Energy distribution and equation of state during and after tachyonic resonance

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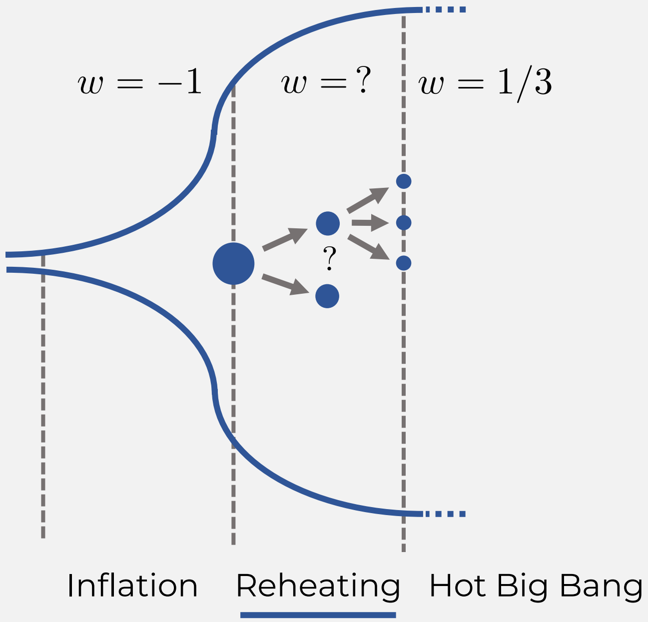
The Reheating Epoch



Why should we study the phase of (p)reheating?

- Energy Transfer
- Expansion History
  - ➜ Number of e-folds N_k of inflation ($N_k = 55 \pm 5$)
  - ➜ Accurate predictions for CMB observables n_s and r

Reheating Model: Motivation

$$V(\phi, X) = \frac{1}{2}\Lambda^4 \tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h^2\phi X^2 + \frac{1}{4}\lambda X^4$$

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- Inflaton ϕ , scalar preheat field X
- Mass scales Λ and M
  - → inflaton mass: $m_\phi^2 = \Lambda^4 / M^2$
- Couplings h^2 and λ
  - → potential bounded form below when: $\lambda > h^4 / 2m_\phi^2$

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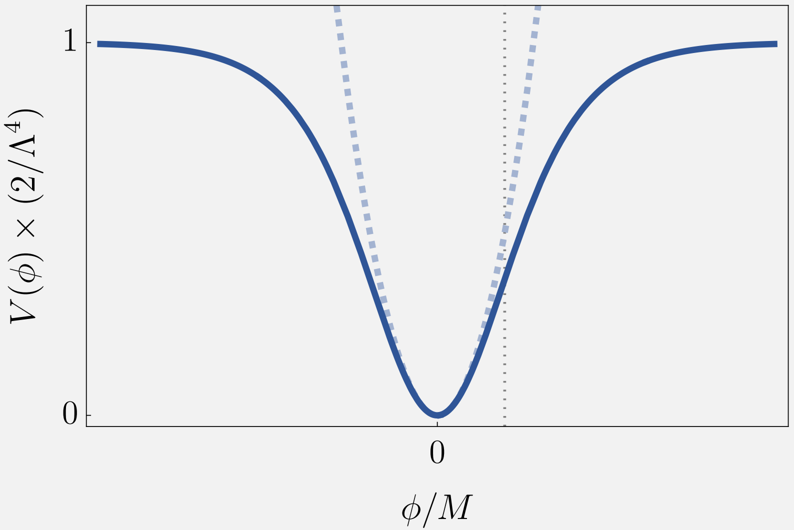
- For interactions of the type $\sim \phi^2 X^2$ we have seen that [1]:
  - ➔ inflaton does <u>not</u> decay completely during the process of parametric resonance
  - ➔ the system does <u>not</u> become radiation dominated
- What happens in case of ϕX^2-interactions?
  - ➔ tachyonic resonance [2]
  - ➔ perturbative decay channel

