Geometric (p)reheating of the universe

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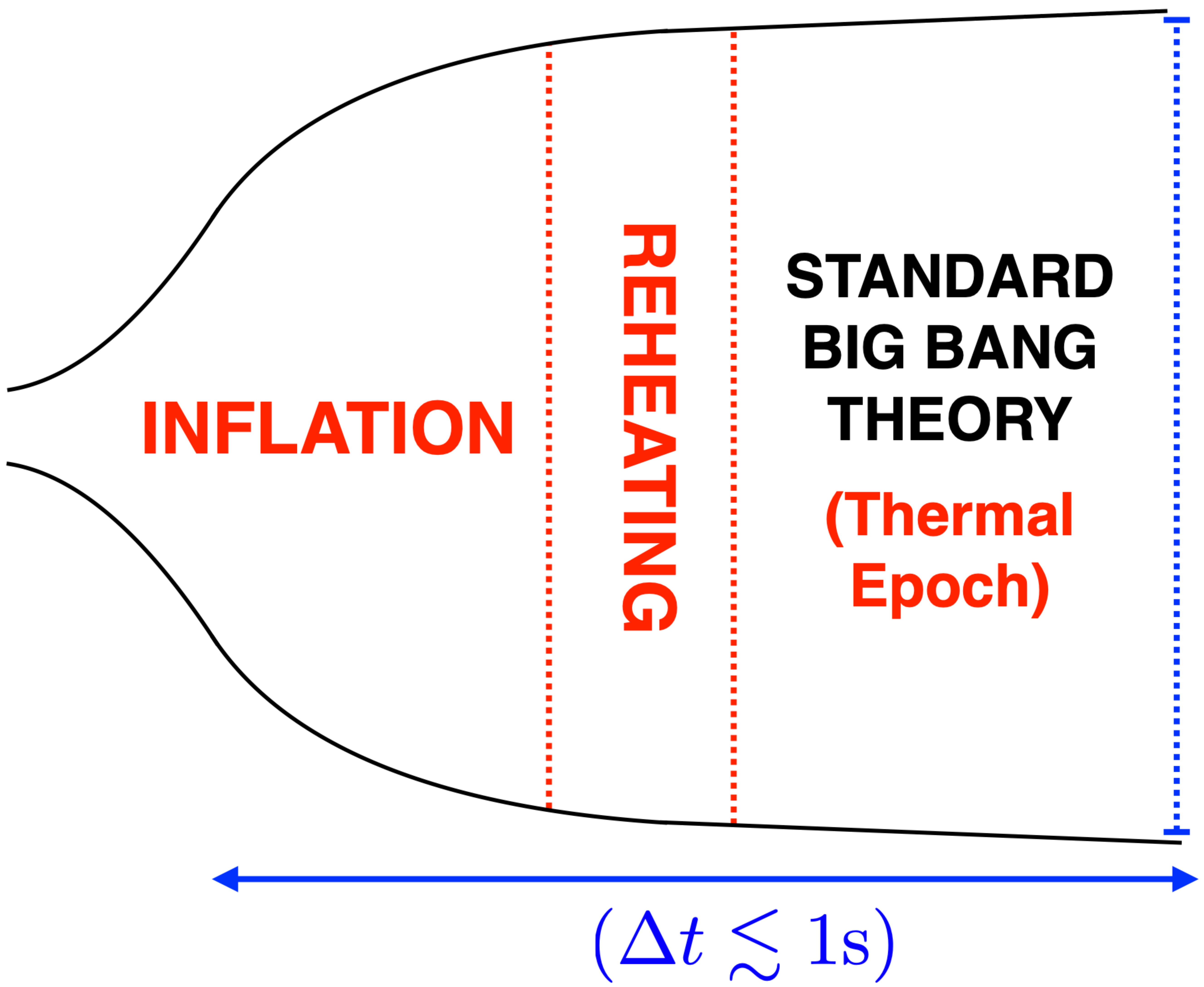
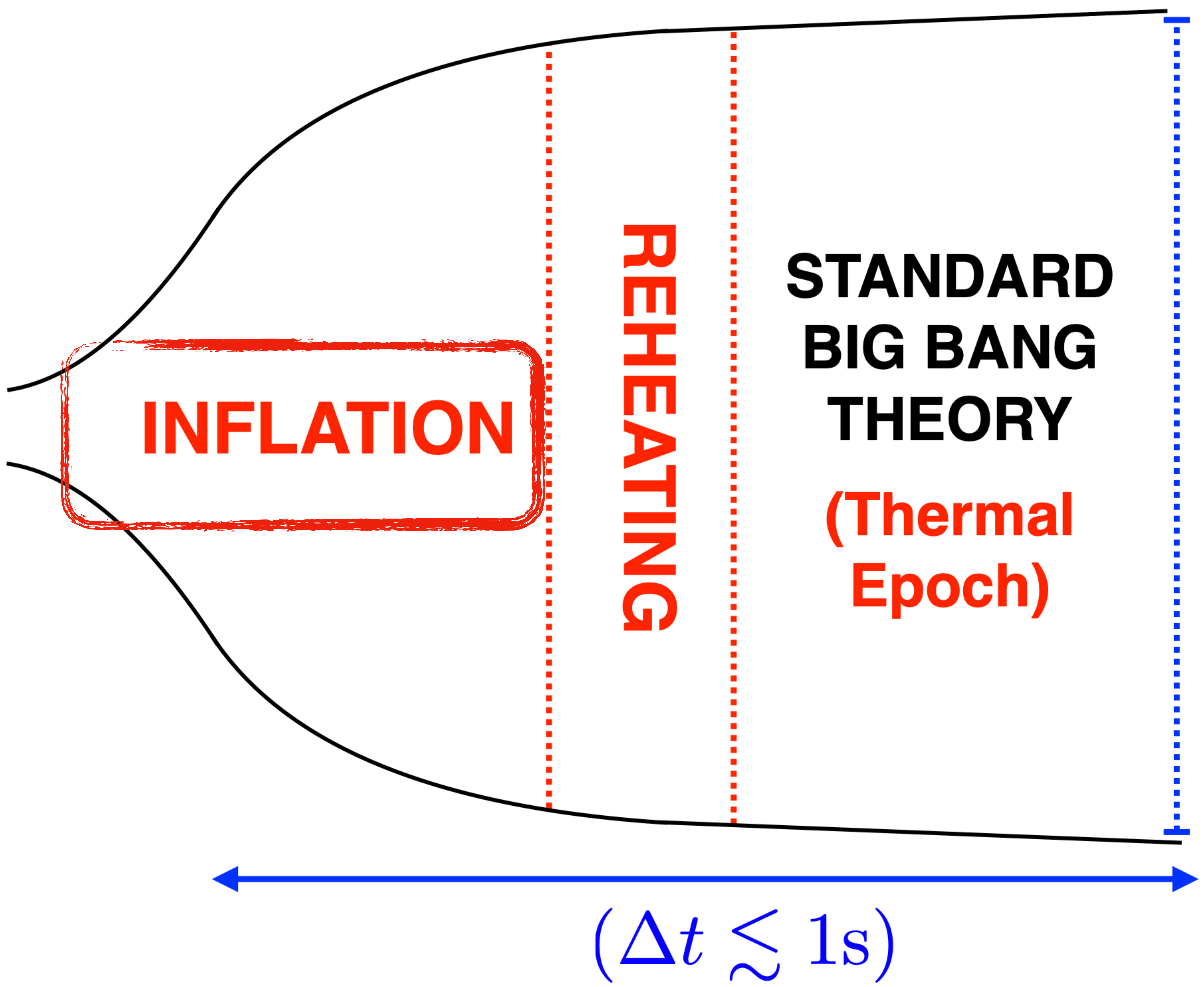
Early Universe

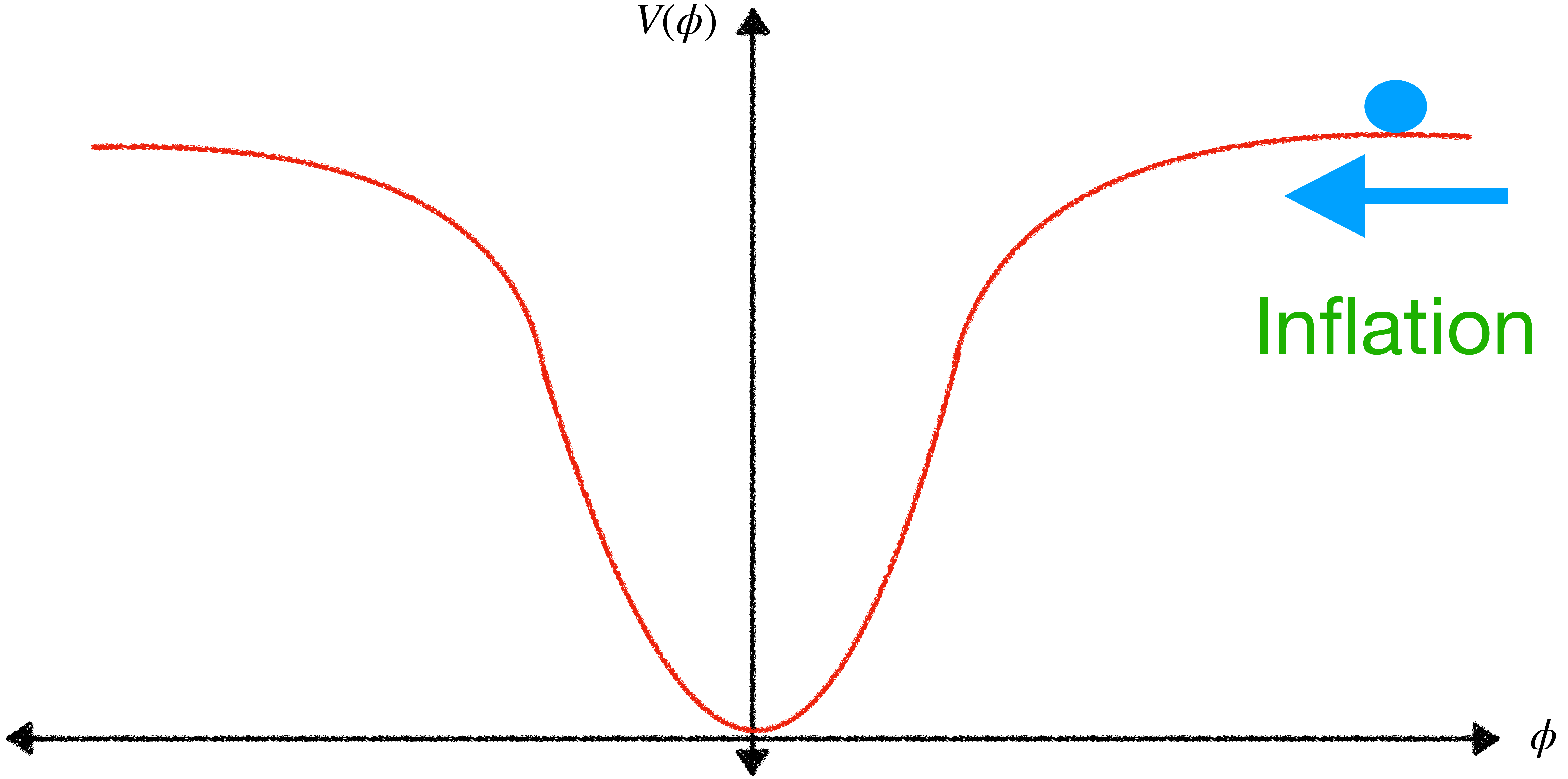


Early Universe



Inflationary Model

$$V(\phi) = \frac{\Lambda^4}{p} \tanh^p \left(\frac{\phi}{M} \right)$$



Inflationary model: ***α-attractor***

Kallosh, Linde '13

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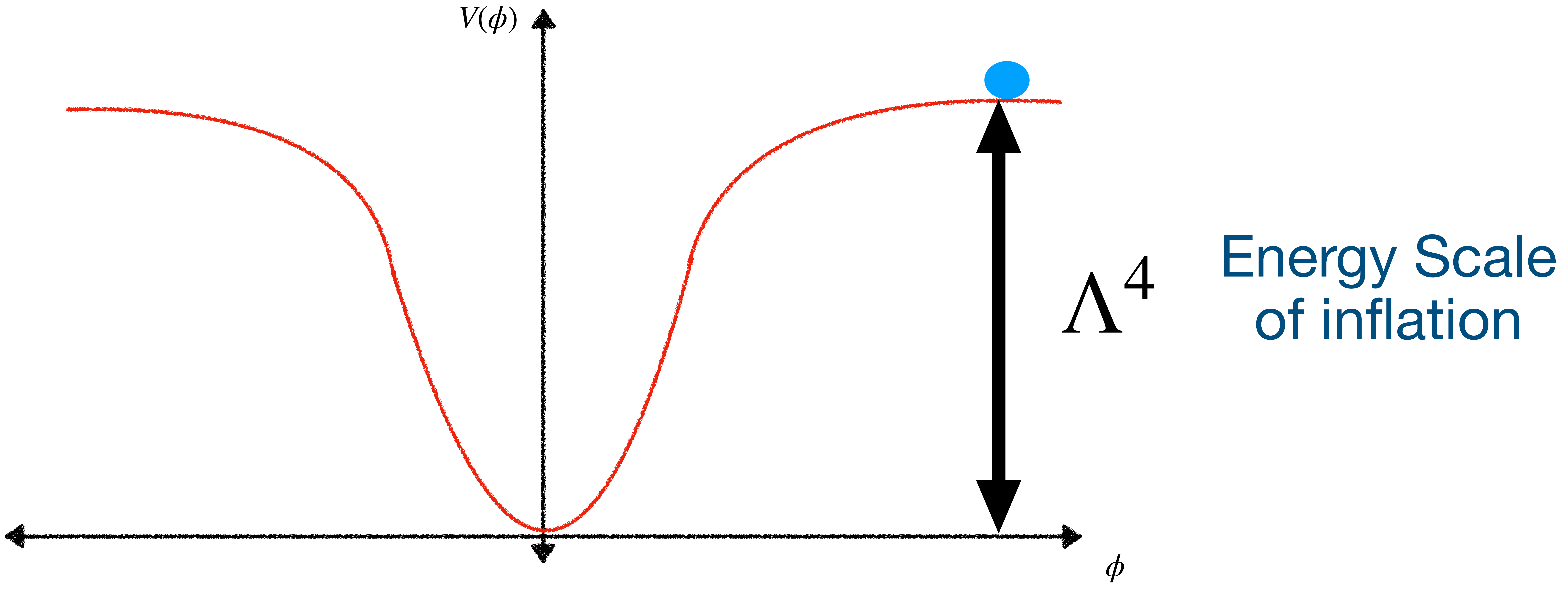

Slow-roll parameters

$$\epsilon_V \equiv \frac{m_p^2}{2}\left(\frac{V'_{\rm inf}(\phi)}{V_{\rm inf}(\phi)}\right)^2 = \frac{2p^2 {\rm csch}^2(\frac{2\phi}{M})}{(M/m_p)^2}$$

$$\eta_V \equiv m_p^2 \frac{V''_{\rm inf}(\phi)}{V_{\rm inf}(\phi)} = \frac{4p(p - \cosh(\frac{2\phi}{M})){\rm csch}^2(\frac{2\phi}{M})}{(M/m_p)^2}$$

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↓

We can relate them with CMB observations

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We can relate them with CMB observations

$$A_s \simeq \frac{1}{24\pi^2\epsilon_{V_k}}\frac{V_{\text{inf}}(\phi_k)}{m_p^4} \qquad n_s - 1 \simeq 2\eta_{V_k} - 6\epsilon_{Vk} \qquad r \simeq 16\epsilon_{V_k}$$

Slow-roll parameters

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We can relate them with CMB observations

$$r < 0.032 \quad (95\,\%\ \mathrm{CL})$$

$$n_s = 0.9649 \pm 0.0042 \quad (68\,\%\ \mathrm{CL})$$

$$A_s = 2.099^{+0.296}_{-0.292} \cdot 10^{-9} \quad (68\,\%\ \mathrm{CL})$$

$$A_s \simeq \frac{1}{24\pi^2 \epsilon_{V_k}}$$

$$r \simeq 16\epsilon_{V_k}$$

CMB constrains for α-attractors

$$A_s(\Lambda, M, p) = \left(\frac{\Lambda}{m_p}\right)^4 \frac{\left[8pN_k^2 + \mathscr{M}^2\left(p - 4N_k\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}}\right)\right]\left[\mathscr{M}^2\left(\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}} - 1\right) - 4pN_k\right]^p}{24\pi^2 p^2 \mathscr{M}^2\left[\mathscr{M}^2\left(\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}} + 1\right) - 4pN_k\right]^p}$$

$$n_s(M, p) = \frac{8p(N_k + 2)N_k - \mathscr{M}^2\left(4(N_k + 1)\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}} + p\right)}{8pN_k^2 + \mathscr{M}^2\left(p - 4N_k\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}}\right)}$$

$$r(M, p) = \frac{16p\mathscr{M}^2}{8pN_k^2 + \mathscr{M}^2\left(p - 4N_k\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}}\right)}$$

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$n_s(M$ … $p)$

r upper value sets a threshold value for M

$$M_{\max} \simeq 8.79 m_p \quad \text{for} \quad p = 2$$
$$M_{\max} \simeq 8.22 m_p \quad \text{for} \quad p = 4$$
$$M_{\max} \simeq 8.08 m_p \quad \text{for} \quad p = 6$$

$$r(M, p) = \frac{16p\mathscr{M}^2}{8pN_k^2 + \mathscr{M}^2\left(p - 4N_k\sqrt{1+\frac{2p^2}{\mathscr{M}^2}}\right)}$$

CMB constrains for α-attractors

They look better in the Asymptotic limit $\mathscr{M} \ll 1$

$$n_s(M,p) \to \frac{29}{30} \simeq 0.967$$

$$r(M,p) \longrightarrow \frac{(M/m_p)^2}{1800} = 5.56 \cdot 10^{-4} \left(\frac{M}{m_p}\right)^2$$

$$\frac{\Lambda(M,p)}{m_p} \longrightarrow \left(\frac{24\pi^2 p A_s}{28800}\right)^{1/4} \sqrt{\frac{M}{m_p}} \simeq 2.04 \cdot 10^{-3}\, p^{1/4} \sqrt{\frac{M}{m_p}}$$

For which we choose $N_k = 60$

$A_s(\Lambda, M, p) =$ … $1\Big) - 4pN_k\Big]^p$ … $8pN_k^2 + \mathscr{M}^2\Big(p - 4N_k\sqrt{1 + \frac{\cdots}{\mathscr{M}^2}}\Big)$

Energy Scale of inflation for α-attractors

$$A_s(\Lambda, M, p) = \Bigg(\frac{\Lambda \ldots}{m_p \ldots} \ldots - 1\Big) - 4pN_k\Big]^p \ldots \Big]^p$$

