WHITE DWARF COOLING: LEPTOPHILIC DARK PHOTONS

Based on **[arXiv: 2405.00094]** in collaboration with **Jaime Hoefken Zink**

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Instituto de Física Teórica UAM-CSIC

WHERE TO LOOK FOR BSM

- Many UV theories predict heavy new states with sizeable couplings (e.g. SUSY, GUTs, String Models, …)

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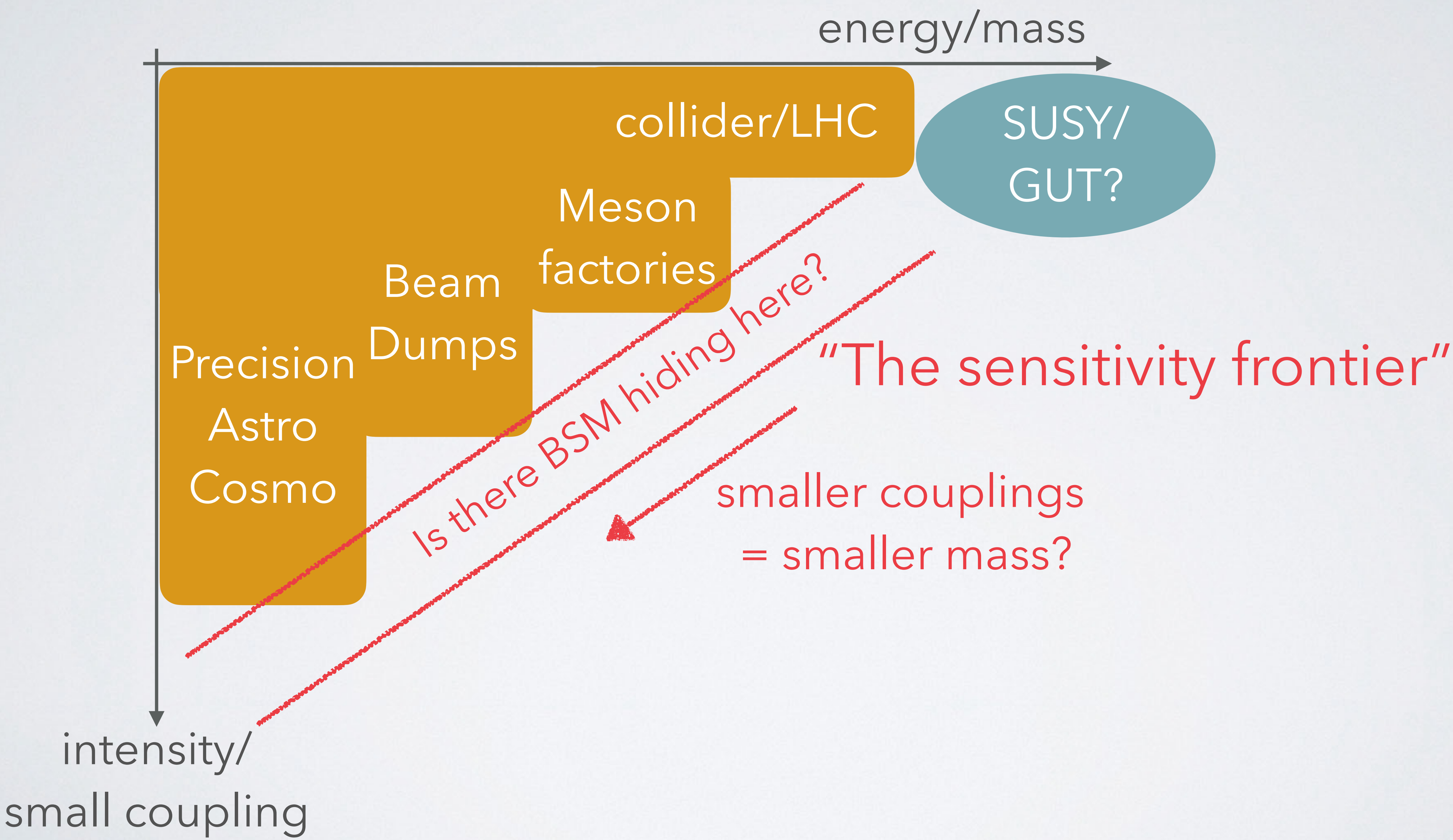

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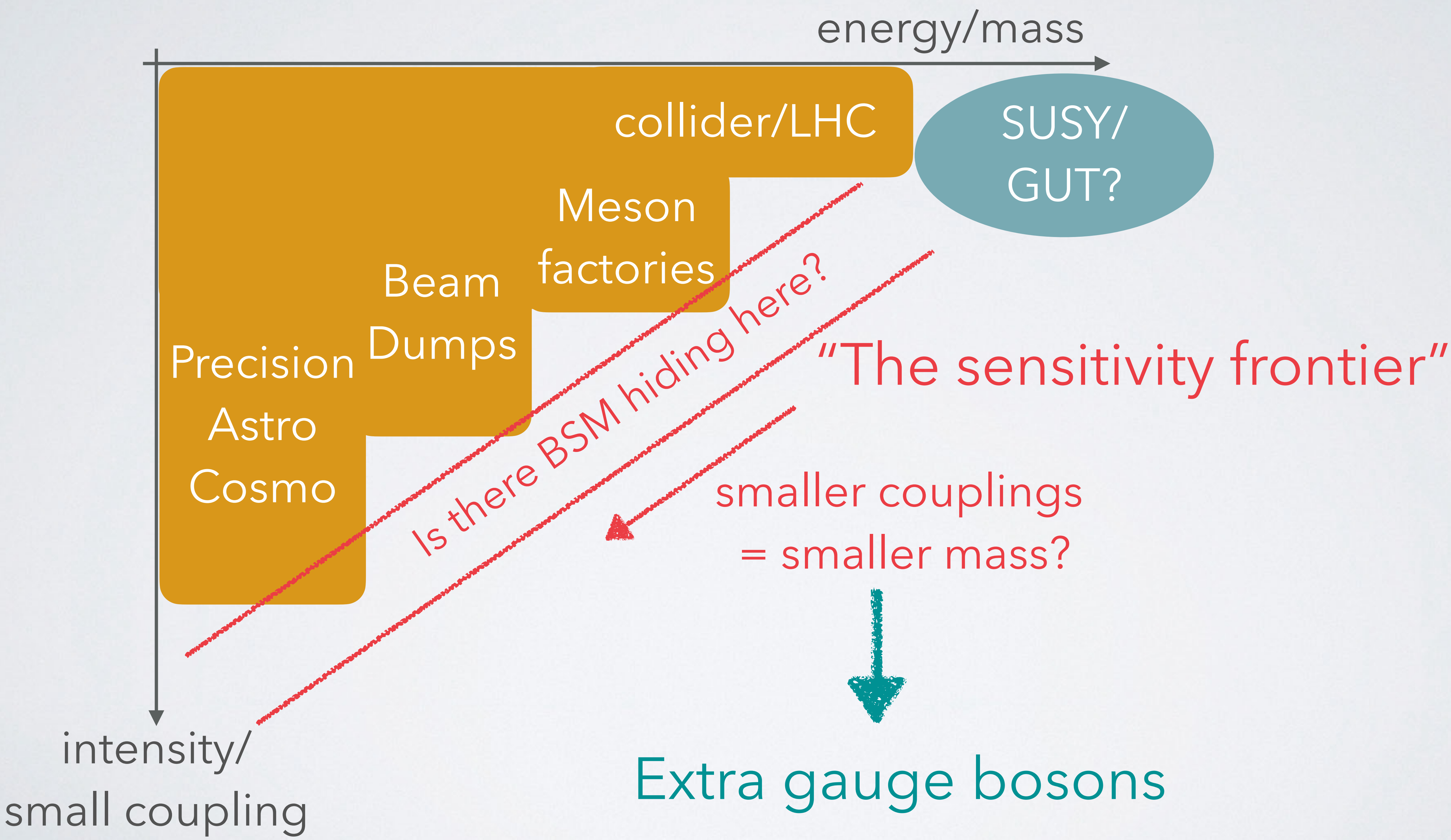


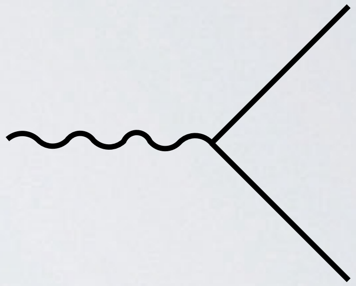
DARK PHOTONS

$$\mathcal{L} \supset -\frac{\epsilon_A}{2} F_{\mu\nu} X^{\mu\nu}$$

[Okun '82; Holdom '86]

- For light mediators $M_X \ll M_Z$ kinetic terms can be diagonalised by simple field redefinition:

$$A^\mu \to A^\mu - \epsilon_A X^\mu \quad \longrightarrow \quad e A_\mu J^\mu_{\mathrm{EM}} - \epsilon_A \, e X_\mu J^\mu_{\mathrm{EM}}$$



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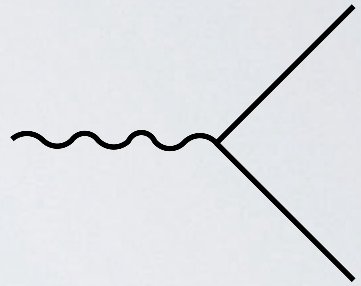

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- If $U(1)_X$ is broken by VEV f of scalar, mass is related to coupling:

$$\mathcal{L} = (D_\mu S)^\dagger D^\mu S \supset g_x^2 f^2\, X_\mu X^\mu$$

$$\Rightarrow \boxed{M_X = g_x f}$$

smaller couplings
$\Rightarrow$ smaller mass!

BEYOND THE MINIMAL

- SM fields can be charged under new $U(1)_X$

$$\mathcal{L}_{\text{int}} = -g_x \, J^\mu_X X_\mu \qquad J^\mu_X = \sum_\psi \bar{\psi} \, Q_\psi \, \gamma^\mu \psi \qquad \psi = Q, L, u, d, \ell, \nu$$

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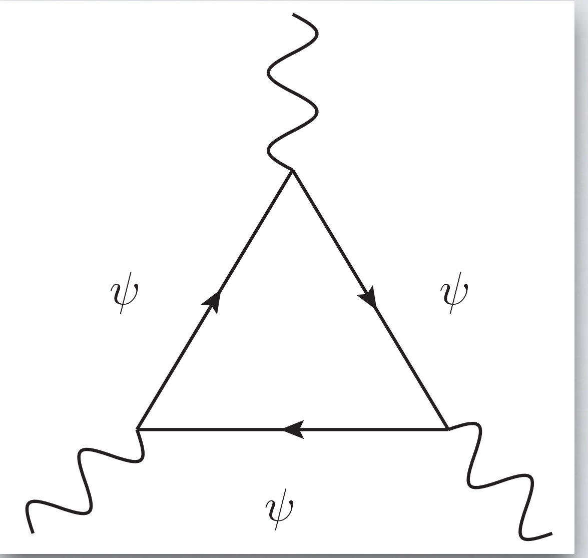
- SM Lagrangian has accidental global symmetries $U(1)_B, U(1)_{L_e}, U(1)_{L_\mu}, U(1)_{L_\tau}$.