Inflaton Potential

$$V(\phi, X) = \underbrace{\frac{1}{2}\Lambda^4 \tanh^2\left(\frac{\phi}{M}\right)}_{\text{'inflation'}} + \ldots$$

$$V(\phi) \sim \begin{cases} \Lambda^4 & |\phi| \gg M \\ \phi^2 & |\phi| \ll M \end{cases}$$

- separated by $\phi_{\rm i} = M \mathrm{arcsinh}(1/\sqrt{2})$



- plateau is fixed by

  $$\Lambda^4 = 6\pi^2 A_s M^2 m_{\rm pl}^2 N_k^{-2} f(M, N_k)$$

- inflation ends at

  $$\phi_* = M \mathrm{arcsinh}(\sqrt{8} m_{\rm pl}/M)/2$$

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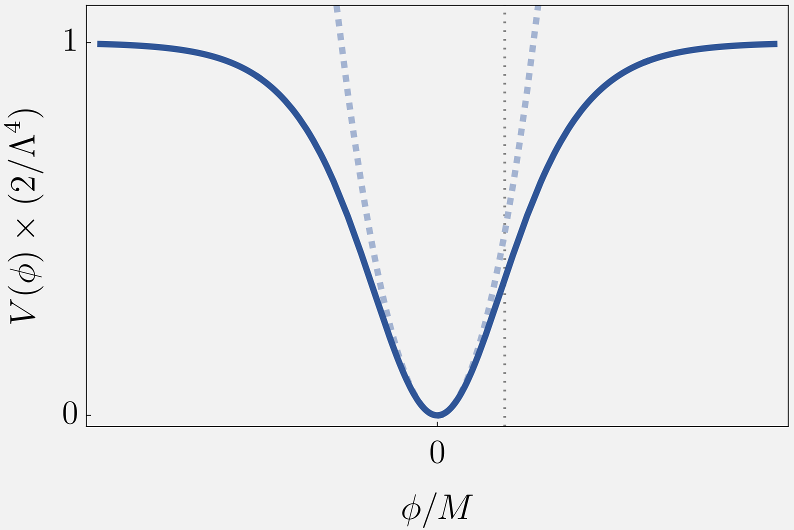

- We fix $M = 5 m_{\rm pl}$ for the following!
  - $V(\phi)$ can be well approximated by a monomial during reheating
- plateau is fixed by

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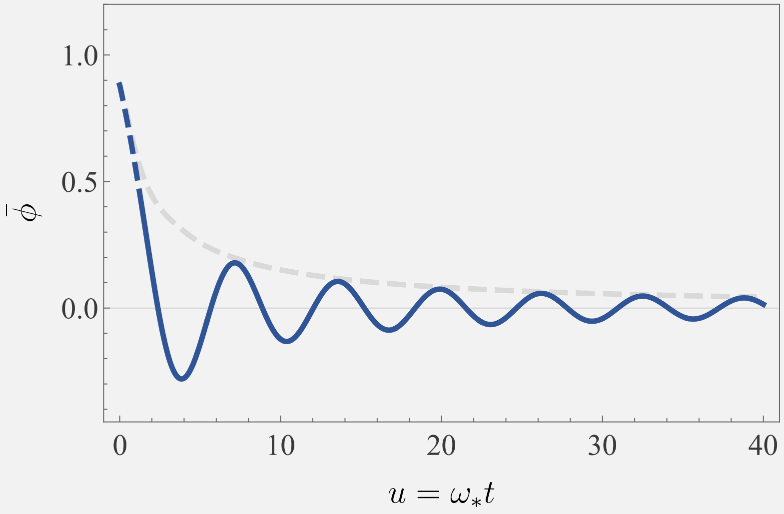
Preheating: Homogeneous Phase

$$V(\phi, X) = \frac{1}{2}\Lambda^4 \tanh^2\left(\frac{\phi}{M}\right) + \dots$$



Evolution of the homogeneous inflaton field is determined by:

$$\left[\begin{array}{l} \ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + \partial_{\bar{\phi}} V = 0 \\ H^2 = \frac{1}{3m_{\text{pl}}^2}\left(\frac{1}{2}\dot{\bar{\phi}}^2 + V\right) \end{array}\right.$$