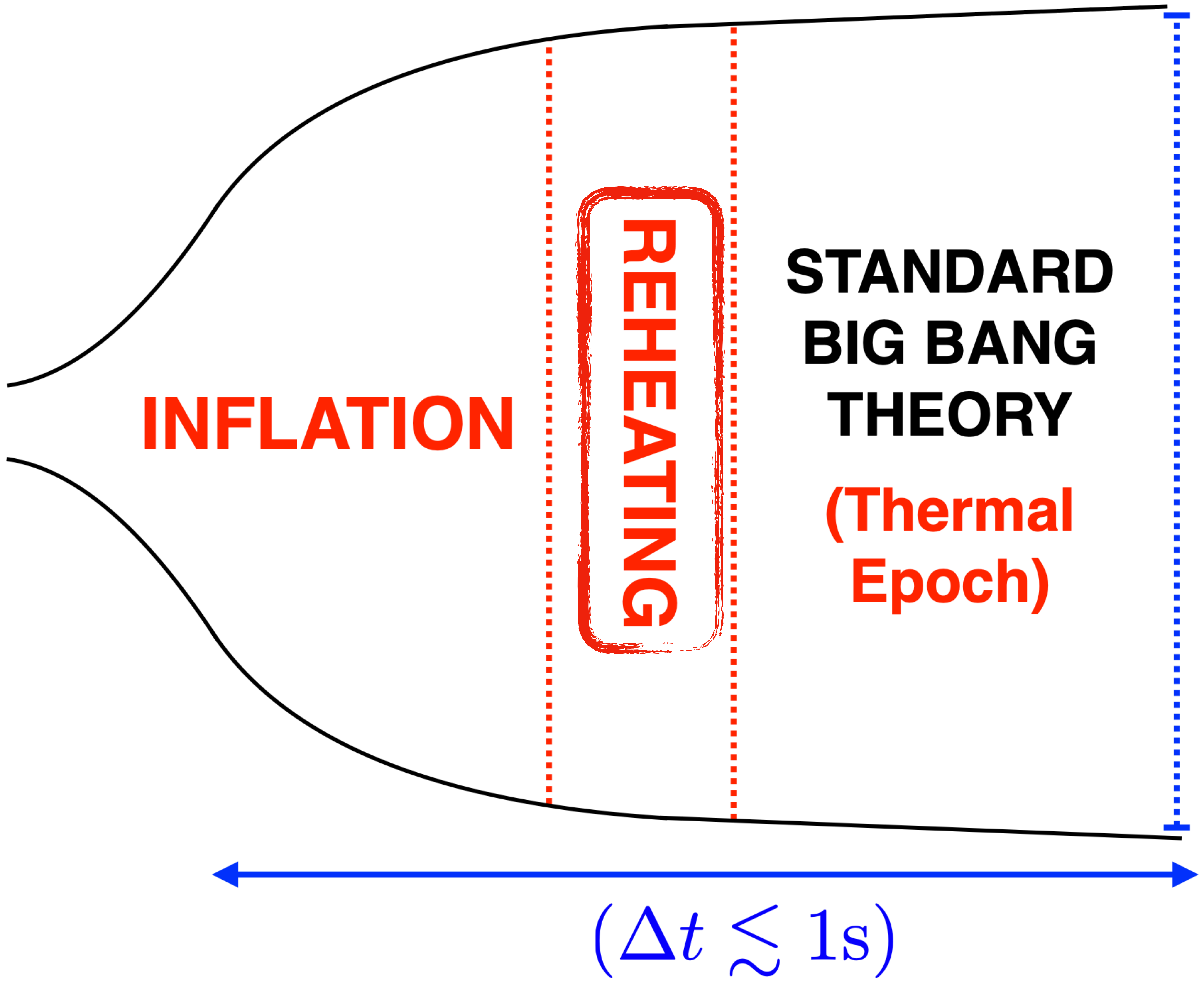
$$\frac{H^2_{\rm inf}(M,p;A_s)}{m_p^2} \simeq \frac{8p\ \mathscr{M}^2 A_s\ \pi^2}{28800\ p + \mathscr{M}^2\left(p + 240\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}}\ \right)}$$

in the Asymptotic limit $\mathscr{M} \ll 1$

$$H_{\rm inf}(M,p) \to 5.8 \times 10^{12} \left(\frac{M}{m_p}\right)\ \text{GeV}$$

$$r(M,p) = \frac{16p\mathscr{M} \ldots}{8pN_k^2 + \mathscr{M}^2\left(p - 4N_k\sqrt{1 + \frac{2p^2}{\mathscr{M}^2}}\ \right)}$$

Early Universe



Inflaton oscillations

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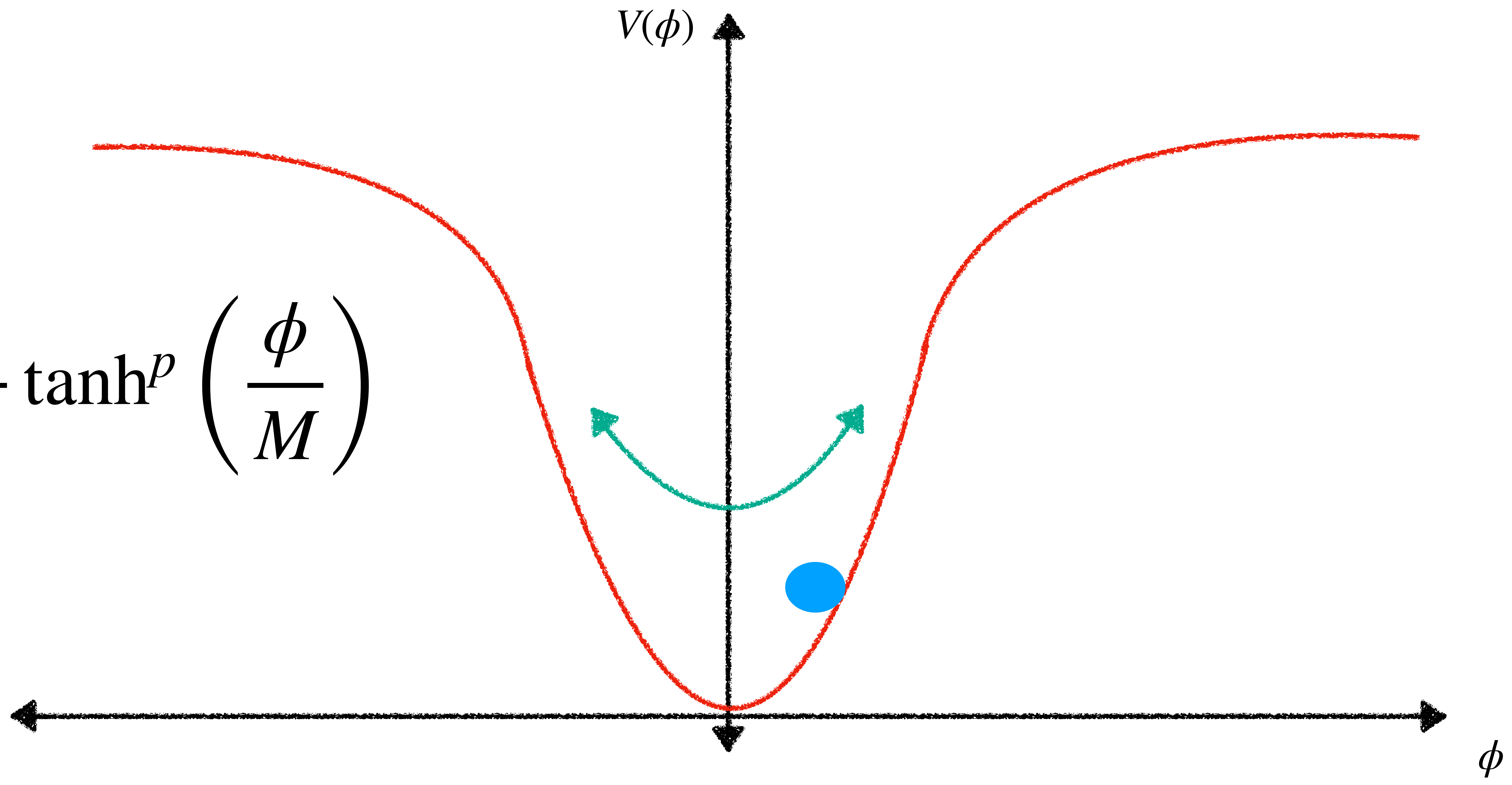
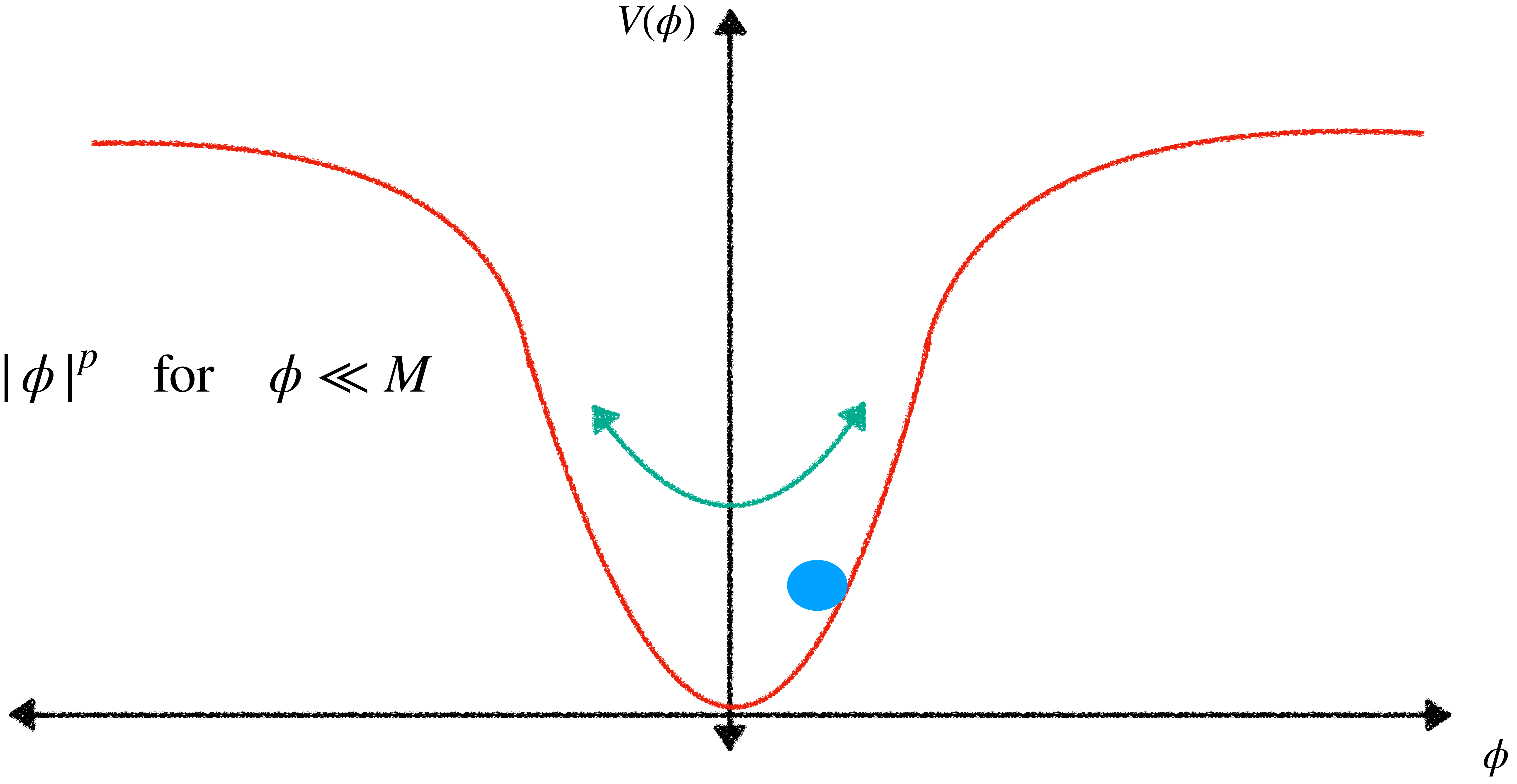


Inflaton oscillations

$$V(\phi) \simeq \frac{1}{p}\lambda\mu^{4-p}|\phi|^p \quad \text{for} \quad \phi \ll M$$



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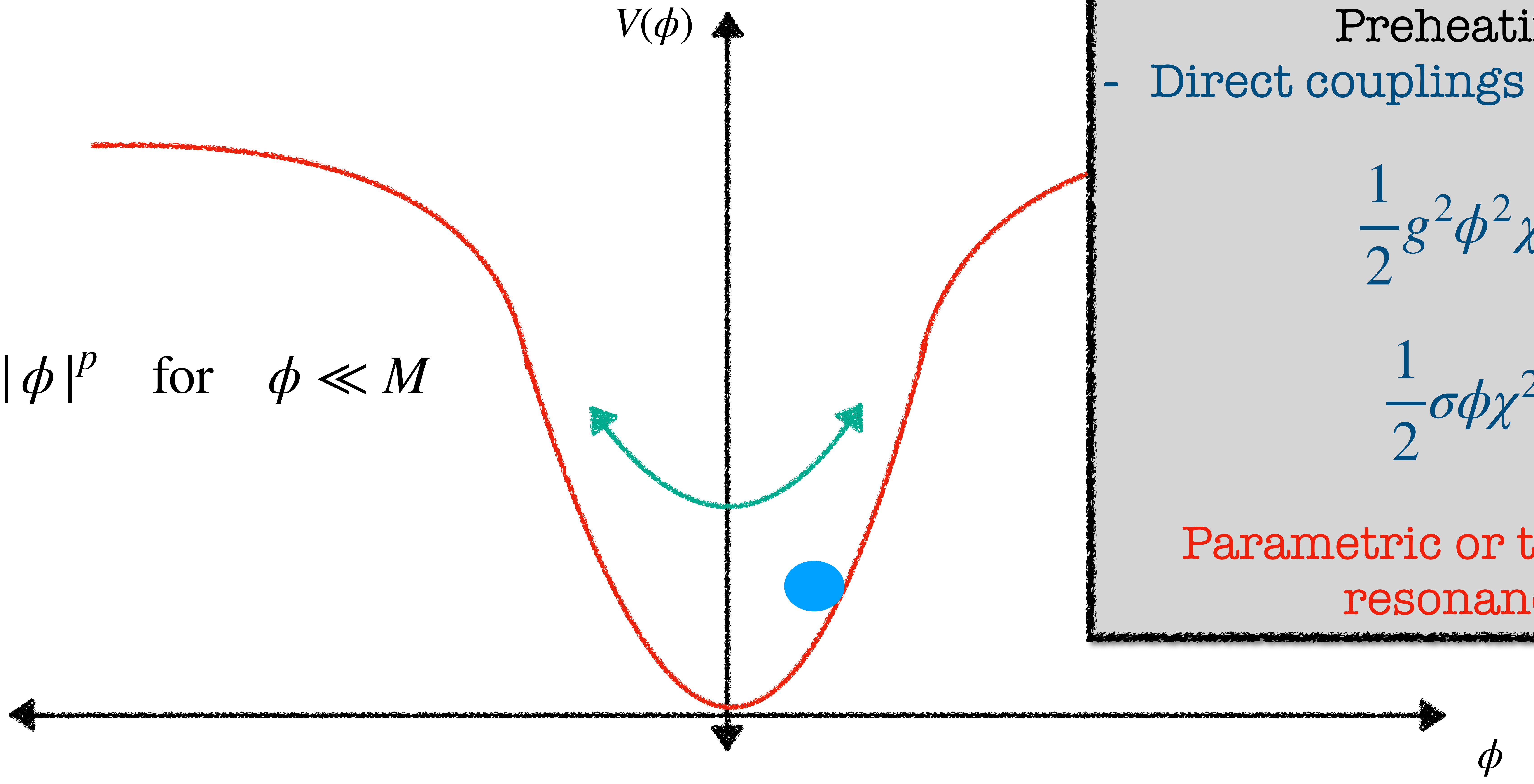

Preheating

- Direct couplings

$$\frac{1}{2}g^2\phi^2\chi^2$$

$$\frac{1}{2}\sigma\phi\chi^2$$

Parametric or tachyonic resonance

Kofman, Linde, Starobinsky '97
Kaiser '97
Peloso, Sorbo '00
…

Inflaton oscillations

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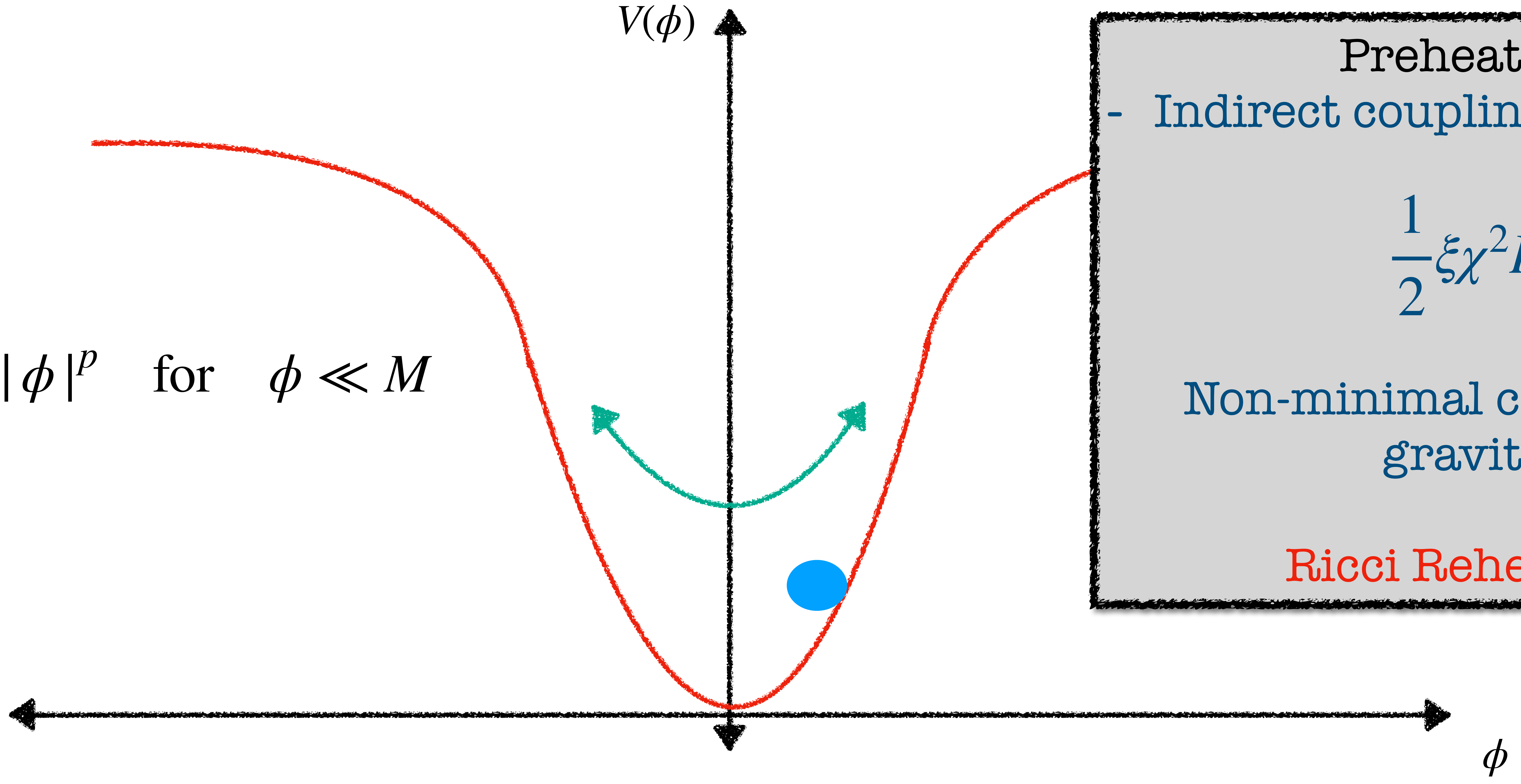

