The Type-I Seesaw Family

Antonio Herrero Brocal

In collaboration with Salva Centelles-Chuliá and Avelino Vicente

arXiv:2404.15415

The Type-I Seesaw family

What we mean by the Type-I Seesaw family?

The Type-I Seesaw family

Three conditions

1) $$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_F \end{pmatrix} ,$$

The Type-I Seesaw family

Three conditions

1) $$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_F \end{pmatrix} ,$$

2) $$\left(M_D\, M_F^{-1}\right)_{ij} \ll 1 \;\forall\, i, j$$

The Type-I Seesaw family

Three conditions

1) $$\mathcal{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_F \end{pmatrix} ,$$

2) $$\left(M_D\, M_F^{-1}\right)_{ij} \ll 1 \;\forall\, i, j$$

3) **A light Majorana mass term for the active neutrinos is generated at tree level.**

The Type-I Seesaw family

We can compute a general formula for M_ν

$$U^T \mathcal{M} U = \widehat{\mathcal{M}}\,; \quad U = \begin{pmatrix} U_l & 0 \\ 0 & U_h \end{pmatrix} \begin{pmatrix} \sqrt{\mathbb{I}_3 - PP^\dagger} & P \\ -P^\dagger & \sqrt{\mathbb{I}_{n_F} - P^\dagger P} \end{pmatrix} \equiv U_2 U_1$$

The Type-I Seesaw family

We can compute a general formula for M_ν

$$U^T \mathcal{M} U = \widehat{\mathcal{M}}\,; \quad U = \begin{pmatrix} U_l & 0 \\ 0 & U_h \end{pmatrix} \begin{pmatrix} \sqrt{\mathbb{I}_3 - P P^\dagger} & P \\ -P^\dagger & \sqrt{\mathbb{I}_{n_F} - P^\dagger P} \end{pmatrix} \equiv U_2 U_1$$

By expanding $P = \sum_{i=1}^{\infty} P_i$ **in powers of** $\varepsilon = \mathcal{O}\left(M_D M_F^{-1}\right)$

$$M_\nu = -M_D M_F^{-1} M_D^T + \mathcal{O}\left(\varepsilon^2\right) \,,$$

The Type-I Seesaw family

Let us assume two different singlets, N and S

$$M_D = \begin{pmatrix} m_D & m_L \end{pmatrix} \,, \qquad M_F = \begin{pmatrix} \mu_N & m_R \\ m_R^T & \mu_S \end{pmatrix} \,,$$

The Type-I Seesaw family

Let us assume two different singlets, N and S

$$M_D = \begin{pmatrix} m_D & m_L \end{pmatrix} \,, \qquad M_F = \begin{pmatrix} \mu_N & m_R \\ m_R^T & \mu_S \end{pmatrix} \,,$$

$$M_\nu = \begin{cases} \left(m_D \left(m_R^T\right)^{-1} \mu_S - m_L\right) m^{-1}\, m_D^T + \left(m_L m_R^{-1} \mu_N - m_D\right) \left(m^T\right)^{-1} m_L^T + \mathcal{O}\left(\varepsilon^2\right) , \text{if } m_R \neq 0 \,, \\ -m_D\, \mu_N^{-1} m_D^T - m_L\, \mu_S^{-1} m_L^T + \mathcal{O}\left(\varepsilon^2\right) , \text{if } m_R = 0 \,. \end{cases}$$

With $\quad \mathrm{m} = \mathrm{m}_R - \mu_N \left(m_R^T\right)^{-1} \mu_S$

The Type-I Seesaw family

We can recover type-I, inverse and linear results by working with different singlet generations and matrix hierarchies

The Type-I Seesaw family

We can recover type-I, inverse and linear results by working with different singlet generations and matrix hierarchies

For instance, if $\mu_S\ ,\mu_N \ll m_R$

$$M_\nu = m_D\,\left(m_R^T\right)^{-1}\,\mu_S m_R^{-1} m_D^T + m_L\, m_R^{-1}\mu_N\,\left(m_R^T\right)^{-1}\,m_L^T$$
$$-m_D\left(m_R^T\right)^{-1}\,m_L^T\ - m_L\, m_R^{-1} m_D^T + \mathcal{O}\left(\varepsilon^2\right)\,,$$

