Modulus stabilisation and modular invariant hilltop inflation

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06/06/2024

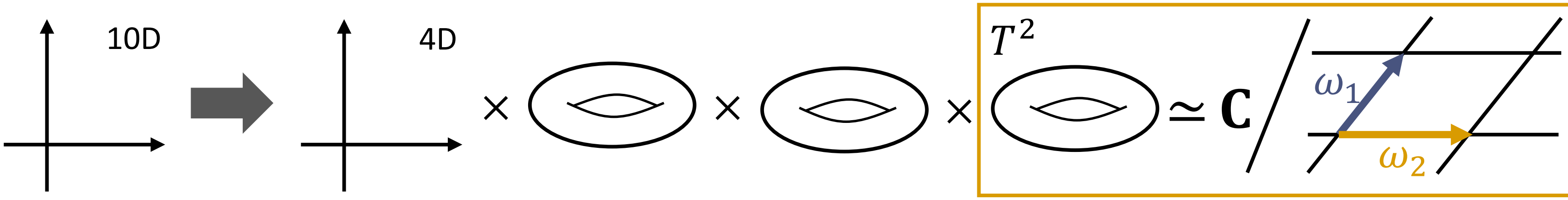
Based on 2310.10369, 2405.08924, in collaboration with Stephen F. King

University of Southampton
THE ROYAL SOCIETY
CFTP Centro de Física Teórica de Partículas
fct Fundação para a Ciência e a Tecnologia
UIDB/00777/2020, UIDP/00777/2020
TÉCNICO LISBOA

Modular flavour symmetry

▪ Modular transformation

Dixon et al., NPB, 1987
Hamidi & Vafa, NPB, 1987



Modulus $\frac{\omega_1}{\omega_2} = \boldsymbol{\tau}, \mathrm{Im}\, \boldsymbol{\tau} > 0$ $\quad a, b, c, d \in Z,$ $ad - bc = 1$

$$\tau \to \gamma\tau = \frac{\omega_1'}{\omega_2'} = \frac{a\tau + b}{c\tau + d}$$

$$\chi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \chi^{(I)}$$

Modular group
$\overline{\Gamma} \simeq \mathrm{PSL}(2, Z)$
$\Gamma \simeq \mathrm{SL}(2, Z)$

Congruence subgroup → $\overline{\Gamma}(N)/\Gamma(N)$

Finite modular group
$\boldsymbol{\Gamma}_2^{(\prime)} \simeq \mathbf{S}_3^{(\prime)}, \boldsymbol{\Gamma}_3^{(\prime)} \simeq \mathbf{A}_4^{(\prime)},$
$\boldsymbol{\Gamma}_4^{(\prime)} \simeq \mathbf{S}_4^{(\prime)}, \boldsymbol{\Gamma}_5^{(\prime)} \simeq \mathbf{A}_5^{(\prime)}$

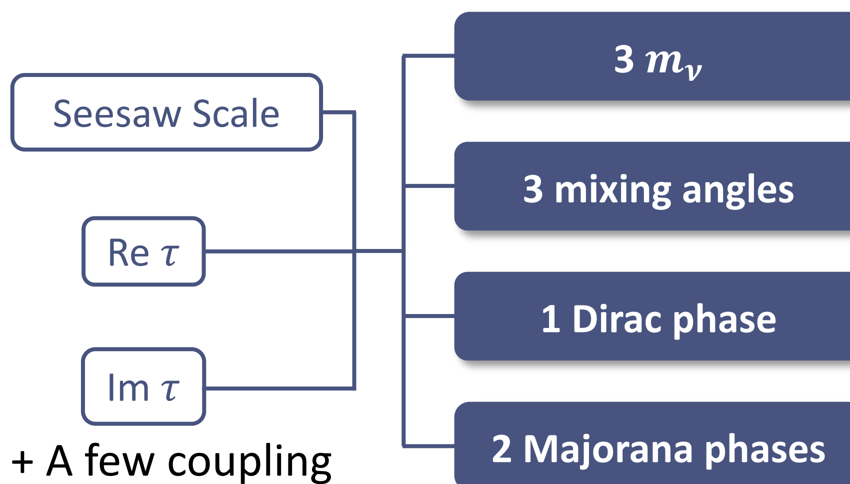
▪ Modular forms

$$Y_{\mathbf{r}}^{(k)}(\gamma\tau) = (c\tau + d)^k \rho_{\mathbf{r}}(\gamma) Y_{\mathbf{r}}^{(k)}(\tau)\,, \quad \gamma \in \Gamma_N$$

Yukawa couplings are modular forms!

Lauer, Mas, Nilles, PLB, 1989
Ferrara et al., PLB, 1989
Feruglio, 2017

$$Y_{\mathbf{r}}^{(k)} = \begin{pmatrix} f_1^{(k)}(\tau) \\ f_2^{(k)}(\tau) \\ \vdots \\ f_r^{(k)}(\tau) \end{pmatrix}$$

