NICPB KBFI

CONNECTING SCIENCES

Natural Metric-Affine Inflation

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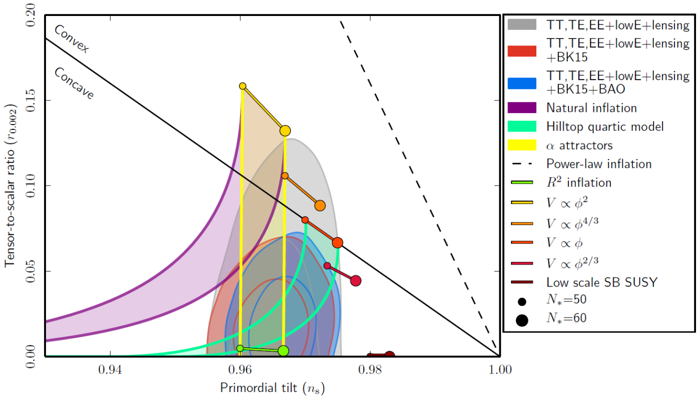
based on
arXiv:2403.18004 (to appear on JCAP)
with
A. Salvio (Univ. Rome Tor Vergata & INFN)

Euroopa Liit
Euroopa Sotsiaalfond
Eesti tuleviku heaks

• NI vs Planck 2018 data •



- NI became strongly disfavored after BICEP/Keck 2018 data
- several proposals to save it by modifying gravity:
  - $\xi[1 + cos(\phi)]R \rightarrow$ OK only at 2σ (Ferreira et al. 1806.05511)
  - $\xi\phi^n R \rightarrow$ OK only at 2σ (Bostan, 2209.02434; dos Santos et al. , 2312.12286)
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  - probably more?

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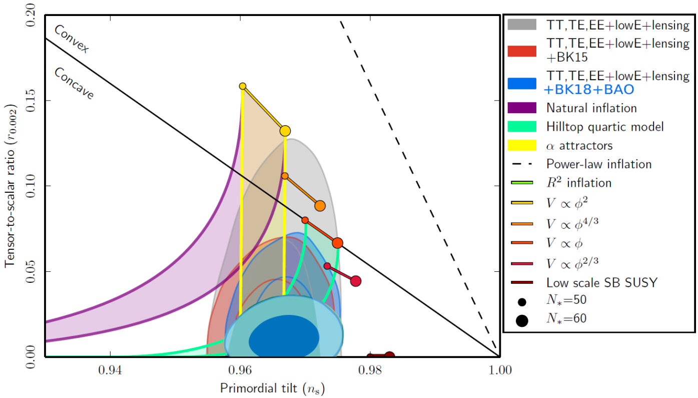

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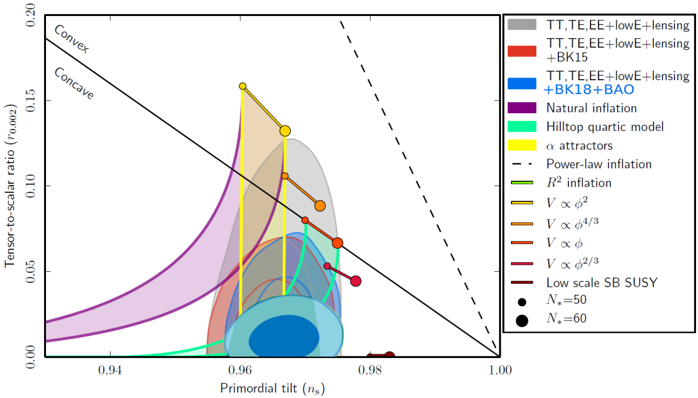

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• Gravity preliminaries •

The properties of torsion-free spacetime are essentially described by:

- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$ → parallel transport
- the metric tensor: $g_{\mu\nu}$ → distance

The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship. If they are to have any relationship, it must derive from

