Functional Matching at Two Loop Order

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PLANCK2024

Effective Field Theory

They represent our best connection between theory and experiment

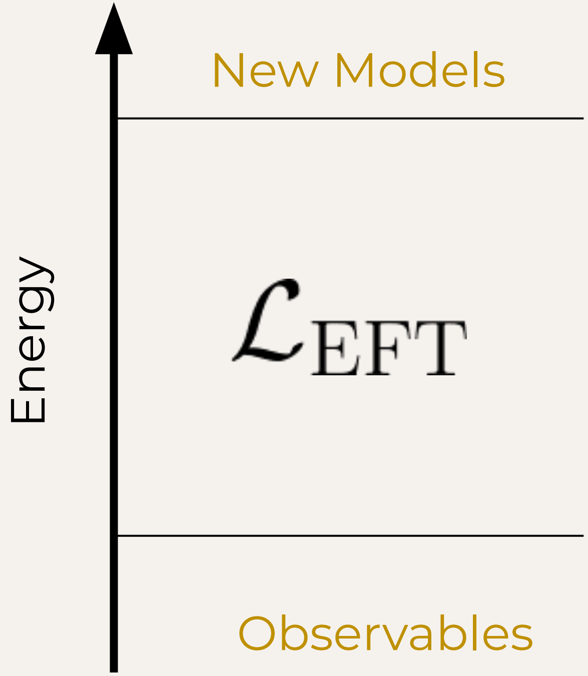
Wilson Coefficients

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{d\leq 4} + \sum_{\ell=0}^{\infty}\sum_{n=5}^{\infty} \frac{C^{(\ell,n)}}{\Lambda^{n-4}} \mathcal{O}_n$$

Bottom up: Parameterize our lack of knowledge

Top Down: Separate scales for precision measurements

Separation of Scales



BSM requires multiscale physics

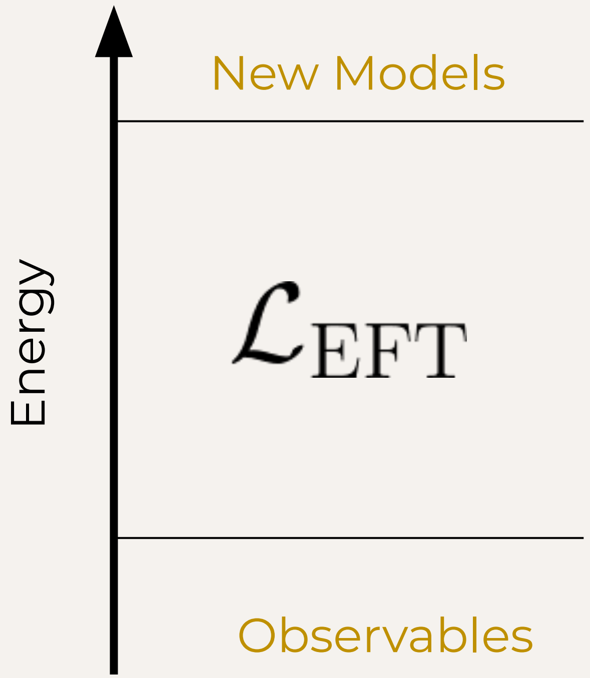
- **Matching** of UV theories to low energy observables
- Process of matching automatized up to one loop

MATCHMAKEREFT [Carmona et al-2112.10787]

MATCHETE [Fuentes-Martín et al-2212.04510]

- Running of the theory via RG evolution

Why Loops



- There is physics we must not lose at the one loop level
- Scheme independence RG evolution at one loop requires two loops
- Precision test of UV models up to two loops are also interesting

Amplitude Matching

$$\mathcal{L}_{\mathrm{UV}}(z_h, z_l) \xrightarrow{q_i << \Lambda} \{\mathcal{A}_{\mathrm{UV}}(q_i)\}$$

Matching:
Determining Wilson Coefficients

$$\mathcal{L}_{\mathrm{EFT}}(z_l) \longrightarrow \{\mathcal{A}_{\mathrm{EFT}}(q_i)\}$$

Feynman Diagrams

- Well-established
- Ansatz: Redundancies, redefinitions...
- Explicit break of Gauge Symmetry in intermediate steps
- Normally uses off-shell computations