- Four independent anomaly-free combinations:

$B - L$	$L_\mu - L_e$	$L_e - L_\tau$	$L_\mu - L_\tau$
charging quarks & leptons	charging 1st & 2nd generation leptons	charging 1st & 3rd generation leptons	charging 2nd & 3rd generation leptons



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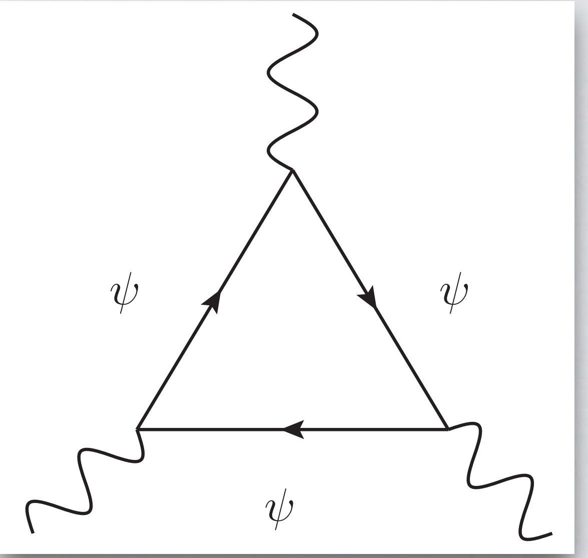
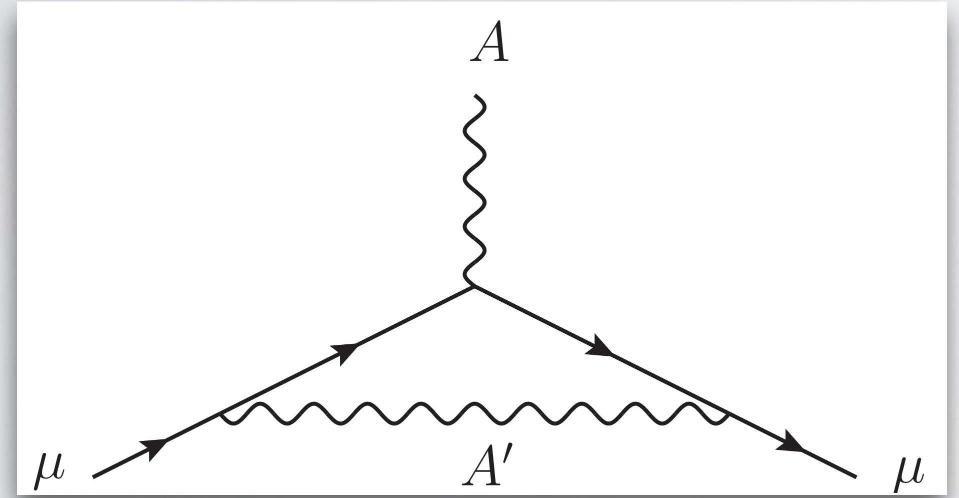


What can these do for us?

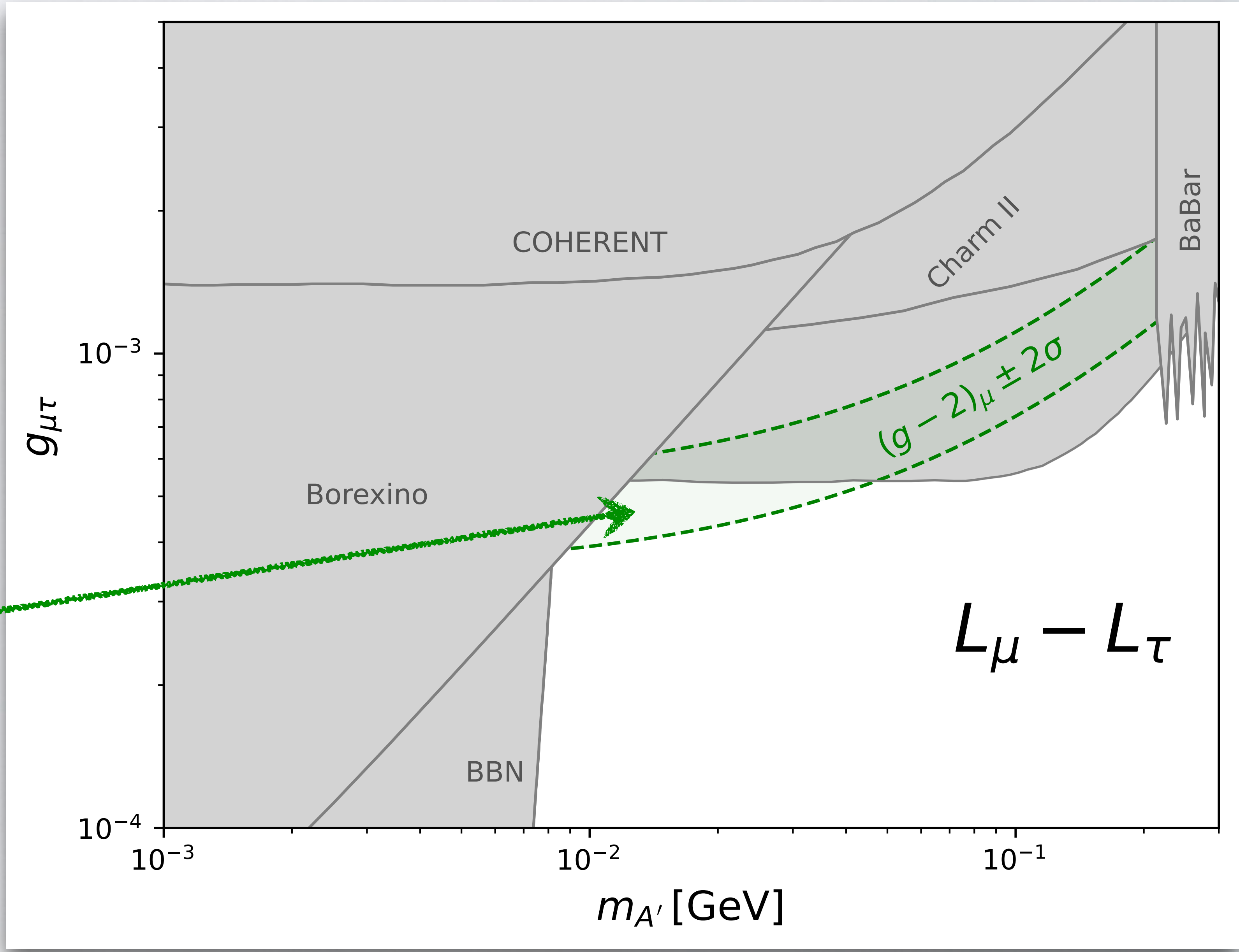
ANOMALOUS MAGNETIC MOMENT

- Muon-philic vectors contribute to $(g-2)_\mu$ at one-loop level

$$\Delta a_\mu = \frac{g_\mu^2}{4\pi^2}\int_0^1 \mathrm{d}u\ \frac{u^2(1-u)}{u^2+\frac{(1-u)}{x_\mu^2}}\,, \qquad \text{where} \quad x_\mu = m_\mu/M_{A'}$$