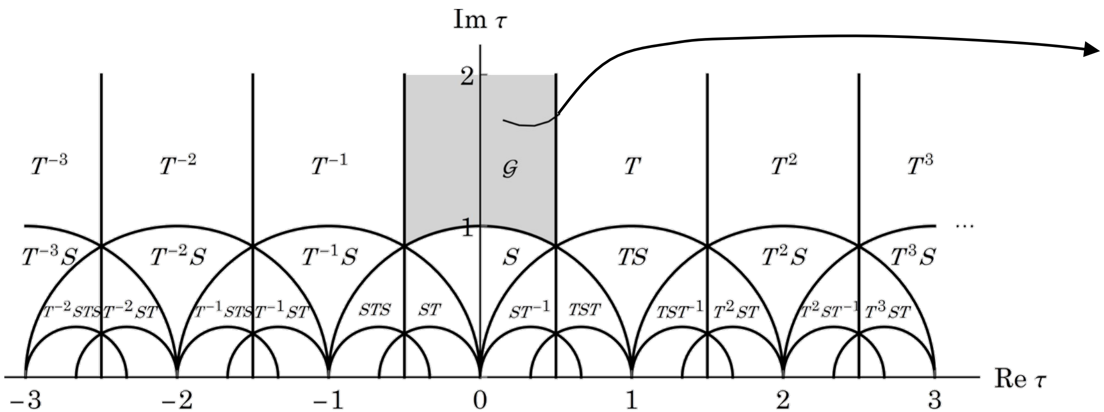


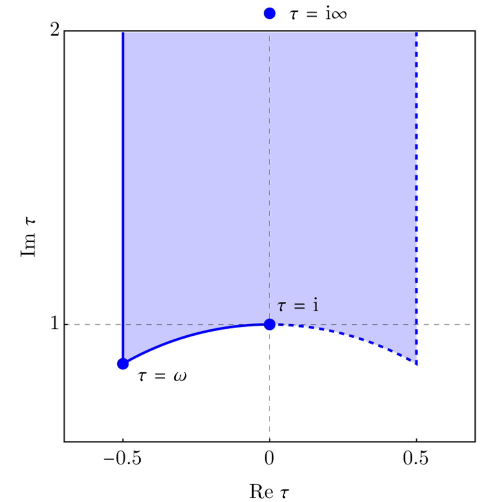
+ A few coupling coefficients

Determine the value of τ

- **Fundamental domain**

$$\mathcal{G} = \left\{\tau \in \mathbb{C}_+ : -\frac{1}{2} \leq \mathrm{Re}\,\tau < \frac{1}{2},\ |\tau| > 1\right\} \cup \left\{\tau \in \mathbb{C}_+ : -\frac{1}{2} \leq \mathrm{Re}\,\tau \leq 0,\ |\tau| = 1\right\}$$





Fixed points

- **Bottom-up or Top-down?**

Bottom-up approach

Free parameter obtained by fitting the experimental data

Top-down approach

Modulus stabilisation

Gaugino condensation

Flux compactification

Ishiguro, Kobayashi, Otsuka, JHEP, 2021, Ishiguro, Okada, Otsuka, JHEP, 2022, Higaki et al., 2024, ...

Non-perturbative effects

Kobayashi et al., PRD, 2019, 2020;
Font et al., PLB, 1990; Gonzalo et al., JHEP, 2018;
Novichkov, Penedo, Petcov, JHEP, 2022;
Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023;
Leedom, Righi, Westphal, JHEP, 2023, ...

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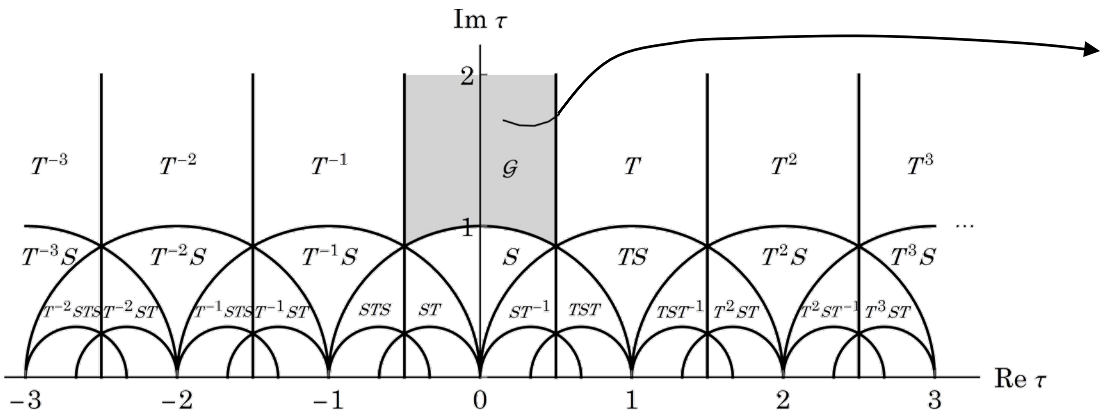

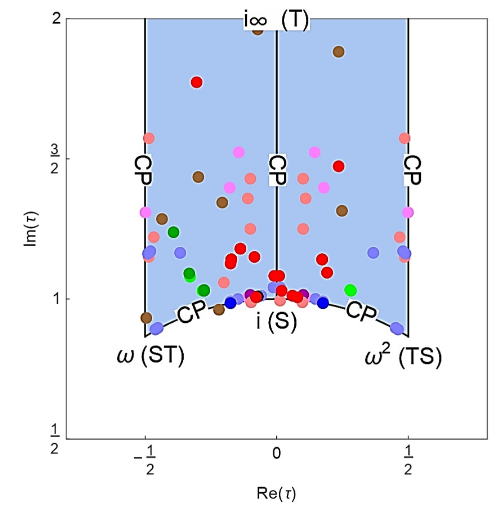

Feruglio, PRL, 2023

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Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023;
Leedom, Righi, Westphal, JHEP, 2023, …

Gaugino condensation

- **Kähler function**

$$G(\tau,\overline{\tau},S,\overline{S}) = \mathcal{K}(\tau,\overline{\tau},S,\overline{S}) + \log|\mathcal{W}(\tau,S)|^2$$

Kähler moduli τ + dilaton S

Modular invariant

- **Kähler potential**

$$\mathcal{K}(\tau,\overline{\tau},S,\overline{S}) = K(S,\overline{S}) - 3\log(2\,\mathrm{Im}\,\tau)$$

Tree level: $K(S,\overline{S}) \propto -\ln(S+\overline{S})$

Loop levels: Non-perturbative effects $\delta K(S,\overline{S})$

Leedom, Righi, Westphal, JHEP, 2023

Crucial for stabilising the dilaton sector

Shenker effects

Shenker, 1990

- **Superpotential**

$$\mathcal{W}(\tau) \to (c\tau+d)^{-3}\mathcal{W}(\tau)$$

Modular function of weight -3

Generate non-trivial superpotential via **gaugino condensation**.
A gauge group undergoing gaugino condensation gives rise to non-perturbative superpotential $\mathcal{W} \sim e^{-f_a/b_a}$

