Charged Lepton Flavour Violation in the Inverse Seesaw

Based on the work "Charged lepton flavour violation from inverse seesaw with flavour and CP symmetries", in collaboration with C. Hagedorn, Soon to be Published

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Flash Introduction to Neutrino Physics

- 1930 : Pauli's hypothesis to explain β-decay
    - 1956: Discovered at Los Alamos (Reines andCowan)
- Supposed massless and only LH

Formulation of GSW theory of EW interactions (approx 1960-1970)

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- 1968 : Detection of Solar Neutrinos (Homestake exp.)
    - Only 1/3 of electron neutrinos observed...
- 1985 : Detection of Atmospheric Neutrinos (Kamiokande and IMB)
    - Smaller ratio of muon neutrinos to electron neutrinos observed
- 1998 : First evidence of Atmospheric Neutrino Oscillations (Super-Kamiokande)
- 2000 : Discovery of Tau neutrino (DONUT collaboration)

Formulation of GSW theory of EW interactions (approx 1960-1970)

Evidences of oscillation in the neutrino sector is incompatible with massless neutrinos!

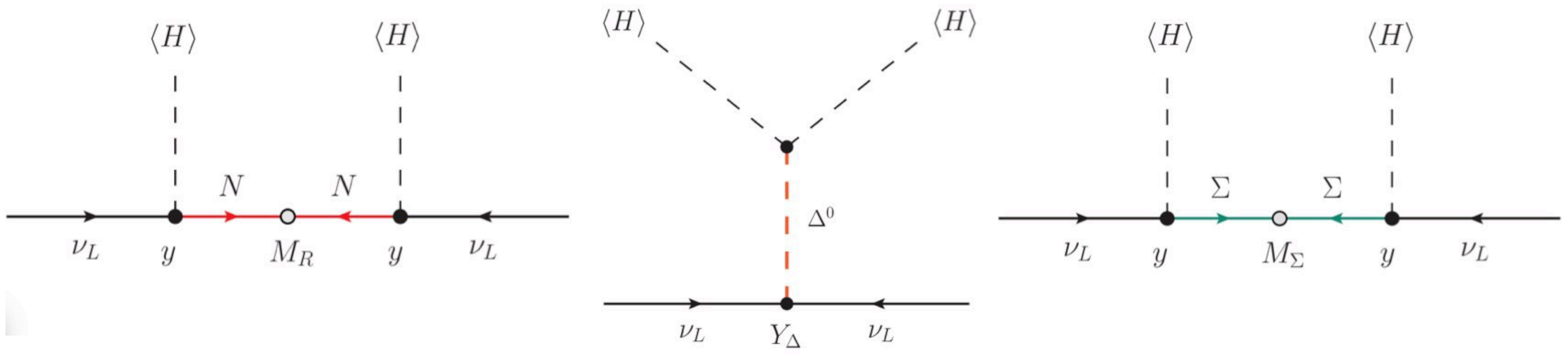
Flavour dynamics needs explanations!!

[https://neutrinos.fnal.gov/history/]

Neutrino Mass Generation

$$\mathcal{O}_W^{(5)} = \frac{1}{\Lambda}\langle LLHH \rangle$$

- Majorana Masses generated via Weinberg operators
- Common implementation: **<u>Seesaw Mechanism</u>**



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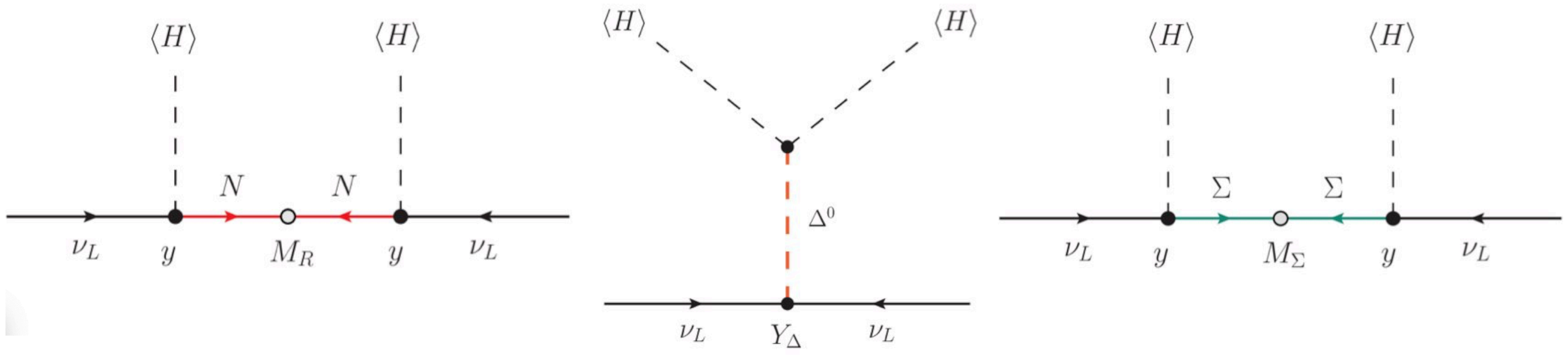

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For natural Yukawa size ($y \sim \mathcal{O}(1)$) small neutrino masses accommodated by

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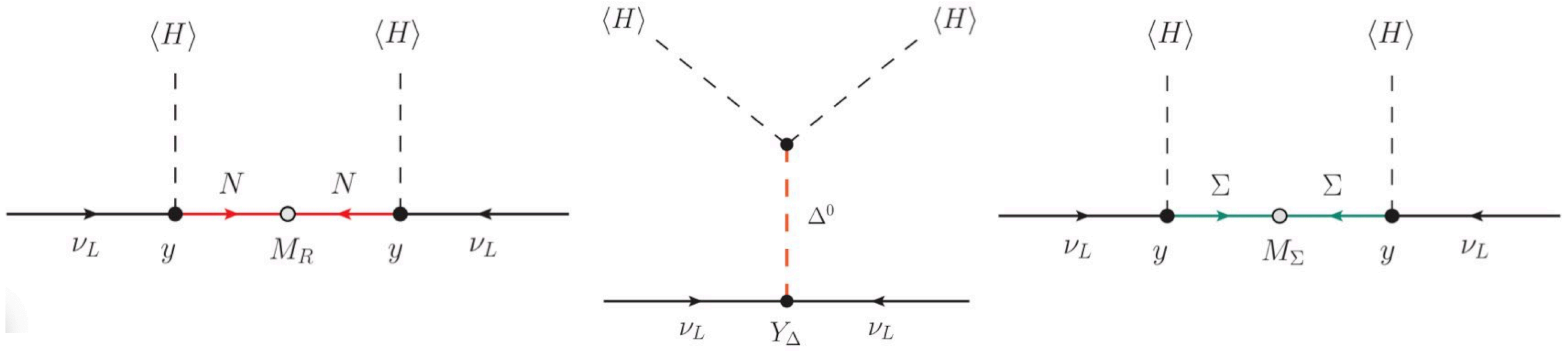
- Can we lower the mass scale?



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Inverse SeeSaw (ISS) Framework

$$\mathscr{L}_m = \bar{L}^c Y_D H N^c + \bar{N} M_{NS} S + \frac{1}{2} \bar{S}^c \mu_S S + h.c. = \left(\bar{\nu}_L \quad \bar{N}^c \quad \bar{S}\right)^c \mathscr{M} \begin{pmatrix} \nu_L \\ N^c \\ S \end{pmatrix}$$

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  - Light neutrino masses generated by the joint action of M_{NS} and small μ_s .
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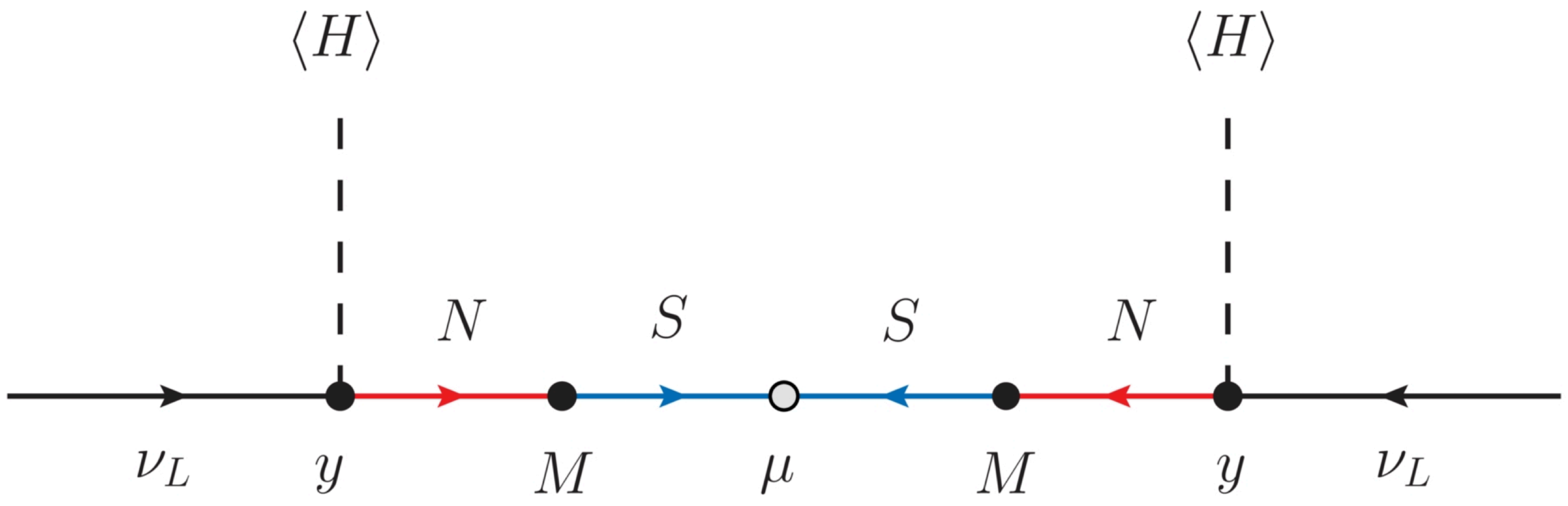

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In the basis in which charged lepton mass matrix is diagonal,
$\tilde{U}_\nu$ is the (**non-unitary**) leptonic mixing matrix

$$\tilde{U}_\nu = (1 - \eta)\, U_0$$

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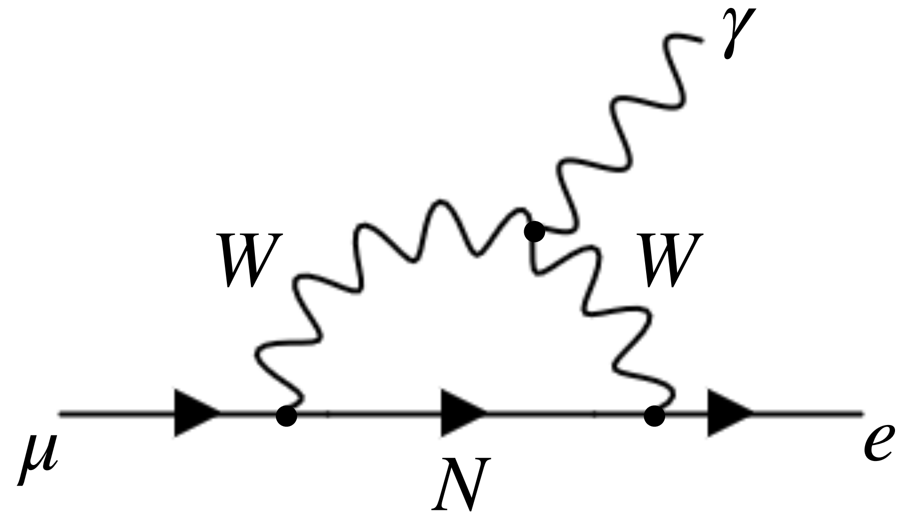
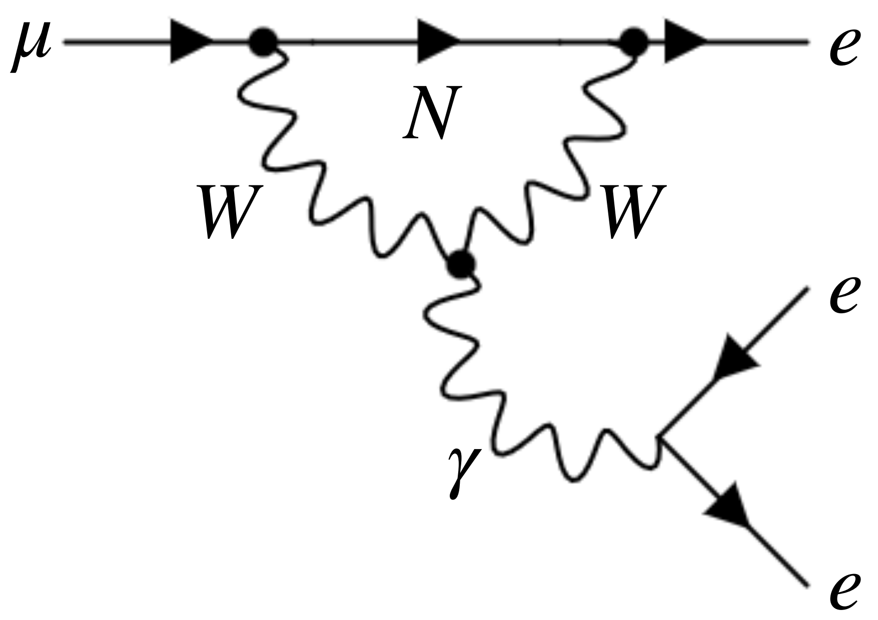
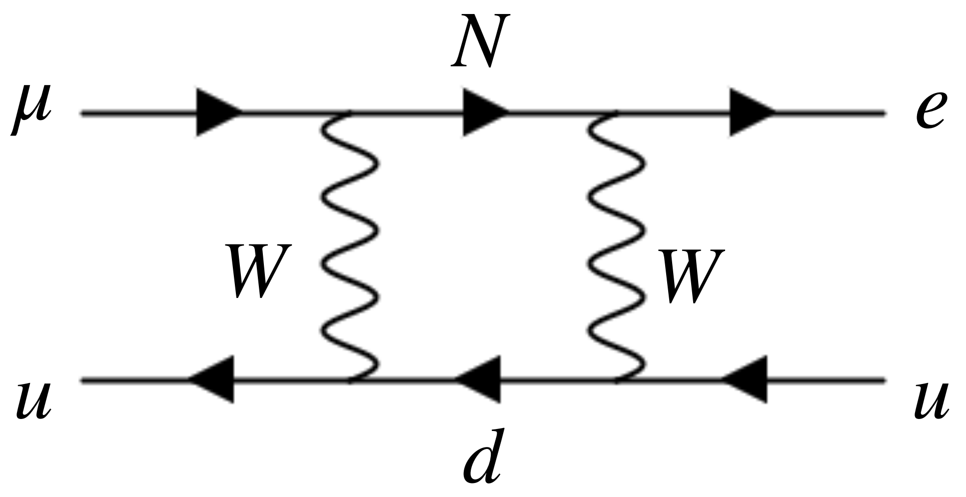
S, T describe the mixing of light and sterile neutrinos