- In $U(1)_{L_\mu - L_\tau}$ this can still explain anomaly

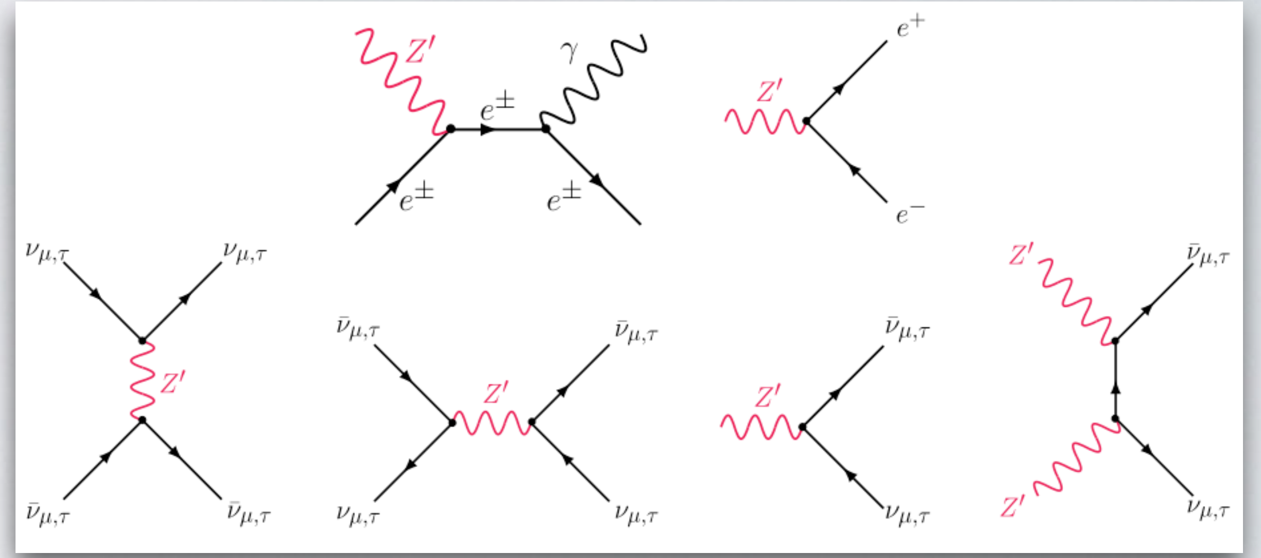


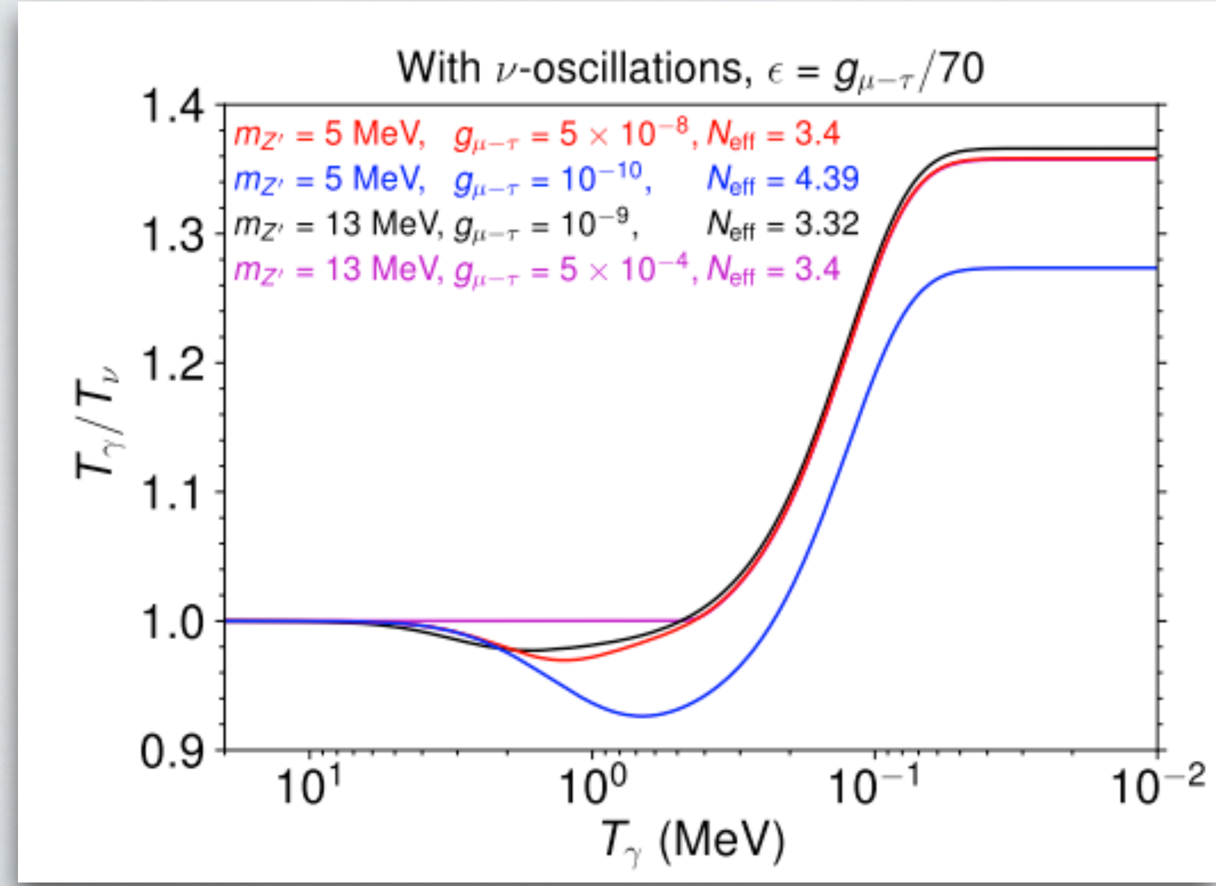
NEUTRINOS AND HUBBLE

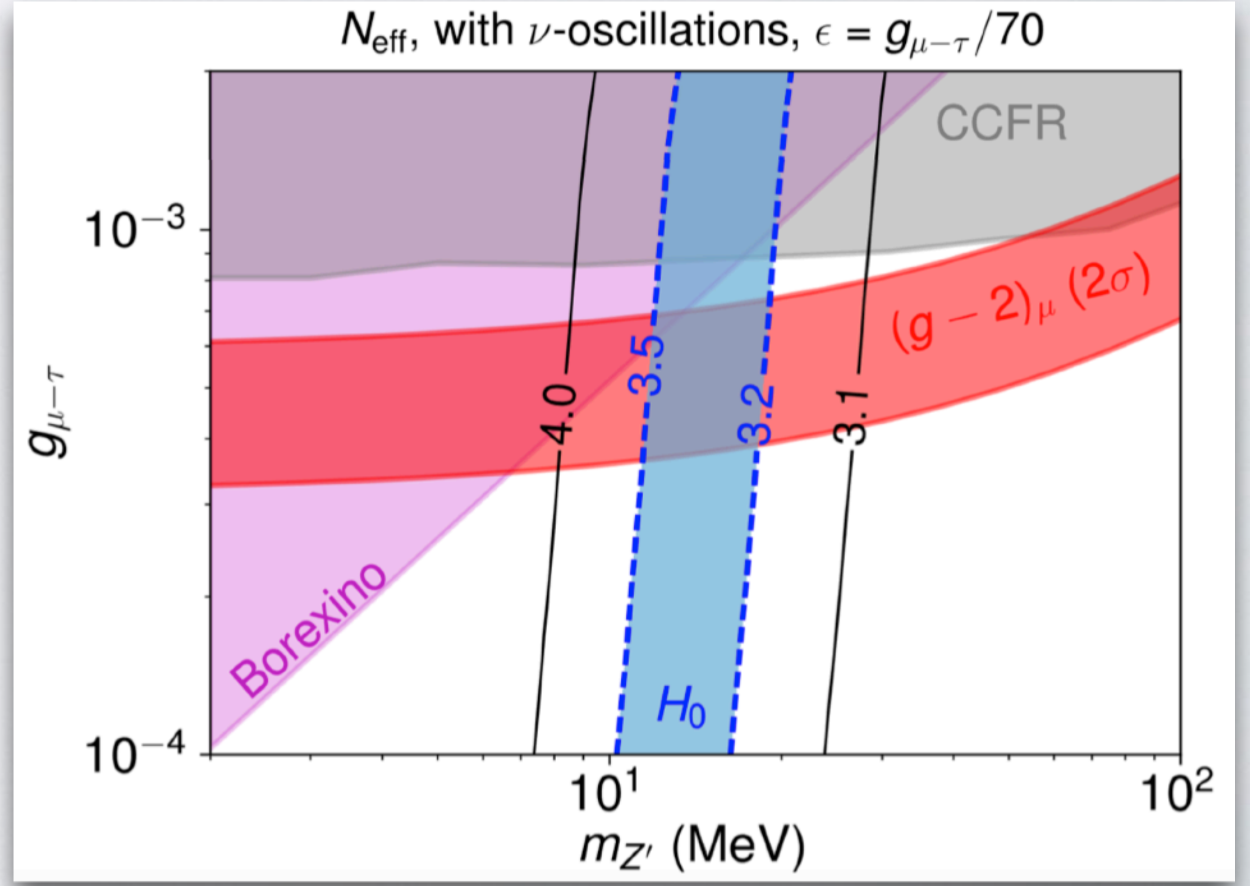
- Decay of A' heats neutrino gas and delays the decoupling

  $\Rightarrow$ increase of $N_{\rm eff}$ at early times

- Leads to larger H_0







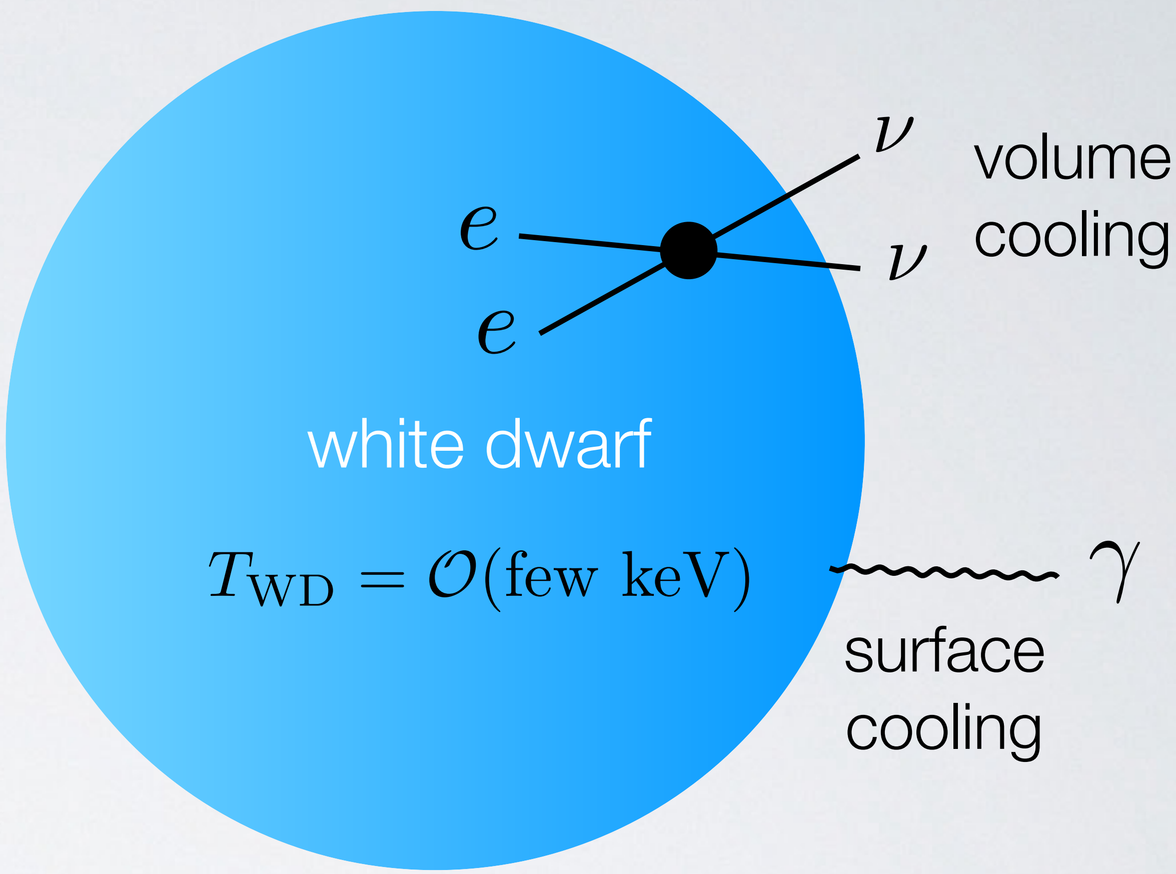
[Escudero, Hooper, Krnjaic, Pierre; JHEP 1903 (2019) 071]

Where could these explanations be tested??

