Type II seesaw: potential minima and metastability

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Talk based on work in progress by K. Kannike, A. K. and L. Marzola

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Type II seesaw model

Scalar fields

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}$$

Neutrino mass term

$$\mathcal{L}_Y \supset -(Y_\nu)_{\alpha\beta}\overline{L^c}_\alpha \epsilon \Delta L_\beta + \text{h.c.}$$

Z and W gauge boson masses

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)(v_H^2 + 4v_\Delta^2), \qquad M_W^2 = \frac{1}{4}g^2(v_H^2 + 2v_\Delta^2),$$

where $0 \leq v_\Delta \leq 2.58$ GeV and $v_H^2 + 2v_\Delta^2 = 246.22$ GeV

Type II seesaw model

Potential

$$V = \mu_H^2 H^\dagger H + \mu_\Delta^2 \,\mathrm{tr}\left(\Delta^\dagger \Delta\right) + \frac{1}{2}\mu_{H\Delta}[H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}]$$
$$+ \lambda_H (H^\dagger H)^2 + \lambda_\Delta [\mathrm{tr}\left(\Delta^\dagger \Delta\right)]^2 + \lambda'_\Delta \,\mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right)$$
$$+ \lambda_{H\Delta} H^\dagger H \,\mathrm{tr}\left(\Delta^\dagger \Delta\right) + \lambda'_{H\Delta} H^\dagger \Delta \Delta^\dagger H$$

Minimisation

$$\mu_H^2 = -\lambda_H v_H^2 + \frac{\mu_{H\Delta} v_\Delta}{\sqrt{2}} - \frac{1}{2}(\lambda_{H\Delta} + \lambda'_{H\Delta}) v_\Delta^2,$$
$$\mu_\Delta^2 = \frac{\mu_{H\Delta} v_H^2}{2\sqrt{2} v_\Delta} - \frac{1}{2}(\lambda_{H\Delta} + \lambda'_{H\Delta}) v_H^2 - (\lambda_\Delta + \lambda'_\Delta) v_\Delta^2$$

Type II seesaw model

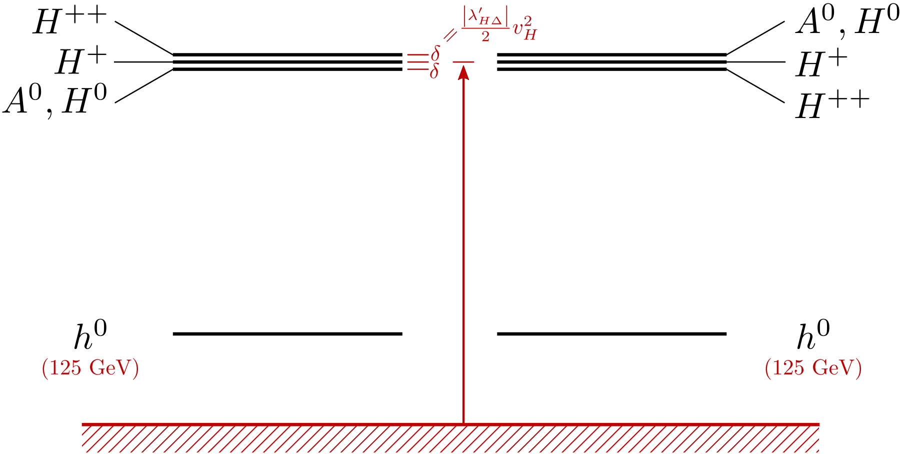
Our neutral extremum $N_{H\Delta}$

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v_H}{\sqrt{2}} \end{pmatrix}, \qquad \langle \Delta \rangle = \begin{pmatrix} 0 & 0 \\ \frac{v_\Delta}{\sqrt{2}} & 0 \end{pmatrix}$$

Scalar mass eigenstates

- Doubly charged field $H^{\pm\pm}$
- Singly charged field $H^{\pm}$
- Charged Goldstone boson $G^{\pm}$
- Neutral scalar fields H^0 & h^0 (SM-like Higgs boson)
- Neutral Goldstone boson G^0
- Neutral pseudoscalar field A^0

Type II seesaw model



Source: C. Bonilla *et al.*, Phys. Rev. D **92** (2015) no.7, 075028 [arXiv:1508.02323 [hep-ph]].

Type II seesaw model

Couplings[1]

$$\lambda_H = \frac{m_{h^0}^2 \cos\alpha + m_{H^0}^2 \sin\alpha}{2v_H^2}, \quad -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$$

$$\lambda'_\Delta = \frac{1}{v_\Delta^2}\left(\frac{2v_H^2}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 - \frac{v_H^2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - m_{H^{++}}^2\right)$$

$$\lambda'_{H\Delta} = \frac{4}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - \frac{4}{v_H^2 + 2v_\Delta^2} m_{H^+}^2$$

$$\mu_{H\Delta} = \frac{2\sqrt{2}v_\Delta}{v_H^2 + 2v_\Delta^2} m_{A^0}^2$$

$$\lambda_\Delta, \lambda_{H\Delta} = \ldots$$

[1] A. Arhrib *et al.*, Phys. Rev. D **84** (2011), 095005 [arXiv:1105.1925 [hep-ph]].