- decaying amplitude: $\Phi \sim a^{-\frac{3}{2}}$
- oscillation frequency: $\omega_* = m_\phi$
- effective equation of state : $\bar{w} = 0$

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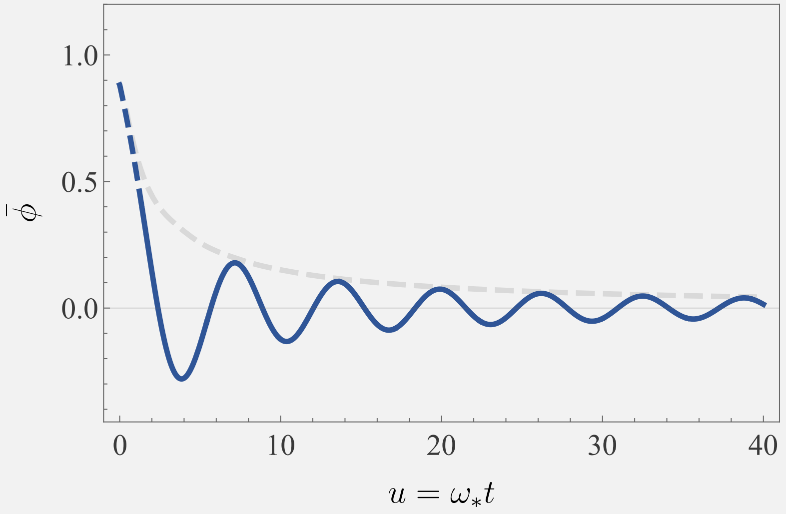

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Natural variables:

$$\begin{cases} \varphi \equiv (\phi/\phi_*)a^{3/2} \\ \chi \equiv (X/\phi_*)a^{3/2} \end{cases} \qquad \begin{cases} u \equiv \omega_* t \\ \vec{y} \equiv \omega_* \vec{x} \end{cases}$$

Natural Variables

The equation of motion in natural variables are then given by:

$$\varphi'' - a^{-2}\nabla^2_{\vec{y}}\varphi + \varphi + F(\varphi) + \tilde{q}^{(h)}\chi^2 = 0$$

$$\chi'' - a^{-2}\nabla^2_{\vec{y}}\chi + \tilde{q}^{(h)}\varphi\chi + \tilde{q}^{(\lambda)}\chi^3 = 0$$

With the effective coupling parameters:

$$\tilde{q}^{(h)} = q_*^{(h)} a^{-3/2} \quad , \qquad q_*^{(h)} \equiv h^2\phi_*/\omega_*^2$$

$$\tilde{q}^{(\lambda)} = q_*^{(\lambda)} a^{-3} \quad , \qquad q_*^{(\lambda)} \equiv \lambda\phi_*^2/\omega_*^2$$

- Potential is bounded from below when: $(q_*^{(h)})^2 > 2q_*^{(\lambda)}$

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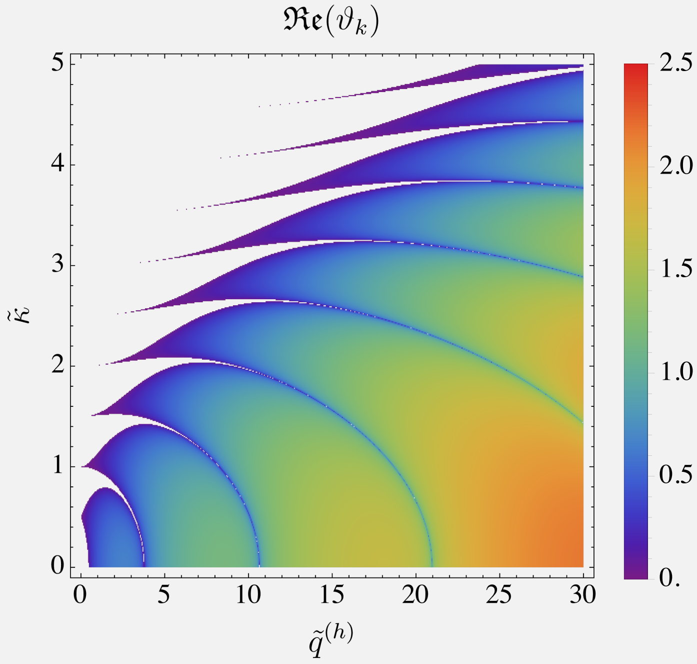
Preheating: Tachyonic Resonance