- additional constraints (metric formalism $\mathcal{A}_{\mu}{}^{\rho}{}_{\sigma} = \Gamma_{\mu}{}^{\rho}{}_{\sigma}$)
- EoM for $\mathcal{A}$ & g (Palatini formalism)
  - minimal theories $\Rightarrow \mathcal{A}_{\mu}{}^{\rho}{}_{\sigma} = \Gamma_{\mu}{}^{\rho}{}_{\sigma}$
    $\Rightarrow$ metric $\sim$ Palatini
  - non-minimal theories $\Rightarrow \mathcal{A}_{\mu}{}^{\rho}{}_{\sigma} \neq \Gamma_{\mu}{}^{\rho}{}_{\sigma}$
    $\Rightarrow$ metric $\neq$ Palatini (e.g. Koivisto & Kurki-Suonio: 0509422)

• Palatini NI action •

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right]$$

$$\alpha(\phi) = \frac{M_P^2}{2}\left[1+\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right)\right] > 0$$

- notation: $\begin{cases} \mathcal{R} \to \text{curvature from generic } \mathcal{A}_\mu{}^\rho{}_\sigma \\ R \to \text{curvature from Levi-Civita } \Gamma_\mu{}^\rho{}_\sigma \end{cases}$
- Einstein frame: $g^E_{\mu\nu} = F(\phi)g^J_{\mu\nu}$, $F \equiv \frac{2\alpha}{M_P^2}$

  N.B. Palatini $\Rightarrow \mathcal{R}_J = F\,R_E$

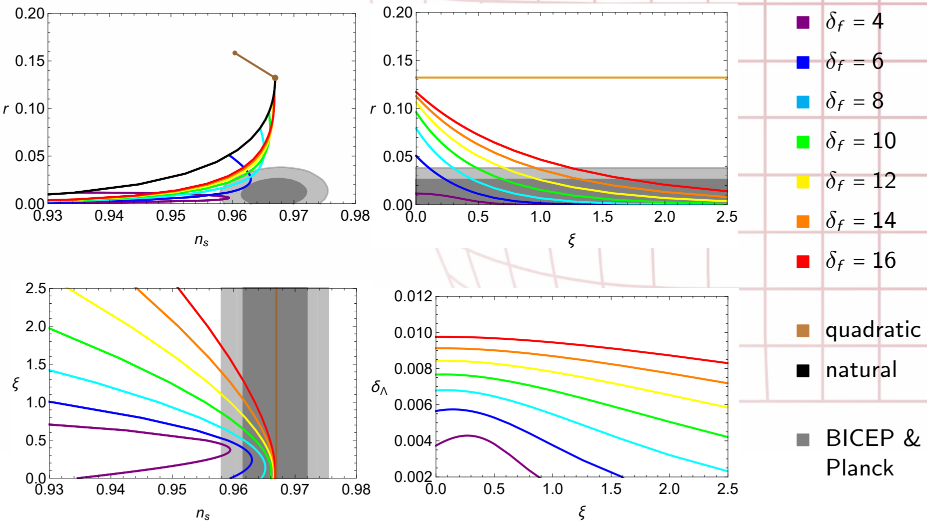
$$S_{\rm NI} = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]$$

$$\frac{d\chi}{d\phi} = \sqrt{\frac{1}{F(\phi)}} = \sqrt{\frac{M_P^2}{2\alpha(\phi)}} \qquad \leftarrow \text{no } \frac{3}{2}\left(\frac{F'}{F}\right)^2 \text{ like in metric}$$

$$U(\chi) = \frac{V(\phi(\chi))}{F^2(\phi(\chi))}$$

• Palatini results •

$N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



• Palatini → Metric-Affine •

- any way to get better results?
- YES! allow for torsion → MAG

  but in a way that new dof's are not generated!