Orbit space

Orbit space[2]

- Field vector transformation under G symmetry group: $\phi \to g \cdot \phi$
- Polynomial invariants transform under G symmetry group: $p_i \to p_i$
- $V(\phi)$ can be expressed as a function of finite set $p = (p_1(\phi), p_2(\phi), ..)$ of basic polynomial invariants.
- $\widehat{V}(p)$ has same range as $V(\phi)$, but is not affected by the same degeneracies.
- We can calculate the full orbit space by the means of the P-matrix formalism with

$$P_{ij} = \frac{\partial p_i}{\partial \Phi_a^\dagger} \frac{\partial p_j}{\partial \Phi^a}$$

[2] G. Sartori and G. Valente, Annals Phys. **319** (2005), 286-325

Orbit space

Rewritten potential

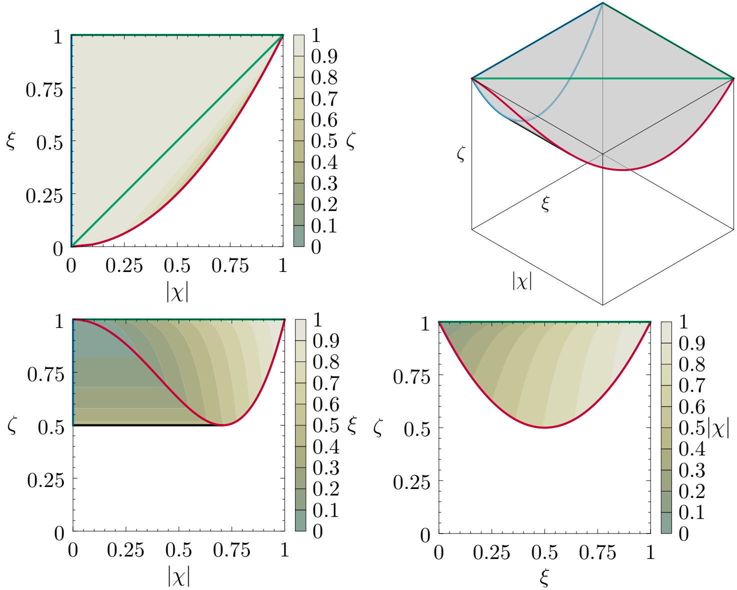
$$V = \frac{1}{2}\mu_H^2 h^2 + \frac{1}{2}\mu_\Delta^2 \delta^2 + \frac{1}{2\sqrt{2}}\mu_{H\Delta}\chi h^2\delta + \frac{1}{4}\lambda_H h^4 + \frac{1}{4}\lambda_\Delta \delta^4 + \frac{1}{4}\lambda'_\Delta \zeta \delta^4 + \frac{1}{4}\lambda_{H\Delta} h^2\delta^2 + \frac{1}{4}\lambda'_{H\Delta}\xi h^2\delta^2$$

Orbit space variables and norms

$$\xi = \frac{H^\dagger \Delta \Delta^\dagger H}{H^\dagger H \,\mathrm{tr}(\Delta^\dagger \Delta)}, \qquad \zeta = \frac{\mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right)}{(H^\dagger H)^2}, \qquad \chi = \frac{\frac{1}{2}[H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}]}{\sqrt{H^\dagger H}\,\mathrm{tr}(\Delta^\dagger \Delta)},$$

$$H^\dagger H = \frac{1}{2}h^2, \quad \mathrm{tr}\left(\Delta^\dagger \Delta\right) = \frac{1}{2}\delta^2$$

Orbit space



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Vacuum structure and metastability

Other minima[3]

- panic vacua $N'_{H\Delta}$: other neutral vacuum solutions to minimisation equations
- Charged extrema $CB_{H\Delta}$: must be on curved edge (except special configurations)