WHITE DWARF COOLING

WHITE DWARFS

- WDs formed after normal star has exhausted fuel
- Only hot dense core of C and O
- Core supported by electron degeneracy pressure
  Mass of the sun, readius of the earth!
  ➞ Very dense: $\sim 10^6$ kg/m^3
  (solar core $\sim 10^5$ kg/m^3)



- Star cools down over billions of years via photons and neutrinos:

$$\frac{dT_{WD}}{dt} = -\frac{L_\gamma}{4\pi R_{WD}\sigma_{SB}T_{WD}} - \frac{L_\nu}{4\pi R_{WD}\sigma_{SB}T_{WD}}$$

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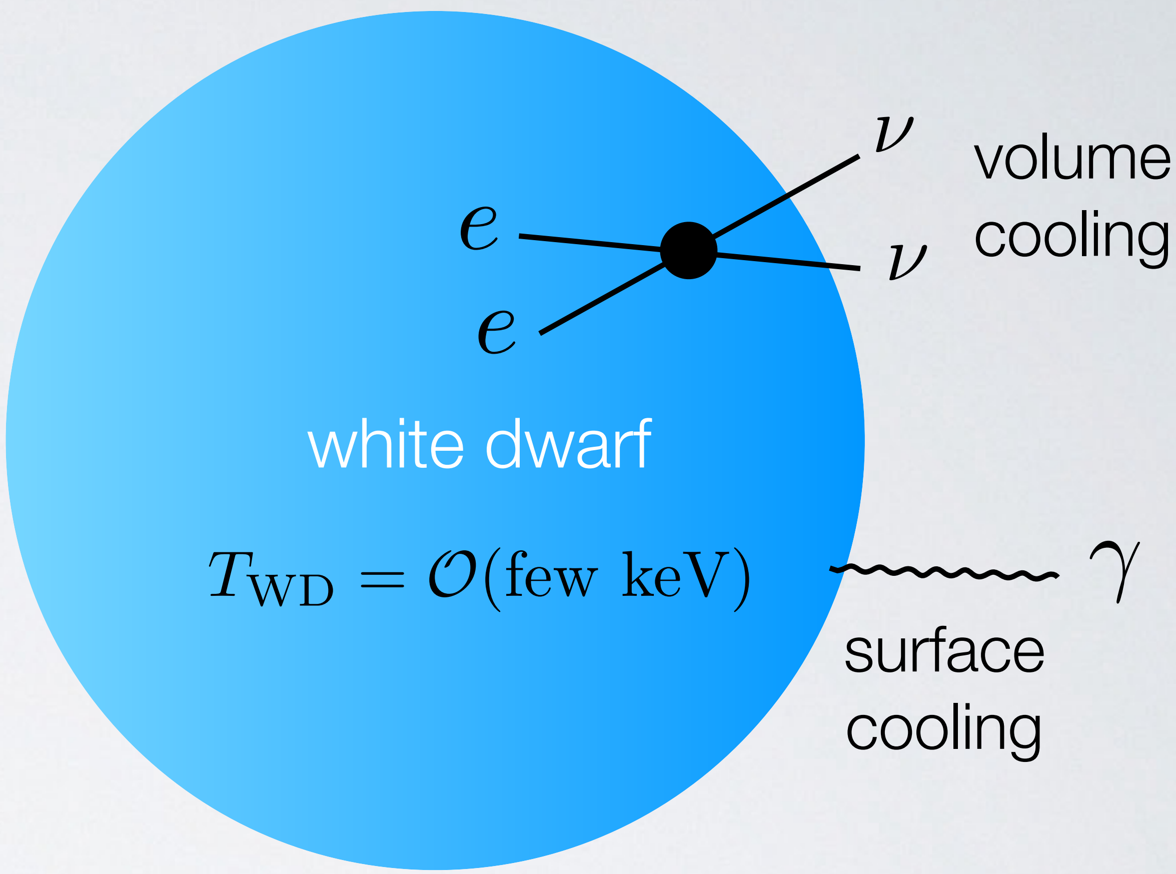


- Star cools down over billions of years via photons and neutrinos:

$$\frac{dT_{WD}}{dt} = -\frac{\overset{\text{COLD}}{L_\gamma}}{4\pi R_{WD}\sigma_{SB}T_{WD}} - \frac{\overset{\text{HOT}}{L_\nu}}{4\pi R_{WD}\sigma_{SB}T_{WD}}$$

EQUATION OF STATE

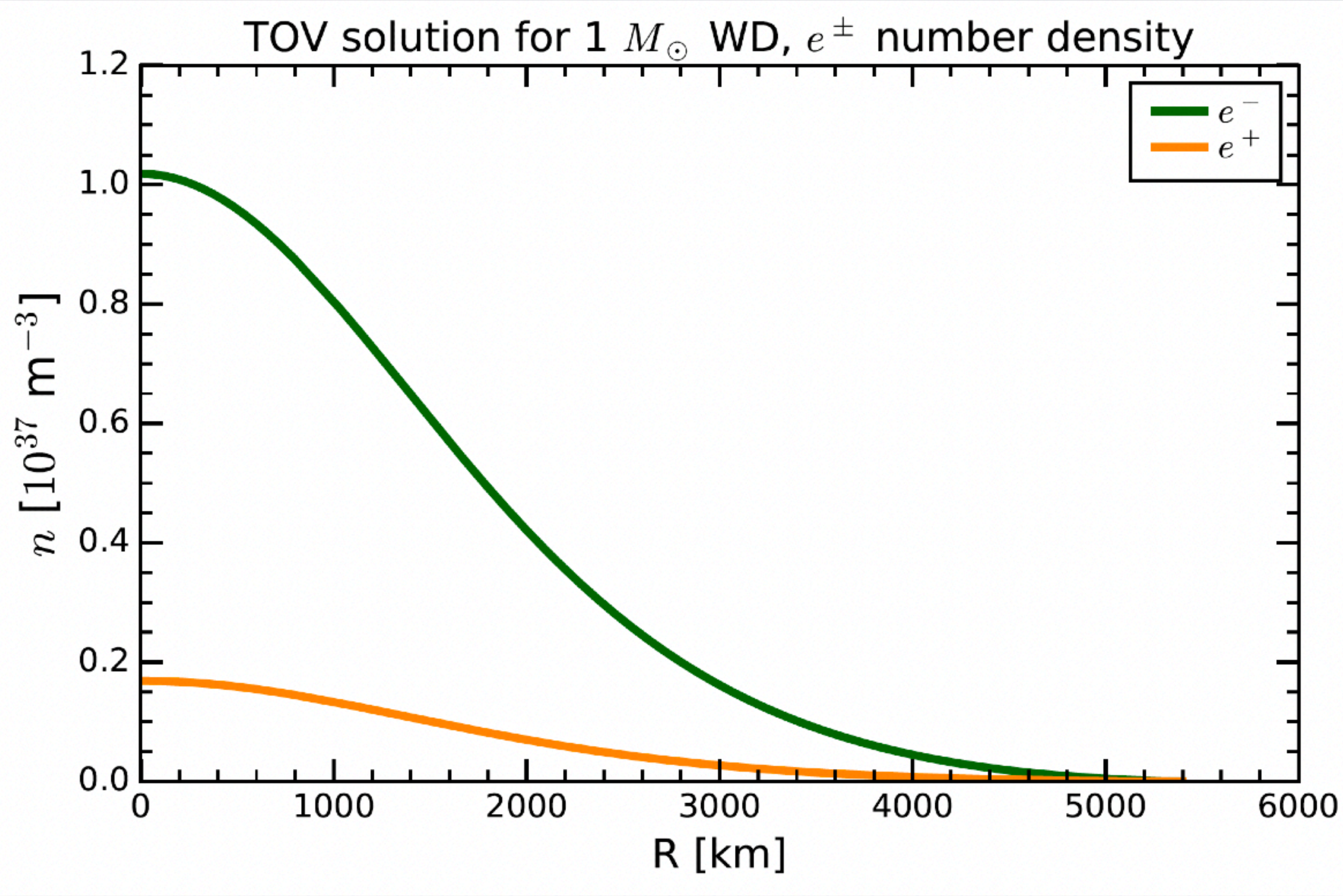
- **EoS of White Dwarfs well-known:**

- Salpeter EoS:
  degenerat ideal gas
  + corrections (non-uniformity, Coulomb potential, …)
  [Salpeter; *Astrophys. J. 134, 669 (1961)*]

- **Tolman-Oppenheim-Volkoff (TOV) equations:** solving the Einstein field equations in Schwarzschild metric with fluid

$$\frac{dp(r)}{dr} = -G\frac{\epsilon(r)+p(r)}{r(r-2G\,m(r))}\left[m(r)+4\pi\,p(r)\,r^3\right]$$

$$\frac{dm(r)}{dr} = 4\pi\epsilon(r)\,r^2$$



Extract density profiles

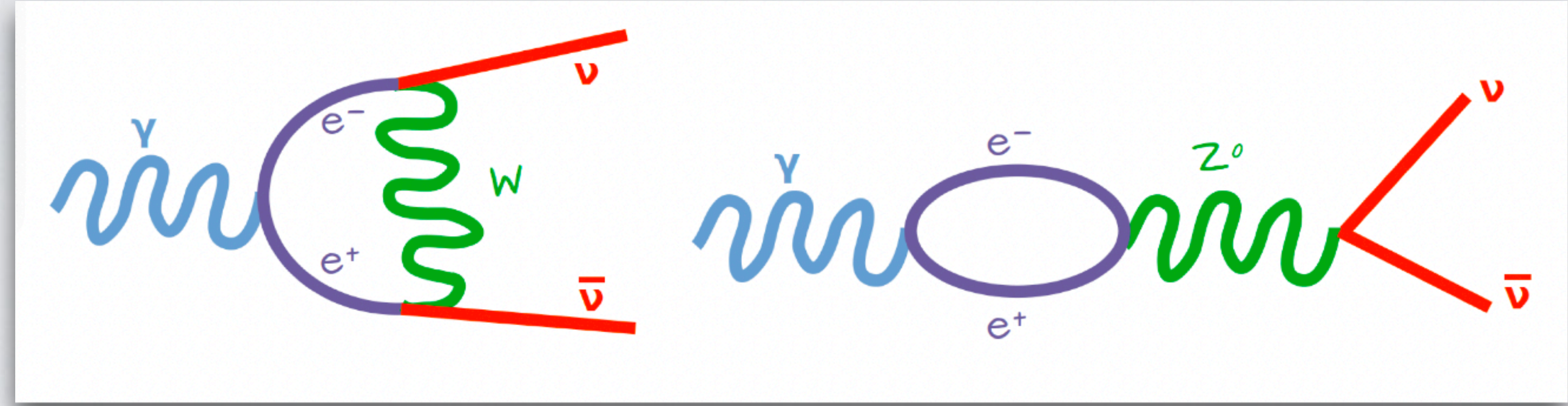
[Tolman, *Phys. Rev., 55, 364 (1939)*]

