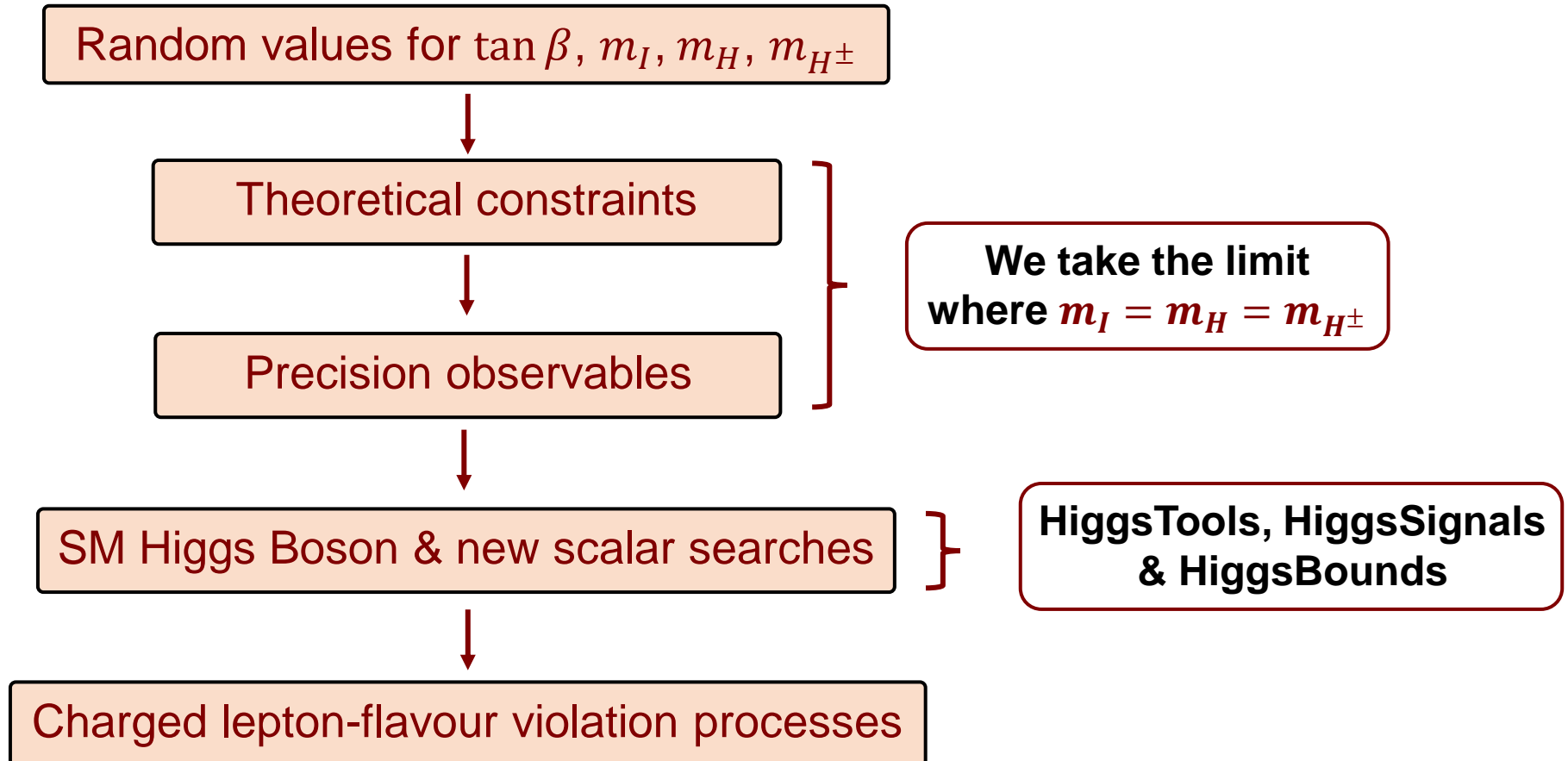
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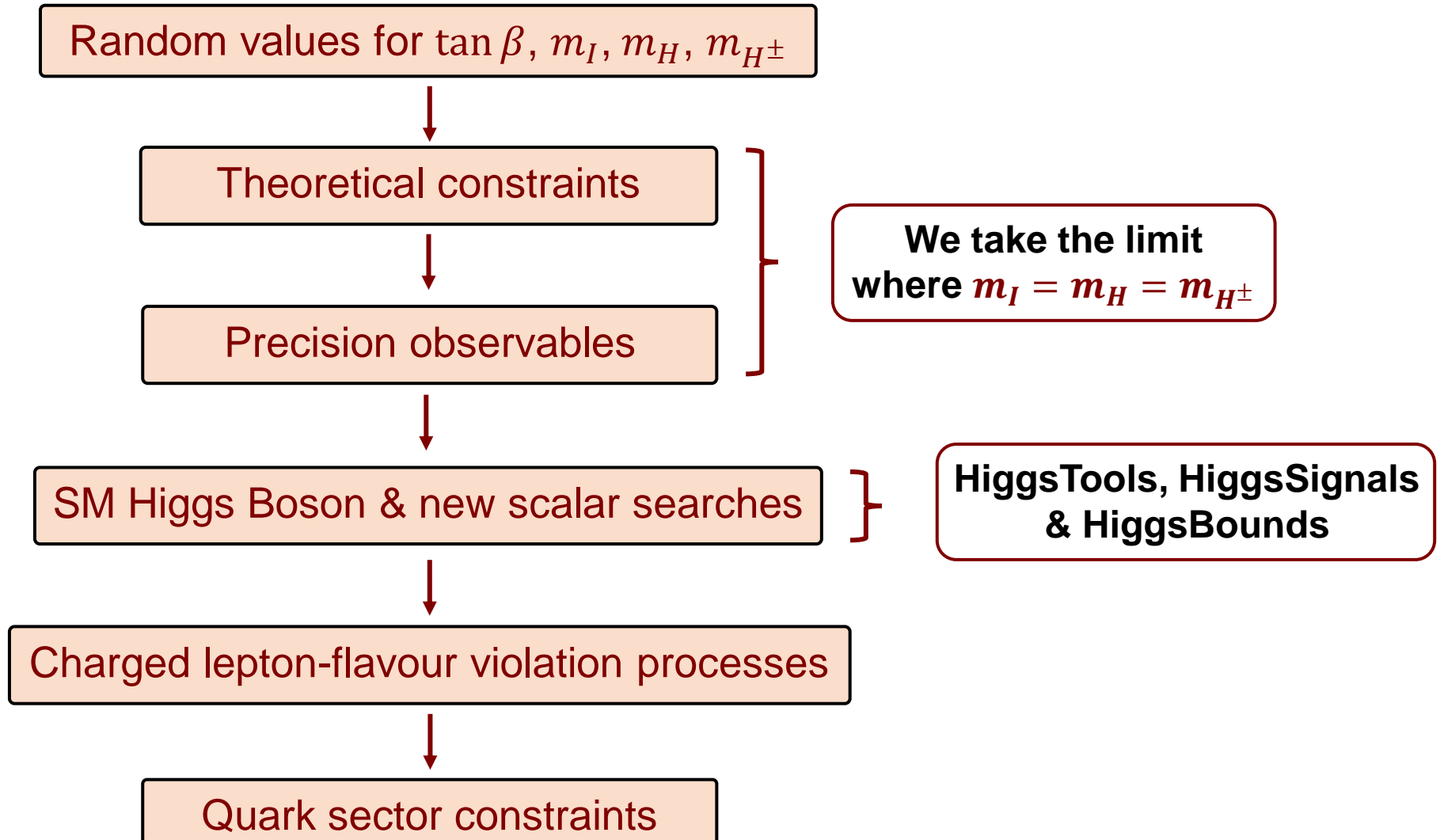