- $\mathcal{A}^{\lambda}_{\alpha\beta} \neq \mathcal{A}^{\lambda}_{\beta\alpha}$

$$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv \partial_\mu \mathcal{A}_\nu{}^{\rho}{}_{\sigma} - \partial_\nu \mathcal{A}_\mu{}^{\rho}{}_{\sigma} + \mathcal{A}_\mu{}^{\rho}{}_{\lambda}\mathcal{A}_\nu{}^{\lambda}{}_{\sigma} - \mathcal{A}_\nu{}^{\rho}{}_{\lambda}\mathcal{A}_\mu{}^{\lambda}{}_{\sigma}$$

$$\text{Holst inv.} \to \tilde{\mathcal{R}} \equiv \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu\rho\sigma}$$

cf. $\mathcal{R} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}$

where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric Levi-Civita symbol with $\epsilon^{0123} = 1$

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• MAG NI action •

$$S_{\rm NI} = \int d^4x\sqrt{-g_J}\left[\alpha(\phi)\mathcal{R}_J + \beta(\phi)\tilde{\mathcal{R}}_J - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$V(\phi) = \Lambda^4\left[1+\cos\left(\frac{\phi}{f}\right)\right] \qquad \alpha(\phi) = \frac{M_P^2}{2}\left[1+\xi\left(1+\cos\left(\frac{\phi}{f}\right)\right)\right] > 0$$

$$\beta(\phi)=\beta_0 + \frac{M_P^2}{2}\tilde{\xi}\left(\cos\left(\frac{\phi}{f}\right)+1\right) \qquad \frac{M_P^2}{4\beta_0} \to \text{Barbero-Irmizzi par.}$$

- it is possible to integrate out the $\tilde{R}$ term
- performing all the computations ...

$$S_{\rm NI} = \int d^4x\sqrt{-g}\left[\alpha\mathcal{R}_J - \left[1+\frac{12(\alpha'\beta+\alpha\beta')^2}{\alpha(\alpha^2+4\beta^2)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- allowing torsion changes the inflaton kinetic term

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• MAG NI action •

Einstein frame:

- $g^E_{\mu\nu} = F(\phi) g^J_{\mu\nu}$, $F \equiv \frac{2\alpha}{M_P^2}$ N.B. MAG $\Rightarrow \mathcal{R}_J = F\, R_E$

$$S_{\rm NI} = \int d^4x \sqrt{-g_E} \left[\frac{M_P^2}{2} R_E - \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \right]$$

$$\frac{d\chi}{d\phi} = M_P \sqrt{\frac{1}{2\alpha} \left[1 + \frac{12(\alpha' \beta + \alpha \beta')^2}{\alpha(\alpha^2 + 4\beta^2)} \right]}$$ ← new term from $\beta \tilde{R}$

$$U(\chi) = \frac{V(\phi(\chi))}{F^2(\phi(\chi))}$$ ← same as before

N.B.

- $\beta < 0$ allowed
- symmetry: $\beta \to -\beta$ $\Rightarrow$ $\tilde{\xi} > 0,\ \beta_0 \gtreqless 0$

