New Constraints on Gauged $U(1)_{L_\mu - L_\tau}$ Models via Z-Z' Mixing

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Based on [arXiv : 2401.17613]

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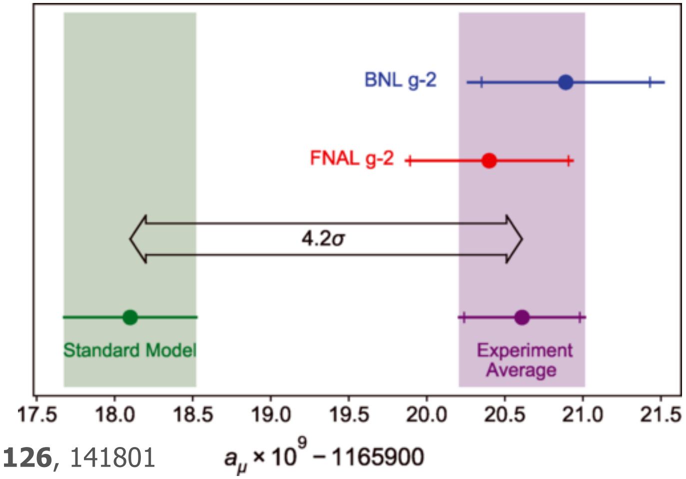
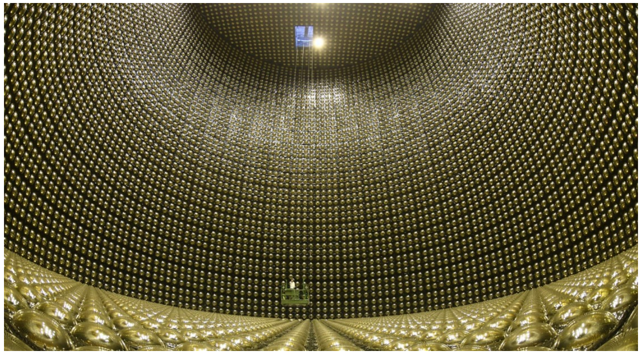
Background

- The discrepancy of muon g-2 between the SM and experimental results.
  → $\mathrm{U}(1)_{L_\mu - L_\tau}$ gauge models can explain.

- The recent experiments of the neutrino oscillation become more precise.
  → Simple $\mathrm{U}(1)_{L_\mu - L_\tau}$ gauge models seem hard to describe the neutrino physics.



Cited from
Phys. Rev. Lett. **126**, 141801



Cited from https://www-sk.icrr.u-tokyo.ac.jp/sk/

Purpose

- To find the $U(1)_{L_\mu - L_\tau}$ gauge models which are consistent to the latest neutrino experiments.

- To get new (model dependent) constraints on the $U(1)_{L_\mu - L_\tau}$ gauge models.

Cited from NuFIT v5.2

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
	$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
	$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
	$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

Minimal $\mathbf{U(1)}_{L_\mu - L_\tau}$ gauge model

- Fields：SM + three right-handed neutrino N_i + one scalar field.
- Symmetry：SM gauge × $\mathrm{U}(1)_{L_\mu - L_\tau}$ gauge.

Lepton	$(\ell_e\ \ell_\mu\ \ell_\tau)$	$(e_R\ \mu_R\ \tau_R)$	$(N_e\ N_\mu\ N_\tau)$
$\mathrm{U}(1)_{L_\mu - L_\tau}$charge	(0 +1 -1)	(0 +1 -1)	(0 +1 -1)

Scalar	Φ_{+1} SU(2) doublet	Φ_{-1} SU(2) doublet	σ SU(2) singlet
charge	+1	-1	+1

Results for Analysis of Neutrino Mass Matrix Structure

- Model independent result set by neutrino mass matrix.
- Each models have their own mass matrix structure.

Our work
(previous work[Phys. Rev. D 99 (2019) 05502])

Model	Normal ordering	Inverted ordering
SM + $\mathrm{N}_i + \sigma_{+1}$	Viable in 2σ (Viable at 3σ)	Excluded (Excluded)
SM + $\mathrm{N}_i + \Phi_{+1}$	Excluded (Excluded)	Viable at 3σ (Excluded)
SM + $\mathrm{N}_i + \Phi_{-1}$	Excluded (Excluded)	Excluded (Excluded)

→ Are there any other constraints on the viable model?

Z-Z' Mixing

- The additional $\mathrm{U}(1)_{L_\mu - L_\tau}$ gauge symmetry induces Z-Z' mixing.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}Z'_{\mu\nu}Z'^{\mu\nu} + \frac{1}{2}\frac{\varepsilon}{\cos\theta_W}B_{\mu\nu}Z'^{\mu\nu}$$

$$\mathcal{L}_{\varepsilon_Z} = \frac{1}{2}\begin{pmatrix} Z_\mu & Z'_\mu \end{pmatrix}\begin{pmatrix} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z'}^2/m_Z^2 \end{pmatrix}\begin{pmatrix} Z^\mu \\ Z'^\mu \end{pmatrix}$$

$$\varepsilon_Z \equiv \frac{m_{Z'}}{m_Z}\delta$$

$$\mathcal{L} \supset Z'_\mu\left(g_{Z'}J^\mu_{L_\mu - L_\tau} + \varepsilon e J^\mu_{\text{em}} + \varepsilon_Z g_Z J^\mu_{\text{NC}}\right)$$

$\rightarrow$ G_F and $\sin^2\theta_W$ are changed.

(Now we ignore the kinetic mixing $\varepsilon \sim g_{Z'}/70$ which is much smaller than ε_Z in our interest parameter space.)