Functional Matching

Background Field Method

$$\phi = \bar{\phi} + \hat{\phi}$$

$\bar{\phi}$ **Classical Configuration:** Tree Level

$\hat{\phi}$ **Quantum Fluctuation:** Loops

Quantum Effective Action: We are going to **"Integrate Out"** the heavy fields

$$e^{i\Gamma_{\mathrm{UV}}[\hat{\phi}]} = \int [D\phi] \exp\left(\int \mathrm{d}^d x \mathcal{L}_{\mathrm{UV}}(\bar{\phi} + \hat{\phi}) \right)$$

Functional Matching

Fluctuation Operator $Q_{ij}(\phi(x), \partial_x)$

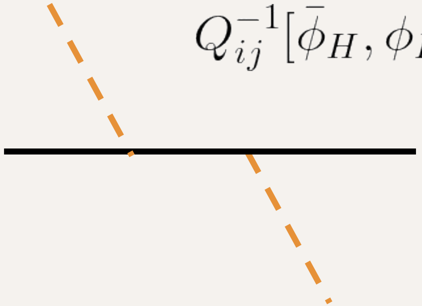
Expanding Lagrangian

$$\mathcal{L}_{\mathrm{UV}}(\bar{\phi}+\hat{\phi}) = \mathcal{L}_{\mathrm{UV}}(\bar{\phi}) + \frac{1}{2}\phi_i\phi_j \frac{\delta^2 \mathcal{L}_{\mathrm{UV}}}{\delta\phi_i\delta\phi_j}\Big|_{\phi=\bar{\phi}}$$

- **Tree Level:**

$$\mathcal{L}_{\mathrm{EFT}}(\phi_L) = \mathcal{L}_{\mathrm{UV}}(\phi_L, \bar{\phi}_H[\phi_L])$$

$Q_{ij}^{-1}[\bar{\phi}_H, \phi_L]$



- **One-loop:**

$$\exp(i\Gamma^{(1)}_{\mathrm{UV}}[\hat{\phi}]) = \int [D\phi] \exp \frac{1}{2}\left(\int \mathrm{d}^d x \phi_i Q_{ij} \phi_j\right)$$

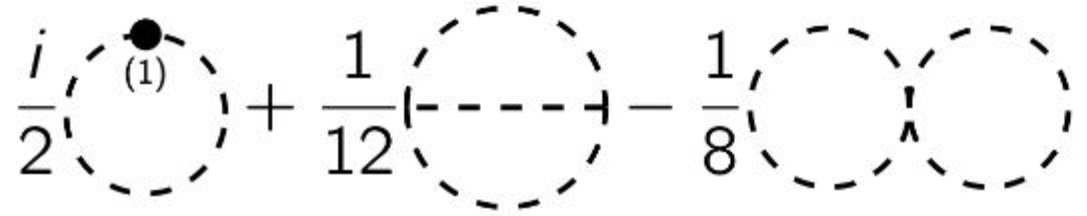
↓ Gaussian Integration

$$\Gamma^{(1)}_{\mathrm{UV}} = \frac{i}{2} \mathrm{STr} \ln Q$$

- **Two Loops:** More Topologies involved

$$\exp(i\Gamma^{(2)}_{\mathrm{UV}}[\hat{\phi}]) = \int [D\phi] \exp\left(\int \mathrm{d}^d x \frac{1}{2}\phi_i Q_{ij}\phi_j\right) \times$$
$$\left[-\frac{1}{2}\phi_i\phi_j B_{ij} - \frac{i}{24}\phi_i\phi_j\phi_k\phi_l D_{ijkl} - \frac{i}{2}\phi_i\phi_j\phi_k\phi_l\phi_m\phi_n C_{ijk}C_{lmn} + \ldots\right]$$

$$\Gamma^{(2)}_{UV}[\bar{\phi}] = \frac{i}{2}Q^{-1}_{ij}B_{ij} - \frac{1}{8}Q^{-1}_{ij}D_{ijkl}Q^{-1}_{kl} + \frac{1}{12}C_{ijk}Q^{-1}_{il}Q^{-1}_{jm}Q^{-1}_{kn}C_{lmn}$$



Method of Regions

[Beneke, Smirnov,-9711391]
[Jantzen-1111.2589]