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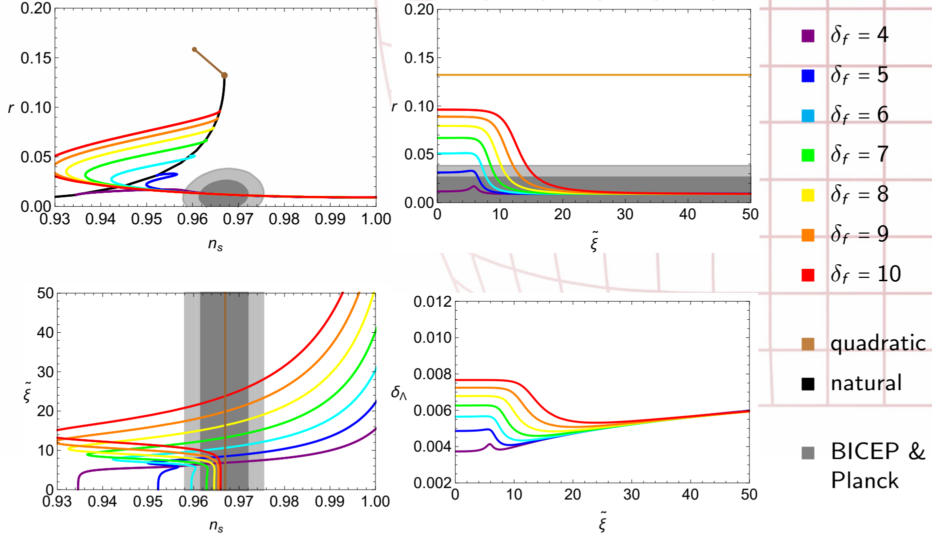
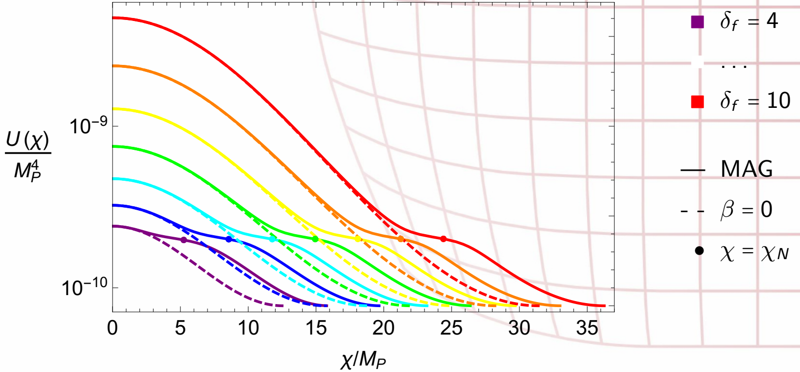
• $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

$\xi = 0 \quad \beta_0 = -6M_P^2 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



$\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$

$\xi = 0 \quad \beta_0 = -6M_P^2 \quad N_e = 60 \quad n_s \simeq 0.97 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



- inflection point inflation!!!

• $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 < 0$ •

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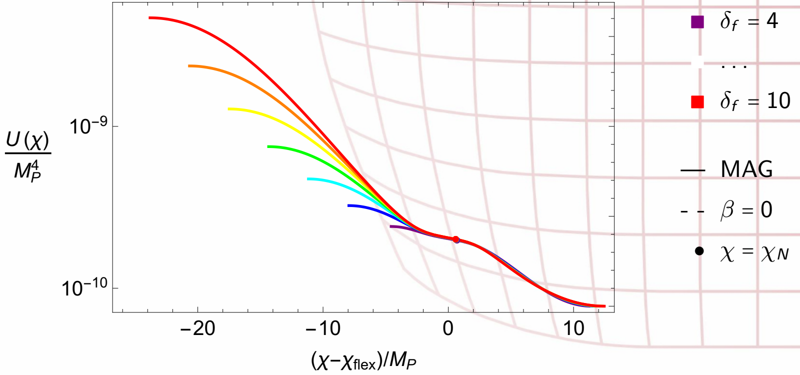
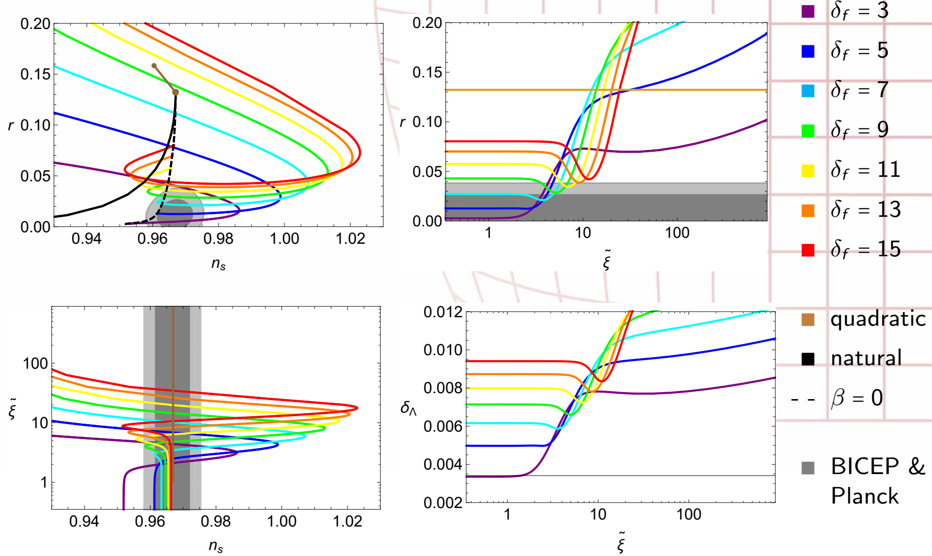


- inflection point inflation!!!