Atomic Parity Violation (APV)

- The weak charge of Cs is given by the measurements of APV;

$$Q_{\mathrm{W}}^{\mathrm{exp}}({}^{133}_{55}\mathrm{Cs}) = -72.94(43)$$

- The weak charge of Cs based on SM is changed by Z-Z' mixing;

$$Q_W({}^{133}_{55}\mathrm{Cs}) \simeq Q_W^{\mathrm{SM}}({}^{133}_{55}\mathrm{Cs})\left(1+\delta^2\right)$$

$$\longrightarrow \quad |\delta|^2 \lesssim 5.7 \times 10^{-3}$$ (90% CL)

Flavor Changing Meson Decay

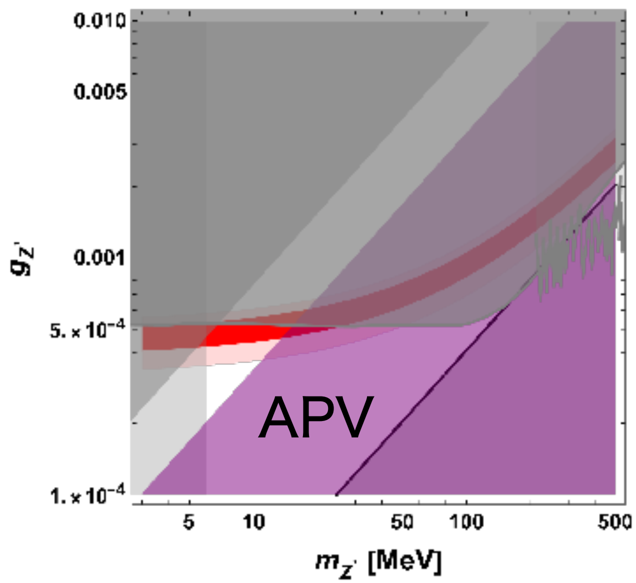
- Flavor changing meson decays provide a good probe of a light Z' boson.

- Branching ratio of $K^+ \to \pi^+ Z'$ is written by;

$$\mathrm{Br}(K^+ \to \pi^+ Z') \simeq 1.6 \times 10^{-4} |\delta|^2$$

$$\longrightarrow \quad |\delta| \lesssim 2.5 \times 10^{-4} \sqrt{\frac{\mathrm{Br}(K^+ \to \pi^+ Z')_{\mathrm{exp}}}{1 \times 10^{-11}}}$$

Constraint on Model with $\boldsymbol{\Phi}_{+1}$



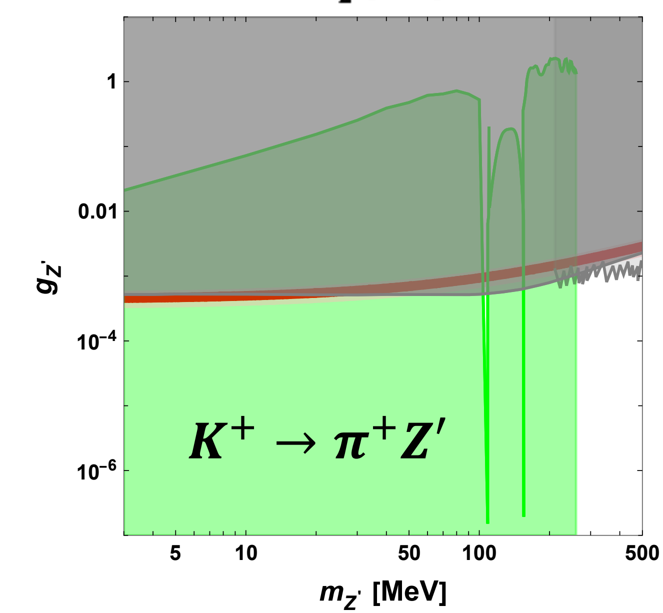
In this model $\delta = \frac{1}{v}\frac{m_{Z'}}{g_{Z'}}$.

Cs APV :

$$g_{Z'} \gtrsim 5.4 \times 10^{-4} \left(\frac{m_{Z'}}{10\,\mathrm{MeV}}\right)$$

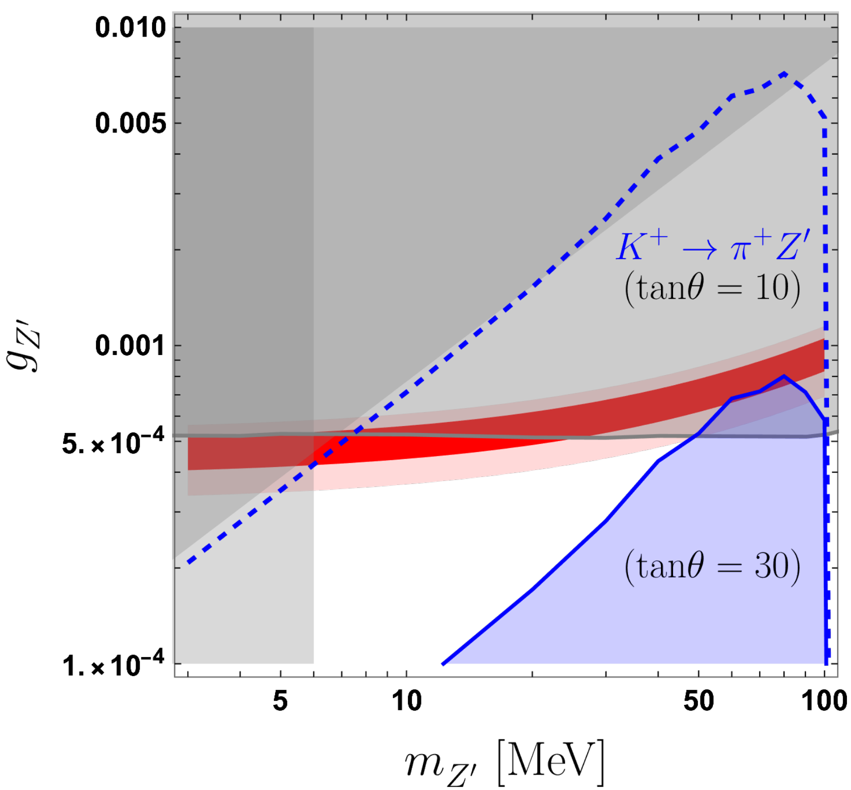
$K^+ \to \pi^+ Z'$:

$$g_{Z'} \gtrsim 1.6 \times 10^{-1} \sqrt{\frac{1 \times 10^{-11}}{\mathrm{Br}(K^+ \to \pi^+ Z')_{\mathrm{exp}}}} \left(\frac{m_{Z'}}{10\,\mathrm{MeV}}\right)$$



- The gray shaded region are excluded by the well-known constraints (from BABAR, NA64μ, white dwarf cooling, and effective number of neutrinos).
- The red region gives the proper correction to muon g-2.
- There is no region which gives proper correction to muon g-2.