Multiscale (m<<M) integrals can be separated in regions

$$\int \frac{d^d k}{(k^2 - m^2)(k^2 - M^2)}$$

$$\int \frac{d^d k}{k^2(k^2 - M^2)} \left[1 - \frac{m^2}{k^2} + \dots\right] \quad \text{soft (k\textasciitilde m)}$$

$$\int \frac{d^d k}{-M^2(k^2 - m^2)} \left[1 - \frac{k^2}{M^2} + \dots\right] \quad \text{hard (k\textasciitilde M)}$$

In our case, we want to integrate out **hard momentum modes**

$$\Gamma_{\mathrm{UV}} = \Gamma_{\mathrm{UV}}\Big|_{\mathrm{hard}} + \Gamma_{\mathrm{UV}}\Big|_{\mathrm{soft}}$$

Matching Condition

[Fuentes-Martín, Palarvric, Eller Thomsen-2311.13630]

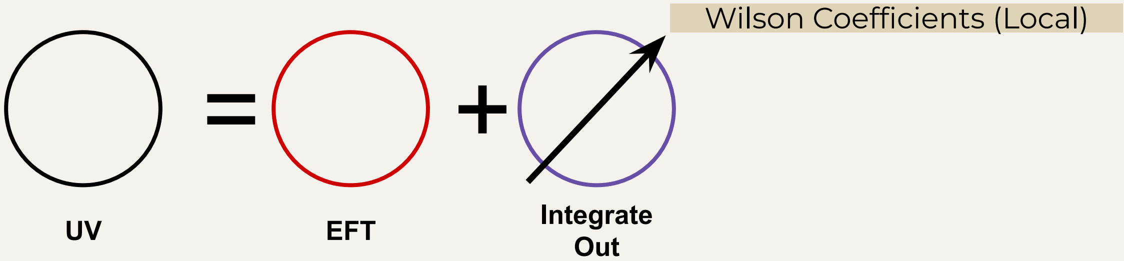
The soft loops explicitly cancel with the EFT matrix elements

$$S_{\mathrm{EFT}} = \Gamma_{\mathrm{UV}}[\bar{\phi}_H[\hat{\phi}_L], \hat{\phi}_L]\Big|_{\mathrm{hard}}$$

$$\frac{\delta\Gamma_{\mathrm{UV}|_{\mathrm{hard}}}}{\delta\phi_H}[\bar{\phi}_H, \phi_L] = 0$$

$$\Gamma_{\mathrm{UV}}\Big|_{\mathrm{soft}} = \Gamma_{\mathrm{EFT}}$$

We just have to consider hard loops (with heavy and light fields)



Evaluation of Traces

[Fuentes-Martín, Palarvric, Eller Thomsen, AM]

Differential operators under a Gauge Symmetry

$$Q_{ij}^{ab}(x,y) = \frac{\delta^2 S}{\delta\phi_i^a(x)\phi_j^b(y)} = \mathcal{Q}_{ij}^{ac}(x,P_x)\delta_c{}^b(x,y)$$

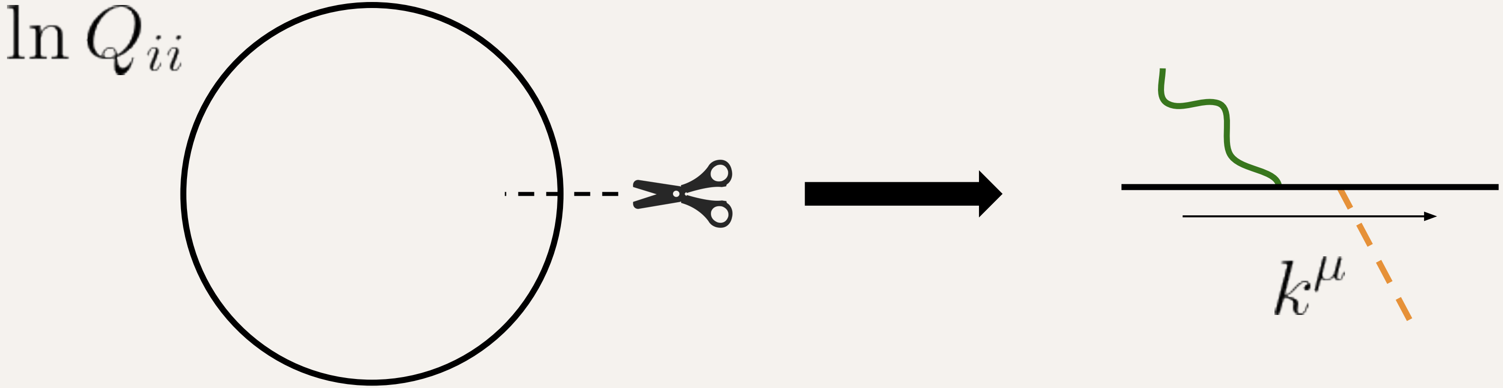
$$\delta_c{}^b(x,y) = \delta(x-y)U_c{}^b(x,y)$$