Nilles, PLB, 1982
Dine et al., PLB, 1985
Ferrara et al., PLB, 1983

$$f_a = k_a S + b_a \ln \eta^6(\tau) + \cdots$$

Dedekind *η* function

One-loop: Threshold corrections,
Anomaly cancellation

$$\mathcal{W}(\tau,S) = \frac{\Omega(S)H(\tau)}{\eta^6(\tau)}$$

Klein *j* function

$$H(\tau) = (j(\tau)-1728)^{m/2} j(\tau)^{n/3} \mathcal{P}(j(\tau))$$

Modulus stabilisation – Single modulus

- **Scalar potential**

Cremmer et al., 1983

$$V = e^{\mathcal{K}}\left(\mathcal{K}^{i\bar{j}} D_i\mathcal{W} D_{\bar{j}}\mathcal{W}^* - 3|\mathcal{W}|^2\right)$$

$$V(\tau,\overline{\tau},S,\overline{S}) = \mathcal{C}(\tau,\overline{\tau},S,\overline{S}) \times \left[\mathcal{M}(\tau,\overline{\tau}) + \left(\mathcal{A}(S,\overline{S}) - 3\right)|H(\tau)|^2\right]$$

Cvetic et al., NPB, 1991,
Leedom, Righi, Westphal, JHEP, 2023

$D_i = \partial_i + (\partial_i\mathcal{K})$ Covariant derivative

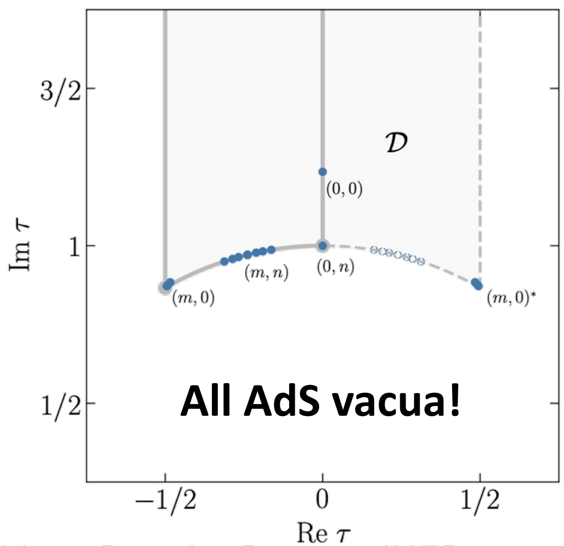
$\mathcal{K}_{i\bar{j}} = \partial_i\partial_{\bar{j}}\mathcal{K}$ Kähler matric

$$\mathcal{C}(\tau,\overline{\tau},S,\overline{S}) = \frac{e^{K(S,\overline{S})}|\Omega(S)|^2}{(2\,\mathrm{Im}\,\tau)^3|\eta(\tau)|^{12}},$$

$$\mathcal{M}(\tau,\overline{\tau}) = \frac{(2\,\mathrm{Im}\,\tau)^2}{3}\left|\mathrm{i}H'(\tau) + \frac{H(\tau)}{2\pi}\widehat{G}_2(\tau,\overline{\tau})\right|^2$$

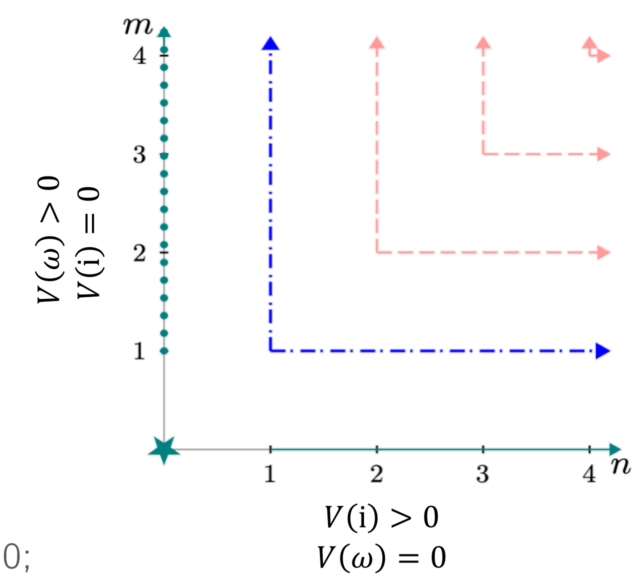
$$\mathcal{A}(S,\overline{S}) = \frac{|\Omega_S + K_S\Omega|^2}{K_{S\overline{S}}|\Omega|^2},$$

Condition A: $\Omega_S + K_S\Omega = 0$



Novichkov, Penedo, Petcov, JHEP, 2022; Font et al., PLB, 1990; Cvetic et al., NPB, 1991; Gonzalo et al., JHEP, 2018;

Condition B: $\Omega_S + K_S\Omega \neq 0$

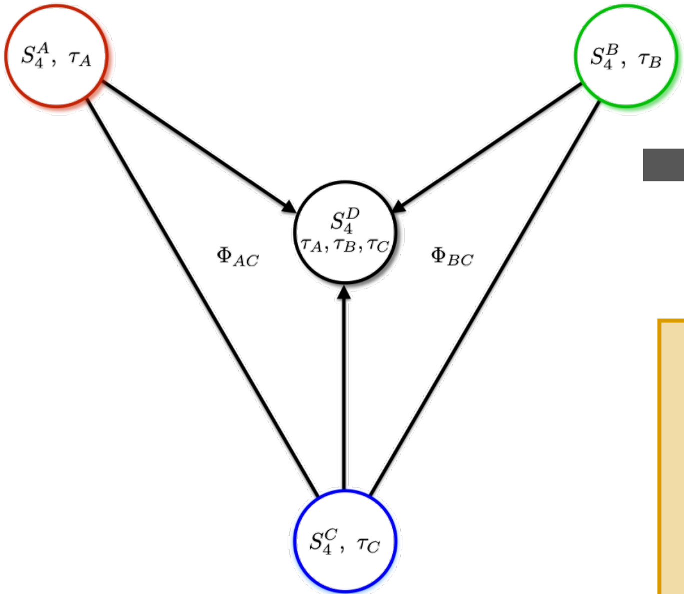


Leedom, Righi, Westphal, JHEP, 2023

Modulus stabilisation – Multiple moduli

Flavour mixing pattern – More than one moduli are required **if τ @ fixed points**.