$(S, T \ll \tilde{U}_\nu) \Rightarrow$ **Can induce cLFV processes**







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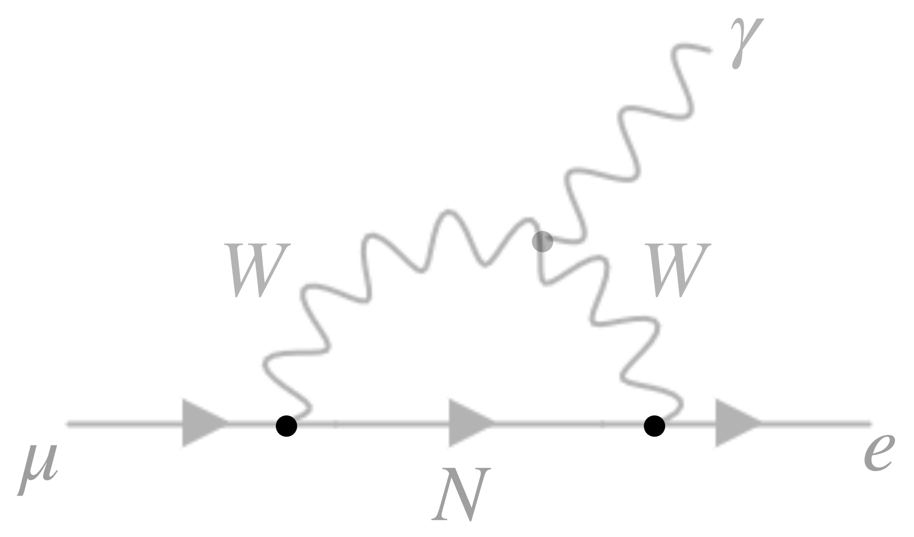
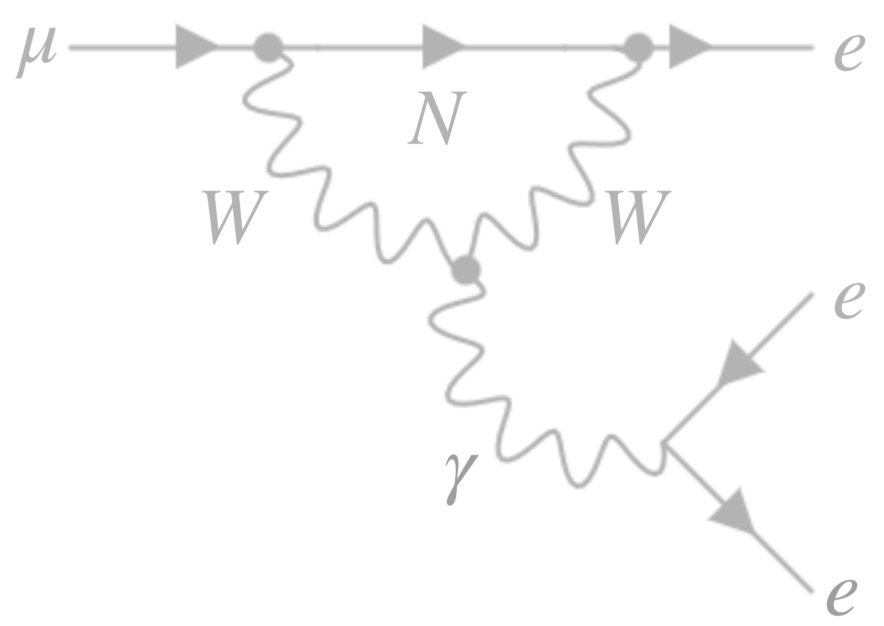
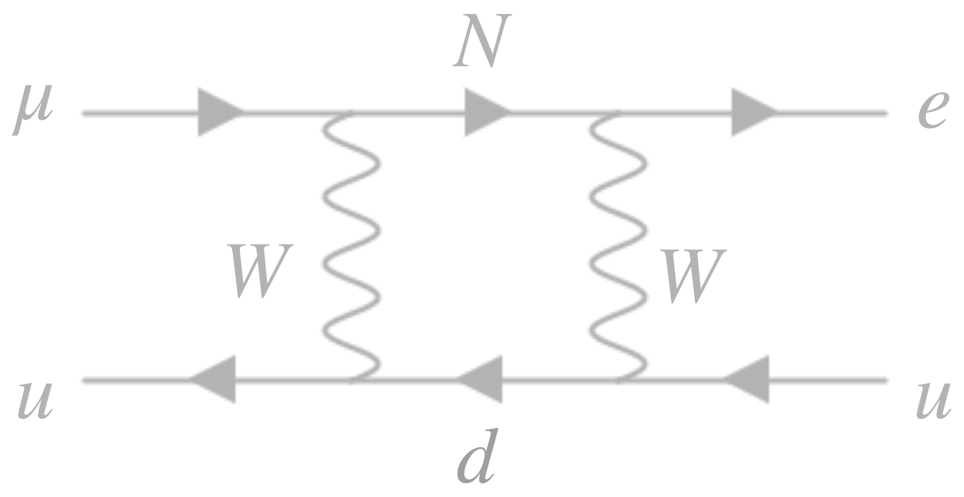
In this talk, we:

- **Reproduce Neutrino Masses and mixing within an ISS framework equipped with some Flavour symmetry group**
- **Show that in such context cLFV processes are predicted, and how future experiment could probe our parameter space**







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$\Delta(6n^2)$ **and** CP

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L, N, S assigned to triplet representations of the flavour group;

Various ways of choosing the residual symmetry are possible

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C. Hagedorn, J. Kriewald, J. Orloff, A. M. Teixeira, 2107.07537

Different choices of $\Delta(6n^2)$ representation result in different phenomenology:

- Option 1 : $L \sim 3 \;;\; N \sim 3 \;;\; S \sim 3$
- Option 2 : $L \sim 3 \;;\; N \sim 3' \;;\; S \sim 3'$
- Option 3 : $L \sim 3 \;;\; N \sim 3 \;;\; S \sim 3'$

$$Y_D = y_0 \frac{\langle H \rangle}{M_0} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M_{NS} = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mu_S = U_S^*(\theta_S) \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} U_S^T(\theta_S)$$

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In this discussion, we focus on **Option 2**

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_D^{(L)*}(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_D^{(R)T}(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_D^{(R)}(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_D^{(L)\dagger}(\theta_L)$$

Heavy neutral states mass matrix:

$$M_h \approx \begin{pmatrix} \varnothing & M_{NS} \\ M_{NS}^T & \mu_S \end{pmatrix}$$

Heavy neutral states spectrum: **Three approximately <u>degenerate</u> Pseudo-Dirac Pairs**

$$m_{(i=4,5,6)} \approx M_0 - \frac{1}{2}\mu_0 \qquad m_{(i=7,8,9)} \approx M_0 + \frac{1}{2}\mu_0$$

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Group theoretical parameters specify the residual symmetry : n, s, t

E.g.:

$$\Omega(3)(u, v) = e^{i\frac{v\pi}{n}} U_{TB} R_{13}\left(-\frac{u\pi}{2n}\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\frac{v\pi}{n}} & 0 \\ 0 & 0 & -i \end{pmatrix}$$

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$R_{ij}(\theta_{L,R})$ is a rotation on the ij plane

Codifies residual freedom in the choice of Ω matrices

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Parameters of the theory:

$$M_0\,,\, \mu_0\,,\, y_i\,,\, \theta_L\,,\, \theta_R$$

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- In our numerical analysis:

$M_0 \in [150\ GeV;\ 10\ TeV]$ $\mu_0 \in [0.1\ keV;\ 10^2\ keV]$ $\theta_R \in [0;\ 2\pi]$

Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_D^{(L)*}(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_D^{(R)T}(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_D^{(R)}(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_D^{(L)\dagger}(\theta_L)$$

- In our numerical analysis:

$$M_0 \in [150\ GeV;\ 10\ TeV] \qquad \mu_0 \in [0.1\ keV;\ 10^2\ keV] \qquad \theta_R \in [0;\ 2\pi]$$

- Fixed by fitting LO predictions of the masses to experimental values

Option 2

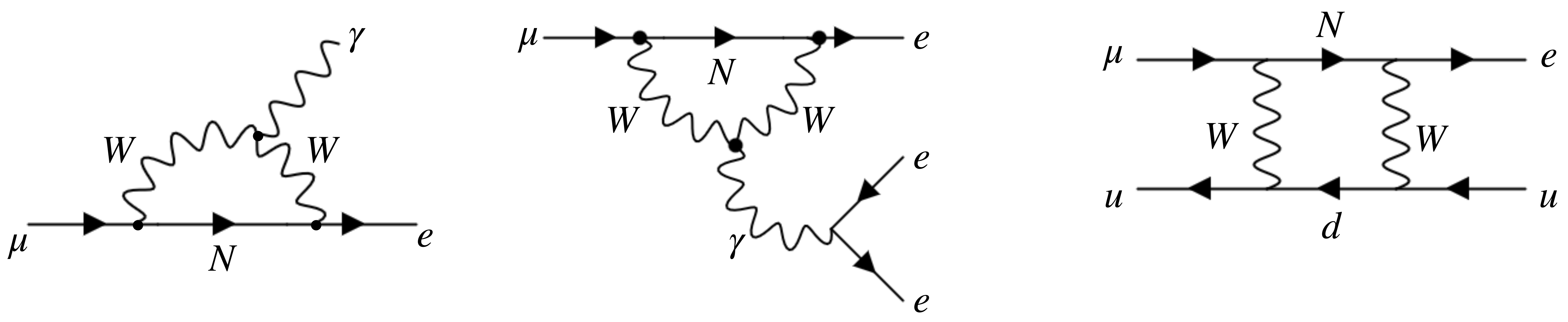
$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T = \langle H \rangle \frac{\mu_0}{M_0^2} U_D^{(L)*}(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_D^{(R)T}(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_D^{(R)}(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_D^{(L)\dagger}(\theta_L)$$

- In our numerical analysis:

$$M_0 \in [150\ GeV;\ 10\ TeV] \qquad \mu_0 \in [0.1\ keV;\ 10^2\ keV] \qquad \theta_R \in [0;\ 2\pi]$$

- Fixed by fitting LO predictions of the masses to experimental values
- Fixed by fitting prediction of mixing to lepton mixing data

cLFV in the ISS



Relevant (approximated) loop functions:

$$G_\gamma^{\beta\alpha} \approx \eta_{\alpha\beta} + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

$$F_\gamma^{\beta\alpha} \approx -2\left(\frac{7}{12} + \frac{1}{6}\log x_0\right)\eta_{\alpha\beta} + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{mu_0}{M_0})$$

$$F_Z^{\beta\alpha} \approx \eta_{\alpha\beta}\left(5 - 3\log x_0\right) + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

$$F_{box}^{\beta 3\alpha} \approx -2\eta_{\alpha\beta} + \mathcal{O}(\frac{Y^2\mu_0}{M_0}, \frac{\mu_0^2}{M_0^2})$$

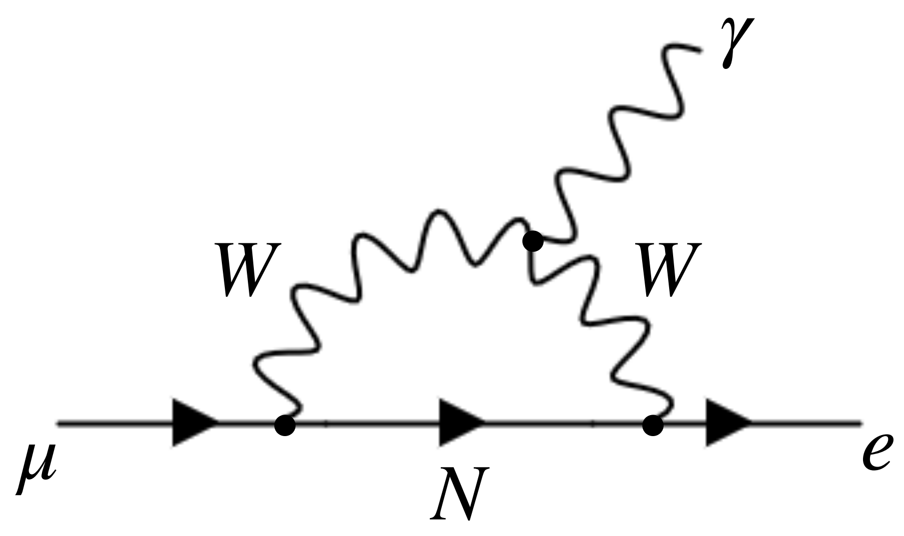
$$F_{box}^{\mu e u u} = 2\eta_{e\mu}\left[-4 - |V_{ub}|^2\left(F_{box}(0,x_b) - F_{box}(x_0, x_b) - 4\right)\right]$$

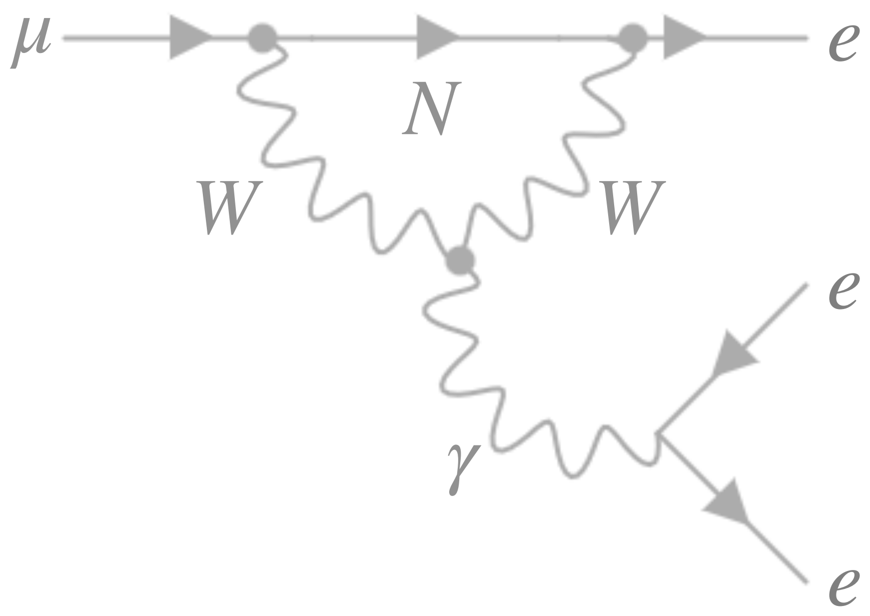
$$F_{box}^{\mu e d d} = 2\eta_{e\mu}\left[1 - |V_{td}|^2\left(F_{Xbox}(0,x_t) - F_{Xbox}(x_0, x_t) + 1\right)\right]$$

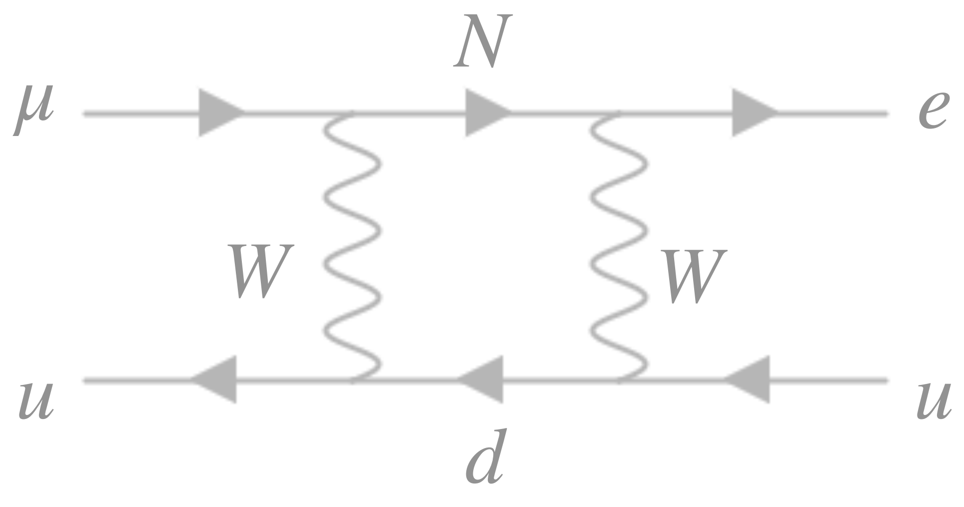
Only LO in ISS framework

Considered degenerate heavy neutral states, **with mass** M_0

cLFV in the ISS







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Photon Penguins

$$F_Z^{\beta\alpha} \approx \eta_{\alpha\beta}\left(5 - 3\log x_0\right) + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

$$F_{box}^{\beta3\alpha} \approx -2\eta_{\alpha\beta} + \mathcal{O}(\frac{Y^2\mu_0}{M_0}, \frac{\mu_0^2}{M_0^2})$$

$$F_{box}^{\mu euu} = 2\eta_{e\mu}\left[-4 - |V_{ub}|^2\left(F_{box}(0,x_b) - F_{box}(x_0, x_b) - 4\right)\right]$$