Locality of the action: Functional Traces are dressed loop integrals

$$\mathrm{Str}\ln Q|_{\mathrm{hard}} = \int_{x,y} \delta_b{}^a(x,y)\ln Q_{ii}^{ab}(x,y)$$
$$= \int_{x,k} \ln \mathcal{Q}_{ii}^{ab}(x,P_x+k)U_b{}^a(x,y)|_{x=y}$$

OPE around $k\sim\mathbf{\Lambda}$
=
Explicit Gauge Invariance

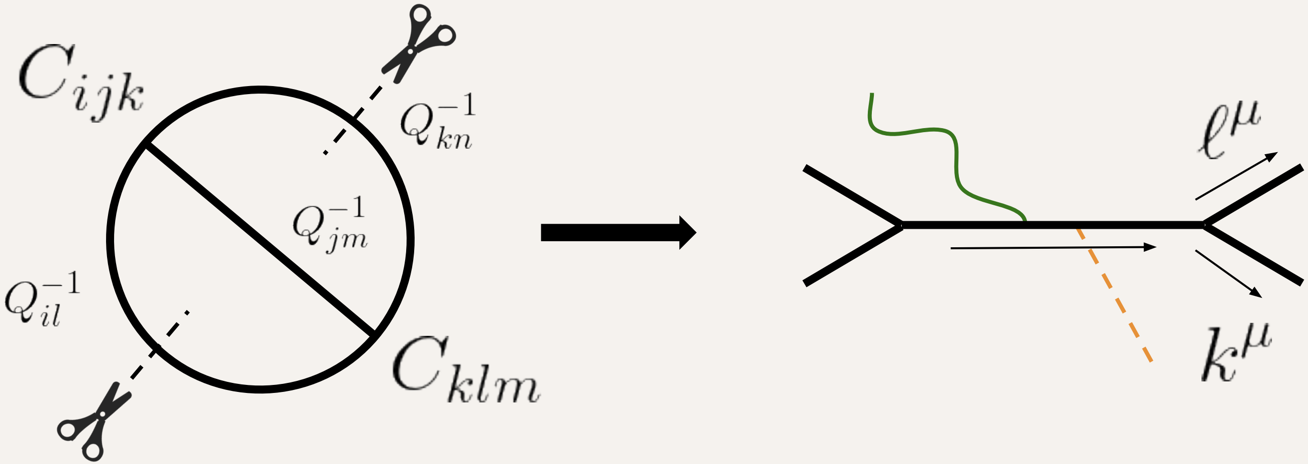
Diagrammatically,



Operators traced in different points of spacetime remain local by a **momentum shift** operation

Two Loops Traces

$$G_{\rm ss}|_{\rm hard} = \sum_{n,m,n'm'} (-1)^{n+m} \int_x \int_{k,\ell} C^{(n,m)}_{abc}\, \mathcal{Q}^{-1}_{aa'}(y, P_y{-}k{-}\ell)\, C^{(n',m')}_{a'b'c'}(y)$$
$$\times\big[(P_x{+}k)^m \mathcal{Q}^{-1}_{be}(x, P_x{+}k)(P_x{+}k)^{m'} U_{b'}{}^{e}(x,y)\big]$$
$$\times\big[(P_x{+}\ell)^n \mathcal{Q}^{-1}_{cf}(x, P_x{+}\ell)(P_x{+}\ell)^{n'} U_{c'}{}^{f}(x,y)\big]\Big|_{x=y}$$



Outlook

- EFT is given by the formalism (not by an ansatz)
- Basis reduction is still needed
- Better for Automation: Matchete
- Future study of applications of interest to the community

Thank You!
Muito Obrigado!