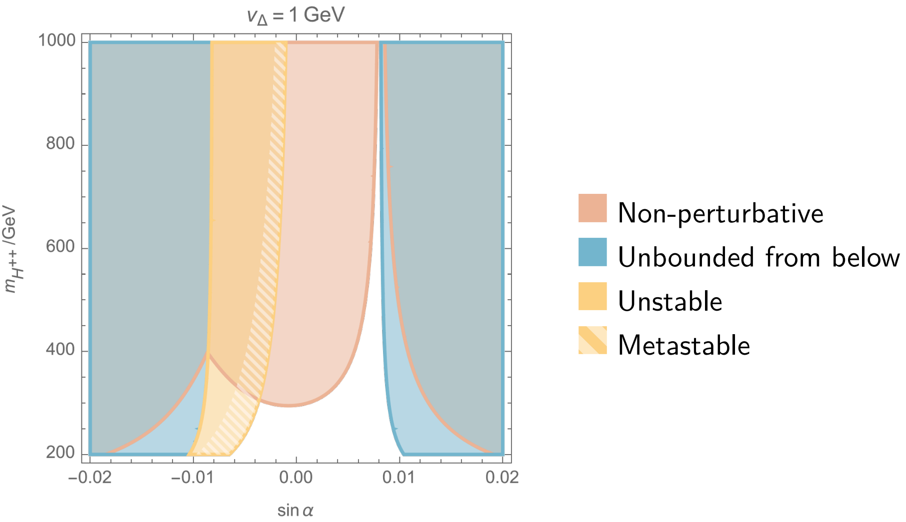
$$\xi = \chi^2, \quad \zeta = 1 - 2\xi + 2\xi^2, \quad -1 \leq \chi \leq 1$$

- Neutral extrema N_Δ or charged extrema CB_Δ ($\langle H \rangle = 0$):

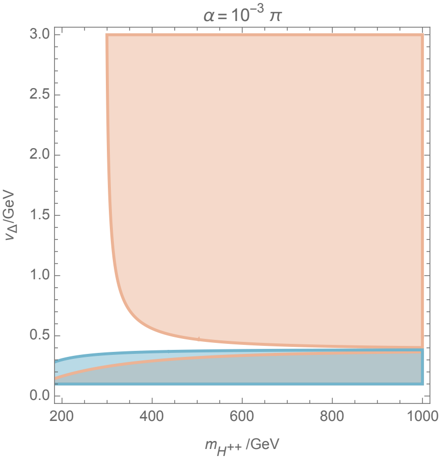
$$\delta^2 = -\frac{\mu_\Delta^2}{\lambda_\Delta + \zeta \lambda'_\Delta}, \qquad V_\Delta = -\frac{1}{4}\frac{\mu_\Delta^4}{\lambda_\Delta + \zeta \lambda'_\Delta}$$

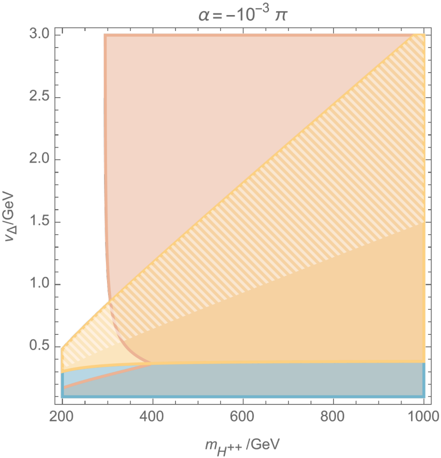
[3] P. M. Ferreira and B. L. Gonçalves, JHEP **02** (2020), 182 [arXiv:1911.09746 [hep-ph]].

Vacuum structure and metastability



Vacuum structure and metastability





Conclusions

- Type II seesaw offers a compelling framework for explaining the observed light neutrino masses.
- Utilizing the orbit space formalism has provided valuable insights into the vacuum structure of the model.
- While panic vacua are absent, regions of instability is present within the allowed parameter space.
- Stay tuned for phase transitions and gravitational waves.

Couplings

$$\lambda_H = \frac{m_{h^0}^2 \cos\alpha + m_{H^0}^2 \sin\alpha}{2v_H^2},$$

$$\lambda_\Delta = \frac{1}{v_\Delta^2}\left(\frac{m_{h^0}^2 \sin^2\alpha + m_{H^0}^2 \cos^2\alpha}{2} + \frac{1}{2}\frac{v_H^2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - \frac{2v_H^2}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 + m_{H^{++}}^2\right),$$

$$\lambda'_\Delta = \frac{1}{v_\Delta^2}\left(\frac{2v_H^2}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 - \frac{v_H^2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - m_{H^{++}}^2\right),$$

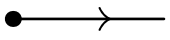
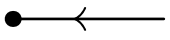
$$\lambda_{H\Delta} = -\frac{2}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 + \frac{4}{v_H^2 + 2v_\Delta^2} m_{H^+}^2 + \frac{\sin\alpha\cos\alpha}{v_H v_\Delta}(m_{h^0}^2 - m_{h^0}^2),$$

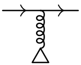
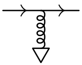
$$\lambda'_{H\Delta} = \frac{4}{v_H^2 + 4v_\Delta^2} m_{A^0}^2 - \frac{4}{v_H^2 + 2v_\Delta^2} m_{H^+}^2, \quad \mu_{H\Delta} = \frac{2\sqrt{2}v_\Delta}{v_H^2 + 2v_\Delta^2} m_{A^0}^2$$