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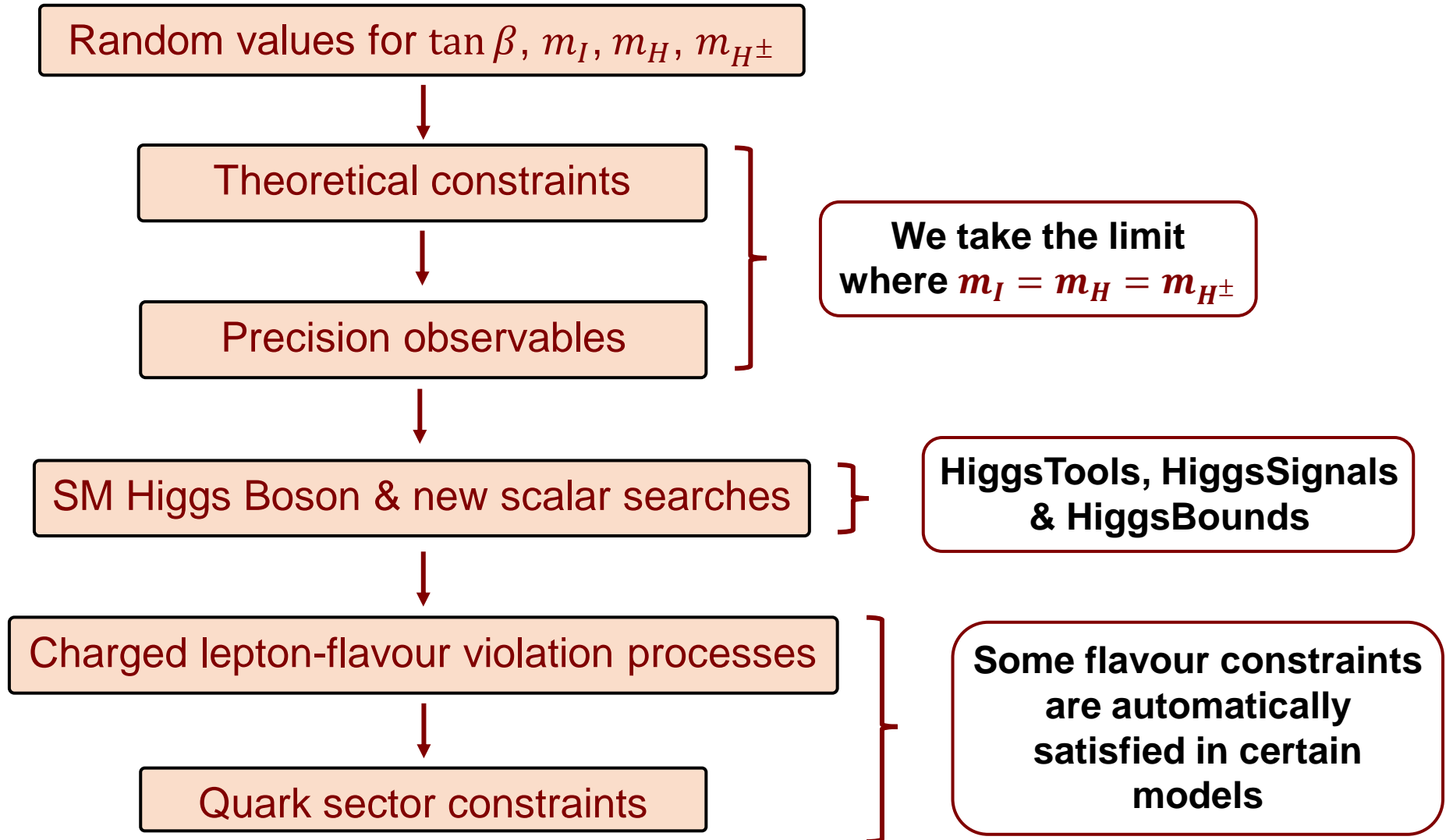
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The mass matrices labelled "5" exhibit an isolated non-zero entry in a given row and column, which coincides with the mass of a fermion translating into:

