Sommerfeld Effect and Bound State Formation of colored mediators in dark matter studies

Martin Napetschnig

Technical University of Munich

Based on a work in preparation with

Mathias Becker, Emanuele Copello

and Julia Harz (JGU Mainz)

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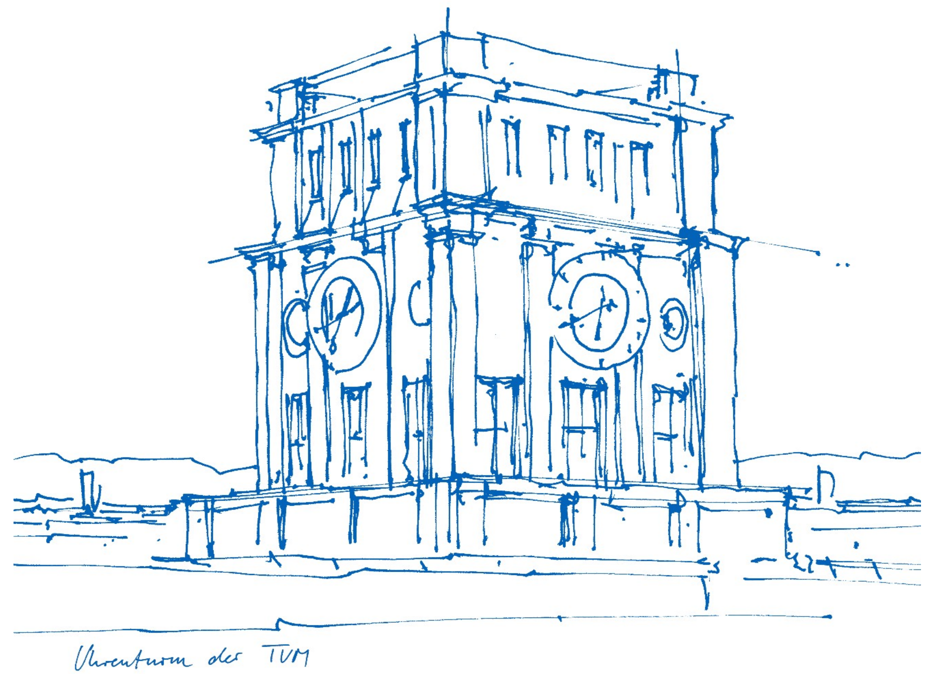
Instituto Superior Técnico

Thursday, June 6th, 2024



Outline

Simplified dark matter models and long-range effects

Sommerfeld effect and bound state formation for colored mediators

Showcases of our computational framework

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Classical WIMP evades detection so far.

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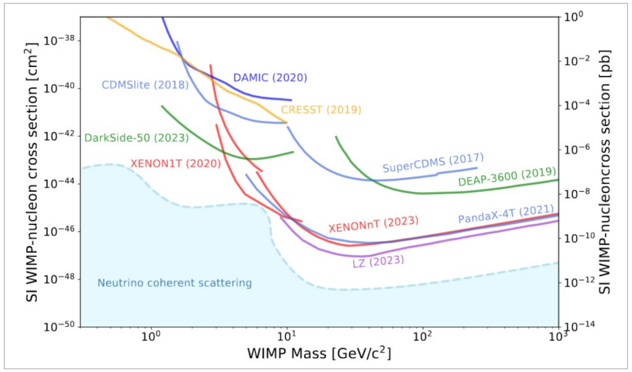

[PDG „Dark Matter“ (2024)]

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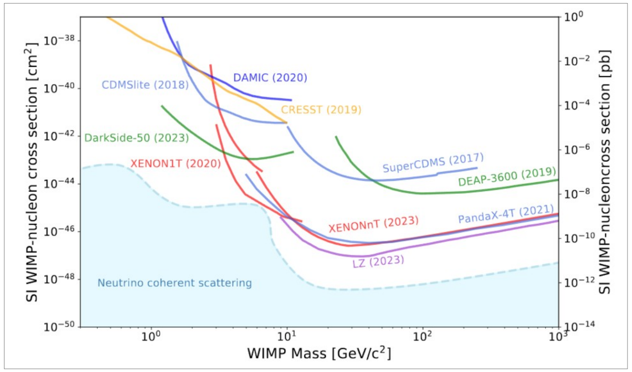

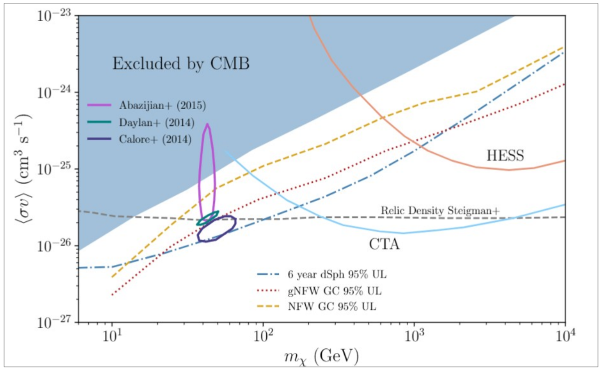

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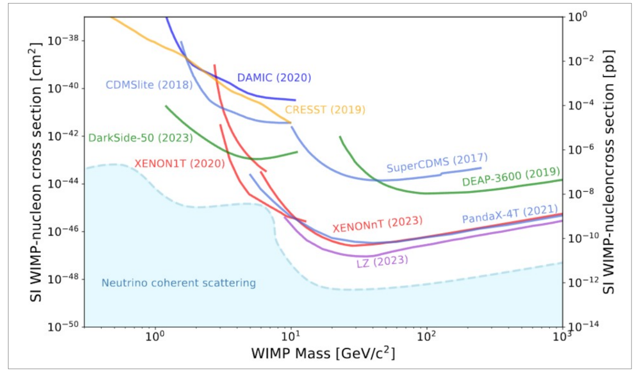

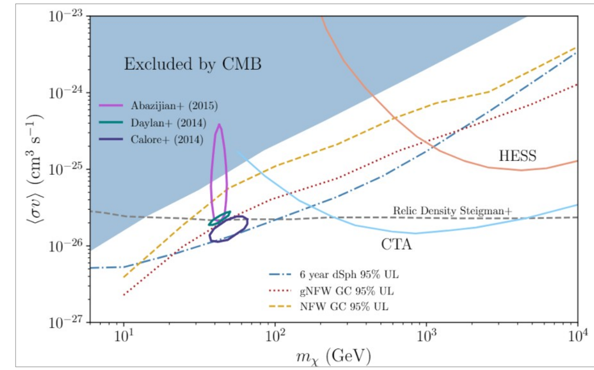

[PDG „Dark Matter“ (2024)]

Nothing either...

DM	Axial-vector mediator (Dirac DM)	0 e, μ	1 – 4 j	Yes	36.1	m_{med} 1.55 TeV
	Colored scalar mediator (Dirac DM)	0 e, μ	1 – 4 j	Yes	36.1	m_{med} 1.67 TeV
	$VV\chi\chi$ EFT (Dirac DM)	0 e, μ	1 J, ≤ 1 j	Yes	3.2	M_* 700 GeV
	Scalar reson. $\phi \to t\chi$ (Dirac DM)	0-1 e, μ	1 b, 0-1 J	Yes	36.1	m_ϕ 3.4 TeV

Cf. P. C. Muino' s talk: ATLAS limits on DM

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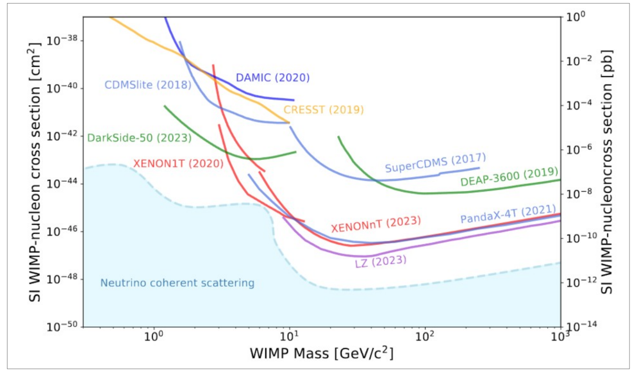

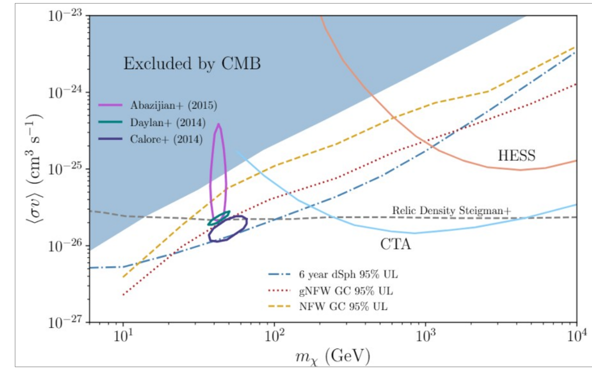

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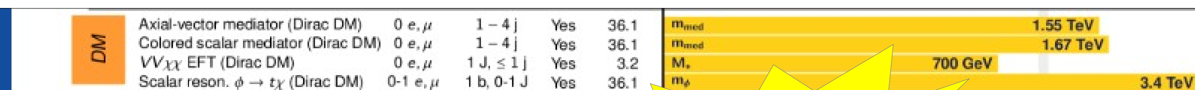


Cf. P. C. Muino' s talk: ATLAS limits on DM

Heavier, **coannihilating** mediators can be the reason.

Many processes and model parameters render the analysis complicated.

PHYSICAL REVIEW D VOLUME 43, NUMBER 10 15 MAY 1991

Three exceptions in the calculation of relic abundances

Kim Griest
Center for Particle Astrophysics and Astronomy Department, University of California, Berkeley, California 94720

David Seckel
Bartol Research Institute, University of Delaware, Newark, Delaware 19716
(Received 15 November 1990)

$$\langle \sigma_{\rm eff} v_{\rm rel} \rangle = \sum_{ij} \langle \sigma_{ij} v_{\rm rel} \rangle \frac{Y_i^{\rm eq} Y_j^{\rm eq}}{\widetilde{Y}_{\rm eq}^2}$$

Pheno toolbox

Experiment needs **minimal models** (few parameters) -
Theory needs precise and reliable **tools**!

micrOMEGAs

DarkSUSY

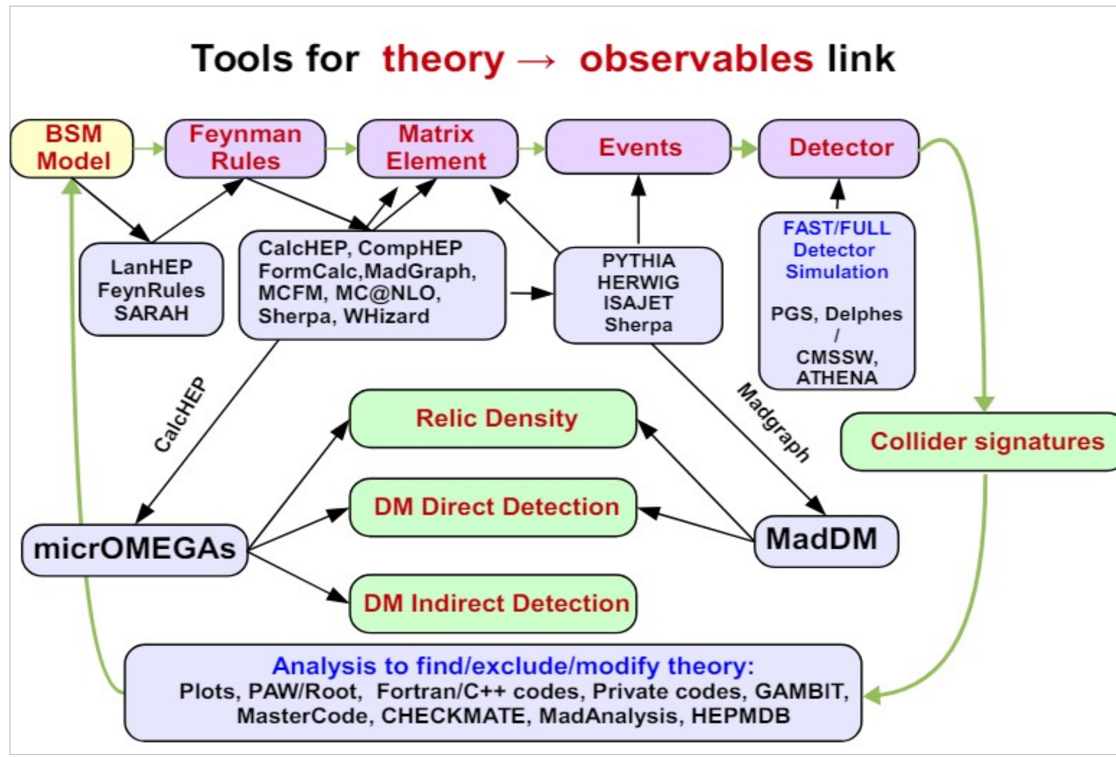
MadDM

DarkPACK

DM@NLO

...