Preheating

- Indirect couplings

$$\frac{1}{2}\xi\chi^2 R$$

Non-minimal coupling to gravity

Ricci Reheating

Figueroa, Stefanek, Opferkuch '23

Inflaton oscillations

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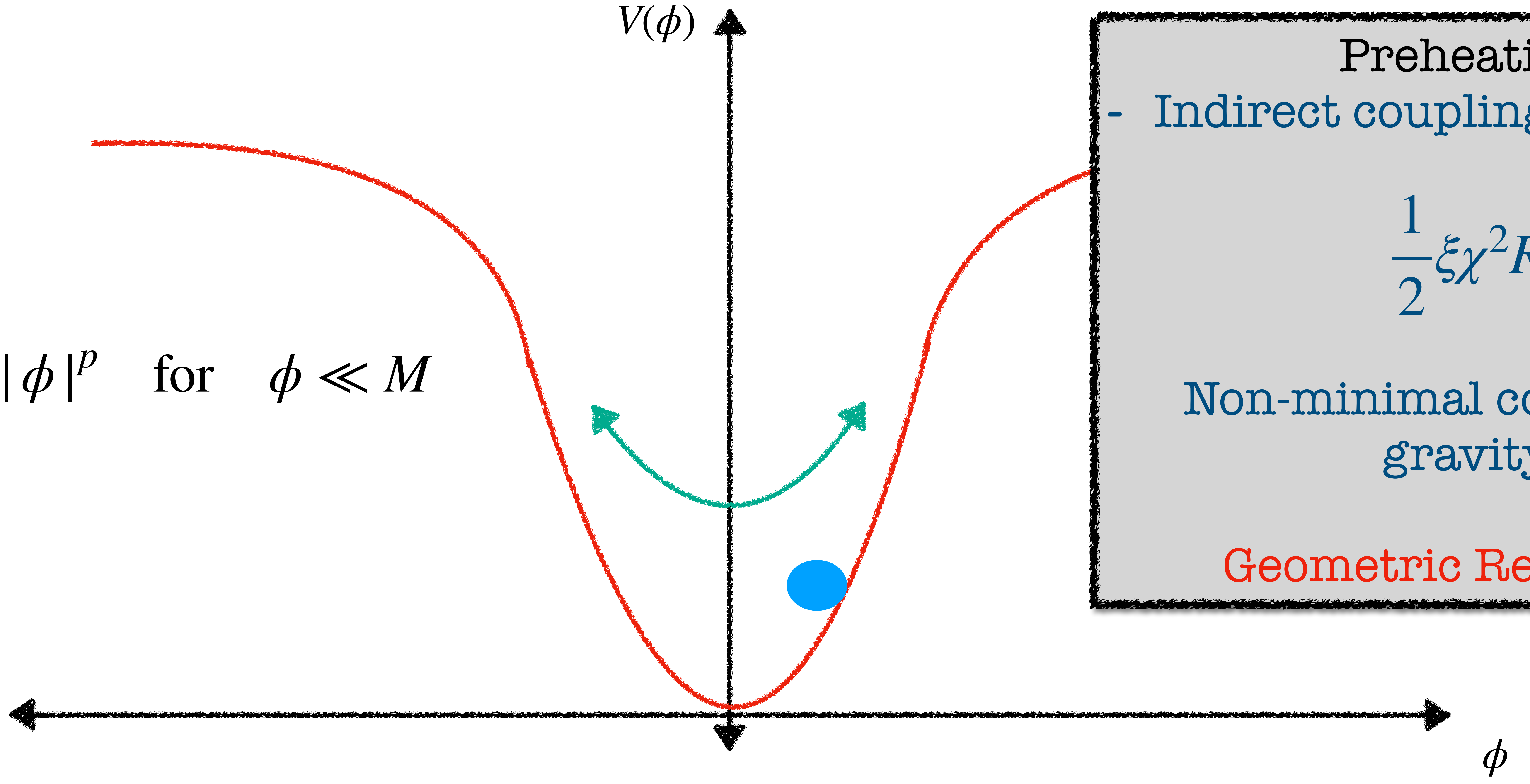

Preheating

- Indirect couplings

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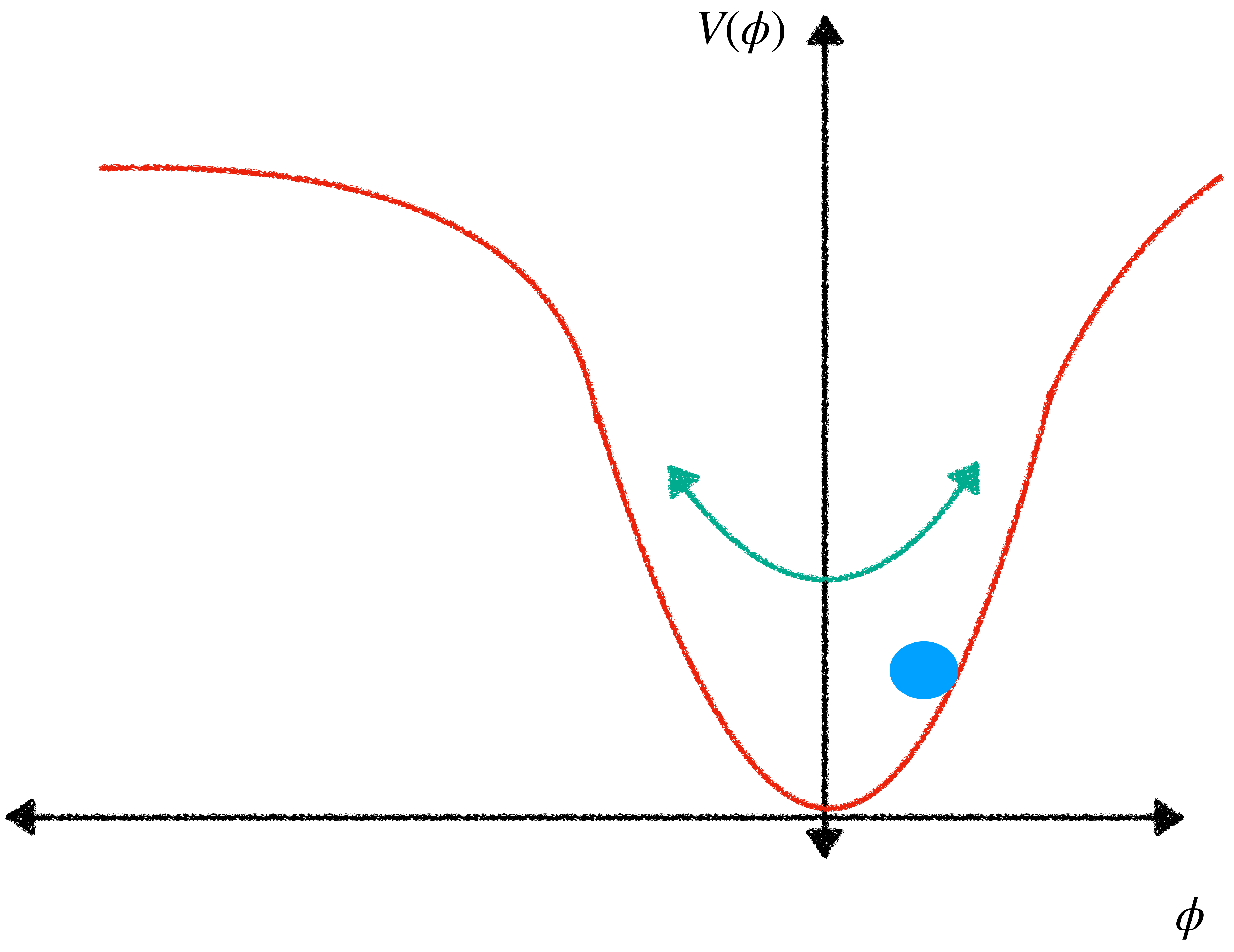
Geometric Reheating

Ford '87
Figueroa, Florio, Opferkuch, Stefanek '23

Inflaton oscillations



Oscillations of the Ricci Scalar

Lets consider the metric

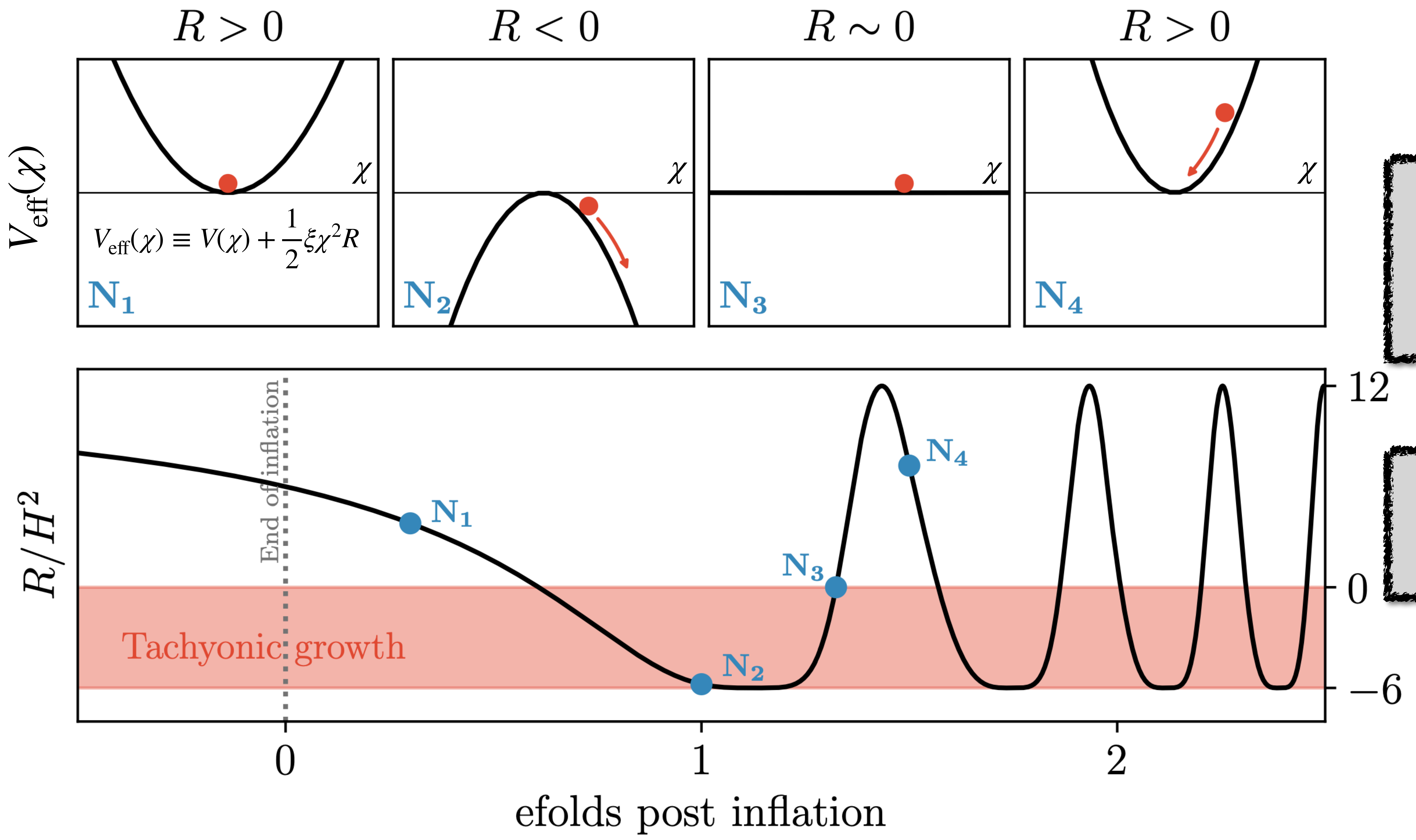
$$g_{\mu\nu} \equiv \mathrm{diag}(-a(\eta)^{2\alpha}, a(\eta)^2, a(\eta)^2, a(\eta)^2)$$

$$R = \frac{6}{a^{2\alpha}}\left[\frac{a''}{a} - (\alpha - 1)\left(\frac{a'}{a}\right)^2\right]$$

If is driven purely by the homogeneous oscillations of the inflation

$$R = \frac{2}{m_p^2}\left(2\langle V_{\mathrm{inf}}(\phi)\rangle - \langle K(\phi)\rangle\right)$$

Briefly description of Geometric (p)reheating



$$\frac{1}{2}\xi\chi^2 R$$

Whenever $R < 0$
Effective mass of χ becomes tachyonic

Whenever $R > 0$
χ oscillates as free field

Adapted from Figueroa, Florio, Opferkuch, Stefanek '23

Inflaton oscillations

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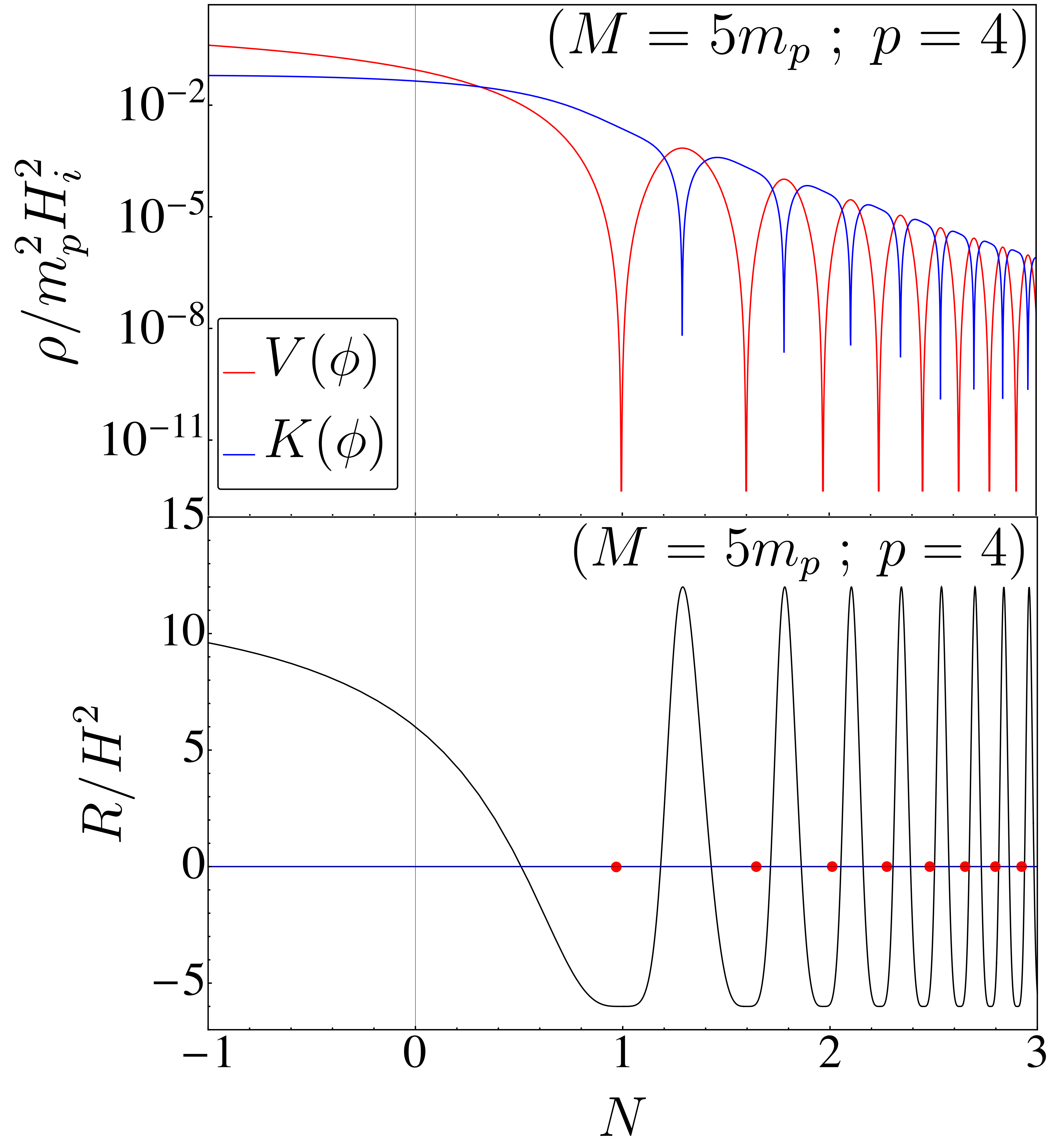

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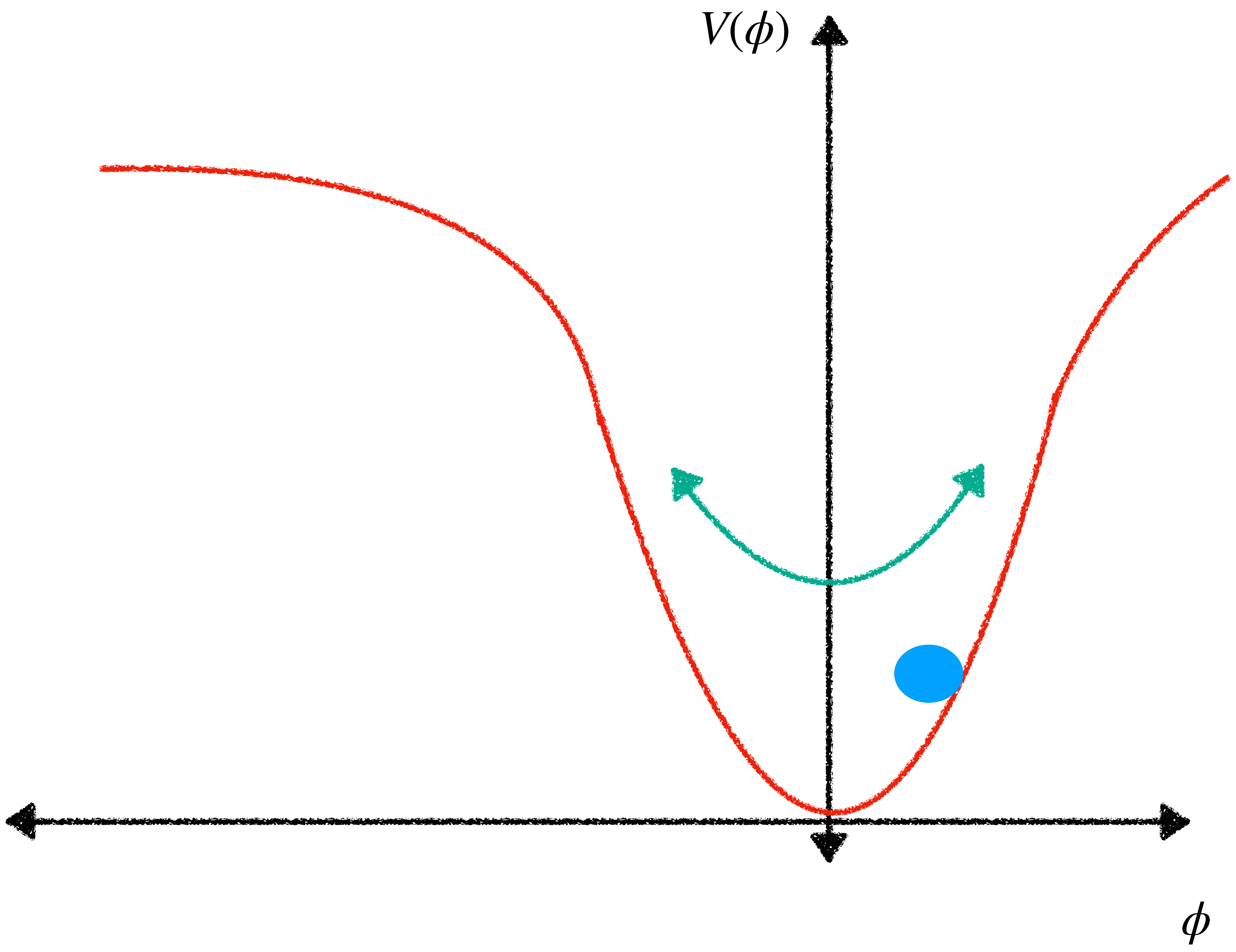

How are inflation oscillations affected by lowering M?