The Type-I Seesaw family with explicit breaking of $U(1)_L$

We can define the multiplet $F = (N \quad S)$

$$-\mathcal{L} = Y\,\bar{L}\tilde{H}F + \frac{M_F}{2}\,\bar{F}^c F + \text{h.c.}$$

The Type-I Seesaw family with explicit breaking of $U(1)_L$

We can define the multiplet $\boldsymbol{F} = (\boldsymbol{N} \quad \boldsymbol{S})$

$$-\mathcal{L} = Y\,\bar{L}\tilde{H}F + \frac{M_F}{2}\,\bar{F}^c F + \text{h.c.}$$

With $Y = \begin{pmatrix} y_N & y_S \end{pmatrix}$, $M_F = \begin{pmatrix} \mu_N & m_R \\ m_R^T & \mu_S \end{pmatrix}$

We can describe all the models with the type-I lagrangian

The Type-I Seesaw family with SSB of $U(1)_L$

The multiplet $F = (N \quad S)$ **is not anymore a well-defined object of the theory in general**

The Type-I Seesaw family with SSB of $U(1)_L$

The multiplet $F = (N \quad S)$ is not anymore a well-defined object of the theory in general

⬇

We cannot use one lagrangian to describe all the models

The Type-I Seesaw family with SSB of $U(1)_L$

The multiplet $F = (N \quad S)$ **is not anymore a well-defined object of the theory in general**

⬇

We cannot use one lagrangian to describe all the models

⬇

We can find different realizations for the same mass mechanism generation

Minimal realization with SSB of $U(1)_L$

By minimal we mean:

- **BSM scalar sector: one singlet and at most one doublet**

Minimal realization with SSB of $U(1)_L$

By minimal we mean:

- **BSM scalar sector: one singlet and at most one doublet**
- **Singlet fermions: N and S. N is the right-handed neutrino** $(\bar{L}HN)$

Minimal realization with SSB of $U(1)_L$

By minimal we mean:

- **BSM scalar sector: one singlet and at most one doublet**
- **Singlet fermions: N and S. N is the right-handed neutrino** $(\bar{L}HN)$
- **Only one majoron, with the singlet being the main component**

Minimal realization with SSB of $U(1)_L$

Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_L$
H	$(\mathbf{2}, \frac{1}{2})$	0
χ	$(\mathbf{2}, \frac{1}{2})$	$q_S - 1$
σ	$(\mathbf{1}, 0)$	q_σ
L	$(\mathbf{2}, -\frac{1}{2})$	1
N	$(\mathbf{1}, 0)$	1
S	$(\mathbf{1}, 0)$	q_S

Minimal realization with SSB of $U(1)_L$

We find 9 different models

- **Mass mechanism ➡ 3 type-I, 2 inverse, 2 linear, and 2 hybrids**

- **Majoron pheno : 6 suppressed by m_ν and 3 enhanced**

Models with enhanced majoron phenomenology

1) Hybrid mechanism (linear + inverse) $q_S = 0 \;\; ; \; q_\sigma = -1$

2) Linear Seesaw $q_S =$ -3 $; \; q_\sigma = -2$

3) Inverse Seesaw $q_\sigma = -1$

Hybrid mechanism with enhanced majoron LFV

Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_L$
H	$(\mathbf{2}, \frac{1}{2})$	0
χ	$(\mathbf{2}, \frac{1}{2})$	-1
σ	$(\mathbf{1}, 0)$	-1
L	$(\mathbf{2}, -\frac{1}{2})$	1
N	$(\mathbf{1}, 0)$	1
S	$(\mathbf{1}, 0)$	0

$$-\mathcal{L} = Y\,\bar{L} H e_R + y_N\,\bar{L}\tilde{H} N + y_S\,\bar{L}\tilde{\chi} S + \lambda\,\sigma \bar{N}^c S + \frac{1}{2}\,\mu_S\,\bar{S}^c S + \text{h.c.}$$

Hybrid mechanism with enhanced majoron LFV

$$-\mathcal{L} = Y\,\bar{L}He_R + y_N\,\bar{L}\tilde{H}N + y_S\,\bar{L}\tilde{\chi}S + \lambda\,\sigma\bar{N}^c S + \frac{1}{2}\,\mu_S\,\bar{S}^c S + \text{h.c.}$$

