CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Quadratic Coupling of Axions to Photons

based on arXiv: 2307.10362

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Outline of this talk

Axion-photon coupling: $\mathcal{L} \supset -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

Quadratic Coupling of Axions to Photons:

Which operator?

$(\partial a)^2 F_{\mu\nu} F^{\mu\nu}$ vs $a^2 F_{\mu\nu} F^{\mu\nu}$

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How to generate $a^2 F_{\mu\nu}F^{\mu\nu}$?

From QCD axion?
From ALP?

$$\mathcal{L}_{a^2F^2} \supset c_{F^2} \frac{\alpha}{16\pi^2} \left(\frac{a^2}{f_a^2}\right) F_{\mu\nu}F^{\mu\nu}$$

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Why do we care $a^2 F_{\mu\nu}F^{\mu\nu}$?

Variation of

fine structure constant

$$\alpha(t) \simeq \alpha\left[1 + c_{_{F^2}}\frac{\alpha}{4\pi^2}\left(\frac{a(t)}{f_a}\right)^2\right]$$

Big Bang Nucleosynthesis (BBN)

Ultra-light DM searches

Fifth-forces and Weak Equivalence Principle

New (more stringent?) constrains

on the axion(QCD/ALP) parameter space

How to generate aaFF: Guidance from QCD axion

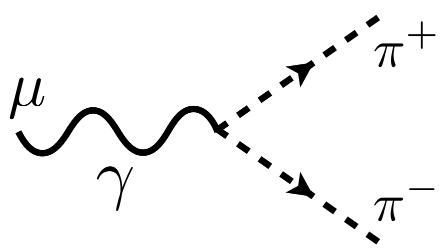
- No tree-level contribution from $\mathcal{L} \supset -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

$$\mathcal{M}_{t;\,+,-} = \frac{g_{a\gamma\gamma}^2}{t} \left[\left(\varepsilon_1^+ \cdot \varepsilon_2^- \right) (p_1 \cdot q) - \left(\varepsilon_1^+ \cdot q \right) \left(\varepsilon_2^- \cdot p_1 \right) \right] (p_2 \cdot q) = 0$$

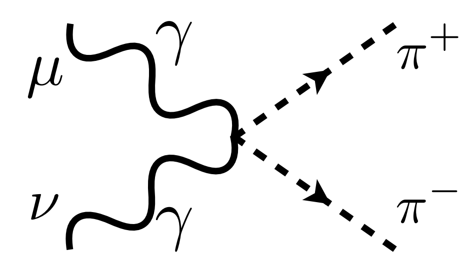
How to generate aaFF: Guidance from QCD axion

- No tree-level contribution from $\mathcal{L} \supset -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$
- 1-loop contribution from QCD Chiral Lagrangian:

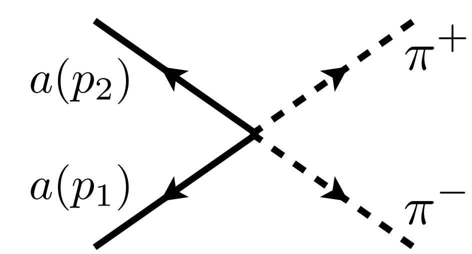
$\mathcal{L}_{\chi PT}^{(p^2)} \supset$



$-i\, e\, (p_+ - p_-)_\mu$



$2\, i\, e^2\, g_{\mu\nu}$



$\frac{i}{f_a^2}\left[\frac{m_u\, m_d\, m_\pi^2}{(m_u + m_d)^2} - \frac{2(m_d - m_u)^2 p_1 \cdot p_2}{(m_u + m_d)^2}\right]$

How to generate aaFF: Guidance from QCD axion

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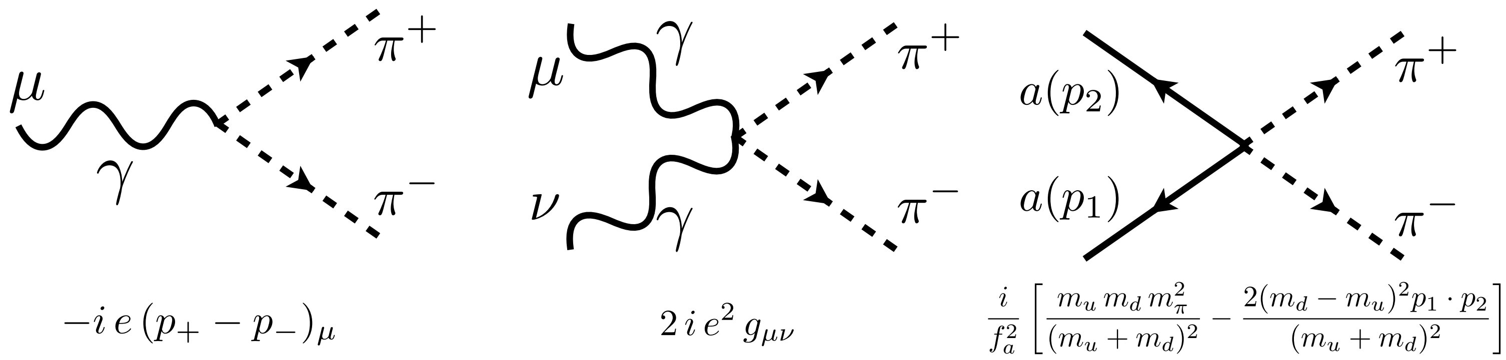
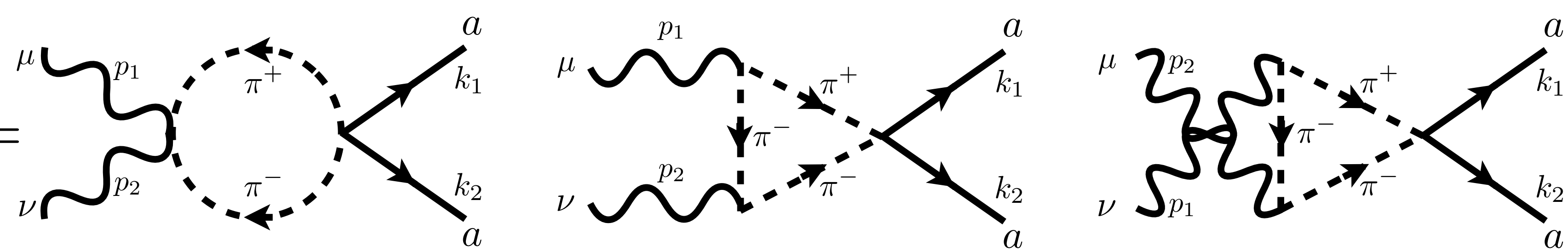