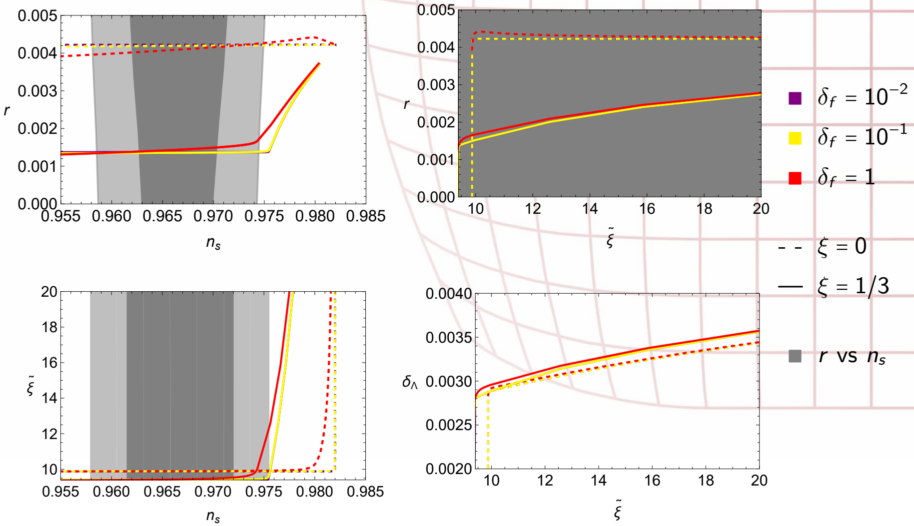
$\xi > 0$ & $\tilde{\xi} > 0$

$\xi = 1/3 \qquad \beta_0 = -2M_P^2 \qquad N_e = 60 \qquad \delta_f = f/M_P \qquad \delta_\Lambda = \Lambda/M_P$



• $\xi > 0$ & $\tilde{\xi} > 0$: low $f \lesssim M_P$ •

$\beta_0 = -10 M_P^2$ $N_e = 60$ $\delta_f = f/M_P$ $\delta_\Lambda = \Lambda/M_P$



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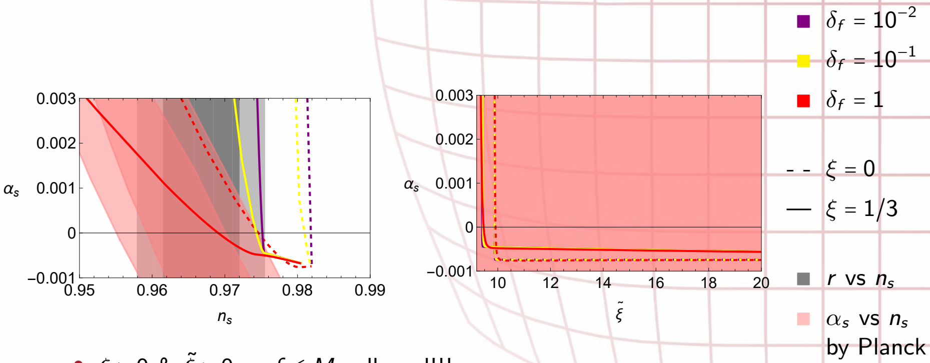


- $\xi > 0$ & $\tilde{\xi} > 0 \Rightarrow f \lesssim M_P$ allowed!!!