Talk by M. Palmiotto



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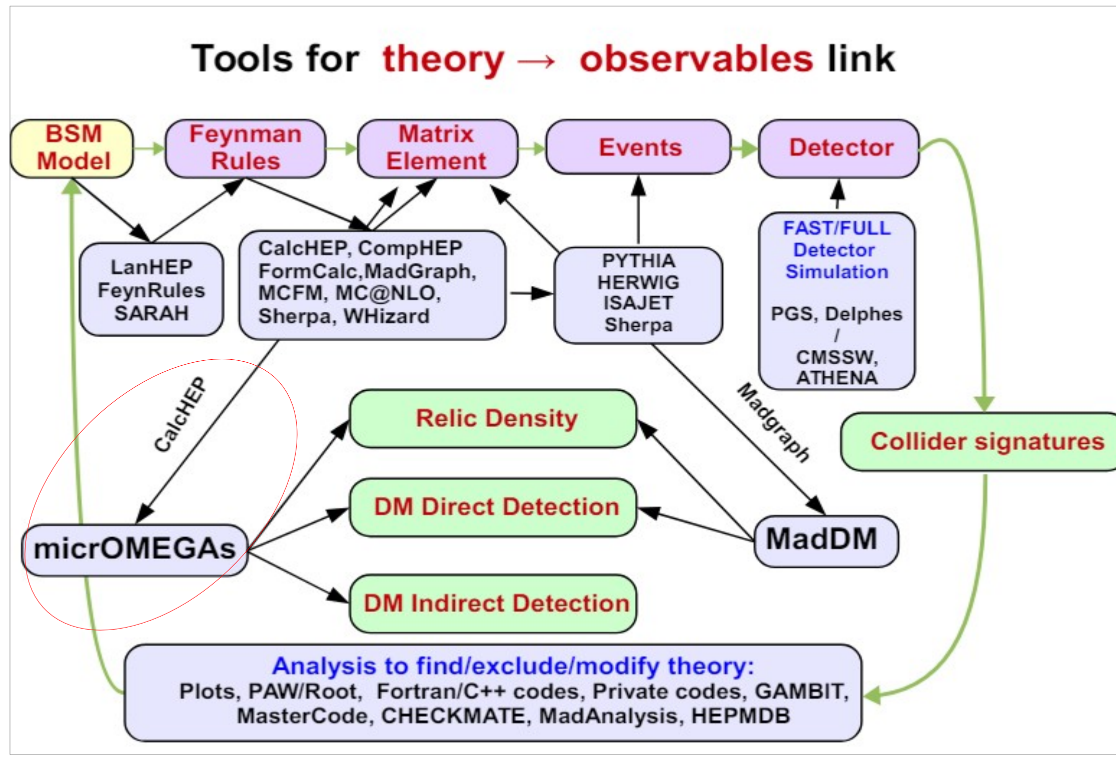

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Long-range effects for dark matter

Dark sector particle charged under a gauge group is subject to non-pertubative effects.

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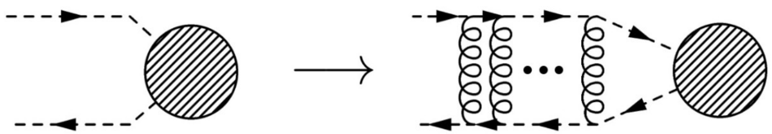

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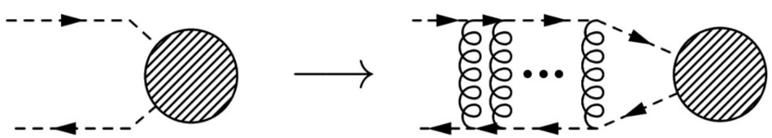

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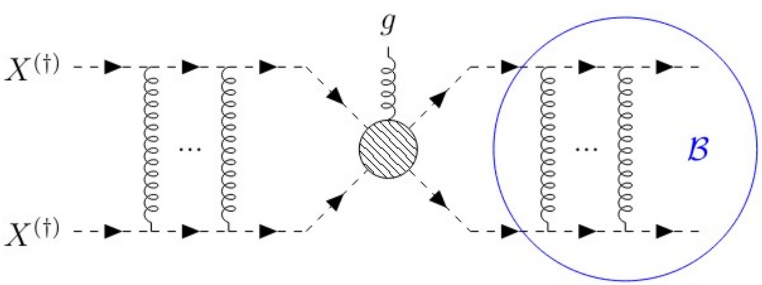

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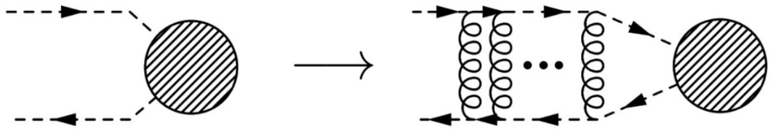

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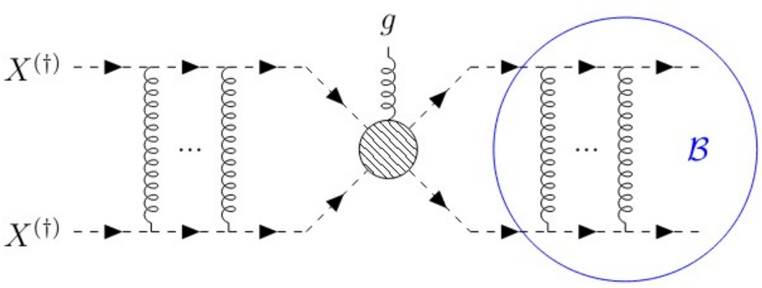

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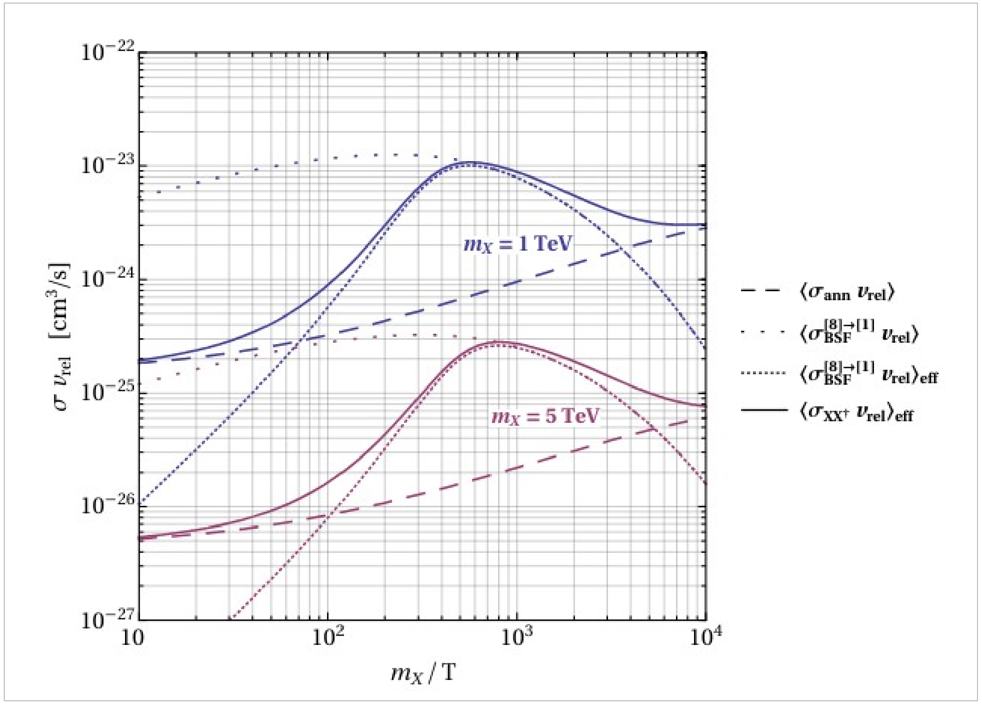

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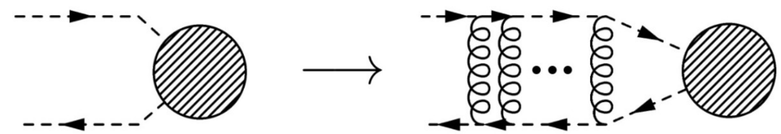

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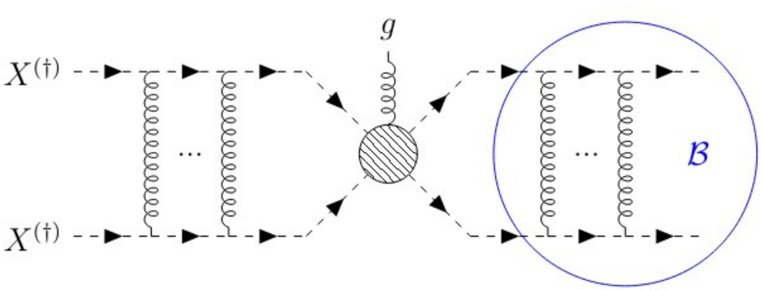

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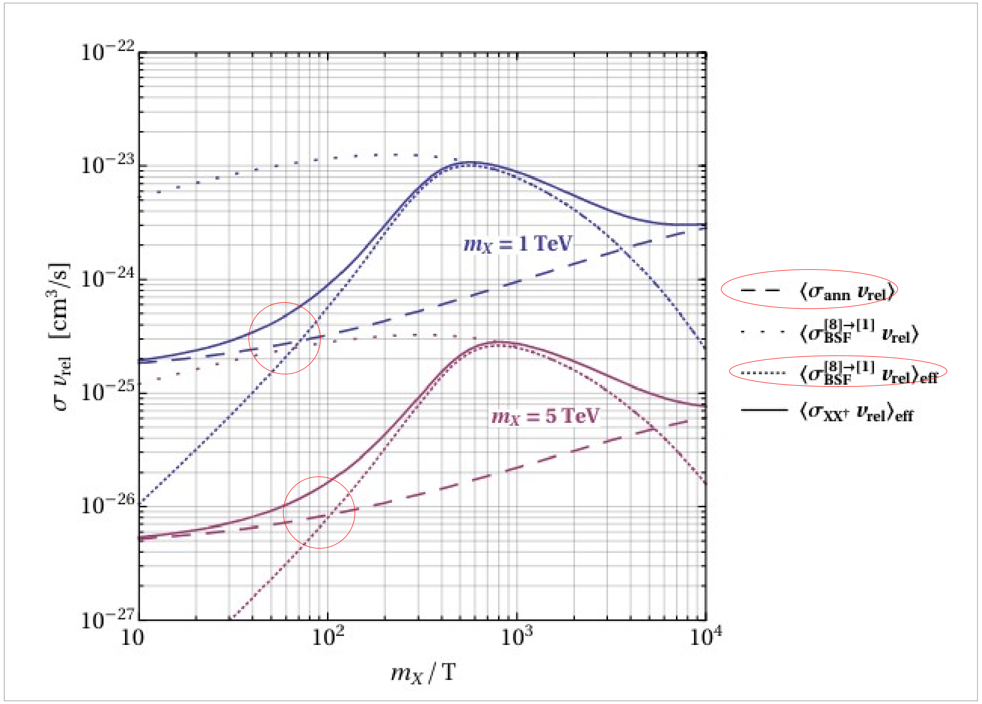
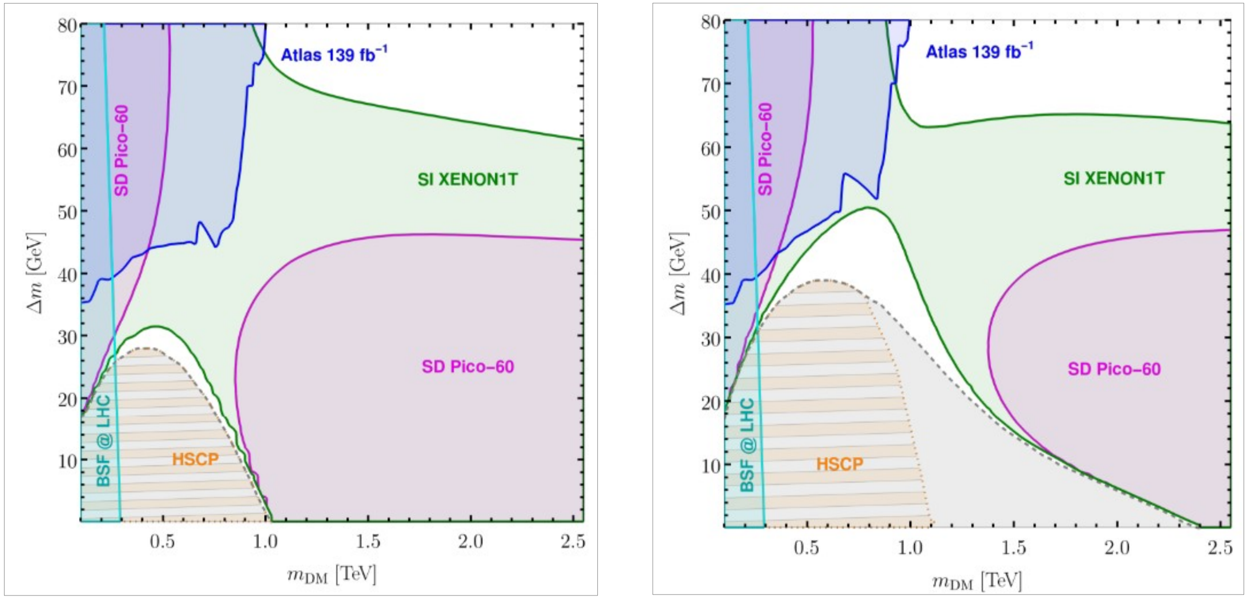


Long-range effects can relax experimental bounds

Pert. vs non-pert.