- Linearised mode equation of χ:
  - $\delta\chi_k'' + \left(\tilde{\kappa}^2 + \tilde{q}^{(h)}\bar{\varphi}\right)\delta\chi_k \simeq 0$

    with $\tilde{\kappa} = k/(\omega_* a)$
  - $\bar{\varphi}'' + \bar{\varphi} \simeq 0$

Preheating: Tachyonic Resonance



$\tilde{q}^{(h)} \gtrsim 0.5$: broad resonance
$\tilde{q}^{(h)} \lesssim 0.5$: narrow resonance

Preheating: Tachyonic Resonance

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    with $\tilde{\kappa} = k/(\omega_* a)$
  - $\bar{\varphi}'' + \bar{\varphi} \simeq 0$
- Inflaton induces a tachyonic mass term each time when $\bar{\varphi} < 0$; modes for which $\tilde{\kappa}^2 < \tilde{q}^{(h)}\varphi$ grow exponentially:

  → $|\delta\chi_k|^2 \sim e^{2\vartheta_k u}$
- Tachyonic resonance is always stronger than parametric resonance ($\sim \phi^2 X^2$)

Preheating: Lattice Simulations

CosmoLattice [3]:

- Simulations have been performed for the T-model:

$$V = \frac{1}{2}\Lambda^4 \tanh^2\left(\frac{\phi}{M}\right) + \frac{1}{2}h^2\phi X^2 + \frac{1}{4}\lambda X^4$$

  with mass scale $M = 5m_{\rm pl}$ and $N_k = 55$, initialised the simulations at ϕ_*

- Details on simulations:
  - For $N = 128 - 512$ grid points
  - Performed in $(2+1)$ dimensions
    - ➔ results almost identical to the ones in $(3+1)$ dimensions!

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- Energy density ratios: $\varepsilon_i \equiv \frac{E_i}{E_{\rm tot}}$

Total energy density:

$$E_{\rm tot} = \sum_f^{\varphi,\chi}\left(E_{\rm k}^f + E_{\rm g}^f + E_{\rm p}^f\right) + E_{\rm i}$$

kinetic gradient potential interaction

Energy density components:

$$E_{\rm k}^f = \frac{1}{2}(f')^2 \qquad E_{\rm p}^\varphi = \frac{1}{2}\varphi^2 + [...] \qquad E_{\rm i} = \frac{1}{2}\tilde{q}^{(h)}\varphi\chi^2$$

$$E_{\rm g}^f = \frac{1}{2a^2}(\nabla_{\vec{y}} f)^2 \qquad E_{\rm p}^\chi = \frac{1}{4}\tilde{q}^{(\lambda)}\chi^4$$