Inflaton oscillations

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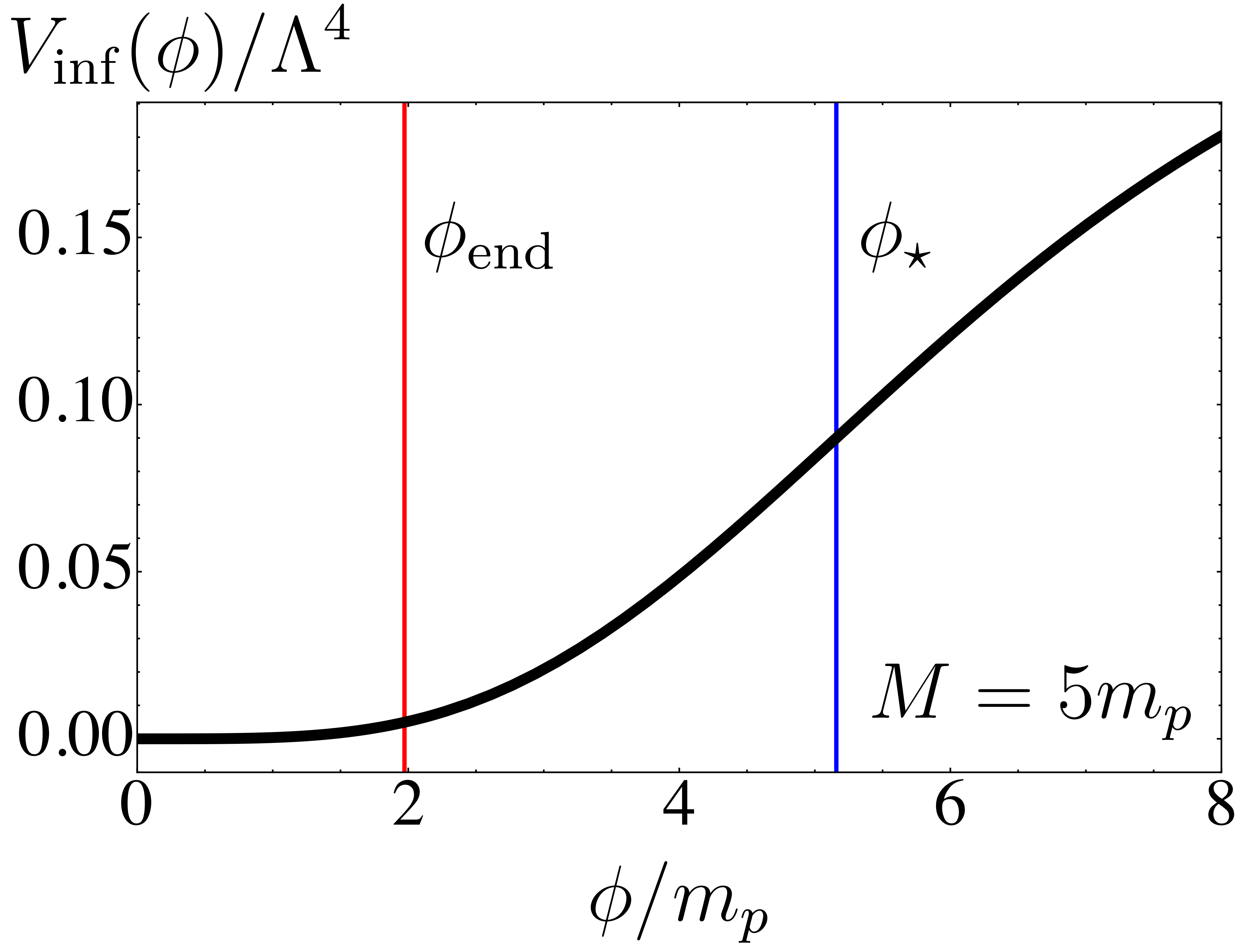
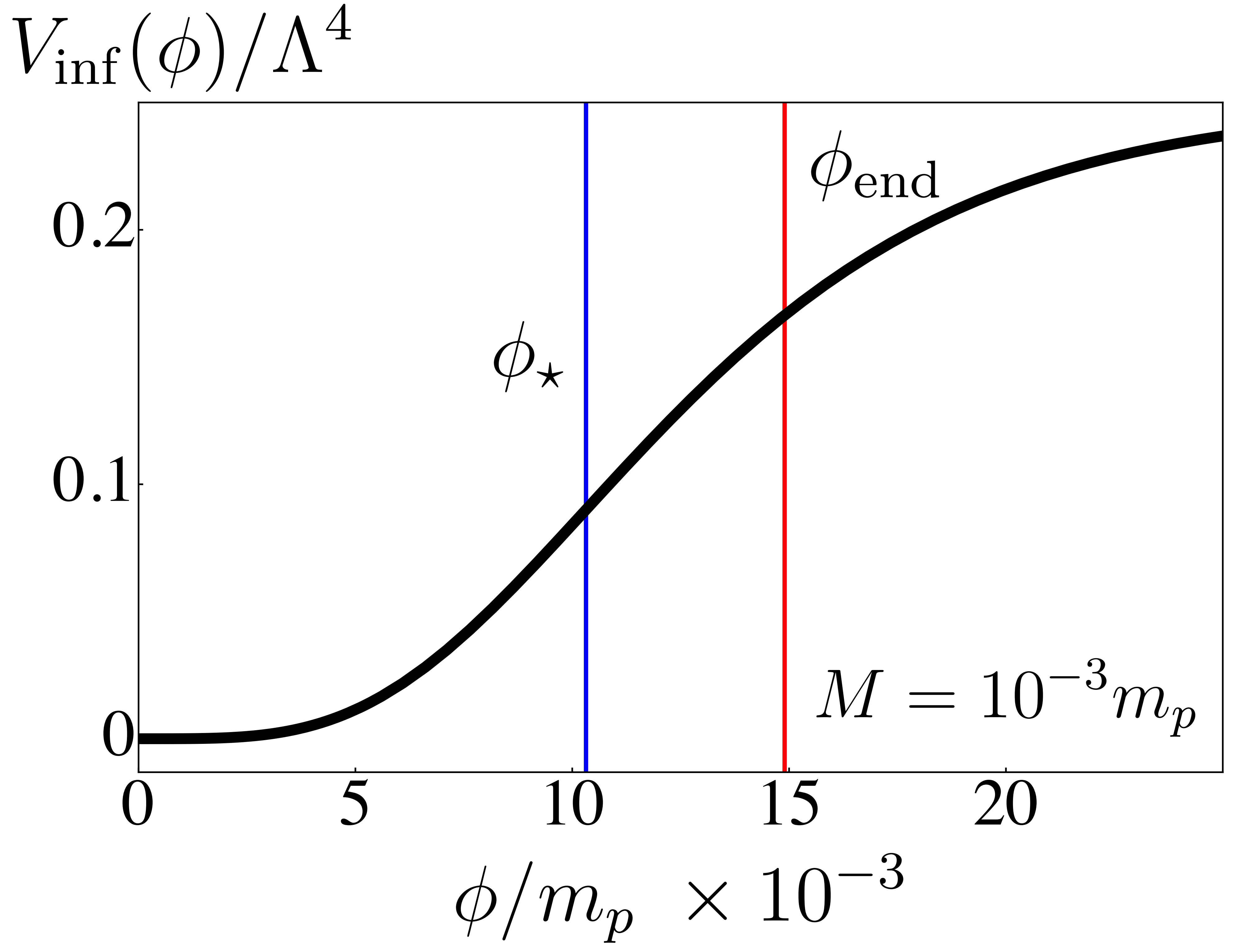
1st: Inflation ends at field value above inflection point





Inflaton oscillations

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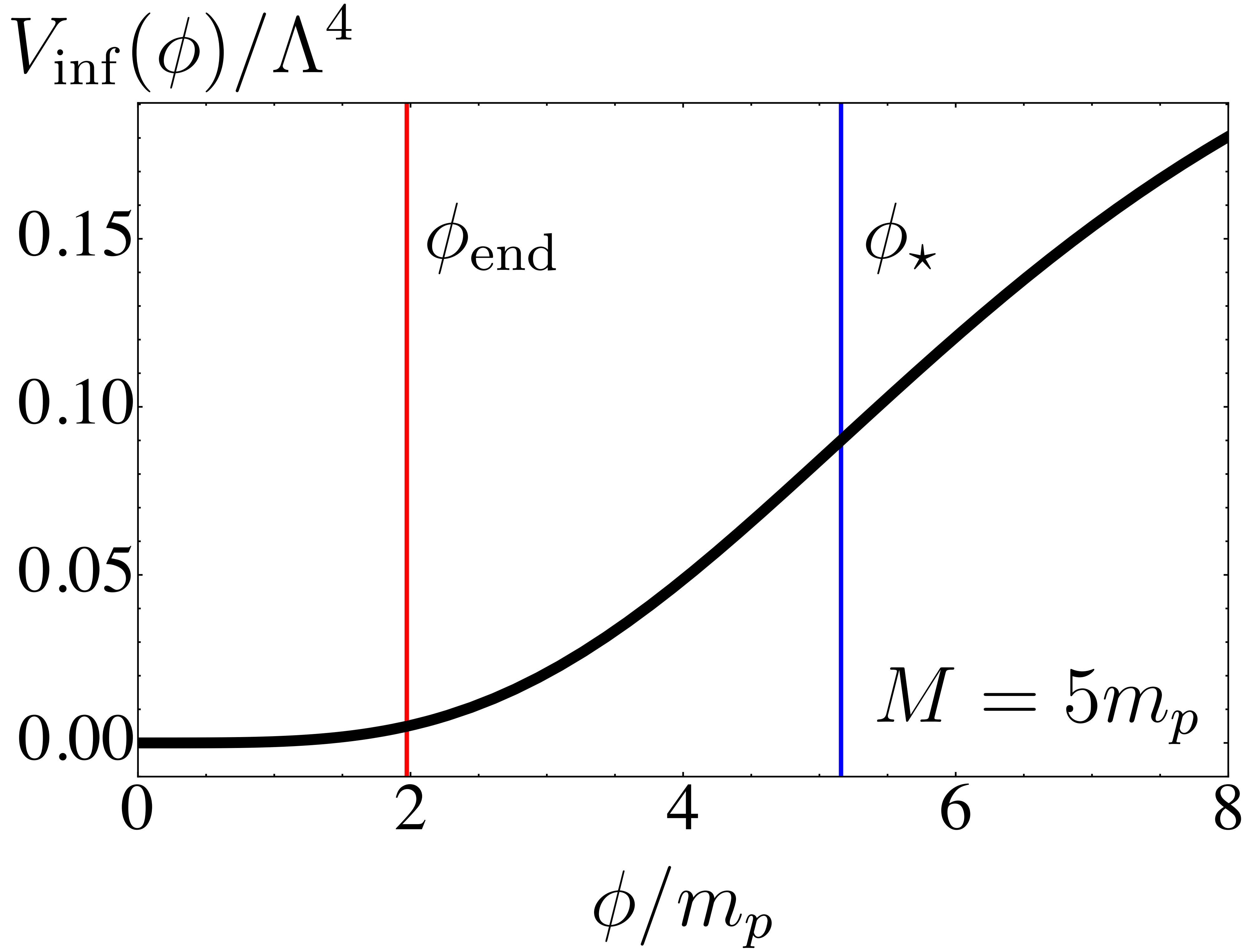
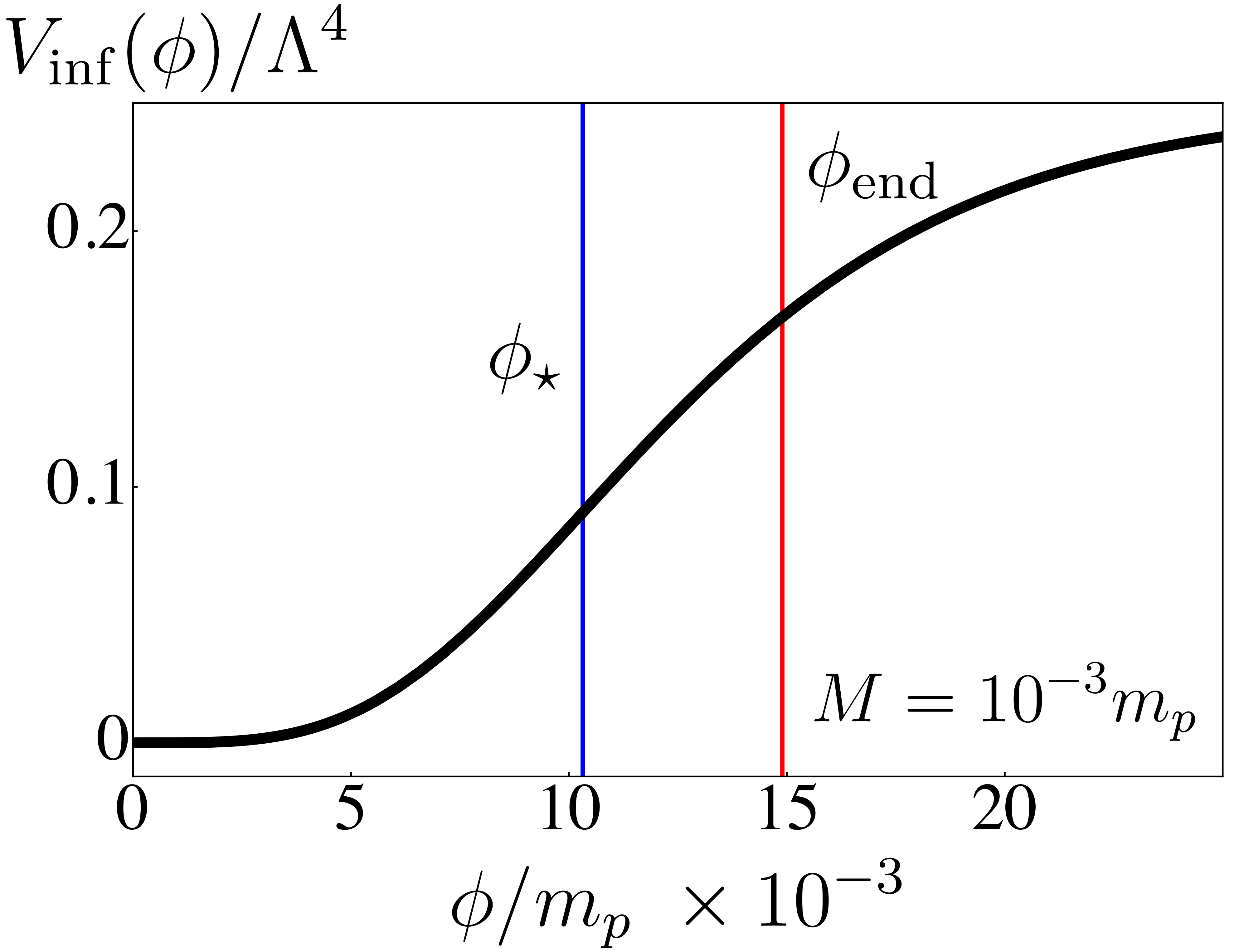
1st: Inflation ends at field value above inflection point

$$\frac{\phi_\star}{M} = \operatorname{arcsinh}\left\{\sqrt{\frac{p-1}{2}}\right\}$$