$$5^{d,u,e} : \mathbf{N}_{d,u,e} \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad 5^{s,c,\mu} : \mathbf{N}_{s,c,\mu} \sim \begin{pmatrix} \times & 0 & \times \\ 0 & \times & 0 \\ \times & 0 & \times \end{pmatrix}, \quad 5^{b,t,\tau} : \mathbf{N}_{b,t,\tau} \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

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To directly observe the effect of flavour symmetries, consider the NP contribution to the $\bar{K}_0 \rightarrow K_0$ transition:

$$M_{21}^{\text{NP}} = \frac{f_k^2 m_K}{96v^2} \left\{ [(\mathbf{N}_d^*)_{ds}^2 + (\mathbf{N}_d)_{sd}^2] \frac{10m_k^2}{(m_s + m_d)^2} \left(\frac{1}{m_I^2} - \frac{c_{\beta-\alpha}^2}{m_h^2} - \frac{s_{\beta-\alpha}^2}{m_H^2} \right) + 4(\mathbf{N}_d^*)_{ds}(\mathbf{N}_d)_{sd} \left[1 + \frac{6m_K^2}{(m_s + m_d)^2} \left(\frac{1}{m_I^2} + \frac{c_{\beta-\alpha}^2}{m_h^2} + \frac{s_{\beta-\alpha}^2}{m_H^2} \right) \right] \right\}$$

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$$\Delta m_K^{\text{NP}} = 2|M_{21}^{\text{NP}}| = 0 \quad \varepsilon_K = \varepsilon_K^{\text{SM}} - \frac{\cancel{\text{Im}(M_{21}^{\text{NP}} \lambda_u^{*2})}}{\sqrt{2}\Delta m_K |\lambda_u|^2}$$

The two constraints associated with K^0 are **inherently satisfied** for d or s decoupled

Yukawa perturbativity bounds

$$\tan^2 \beta \leq \frac{2\pi v^2}{|(\mathbf{M}_1^x)_{ij}|^2} - 1, \quad \tan^2 \beta \geq 1 / \left(\frac{2\pi v^2}{|(\mathbf{M}_2^x)_{ij}|^2} - 1 \right)$$

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Lepton sector constraints

We only consider the lepton model $(5_1^e, 2_3^v)_{\text{NO}}$, as the conclusions do not differ with a more detailed analysis.

The only exception is for the $(5_1^d, \mathbf{P}_{123} 4_3^u \mathbf{P}_{12})$ model.

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Lepton sector constraints

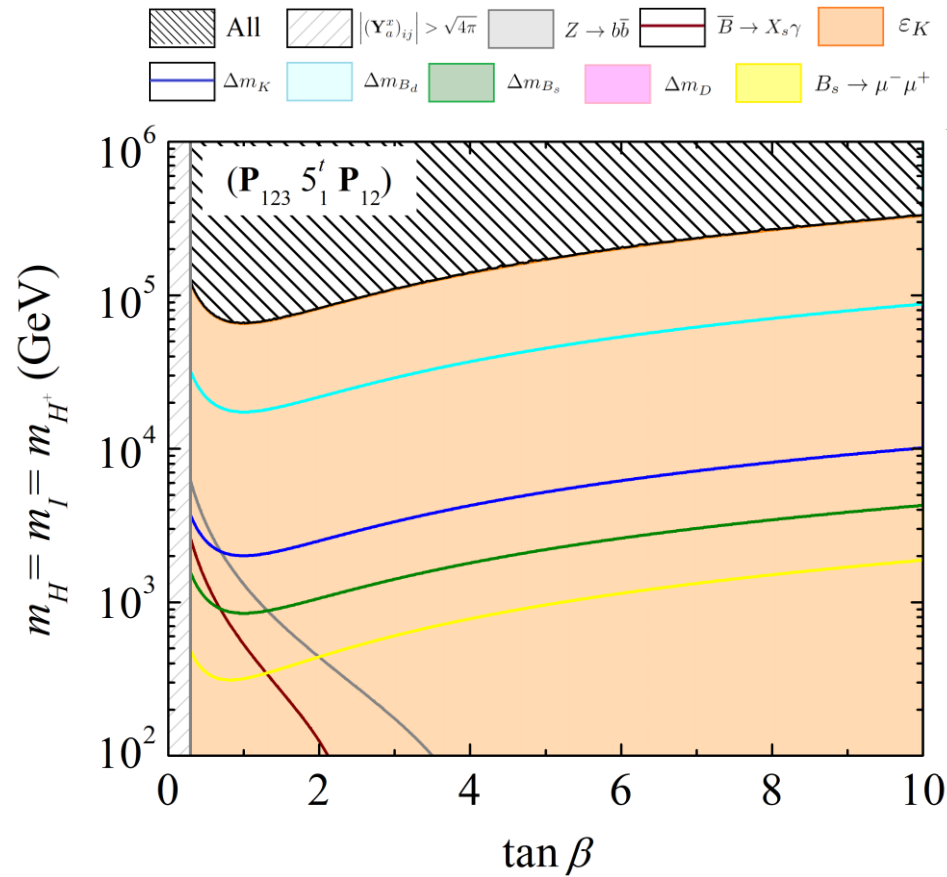
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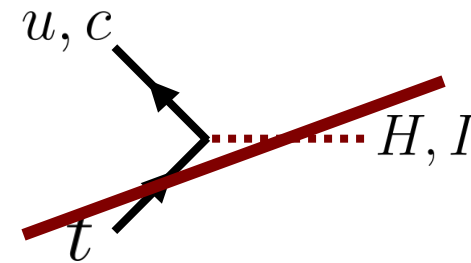
Most restrictive constraints