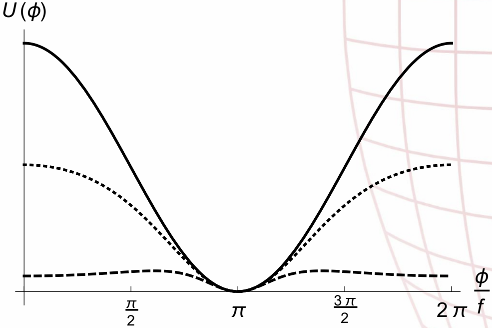
• Summary & Conclusions •

- NI strongly disfavored after Planck+BICEP 2018 data
- introducing a non-minimal coupling to gravity
  - compatible at 2σ in the Palatini formulation
- allowing for torsion (i.e. $\tilde{\mathcal{R}}$) (MAG formalism)
  - compatible at 1σ with data
  - allows also for subPlanckian f!!!

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

• Jordan frame potential •



$$U(\chi) = \frac{\Lambda^4\left(1+\cos\left(\frac{\phi(\chi)}{f}\right)\right)}{\left[1+\xi\left(1+\cos\left(\frac{\phi(\chi)}{f}\right)\right)\right]^2}$$

— $\xi = 0$

· · · $0 < \xi < \frac{1}{2}$

- - $\xi > \frac{1}{2}$

Stationary points:

$$\phi_1 = 0 \qquad U_{\phi\phi}(\phi_1) = \frac{\Lambda^4(2\xi-1)}{f^2(2\xi+1)^3} \rightarrow \begin{cases} \xi < \frac{1}{2} & \text{max} \\ \xi > \frac{1}{2} & \text{min} \end{cases}$$

$$\phi_2 = \pi f \qquad U_{\phi\phi}(\phi_2) = \frac{\Lambda^4}{f^2} \rightarrow \text{(absolute) min}$$

$$\phi_3 = f\arccos\left(\frac{1-\xi}{\xi}\right) \qquad U_{\phi\phi}(\phi_3) = \frac{\Lambda^4(1-2\xi)}{8f^2\xi} \rightarrow \begin{cases} \xi < \frac{1}{2} & \text{NA} \\ \xi > \frac{1}{2} & \text{max} \end{cases}$$

• Inflationary observables •

- SR parameters

$$\epsilon_U(\chi) = \frac{M_P^2}{2}\left(\frac{U'(\chi)}{U(\chi)}\right)^2$$

$$\eta_U(\chi) = M_P^2 \frac{U''(\chi)}{U(\chi)}$$

$$\xi_U^2(\chi) = M_P^4 \frac{U'(\chi)U'''(\chi)}{U(\chi)^2}$$

- observables

$$N_e = \frac{1}{M_P^2}\int_{\chi_{\rm end}}^{\chi_N} \mathrm{d}\chi \, \frac{U(\chi)}{U'(\chi)}$$

$$r = 16\epsilon_U(\chi_N)$$

$$n_{\rm s} = 1 + 2\eta_U(\chi_N) - 6\epsilon_U(\chi_N)$$

$$\alpha_{\rm s} \equiv \mathrm{d}n_{\rm s}/\mathrm{d}\ln k = 16\epsilon_U(\chi_N)\eta_U(\chi_N) - 24\epsilon_U^2(\chi_N) - 2\xi_U^2(\chi_N)$$

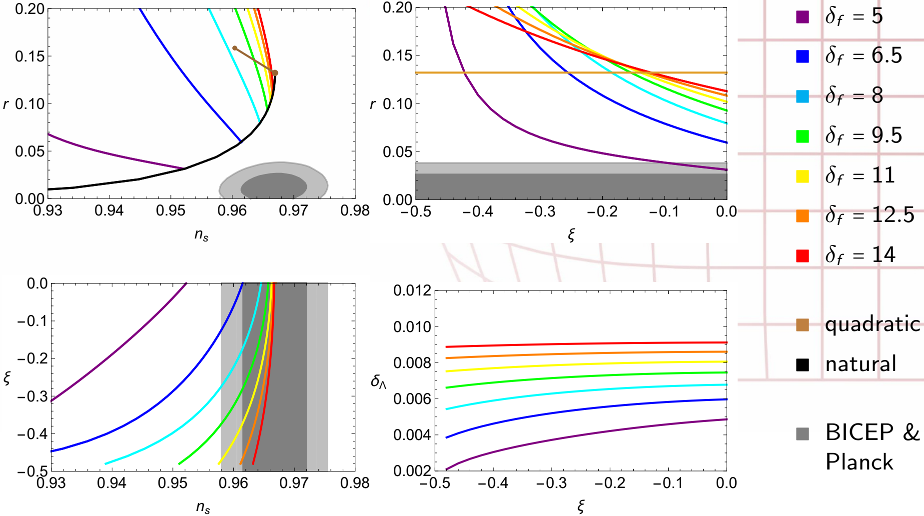
$$A_{\rm s} = \frac{1}{24\pi^2 M_P^4}\frac{U(\chi_N)}{\epsilon_U(\chi_N)}$$