1-loop matching

$\mathcal{A}(\gamma\gamma \to aa) =$



$$\mathcal{L}_{a^2F^2} \supset c_{F^2} \frac{\alpha}{16\pi^2}\left(\frac{a^2}{f_a^2}\right) F_{\mu\nu}F^{\mu\nu}$$

$$c_{F^2} = \pi\, m_u\, m_d / 3\,(m_u + m_d)^2 \sim 0.2$$

How to generate aaFF: Guidance from QCD axion

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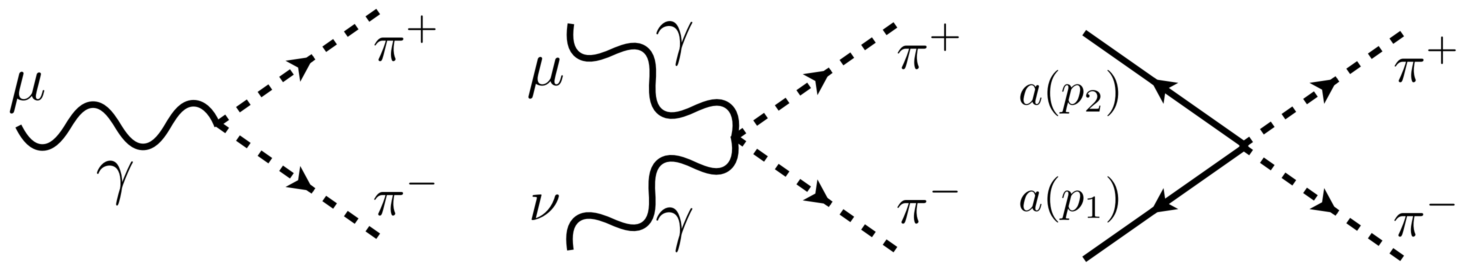

$-i\, e\, (p_+ - p_-)_\mu$ $\quad$ $2\, i\, e^2\, g_{\mu\nu}$ $\quad$ $\frac{i}{f_a^2}\left[\frac{m_u\, m_d\, m_\pi^2}{(m_u+m_d)^2} - \frac{2(m_d-m_u)^2 p_1\cdot p_2}{(m_u+m_d)^2}\right]$

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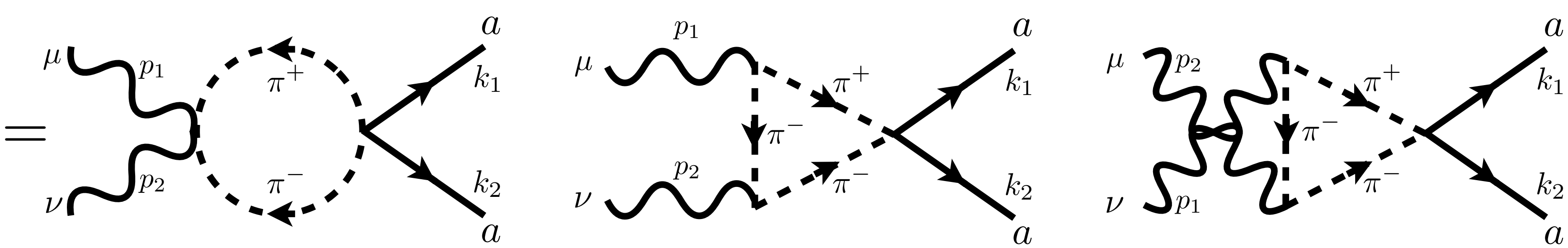

$$\mathcal{L}_{a^2F^2} \supset c_{F^2} \frac{\alpha}{16\pi^2}\left(\frac{a^2}{f_a^2}\right) F_{\mu\nu}F^{\mu\nu} \quad \cdots\cdots\to \quad \mathcal{L}_{a^2F^2} \supset \frac{\alpha}{16\pi^2}\,\frac{\pi}{3}\,\frac{m_a^2}{\epsilon\, m_\pi^2\, f_\pi^2}\, a^2\, F_{\mu\nu}F^{\mu\nu}$$

$$c_{F^2} = \pi\, m_u\, m_d / 3\,(m_u + m_d)^2 \sim 0.2$$

$$m_a^2 = \epsilon \frac{m_u m_d}{(m_u+m_d)^2}\,\frac{m_\pi^2 f_\pi^2}{f_a^2}$$

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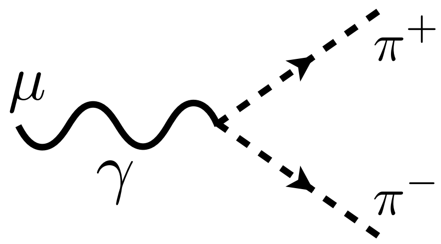

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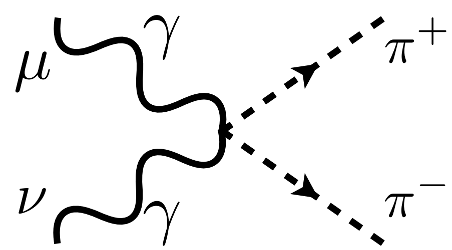

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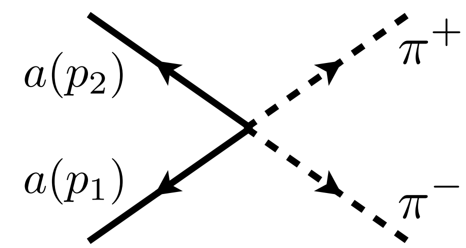

$\frac{i}{f_a^2}\left[\frac{m_u\, m_d\, m_\pi^2}{(m_u+m_d)^2} - \frac{2(m_d-m_u)^2 p_1\cdot p_2}{(m_u+m_d)^2}\right]$

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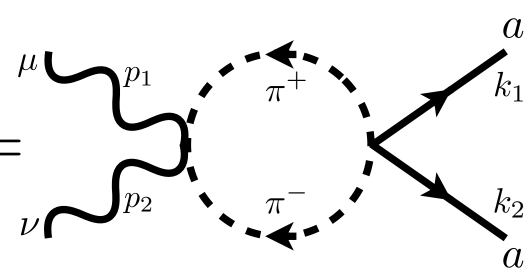