Only some constraints shape the allowed region $(\tan \beta, \{m_H = m_I = m_{H^\pm}\})$, which we refer to as the **most restrictive constraints**.

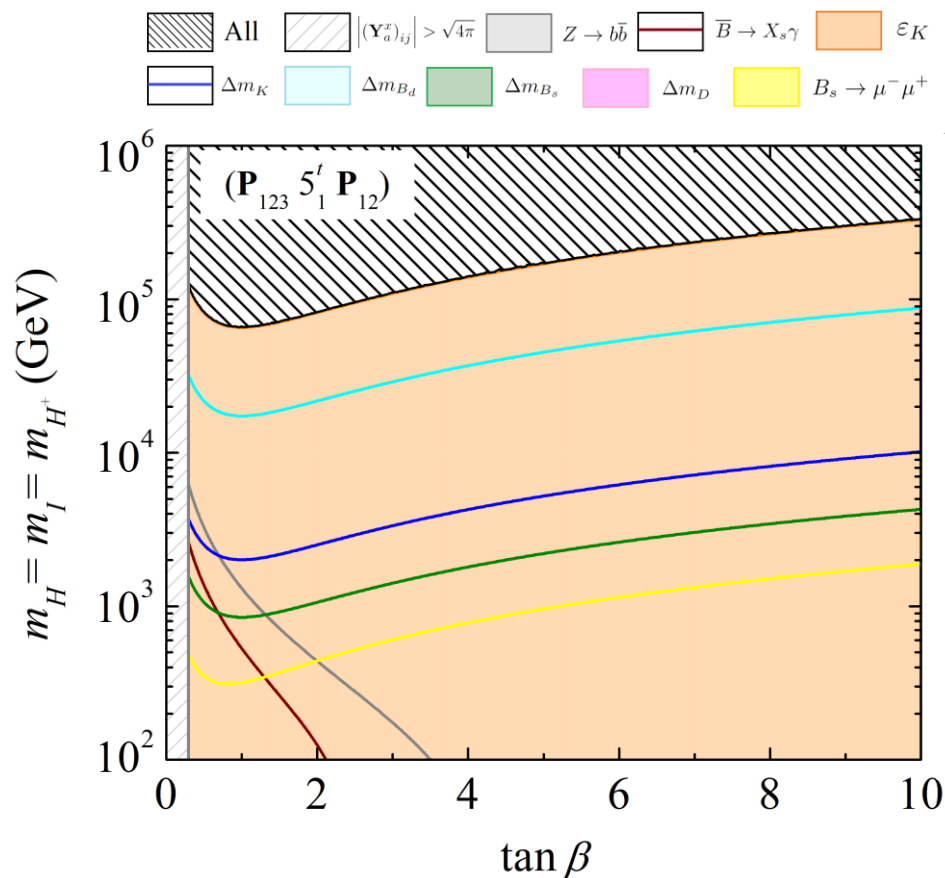
Numerical procedure and phenomenological analysis