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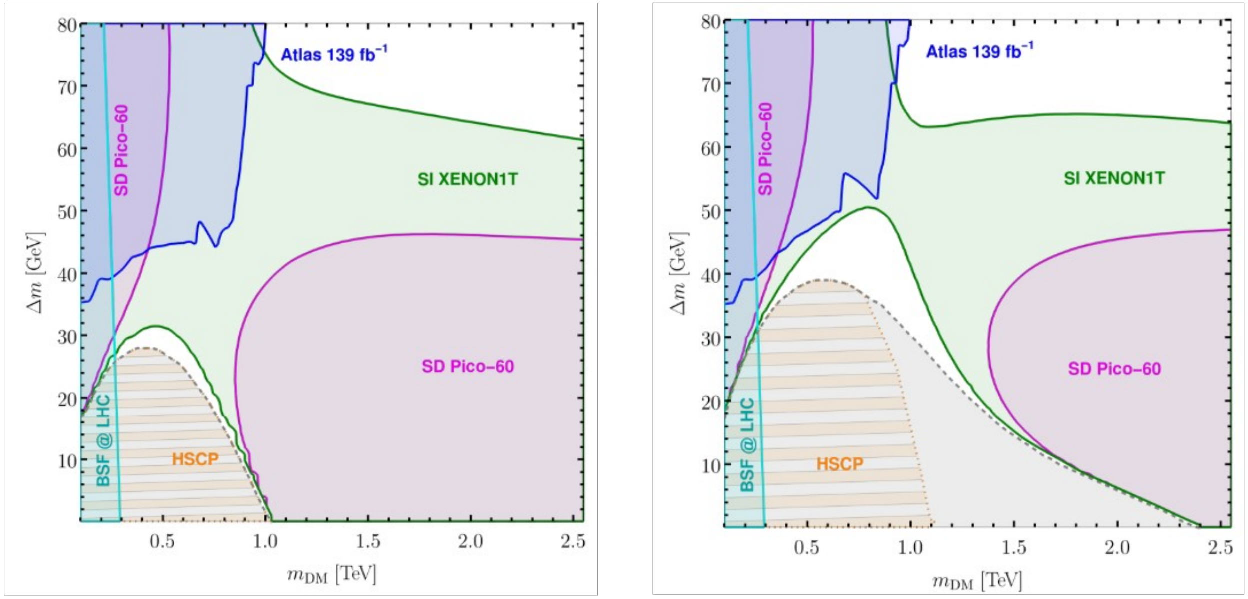

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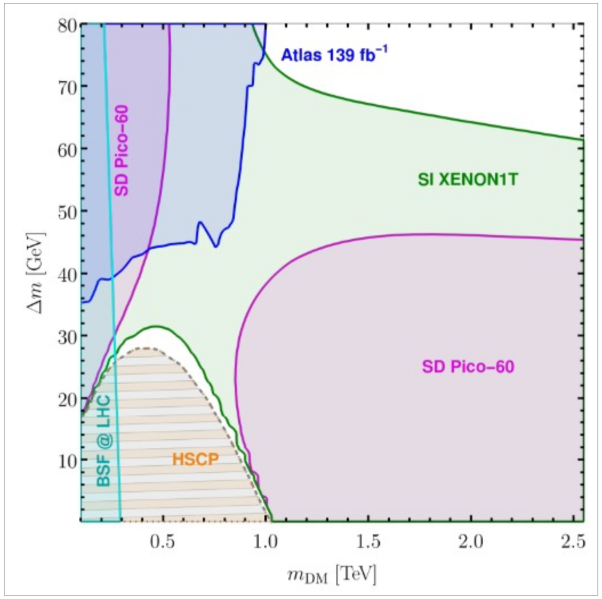

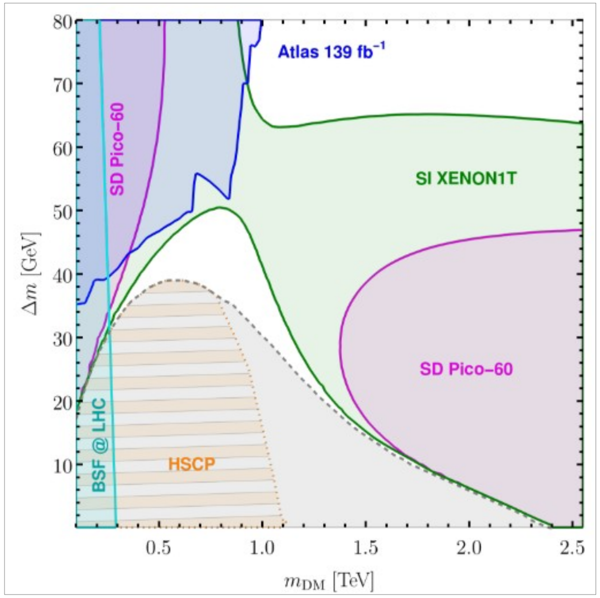

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We incorporate long-range effects into micrOMEGAs!

Simplified dark matter models and non-perturbative effects

General class of simplified models, studied vastly in the literature. In t-channel models → mediators are colored.
A phenomenological toolbox exists (DMSimpt).

[Arina et al. (2021)]
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$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm kin} + \mathcal{L}_F(\chi) + \mathcal{L}_F(\tilde{\chi}) + \mathcal{L}_S(S) + \mathcal{L}_S(\tilde{S}) + \mathcal{L}_V(V) + \mathcal{L}_V(\tilde{V})$$

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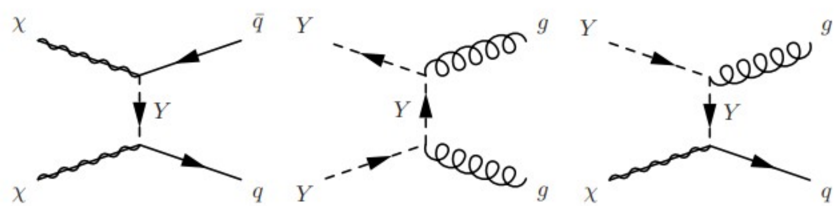

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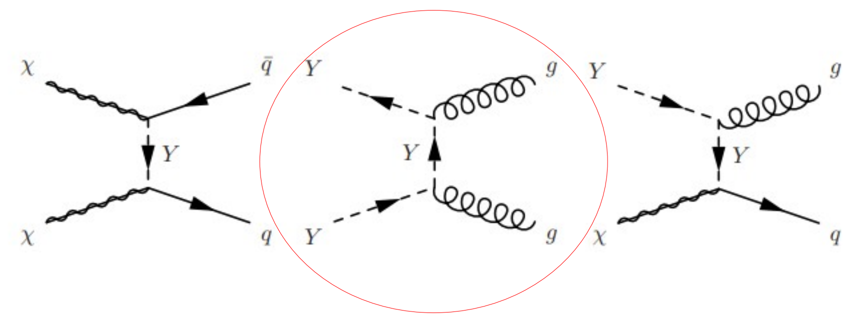

Subject to non-perturbative effects

Simplified dark matter models and non-perturbative effects (II)

Tools for relic density calculation with perturbative cross sections exist abundantly.

→ **Need for an automated framework for the inclusion of non-perturbative effects.**

We provide such a framework for the relic density calculation for colored particles.

Name	DM	Mediators	Parameters
S3M_uni	$\tilde{\chi}$	$\varphi_{Q_f}, \varphi_{u_f}, \varphi_{d_f}$	$M_\varphi, M_\chi, \lambda_\varphi$
S3D_uni	χ		
S3M_3rd	$\tilde{\chi}$	$\varphi_{Q_3}, \varphi_{u_3}, \varphi_{d_3}$	
S3D_3rd	χ		
S3M_uR	$\tilde{\chi}$	φ_{u_1}	
S3D_uR	χ		
F3S_uni	$\tilde{S}$	$\psi_{Q_f}, \psi_{u_f}, \psi_{d_f}$	$M_S, M_\psi, \hat{\lambda}_\psi$
F3C_uni	S		
F3S_3rd	$\tilde{S}$	$\psi_{Q_3}, \psi_{u_3}, \psi_{d_3}$	
F3C_3rd	S		
F3S_uR	$\tilde{S}$	ψ_{u_1}	
F3C_uR	S		
F3V_uni	$\tilde{V}_\mu$	$\psi_{Q_f}, \psi_{u_f}, \psi_{d_f}$	$M_V, M_\psi, \hat{\lambda}_\psi$
F3W_uni	V_μ		
F3V_3rd	$\tilde{V}_\mu$	$\psi_{Q_3}, \psi_{u_3}, \psi_{d_3}$	
F3W_3rd	V_μ		
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F3W_uR	V_μ		

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This talk: Two representative examples

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F3S_uni	$\tilde{S}$	$\psi_{Q_f}, \psi_{u_f}, \psi_{d_f}$	$M_S, M_\psi, \hat{\lambda}_\psi$
F3C_uni	S		
F3S_3rd	$\tilde{S}$	$\psi_{Q_3}, \psi_{u_3}, \psi_{d_3}$	
F3C_3rd	S		
F3S_uR	$\tilde{S}$	ψ_{u_1}	
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F3W_uni	V_μ		
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Showcases of our computational framework

Setup of the computation

$$< \sigma v >_{\text{total}} = < \mathcal{S} \sigma v >_{\text{eff}} + < \sigma_{\text{BSF}} v >_{\text{eff}}$$

Setup of the computation

All perturbative (co-)annihilations automatically calculated by micrOMEGAs

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Sommerfeld enhancement for s- and p-wave annihilations with the color structure

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$

Setup of the computation

All perturbative (co-)annihilations automatically calculated by micrOMEGAs

Bound state effects are covered via an effective cross section.

[Ellis et al. (2015)]
[Petraki et al. (2015)]
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Cf. Andi Trautner's talk: „birdstrack technique“

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Sommerfeld effect for colored particles

Color decomposition splits the cross section into an enhanced (attractive configuration) and a suppressed part (repulsive configuration).

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[El Hedri et al. (2017)]

We use explicitly

$$\mathcal{S}\sigma = \sum_{l=0}^{1}\left[c_{l,[\mathbf{1}]} S_l\left(\frac{4}{3}\frac{\alpha_s}{v_{\rm rel}}\right) + c_{l,[\mathbf{8}]} S_l\left(-\frac{1}{6}\frac{\alpha_s}{v_{\rm rel}}\right) + c_{l,[\bar{\mathbf{3}}]} S_l\left(\frac{2}{3}\frac{\alpha_s}{v_{\rm rel}}\right) + c_{l,[\mathbf{6}]} S_l\left(-\frac{1}{3}\frac{\alpha_s}{v_{\rm rel}}\right)\right]\sigma_l + \dots$$

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Coefficients for the color decomposition are not uniquely determined by the inital and final state representations.

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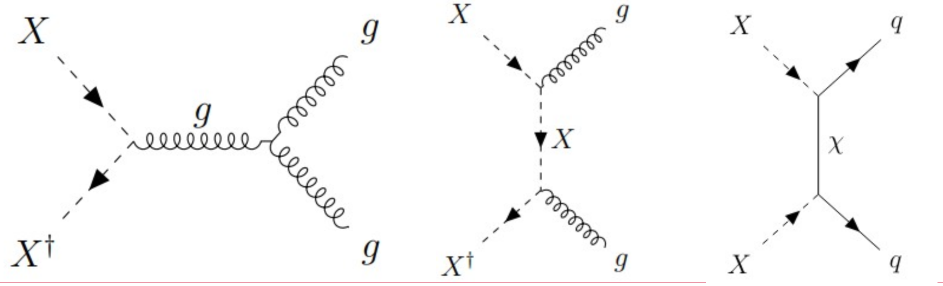

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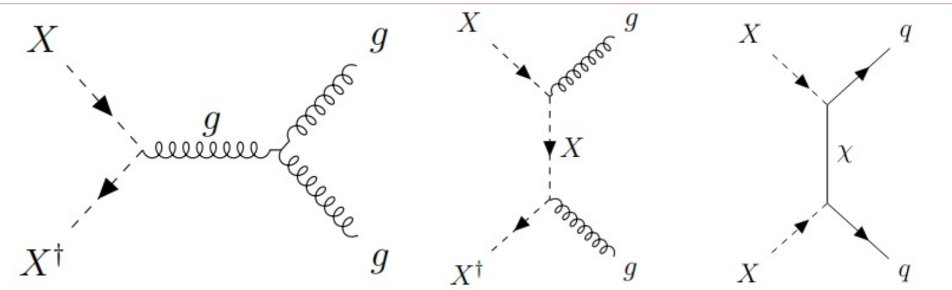

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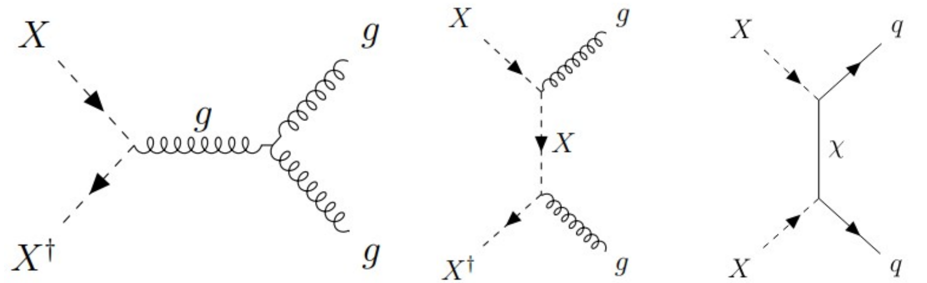

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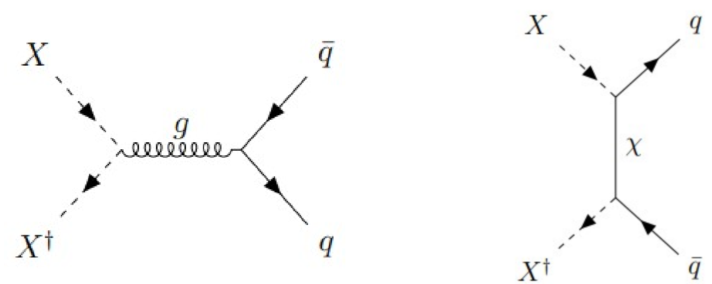

$[\mathbf{8}]$ $[\mathbf{1}] + [\mathbf{8}]$

(Excited) bound states

Network of Boltzmann equations for excited states can be **simplified to one** and an **effective bound state formation cross section** can be obtained.