Novichkov, Penedo, Petcov, Titov, JHEP, 2019
Novichkov, Petcov, Tanimoto, PLB, 2019



$S_4^A \times S_4^B \times S_4^C \to S_4^D$
by two bi-triplets

Varzielas, King, Zhou, PRD, 2020

Modulus stabilisation in the multiple-modulus framework

- Gaugino condensation in heterotic string.
- 3 factorised tori: $\mathrm{SL}(2,\mathrm{Z}) \to \mathrm{SL}(2,\mathrm{Z})_1 \times \mathrm{SL}(2,\mathrm{Z})_2 \times \mathrm{SL}(2,\mathrm{Z})_3$.
- Uplift the vacua to dS vacua by considering non-trivial dilaton sector.
- Focus on minima at fixed points.

$$\mathcal{K}(\tau,\overline{\tau},S,\overline{S}) = K(S,\overline{S}) - 3\log(2\,\mathrm{Im}\,\tau)$$

$$\mathcal{W}(\tau,S) = \frac{\Omega(S)H(\tau)}{\eta^6(\tau)}$$

$$\mathcal{K}(\tau_i,\overline{\tau}_i,S,\overline{S}) = K(S,\overline{S}) - \log[(2\,\mathrm{Im}\,\tau_1)(2\,\mathrm{Im}\,\tau_2)(2\,\mathrm{Im}\,\tau_3)]\ ,$$

$$\mathcal{W}(\tau_i,S) = \frac{\Omega(S)[H^{(m_1,n_1)}(\tau_1) + H^{(m_2,n_2)}(\tau_2) + H^{(m_3,n_3)}(\tau_3)]}{\eta^2(\tau_1)\eta^2(\tau_2)\eta^2(\tau_3)}$$

King, XW, 2310.10369

Modulus stabilisation – Multiple moduli

(m_1,n_1)	(m_3,n_3)	τ_1	τ_3	$\mathcal{A}(S,\overline{S})$	(m_1,n_1)	(m_3,n_3)	τ_1	τ_3	$\mathcal{A}(S,\overline{S})$
$(0,0)$	$(0,3)$	i	i	$(3,3.596)$	$(0,3)$	$(0,0)$	i	i	$(3,3.596)$
		i	ω	$(3,3.596)$			i	ω	$(3,60.43)$
		ω	i	$(3,117.2)$			ω	i	$(3,3.596)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
$(0,0)$	$(2,0)$	i	i	$(3,3.596)$	$(2,0)$	$(0,0)$	i	i	$(3,3.596)$
		i	ω	$(3,3.596)$			i	ω	$(3,198624)$
		ω	i	$(3,99314)$			ω	i	$(3,3.596)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
$(0,0)$	$(2,3)$	i	i	$(3,3.596)$	$(2,3)$	$(0,0)$	i	i	$(3,3.596)$
		i	ω	$(3,3.596)$			i	ω	$(3,3.43\times10^8)$
		ω	i	$(3,1.72\times10^8)$			ω	i	$(3,3.596)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
$(2,0)$	$(0,3)$	i	i	$(3,117.3)$	$(0,3)$	$(2,0)$	i	i	$(3,60.45)$
		i	ω	$[3,+\infty)$			i	ω	$[3,117.3)$
		ω	i	$(3,114.1)$			ω	i	$[3,+\infty)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
$(2,0)$	$(2,3)$	i	i	$[3,+\infty)$	$(2,3)$	$(2,0)$	i	i	$[3,+\infty)$
		i	ω	$[3,+\infty)$			i	ω	$(3,+\infty)$
		ω	i	$(3,+\infty)$			ω	i	$[3,+\infty)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
$(0,3)$	$(2,3)$	i	i	$(3,60.45)$	$(2,3)$	$(0,3)$	i	i	$(3,117.3)$
		i	ω	$(3,60.45)$			i	ω	$[3,+\infty)$
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King, XW, 2310.10369

$(m_1,n_1)=(m_2,n_2)\neq(m_3,n_3)$

$\tau_1=\tau_2$ for the global minimum.

$(\tau_1,\tau_3)=(\omega,\omega)$ is always the vacuum.

$\tau=\mathrm{i}$ is not always the minimum when $m>1$.

At least one $(m_i,n_i)=(0,0)$, global dS vacua; All $(m_i,n_i)\neq(0,0)$, Minkowski vacua.

CP-violating vacuum near $\tau=\omega$ may still exist.

Modulus stabilisation – Multiple moduli

(m_1,n_1)	(m_3,n_3)	τ_1	τ_3	$\mathcal{A}(S,\overline{S})$	(m_1,n_1)	(m_3,n_3)	τ_1	τ_3	$\mathcal{A}(S,\overline{S})$
$(0,0)$	$(0,3)$	i	i	$(3,3.596)$	$(0,3)$	$(0,0)$	i	i	$(3,3.596)$
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		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
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		i	ω	$(3,3.596)$			i	ω	$(3,3.43\times10^8)$
		ω	i	$(3,1.72\times10^8)$			ω	i	$(3,3.596)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
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$(2,0)$	$(2,3)$	i	i	$[3,+\infty)$	$(2,3)$	$(2,0)$	i	i	$[3,+\infty)$
		i	ω	$[3,+\infty)$			i	ω	$(3,+\infty)$
		ω	i	$(3,+\infty)$			ω	i	$[3,+\infty)$
		ω	ω	$(3,+\infty)$			ω	ω	$(3,+\infty)$
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		ω	ω	$[3,+\infty)$			ω	ω	$[3,+\infty)$