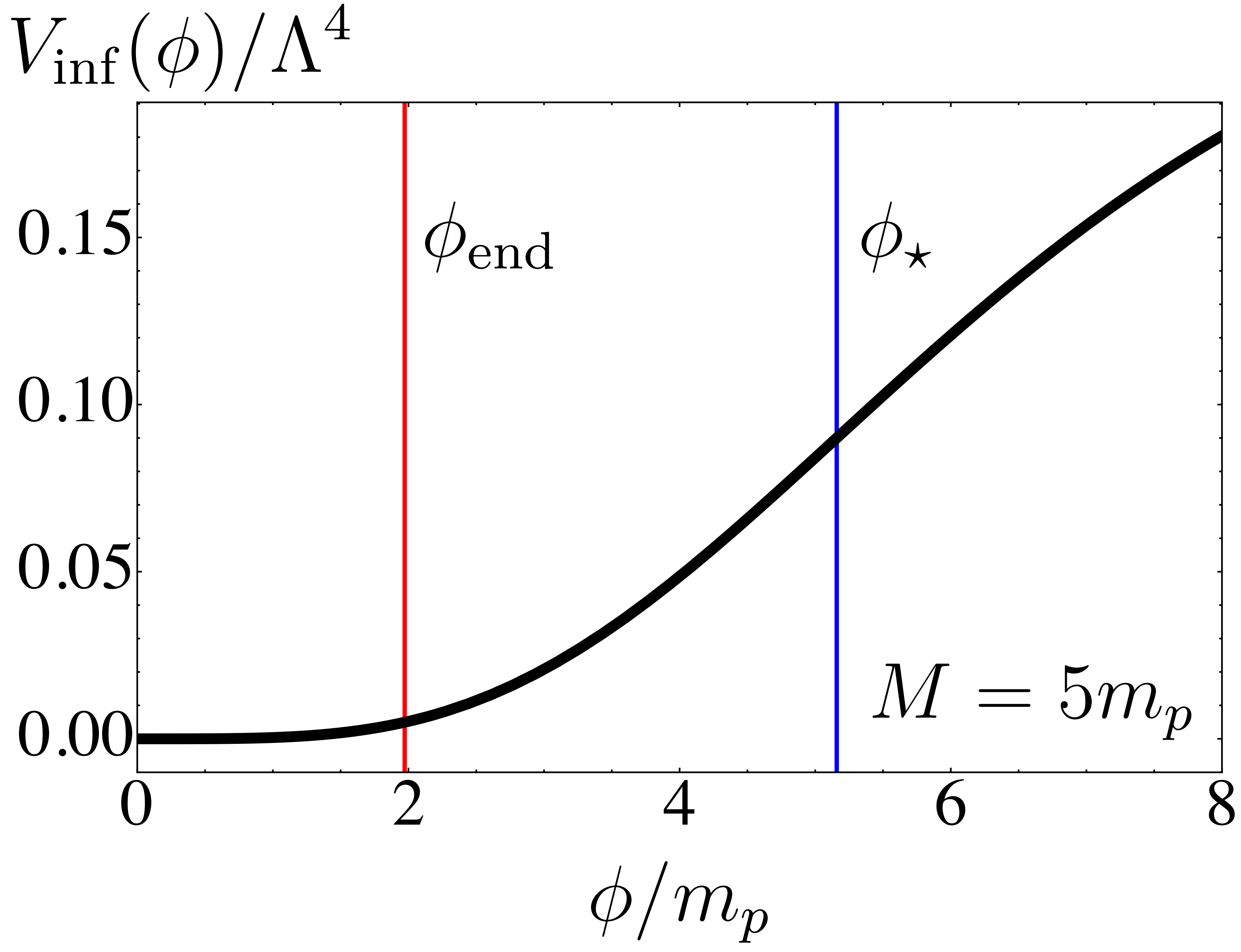




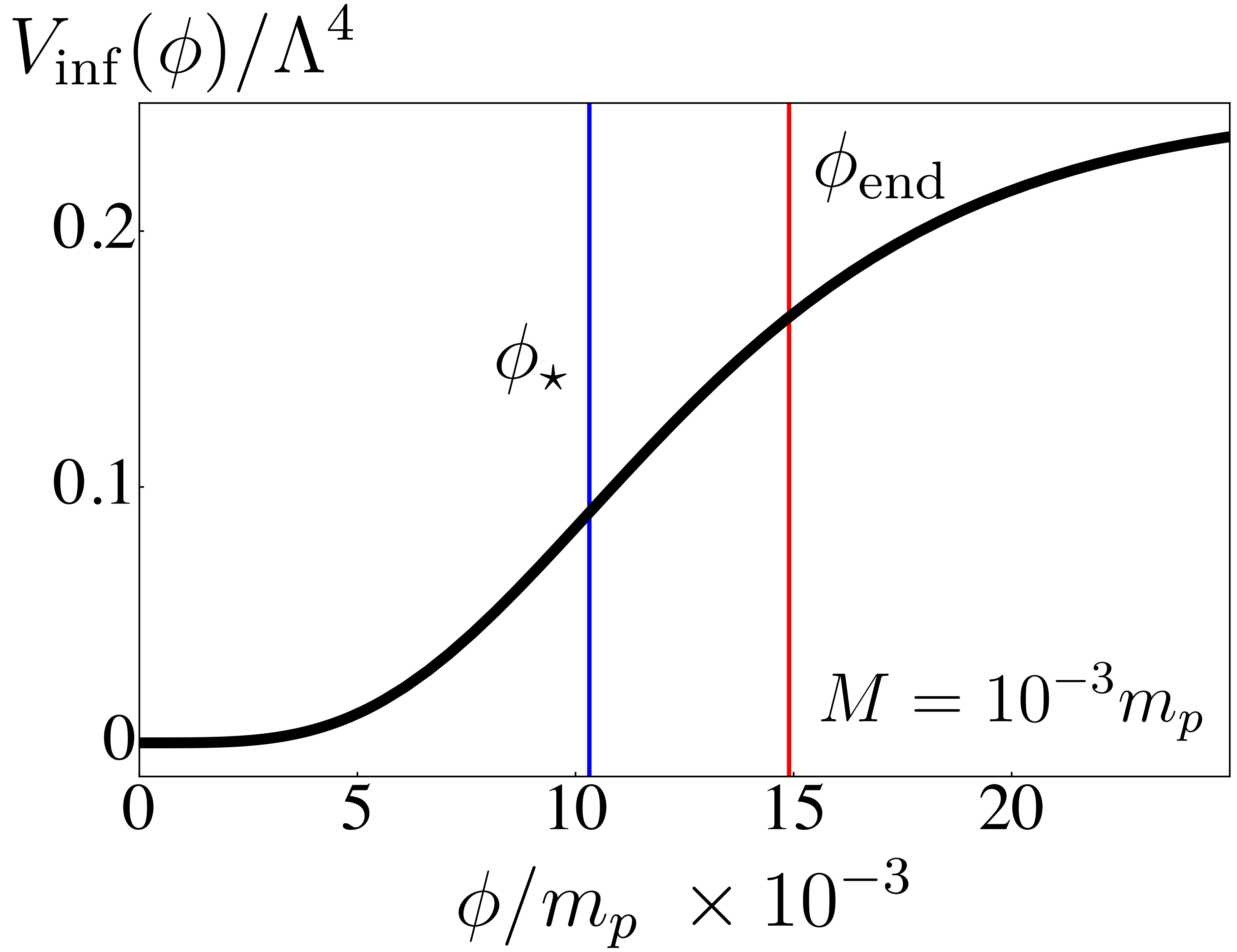
Inflaton oscillations

How are inflation oscillations affected by lowering M?

oscillations dominated by monomial potential $\sim \phi^p$



First oscillations dominated by potential energy



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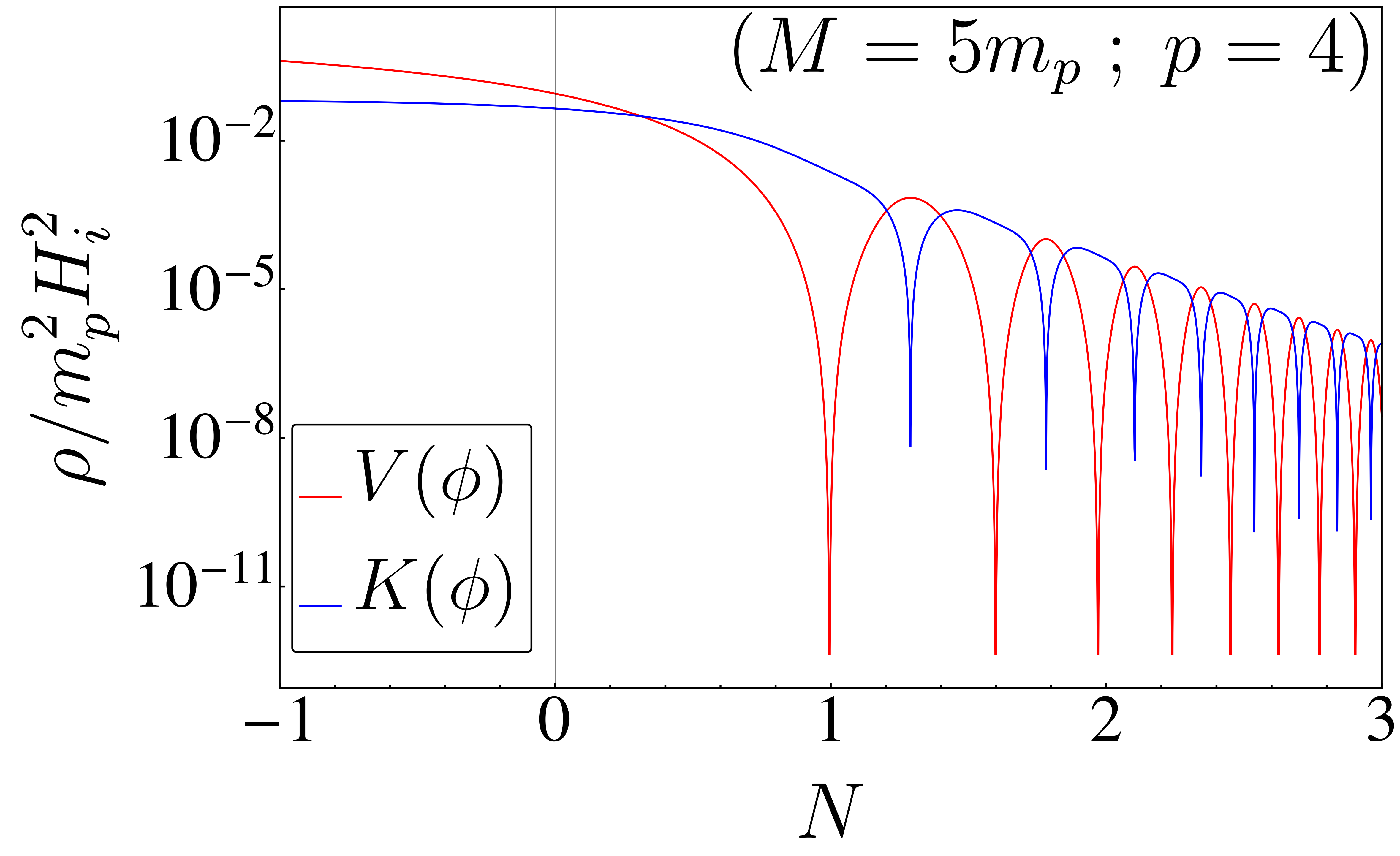

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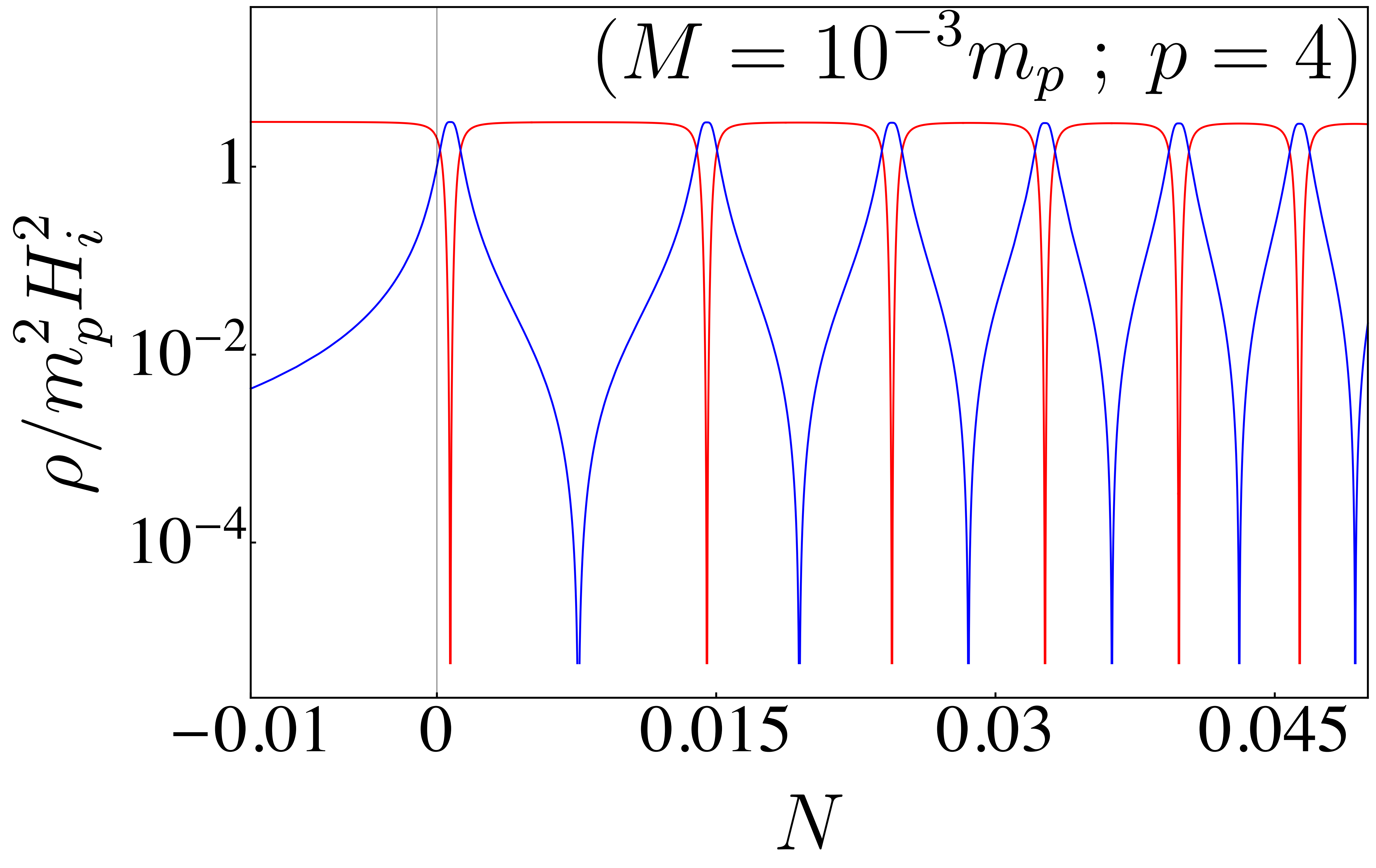

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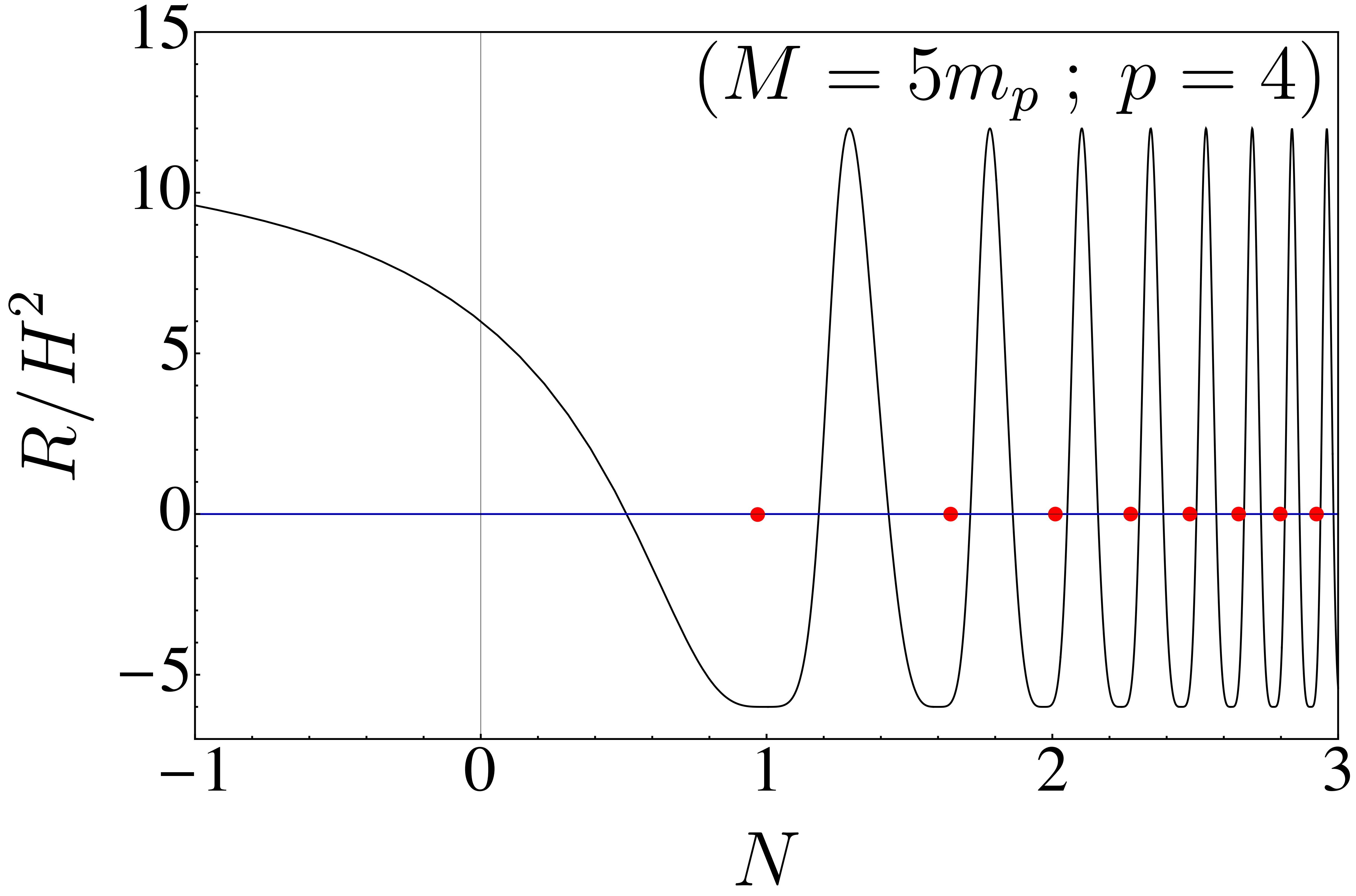