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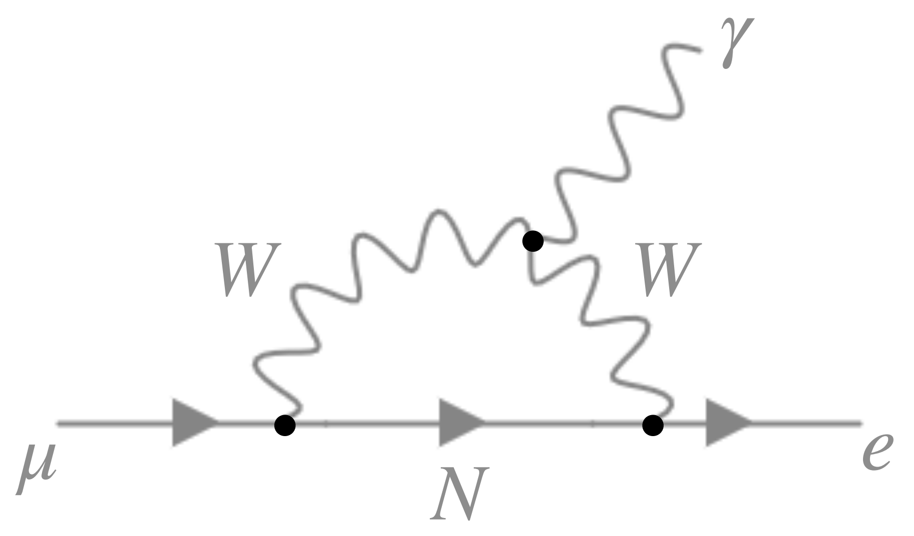
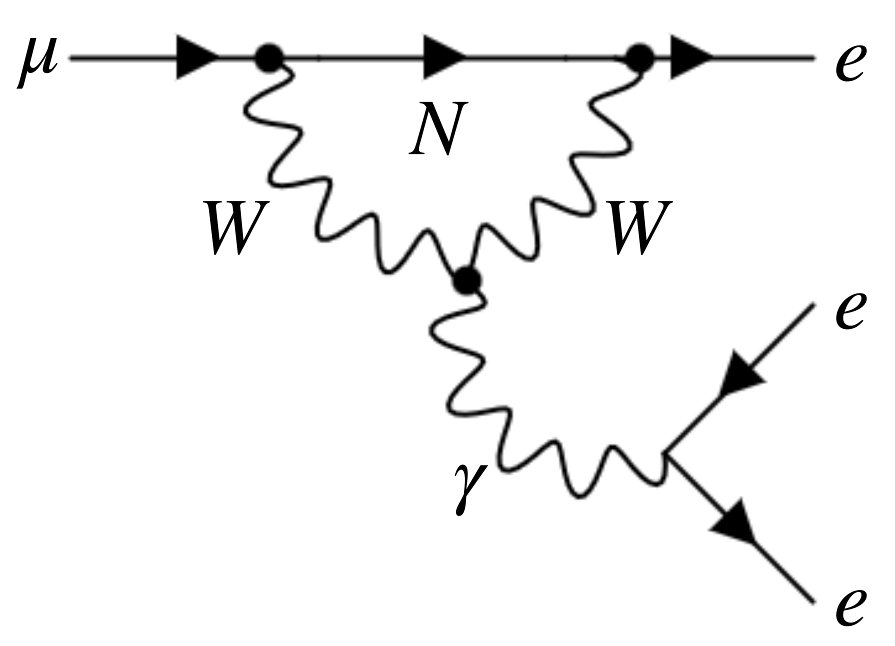
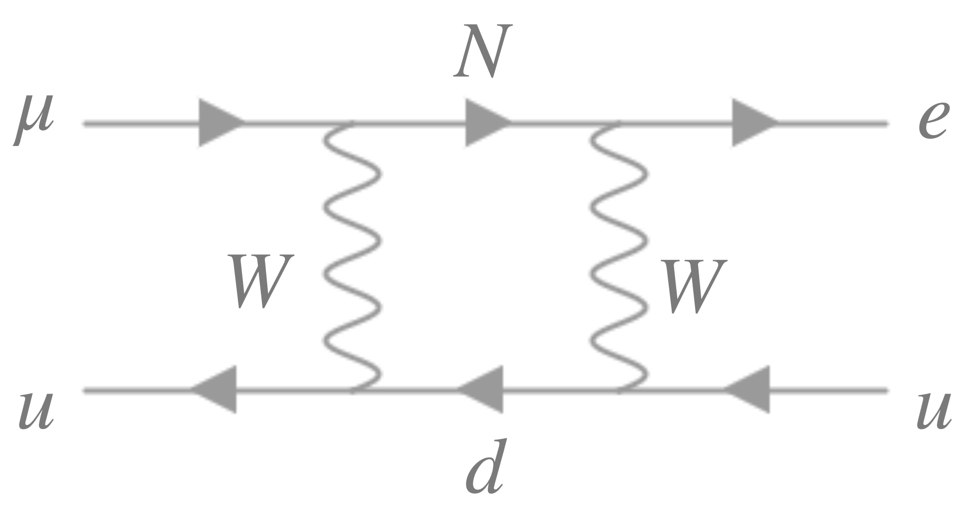
cLFV in the ISS







Relevant (approximated) loop functions:

$$G_\gamma^{\beta\alpha} \approx \eta_{\alpha\beta} + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

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Photon Penguins

$$F_Z^{\beta\alpha} \approx \eta_{\alpha\beta}\left(5 - 3\log x_0\right) + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

Z **Penguin**

$$F_{box}^{\beta 3\alpha} \approx -2\eta_{\alpha\beta} + \mathcal{O}(\frac{Y^2\mu_0}{M_0}, \frac{\mu_0^2}{M_0^2})$$

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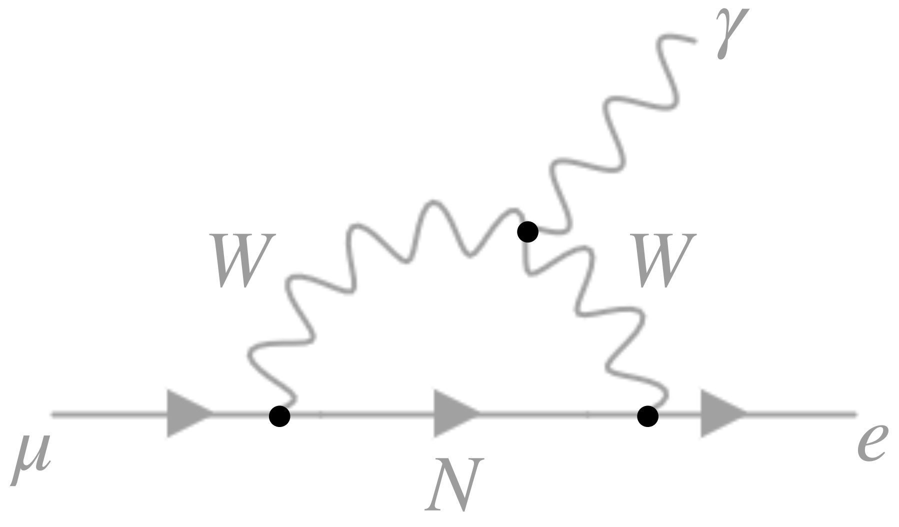
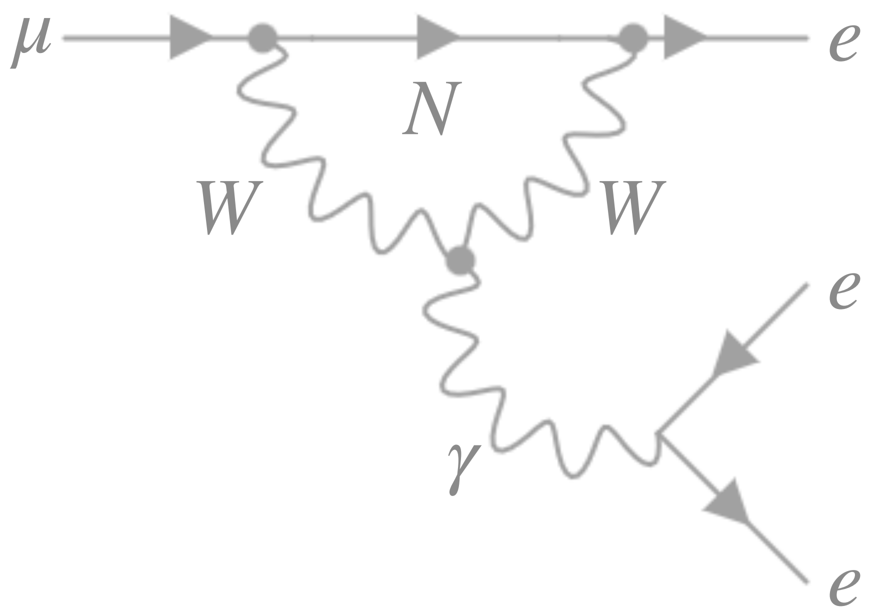
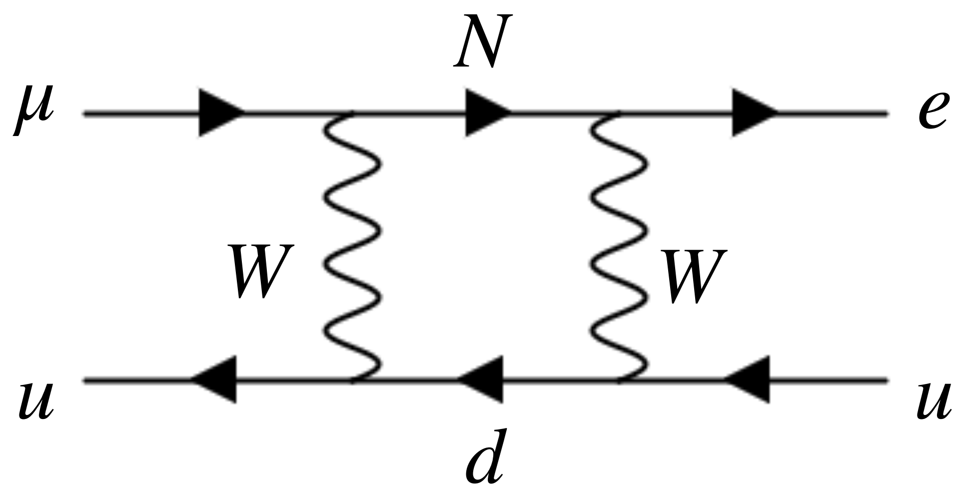
cLFV in the ISS







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Photon Penguins

$$F_Z^{\beta\alpha} \approx \eta_{\alpha\beta}\left(5 - 3\log x_0\right) + \mathcal{O}(\frac{Y\mu_0}{M_0}, \frac{\mu_0}{M_0})$$

Z Penguin

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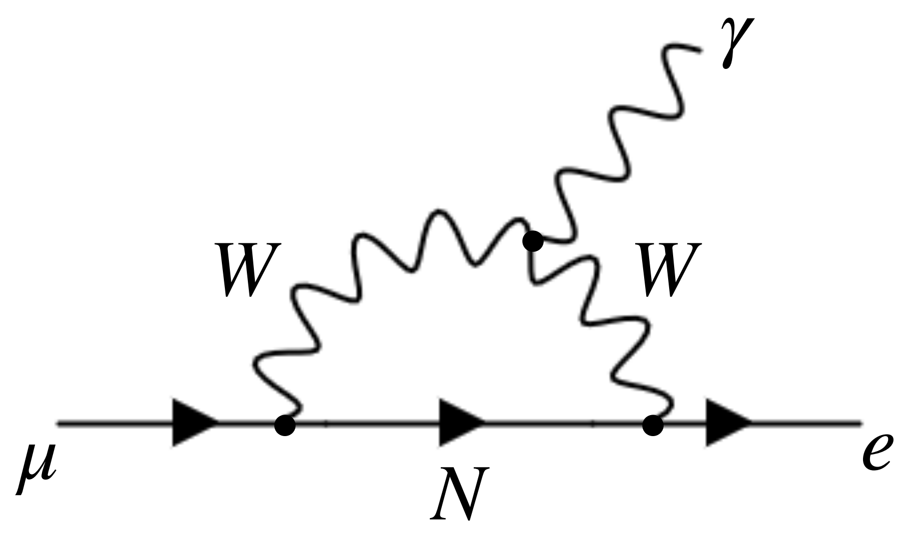
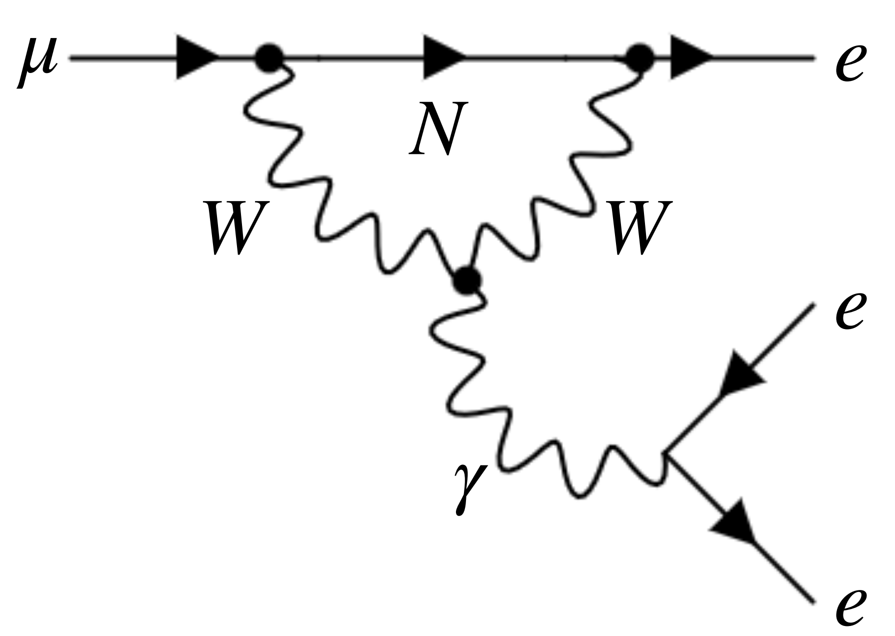
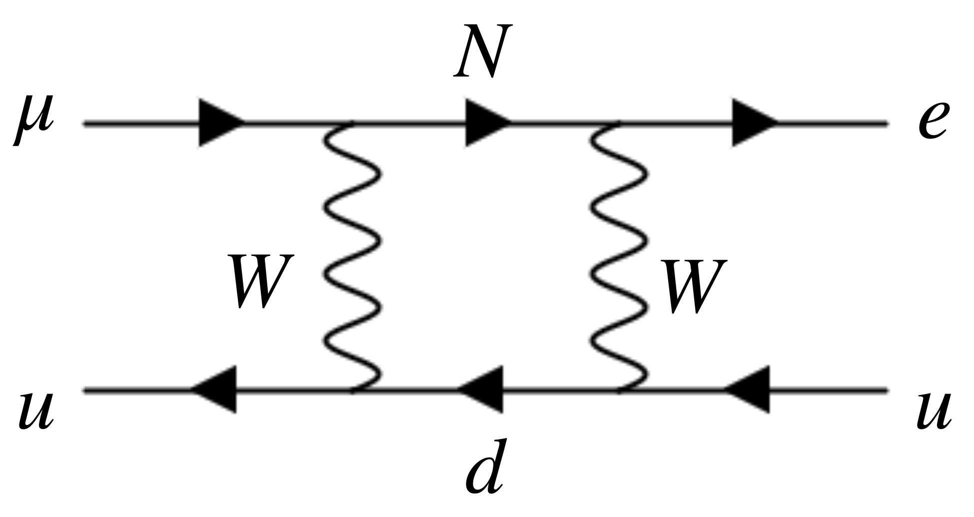
Box Diagrams

cLFV in the ISS







Relevant (approximated) loop functions:

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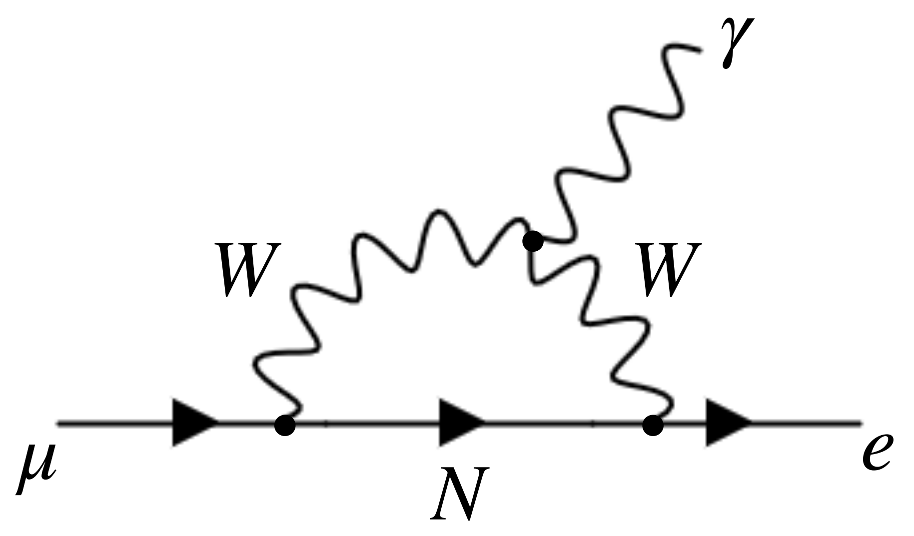
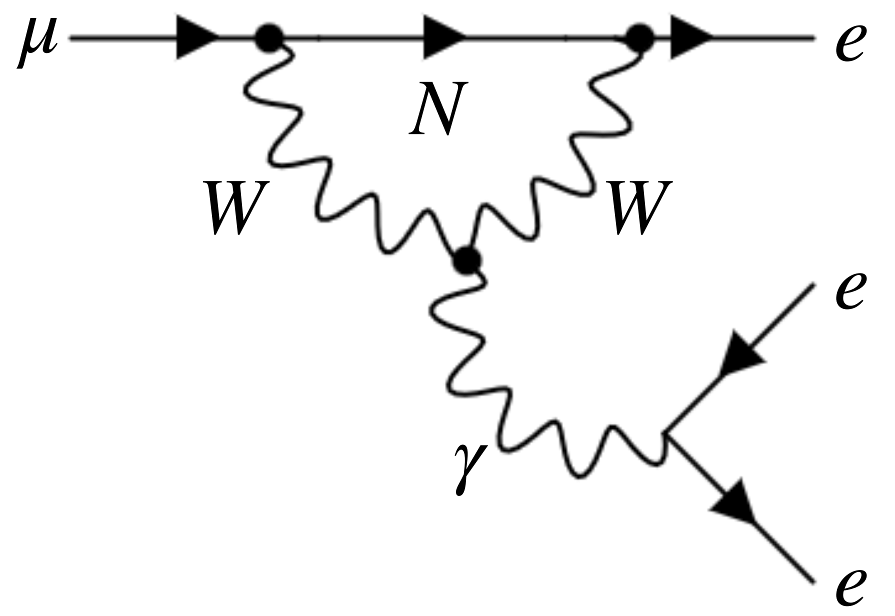
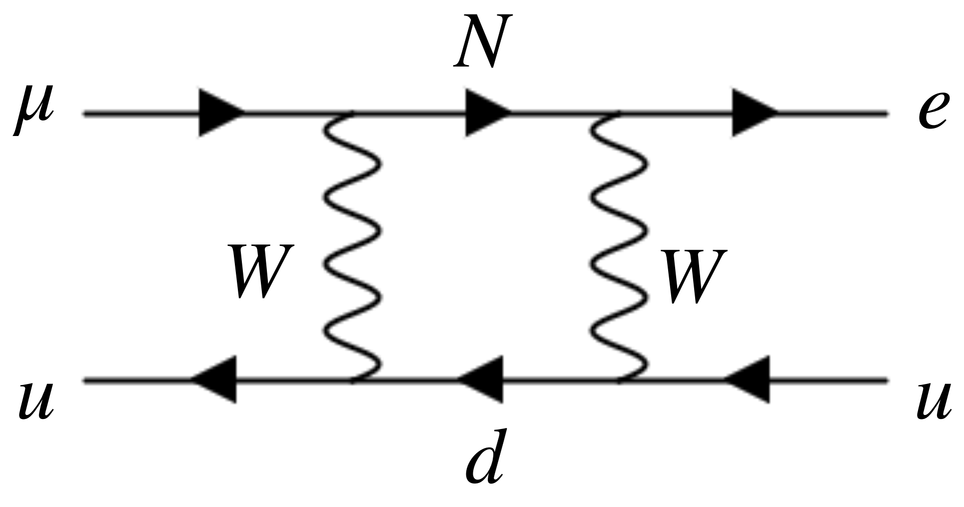
Rates of the processes are proportional to η at LO in ISS framework

cLFV in the ISS







Relevant (approximated) loop functions:

For a particular choice of residual symmetry (Case 2):

$$\eta_{\mu e} = \frac{1}{6}\eta_0' \left[2\Delta y_{21}^2 - \Delta y_{31}^2 \left(1 - \cos\left(2\theta_L\right)\cos\phi_u - \sqrt{3}\left(\cos\left(2\theta_L\right)\sin\phi_u + i\sin\left(2\theta_L\right)\right)\right)\right]$$

$$F_{box}^{\mu euu} = 2\eta_{e\mu}\left[-4 - |V_{ub}|^2\left(F_{box}(0,x_b) - F_{box}(x_0,x_b) - 4\right)\right]$$

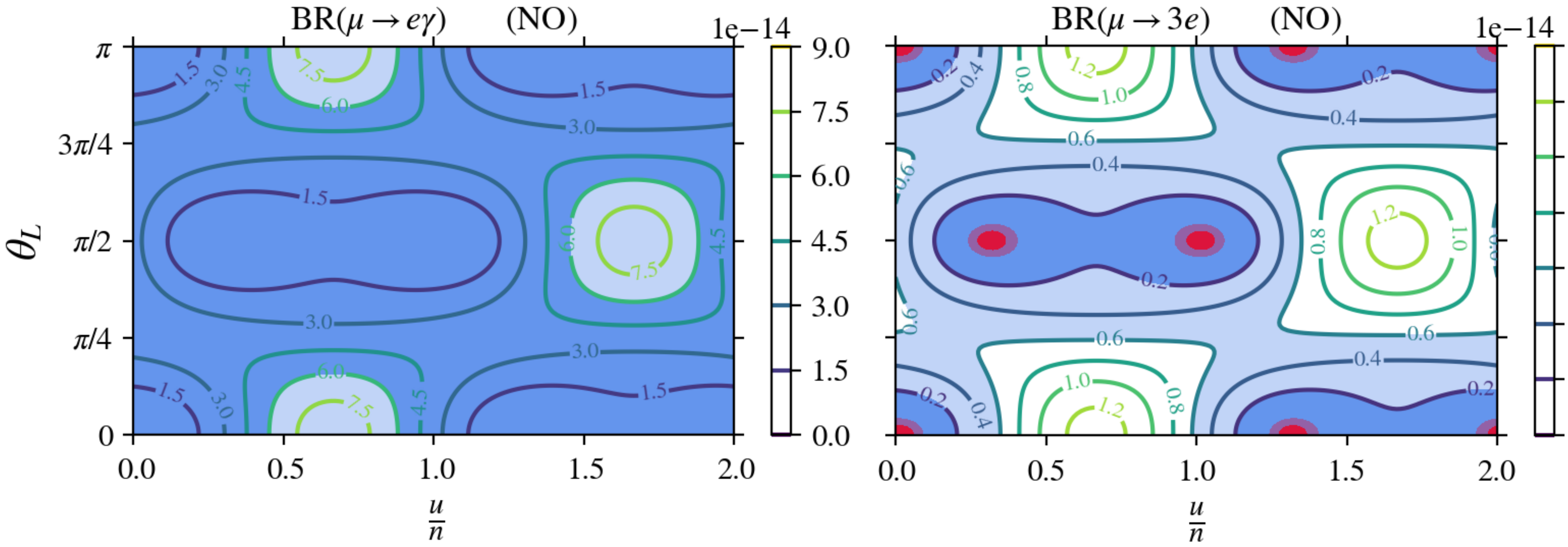
$$F_{box}^{\mu edd} = 2\eta_{e\mu}\left[1 - |V_{td}|^2\left(F_{Xbox}(0,x_t) - F_{Xbox}(x_0,x_t) + 1\right)\right]$$

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

($u = 2s - t$)

t **even**



$BR(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$ **Meg-II**

$BR(\mu \to 3e) \lesssim 20(1) \times 10^{-16}$ **Mu4E Phase-I (II)**

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **odd**

$BR(\mu \to e\gamma) \lesssim 6 \times 10^{-14}$ **Meg-II**

$BR(\mu \to 3e) \lesssim 20 \times 10^{-16}$ **Mu4E Phase-I (II)**

Option 2: Case 2

$M_0 = 3\ TeV$
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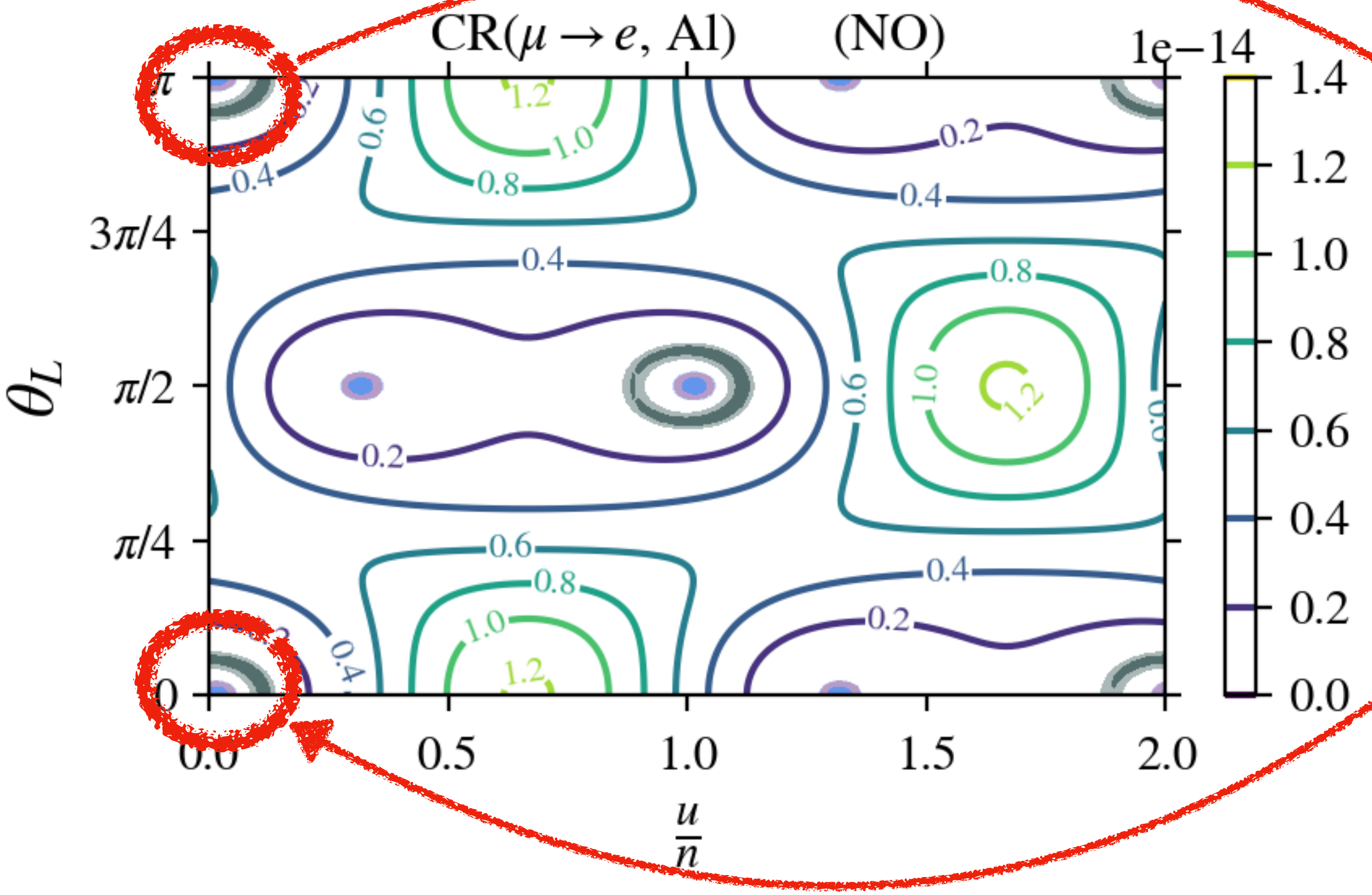
t **even**



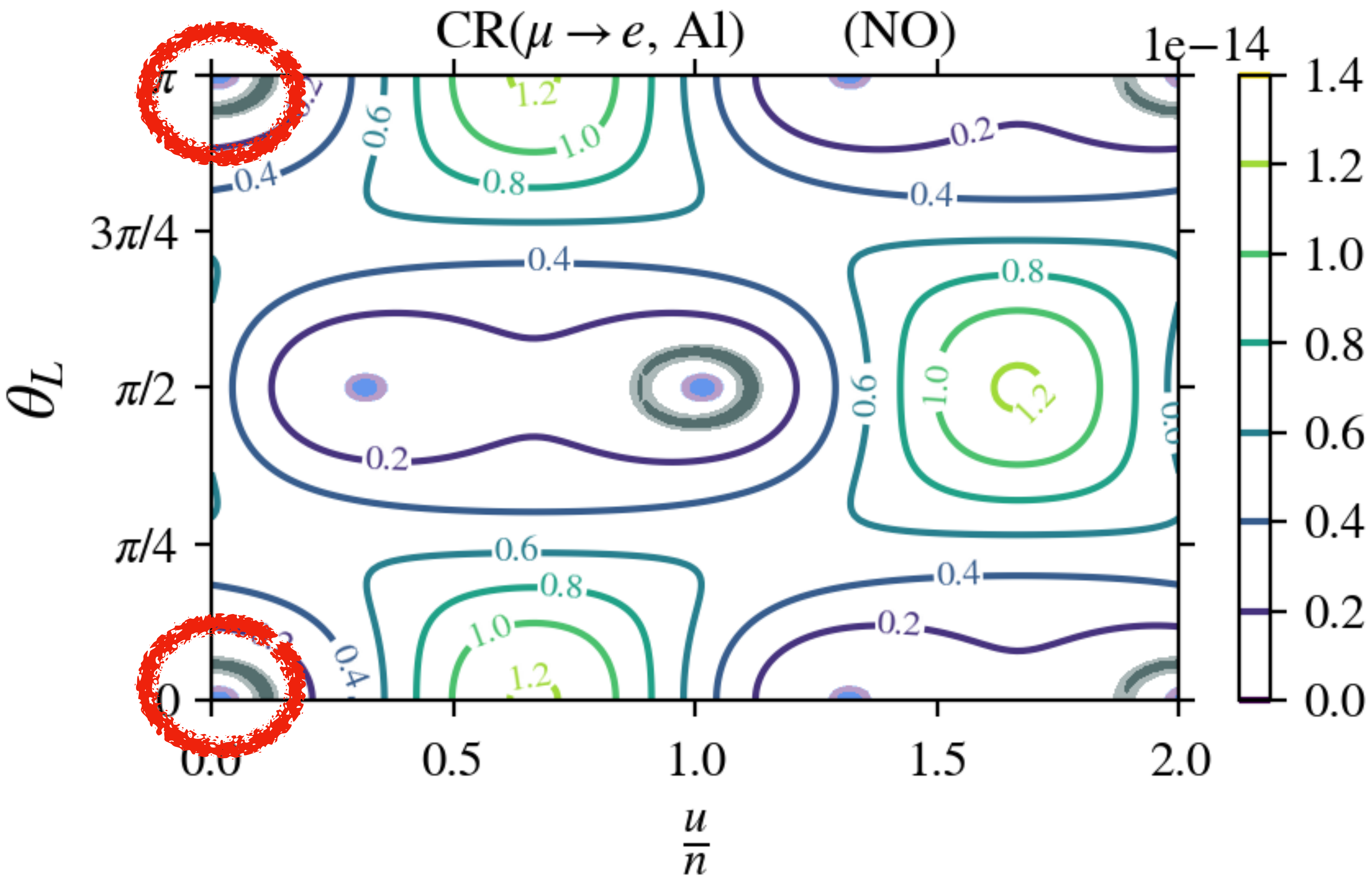
Double Solution for θ_L

$CR(\mu \to e\gamma) \lesssim 2.6(2.9) \times 10^{-17}$ **COMET Phase-II (Mu2E)**