[Oppenheimer & Volkoff, *Phys. Rev., 55, 374 (1939)*]

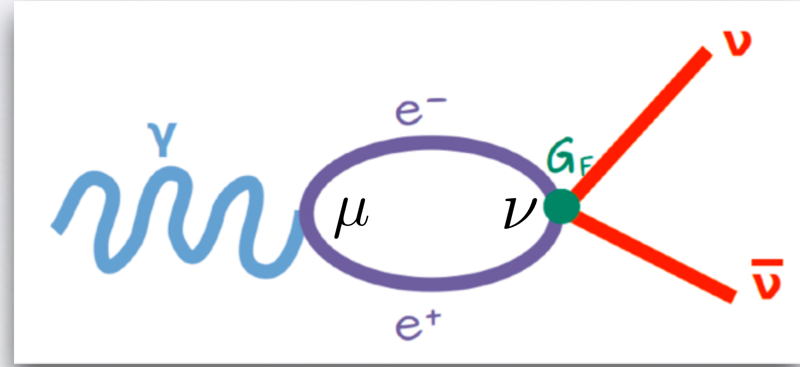
[Mathew & Nandy, *Res. Astron. Astrophys.* **17** 061]

PLASMON DECAY

- Early WD cooling via ``on-shell" photon decay in plasma into neutrinos



- Since in WDs the typical $q^2 \ll M_W^2, M_Z^2$ we can compute this as



$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\Gamma_\lambda^{\mu\nu} \, \varepsilon_\mu(\boldsymbol{q}, \lambda) \right) \, \bar{u}(p_1)\gamma_\nu(1-\gamma_5)u(p_2)$$

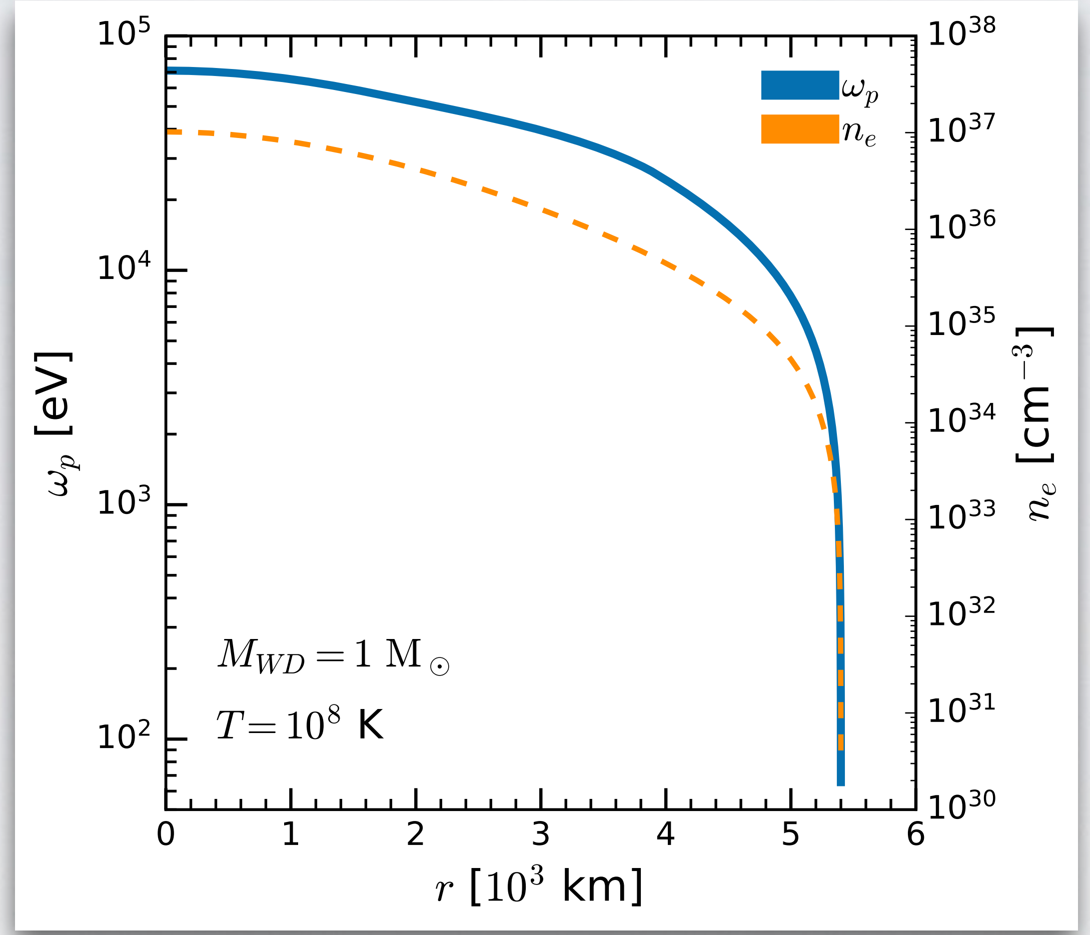
with effective vertex $\Gamma_\lambda^{\mu\nu}$ for each photon polarization with couplings C_V^{SM}, C_A^{SM}

[Braaten & Segel; *Phys.Rev.D* 48 (1993) 1478]

WD NEUTRINO LUMINOSITY

- **Plasmon decay width** in terms of effective vertex $\Gamma^{\mu\nu}_{\lambda}$ and **plasmon frequencies** $\omega_\lambda(q)$.

$$\Gamma_\lambda(q) = -\frac{G_F^2}{12\pi}\frac{\omega_\lambda(q)^2 - q^2}{\omega_\lambda(q)}\left(\Gamma^{\alpha\mu}_{\lambda}\varepsilon_\mu(q,\lambda)\right)\left(\Gamma^{\lambda}_{\alpha\rho}\varepsilon^\rho(q,\lambda)\right)^*$$



- **Neutrino emissivity & total luminosity:**

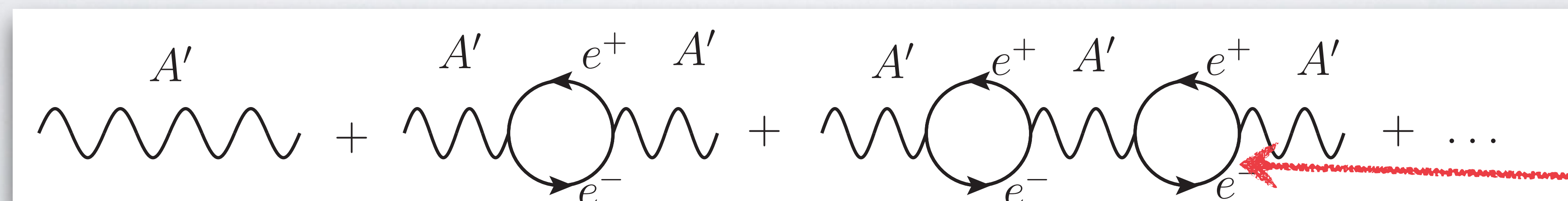
$$\mathcal{Q}_\lambda \equiv \int d^3\vec{q}\ \ \Gamma_\lambda(q)\ \omega_\lambda(q)\ n_B\left(\omega_\lambda(q), T\right) \qquad L_\nu = 4\pi\int_0^{R_{\mathrm{WD}}}\sum_\lambda \mathcal{Q}_\lambda(r) r^2 dr$$

PLASMON DECAY - DARK PHOTONS

- Leptophilic dark photons contribute

- Since **dark photon couples to plasma electrons** have to compute full thermal propagtor (Dyson sum)

$D^{\mu\nu}_{A'} =$



Thermal loops!

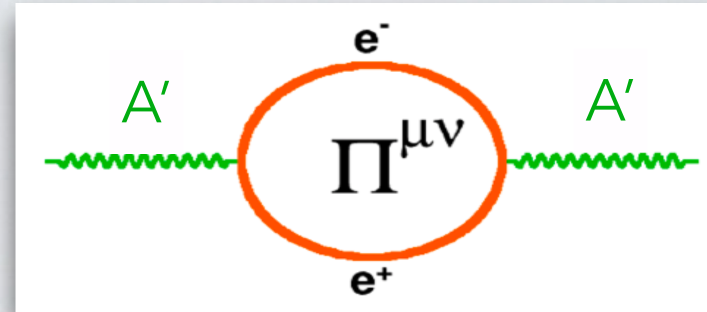
$$= \frac{-i\,(g^{\mu\nu} - q^{\mu}q^{\nu}/m^2_{A'})}{q^2 - m^2_{A'}} + \frac{-i\,(g^{\mu}_{\lambda} - q^{\mu}q_{\lambda}/m^2_{A'})}{q^2 - m^2_{A'}}\,(i\,\Pi^{\lambda\sigma}_{A'})\,\frac{-i\,(g^{\nu}_{\sigma} - q_{\sigma}q^{\nu}/m^2_{A'})}{q^2 - m^2_{A'}} + \ldots$$

$$= \frac{-i\,g^{\mu\lambda}}{q^2 - m^2_{A'} - F_{A'}}\,P^{\nu}_{L\lambda} + \frac{-i\,g^{\mu\lambda}}{q^2 - m^2_{A'} - G_{A'}}\,P^{\nu}_{T\lambda}$$

Longitudinal & transverse components

with $F_{A'} = \frac{q^2}{\boldsymbol{q}^2}\,\Pi^{00}_{A'}$ $\quad G_{A'} = \Pi^{xx}_{A'}$

DARK PHOTON SELF ENERGY



- Evaluate A' self-energy in **thermal background — a beast!**

$$\Pi^{\mu\nu}_{A'}(q) = -\,\epsilon_A^2 \int \frac{d^4k}{(2\pi)^4}\, \mathrm{tr}\big[\gamma^\mu(\not{k}+m_e)\gamma^\nu(\not{q}-\not{k}-m_e)\big]$$

$$\times \left\{ \frac{i}{k^2-m_e^2} - 2\pi\big[\theta(-k^0)+\mathrm{sign}(k^0)\,\tilde{f}(k^0-\mu)\big]\delta\big(k^2-m_e^2\big) \right\}$$

$$\times \left\{ \frac{i}{(q-k)^2-m_e^2} - 2\pi\big[\theta(-q^0+k^0)+\mathrm{sign}(q^0-k^0)\,\tilde{f}(q^0-k^0+\mu)\big]\delta\big((q-k)^2-m_e^2\big) \right\}$$

with $\tilde{f}(x) = (1+e^{\beta x})^{-1}$

Thermal fermion propagators

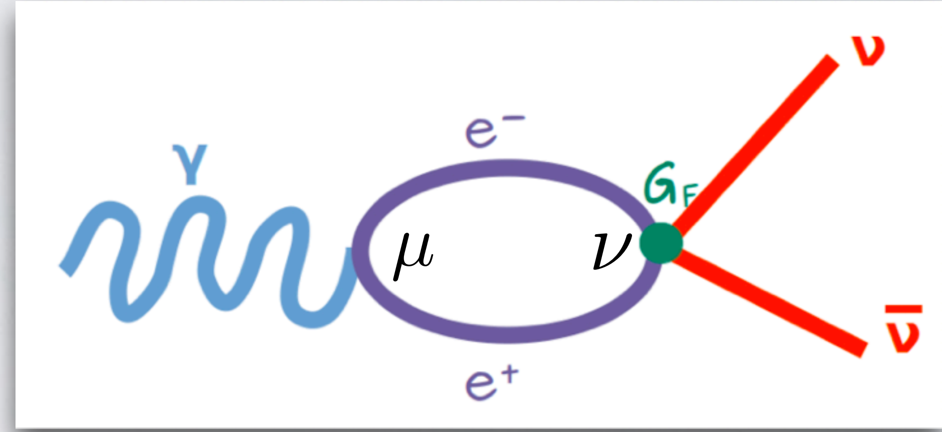
- **But, this is essentially the plasmon self-energy**: $\epsilon_A^2 \times \Pi_\gamma^{\mu\nu}$!