Assuming $v_\chi \ll v_H \ll v_\sigma$

$$M_\nu \approx \frac{v_H^2}{v_\sigma^2}\left[y_N\,\left(\lambda^T\right)^{-1}\mu_S\lambda^{-1}y_N^T\right] - \frac{v_H\,v_\chi}{v_\sigma}\left[\left(y_N\,\left(\lambda^T\right)^{-1}\,y_S^T\right) + \text{tr.}\right],$$

Hybrid mechanism with enhanced majoron LFV

$$-\mathcal{L} = Y\bar{L}He_R + y_N\,\bar{L}\tilde{H}N + y_S\,\bar{L}\tilde{\chi}S + \lambda\,\sigma\bar{N}^cS + \frac{1}{2}\mu_S\,\bar{S}^cS + \text{h.c.}$$

Assuming $v_\chi \ll v_H \ll v_\sigma$

$$M_\nu \approx \frac{v_H^2}{v_\sigma^2}\left[y_N\,\left(\lambda^T\right)^{-1}\mu_S\lambda^{-1}y_N^T\right] - \frac{v_H\,v_\chi}{v_\sigma}\left[\left(y_N\,\left(\lambda^T\right)^{-1}\,y_S^T\right) + \text{tr.}\right],$$

⬇

Inverse + linear ➡ **Hybrid mechanism**

Hybrid mechanism with enhanced majoron LFV

We can compute the majoron in the basis $\mathrm{Im}\left[(H,\, \chi,\, \sigma)\right]$

Hybrid mechanism with enhanced majoron LFV

We can compute the majoron in the basis $\mathrm{Im}\left[(H,\, \chi,\, \sigma)\right]$

$$J = \frac{1}{\mathcal{N}}\left(-\frac{v_H v_\chi^2}{v_\sigma v^2},\, \frac{v_H^2 v_\chi}{v_\sigma v^2},\, 1\right) \;\; ; \qquad \mathcal{N}^2 = 1 + \frac{v_H^2\, v_\chi^2}{v_\sigma^2\, v^2}$$

Hybrid mechanism with enhanced majoron LFV

We can compute the majoron in the basis $\text{Im}\left[(H,\,\chi,\,\sigma)\right]$

$$J = \frac{1}{\mathcal{N}}\left(-\frac{v_H v_\chi^2}{v_\sigma v^2},\, \frac{v_H^2 v_\chi}{v_\sigma v^2},\, 1\right) \; ; \qquad \mathcal{N}^2 = 1 + \frac{v_H^2\, v_\chi^2}{v_\sigma^2\, v^2}$$

$$\mathcal{L}_{\ell\ell J}^{\text{tree-level}} = i\,\frac{v_\chi^2 v_H}{\mathcal{N}\, v^3 v_\sigma}\, J\bar{\ell} M_\ell\, \gamma_5\, \ell \lesssim \frac{v_\chi\, m_\ell\, m_\nu}{v^3}\, J\bar{\ell}\gamma_5\, \ell\,.$$

Hybrid mechanism with enhanced majoron LFV

At one-loop **(AHB, A.Vicente, arXiv:2311.10145)**

$$\mathcal{L}_{\ell\ell J} = \frac{iJ}{32\pi^2 v_\sigma}\bar{\ell}\left[M_\ell\ \mathrm{Tr}(y_N\, y_N^\dagger)\,\gamma_5 + 2M_\ell\ \left(y_N\, y_N^\dagger - y_S\,\Theta\, y_S^\dagger\right) P_L - 2\ \left(y_N\, y_N^\dagger - y_S\,\Theta\, y_S^\dagger\right) M_\ell P_R\right]\ell\,,$$

With

$$\Theta_{sp} \equiv \frac{(m_R^2)_s}{\left((m_R^2)_s - m_{\eta^+}^2\right)^2}\left((m_R^2)_s - m_{\eta^+}^2 + m_{\eta^+}^2 \log\frac{m_{\eta^+}^2}{(m_R^2)_s}\right)\delta_{sp}\,.$$

Hybrid mechanism with enhanced majoron LFV

$$\mathcal{L}_{\ell\ell J} = \frac{iJ}{32\pi^2 v_\sigma}\bar{\ell}\left[M_\ell\,\mathrm{Tr}(y_N\,y_N^\dagger)\,\gamma_5 + 2M_\ell\,\left(y_N\,y_N^\dagger - y_S\,\Theta\,y_S^\dagger\right)P_L - 2\,\left(y_N\,y_N^\dagger - y_S\,\Theta\,y_S^\dagger\right)M_\ell P_R\right]\ell\,,$$