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **even**



	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
$n=14;\ s=t=1$	0.34	0.021(0.022)	0.559(0.561)
$n=14;\ s=t=0$	0.341	0.022	0.5
$n=14;\ s=0;\ t=1$	0.34	0.022	0.436
$n=14;\ s=1;\ t=2$	0.341	0.022	0.5

$\theta_L \approx 0.183\ (2.959)$

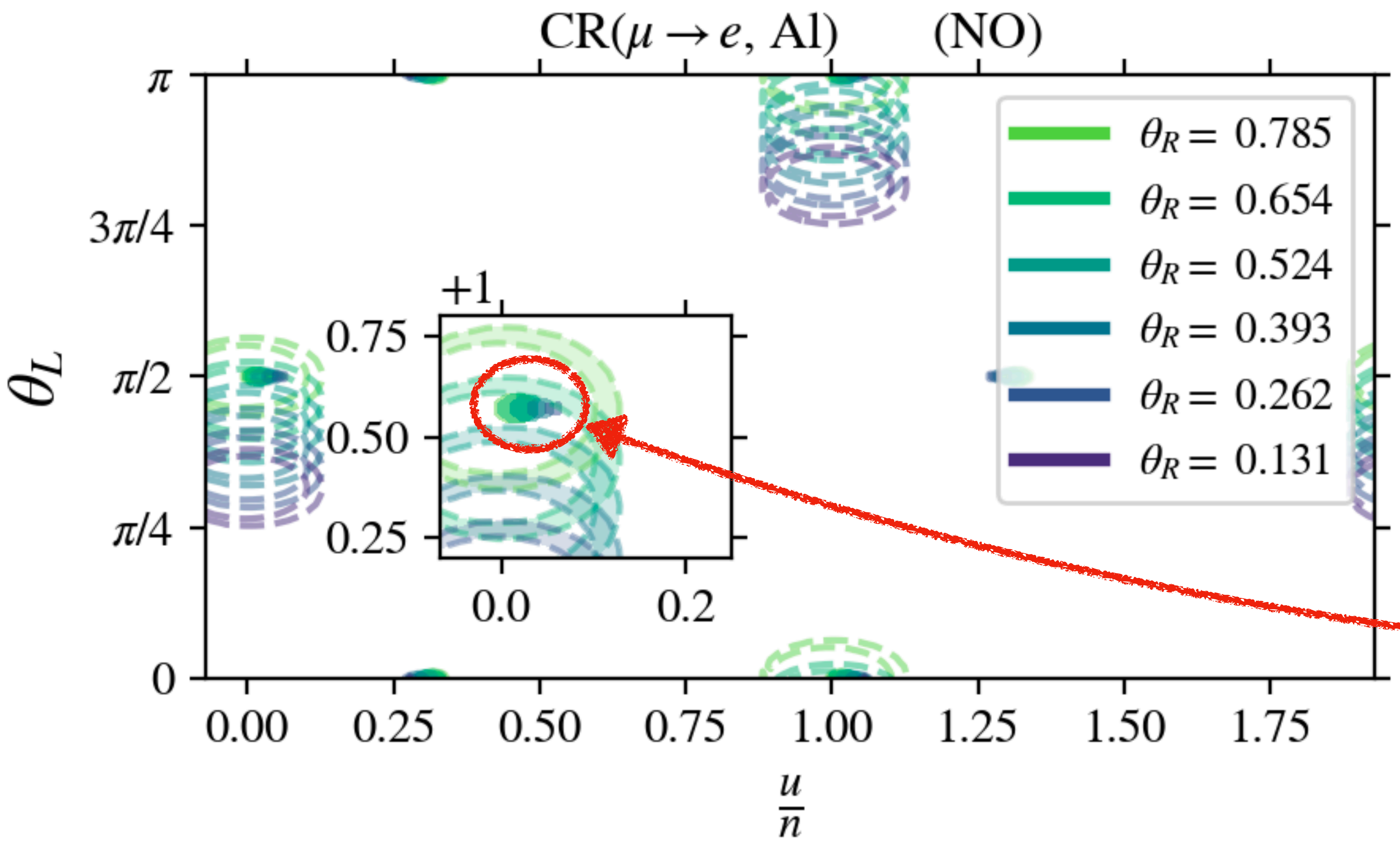
Best fit results: ($\chi^2_{mix} \leq 12$)

$CR(\mu \to e\gamma) \lesssim 2.6(2.9) \times 10^{-17}$ **COMET Phase-II (Mu2E)**

Option 2: Case 2

$M_0 = 3\ TeV$
$\mu_0 = 1\ keV$

t **odd**



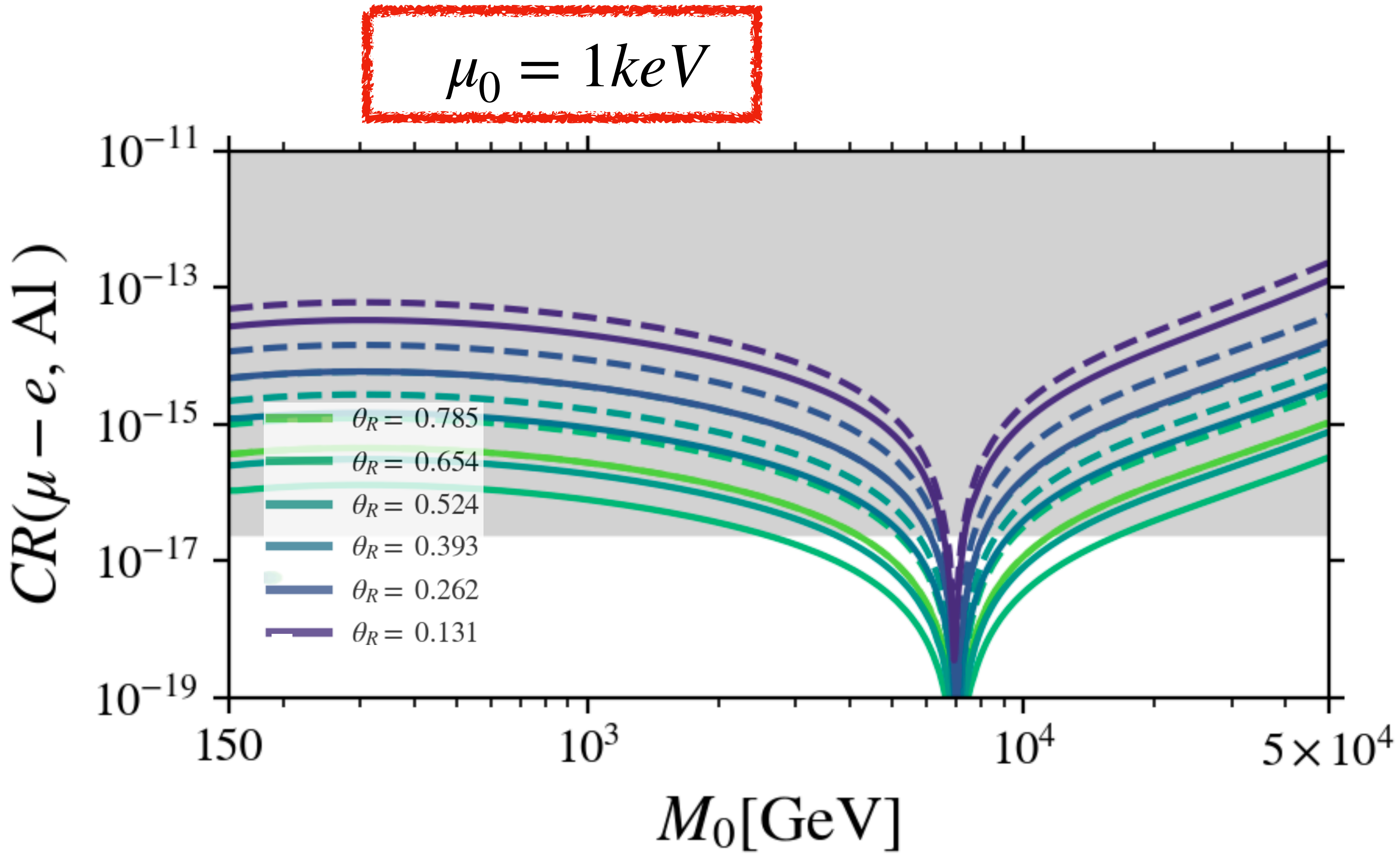
$CR(\mu \to e\gamma) \lesssim 2.6 \times 10^{-17}$ **COMET Phase-II**

θ_R Shifts best-fit regions upward:

$$\tan(\delta_\theta) = -\frac{y_i y_j \cot(2\theta_R)}{y_i^2 + y_j^2}$$

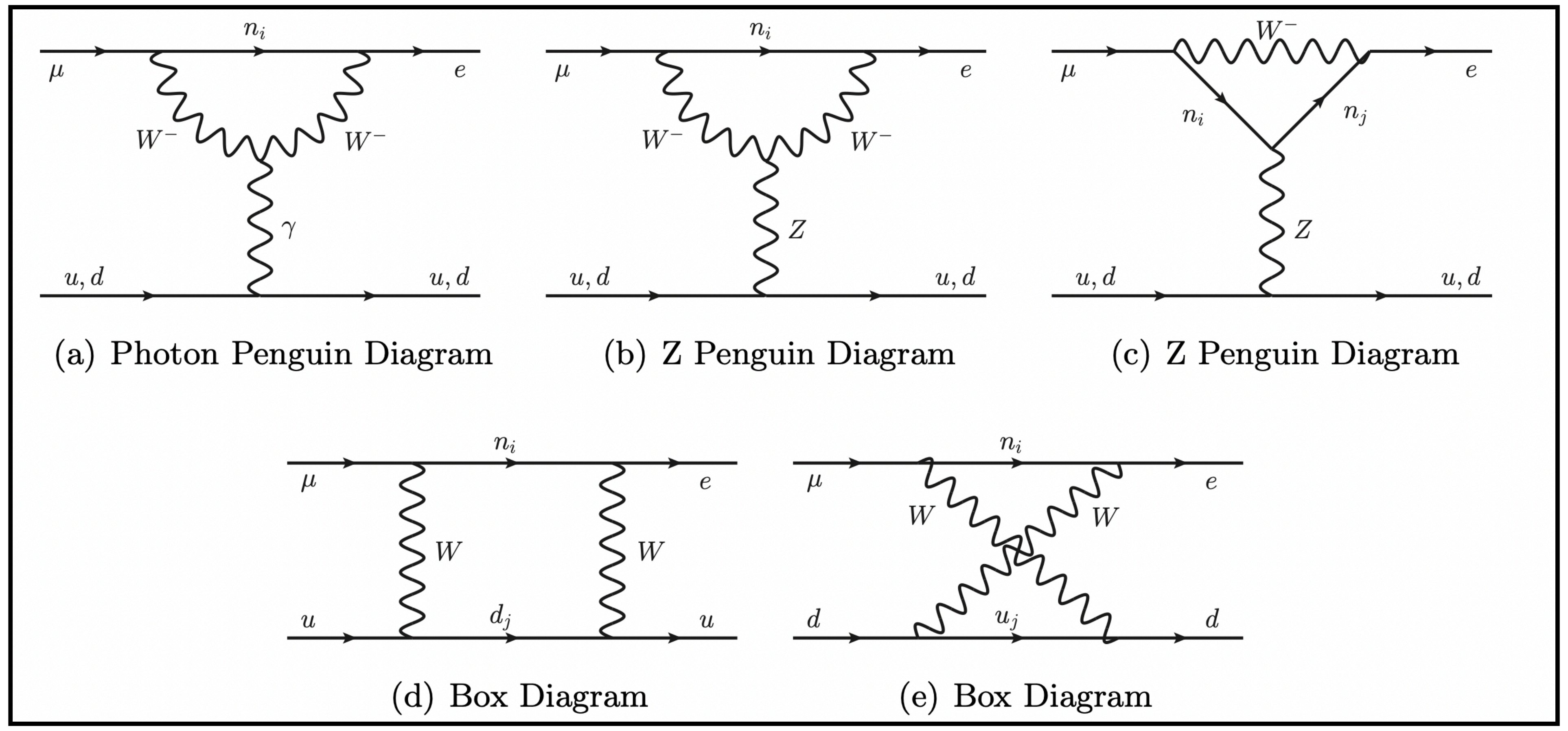
Different choices of θ_R give different predictions for cLFV

Option 2: Case 2



$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}}\left|4V^{(p)}\left(2\tilde{F}_u^{\mu e} + \tilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\tilde{F}_u^{\mu e} + 2\tilde{F}_d^{\mu e}\right) + s_w^2\frac{G_\gamma^{\mu e}D}{2e}\right|^2$$