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$$\langle \sigma_{BSF} v \rangle_{\rm eff} = \sum_i \langle \sigma_{\rm BSF,i} v \rangle \left(1 - (M)^{-1}_{ij} \frac{\langle \Gamma^j_{\rm ion} \rangle}{\langle \Gamma^j \rangle} \right)$$

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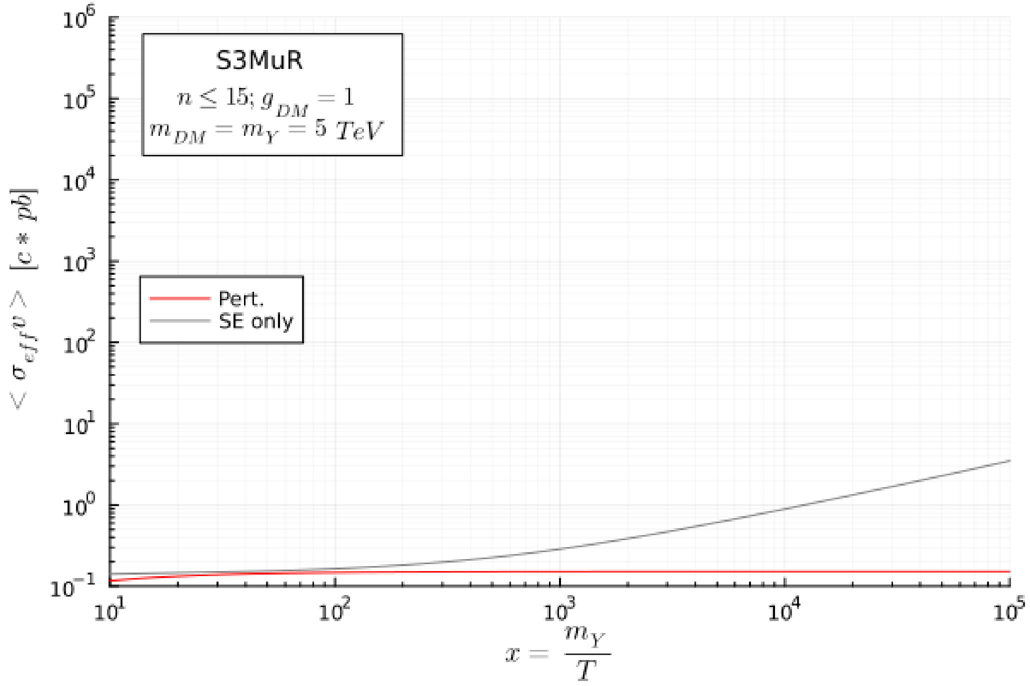
In the coannihilation regime, including only the **ground state is usually sufficient**.

$$\langle\sigma_{BSF}v\rangle_{\rm eff} = \langle\sigma_{BSF,n=1}v\rangle \frac{\langle\Gamma^{n=1}_{\rm dec}\rangle}{\langle\Gamma^{n=1}_{\rm ion}\rangle + \langle\Gamma^{n=1}_{\rm dec}\rangle}$$

with $$v_{\rm rel}\frac{d\sigma_{\mathbf{k}\to\{100\}}}{d\Omega} = \frac{|\mathbf{P}_g|}{64\pi^2 M^2\mu}\left(|\boldsymbol{\mathcal{M}}_{\mathbf{k}\to\{100\}}|^2 - |\hat{\mathbf{P}}_g\cdot\boldsymbol{\mathcal{M}}_{\mathbf{k}\to\{100\}}|^2\right)$$

Comparison of non-perturbative effects



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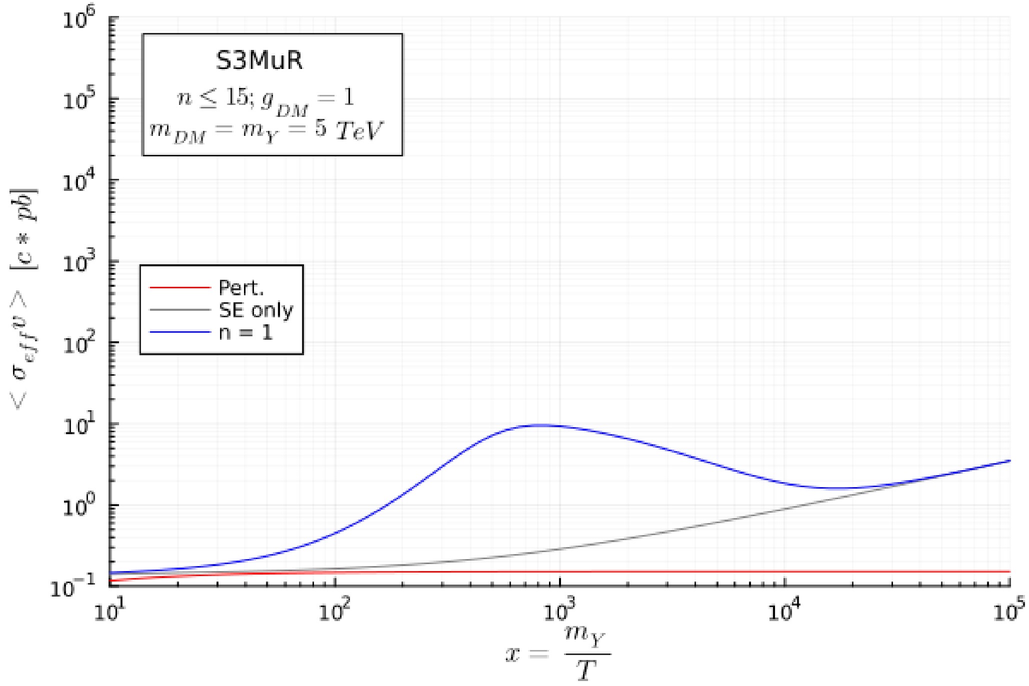

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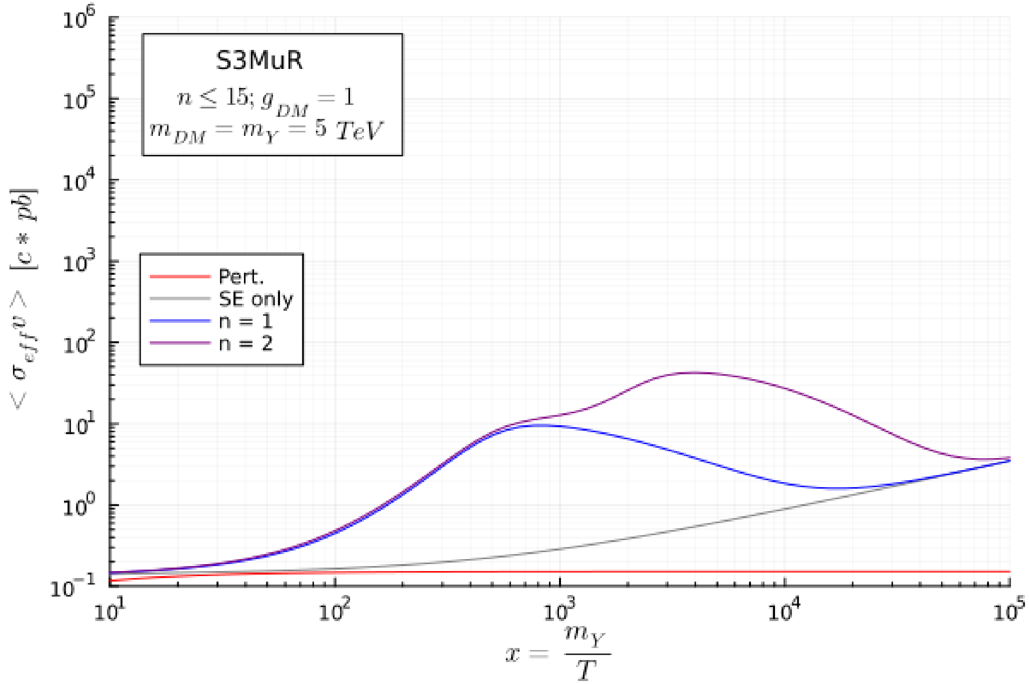

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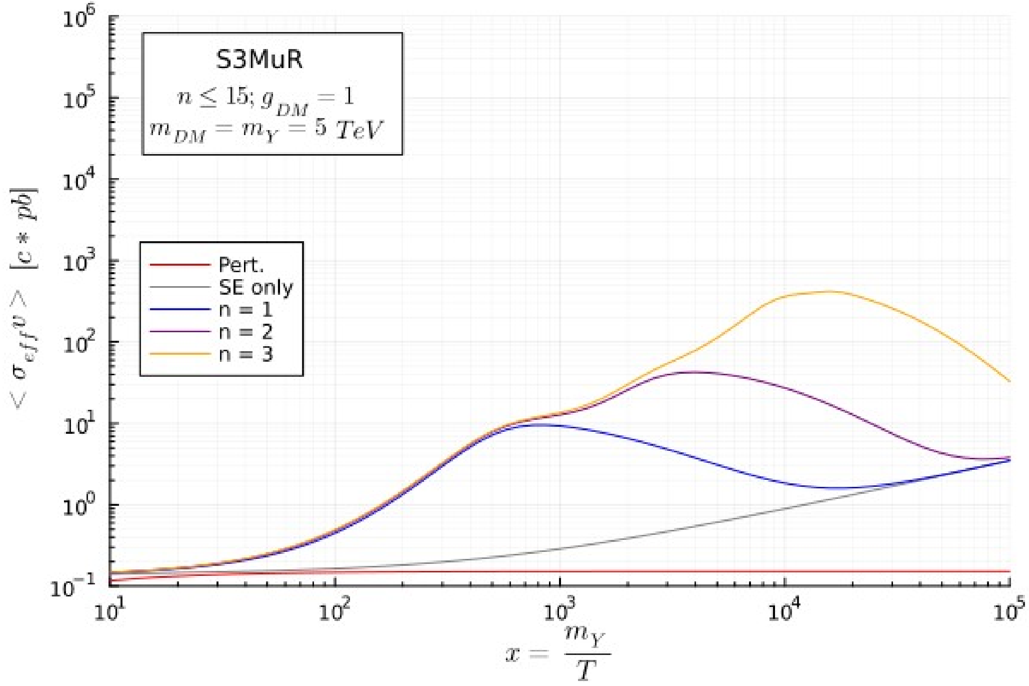

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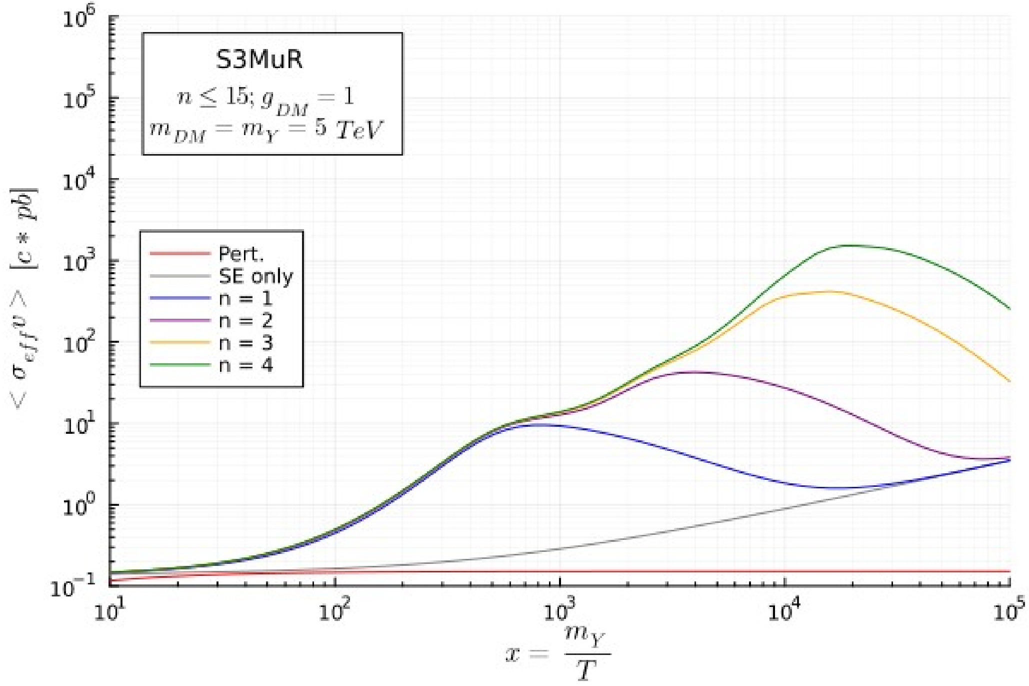

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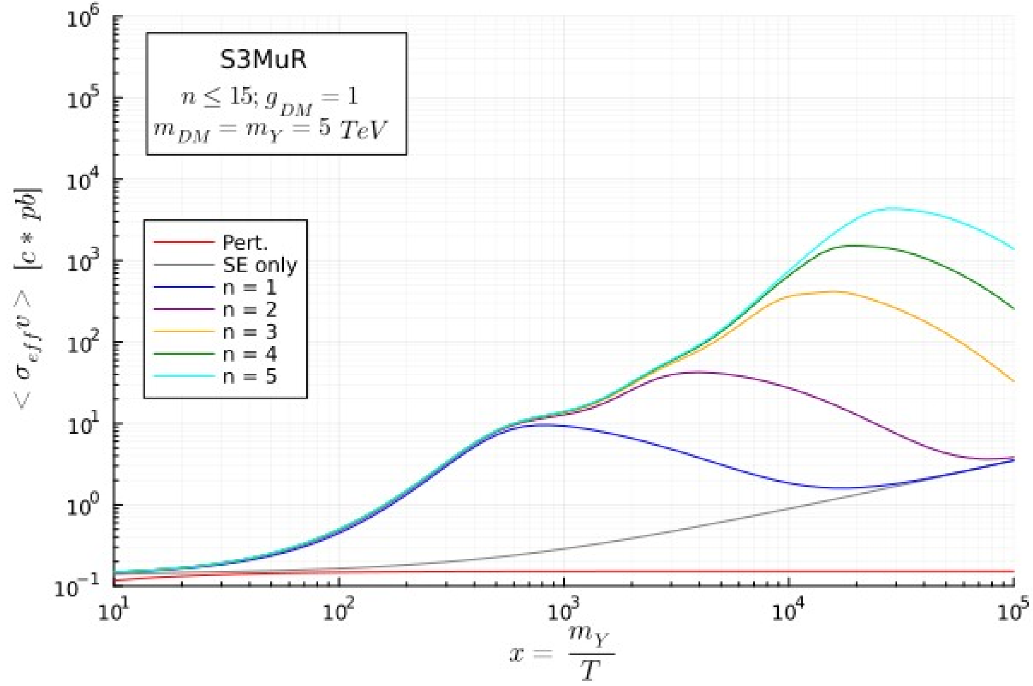