King, XW, 2310.10369

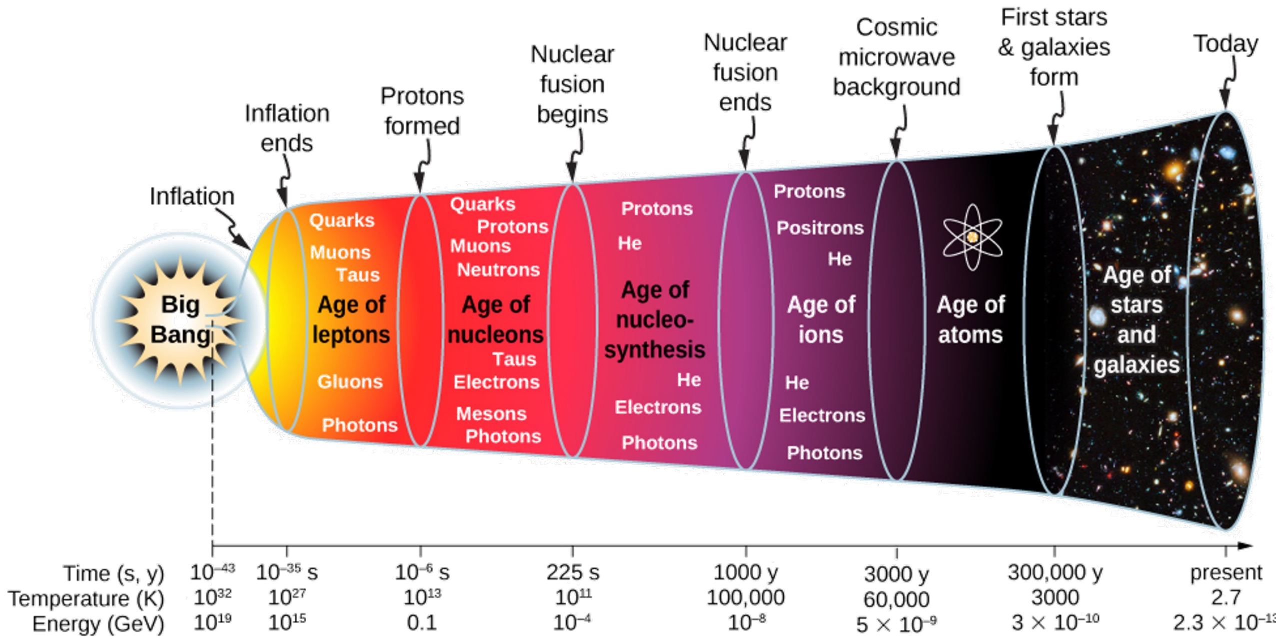
$(m_1,n_1)=(0,0)$
$(m_3,n_3)=(2,0)$ **TM1 mixing**

Varzielas, King, Zhou, PRD, 2020,
King, Zhou, PRD, 2020, …

$(m_1,n_1)=(2,0)$
$(m_3,n_3)=(0,0)$ **Littlest Seesaw**

Varzielas, King, Levy, JHEP, 2022,
de Anda, King, JHEP, 2023, …

Modular inflation



Brout, Englert, Gunzig, 1978
Starobinsky, 1980
Kazanas, 1980
Sato, 1981
Guth, 1981
Linde, 1982
Albrecht & Steinhardt, 1982
Linde, 1983

Inflation

Several issues in hot big-bang cosmology

Horizon problem; Flatness problem; Entropy problem; Primordial perturbation problems; …

Moduli as natural candidates for inflaton in SUGRA!

A cosmological probe of modular symmetries

Kobayashi, Nitta, Urakawa, JCAP, 2016; Abe et al., JHEP, 2023; Gunji, Ishiwata, Yoshida, JHEP, 2022

Modular inflation

King, XW, 2405.08924

$$\mathcal{K}(\tau,\overline{\tau},S,\overline{S}) = K(S,\overline{S}) - 3\log(2\,\mathrm{Im}\,\tau)$$

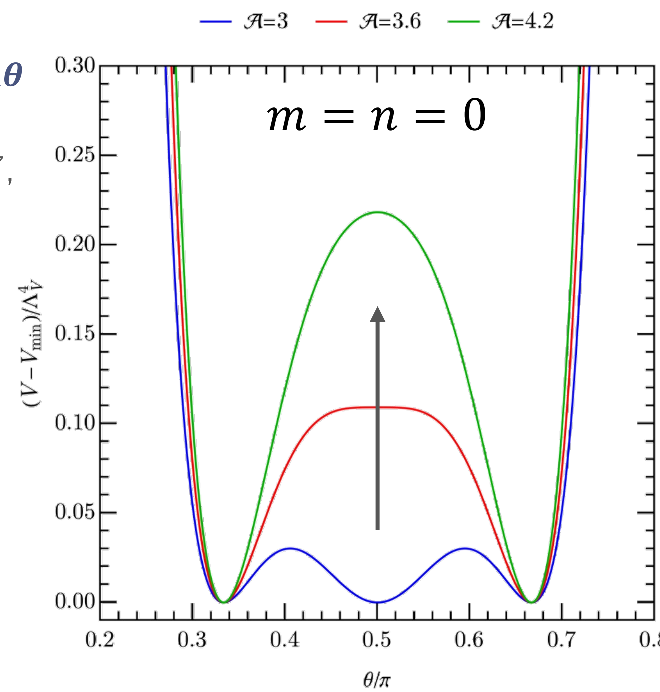
$$\mathcal{W}(\tau,S) = \frac{\Omega(S)H(\tau)}{\eta^6(\tau)}$$