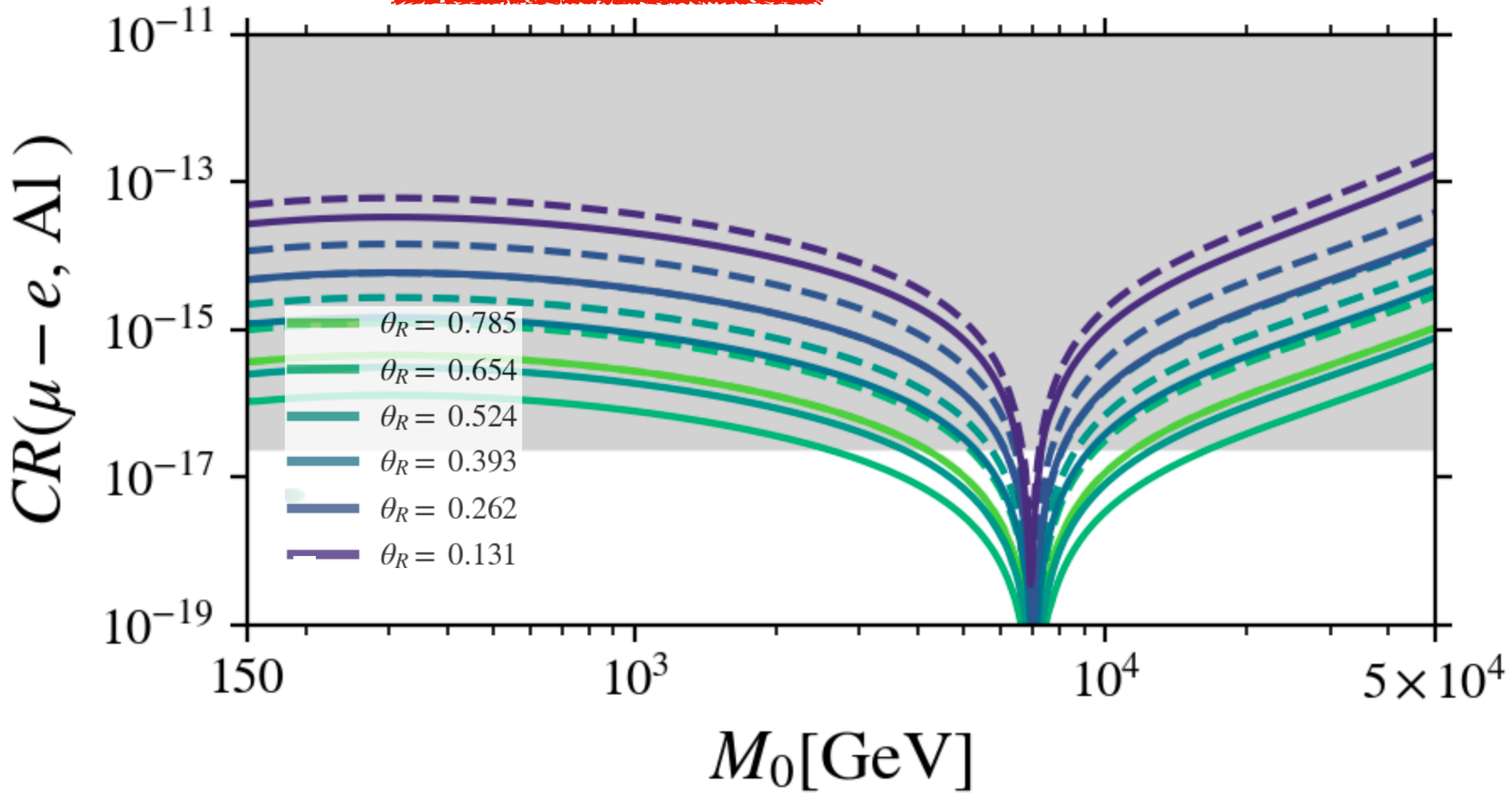
Different solutions for θ_L (at fixed θ_R)
also give **<u>different predictions for cLFV</u>**



R. Alonso, M. Dhen, M. B. Gavela, T. Hambye; 1209.2679

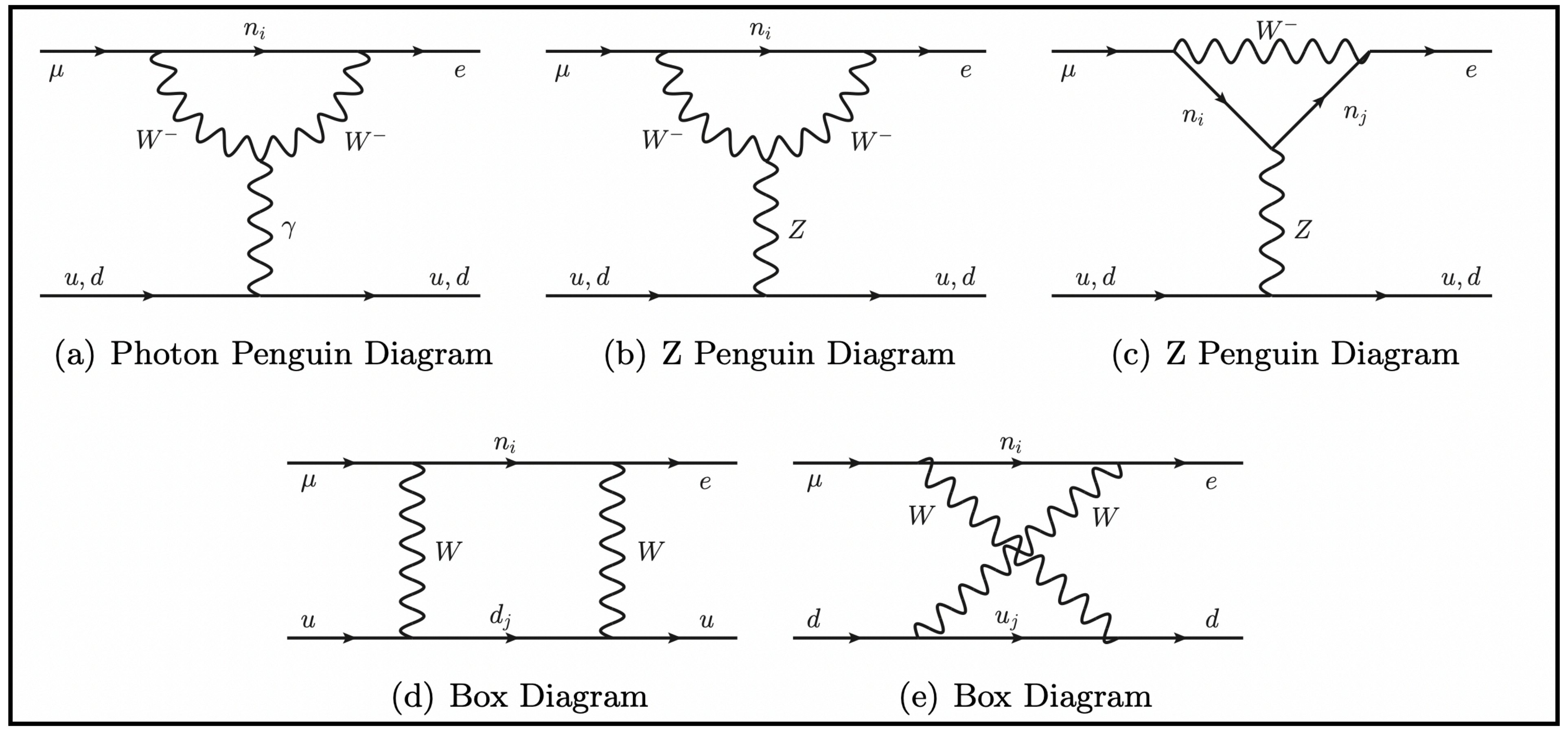
Option 2: Case 2

$\mu_0 = 1keV$



$$CR(\mu - e, Al) = \frac{2G_F^2\alpha_w^2 m_\mu^5}{(4\pi)^2\Gamma_{capt}} \left| 4V^{(p)}\left(2\tilde{F}_u^{\mu e} + \tilde{F}_d^{\mu e}\right) + 4V^{(n)}\left(\tilde{F}_u^{\mu e} + 2\tilde{F}_d^{\mu e}\right) + s_w^2 \frac{G_\gamma^{\mu e} D}{2e} \right|^2$$

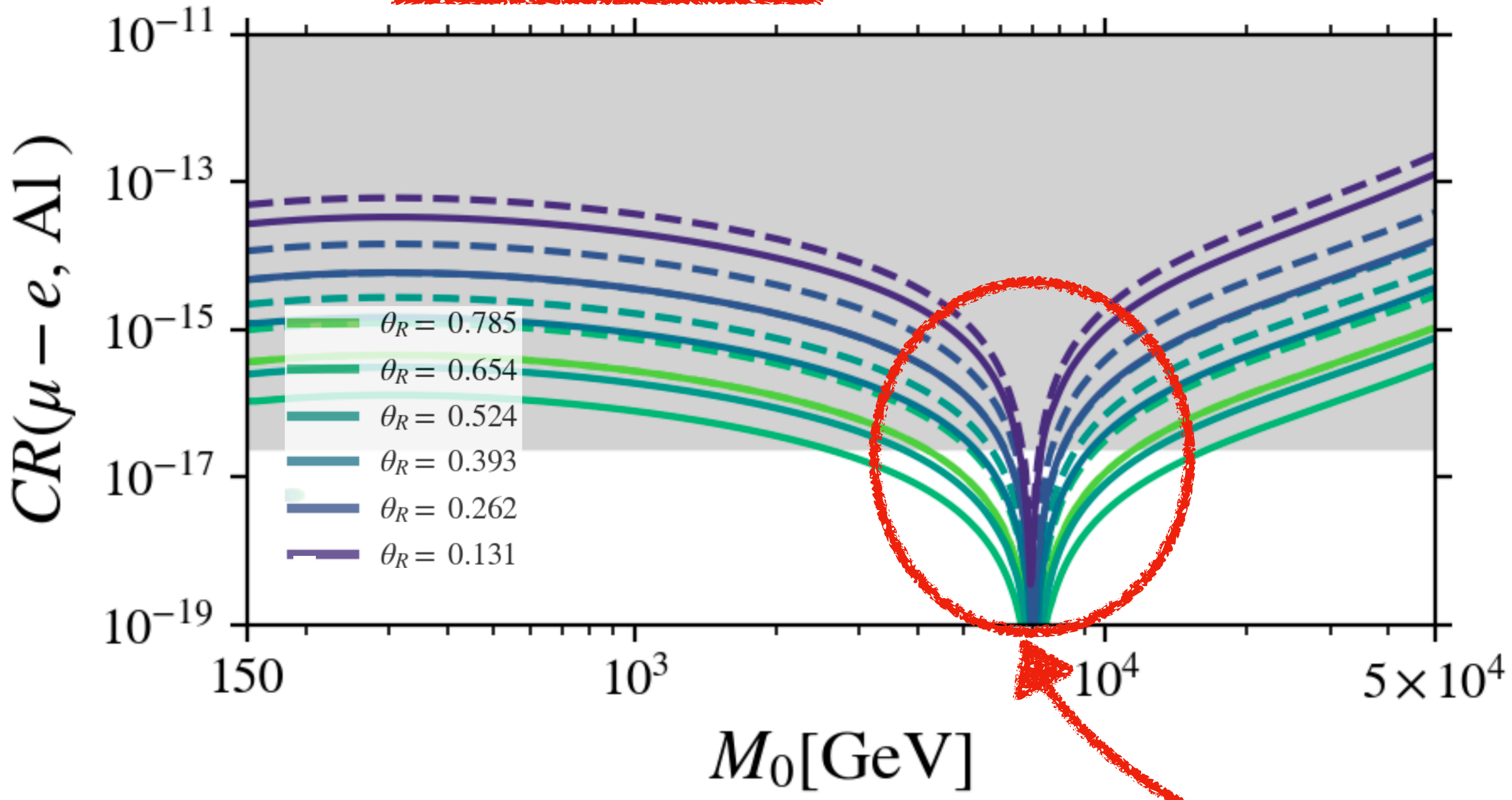
Up and down quark contributions have different sign due to different charge and weak isospin



R. Alonso, M. Dhen, M. B. Gavela, T. Hambye; 1209.2679

Option 2: Case 2

$\mu_0 = 1keV$



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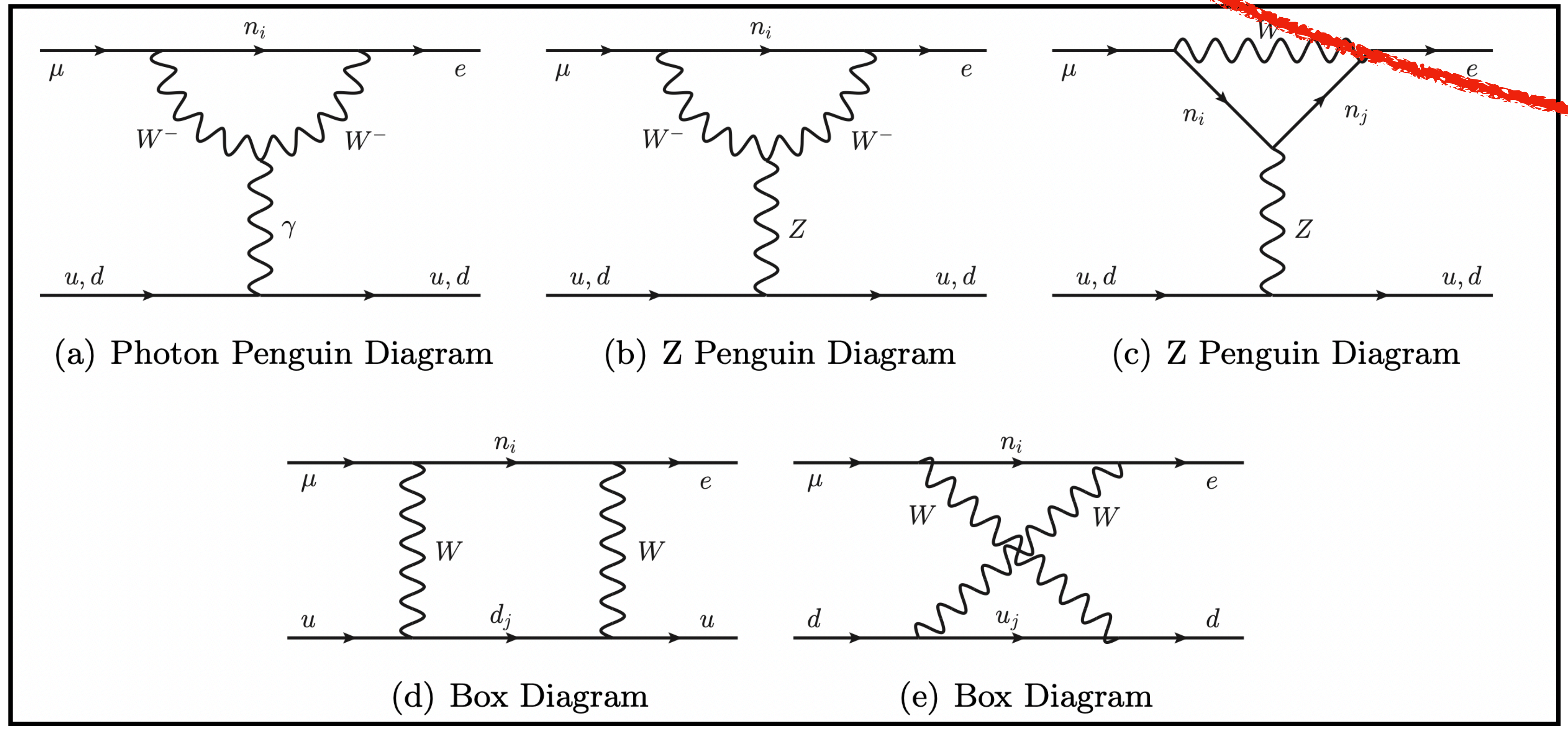
Up and down quark contributions have different sign due to different charge and weak isospin

Cancellation only depend on M_0:

$$M_0^2 = exp\left(\frac{\frac{9}{8}V^{(n)} + \left(\frac{9}{8} + \frac{37}{12}s_w^2\right)V^{(p)} - \frac{s_w^2}{16e}D}{\frac{3}{8}V^{(n)} + \left(\frac{4s_w^2}{3} - \frac{3}{8}\right)V^{(p)}}\right)M_W^2$$



(a) Photon Penguin Diagram (b) Z Penguin Diagram (c) Z Penguin Diagram (d) Box Diagram (e) Box Diagram