Identify $F_{A'} = \dfrac{q^2}{\boldsymbol{q}^2}\,\Pi^{00}_{A'} = \epsilon_A^2\,\dfrac{q^2}{\boldsymbol{q}^2}\,\Pi^\gamma_L \qquad G_{A'} = \Pi^{xx}_{A'} = \epsilon_A^2\,\Pi^\gamma_T$ with known results!

[Hoefken Zink, Ramirez-Quezada; *PRD* 108 (2023) 4, 043014]

PLASMON DECAY - DARK PHOTONS

- Repeat computation as before



but shifting the couplings by the A' coupling and full propagator:

$$C_{V,L}(q) \to C_V^{SM} + \frac{\sqrt{2}}{2\,G_F}\frac{e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - F_{A'}} \qquad \Pi_L^\gamma$$

$$C_{V,T}(q) \to C_V^{SM} + \frac{\sqrt{2}}{2\,G_F}\frac{e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - G_{A'}}$$

$$\Pi_T^\gamma$$

$$C_A(q) \to C_A^{SM} - \frac{\sqrt{2}}{16\,G_F}\frac{\tan^2\theta_W\;e\,\epsilon_A\,g_x\,Q_{\nu_\alpha}}{q^2 - m_{A'}^2 - G_{A'}}$$

THREE REGIMES

- **Heavy regime** $(m_{A'} \gg T, \omega_P)$

$$\frac{1}{q^2 - m_{A'}^2} \sim \frac{-1}{m_{A'}^2}$$

- **Ultra-light regime** $(m_{A'} \ll T, \omega_P)$:

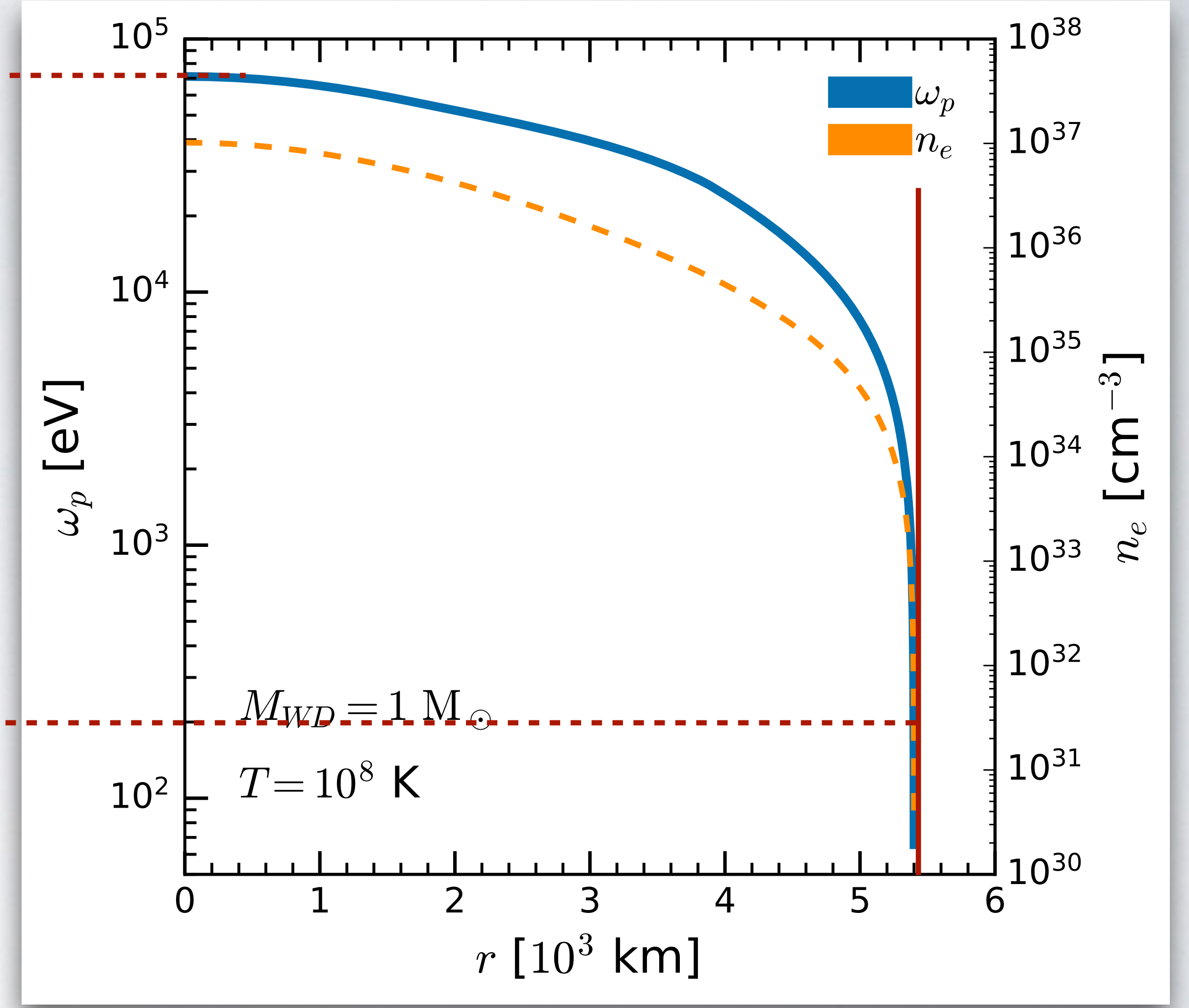
$$\frac{1}{q^2 - m_{A'}^2} \sim \frac{1}{q^2}$$

Mass independent!

- **Resonant regime** $(m_{A'} \sim T, \omega_P)$:
  - dark photon goes on resonance w/ plasma frequency $\omega_P(r)$!
  - *regulate pole via Breit-Wigner propagator!*

100 keV

200 eV



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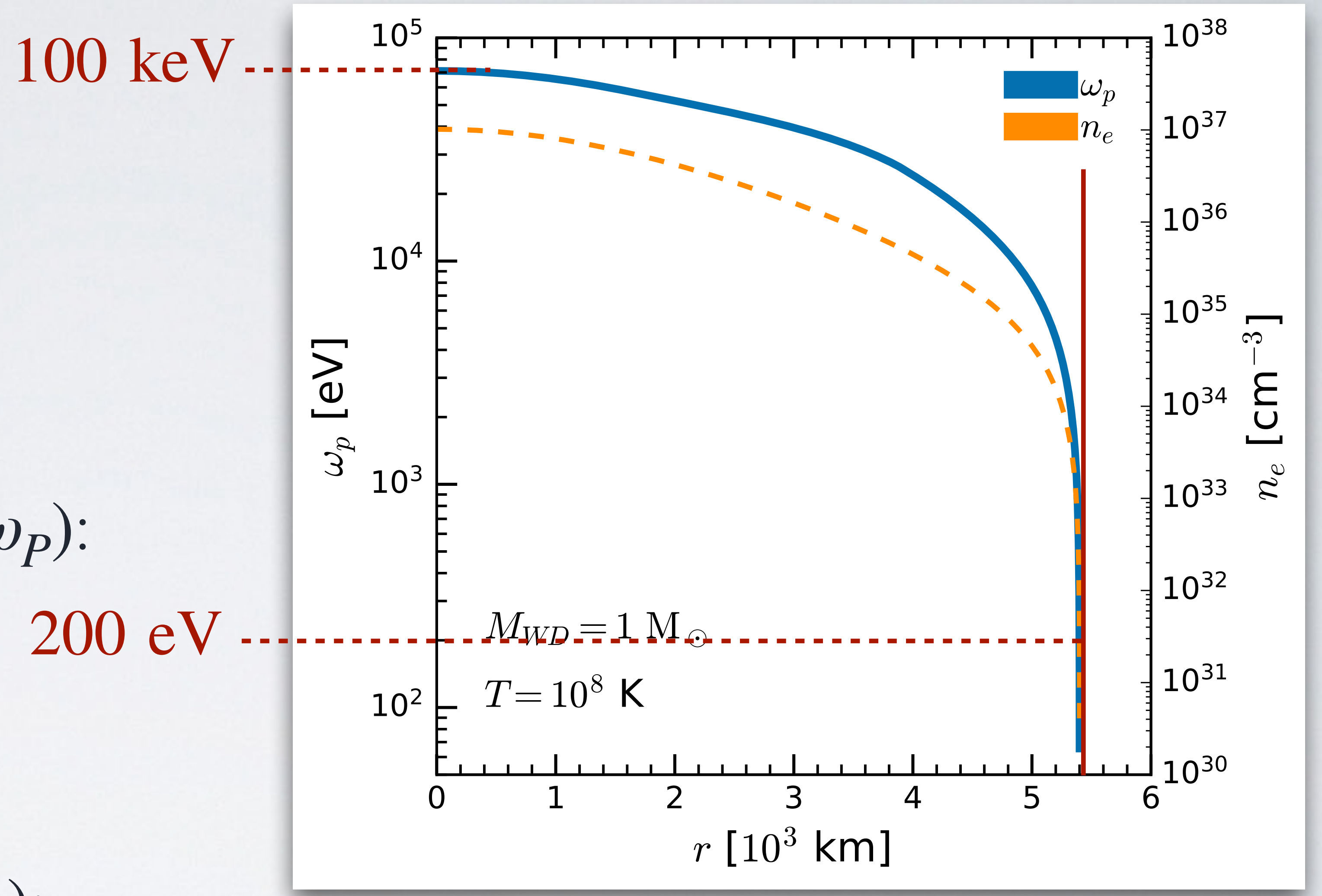