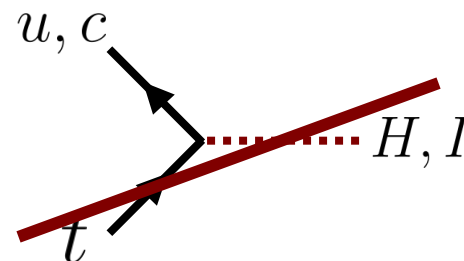
$$\mathbf{N}_t \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$



Numerical procedure and phenomenological analysis



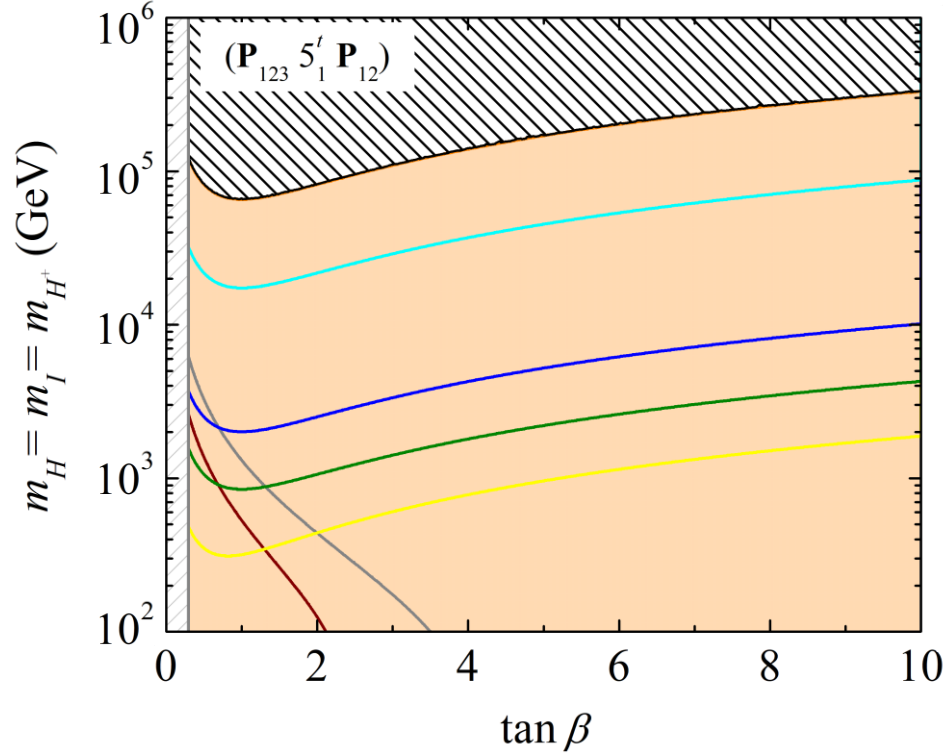
$$\mathbf{N}_t \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$



\downarrow

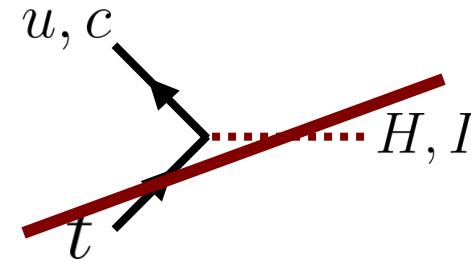
None of the most restrictive constraints are automatically satisfied.