Constraint on Model with $\boldsymbol{\Phi}_{+1}$ and a SU(2) singlet scalar $\boldsymbol{\sigma}_{+1}$



In this model, $\delta = \dfrac{\mathrm{sign}(Q_\Phi)}{1+\tan^2\theta}\dfrac{1}{v}\dfrac{m_{Z'}}{g_{Z'}}$.

$\tan\theta \equiv \dfrac{v_\sigma}{v_\Phi}$ ($v_{\Phi(\sigma)}$means VEV of $\Phi(\sigma)$).

Cs APV : much smaller than the flavor changing meson decay.

$$g_{Z'} \gtrsim \frac{5.4\times10^{-4}}{1+\tan^2\theta}\left(\frac{m_{Z'}}{10\,\mathrm{MeV}}\right)$$

$K^+ \to \pi^+ Z'$:

$$g_{Z'} \gtrsim \frac{1.6\times10^{-1}}{1+\tan^2\theta}\sqrt{\frac{1\times10^{-11}}{\mathrm{Br}(K^+\to\pi^+Z')_{\mathrm{exp}}}}\left(\frac{m_{Z'}}{10\,\mathrm{MeV}}\right)$$

- Model gives proper correction to the muon g-2 discrepancy when $\tan\theta \equiv \dfrac{v_\sigma}{v_2} \gtrsim 10$.

Conclusion

- We revisited the minimal $\mathrm{U}(1)_{L_\mu - L_\tau}$ gauge model based on the latest NuFITv5.2 data. As the results, the model with SU(2) doublet scalar $\mathbf{\Phi_{+1}}$ was viable at 3σ in case of Inverted ordering while the model was excluded in the previous work.
- Considering the constraints from Z-Z' mixing (APV and flavor changing meson decay process), the model with $\mathbf{\Phi_{+1}}$ is completely excluded in the region which give the explanation to muon g-2.
- The model with $\mathbf{\Phi}$ and $\boldsymbol{\sigma}$ is viable when $\tan\theta \equiv \frac{v_\sigma}{v_2} \gtrsim 10$.

BACKUP

NuFITv4.0

with SK atmospheric data	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.75}$	$31.62 \to 36.27$
$\sin^2\theta_{23}$	$0.582^{+0.015}_{-0.019}$	$0.428 \to 0.624$	$0.582^{+0.015}_{-0.018}$	$0.433 \to 0.623$
$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	$40.9 \to 52.2$	$49.7^{+0.9}_{-1.0}$	$41.2 \to 52.1$
$\sin^2\theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	$0.02044 \to 0.02437$	$0.02263^{+0.00065}_{-0.00066}$	$0.02067 \to 0.02461$
$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	$8.22 \to 8.98$	$8.65^{+0.12}_{-0.13}$	$8.27 \to 9.03$
$\delta_{CP}/^\circ$	217^{+40}_{-28}	$135 \to 366$	280^{+25}_{-28}	$196 \to 351$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$	$7.39^{+0.21}_{-0.20}$	$6.79 \to 8.01$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.525^{+0.033}_{-0.031}$	$+2.431 \to +2.622$	$-2.512^{+0.034}_{-0.031}$	$-2.606 \to -2.413$

From http://www.nu-fit.org/?q=node/177

Neutrino Mass Matrix

- In general,

$$\mathcal{M}_{\nu_L} = U_{\mathrm{PMNS}}\, \mathrm{diag}(m_1\ m_2\ m_3)\, U_{\mathrm{PMNS}}^T \equiv \mathcal{M}_{\nu_L}^{\mathrm{gen}}.$$

$$U_{\mathrm{PMNS}} \equiv \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{\frac{i\alpha_2}{2}} & \\ & & e^{\frac{i\alpha_3}{2}} \end{pmatrix}.$$

m_i:light neutrino mass α_i:Majorana phase

V_{ij}:matrix component including mixing angles and CP phase

- Through the seesaw mechanism

$$\mathcal{M}_{\nu_L} \simeq -\mathcal{M}_D\, \mathcal{M}_R^{-1} \mathcal{M}_D^T.$$

→ Some equations arise by comparing these.

Two Zero Texture (Minor) Structure Mass Matrix

- Classification of structures;

$$\mathbf{B_3}:\begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix},\ \mathbf{B_4}:\begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},\ \mathbf{C}:\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

- Thorough the seesaw mechanism, the neutrino mass matrix (or its inverted one) often has such structure.
  - Two components of $\mathcal{M}_{\nu_L}$ are zero → Two zero texture
  - Two components of $\mathcal{M}_{\nu_L}^{-1}$ are zero → Two zero minor

The mass matrix with such structures give us two equations. → Predictions

Light Neutrino Mass

$$m_3 = \sqrt{\frac{\Delta m_{31}^2}{1 - \frac{1}{|R_3(\theta_{12},\theta_{13},\theta_{23},\delta_{CP})|^2}}}$$

Neutrino mass in case of NO.

$$m_1 = \sqrt{m_3^2 - \Delta m_{31}^2}$$

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} = \sqrt{m_3^2 + \Delta m_{21}^2 - \Delta m_{31}^2}$$

- These masses can be described in terms of θ_{23}.