And we obtain an enhanced coupling since

$$M_\nu \sim \mu_S\frac{v_H^2}{v_\sigma^2} - 2\frac{v_H v_\chi}{v_\sigma} \qquad\qquad g_{J\ell\ell} \sim \frac{M_\ell}{v_\sigma}$$

Linear Seesaw with enhanced majoron LFV

Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_L$
H	$(\mathbf{2}, \frac{1}{2})$	0
χ	$(\mathbf{2}, \frac{1}{2})$	-4
σ	$(\mathbf{1}, 0)$	-1
L	$(\mathbf{2}, -\frac{1}{2})$	1
N	$(\mathbf{1}, 0)$	1
S	$(\mathbf{1}, 0)$	-3

$$-\mathcal{L} = Y \bar{L} H e_R + y_N \, \bar{L} \tilde{H} N + y_s \, \bar{L} \tilde{\chi} S + \lambda \, \sigma^* \, \bar{N}^c S + \frac{1}{2} \, \lambda_N \, \sigma \, \bar{N}^c N + \text{h.c.}$$

Linear Seesaw with enhanced majoron LFV

$$-\mathcal{L} = Y\bar{L}He_R + y_N\,\bar{L}\tilde{H}N + y_s\,\bar{L}\tilde{\chi}S + \lambda\,\sigma^*\,\bar{N}^cS + \frac{1}{2}\,\lambda_N\,\sigma\,\bar{N}^cN + \text{h.c.}$$

Assuming $v_\chi \ll v_H \ll v_\sigma$

$$M_\nu \approx -\frac{v_\chi v_H}{v_\sigma}\,y_S\,\lambda^{-1}\,y_N^T + \text{tr.}$$

Linear seesaw

Linear Seesaw with enhanced majoron LFV

We can compute the majoron in the basis $\mathrm{Im}\left[(H,\, \chi,\, \sigma)\right]$

Linear Seesaw with enhanced majoron LFV

We can compute the majoron in the basis $\mathrm{Im}\left[(H,\,\chi,\,\sigma)\right]$

$$J = \frac{1}{\mathcal{N}}\left(-2\,\frac{v_\chi^2\,v_H}{v_\sigma\,v^2},\,2\,\frac{v_H^2\,v_\chi}{v_\sigma\,v^2},\,1\right)\;; \qquad \mathcal{N}^2 = 1 + 4\,\frac{v_H^2\,v_\chi^2}{v_\sigma^2\,v^2}$$

Linear Seesaw with enhanced majoron LFV

We can compute the majoron in the basis $\mathrm{Im}\left[(H,\,\chi,\,\sigma)\right]$

$$J = \frac{1}{\mathcal{N}}\left(-2\,\frac{v_\chi^2\,v_H}{v_\sigma\,v^2},\,2\,\frac{v_H^2\,v_\chi}{v_\sigma\,v^2},\,1\right)\,; \qquad \mathcal{N}^2 = 1+4\,\frac{v_H^2\,v_\chi^2}{v_\sigma^2\,v^2}$$

$$\mathcal{L}_{\ell\ell J}^{\text{Tree-Level}} = 2i\frac{v_\chi^2 v_H}{\mathcal{N}v^3 v_\sigma}J\bar{\ell}M_\ell\gamma_5\ell \sim \frac{m_\nu\,m_\ell\,v_\chi}{v^3}\,J\bar{\ell}\,\gamma_5\,\ell\,.$$

Linear Seesaw with enhanced majoron LFV

At one-loop (AHB, A.Vicente, arXiv:2311.10145) , defining $R \equiv \lambda_N^\dagger \lambda_N \left(\lambda^T\right)^{-1} \left(\lambda^*\right)^{-1}$

$$\mathcal{L}_{\ell\ell J} = -\frac{iJ}{32\pi^2 v_\sigma}\,\bar{\ell}\left\{M_\ell\ \left[\mathrm{Tr}\left(y_N\ \left(\mathbb{I}_3 - \frac{1}{3}R\right) y_N^\dagger\right)\right]\gamma_5 + M_\ell\, y_N\ \left(2\,\mathbb{I}_3 - \frac{5}{12}R\right) y_N^\dagger P_L \right.$$
$$\left. - 2 y_N\ \left(2\,\mathbb{I}_3 - \frac{5}{12}R^\dagger\right) y_N^\dagger\ M_\ell P_R\right\}\ell\,,$$