Numerical procedure and phenomenological analysis



The decoupled state could be picked to satisfy some constraints, for example d

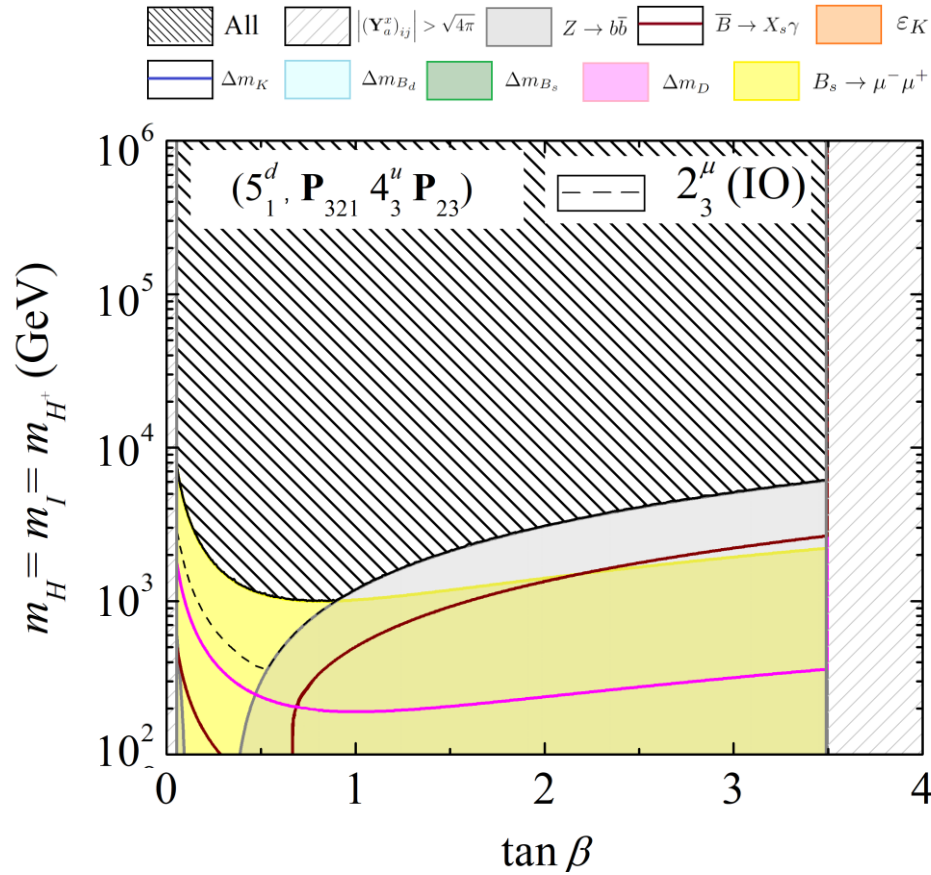
$$\mathbf{N}_t \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$



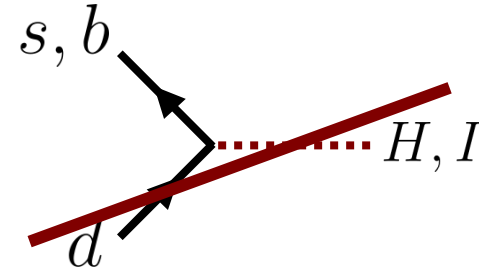
↓
None of the most restrictive constraints are automatically satisfied.

Observable	Constraint	Decoupled state
$ \varepsilon_K $	$(2.228 \pm 0.011) \times 10^{-3}$	(u, d, s)
Δm_K^{NP}	$< 3.484 \times 10^{-15} \text{ GeV}$	(d, s)
Δm_{B_d}	$(3.334 \pm 0.013) \times 10^{-13} \text{ GeV}$	(d, b)
Δm_{B_s}	$(1.1693 \pm 0.0004) \times 10^{-11} \text{ GeV}$	(s, b)
Δm_D^{NP}	$< 6.56 \times 10^{-15} \text{ GeV}$	(u, c)