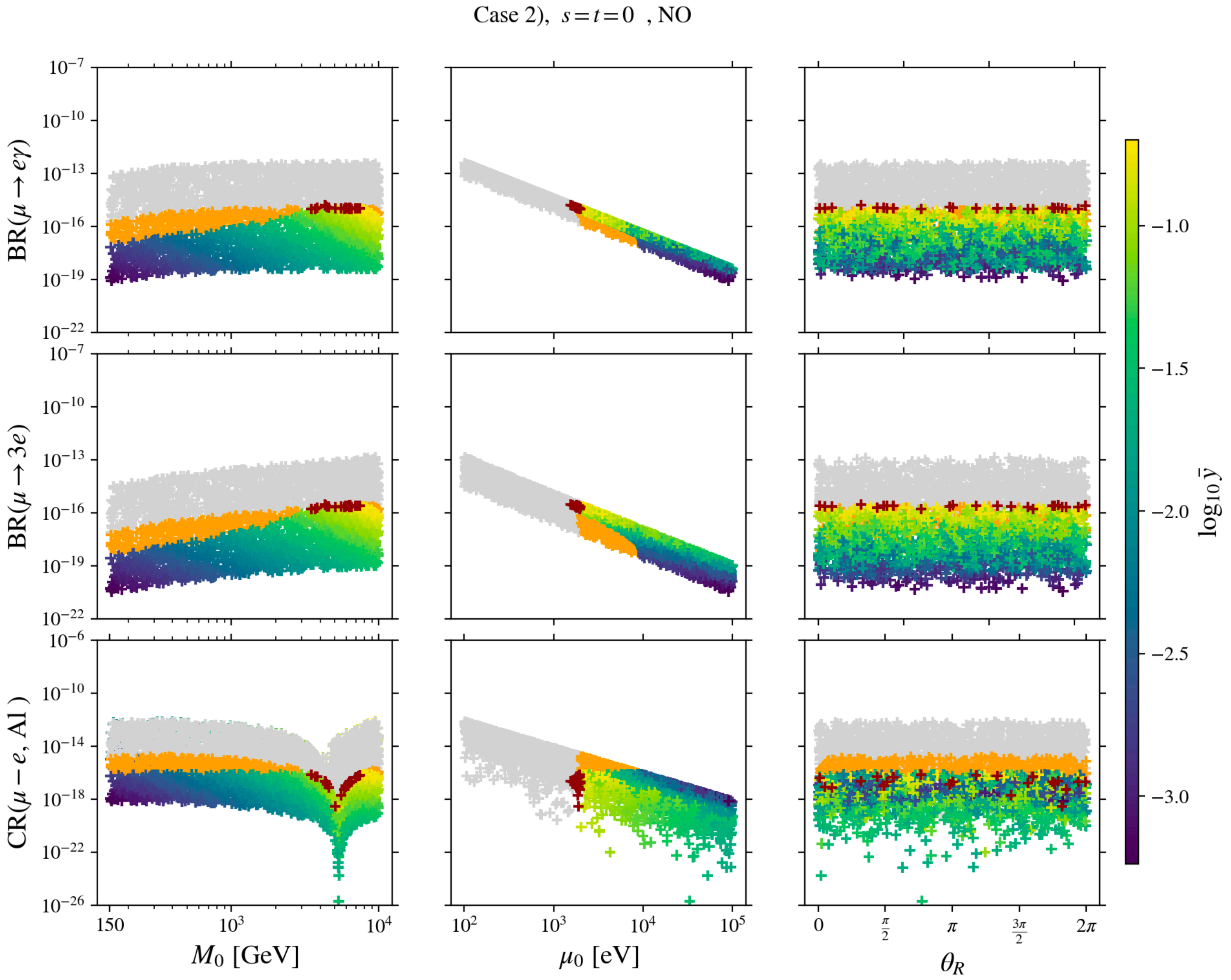
R. Alonso, M. Dhen, M. B. Gavela, T. Hambye; 1209.2679

Option 2: Case 2

Predictions are bounded by both experimental bounds on **cLFV processes** as by bounds on **unitarity violation**

$$\eta \lesssim \begin{pmatrix} 1.3 \times 10^{-3} & & \\ 1.2 \times 10^{-5} & 1.1 \times 10^{-5} & \\ 9.0 \times 10^{-4} & 5.7 \times 10^{-5} & 1.0 \times 10^{-3} \end{pmatrix}$$

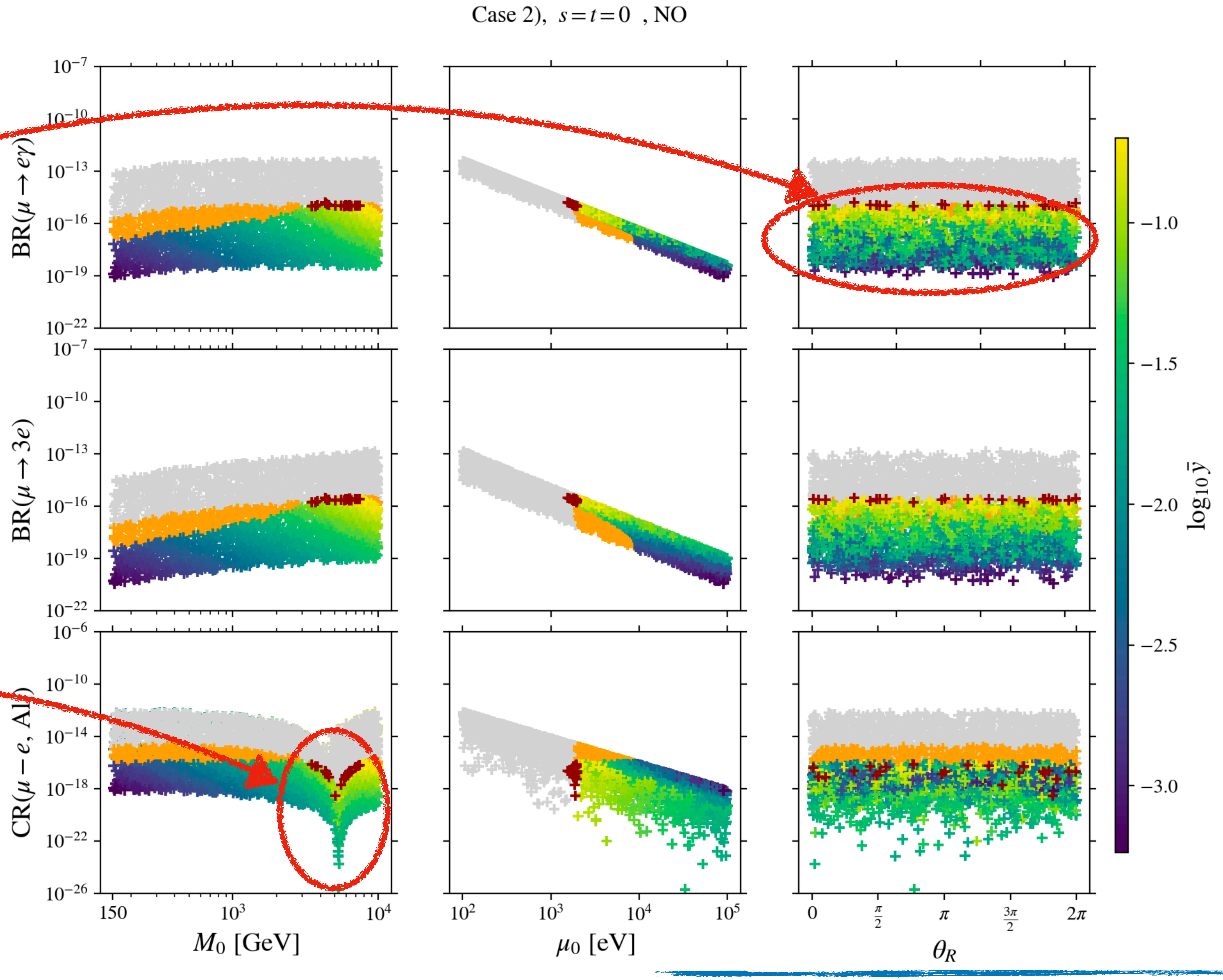
M. Blennow, E. Fernández-Martínez, J. Hernández-García, J. López-Pavón, X., D. Naredo-Tuero; 2306.01040



Option 2: Case 2

No dependence from θ_R

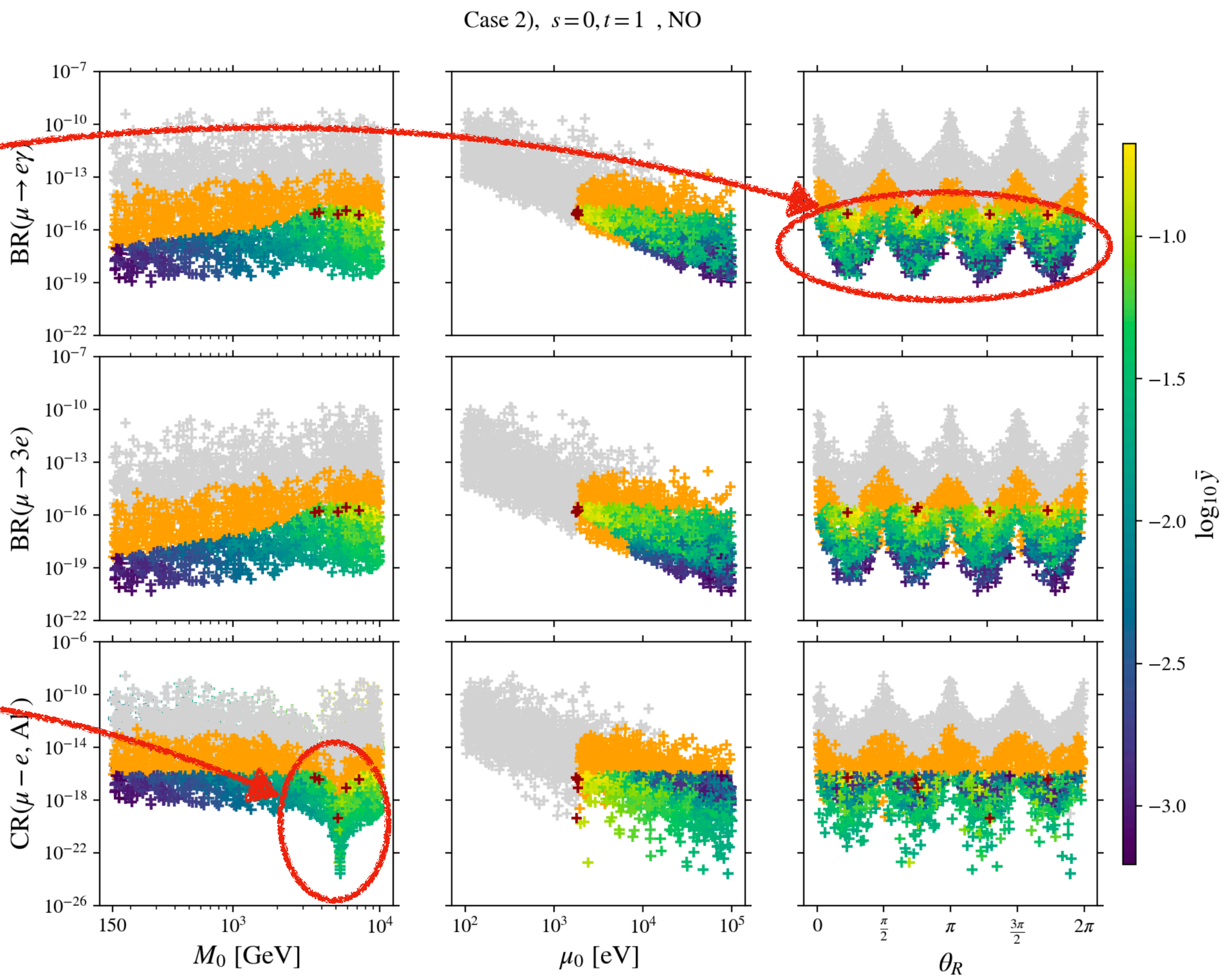
Cancellation of contributions



Option 2: Case 2

Modulation in function of θ_R

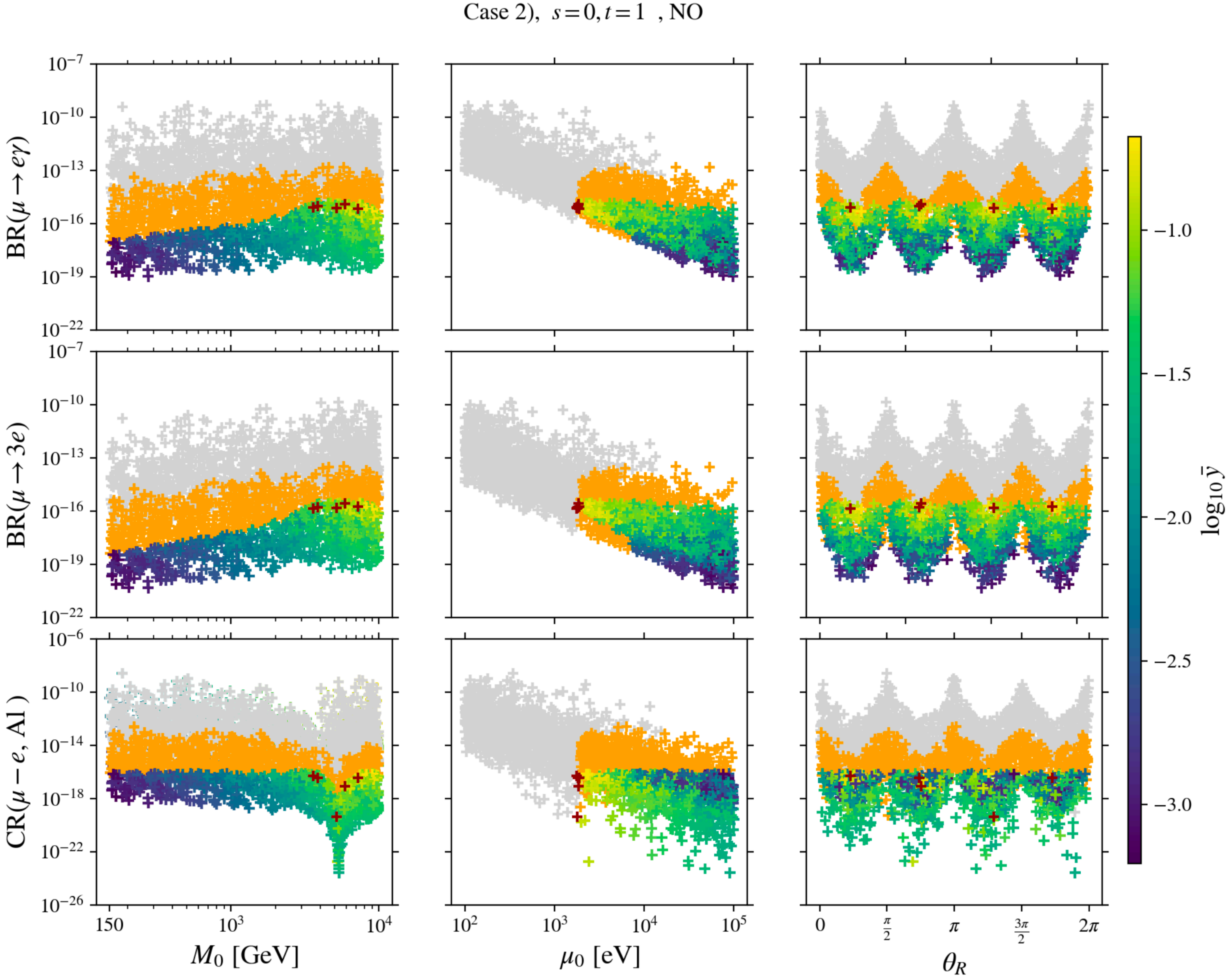
Cancellation of contributions



Option 2: Case 2

Predictions are compatible with future bounds on $\mu - e$ transitions!