($\theta_{12}, \theta_{13}, \Delta m_{21}^2$, Δm_{31}^2 are fixed as best fit value of NuFITv5.2)

Analysis of SM+N_i+Φ_{+1} Model

- **$\mathbf{B_3}$ texture** : $\left(\mathcal{M}_{\nu_L}\right)_{[1,2],[2,2]} = 0$

$$
\begin{cases}
\left(\mathcal{M}_{\nu}^{\mathrm{gen}}\right)_{12} = m_1 V_{11} V_{21} + m_2 e^{i\alpha_2} V_{12} V_{22} + m_3 e^{i\alpha_3} V_{13} V_{23} = 0 \left(= \left(\mathcal{M}_{\nu}\ \right)_{12}\right). \\
\left(\mathcal{M}_{\nu}^{\mathrm{gen}}\right)_{22} = m_1 V_{21}^2 + m_2 e^{i\alpha_2} V_{22}^2 + m_3 e^{i\alpha_3} V_{23}^2 = 0 \left(= \left(\mathcal{M}_{\nu}\ \right)_{22}\right).
\end{cases}
$$

$$
\Rightarrow
\begin{cases}
e^{i\alpha_2} \equiv \dfrac{m_1}{m_2} R_2(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \equiv \dfrac{R_2}{|R_2|} \\
e^{i\alpha_3} \equiv \dfrac{m_1}{m_3} R_3(\theta_{12}, \theta_{13}, \theta_{23}, \delta) \equiv \dfrac{R_3}{|R_3|}
\end{cases}
$$

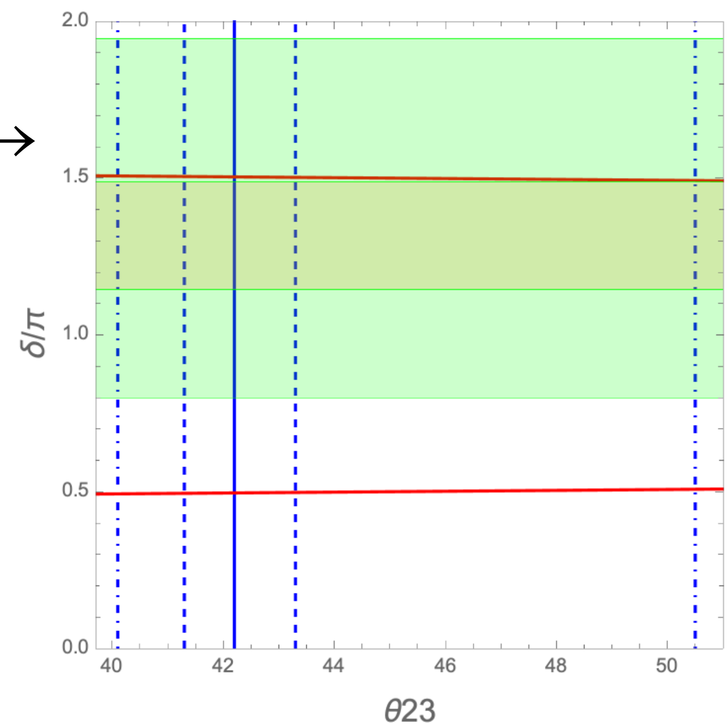
V_{ij} : components of PMNS matrix , θ_{ij} : mixing angle, δ : CP phase,
α : Majorana phase

- To rewrite mass-squared difference

  in case of **Normal ordering(NO)**

$$\begin{cases} \Delta m_{31}^2 = m_3^2 - m_1^2 = m_1^2(|R_3|^2 - 1) \\ \Delta m_{21}^2 = m_2^2 - m_1^2 = m_1^2(|R_2|^2 - 1) \end{cases}$$

$$(|R_2|^2 - 1) = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}(|R_3|^2 - 1) \quad \rightarrow$$

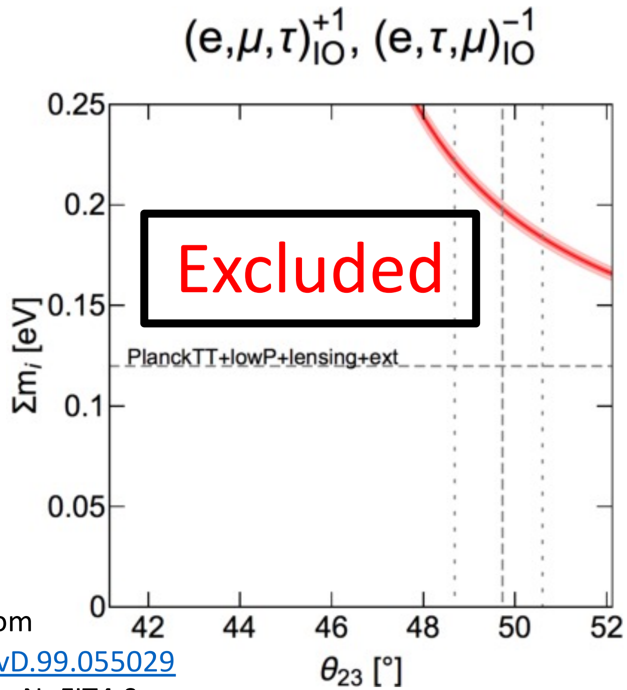


By fixing $\theta_{12}, \theta_{13}, \Delta m_{21}^2$, Δm_{31}^2 as the best-fit value of NuFITv5.2, θ_{23} -dependence of δ are found.

Neutrino mass and Majorana phase can be written by θ_{23}!