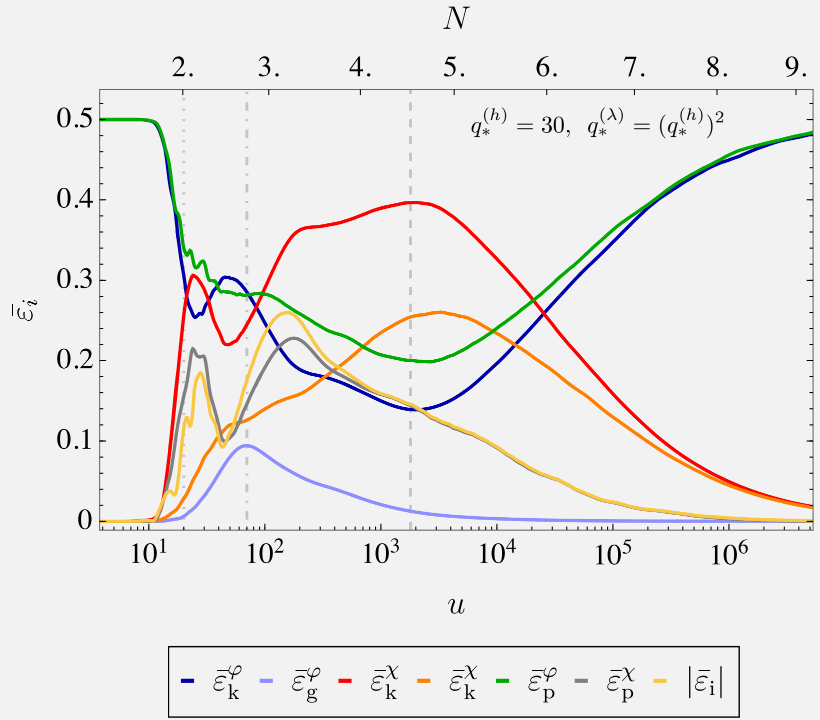
- Equation of state: $w = \varepsilon_{\rm k} - \frac{1}{3}\varepsilon_{\rm g} - \varepsilon_{\rm p}$

Lattice Simulations: Effective Energy Ratios

Equations of motion:

$$\varphi'' - a^{-2}\nabla^2_{\vec{y}}\varphi + \varphi + F(\varphi) + \tilde{q}^{(h)}\chi^2 = 0$$
$$\chi'' - a^{-2}\nabla^2_{\vec{y}}\chi + \tilde{q}^{(h)}\varphi\chi + \tilde{q}^{(\lambda)}\chi^3 = 0$$

- Homogeneous phase: $\bar{\varepsilon}^{\varphi}_{\rm k} \simeq \bar{\varepsilon}^{\varphi}_{\rm p} \simeq 0.5$
- System starts to become non-linear at $u_{\rm br} \approx 20$
- Backreaction process terminates when $\tilde{q}^{(h)} < 0.5$ at $u \approx 70$
- Self-resonance process continues until $u \approx 1800$
- Eventually: $\bar{\varepsilon}^{\varphi}_{\rm k} \simeq \bar{\varepsilon}^{\varphi}_{\rm p} \to 0.5$



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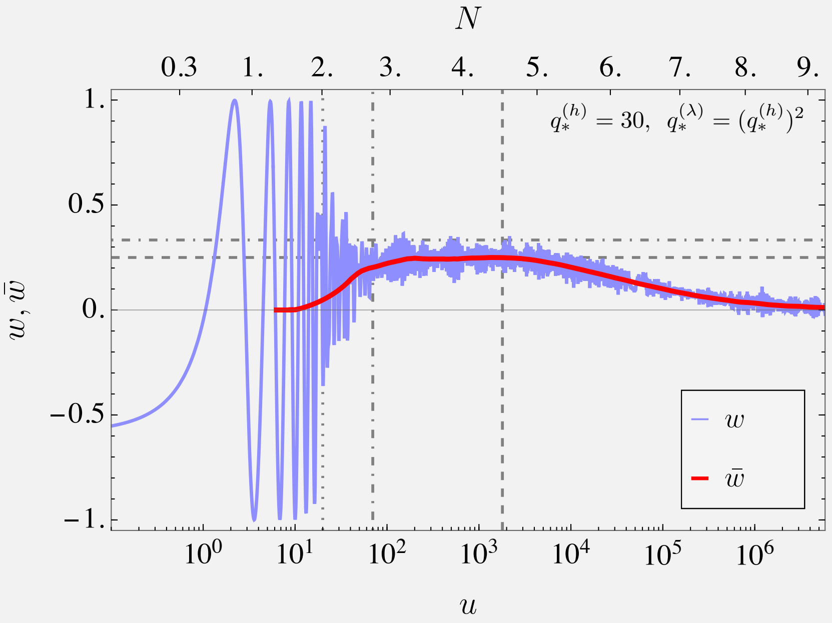
- Homogeneous phase: $\bar{w} \simeq 0$
- System starts to become non-linear and equation of state grows
- Backreaction process terminates and $\bar{w} \to 0.25$
- Self-resonance process continues and equation of state stays at $\bar{w} \simeq 0.25$
- Eventually: $\bar{w} \to 0$