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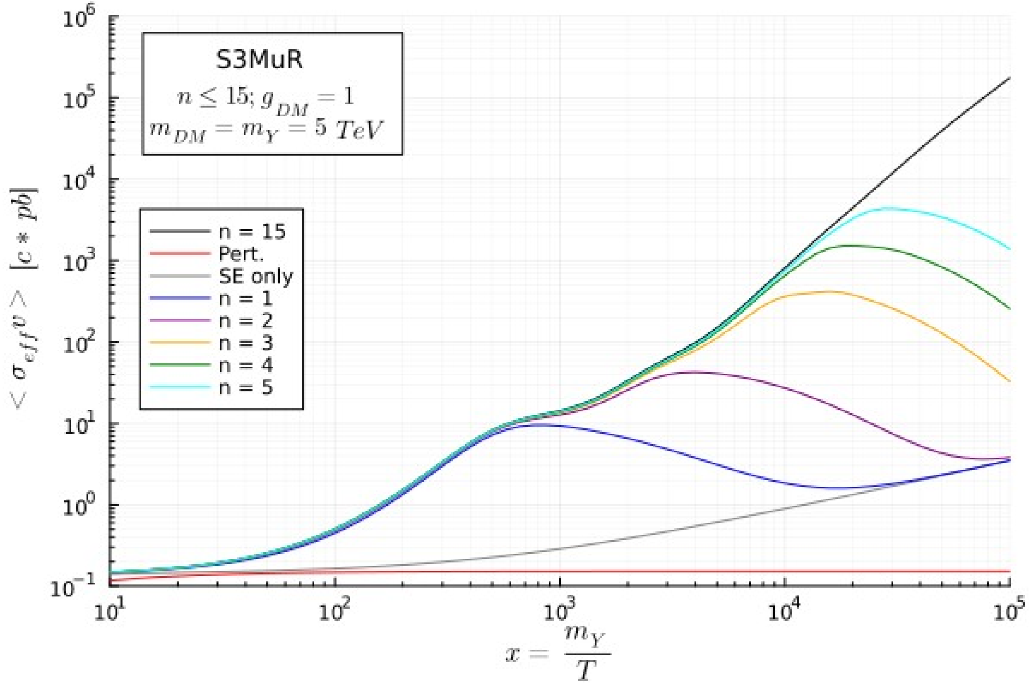
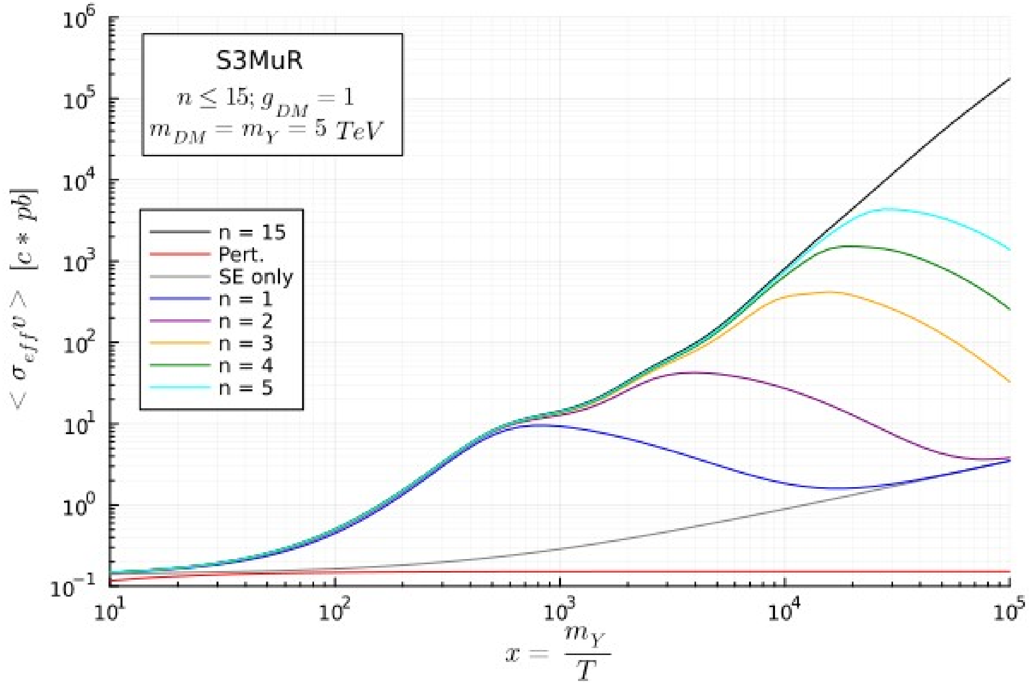


Comparison of non-perturbative effects

Bound state formation cross section **never freezes-out** for colored DM candidates (but they do for coannihilation).

[Binder et al. (2023)]



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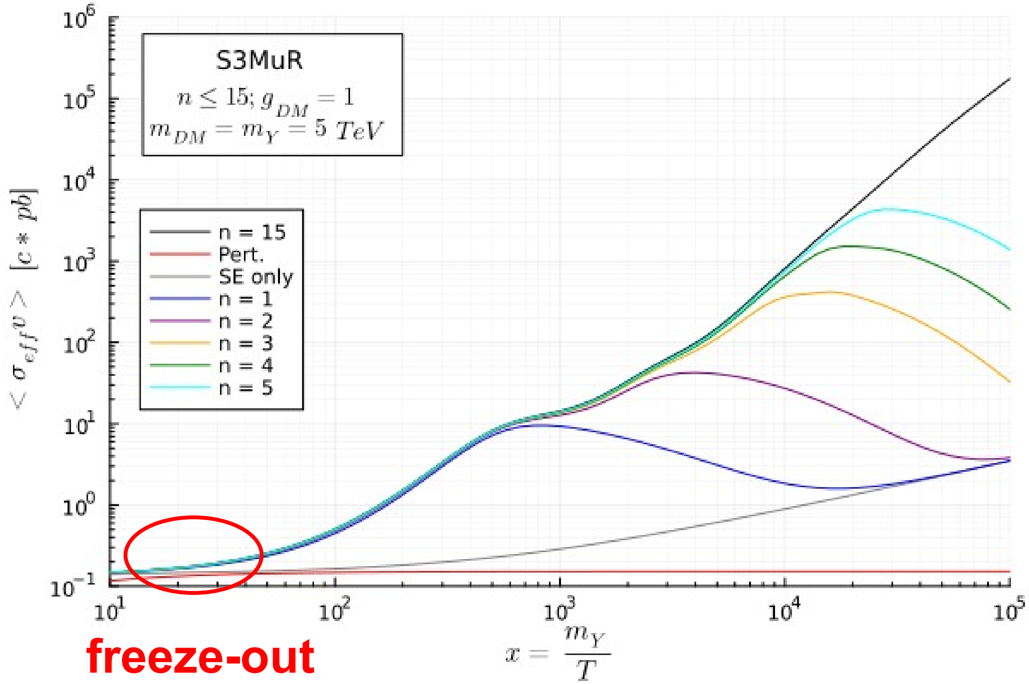


Dominant contribution during freeze-out comes from the **ground state (n = 1)**

$$\langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle = \sum_{ij} \langle \sigma_{ij} v_{\mathrm{rel}} \rangle \frac{Y_i^{\mathrm{eq}} Y_j^{\mathrm{eq}}}{\widetilde{Y}_{\mathrm{eq}}^2} \propto e^{-2\delta x}$$

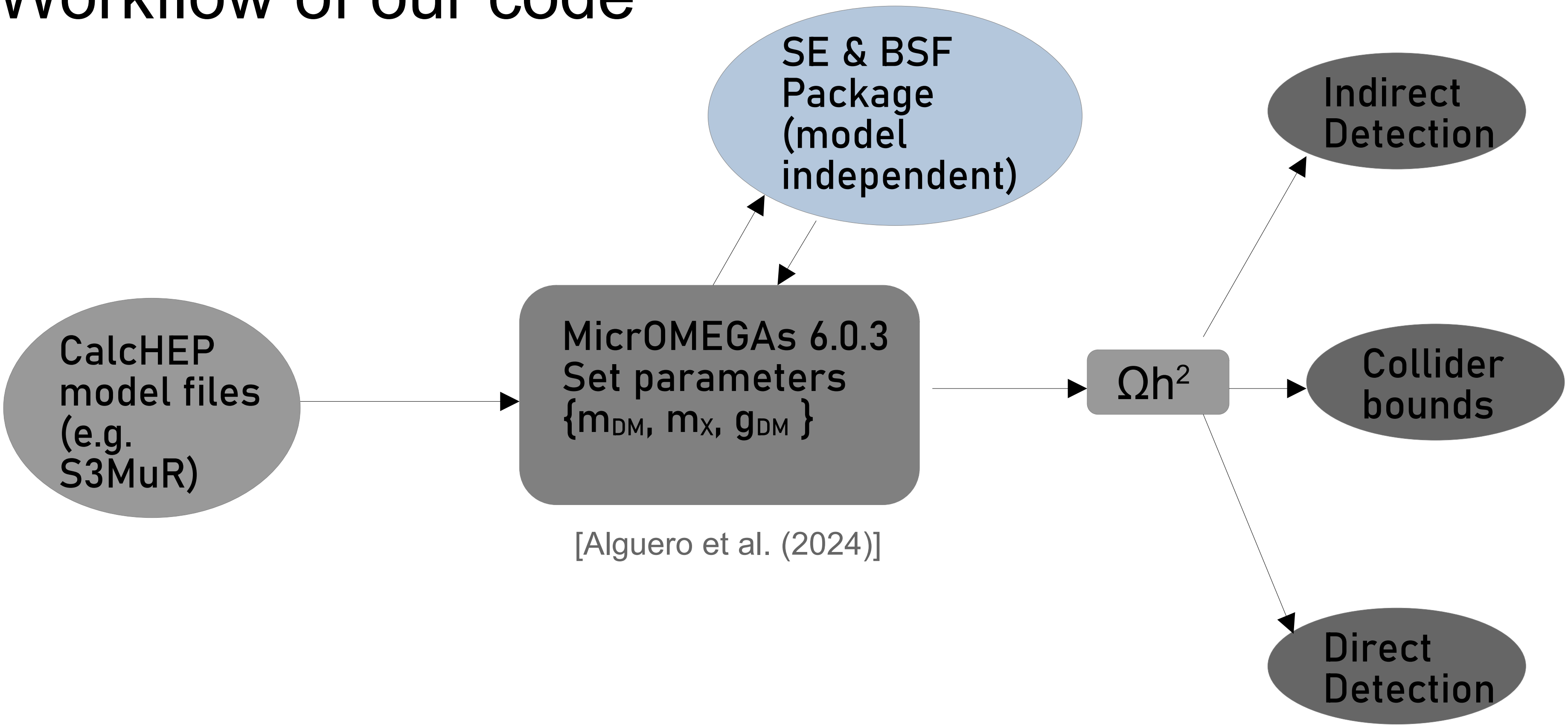
Outline

Simplified dark matter models and long-range effects

Sommerfeld effect and bound state formation for colored mediators

Showcases of our computational framework

Workflow of our code



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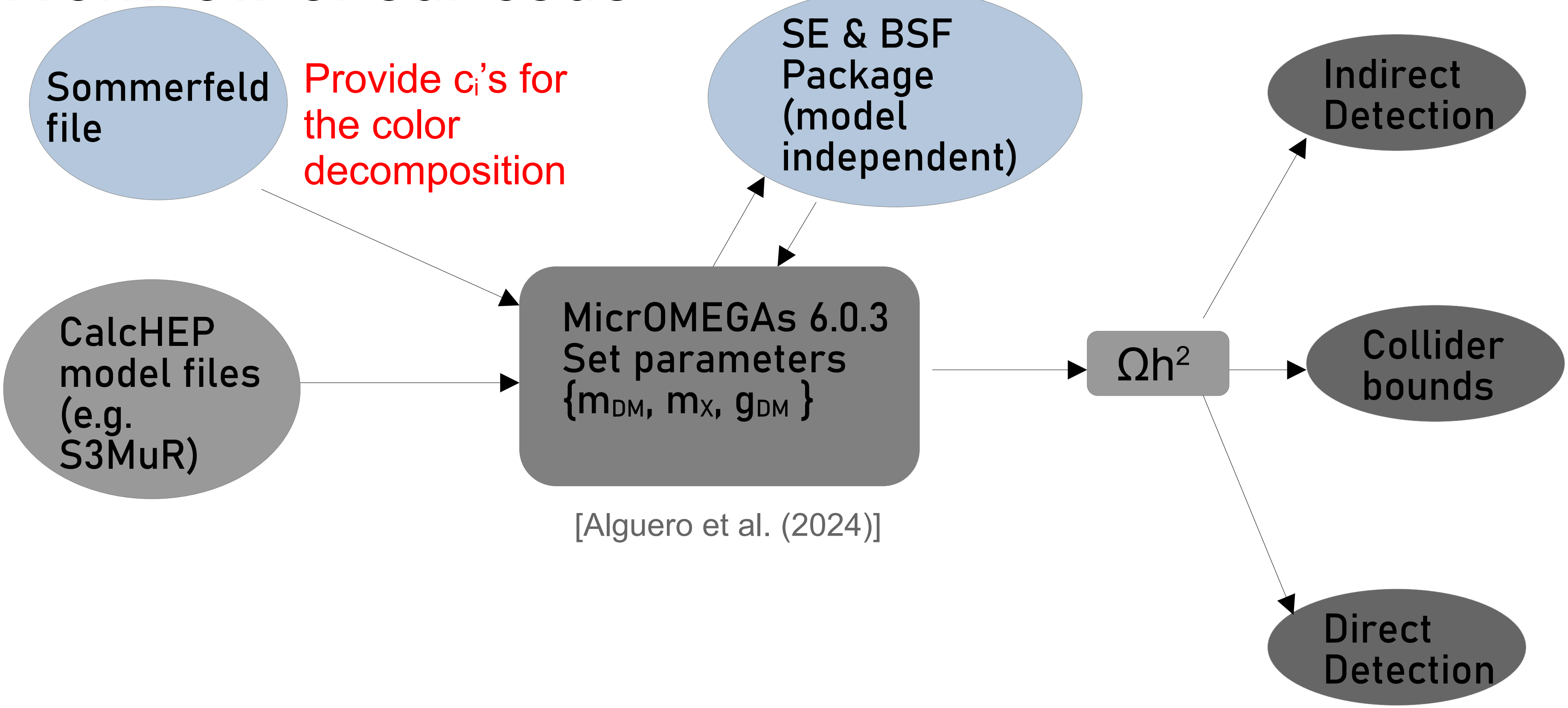