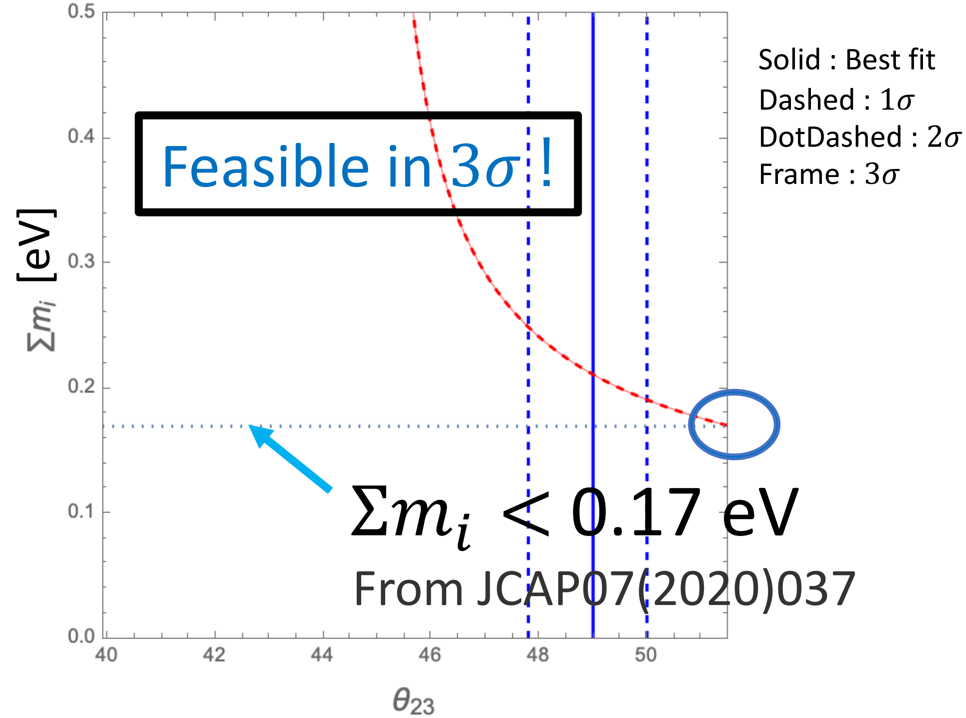
Result of Analysis ($\mathbf{B_3}$ Texture)

- Previous Work



Cited from
PhysRevD.99.055029
Based on NuFIT4.0

- Our Result

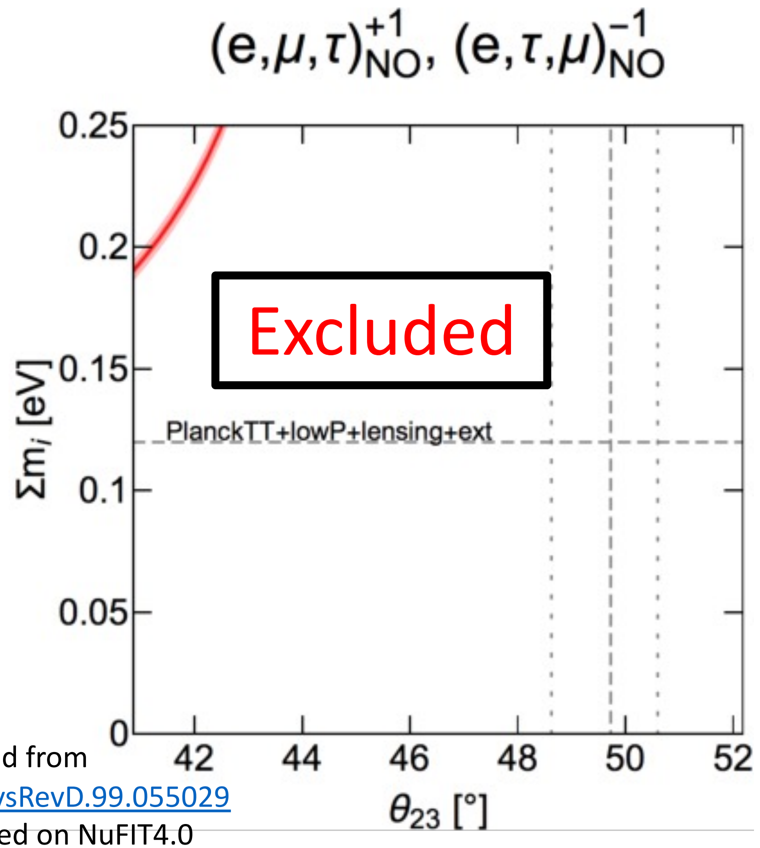


- $\mathbf{B_3}$-type mass matrix in Inverted ordering is revived.
- The range of θ_{23} shift to left in the latest NuFITv5.2.
- The mass sum constraint is relaxed because of being had considered mass ordering in the analysis.

Analysis Result (NO)

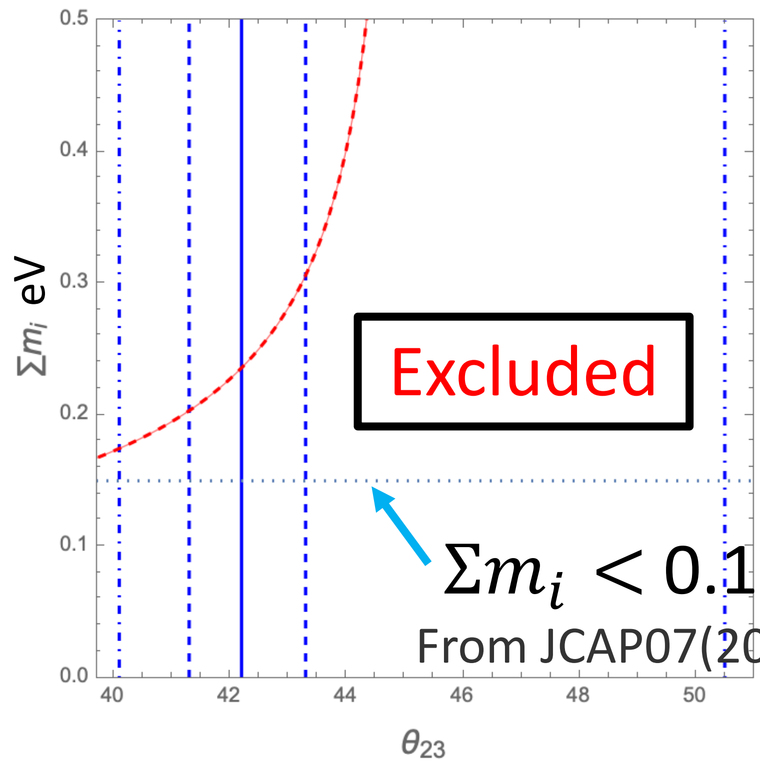
Solid : Best fit
Dashed : 1σ
DotDashed : 2σ
Frame : 3σ