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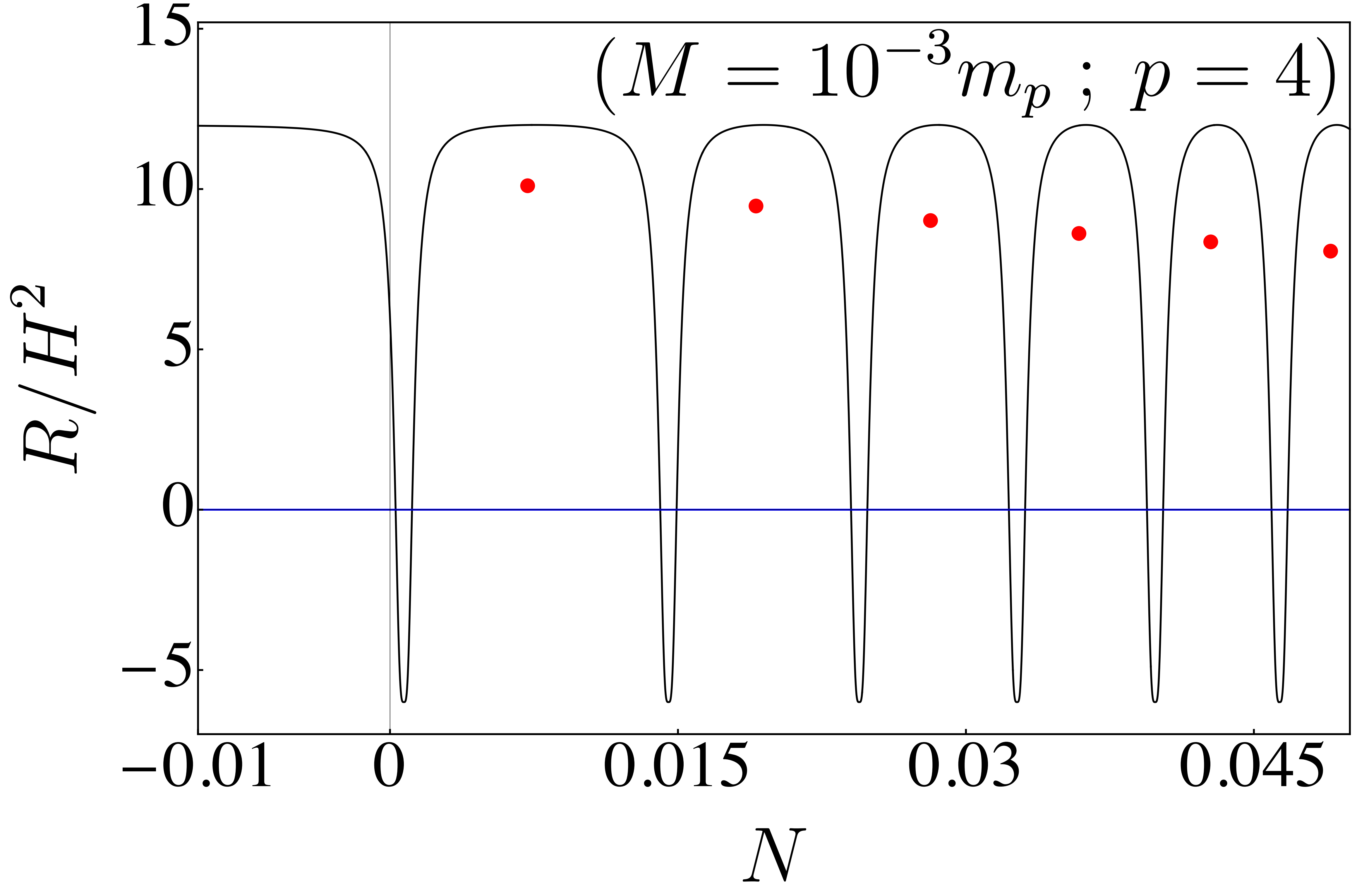


Inflaton oscillations

We can relate Ricci oscillations to the Equation of State (EoS)
w

For potentials of the kind $\sim \phi^p$

$$w_{\rm osc} \equiv \frac{\overline{p_\phi}}{\overline{\rho_\phi}} \simeq \frac{p-2}{p+2}$$

Ricci scalar normalized by Hubble square is

$$\frac{\overline{R}}{\overline{H^2}} = 3(1-3w_{\rm osc}) \simeq 6\frac{(4-p)}{(p+2)}$$

Inflaton oscillations

We can relate Ricci oscillations to the Equation of State (EoS) w

For potential α-attractors

$$w_{\rm osc} \simeq \frac{p-2}{p+2} \quad \text{for} \quad M \gtrsim m_p$$

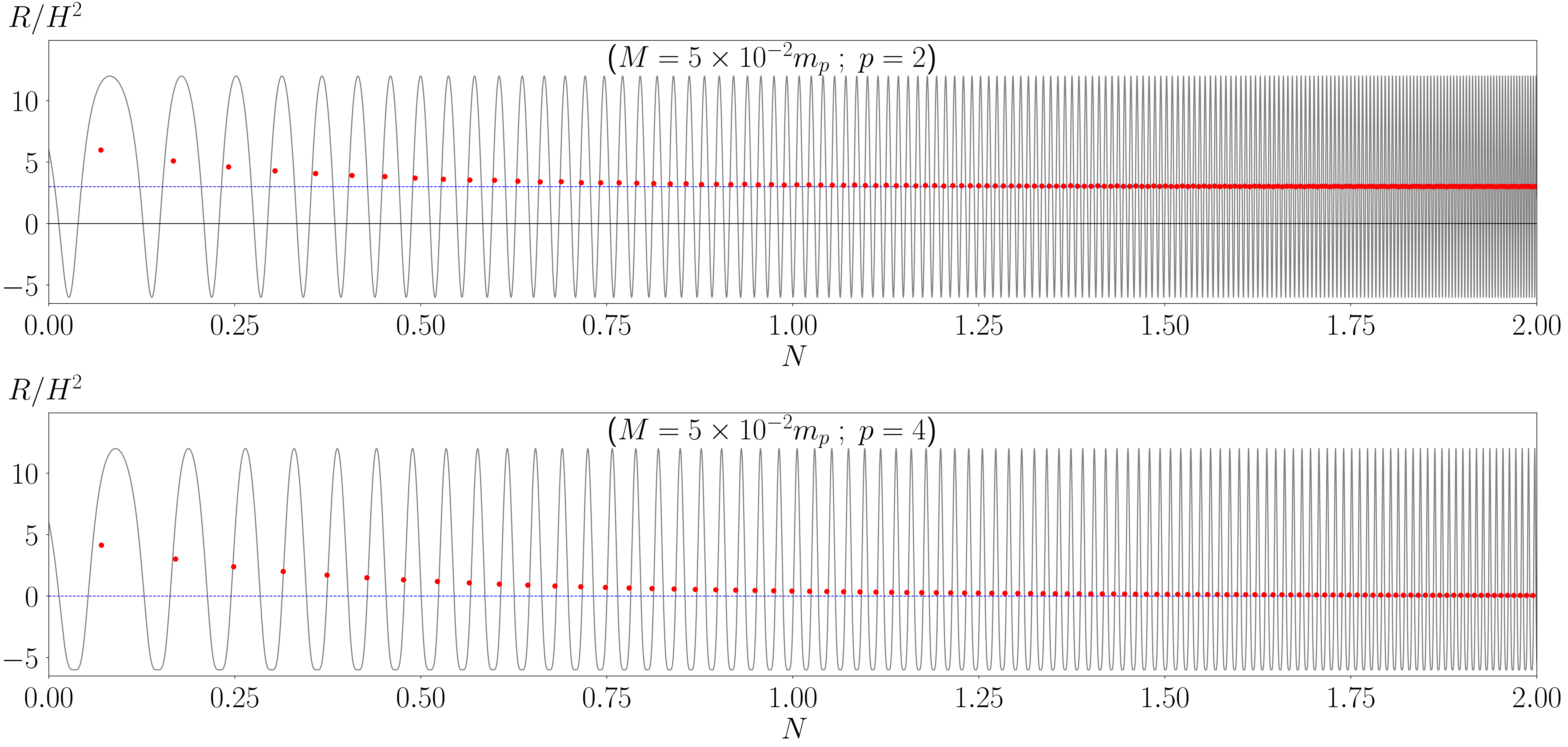
$$w_{\rm osc} \simeq \text{(increasing function)} \to \frac{p-2}{p+2} \quad \text{for} \quad M \lesssim m_p$$

Ricci scalar normalized by Hubble square is

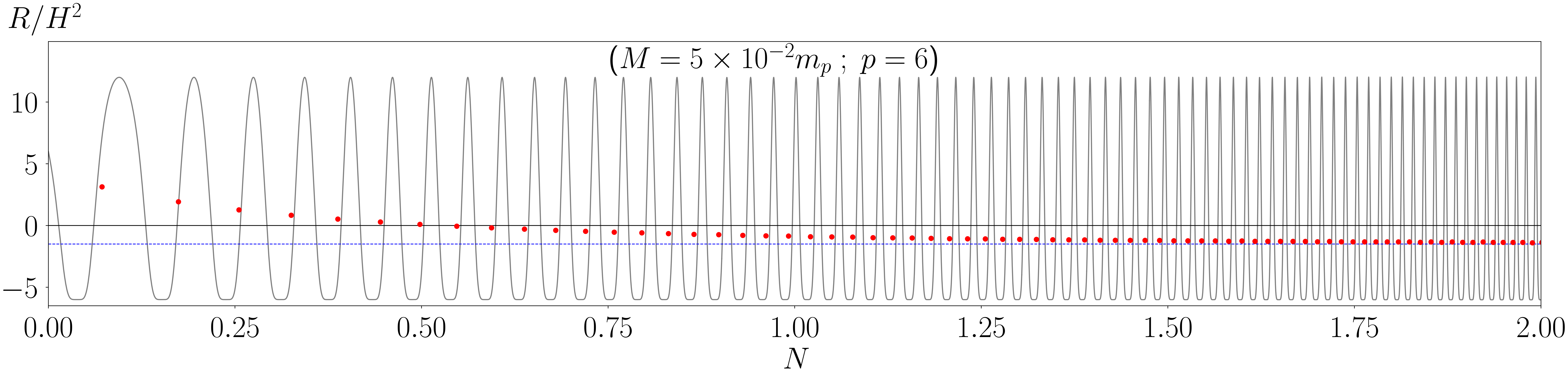
$$\frac{\overline{R}}{\overline{H^2}} \simeq 6\frac{(4-p)}{(p+2)} \quad \text{for} \quad M \gtrsim m_p$$

$$\frac{\overline{R}}{\overline{H^2}} \simeq \text{(decreasing function)} \to 6\frac{(4-p)}{(p+2)} \quad \text{for} \quad M \lesssim m_p$$

Inflaton oscillations

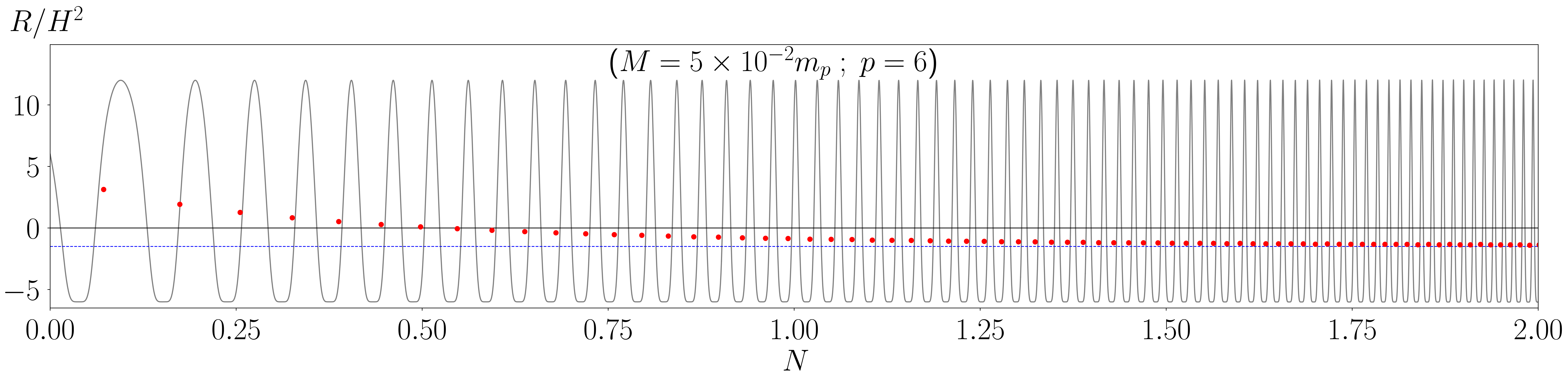


Inflaton oscillations



We are interested in those cases that lead to NEGATIVE Ricci oscillations

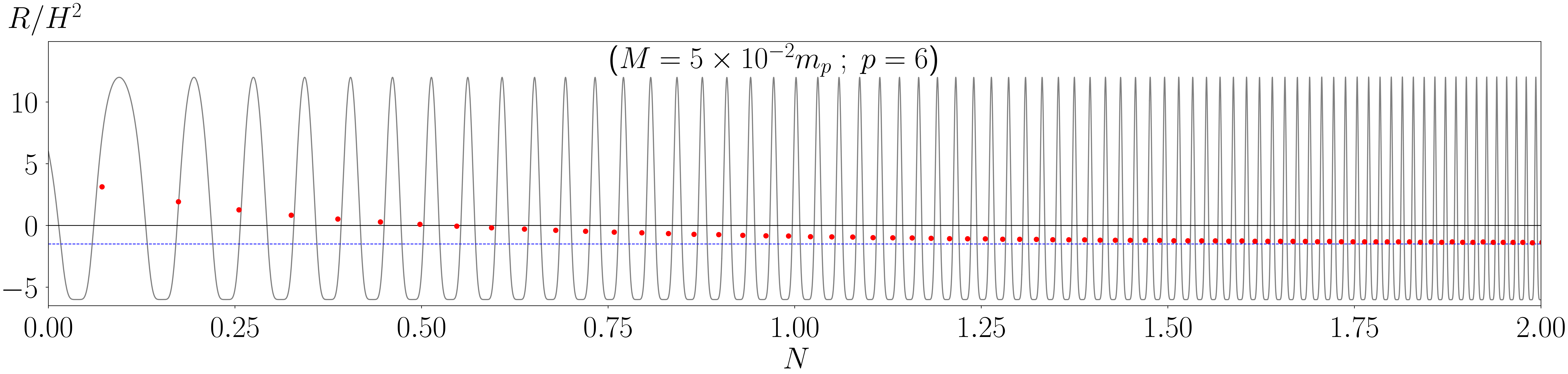
Inflaton oscillations



We are interested in those cases that lead to NEGATIVE Ricci oscillations

$$\frac{\overline{R}}{\overline{H^2}} < 0 \implies p > 4$$

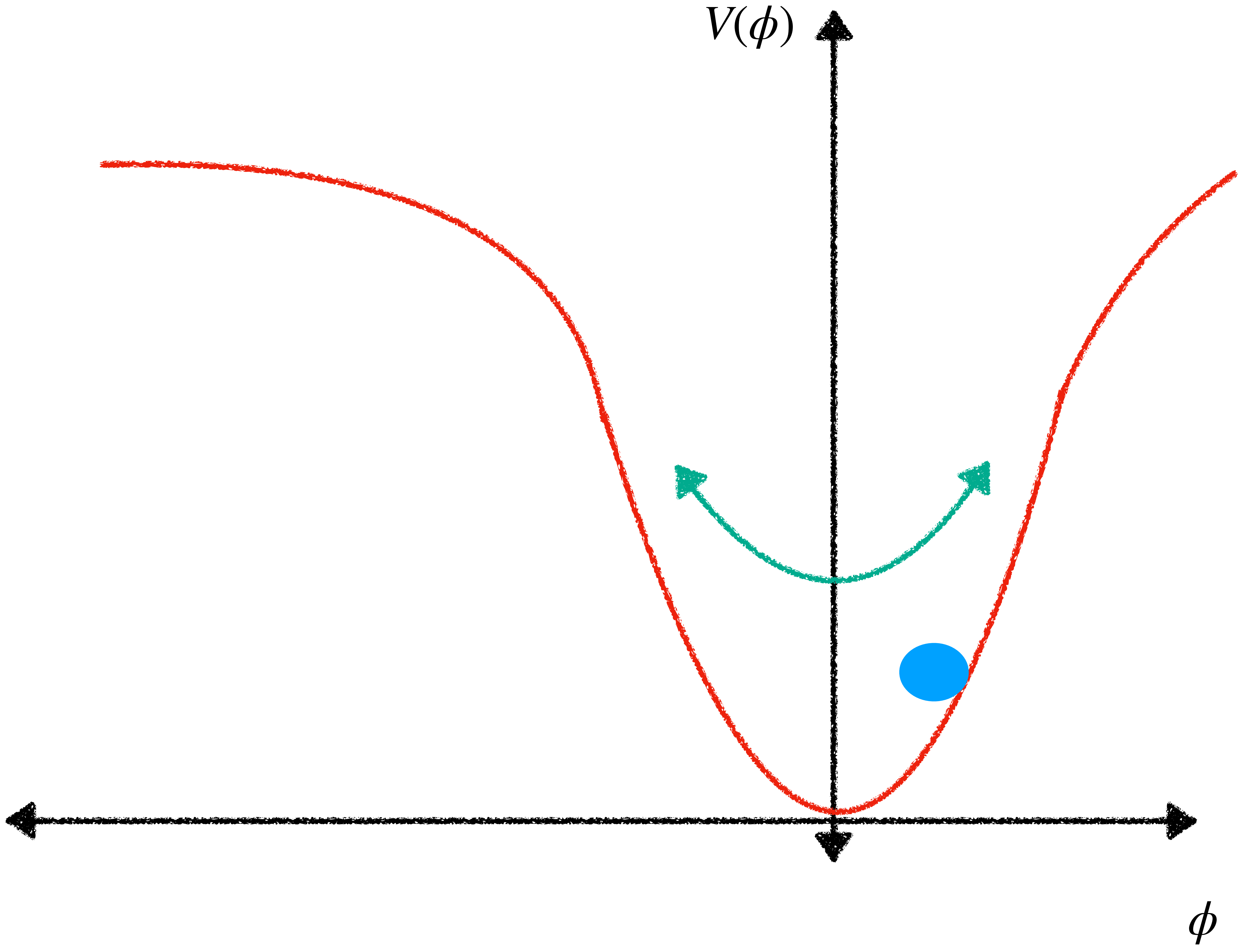
Inflaton oscillations



We are interested in those cases that lead to NEGATIVE Ricci oscillations

$$\frac{\overline{R}}{\overline{H^2}} < 0 \implies \bar{w} > 1/3$$

Inflaton fragmentation



Inflaton inhomogeneous perturbations $\delta\phi(\mathbf{x}, t)$ couple to the background

$$\delta\ddot{\phi}_k + \left[\kappa^2 + (p-1)\,|\,\phi\,|^{p-2}\right]\delta\phi_k = 0$$

Inefficient effect

Fragmentation time scale very long $N_{\rm frag} \sim \mathcal{O}(1-10)$

$$\delta\ddot{\phi}_k + \left[\kappa^2 + \frac{\partial^2 V(\phi)}{\partial\phi^2}\bigg|_{\phi_{\rm end}}\right]\delta\phi_k = 0$$