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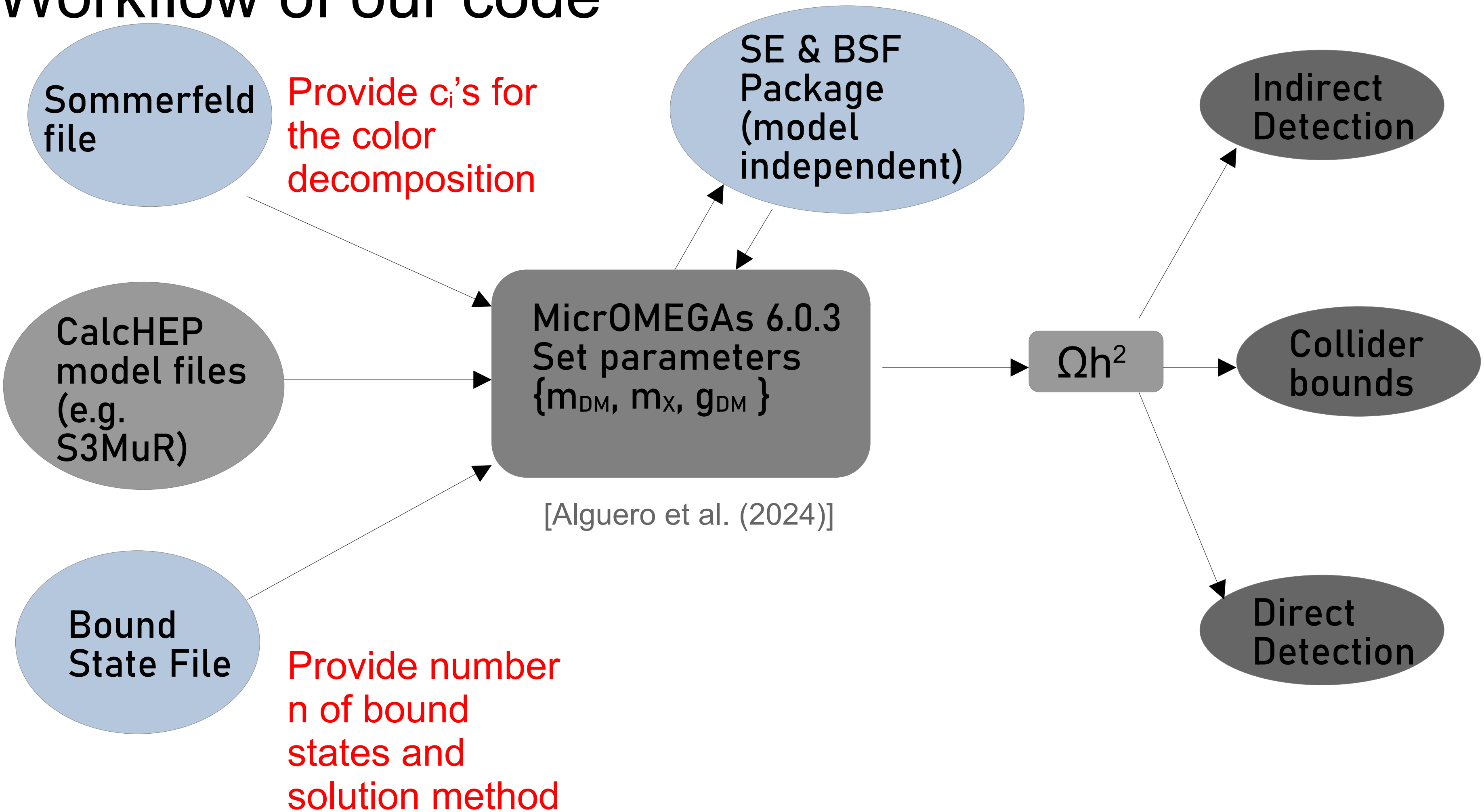
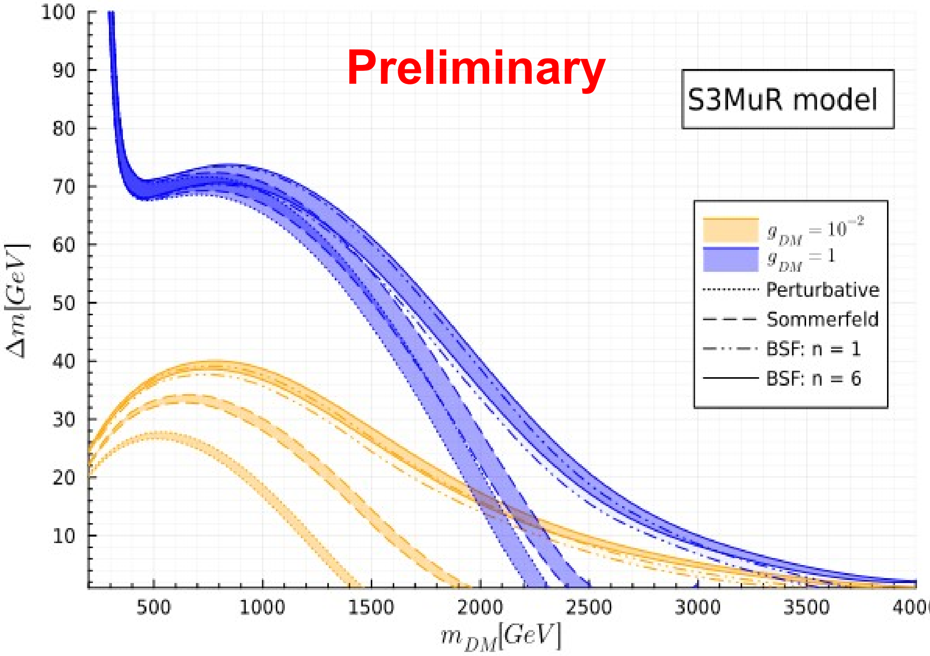
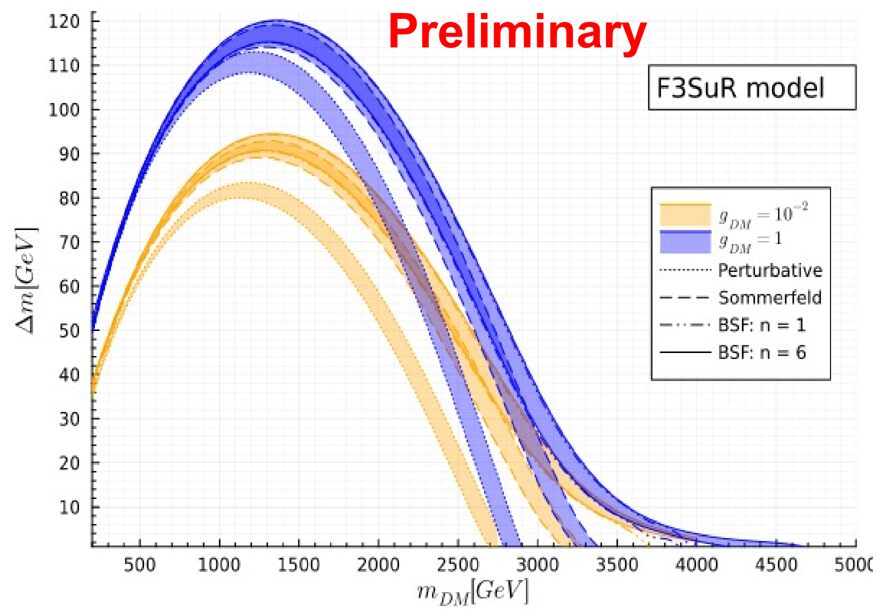


Impact for two types of mediators: SE + $BSF_{n=6}$

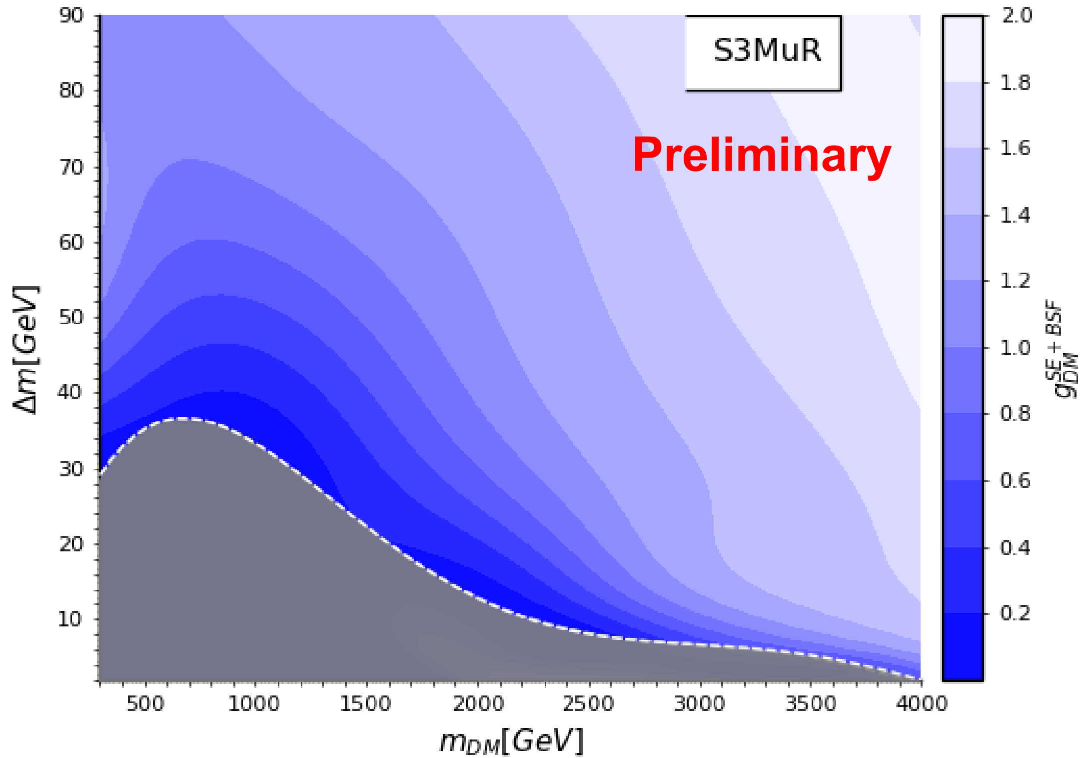
Allowed bands for Ω_{DM} = 0.1200 ± 0.0050 (5σ)





First scan for scalar mediators: SE + $BSF_{n=6}$

Upper limit on g_{DM} for $\Omega_{DM} = 0.1200 \pm 0.0050$ (5σ)



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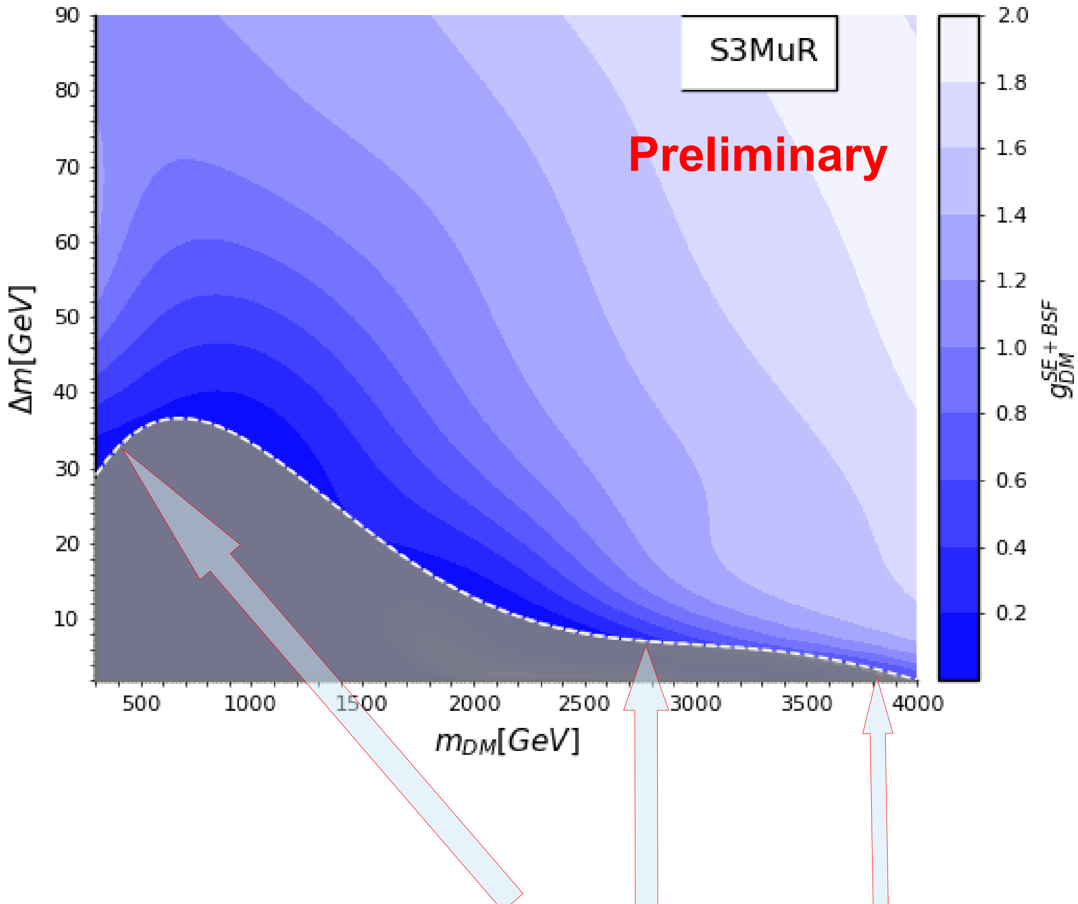