And we obtain an enhanced coupling since

$$M_\nu \sim \frac{v_H v_\chi}{v_\sigma} \qquad\qquad g_{J\ell\ell} \sim \frac{M_\ell}{v_\sigma}$$

Lepton Flavor Violation Phenomenology

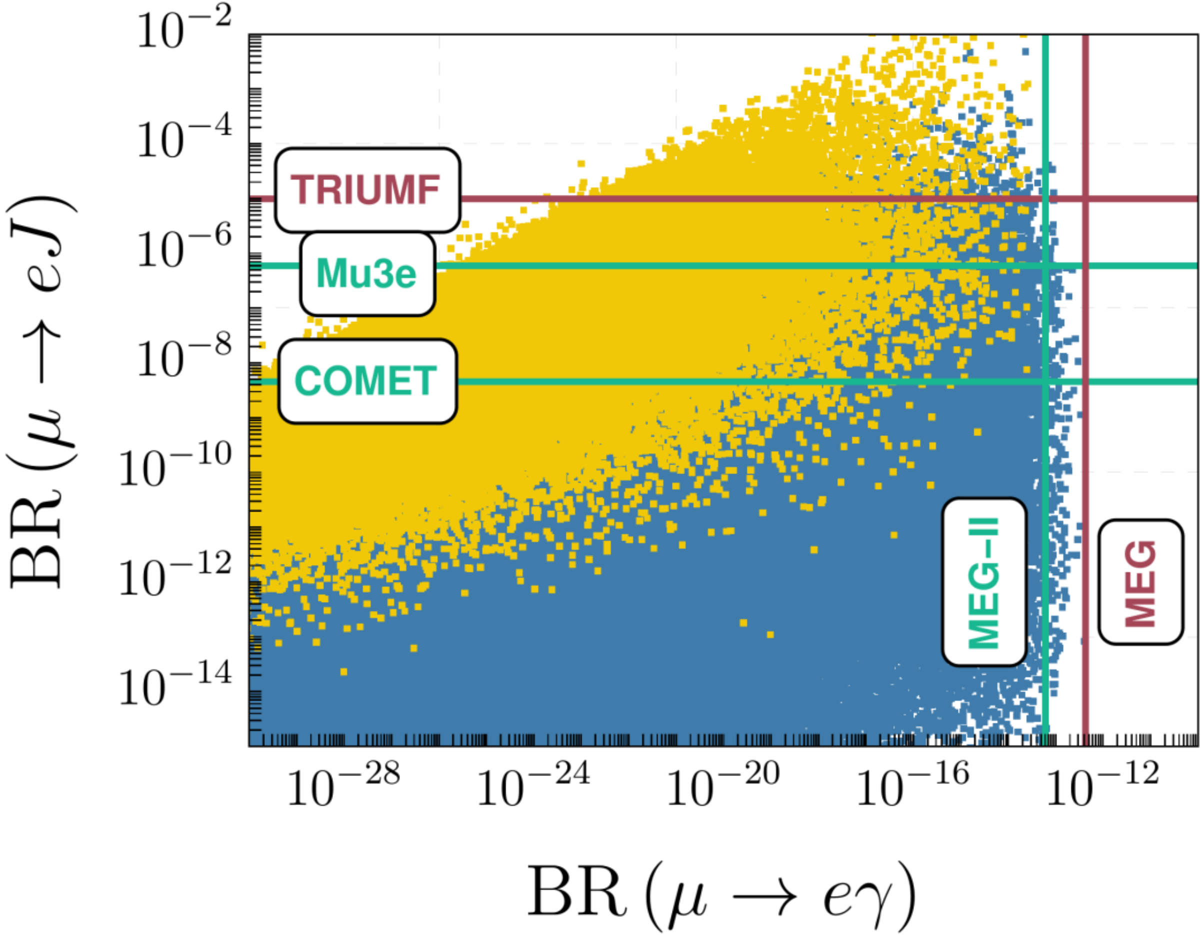
We can study the models in the limit where the majoron interaction is the same for both models