$\boldsymbol{H(\tau) = 1}$

$\boldsymbol{\tau = \rho e^{i\theta}}$

Ding, Jiang, Zhao, 2405.06497, different choices of m and n

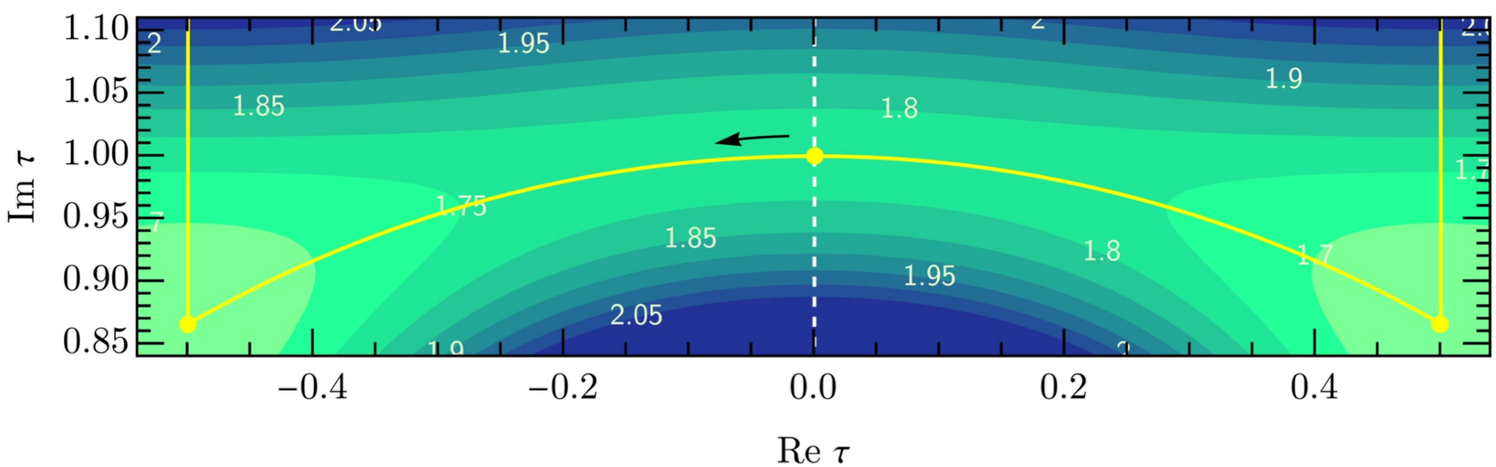
$$V = \frac{\Lambda_V^4 e^{K(S,\overline{S})}|\Omega(S)|^2}{(2\,\mathrm{Im}\,\tau)^3|\eta(\tau)|^{12}}\left[\frac{(2\,\mathrm{Im}\,\tau)^2}{3}\left|\frac{3}{2\pi}\widehat{G}_2(\tau,\overline{\tau})\right|^2 + \left(\mathcal{A}(S,\overline{S}) - 3\right)\right]$$



The dilaton term can uplift the potential.

Transition: $\tau = \mathrm{i}$ from local minimum to saddle point as $\mathcal{A}$ increases.

Hessian = 0 ➡ $\mathcal{A} = 2 + \frac{\Gamma^8(1/4)}{192\pi^4} \approx 3.596$



Slow-roll behaviour near $\tau = \mathrm{i}$

King, XW, 2405.08924

- **Canonical normalisation**

$$\mathcal{L}_{\rm kin} = 3\,(\partial_\mu t\ \partial_\mu \theta)\begin{pmatrix} t^{-2} & \\ & \csc^2\theta \end{pmatrix}\begin{pmatrix} \partial^\mu t \\ \partial^\mu \theta \end{pmatrix} \xrightarrow[\;t \,\equiv\, e^{-\rho/\sqrt{3}}\;]{\;\theta \,\equiv\, 2\arctan(e^{x/\sqrt{3}})\;} \mathcal{L}_{\rm kin} = (\partial_\mu\rho\partial^\mu\rho + \partial_\mu x \partial^\mu x)/2$$

- **Scalar potential: quartic-hilltop approximation**

$$\mathcal{A} = 2 + \frac{\Gamma^8(1/4)}{192\pi^4} \approx 3.596$$

$$\frac{V(x)}{\Lambda_V^4} = \frac{512\pi^9(\mathcal{A}-3)}{\Gamma^{12}(1/4)} + \frac{\pi(192\pi^4 - \Gamma^8(1/4))(192(\mathcal{A}-2)\pi^4 - \Gamma^8(1/4))}{288\Gamma^{12}(1/4)}x^2 + \left[\left(\frac{128\pi^9}{9\Gamma^{12}(1/4)} - \frac{7\pi^5}{54\Gamma^4(1/4)} + \frac{\pi\Gamma^4(1/4)}{1152}\right)\mathcal{A} - \left(\frac{160\pi^9}{9\Gamma^{12}(1/4)} - \frac{2\pi^5}{27\Gamma^4(1/4)} - \frac{5\pi\Gamma^4(1/4)}{5184} + \frac{\Gamma^{12}(1/4)}{73728\pi^3}\right)\right]x^4 + \mathcal{O}(x^6)\,,$$

Slow-roll parameters $\epsilon \equiv \frac{1}{2}\left(\frac{V'}{V}\right)^2\,,\quad \eta \equiv \frac{V''}{V}$

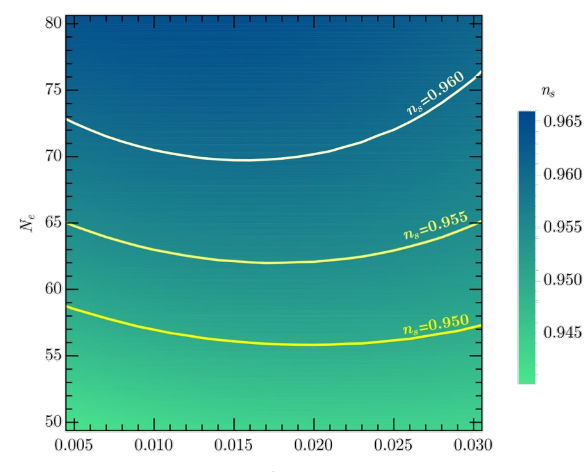
Observables $r \simeq 16\epsilon\,,\quad n_s \simeq 1 - 6\epsilon + 2\eta\,,\quad A_s = \frac{1}{24\pi}\frac{V}{\epsilon}$

$$r \approx \left(\frac{3.78}{\widehat{N}_e^6} + \frac{113}{\widehat{N}_e^4}\mathfrak{a} - \frac{851}{\widehat{N}_e^2}\mathfrak{a}^2\right) \times 10^{-6}\,,$$

$$n_s \approx 1 - \frac{0.05}{\widehat{N}_e^2} + 0.5\mathfrak{a} - 11.25\widehat{N}_e^2\mathfrak{a}^2\,.$$

$$\mathfrak{a} \equiv \mathcal{A} - 3.596$$
$$\hat{N}_e \equiv N_e/60$$

$n_s < 0.956$ **for** $N_e < 60$



$n_s = 0.9661 \pm 0.0040$ (68% CL), $r < 0.036$ (95% CL) Planck (2020), BICEP/KECK (2021)