Efficient effect

Fragmentation time scale very short $N_{\rm frag} \sim \mathcal{O}(10^{-2})$

Fragmentation $\Longrightarrow$ $w = 1/3$

Lozanov, Amin '17
Antusch, Figueroa, Marschall, Torrenti '20

Geometric preheating: Linear regime

Let us consider the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}m_p^2 R - \frac{1}{2}\xi R\chi^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi,\chi)\right)$$

Non-minimal coupled field EoM in α-time

$$\chi'' + (3-\alpha)\mathcal{H}\chi' - a^{-2(1-\alpha)}\nabla^2\chi + a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right) = 0$$

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Non-minimal coupled field EoM in α-time

$$\chi'' + (3-\alpha)\mathcal{H}\chi' - a^{-2(1-\alpha)}\nabla^2\chi + a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right) = 0$$

In the linear regime we can focus on the χ_k modes

$$\tilde{\chi}_k'' + \omega_k^2\,\tilde{\chi}_k = 0\,, \qquad \text{with} \quad \omega_k^2 \equiv k^2 + a^2\left(\xi - \frac{1}{6}\right)R$$

Geometric preheating: Linear regime

$$\tilde{\chi}_k'' + \omega_k^2 \, \tilde{\chi}_k = 0 \,, \qquad \text{with} \quad \omega_k^2 \equiv k^2 + a^2\left(\xi - \frac{1}{6}\right)R$$