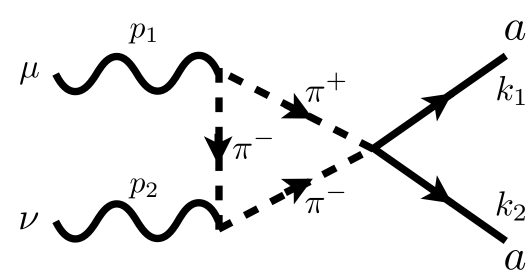

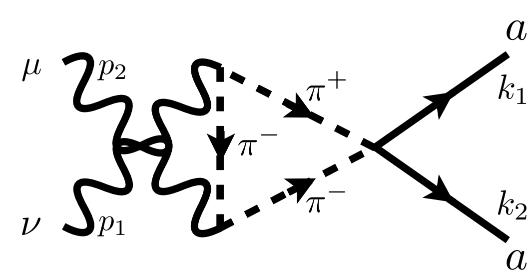

$$\mathcal{L}_{a^2F^2} \supset c_{_{F^2}} \frac{\alpha}{16\pi^2}\left(\frac{a^2}{f_a^2}\right) F_{\mu\nu}F^{\mu\nu} \quad \longrightarrow \quad \alpha \simeq \alpha_0\left(1 + \frac{\alpha_0\, m_u\, m_d\, a^2}{12\,\pi\, f_a^2\,(m_u+m_d)^2}\right)$$

$$c_{_{F^2}} = \pi\, m_u\, m_d / 3\,(m_u+m_d)^2 \sim 0.2$$

The same result can be obtained from threshold corrections
H. Kim, A. Lenoci, G. Perez, W. Ratzinger (2307.14962)

How to generate aaFF: Guidance from QCD axion

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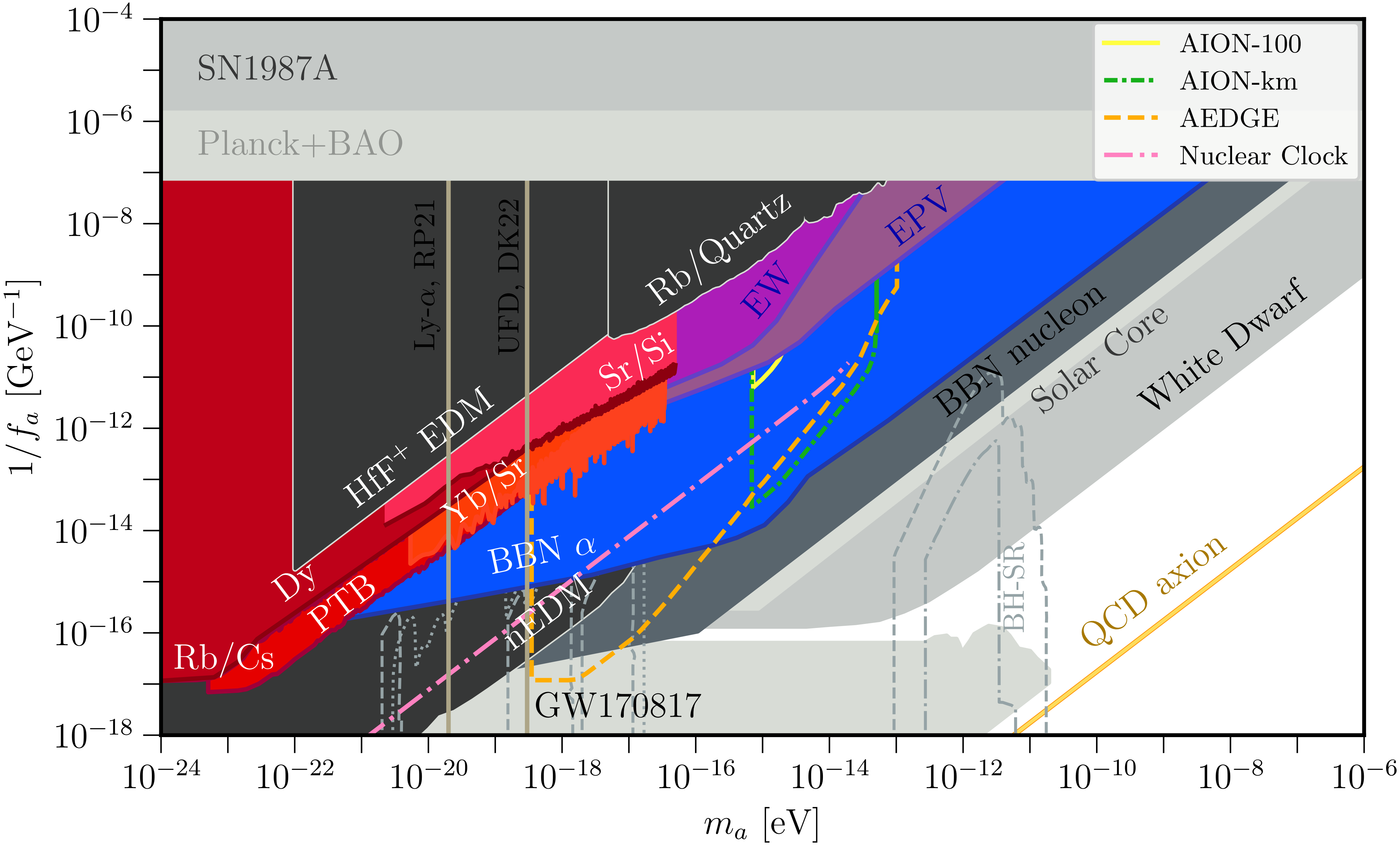
- If axion make up dark matter:

Temporal variation of alpha

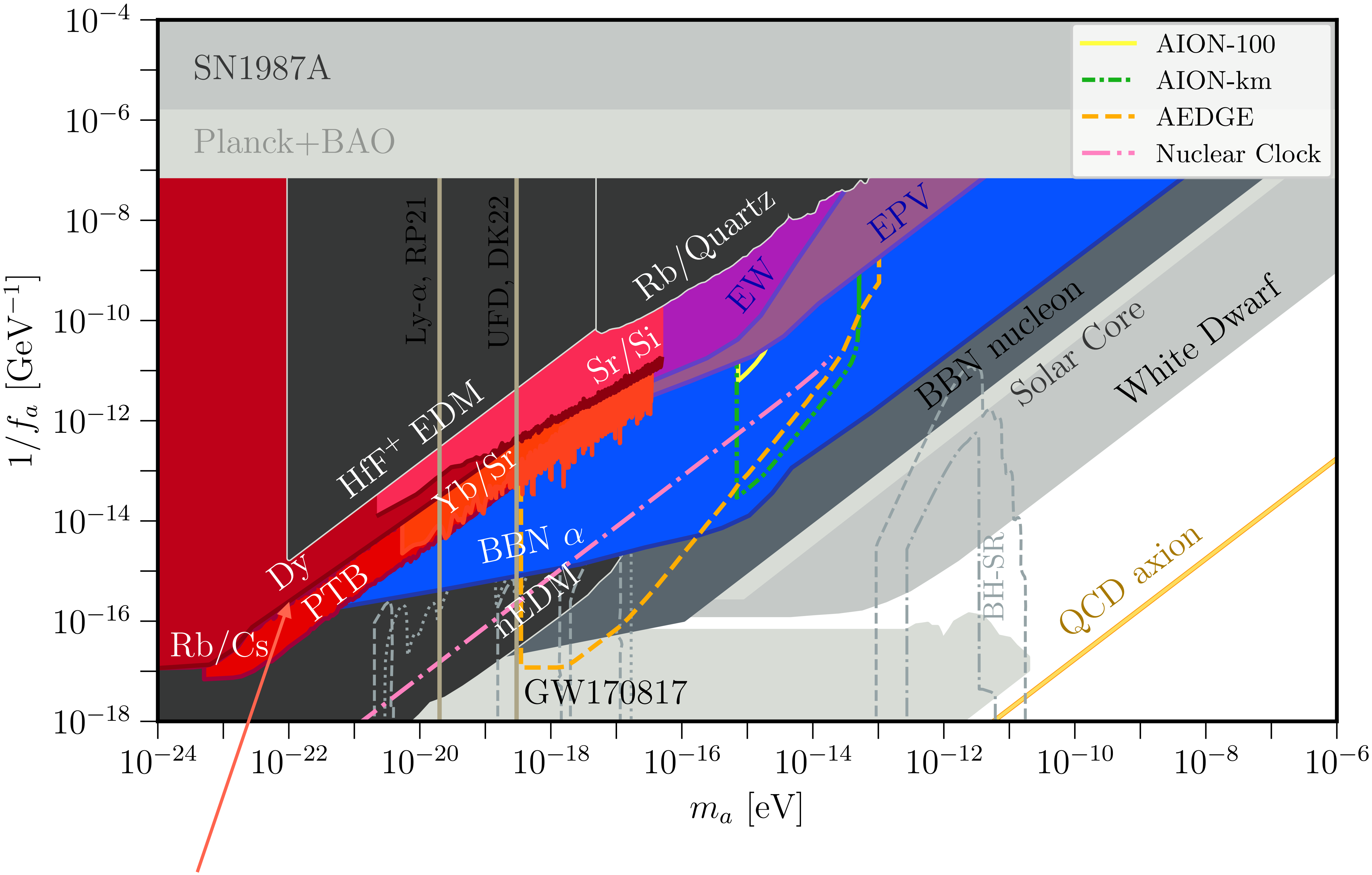
$$a(t) \simeq \frac{\sqrt{2\rho_{DM}}}{m_a} \cos(m_a t + \varphi) X(r) \quad \longrightarrow \quad \frac{\Delta\alpha}{\alpha} \simeq c_{_{F^2}} \frac{\alpha}{4\pi^2} \frac{2\rho_{DM}}{m_a^2\, f_a^2} \cos^2(m_a t + \varphi) X(r)^2$$

Constrains on axion parameter space: Guidance from QCD axion



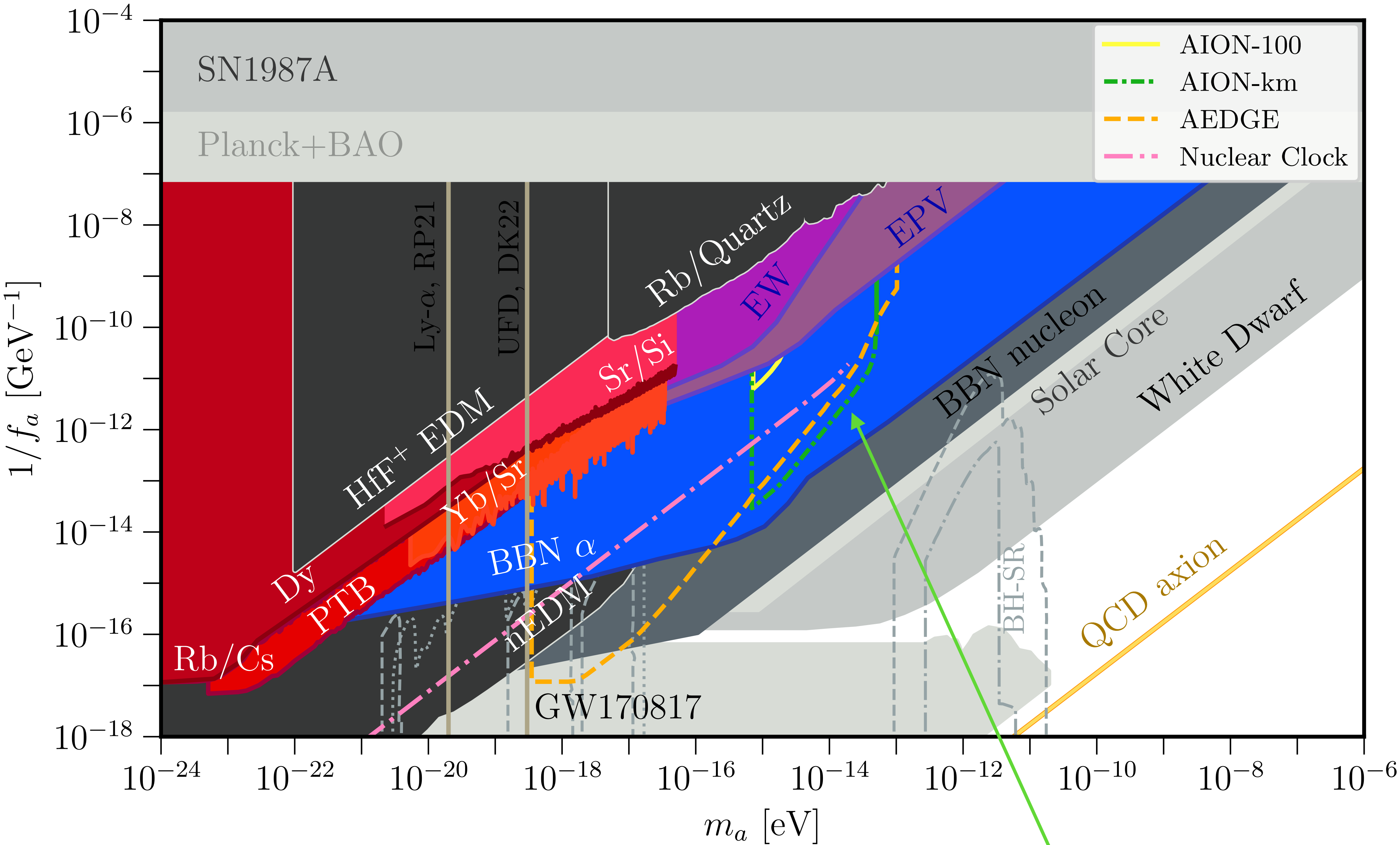
See also H. Kim, A. Lenoci, G. Perez, W. Ratzinger (2307.14962)