• Palatini results •

- $\alpha(\phi) = \frac{M_P^2}{2}\left[1 + \xi\left(1 + \cos\left(\frac{\phi}{f}\right)\right)\right] > 0 \Rightarrow$ otherwise repulsive gravity
- $0 \leq (1 + \cos) \leq 2 \Rightarrow \xi \geq -1/2$
- $-1/2 \leq \xi < 0$ predictions very disfavored → Q & A
- $\xi > 0$ → now

Palatini results: $\xi < 0$

$\xi < 0 \quad \alpha = 0 \quad N_e = 60 \quad \delta_f = f/M_P \quad \delta_\Lambda = \Lambda/M_P$



• Palatini: α_s vs. n_s •

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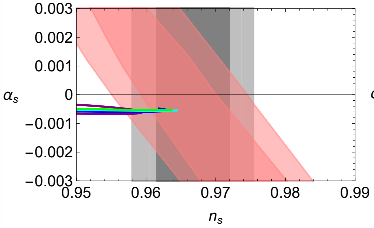
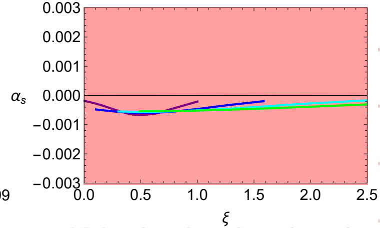




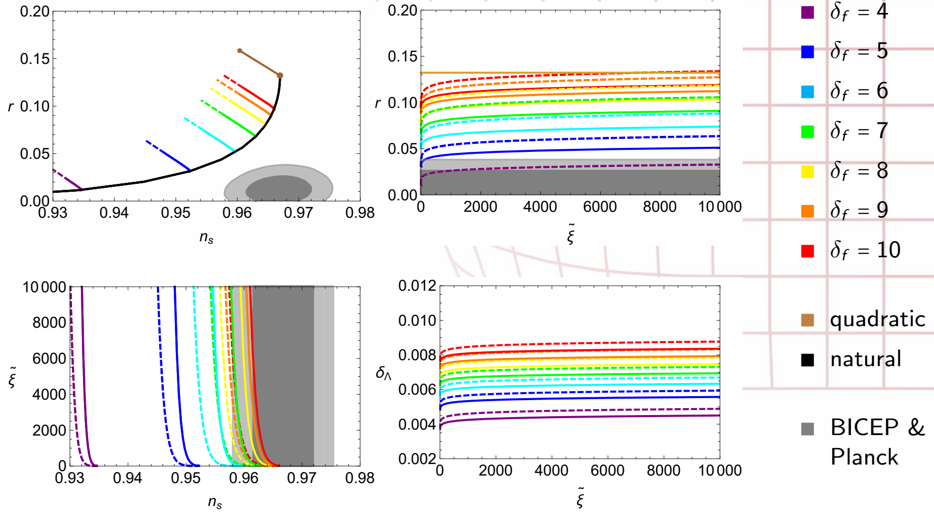
- $\delta_f = 4$
- $\delta_f = 6$
- $\delta_f = 8$
- $\delta_f = 10$
- $\delta_f = 12$
- $\delta_f = 14$
- $\delta_f = 16$