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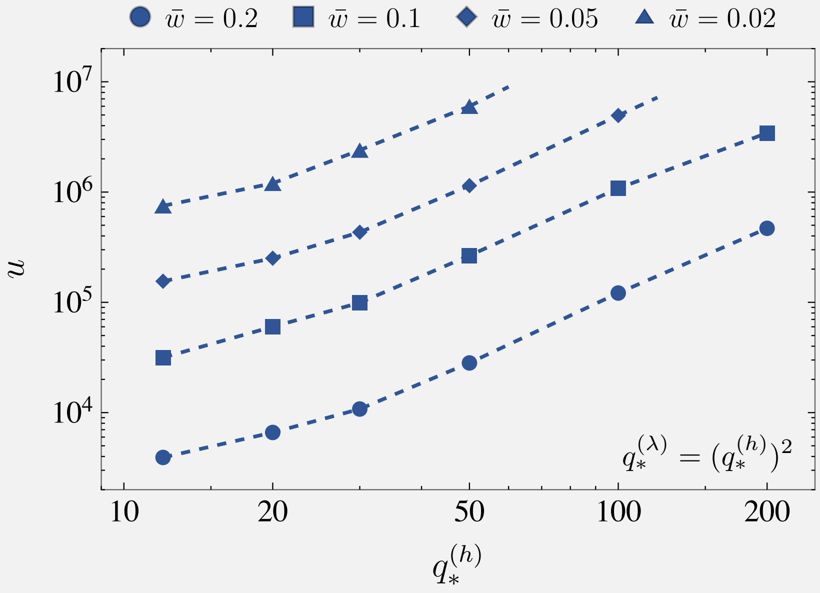
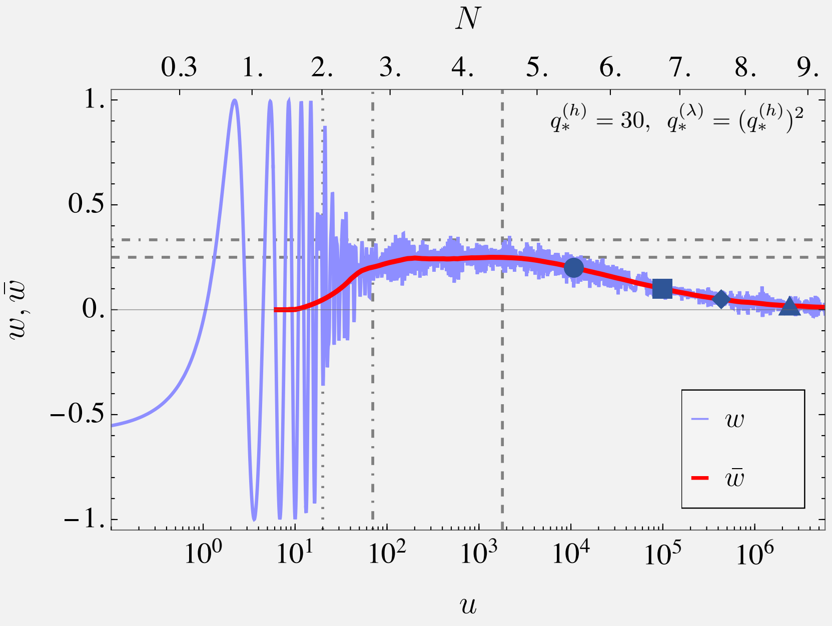