Constraints on axion parameter space: Guidance from QCD axion



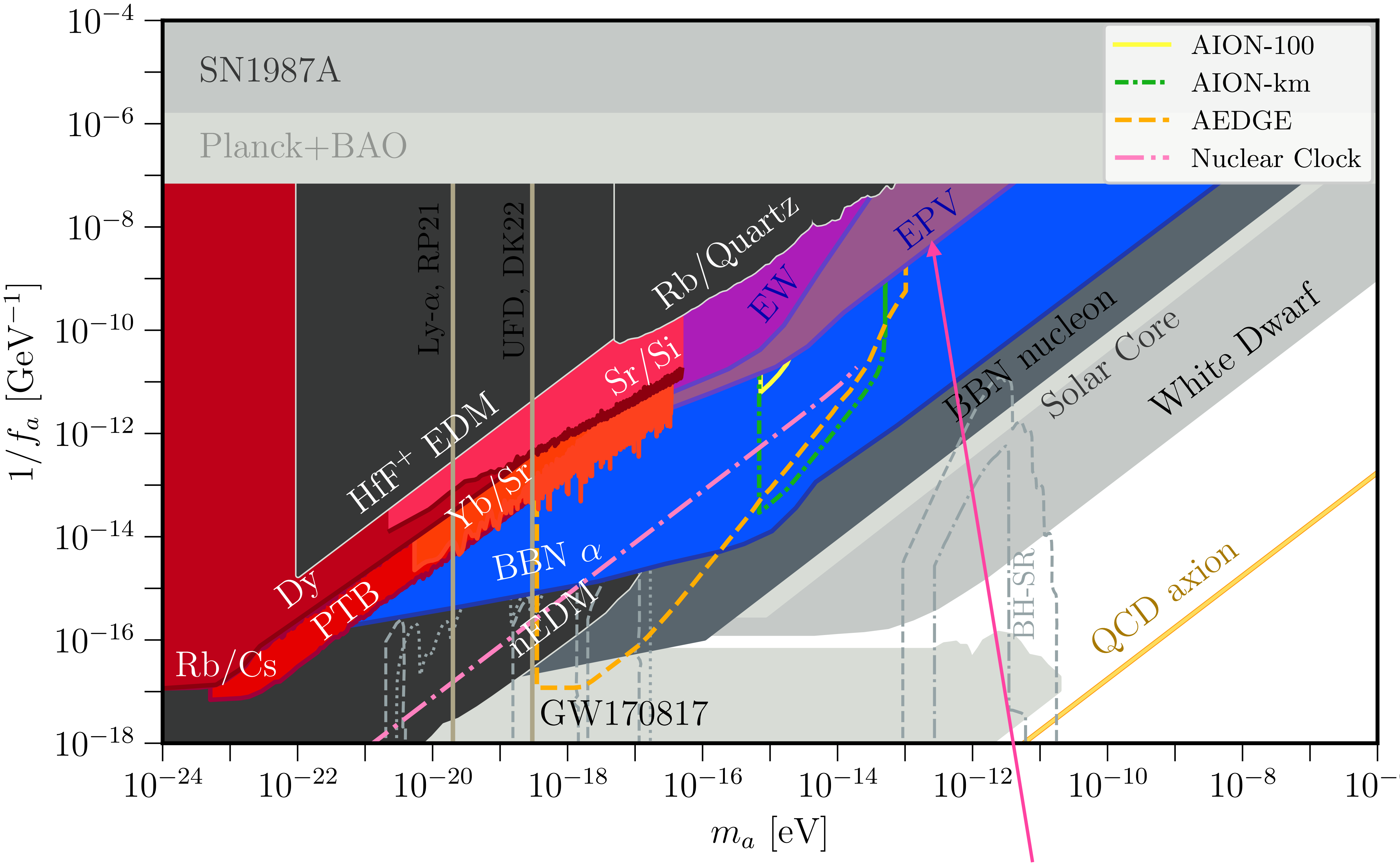
Constraints from atomic clocks

Constraints on axion parameter space: Guidance from QCD axion



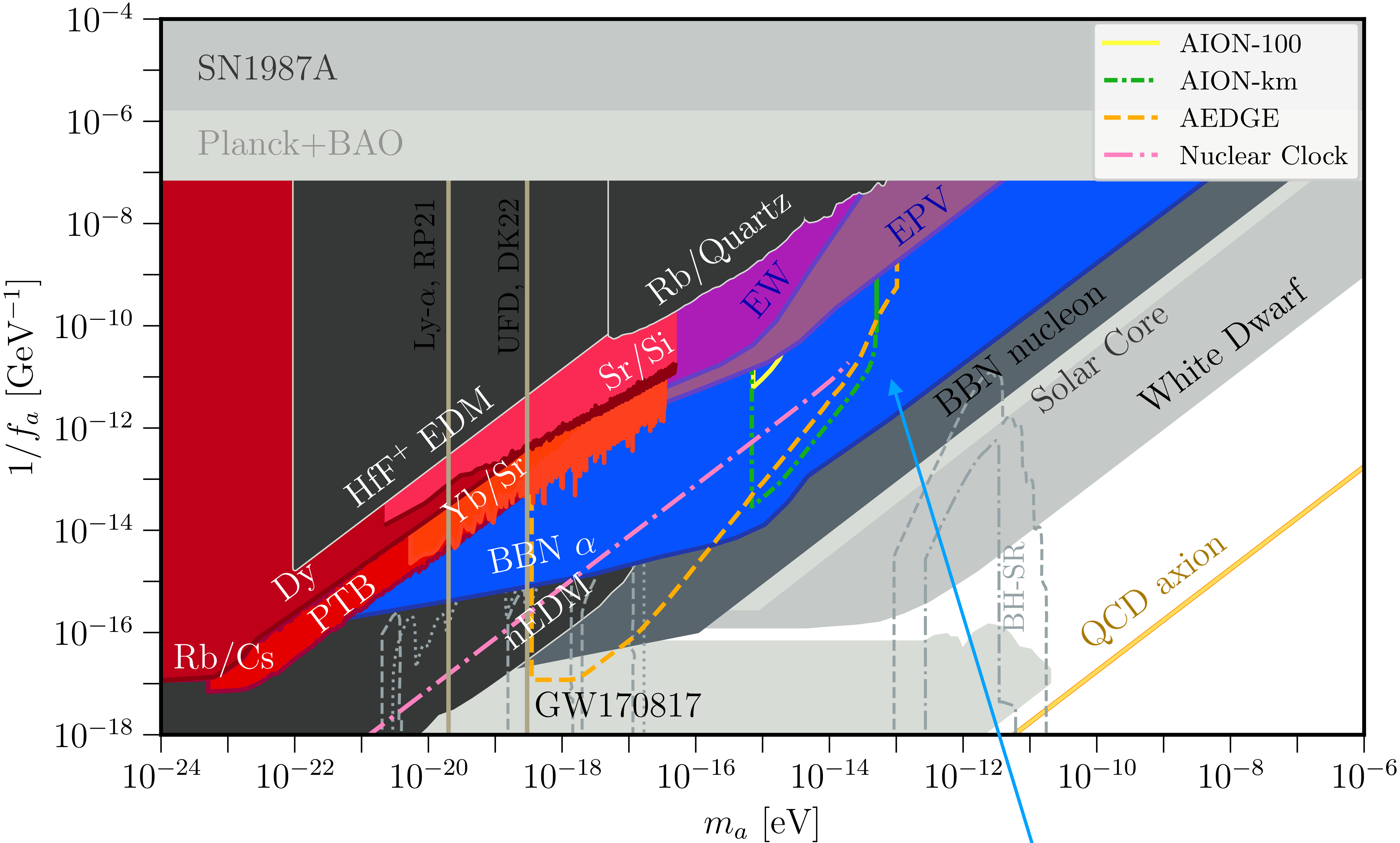
Constraints from atomic interferometers (AION)