- Previous Work



Cited from
PhysRevD.99.055029
Based on NuFIT4.0

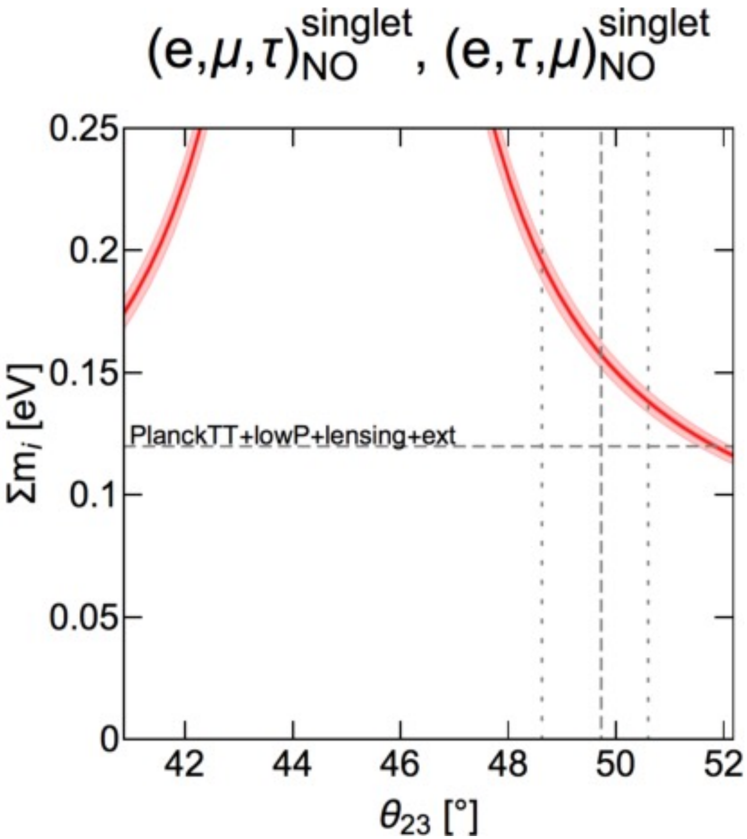
- Our Result

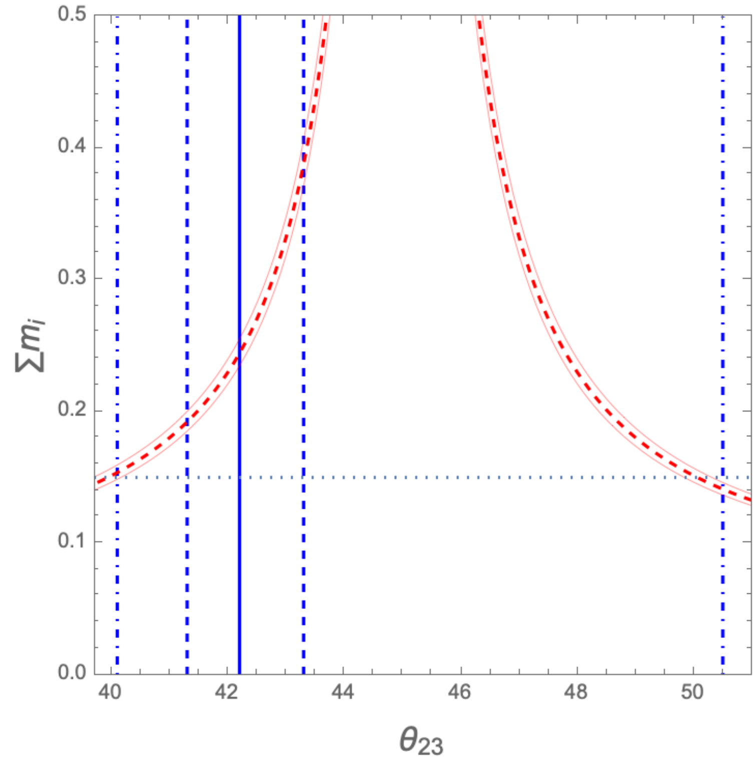


$\Sigma m_i < 0.15$ eV
From JCAP07(2020)037

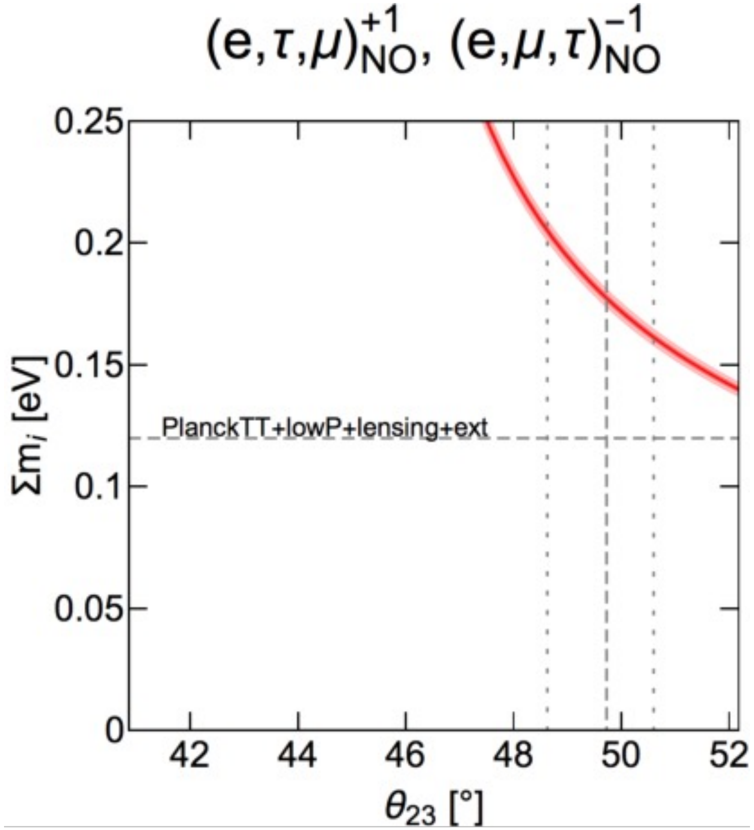
- The range of θ_{23} shift to left in the latest NuFITv5.2.