Slow-roll behaviour near $\tau = \mathrm{i}$

King, XW, 2405.08924

- **Modification from non-minimal $H(\tau)$**

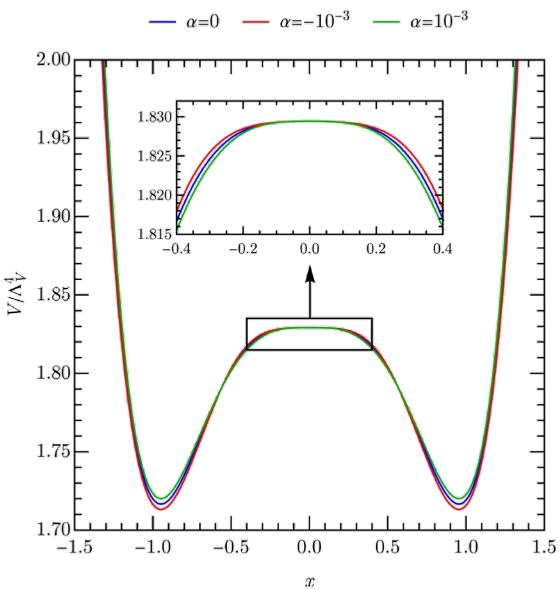
$$H(\tau) = 1 + \alpha\underbrace{[j(\tau)/1728 - 1]^2}_{\delta(\tau)}$$

$$V(\tau,\bar\tau) = \frac{\Lambda_V^4}{(2\,\mathrm{Im}\,\tau)^3|\eta(\tau)|^{12}}\left[\left|\frac{(2\,\mathrm{Im}\,\tau)^2}{3}\mathrm{i}\alpha\delta' + \frac{3(1+\alpha\delta)}{2\pi}\widehat{G}_2(\tau,\bar\tau)\right|^2 + (\mathcal{A}-3)\,|1+\alpha\delta|^2\right]$$

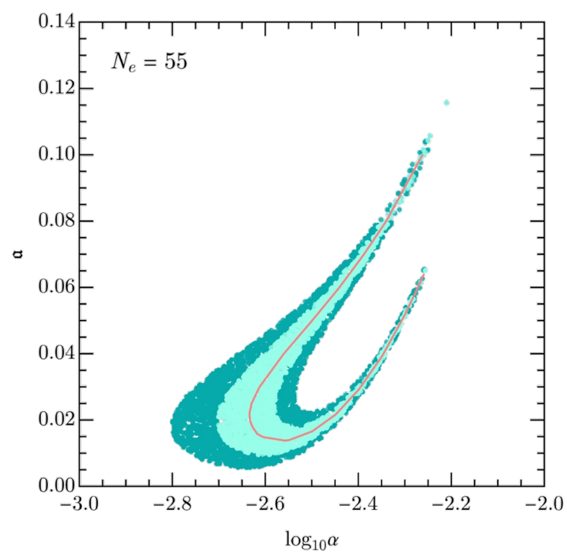
Series expansion

$$V(x) \approx \widetilde{\Lambda}_V^4\left[1 - 0.25\mathfrak{a}x^2 - f_4(\mathfrak{a},\alpha)x^4 + f_6(\mathfrak{a},\alpha)x^6 + \mathcal{O}(x^8)\right]$$

with $f_4(\mathfrak{a},\alpha) \equiv (0.306 - 0.697\mathfrak{a}) + (138 - 308\mathfrak{a})\alpha$, $f_6(\mathfrak{a},\alpha) \equiv (0.389 - 0.702\mathfrak{a}) + (1231 - 2397\mathfrak{a})\alpha + (56455 - 94656\mathfrak{a})\alpha^2$.



$\alpha \sim -10^{-3}$, flatter potential



1σ 2σ best-fit

$$\chi^2(\alpha,\mathfrak{a}) = \frac{(n_s - n_s^{\mathrm{bf}})^2}{\sigma_{n_s}^2}$$

($r < 10^{-5}$ very small)

$$\ln(10^{10}A_s) = 3.040$$

Best-fit from Planck (2020)

$\Lambda_V \simeq 5.7\times10^{-4}M_{\mathrm{Pl}}$

$\Lambda_W \simeq 6.9\times10^{-3}M_{\mathrm{Pl}}$

Nilles, PLB, 1982

Summary

- Modulus stabilisation is crucial for determining the value of τ.
- Modular invariant inflation may be a possible probe of modular symmetries in the early universe.
- Fixed points are special: Special flavour structure; Modulus stabilisation; Orbifold compactification; Inflationary trajectory, …

Obrigado!

Backup

Hessian matrix

$$\frac{\partial^2 V}{\Lambda_V^4 \partial\tau_i^2} = \frac{\partial^2 \widetilde{\mathcal{C}}}{\partial\tau_i^2}\left[\widetilde{\mathcal{M}} + (\mathcal{A}-3)|\mathcal{H}|^2\right] + \widetilde{\mathcal{C}}\left[\frac{\partial^2 \widetilde{\mathcal{M}}}{\partial\tau_i^2} + (\mathcal{A}-3)\mathcal{H}^* \frac{\partial^2 H^{(m_i,n_i)}}{\partial\tau_i^2}\right] ,$$

$$\frac{\partial^2 V}{\Lambda_V^4 \partial\tau_i \partial\overline{\tau}_j} = \frac{\partial^2 \widetilde{\mathcal{C}}}{\partial\tau_i \partial\overline{\tau}_j}[\widetilde{\mathcal{M}} + (\mathcal{A}-3)|\mathcal{H}|^2] + \widetilde{\mathcal{C}}\left[\frac{\partial^2 \widetilde{\mathcal{M}}}{\partial\tau_i \partial\overline{\tau}_j} + (\mathcal{A}-3)\left|\frac{\partial H^{(m_i,n_i)}}{\partial\tau_i}\right|^2\right] ,$$