Numerical procedure and phenomenological analysis



$$\mathbf{N}_d \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

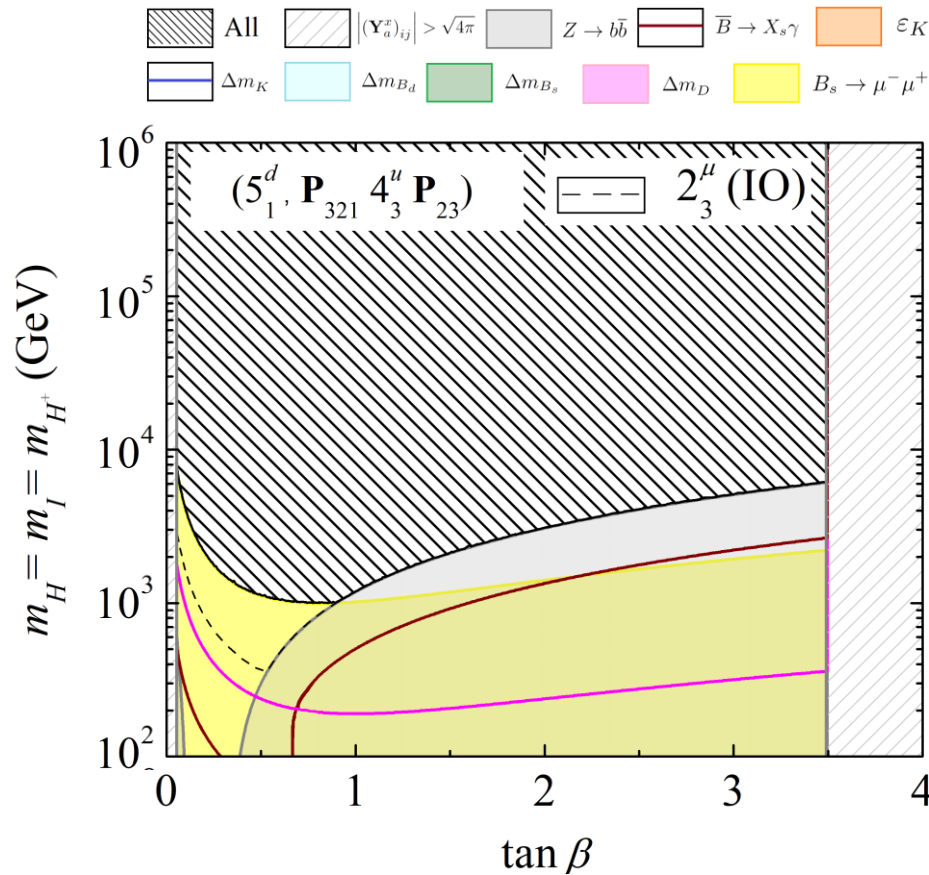


✓ K mesons: $K^0(d\bar{s})$: $\Delta m_K, \epsilon_K$

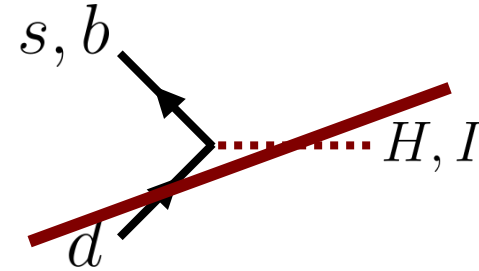
✓ B mesons:

$B_d^0(d\bar{b})$: $\Delta m_{B_d}, \text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)$

Numerical procedure and phenomenological analysis



$$\mathbf{N}_d \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$



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✓ B mesons:

$B_d^0(d\bar{b})$: $\Delta m_{B_d}, \text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)$

This model highlights the effectiveness of Abelian flavour symmetries in aligning theoretical frameworks with highly constrained experimental observations.

Summary and outlook

Work done:

- ✓ Study of the theoretical framework of the **minimal U(1) 2HDM for flavour**;
- ✓ Identification of the **maximally-restrictive pairs of quark and lepton mass matrices** compatible with current masses, mixing and CP violation data;
- ✓ Lepton sector **predictions**;
- ✓ **Phenomenological study** (analytical and numerical) of the quark and charged lepton sectors.

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Thank you !