Rapid change from $g_{DM} = 10^{-2}$ to 10^{-7}

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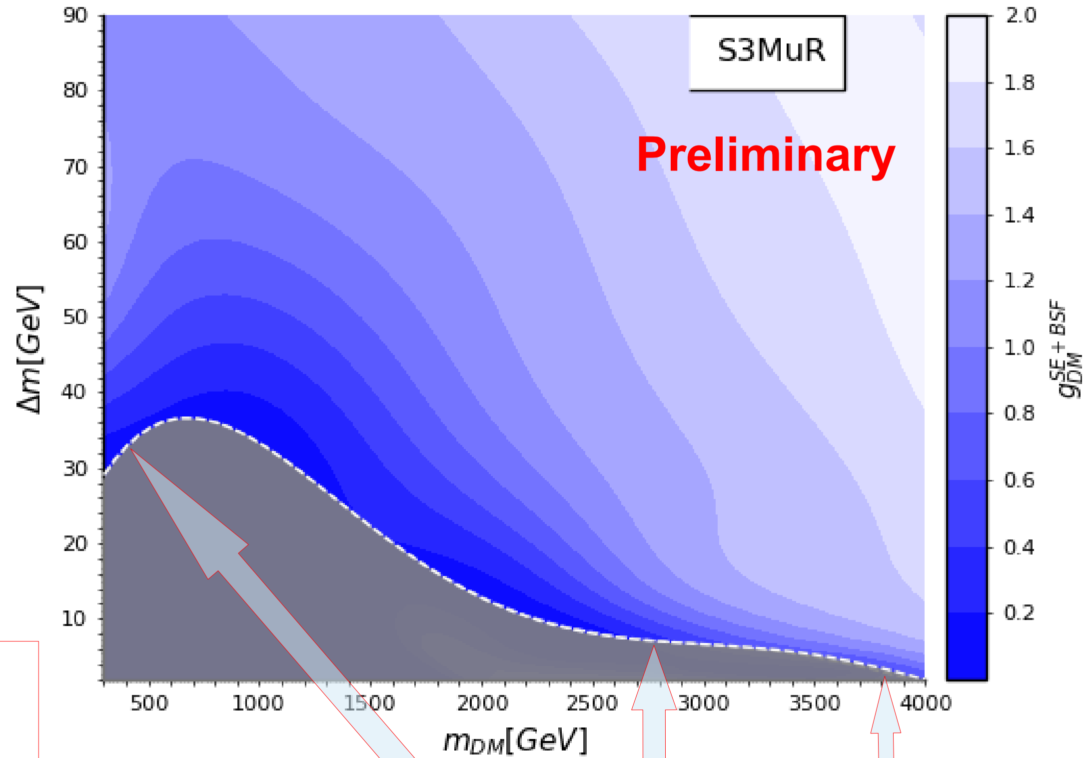

Gray region:

$$\Gamma_X \frac{Y_X^{eq}}{Y_\chi^{eq}} < H$$

No freeze-out

Rapid change from $g_{DM} = 10^{-2}$ to 10^{-7}

Conclusions

Non-perturbative long range effects have **a sizeable impact** on the predicted relic abundance.

Simplified dark matter models allow for a universal treatment of these effects, which can be **efficiently incorporated by our framework**.

Impact of Sommerfeld enhancement depends on the dominant annihilation channels and **spin of the mediator**.

The inclusion of bound state formation **lifts the predicted DM mass and (re-)opens parameter space**.
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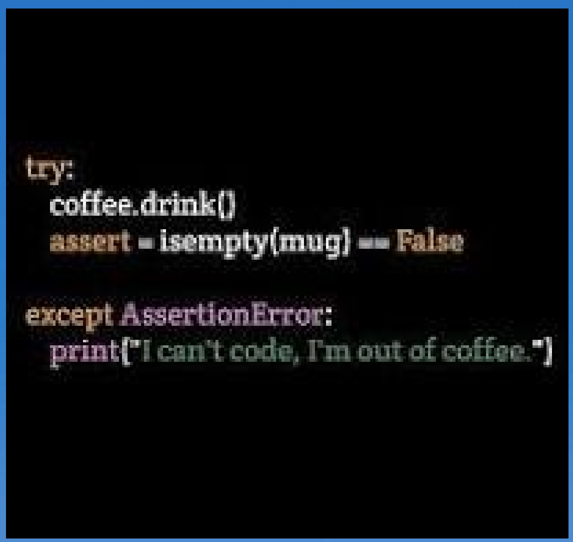
Our code will be publicly available soon!

Thank you for your attention!
Obrigado pela sua atenção!



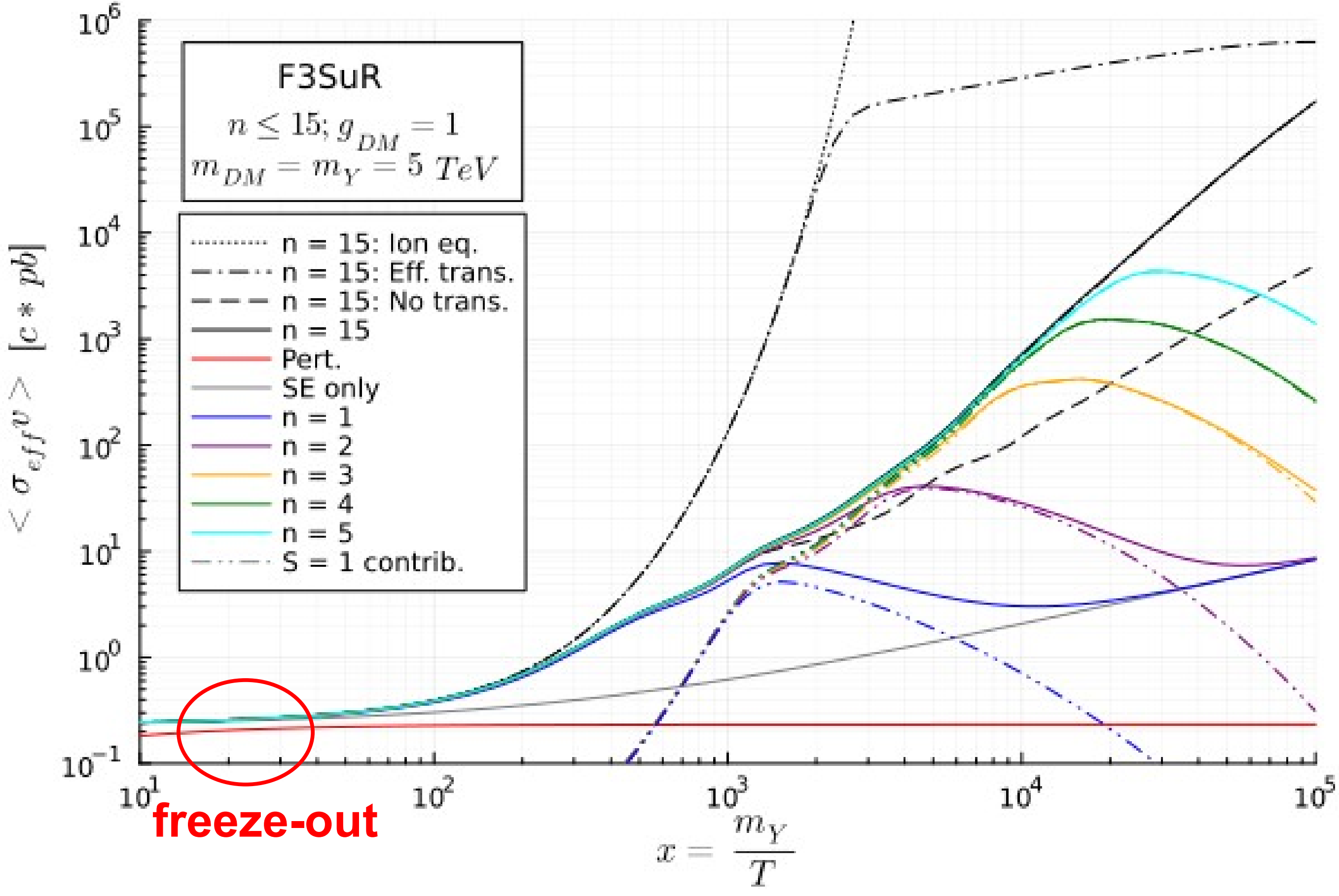
Backup

Running coupling at different scales

Vertices	α_s	α_g	Average momentum transfer Q
Wavefunction (ladder diagrams) of scattering state in colour rep. $\hat{\mathbf{R}}$	α_s^S	$\alpha_{g,[\hat{\mathbf{R}}]}^S = (\alpha_s^S/2)\times$ $\times\,[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\hat{\mathbf{R}})]$	$k \equiv \mu\, v_{\rm rel}$
Wavefunction (ladder diagrams) of bound state in colour rep. $\hat{\mathbf{R}}$	$\alpha_{s,[\hat{\mathbf{R}}]}^B$	$\alpha_{g,[\hat{\mathbf{R}}]}^B = (\alpha_{s,[\hat{\mathbf{R}}]}^B/2)\times$ $\times\,[C_2(\mathbf{R_1}) + C_2(\mathbf{R_2}) - C_2(\hat{\mathbf{R}})]$	$\kappa_{\hat{\mathbf{R}}} \equiv \mu\, \alpha_{g,[\hat{\mathbf{R}}]}^B$
Formation of bound states of colour rep. $\hat{\mathbf{R}}$: gluon emission	$\alpha_{s,[\hat{\mathbf{R}}]}^{\rm BSF}$		$\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{n\ell} = \frac{\mu}{2}\left[v_{\rm rel}^2 + (\alpha_{g,[\hat{\mathbf{R}}]}^B/n)^2\right]$
$gX_i^\dagger X_i$ vertices in non-Abelian diagram for capture in colour rep. $\hat{\mathbf{R}}$	$\alpha_{s,[\hat{\mathbf{R}}]}^{\rm NA}$		$\mu\sqrt{v_{\rm rel}^2 + {\alpha_{g,[\hat{\mathbf{R}}]}^B}^2}$

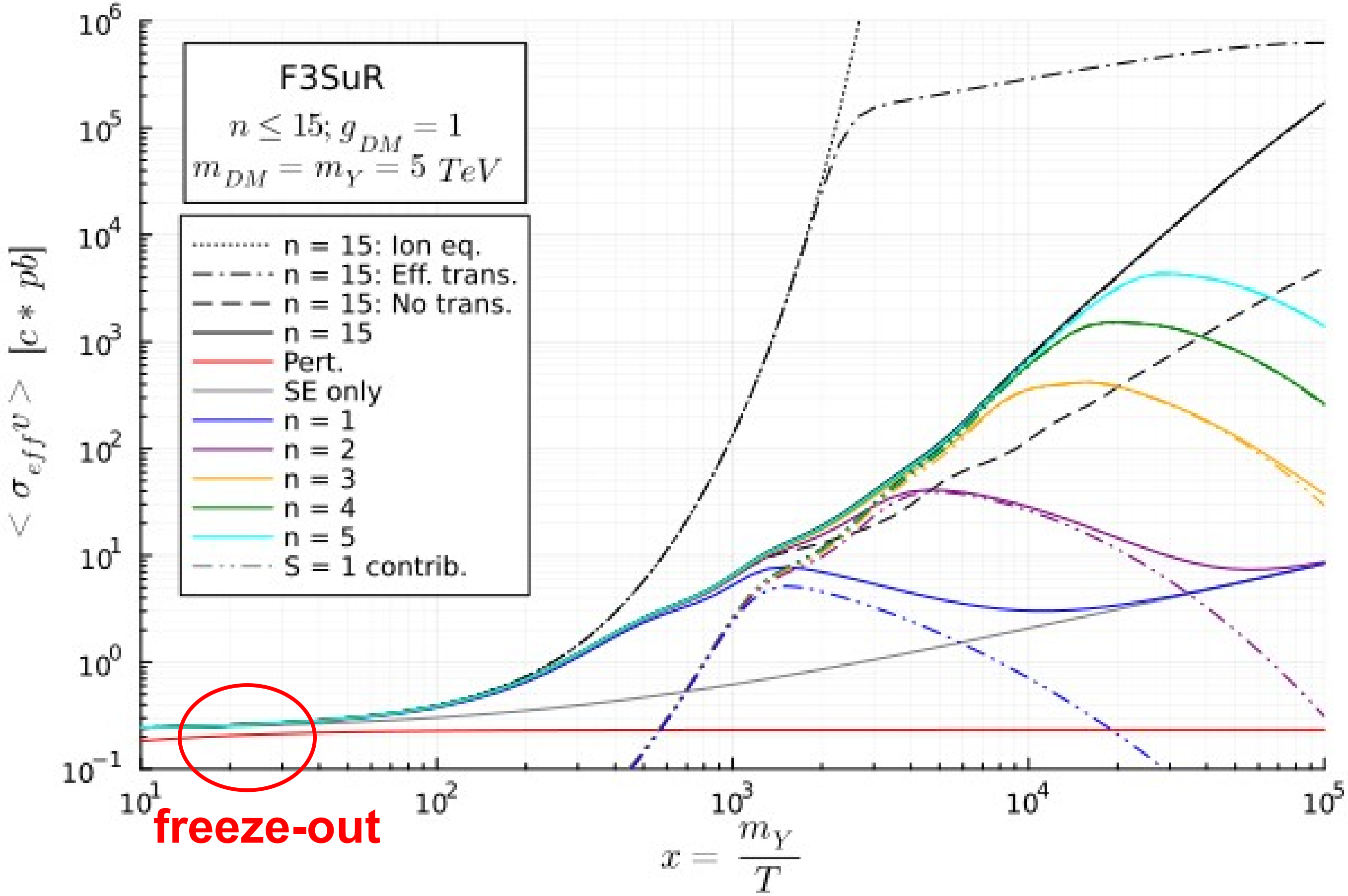
[J. Harz and K. Petraki (2018)]