- quadratic
- natural

- r vs n_s
- α_s vs n_s by Planck

• $\xi = 0$ & $\tilde{\xi} > 0$ & $\beta_0 > 0$ •

$\tilde{\xi} > 0$ $\beta_0 \geq 0$ $N_e = 60$ $\delta_f = f/M_P$ $\delta_\Lambda = \Lambda/M_P$ — $\beta_0 = 2M_P^2$ - - $\beta_0 = 0$



• MAG $\xi = 0$: α_s vs. n_s •

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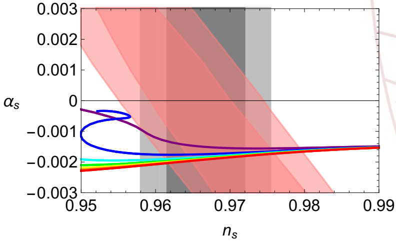

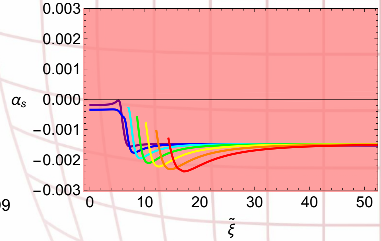

- $\delta_f = 4$
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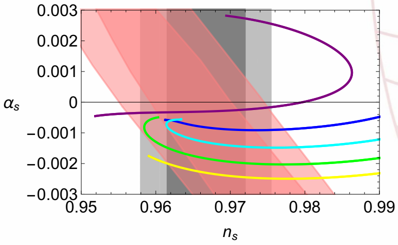

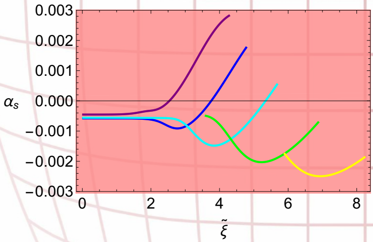

$\delta_f = 3$
$\delta_f = 5$
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$\delta_f = 9$
$\delta_f = 11$
$\delta_f = 13$
$\delta_f = 15$

quadratic
natural

r vs n_s
α_s vs n_s by Planck

• Inflaton model completion •

- dark QCD ($SU(3)_f$) with confinement scale f
- $\tilde{q}$ mass terms break the axial part of $SU(3)_f$

$$\mathcal{L}_{\mathrm{mass}} = \bar{\tilde{q}} M_q \tilde{q} = \bar{\tilde{q}}' \exp\Big(-i\gamma_5 B/(\sqrt{2}f)\Big) M_q \exp\Big(-i\gamma_5 B/(\sqrt{2}f)\Big)\tilde{q}'$$

where $\tilde{q}_i'$ are the Goldstone-free quark fields $\tilde{q}' = \exp\big(i\gamma_5 B/(\sqrt{2}f)\big)\tilde{q}$

- tilde-mesons as PNGBs

$$B \equiv \begin{pmatrix} \frac{\tilde{\pi}^0}{\sqrt{2}} + \frac{\tilde{\eta}^0}{\sqrt{6}} & \tilde{\pi}^+ & \tilde{K}^+ \\ (\tilde{\pi}^+)^\dagger & -\frac{\tilde{\pi}^0}{\sqrt{2}} + \frac{\tilde{\eta}^0}{\sqrt{6}} & \tilde{K}^0 \\ (\tilde{K}^+)^\dagger & (\tilde{K}^0)^\dagger & -\sqrt{\frac{2}{3}}\tilde{\eta}^0 \end{pmatrix}$$

- the lightest acts as the inflaton
- natural inflation potential arising from the $\tilde{q}$ mass terms
- ~~minimal~~ couplings $\tilde{q}$'s with gravity $\Rightarrow$ ~~minimal~~ couplings of ϕ with gravity

$$\bar{\tilde{q}} J \tilde{q} \mathcal{R}\,,\ \bar{\tilde{q}} J' \tilde{q} \tilde{\mathcal{R}} \Rightarrow \alpha\mathcal{R}\,,\ \beta\tilde{\mathcal{R}}$$