Lattice Simulations: Number of e-folds of Reheating

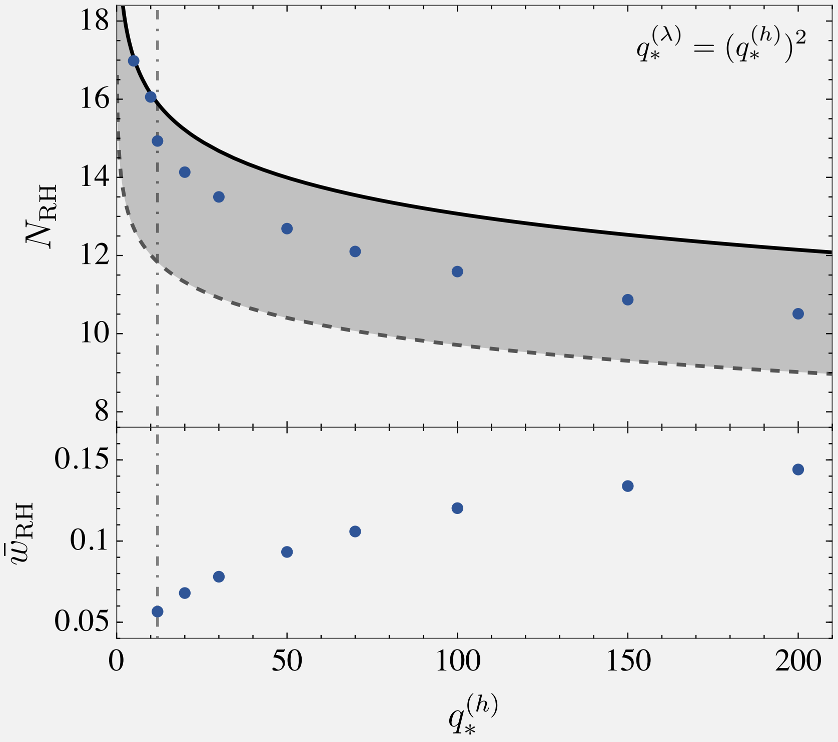
The trilinear interaction $\sim \phi X^2$ provides a perturbative decay channel, with decay rate:

$$\Gamma_\phi = \frac{h^4}{32\pi m_\phi}$$

Thus reheating may be completed by the decay process $\phi \to XX$, when $H_{\rm RH} \sim \Gamma_\phi$.

We can then determine:

- $$N_{\rm RH} = \frac{1}{3(1+\bar{w}_{\rm RH})} \ln \frac{\rho_{\rm end}}{\rho_{\rm RH}}$$
- $$\bar{w}_{\rm RH} = \frac{1}{\Delta N} \int_{N_{\rm end}}^{N_{\rm RH}} w(N) dN$$



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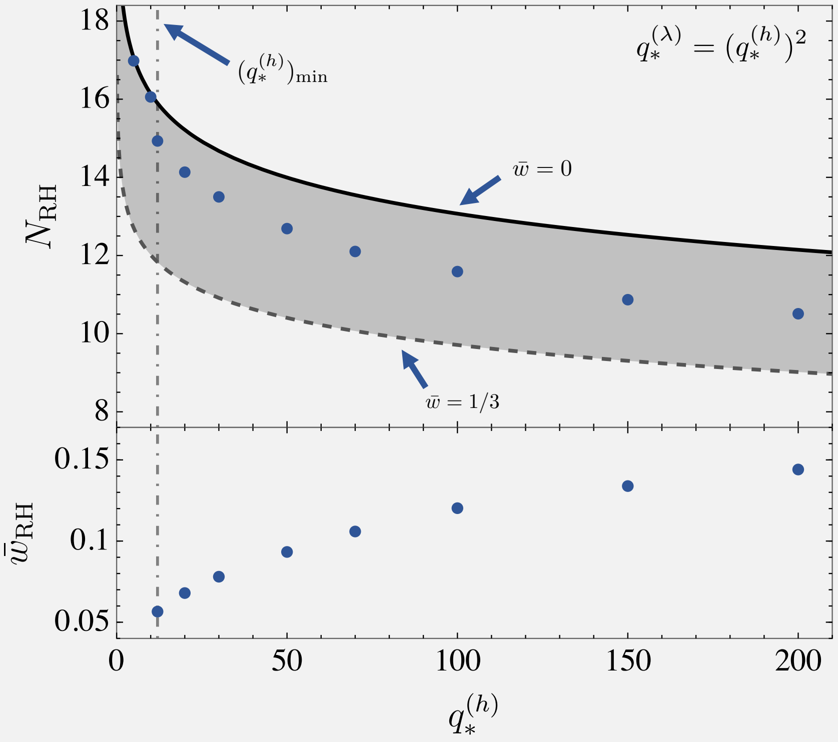