Cross sections for fermionic mediators



Cross sections for fermionic mediators

Triplet contributions negligible as expected → Only relevant at very **late times**



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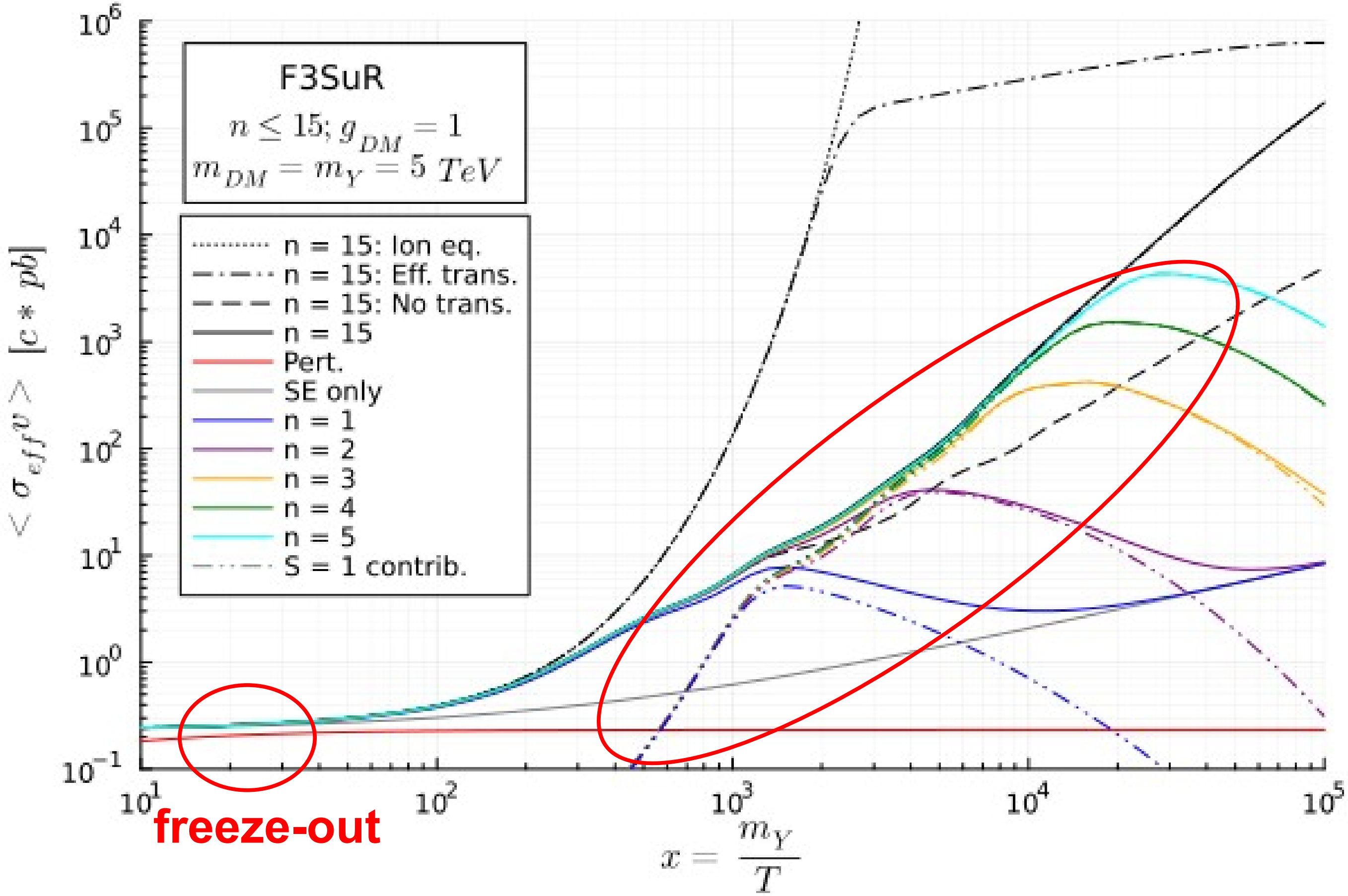
Transitions are **much more efficient** for triplet states



Limiting scenarios for excited bound states

1) At early times: **Ionization equilibrium**: $\Gamma^i_{\text{ion}} >> \Gamma^i_{\text{dec}}, \Gamma^{ij}_{\text{trans}}$

[Garny & Heisig (2022)]

$$< \sigma_{BSF} v >_{\text{eff}} = \sum_i \frac{g_{\mathcal{B}_i}}{g_X^2} \left(\frac{2\pi m_{\mathcal{B}_i}}{T m_X^2} \right)^{3/2} e^{E_{\mathcal{B}_i}/T} \Gamma^i_{\text{dec}}$$

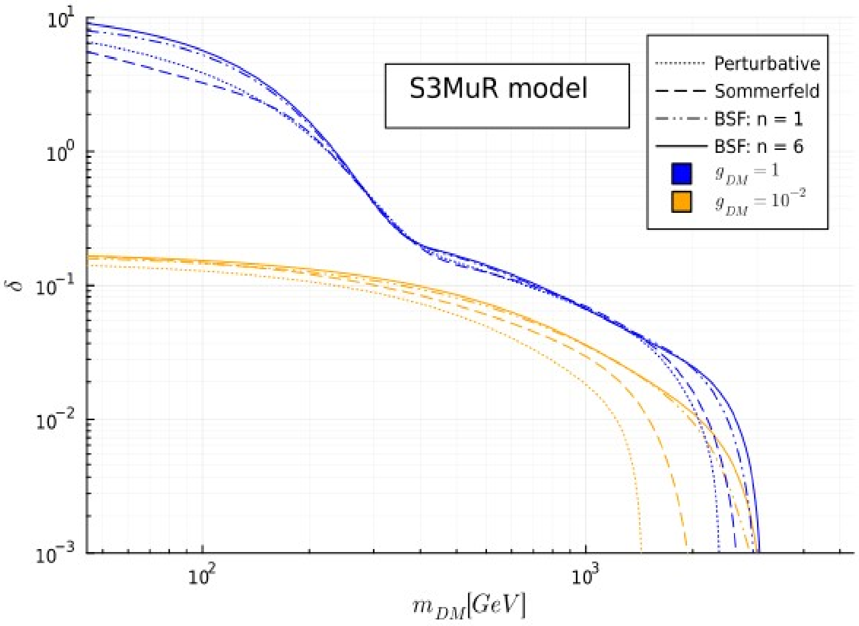
2) **Efficient transition limit**: $\Gamma^{ij}_{\text{trans}} >> \Gamma^i_{\text{dec}}, \Gamma^i_{\text{ion}}$

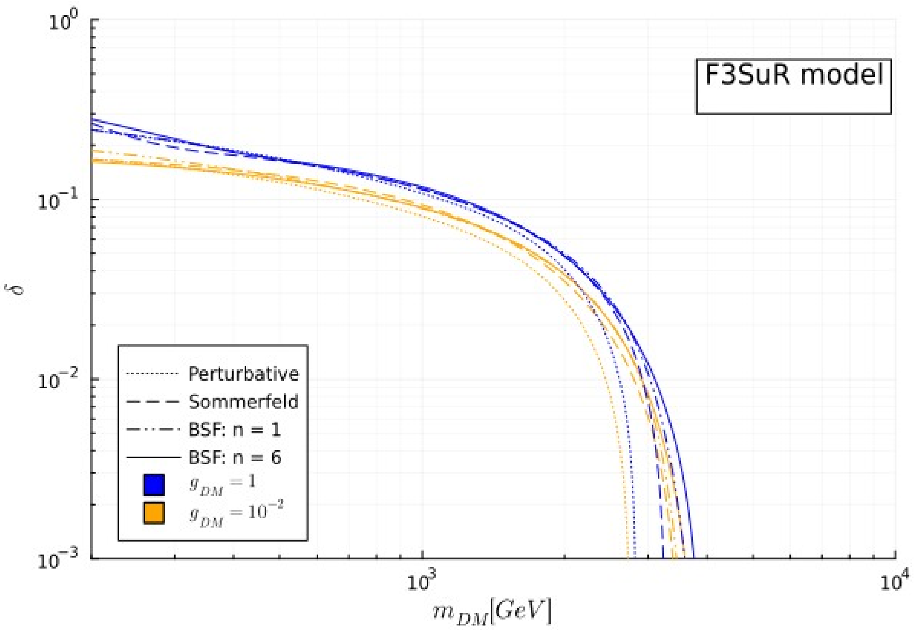
$$< \sigma_{BSF} v >_{\text{eff}} = < \sigma_{BSF} v >_{\text{sum}} \frac{\Gamma^{\text{eff}}_{\text{dec}}}{\Gamma^{\text{eff}}_{\text{ion}} + \Gamma^{\text{eff}}_{\text{dec}}} \qquad \Gamma^{\text{eff}}_{\text{ion/dec}} = \frac{\sum_i \Gamma^i_{\text{ion/dec}} Y^{\text{eq}}_{\mathcal{B}_i}}{Y^{\text{eq}}_{\mathcal{B}}}$$

3) **No transition limit**: $\Gamma^i_{\text{dec}} >> \Gamma^i_{\text{ion}}, \Gamma^{ij}_{\text{trans}}$

$$< \sigma_{BSF} v >_{\text{eff}} = \sum_i < \sigma_{BSF,i} v > \frac{\Gamma^i_{\text{dec}}}{\Gamma^i_{\text{ion}} + \Gamma^i_{\text{dec}}}$$

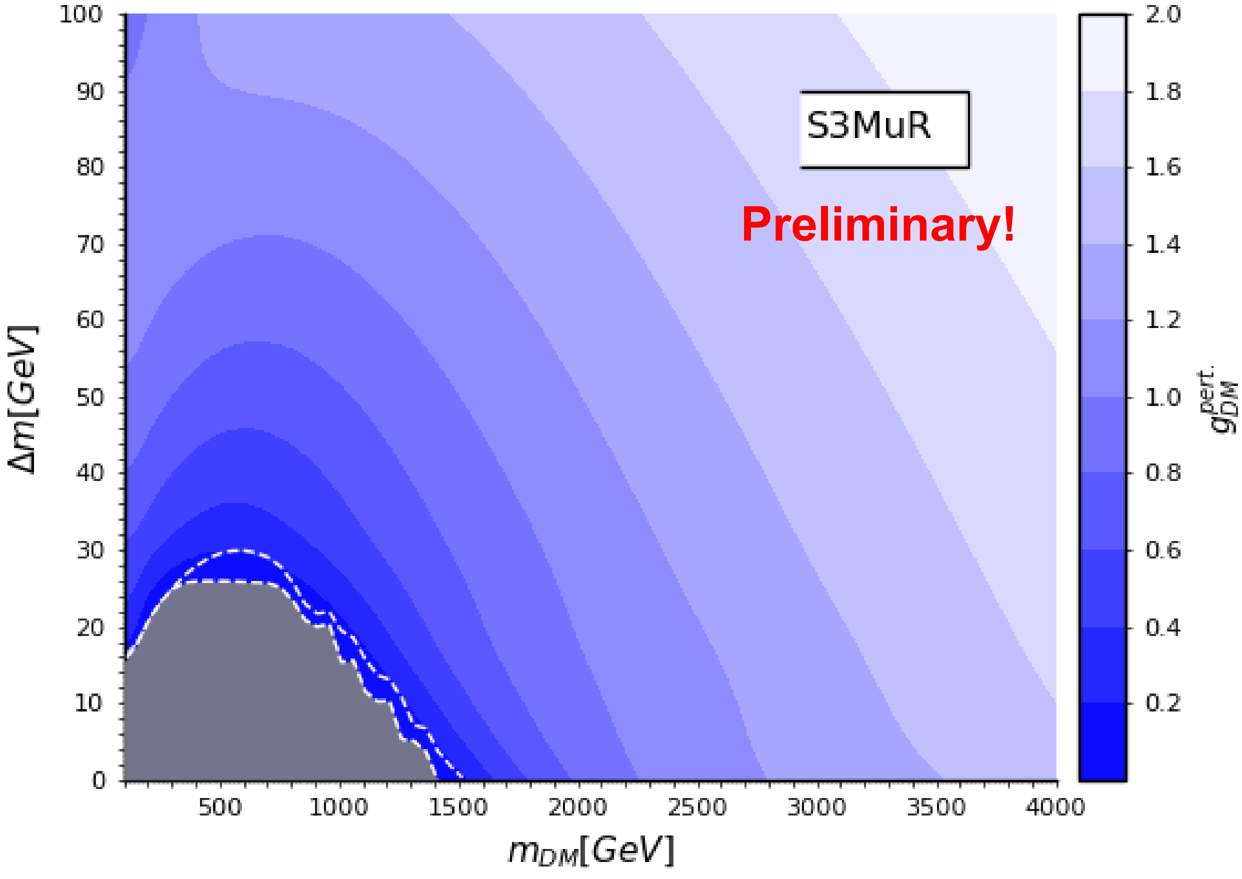
Bandscans on a logarithmic scale





Preliminary!

Perturbative results for S3MuR



Relative impact of non-perturbative effects for S3MuR