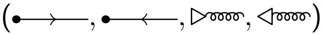
Invariants

$$
\begin{aligned}
p_1 &= H^\dagger H, \\
p_2 &= \mathrm{tr}\left(\Delta^\dagger \Delta\right), \\
p_3 &= \mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right), \\
p_4 &= H^\dagger \Delta \Delta^\dagger H, \\
p_5 &= \frac{1}{2}[H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}], \\
p_6 &= \frac{1}{2}(\mathrm{tr}\,\Delta^{\dagger 2}\, H^T \epsilon \Delta H + \mathrm{tr}\,\Delta^2\, H^\dagger \Delta^\dagger \epsilon^\dagger H^*), \\
p_7 &= H^\dagger \Delta^\dagger H H^\dagger \Delta H.
\end{aligned}
$$

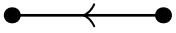
Birdtracks

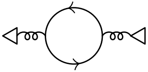
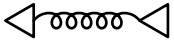
$H^i =$  $i,$ $\quad H_i^\dagger =$  i

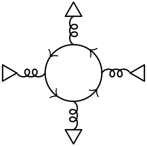
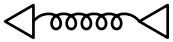
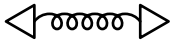
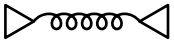
$\Delta_j^i = \Delta^a (T^a)_j^i =$ j  i , $\quad (\Delta^\dagger)_j^i = \Delta^{*a} (T^a)_j^i =$ j  i

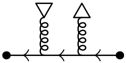
$\Phi = (H^i, H_j^\dagger, \Delta^a, \Delta^{*b}) =$  $,$

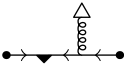
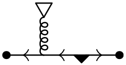
Birdtracks

$p_1 = H^\dagger H =$ ,

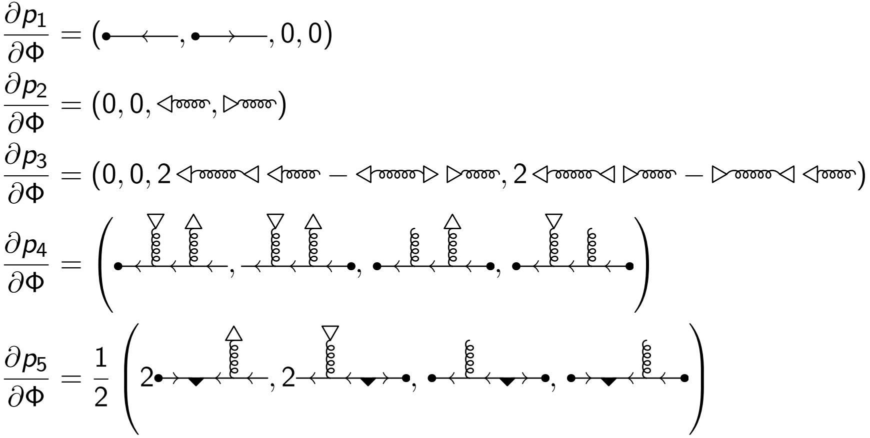
$p_2 = \mathrm{tr}\left(\Delta^\dagger \Delta\right) =$  $=$ 

$p_3 = \mathrm{tr}\left(\Delta^\dagger \Delta \Delta^\dagger \Delta\right) =$  $=$  $^2 - \frac{1}{2}$  

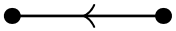
$p_4 = H^\dagger \Delta \Delta^\dagger H =$ 

$p_5 = \frac{1}{2}(H^T \epsilon \Delta^\dagger H + \mathrm{h.c.}) = \frac{1}{2}\Bigg($  $+$  $\Bigg)$

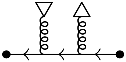
Birdtracks

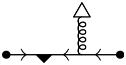
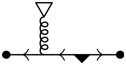


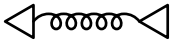
Examples of P-matrix elements

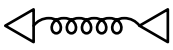
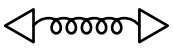
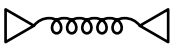
$P_{11} = 2$  $= 2p_1$

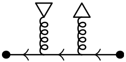
$P_{12} = P_{13} = 0$

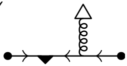
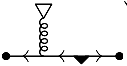
$P_{14} = 2$  $= 2p_4$

$P_{15} =$  $+$  $= 2p_5$

$P_{22} = 2$  $= 2p_2$

$P_{23} = 4 \Big($  $^2 - \frac{1}{2}$   $\Big) = 4p_3$

$P_{24} = 2$  $= 2p_4$

$P_{25} = \frac{1}{2} \Big($  $+$  $\Big) = p_5$