Constraints on axion parameter space: Guidance from QCD axion



Constraints from MICROSCOPE mission
(searching for violation of weak equivalent principle)

Constraints on axion parameter space: Guidance from QCD axion



Constraints from the standard BBN
(Yield of ^4He)

How to generate aaFF: Generalise to Axion-Like Particles(ALPs)

- Toy model 1: Shift symmetry-breaking EFT resulting from KSVZ-like setup

$$\mathcal{L}_{\rm UV} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R + \left(y\phi\bar{\psi}_L\psi_R + \text{h.c.}\right) + \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi^\dagger\phi)$$

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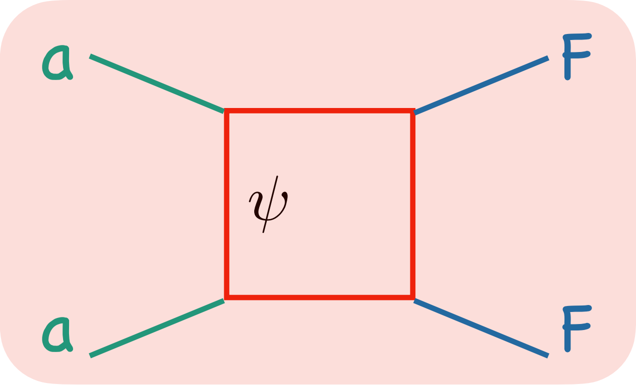
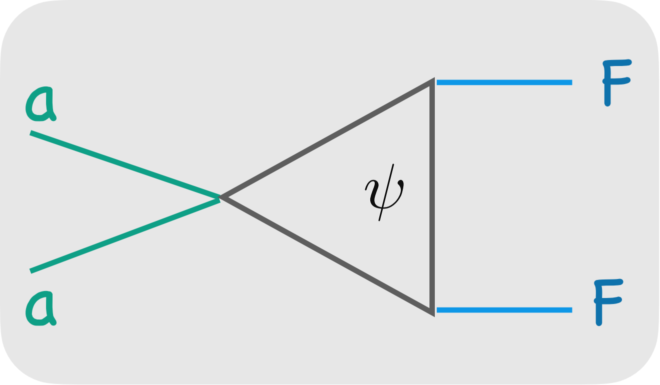
Without explicit shift-symmetry-breaking terms:

$$V(\phi^\dagger\phi) = \lambda\left(\phi^\dagger\phi - \frac{f_a^2}{2}\right)^2$$ and $$\phi = \frac{1}{\sqrt{2}}(\rho + f_a)e^{ia/f_a}$$

$$\mathcal{L}^{\text{1-loop}}_{a^2F^2} = \frac{i^2}{16\pi^2}\frac{1}{3M_\psi^2}\left[M_\psi^2\frac{a^2}{f_a^2}\right](iQ_\psi e)^2 F_{\mu\nu}F^{\mu\nu} + \frac{i^2}{16\pi^2}\frac{2}{3M_\psi}\left[M_\psi\frac{(ia)^2}{2f_a^2}\right](iQ_\psi e)^2 F_{\mu\nu}F^{\mu\nu} = 0$$





How to generate aaFF: Generalise to Axion-Like Particles(ALPs)

- Toy model 1: Shift symmetry-breaking EFT resulting from KSVZ-like setup

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Adding a shift-symmetry-breaking terms to the potential:

$$V(\phi^\dagger\phi) \supset \lambda\left(\phi^\dagger\phi - \frac{f_a^2}{2}\right)^2 + g^2\left(\phi^\dagger\phi - \frac{f_a^2}{2}\right)\left(1-\cos\left(\frac{a}{f_a}\right)\right) \quad \text{and} \quad \phi = \frac{1}{\sqrt{2}}(\rho + f_a)e^{ia/f_a}$$

Preserves CP, $\mathbb{Z}_n$ symmetry, no mass term of an ALP at tree-level

How to generate aaFF: Generalise to Axion-Like Particles(ALPs)

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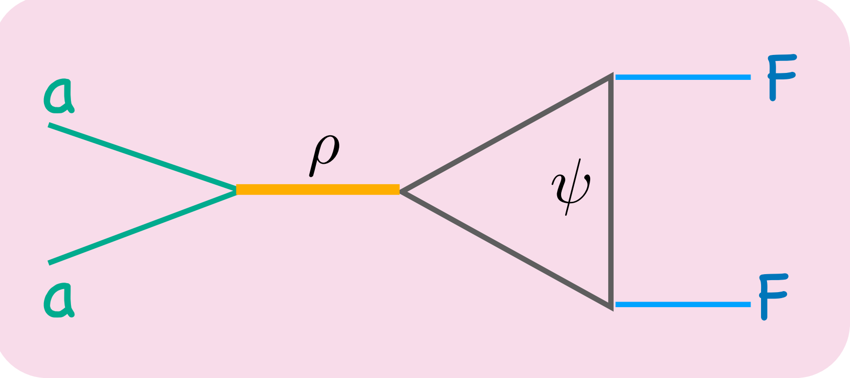
Integrating out: ρ, ψ



$$\mathcal{L}^{1-\text{loop}}_{a^2F^2} \supset \frac{i^2}{16\pi^2}\frac{2}{3M_\psi}\left[M_\psi\frac{\rho}{f_a}\right](iQ_\psi e)^2 F_{\mu\nu}F^{\mu\nu}\Big|_{\rho=\rho_c(a)}$$

$$\supset \frac{1}{48\pi^2}(Q_\psi e)^2\frac{g^2}{f_a^2 M_\rho^2}a^2F_{\mu\nu}F^{\mu\nu}$$