- The mass sum constraint are relaxed by considering mass ordering.

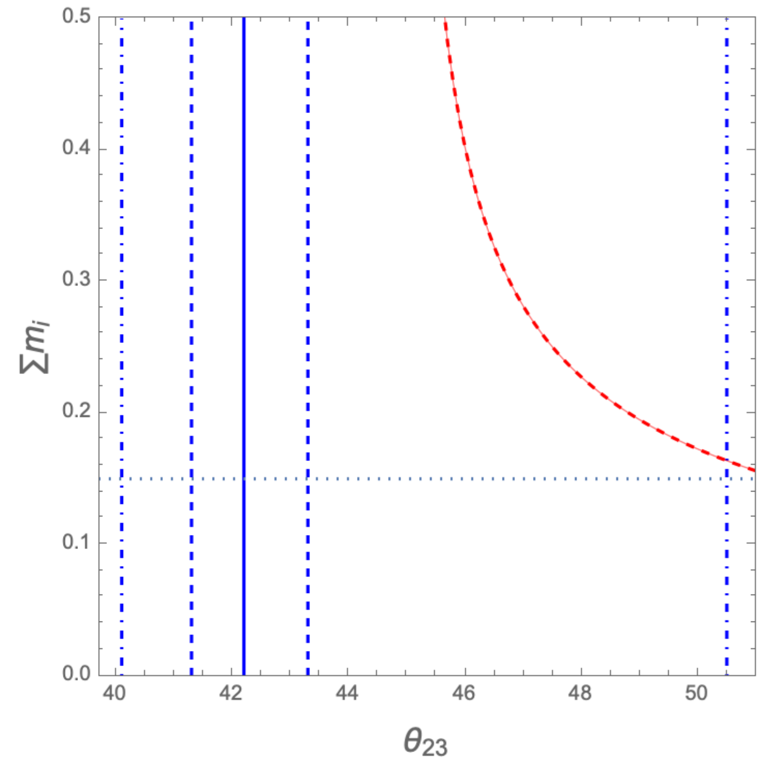
C Minor (NO)


$(e,\mu,\tau)^{\text{singlet}}_{\text{NO}}$, $(e,\tau,\mu)^{\text{singlet}}_{\text{NO}}$
0.25
0.2
0.15
0.1
0.05
0
Σm_i [eV]
PlanckTT+lowP+lensing+ext
42
44
46
48
50
52
θ_{23} [°]