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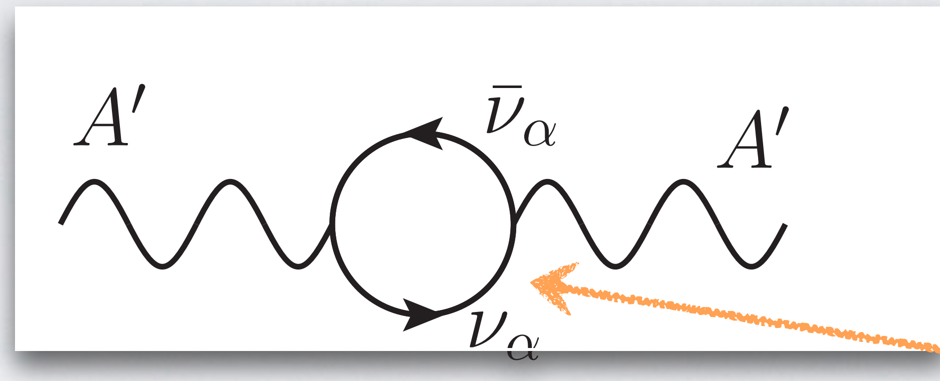
$$G_{\mathrm{BW}}^{\mu\nu}(q^2) = \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{q^2 - m^2 - \mathrm{Re}(F) - i\,\mathrm{Im}(F)} P_{L\lambda}^{\nu} + \frac{-i(g^{\mu\lambda} - q^\mu q^\lambda/m^2)}{q^2 - m^2 - \mathrm{Re}(G) - i\,\mathrm{Im}(G)} P_{T\lambda}^{\nu}$$

BREIT WIGNER REGULATOR

- Compute the imaginary part of dark photon self-energy
  → In resonant region only due to neutrinos



Neutrinos are non-thermal!

- We find the typical relation

$$\bar{\Pi}^{\mu\nu}_{A'}(q^2) = -\frac{\left(k^\alpha_\nu\right)^2}{4\pi^2} q^2 g^{\mu\nu} \int_0^1 dx\, x\,(1-x)\ \log\left(\frac{m_\alpha^2}{m_\alpha^2 - x(1-x)q^2}\right)$$

- So the regulators

$$\mathrm{Im}(\bar{\Pi}^{\mu\nu}_{A'})(q^2) = \frac{\left(k^\alpha_\nu\right)^2}{24\pi} \frac{(\omega_l^2 - \boldsymbol{q}^2)^2}{\boldsymbol{q}^2} P_L^{\mu\nu} - \frac{\left(k^\alpha_\nu\right)^2}{24\pi}(\omega_t^2 - \boldsymbol{q}^2) P_T^{\mu\nu}$$

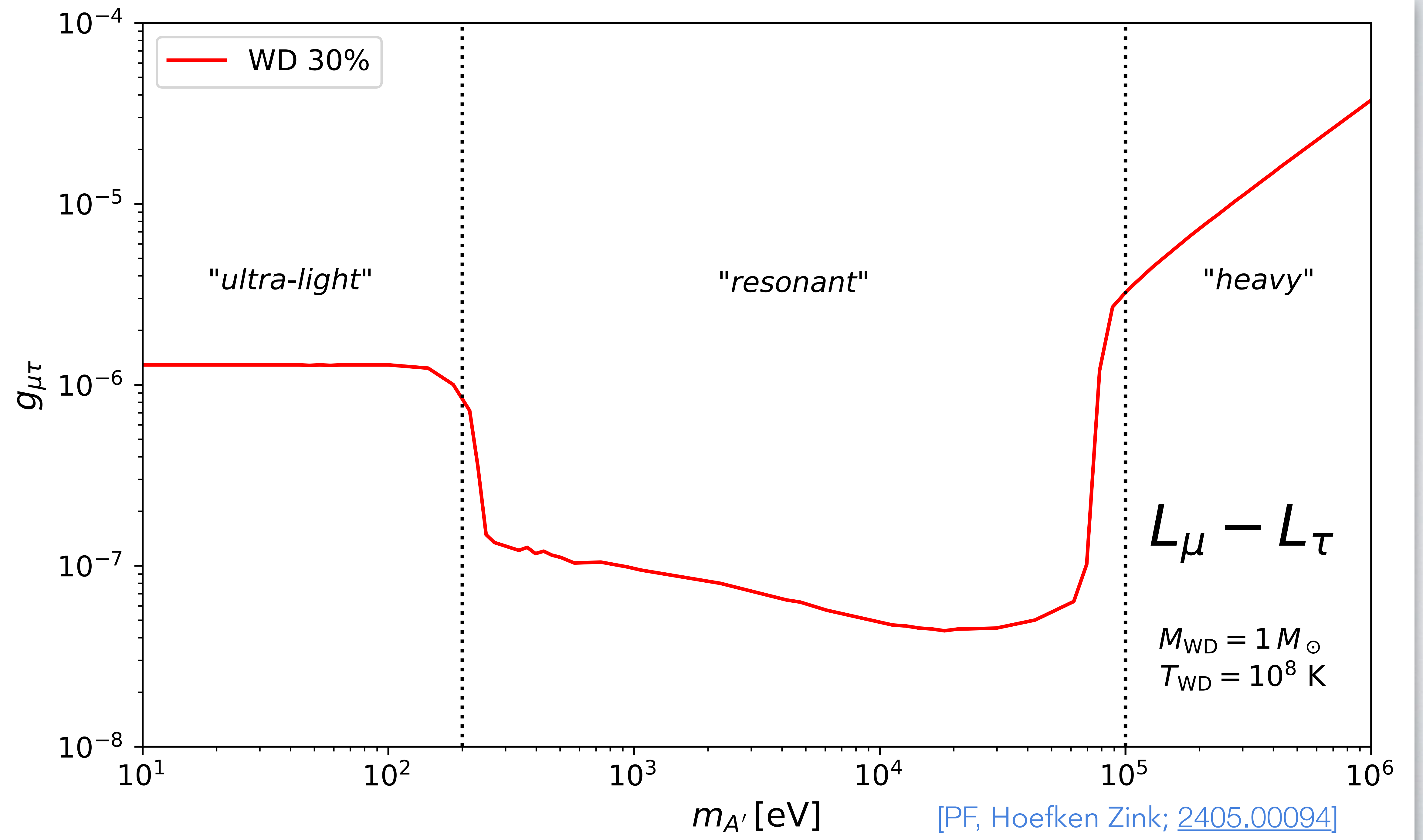
$\mathrm{Im}(F)$ $\mathrm{Im}(G)$

WD COOLING SENSITIVITIES

- Fraction of extra cooling $\varepsilon^{\mathrm{BSM}} = L_\nu^{\mathrm{BSM}}/L_\nu^{\mathrm{SM}} - 1$
- Existing bounds at 70% extra cooling @ 95% CL

[Hansen et al., *Astrophys. J. 809 (2015) no. 2, 141*]

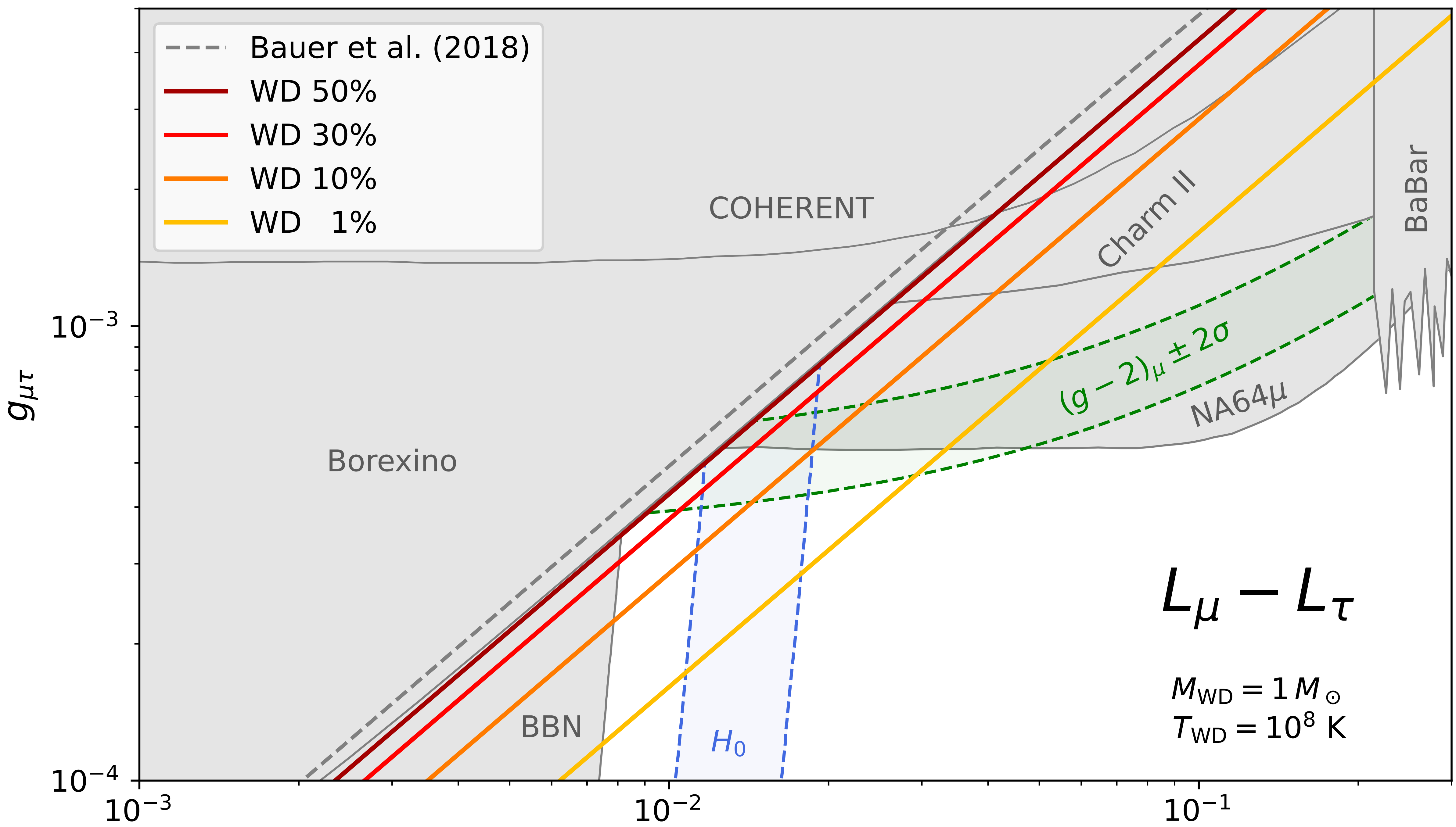
Finally some plots!