A lower limit for μ_0 can be extrapolated

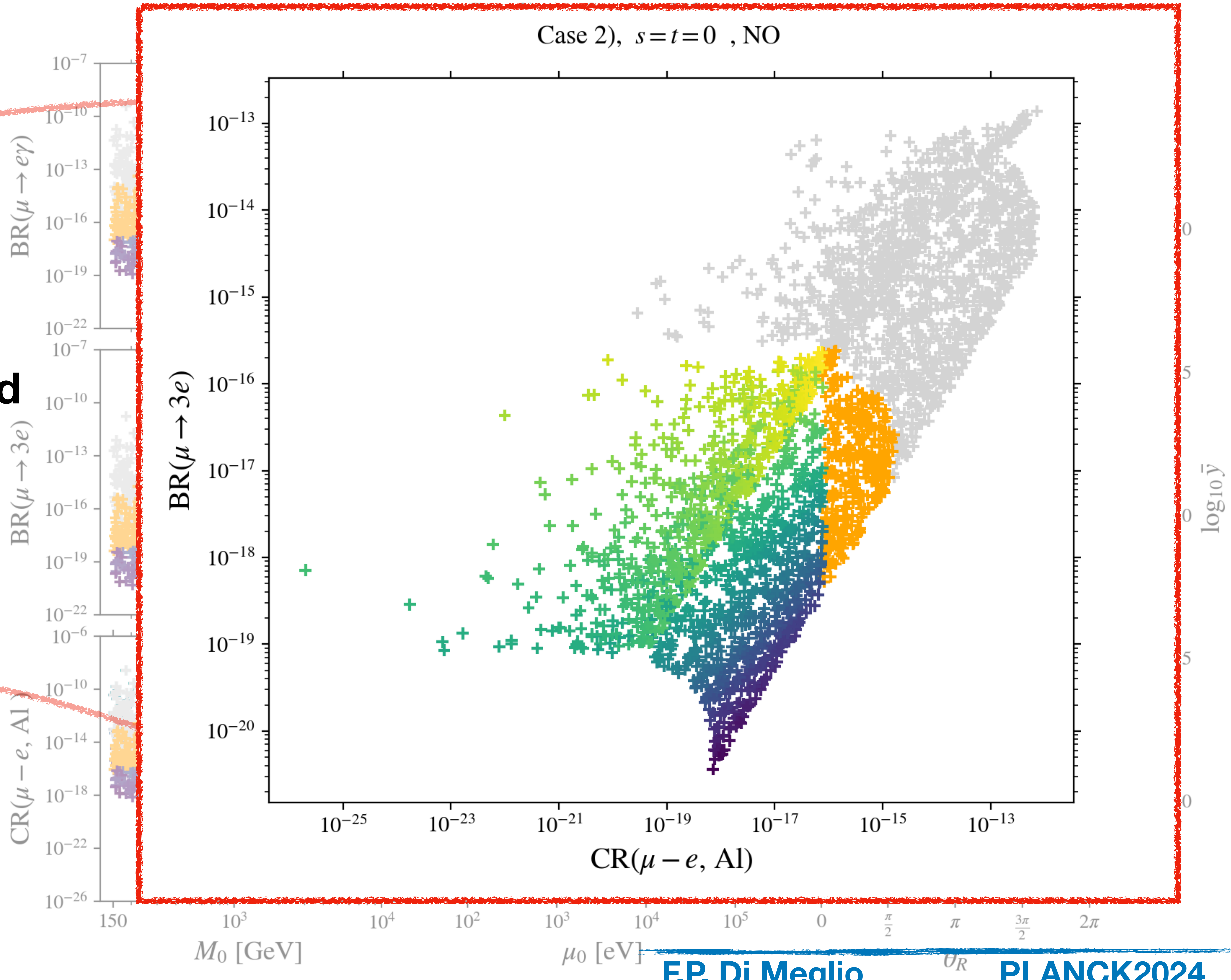


Option 2: Case 2

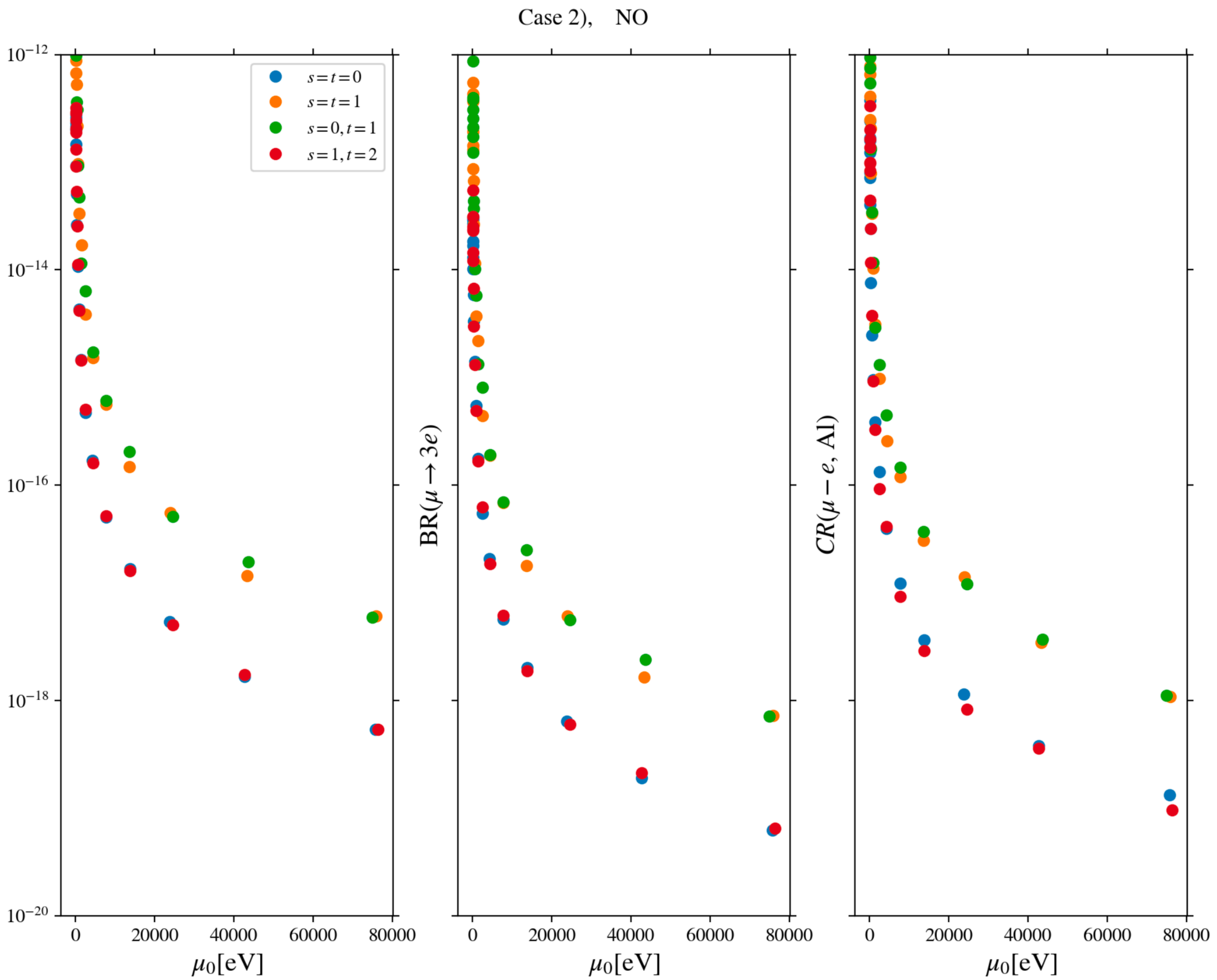
Modulation in function of θ_R

Strong correlation is related to Z penguin domination!

Cancellation of contributions



Option 2: Case 2

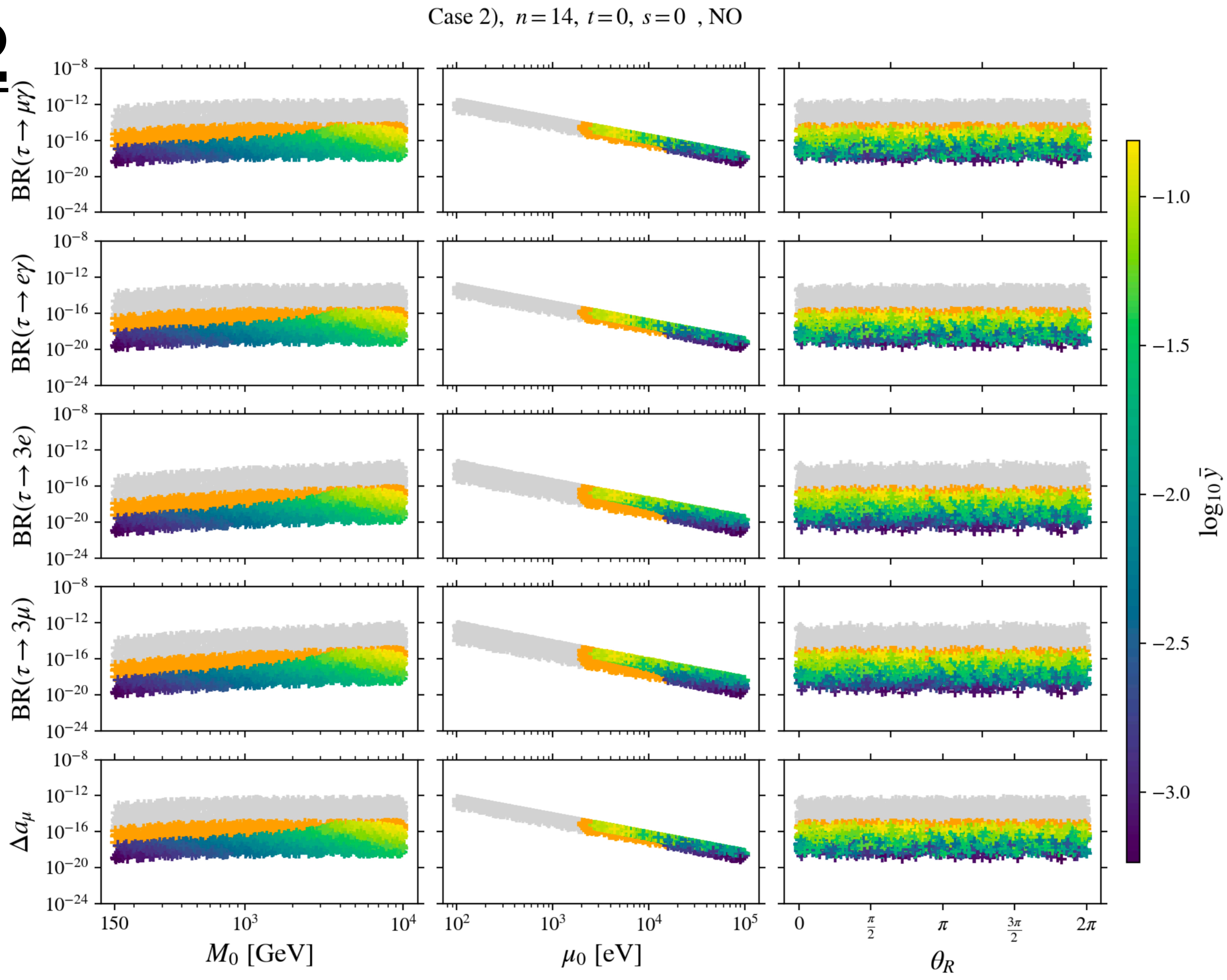


Binned averages of the scans:

- $\theta_R -$dependent case predicts slightly larger rates due to larger number of contributions to $\eta_{e\mu}$
- Different choices of residual symmetry can lead to different predictions

Option 2: Case 2

$\tau - \mu$ and $\tau - e$ processes are non-constraining



Conclusions:

- ISS Framework is promising framework, due to the possibility of testing it at future experiments
- Flavour and CP symmetries are successful in reproducing lepton mixing data and light n
- cLFV p mework analys parameter space
- Other cLFV processes such as $\tau - \mu$ and $\tau - e$ transitions are still far from probing our parameter space

Thank You For the Attention!
Questions? Doubts?

Backup Slides

In our theory:

- e_R, μ_R, τ_R transform under an auxiliary Z_3^{aux}
- L, N, S transform as triplets of $G_f = \Delta(6n^2)$
- CP acts non trivially on flavour

Symmetry Breaking

Breaking mechanism is irrelevant. After Breaking:

- e_R, μ_R, τ_R transform under $G_l = diag\left(\left(Z_3 \subset \Delta(6n^2)\right) \times Z_3^{aux}\right)$
- L, N, S transform under $G_\nu = \left(Z_2 \subset \Delta(6n^2)\right) \times CP$

$$\begin{cases} e_R \to e_R \\ \mu_R \to \omega\mu_R \\ \tau_R \to \omega^2\tau_R \end{cases}$$ **with** $\omega = e^{\frac{2\pi i}{3}}$

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[0809.0639]

$$\Delta(6n^2) \sim (\mathcal{Z}_n \times \mathcal{Z}_n) \rtimes \mathcal{S}_3$$

- Non-abelian subgroup of $SU(3)$
- Spanned by four generators a, b, c, d

$$a^n = b^n = (ab)^2 = c^n = d^n = 1$$

$$cd = dc$$

$$aca^{-1} = c^{-1}d^{-1} \qquad ada^{-1} = d$$

$$bcb^{-1} = d^{-1} \qquad bsdb^{-1} = c^{-1}$$

- General element of the group is written:

$g = a^\alpha b^\beta c^\gamma d^\delta$ **with** $\alpha, \beta, \gamma, \delta \in N$

- Equipped with a variety of independent triplet representations

$3_l^\pm$

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- CP represented by $X(\mathrm{r})$ that acts on flavour
- Consistency requires : $X(\mathrm{r})$ must be an automorphism of the flavour group:

for $g(\mathrm{r}) \in G_f \Rightarrow X(\mathrm{r})^* g(\mathrm{r}) X(\mathrm{r}) = g'(\mathrm{r}) \in G_f$

- We always choose:

$$X(\mathrm{r})^* X(\mathrm{r}) = X(\mathrm{r}) X(\mathrm{r})^* = 1$$

About Option 2

$$m_\nu \approx \frac{1}{M_0^2} m_D \mu_S m_D^T =$$

$$= \langle H \rangle \frac{\mu_0}{M_0^2} U_D^{(L)*}(\theta_L) \left[\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_D^{(R)T}(\theta_R) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_D^{(R)}(\theta_R) \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \right] U_D^{(L)\dagger}(\theta_L)$$

$$U_D^{(L)}(\theta_L) = \Omega(3) R_{ij}(\theta_L)$$

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The Ω matrices:

- **Unitary**
- Depend on the choice of **residual symmetry and** CP

Parameters:

- Mass scales: M_0, μ_0
- Yukawas y_1, y_2, y_3
- Angles θ_L; θ_R

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If it is diagonal :

- **$U_D^{(L)}(\theta_L)$ diagonalises m_ν**
- **Mass eigenvalues are $m_i = \frac{\langle H \rangle^2}{M_0^2} \mu_0$**

If not diagonal:

- **Redefinition $\theta_L \to \tilde{\theta}_L \Rightarrow U_D^{(L)}(\tilde{\theta}_L)$ diagonalises m_ν**

$$\tan(\tilde{\theta}_L - \theta_L) = - \frac{y_i y_j \tan(2\theta_R)}{y_i^2 + y_j^2}$$

- **Mass eigenvalues are :**

$$m_{i,j} = \frac{v^2}{4M_0^2} \mu_0 \left| \left(y_i^2 - y_j^2 \right) \cos\left(2\theta_R\right) \pm \sqrt{4 y_i^2 y_j^2 + \left(y_i^2 - y_j^2 \right)^2 \cos^2\left(2\theta_R\right)} \right|$$

$$m_{k \neq i,j} = \frac{v^2}{2M_0^2} \mu_0 y_k^2$$

Parameters:

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Outline of numerical study:

- Varied in ranges:
  $M_0 \in [150\ GeV;\ 10\ TeV]$ $\mu_0 \in [0.1\ keV;\ 10^2\ keV]$ $\theta_R \in [0;\ 2\pi]$

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$$\Omega(\mathbf{3})(u, v) = e^{i\frac{v\pi}{n}} U_{TB} R_{13} \left(-\frac{u\pi}{2n} \right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\frac{v\pi}{n}} & 0 \\ 0 & 0 & -i \end{pmatrix}$$

s even, t odd:

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Not diagonal for t odd!

Predictions are independent of θ_R !

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Diagonal for t even!

Predictions depend on θ_R !

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