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 V}{\partial s_1^2} & 0 & 0 & 0 \\ 0 & \frac{\partial^2 V}{\partial t_1^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2 V}{\partial s_3^2} & 0 \\ 0 & 0 & 0 & \frac{\partial^2 V}{\partial t_3^2} \end{pmatrix}$$

with

$$\frac{\partial^2 \widetilde{\mathcal{C}}}{\partial\tau_i \partial\tau_j} = -\mathrm{i}\delta_{ij}\widetilde{\mathcal{C}}\frac{\partial}{\partial\tau_i}\frac{\widehat{G}_2(\tau_i,\overline{\tau}_i)}{2\pi} ,$$

$$\frac{\partial^2 \widetilde{\mathcal{C}}}{\partial\tau_i \partial\overline{\tau}_j} = \mathrm{i}\delta_{ij}\widetilde{\mathcal{C}}\frac{\partial}{\partial\tau_i}\frac{[\widehat{G}_2(\tau_j,\overline{\tau}_j)]^*}{2\pi} ,$$

$$\frac{\partial^2 \widetilde{\mathcal{M}}}{\partial\tau_i \partial\tau_j} = \delta_{ij}(2\,\mathrm{Im}\,\tau_i)^2\left[\mathrm{i}\frac{\partial^2 H^{(m_i,n_i)}(\tau_i)}{\partial\tau_i^2} + \frac{\mathcal{H}(\tau_i)}{2\pi}\frac{\partial\widehat{G}_2(\tau_i,\overline{\tau}_i)}{\partial\tau_i}\right]\frac{\mathcal{H}^*(\tau_i)}{\pi}\frac{\partial\widehat{G}_2^*(\tau_i,\overline{\tau}_i)}{\partial\tau_i} ,$$

$$\frac{\partial^2 \widetilde{\mathcal{M}}}{\partial\tau_i \partial\overline{\tau}_j} = \delta_{ij}(2\,\mathrm{Im}\,\tau_i)^2\left[\left|\mathrm{i}\frac{\partial^2 H^{(m_i,n_i)}(\tau_i)}{\partial\tau_i^2} + \frac{\mathcal{H}(\tau_i)}{2\pi}\frac{\partial\widehat{G}_2(\tau_i,\overline{\tau}_i)}{\partial\tau_i}\right|^2 + \left|\frac{\mathcal{H}(\tau_i)}{2\pi}\frac{\partial\widehat{G}_2(\tau_i,\overline{\tau}_i)}{\partial\overline{\tau}_i}\right|^2\right] ,$$

Series expansion of the scalar potential

Ramanujan identities indicate that the derivatives of Eisenstein series can be expressed as the linear combinations of the first few Eisenstein series.

$$G_2|_{\tau=\mathrm{i}} = \pi \,,$$

$$\left.\frac{\mathrm{d}G_2}{\mathrm{d}\tau}\right|_{\tau=\mathrm{i}} = \frac{\pi \mathrm{i}}{6}\left[3 - \frac{\Gamma^8(1/4)}{64\pi^4}\right] ,$$

$$\left.\frac{\mathrm{d}^2G_2}{\mathrm{d}\tau^2}\right|_{\tau=\mathrm{i}} = -\frac{\pi}{2}\left[1 - \frac{\Gamma^8(1/4)}{64\pi^4}\right] ,$$

$$\left.\frac{\mathrm{d}^3G_2}{\mathrm{d}\tau^3}\right|_{\tau=\mathrm{i}} = -\frac{\pi \mathrm{i}}{4}\left[3 - \frac{3\Gamma^8(1/4)}{32\pi^4} - \frac{\Gamma^{16}(1/4)}{4096\pi^8}\right] ,$$

$$\left.\frac{\mathrm{d}^4G_2}{\mathrm{d}\tau^4}\right|_{\tau=\mathrm{i}} = \frac{\pi}{2}\left[3 - \frac{5\Gamma^8(1/4)}{32\pi^4} - \frac{5\Gamma^{16}(1/4)}{4096\pi^8}\right] ,$$

$$\frac{dE_2}{d\tau} = 2\pi \mathrm{i}\frac{E_2^2 - E_4}{12} ,$$

$$\frac{dE_4}{d\tau} = 2\pi \mathrm{i}\frac{E_2E_4 - E_6}{3} ,$$

$$\frac{dE_6}{d\tau} = 2\pi \mathrm{i}\frac{E_2E_6 - E_4^2}{2} .$$

$$\left.\frac{\mathrm{d}\eta}{\mathrm{d}\tau}\right|_{\tau=\mathrm{i}} = \frac{\mathrm{i}\Gamma(1/4)}{8\pi^{3/4}}$$

$$\left.\frac{\mathrm{d}^2\eta}{\mathrm{d}\tau^2}\right|_{\tau=\mathrm{i}} = -\frac{\Gamma(1/4)[288\pi^4 - \Gamma^8(1/4)]}{3072\pi^{19/4}} ,$$

$$\left.\frac{\mathrm{d}^3\eta}{\mathrm{d}\tau^3}\right|_{\tau=\mathrm{i}} = -\frac{5\mathrm{i}\Gamma(1/4)[96\pi^4 - \Gamma^8(1/4)]}{4096\pi^{19/4}} ,$$

$$\left.\frac{\mathrm{d}^4\eta}{\mathrm{d}\tau^4}\right|_{\tau=\mathrm{i}} = \frac{\Gamma(1/4)[322560\pi^8 - 6720\pi^4\Gamma^8(1/4) - 11\Gamma^{16}(1/4)]}{1572864\pi^{35/4}} .$$

Evading the cosmological constant problem

Large vacuum energy after inflation can be eliminated by introducing a "waterfall" field direction.

$$\sigma \equiv y\Lambda_V$$

Toy model

$$\frac{\Delta V(x,y)}{\Lambda_V^4} = \frac{1}{2}[-\widehat{\mu}^2 + h(x)]y^2 + \frac{\lambda}{4}y^4$$

$$V_{\rm tot} = V(x) + \Delta V(x,y)$$

Choosing

$\hat{\mu}^2 = 1.3,$
$\lambda = 0.32,$
$h(x) = 2.4(x^2 - 0.925)^2$