[PF, Hoefken Zink; 2405.00094]

WD COOLING & $(g-2)_\mu$

- Sensitivity to unprobed parameter space at 50% sensitivity or better



[PF, Hoefken Zink; 2405.00094]

CONCLUSIONS

- Neutrino cooling of White Dwarfs is sensitive laboratory for (light) leptophilic mediators
- **Full computation of A′ induced plasmon decay in resonant domain**
- Already at current sensitivities WD cooling excludes unconstraint parameter space of $U(1)_{L_\mu - L_\tau}$
- **Improved measurements of hot WD neutrino luminosity function can exclude $(g-2)_\mu$ explanation within $U(1)_{L_\mu - L_\tau}$!**
- For all the fun details ask me and check out our paper :)

[arXiv:2405.00094]

MUITO OBRIGADO!

BACKUP

PLASMON PROPAGATOR

- Photon in plasma is on-shell with plasmon frequencies $\omega_\lambda(q)$
- Can extract field strength normalisations $Z_l(q)$ & $Z_t(q)$

Longitudinal: $D^{00} = \frac{1}{q^2 - \Pi_L(Q)}$

$$\lim_{q_0 \to \omega_l(q)} D^{00} = \frac{\omega_l^2(q)}{q^2} \frac{Z_l(q)}{q_0^2 - \omega_l(q)^2}$$

Transverse: $D^{xx} = \frac{1}{q_0^2 - q^2 - \Pi_T(Q)}$

$$\lim_{q_0 \to \omega_t(q)} D^{xx} = \frac{Z_t(q)}{q_0^2 - \omega_t(q)^2}$$

Solution

$$Z_l(q) = \frac{q^2}{\omega_l(q)^2} \left[-\frac{\partial \Pi_L}{\partial q_0^2} (\omega_l(q), q) \right]^{-1}$$

$$Z_t(q) = \left[1 - \frac{\partial \Pi_T}{\partial q_0^2} (\omega_t(q), q) \right]^{-1}$$

[J. Hoefken Zink]

PLASMON PROPAGATOR

The residue of a pole in q_0^2 of $D^{\mu\nu}(q_0, q)$ can be identified with $\varepsilon^\mu(q)\varepsilon^\nu(q)^*$. So we have:

$$\mathrm{Res}\, D^{00} = \mathrm{Res}\left(\frac{\omega_l(q)^2}{q^2}\frac{Z_l(q)}{q_0^2 - \omega_l(q)^2}\right) = \frac{\omega_l(q)^2}{q^2} Z_l(q)$$

$$\mathrm{Res}\, D^{xx} = \mathrm{Res}\left(\frac{Z_t(q)}{q_0^2 - \omega_t(q)^2}\right) = Z_t(q)$$

From these expressions, we can find the polarization 4-vectors:

$$\varepsilon^\mu(q, \lambda = 0) = \frac{\omega_l(q)}{q}\sqrt{Z_l(q)}(1, 0)^\mu$$

$$\varepsilon^\mu(q, \lambda = \pm 1) = \sqrt{Z_t(q)}(0, \varepsilon_\pm(q))^\mu$$

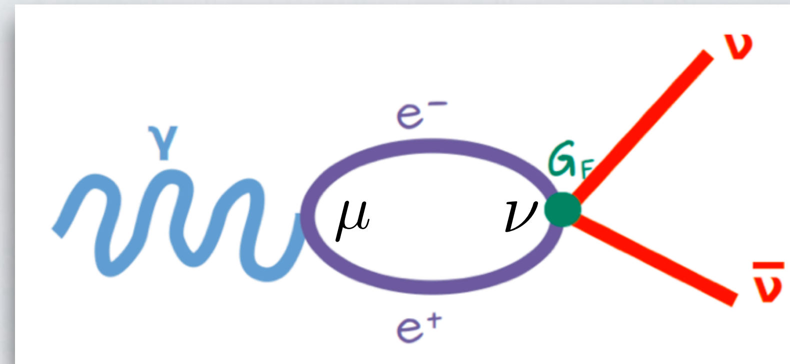
[J. Hoefken Zink]

- Obtain dispersion relations

$$\omega_l(q)^2 = \frac{\omega_l(q)^2}{q^2}\Pi_L(\omega_l(q), q)$$

$$\omega_t(q)^2 = q^2 + \Pi_T(\omega_t(q), q)$$

PLASMON DECAY AMPLITUDE



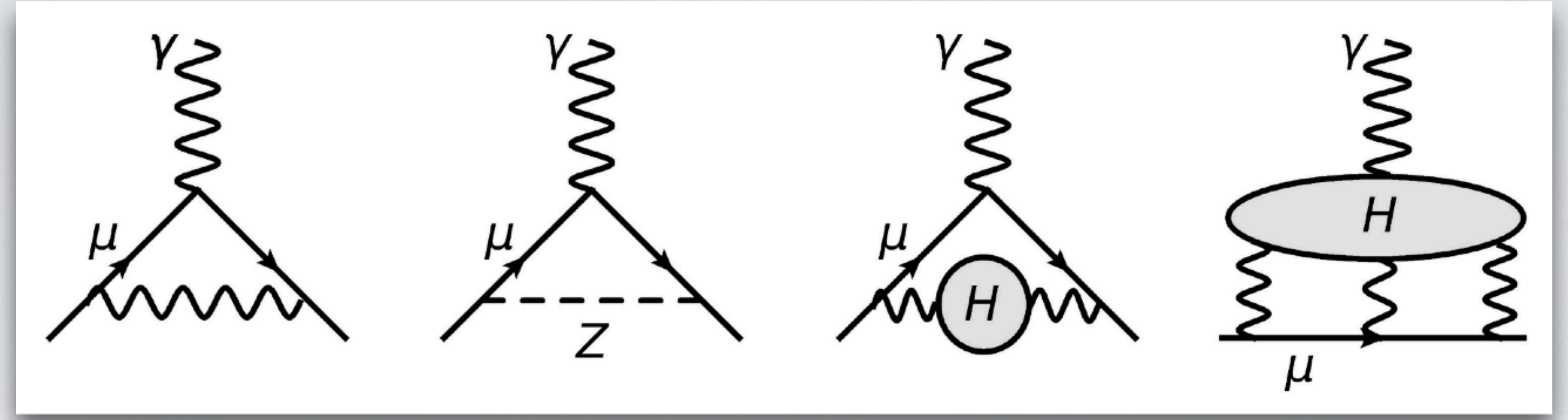
- In effective theory can write the plasmon decay amplitude in the SM as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \frac{1}{\sqrt{4\pi\alpha}} \Bigg[\varepsilon_\mu(\omega_l, q) C_V \left(\Pi_L(\omega_l, q) \left(1, \frac{\omega_l}{q}\hat{q}\right)^\mu \left(1, \frac{\omega_l}{q}\hat{q}\right)^\nu \right)$$
$$+ \varepsilon_\mu(\omega_t, q) g^{\mu i} \Big(C_V \Pi_T(\omega_t, q) \left(\delta^{ij} - \hat{q}^i \hat{q}^j\right)$$
$$+ C_A \Pi_A(\omega_t, q)(i\varepsilon^{ijm}\hat{q}^m) \Big) g^{\nu j} \Bigg] \overline{u}(p_1)\gamma_\nu(1-\gamma_5)v(p_2)$$

- Write this in terms of effective vertex $\Gamma^{\mu\nu}_\lambda$ as function of couplings C_V & C_A

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\Gamma^{\mu\nu}_\lambda \, \varepsilon_\mu(\boldsymbol{q}, \lambda) \right) \bar{u}(p_1)\gamma_\nu(1-\gamma_5)u(p_2)$$

MUON MAGNETIC MOMENT



- The theoretical prediction for $(g-2)_\mu$ within the SM has been determined to

$$a_\mu^{\mathrm{SM}} = 116\ 591\ 810(43) \times 10^{-11}$$

[Aoyama et al; *Phys.Rept.* 887 (2020) 1-166]

- The recent Fermilab E989 result

$$a_\mu^{\mathrm{FNAL}} = 116\ 592\ 059(22) \times 10^{-11}$$

[MUON g-2; *PRL* 131 (2023), 161802]

when combined with the previous BNL results leads to the **5.2σ excess** of

$$\Delta a_\mu = 249(48) \times 10^{-11}$$

FULL MIXING LAGRANGINA

- Starting from the low-energy Lagrangian

One-loop corrections: vacuum polarization

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}, Z_{\mu\nu}, X_{\mu\nu}\right)\left[\begin{pmatrix} 1 & 0 & \epsilon_A \\ 0 & 1 & \epsilon_Z \\ \epsilon_A & \epsilon_Z & 1 \end{pmatrix} + \mathbf{\Pi}\right]\begin{pmatrix} F^{\mu\nu} \\ Z^{\mu\nu} \\ X^{\mu\nu} \end{pmatrix}$$

- Diagonalise tree-level kinetic terms to get effective one-loop action

vacuum polarizations in canonical normalisation

$$\left.\begin{aligned} \mathbf{\Pi} &= \begin{pmatrix} \Pi_{\gamma\gamma} & \Pi_{\gamma Z} & \Pi_{\gamma X} \\ \Pi_{\gamma Z} & \Pi_{ZZ} & \Pi_{ZX} \\ \Pi_{\gamma X} & \Pi_{ZX} & \Pi_{XX} \end{pmatrix} \\ G &= \begin{pmatrix} 1 & 0 & -\frac{\epsilon_A}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \\ 0 & 1 & -\frac{\epsilon_Z}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \\ 0 & 0 & \frac{1}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \end{pmatrix} \end{aligned}\right\} \quad G^T\,\mathbf{\Pi}\,G = \mathbf{\Pi} - \begin{pmatrix} 0 & 0 & \epsilon_A\,\Pi_{\gamma\gamma} + \epsilon_Z\,\Pi_{\gamma Z} \\ \cdot & 0 & \epsilon_A\,\Pi_{\gamma Z} + \epsilon_Z\,\Pi_{ZZ} \\ \cdot & \cdot & 2\epsilon_A\Pi_{\gamma X} + 2\epsilon_Z\Pi_{ZX} \end{pmatrix}$$