Lepton Flavor Violation Phenomenology

We can study the models in the limit where the majoron interaction is the same for both models

$$\mathcal{L}_{\ell\ell J} = \frac{iJ}{32\pi^2 v_\sigma}\bar{\ell}\left[M_\ell \; \mathrm{Tr}(y_N\, y_N^\dagger)\gamma_5 + 2M_\ell\, y_N\, y_N^\dagger\, P_L - 2\, y_N\, y_N^\dagger M_\ell\, P_R\right]\ell\,.$$

Lepton Flavor Violation Phenomenology



Conclusions

- **In the case of explicitly lepton number breaking all Type-I family models are equivalent since singlet fermions are indistinguishable**
- **In the case of SSB a lot of different realizations are possible**
- **We can build models with majoron enhanced phenomenology**
- **In these models we expect to see** $\mu \to e\, J$ **over** $\mu \to e\, \gamma$

Inverse mechanism with enhanced majoron LFV

$$-\mathcal{L} = Y \bar{L} H e_R + y_N \, \bar{L} \tilde{H} N + \lambda \, \sigma \bar{N}^c S + \frac{1}{2} \mu_S \, \bar{S}^c S + \text{h.c.}$$

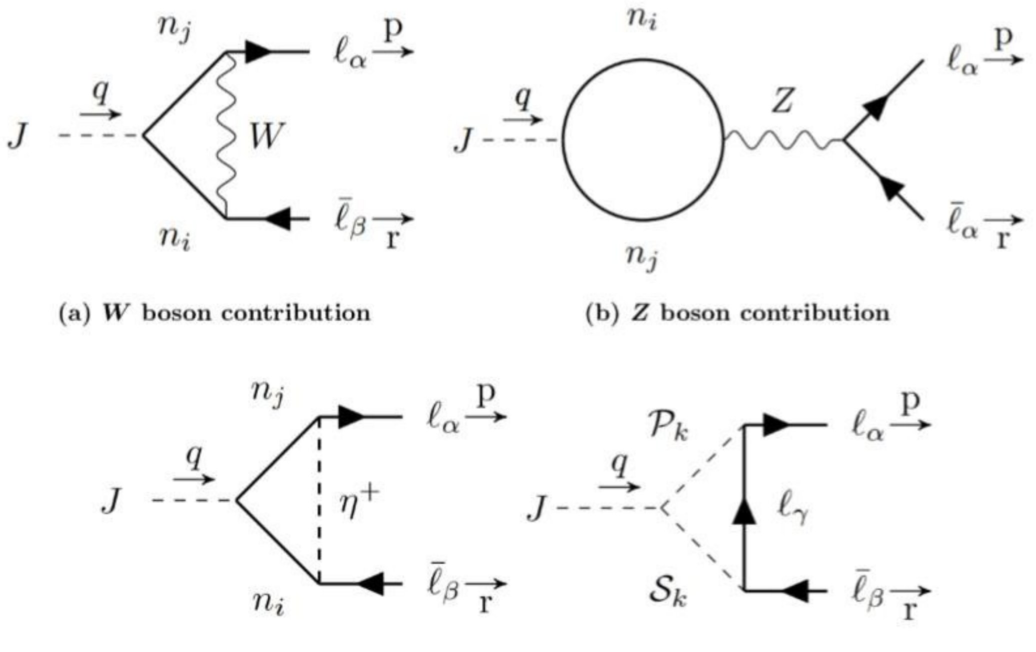
Assuming $v_H \ll v_\sigma$

$$M_\nu \approx \frac{v_H^2}{v_\sigma^2} \left[y_N \, \left(\lambda^T \right)^{-1} \mu_S \lambda^{-1} y_N^T \right]$$

The coupling at one-loop (AHB,A.Vicente, arXiv:2311.10145)

$$\mathcal{L}_{\ell\ell J} = \frac{i J}{32 \pi^2 v_\sigma} \bar{\ell} \left[M_\ell \, \text{Tr}(y_N \, y_N^\dagger) \gamma_5 + 2 M_\ell \, y_N \, y_N^\dagger \, P_L - 2 \, y_N \, y_N^\dagger M_\ell \, P_R \right] \ell \, .$$

Diagrams leading to the one-loop majoron coupling



(a) W boson contribution

(b) Z boson contribution

(c) η contribution

(d) Scalar contribution

Figure 1: Feynman diagrams leading to the 1-loop coupling of the majoron to a pair of charged leptons.