One can derive the variation of $\alpha(a)$:

$$c_{F^2} = \frac{4\pi}{3}Q_\psi^2\frac{g^2}{M_\rho^2}\,, \quad \alpha(a) = \alpha\left(1 + \frac{Q_\psi^2\alpha}{3\pi}\frac{g^2a^2}{M_\rho^2 f_a^2}\right)$$

How to generate aaFF: Generalise to Axion-Like Particles(ALPs)

- Toy model 2: QCD-like dynamics for an ALP

Main ingredients: Dark QCD-like & Dark photon sectors: $SU(N)' \otimes U(1)'$
Chiral fermions charged under the symmetries of the dark sectors

Using SU(N)' instanton to break the shift-symmetry of an ALP:

$$V(a) = -m_{\pi'}^2 f_{\pi'}^2 \cos\left(\frac{a}{2f_a}\right) \simeq -\frac{1}{2} m_a^2 a^2$$

(assuming: $m_{u'} = m_{d'}$)

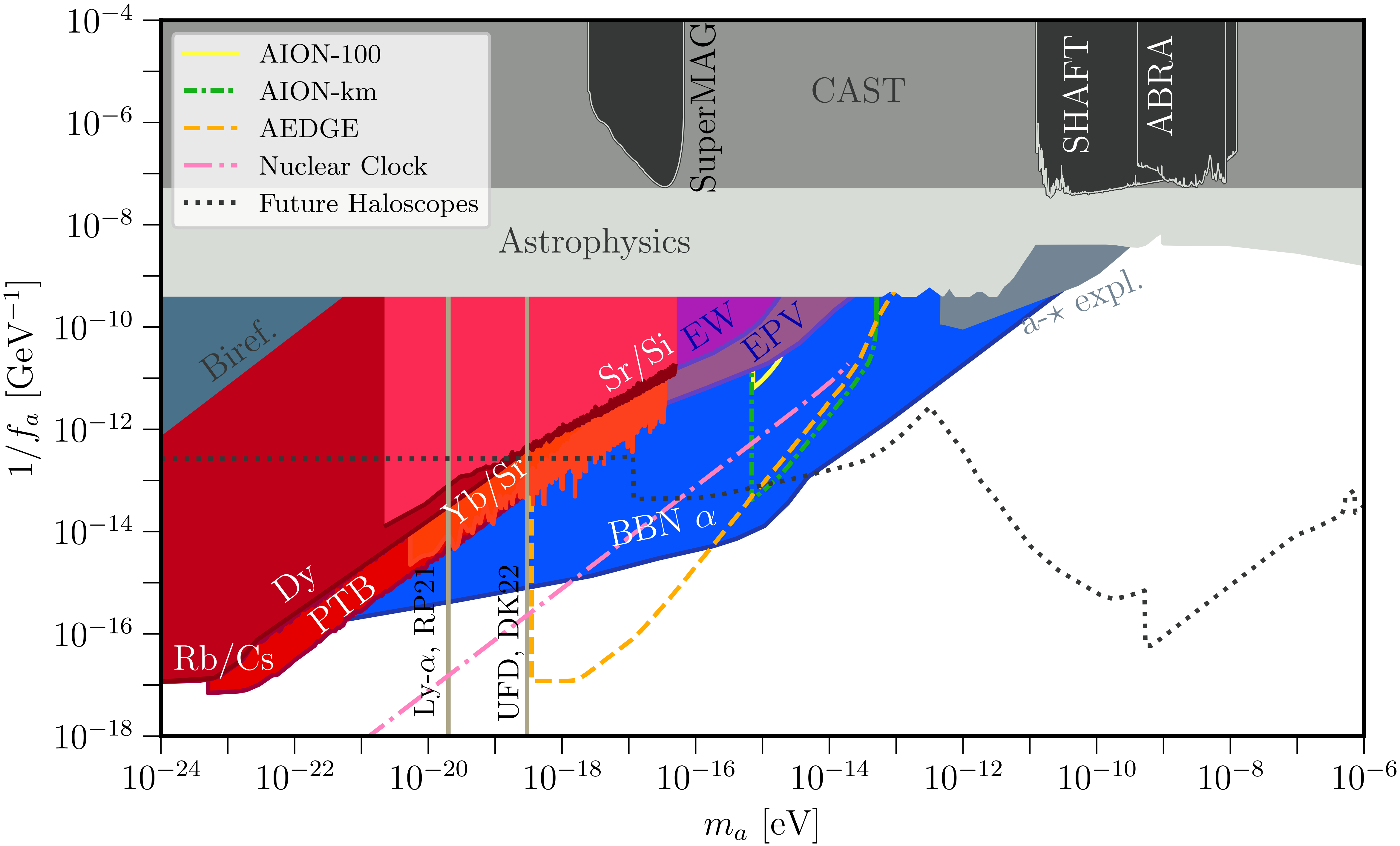
Analogously to QCD axion computations, the fine structure constant in the dark sector is also modified:

$$\alpha' \simeq \alpha'_0 \left(1 + \frac{\alpha'_0\, a^2}{48\,\pi\, f_a^2}\right)$$