Lattice Simulations: Number of e-folds of Inflation

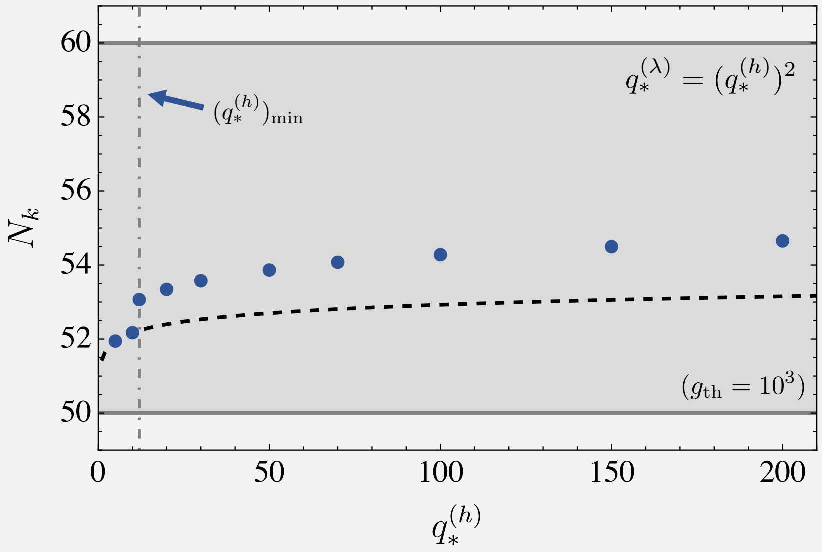
The number of e-folds of inflation are given by:

$$N_k \simeq 67 - \ln \frac{k_{\rm CMB}}{a_0 H_0} - \frac{1}{12} \ln g_{\rm th} + \ldots$$
$$\ldots + \frac{1}{4} \ln \frac{V_k^2}{m_{\rm pl}^4 \rho_{\rm end}} + \frac{1 - 3\bar{w}_{\rm RH}}{12(1+\bar{w}_{\rm RH})} \ln \frac{\rho_{\rm RH}}{\rho_{\rm end}}$$

Assuming the system stays in RD until BBN, followed by the standard expansion history of the universe (RD ➔ MD ➔ $\Lambda\mathrm{D}$)

- ➔ with $N_{\rm RH}$ and $\bar{w}_{\rm RH}$ we can determine N_k
- ➔ since V_k depends on N_k we use an iterative scheme

N_k for the *pivot scale* $k_{\rm CMB} = 0.05 \mathrm{Mpc}^{-1}$:



Concluding Remarks

- Equation of state eventually returns to $\bar{w} \to 0$ and the total energy is distributed as $\bar{\varepsilon}_{\rm k}^{\varphi} \simeq \bar{\varepsilon}_{\rm p}^{\varphi} \simeq 0.5$

- Thus, the reheating process is not completed after the initial preheating phase.

- The inflaton decays at latest perturbatively via the trilinear interaction. This allows us to determine the maximal possible number of e-folds and $N_{\rm RH}$, as well as the according n_s and r.

References

[1]: References on quadratic-quadratic interactions

- Energy distribution and equation of state of the early Universe: matching the end of inflation and the onset of radiation domination (2005.07563)
- Characterizing the post-inflationary reheating history, Part I: single daughter field with quadratic-quadratic interaction (2112.11280)
- Characterizing the post-inflationary reheating history, Part II: Multiple interacting daughter fields (2206.06319)

[2]: References on tri-linear interactions:

- Preheating with trilinear interactions: Tachyonic resonance (0602144)
- Gravitational wave production from preheating with trilinear interactions (2206.1472)
- ...

[3]: CosmoLattice:

- CosmoLattice: A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe (2102.01031)

Thank you