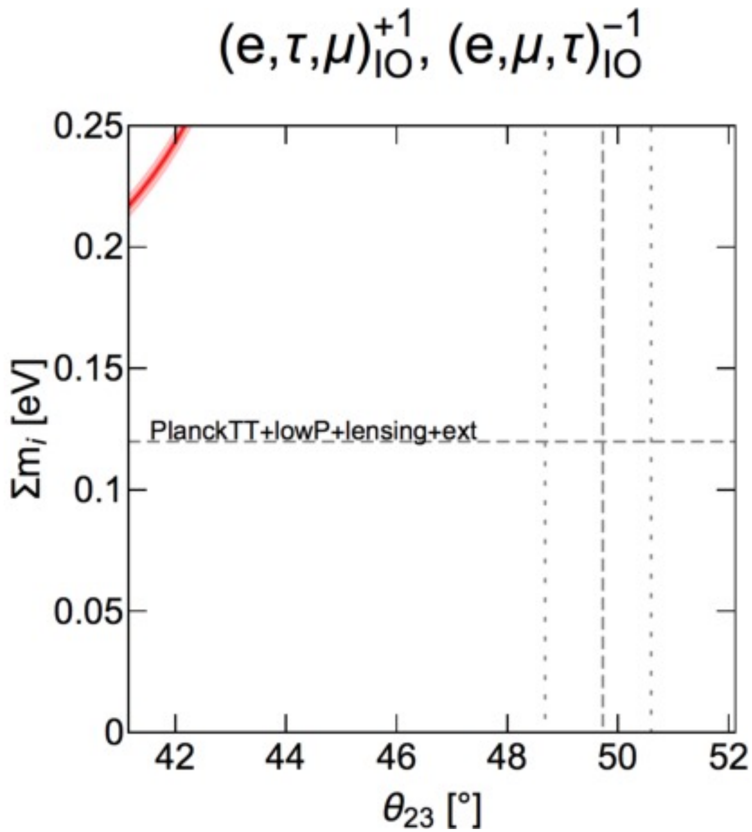

0.5
0.4
0.3
0.2
0.1
0.0
Σm_i
40
42
44
46
48
50
θ_{23}

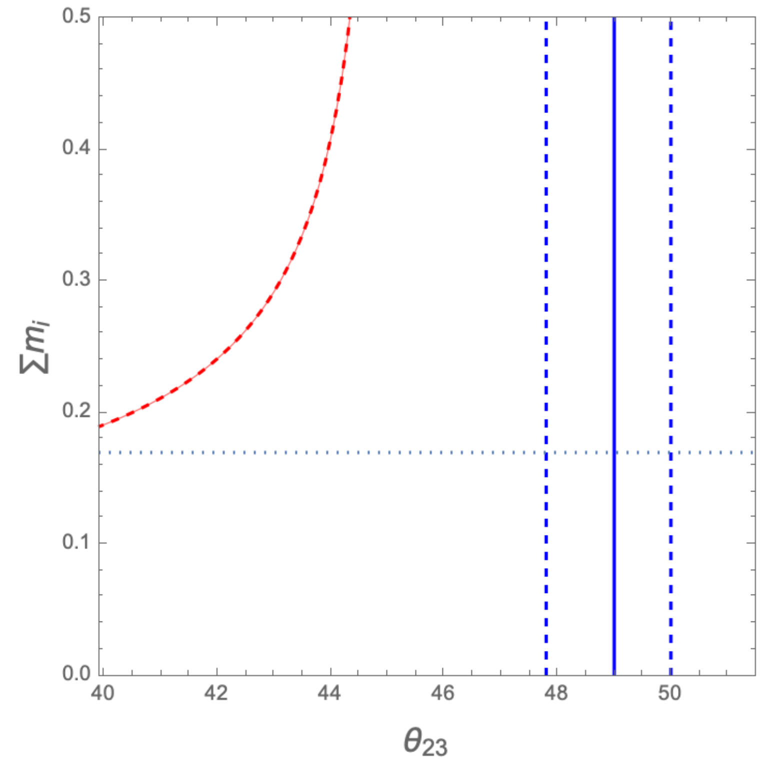
B4 Texture (NO)



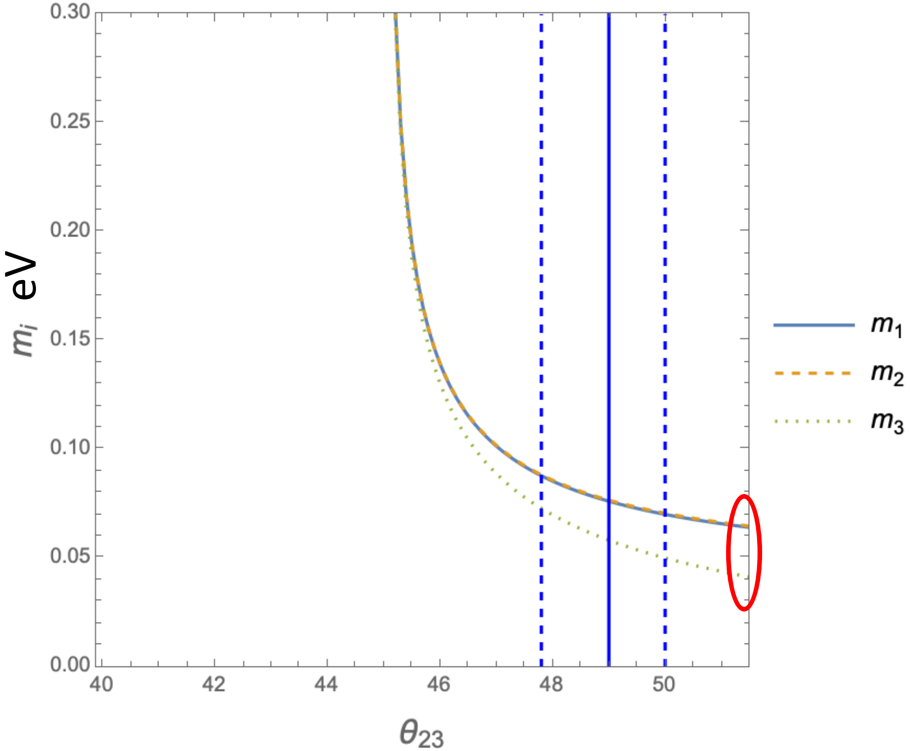


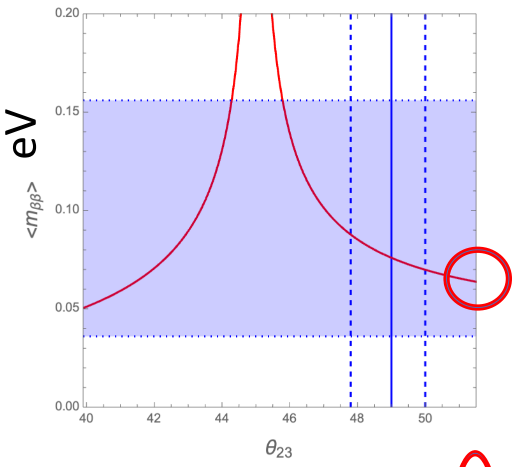
B4 Texture (IO)

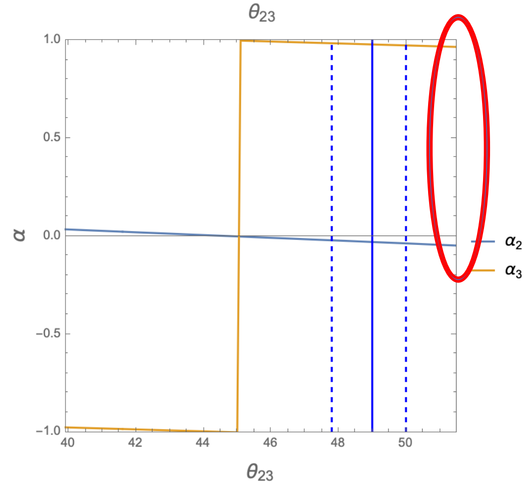




Result of analysis 2







	m_1 [eV]	m_2 [eV]	m_3 [eV]	α_2/π	α_3/π	$< m_{\beta\beta} >$ [eV]
$\mathbf{B}_3$ texture (IO)	0.064	0.065	0.041	-0.05	0.96	0.064