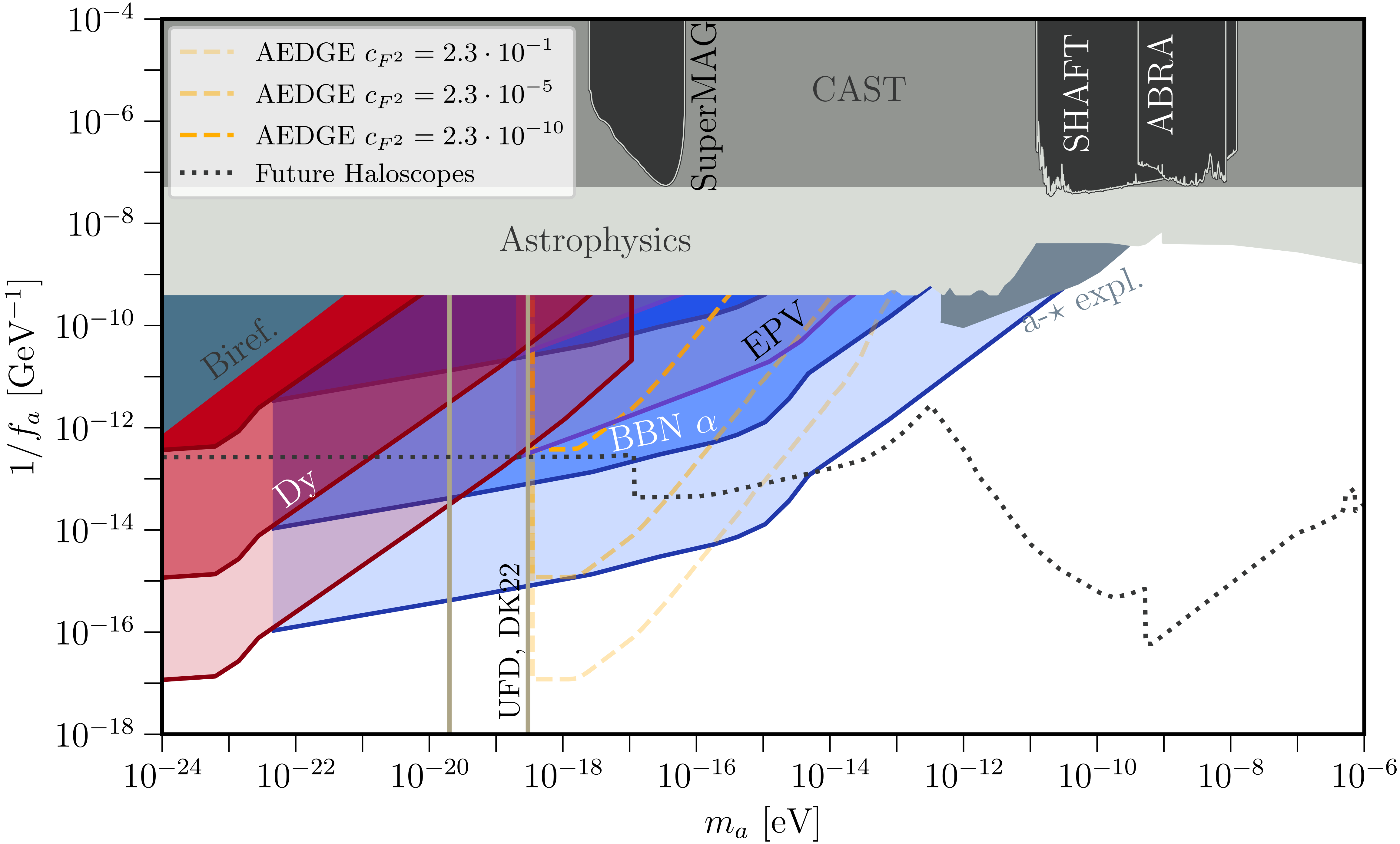
Using kinetic mixing χ between photon & dark photon:

$$\alpha \simeq \alpha_0 \left(1 + \chi^2 \alpha'_0 \frac{a^2}{48\,\pi\, f_a^2}\right)$$

Bounds in axion-photon couplings parameter space: ALPs



Sensitivity of the quadratic coupling: ALPs



Quadratic coupling of Axions to Photons: Summary

- **For the QCD axion:**

  1.) Dynamics endowing axion with a mass can also lead to a quadratic coupling of axions to photons

  2.) Other pre-existing constraints are stronger, but quadratic coupling offers new ways to probe parts of parameter space and exploit the precision of table-top experiments of fundamental constants.

- **For Axion-like-Particles:**

  1.) Resulting constrain may be the strongest bounds in large regions of parameter space

  2.) Constructing a UV completion for the shift-breaking coupling with less fine-tuning of the ALP mass is not easy

Backup slides

Backup slides: One-Loop Effective Action

Path integral formalism: $$e^{iS_{eff}[\psi^L_{SM}](\mu)} = \int \mathcal{D}\psi^H_{BSM}\, e^{iS[\psi^H_{BSM},\psi^L_{SM}](\mu)}$$

Find classical solution by solving EOM:

$$\left.\frac{\delta S\left[\psi^H_{BSM},\psi^L_{SM}\right]}{\delta\psi^H_{BSM}}\right|_{\psi^H_{BSM}=\psi^H_{BSM,c}} = 0 \Rightarrow \psi^H_{BSM,c}(\psi^L_{SM})$$

Expand action around minimum:

$$S\left[\psi^H_{BSM}\right] = S\left[\psi^H_{BSM,c}+\eta\right] = S\left[\psi^H_{BSM,c}\right] + \frac{1}{2}\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\eta^2 + \mathcal{O}(\eta^3)$$

Integrate over quantum fluctuation η :

$$e^{iS_{eff}[\psi^L_{SM}]} = e^{iS[\psi^H_{BSM,c}]}\left[\det\left(-\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\right)\right]^{-c_s}$$

c_s is spin factor ($c_s = +1/2$ for real scalar, -1 for Dirac fermion)

Re-write the determinant, $\det(A) = e^{\mathrm{Tr}\log A}$:

$$S_{eff}\left[\psi^L_{SM}\right] = S\left[\psi^H_{BSM,c}\left(\psi^L_{SM}\right),\psi^L_{SM}\right] + ic_s\mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta(\psi^H_{BSM})^2}\right|_{\psi^H_{BSM,c}}\right)$$

Tree-level One-loop level

Backup slides—Building axion EFTs: One-loop matching using functional method

We parametrise the shape of UV Lagrangian as follows:

$$\mathcal{L}_{\rm UV}^{\rm fermion}\left[\Psi_H,\phi\right] \supset \bar{\Psi}_H\left[iD_\mu\gamma^\mu - M + X[\phi]\right]\Psi_H$$

General coupling with background fields

Example: $X[\phi] = V_\mu[\phi]\gamma^\mu - A_\mu[\phi]\gamma^\mu\gamma^5 - W_1[\phi]i\gamma^5$

Path Integral: extract the one-loop (heavy-only) piece: $e^{iS_{eff}[\phi_L]} = \int \mathcal{D}\bar{\Psi}_H\mathcal{D}\Psi_H\, e^{iS_{\rm UV}[\Psi_H,\phi_L]}$

$$\mathcal{S}_{eff}^{1-loop} = -i\,\mathrm{Tr}\log\left(-\left.\frac{\delta^2 S}{\delta\Psi_H^2}\right|_{\Psi_{H,c}}\right) = -i\,\mathrm{Tr}\log\left(iD_\mu\gamma^\mu - M + X[\phi]\right)$$

Evaluating the functional trace: $\mathrm{Tr}\,\mathcal{O}(i\not{D},X) = \int d^4x\int\frac{d^4q}{(2\pi)^4}\,\mathrm{tr}\,\mathcal{O}(i\not{D}-\not{q},X)$

$$\mathcal{L}_{\rm EFT}^{\rm 1loop} = i\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^4q}{(2\pi)^4}\left[\frac{-1}{\not{q}+M}\left(-iD_\mu\gamma^\mu - V_\mu[\phi]\gamma^\mu + A_\mu[\phi]\gamma^\mu\gamma^5 + W_1[\phi]i\gamma^5\right)\right]^n$$

Encapsulate axion derivative couplings

- Expanding order by order (ex: up to n=6)
- Integrating over momentum q (use Dimensional Regularisation for divergence integrals)
- Evaluating the Dirac traces (careful with γ^5)

 S. A. R. Ellis, J. Quevillon, P. N. H. Vuong, T. You, Z. Zhang (2006.16260)