Whenever $\omega_k^2 < 0$ modes grow exponentially

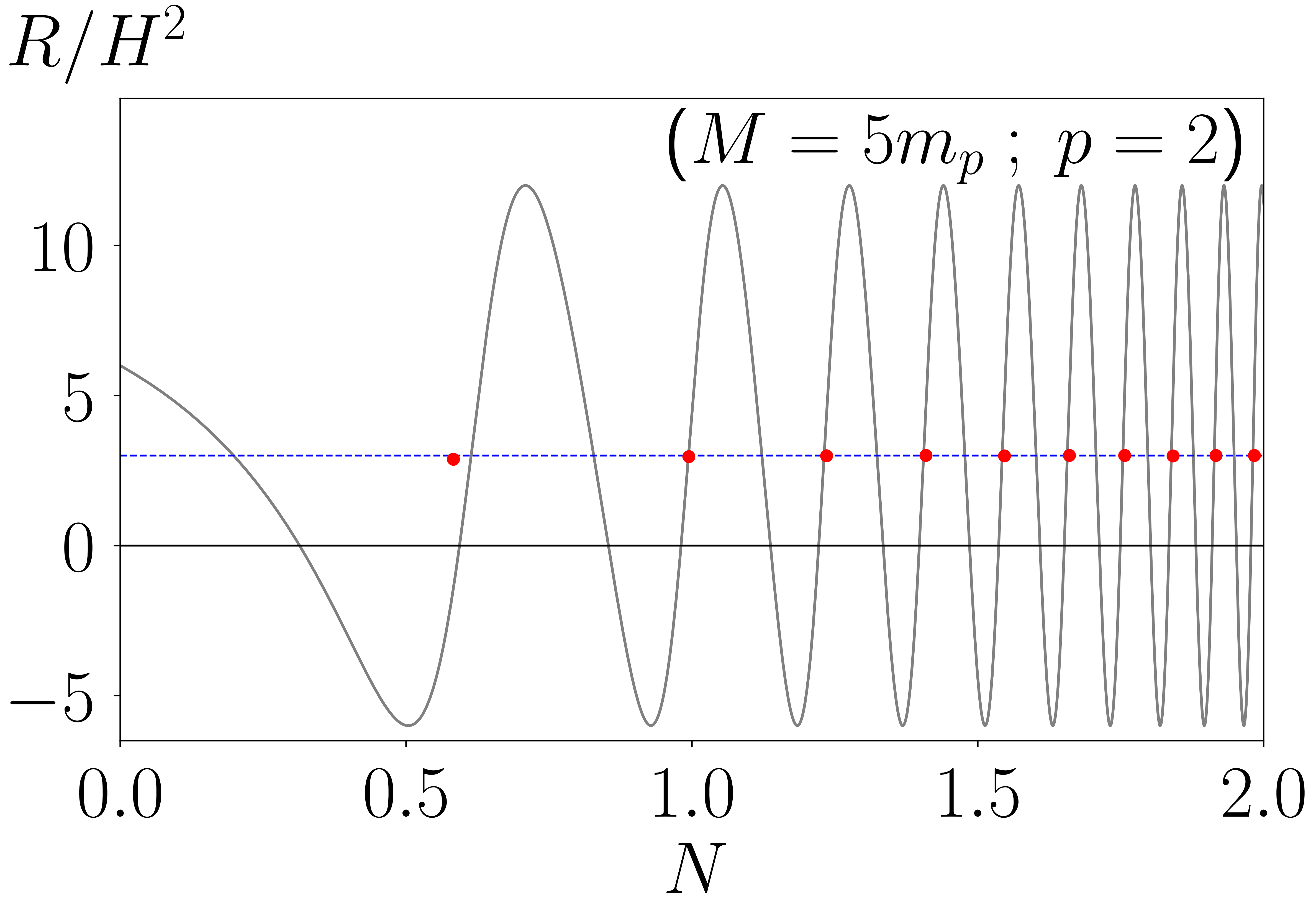
$$\frac{k_*}{aH} < \sqrt{6\xi - 1}$$

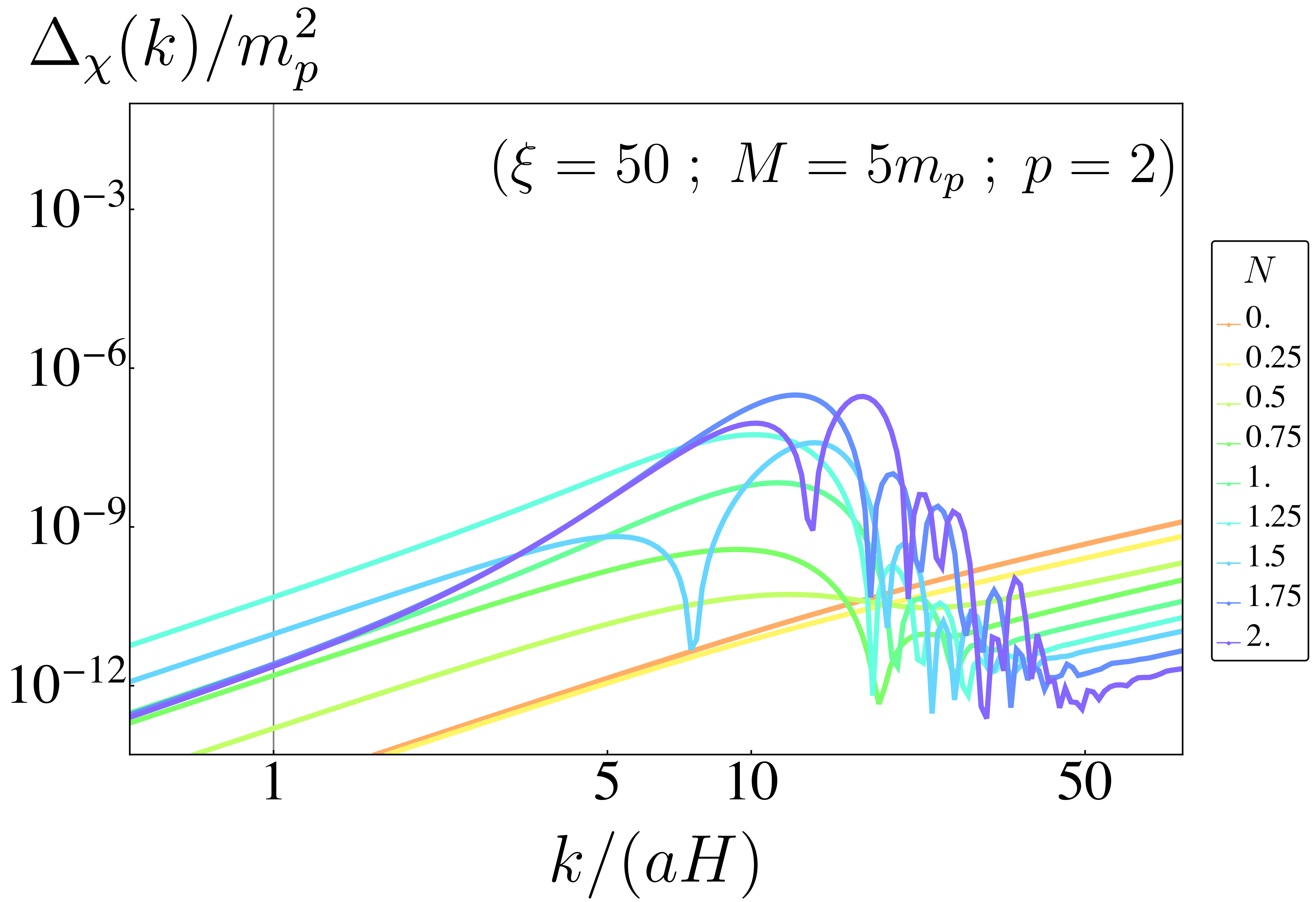
k_* is a treshold momenta

Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 2$

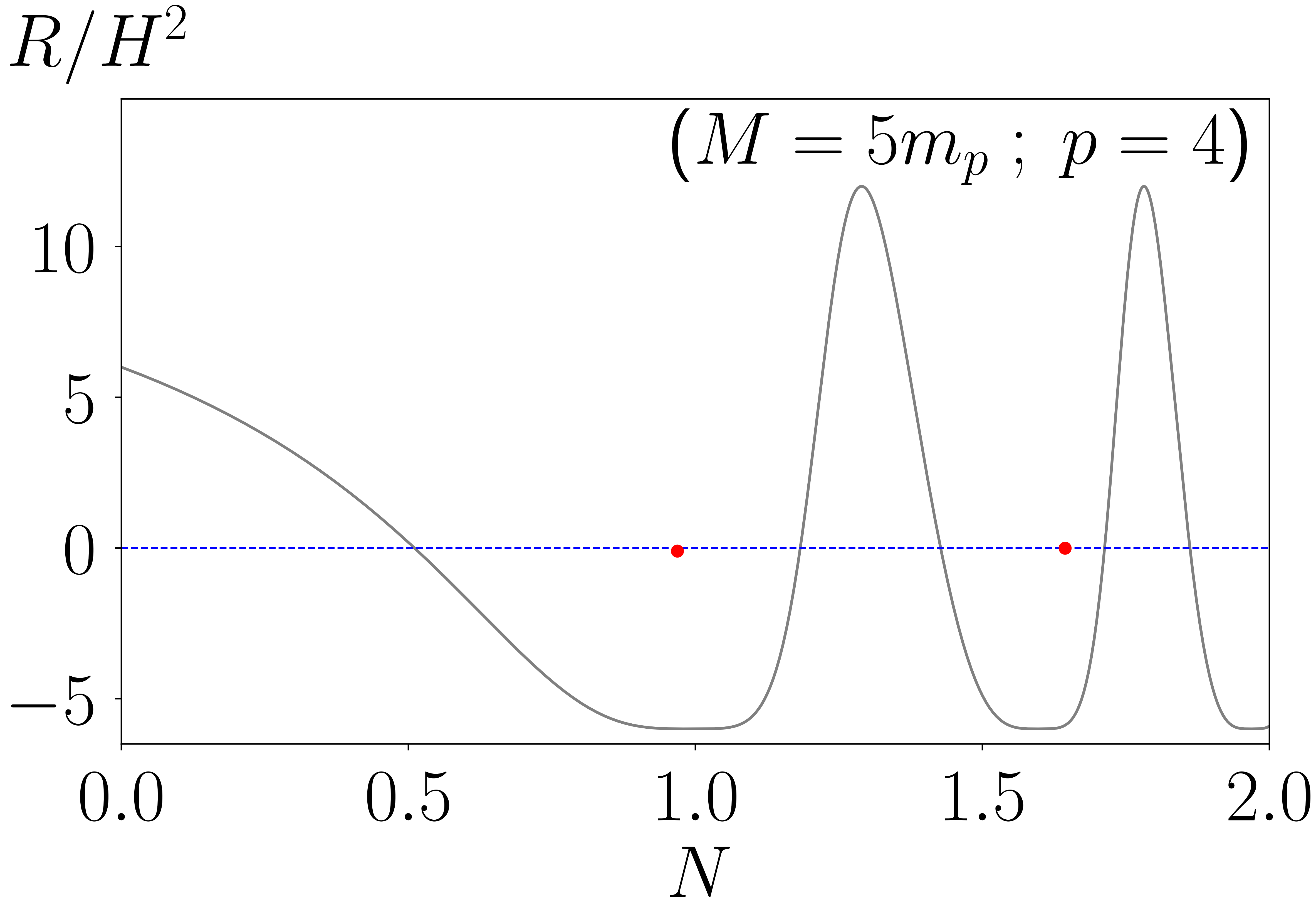


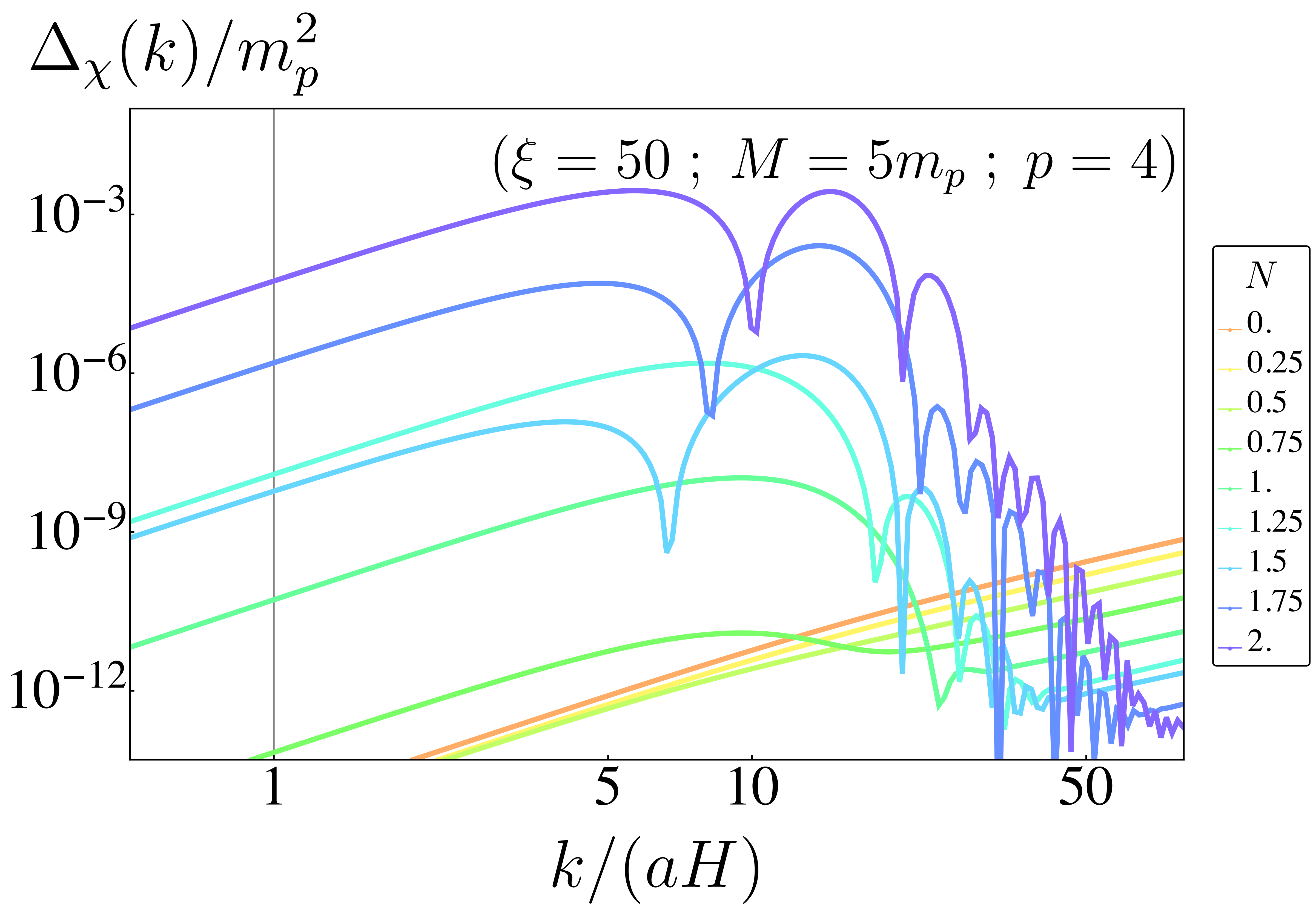


Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 4$

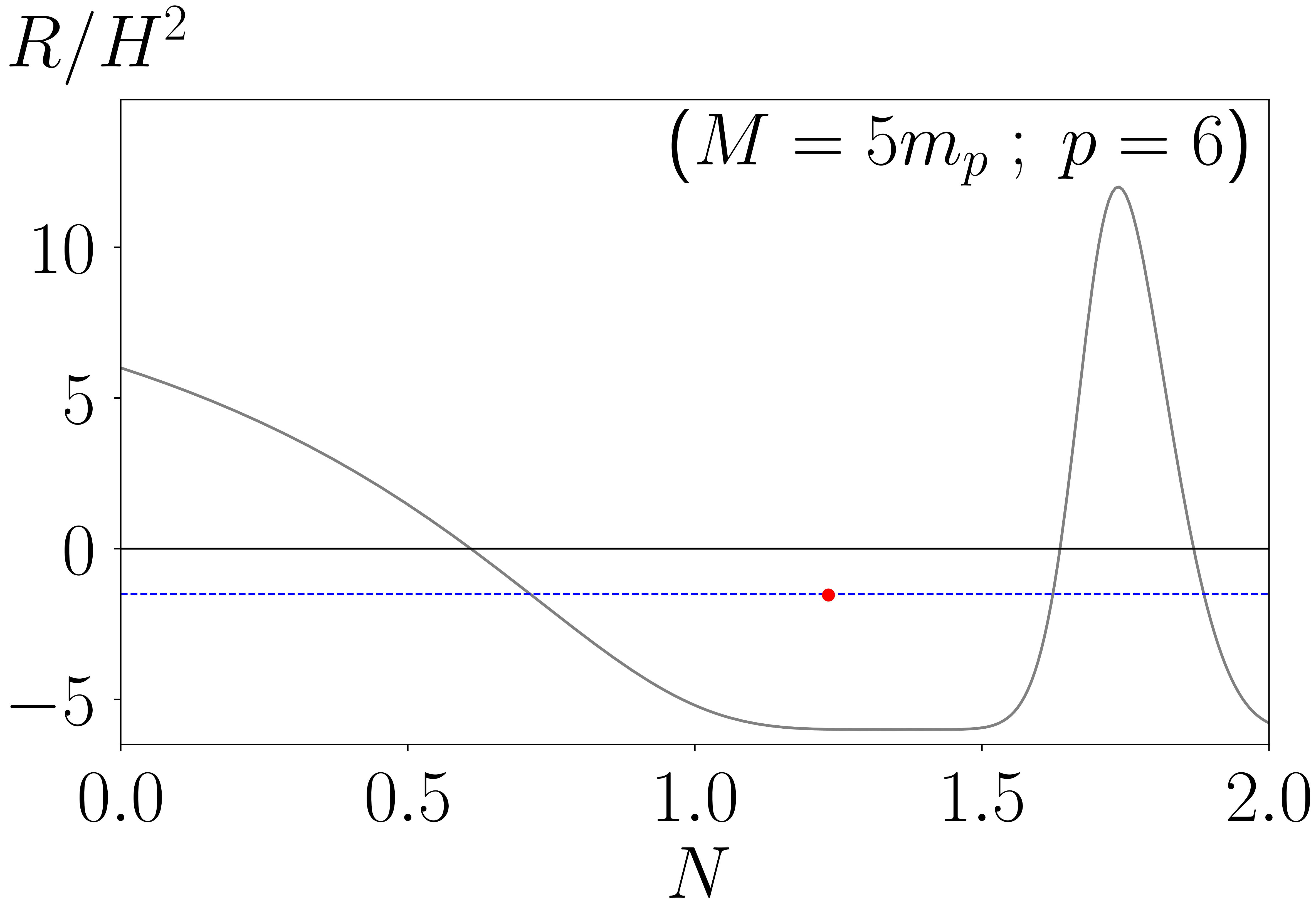


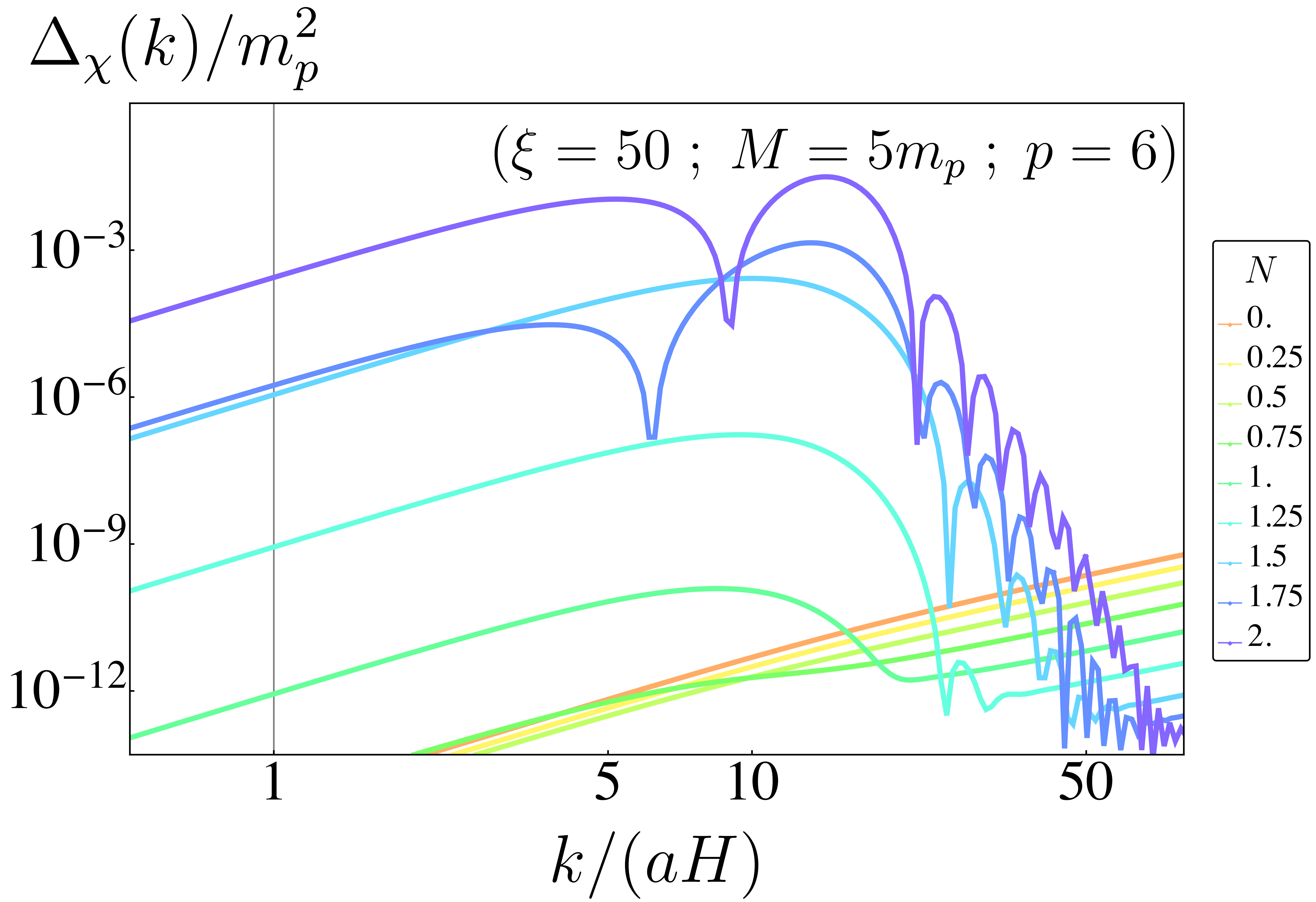


Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 6$

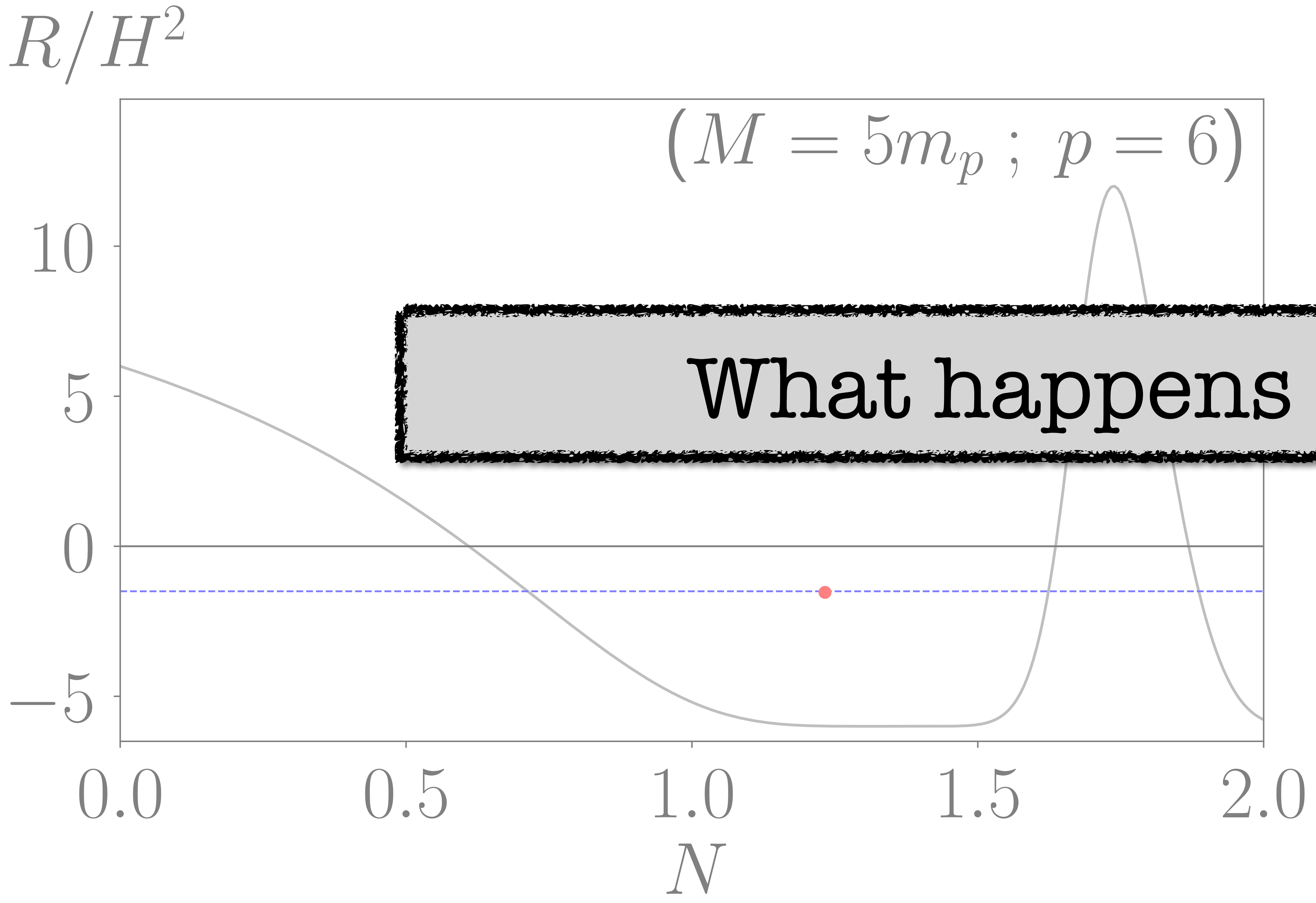


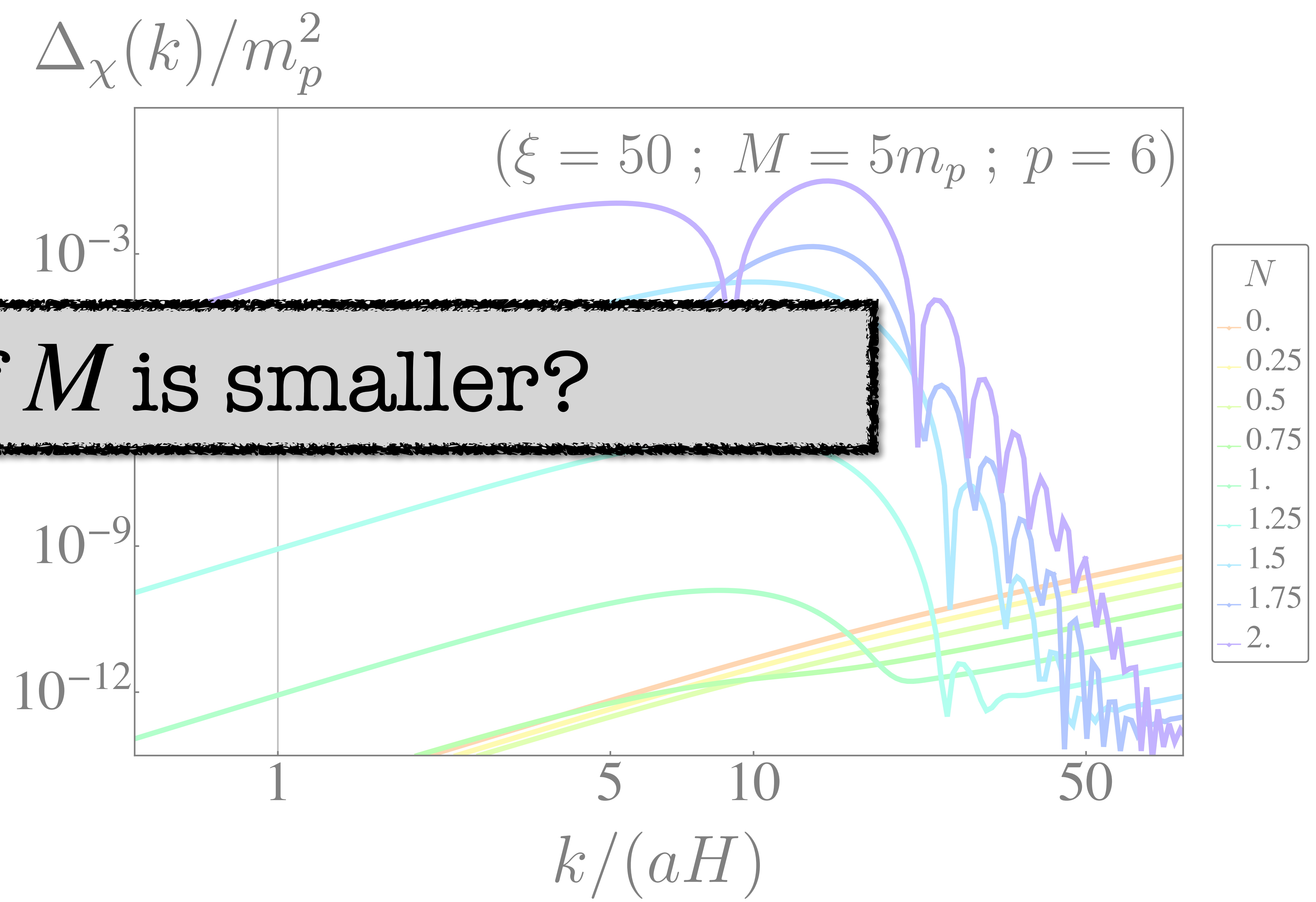


Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 6$



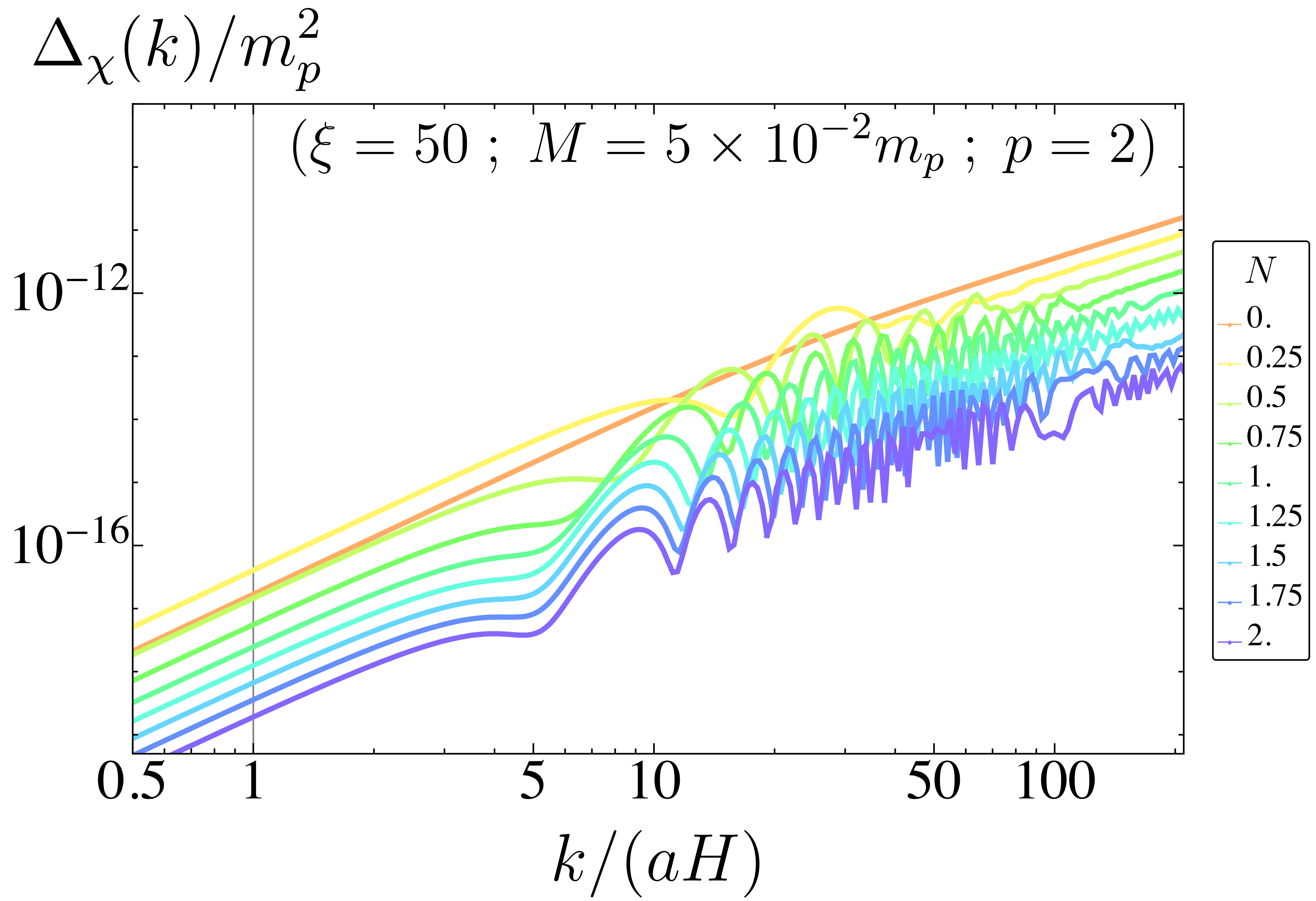


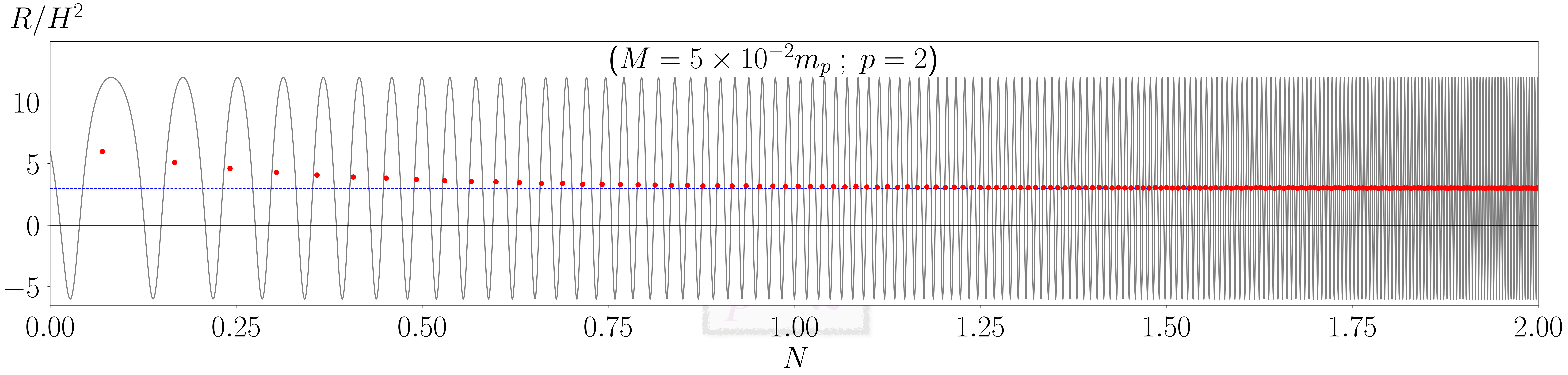
What happens if M is smaller?

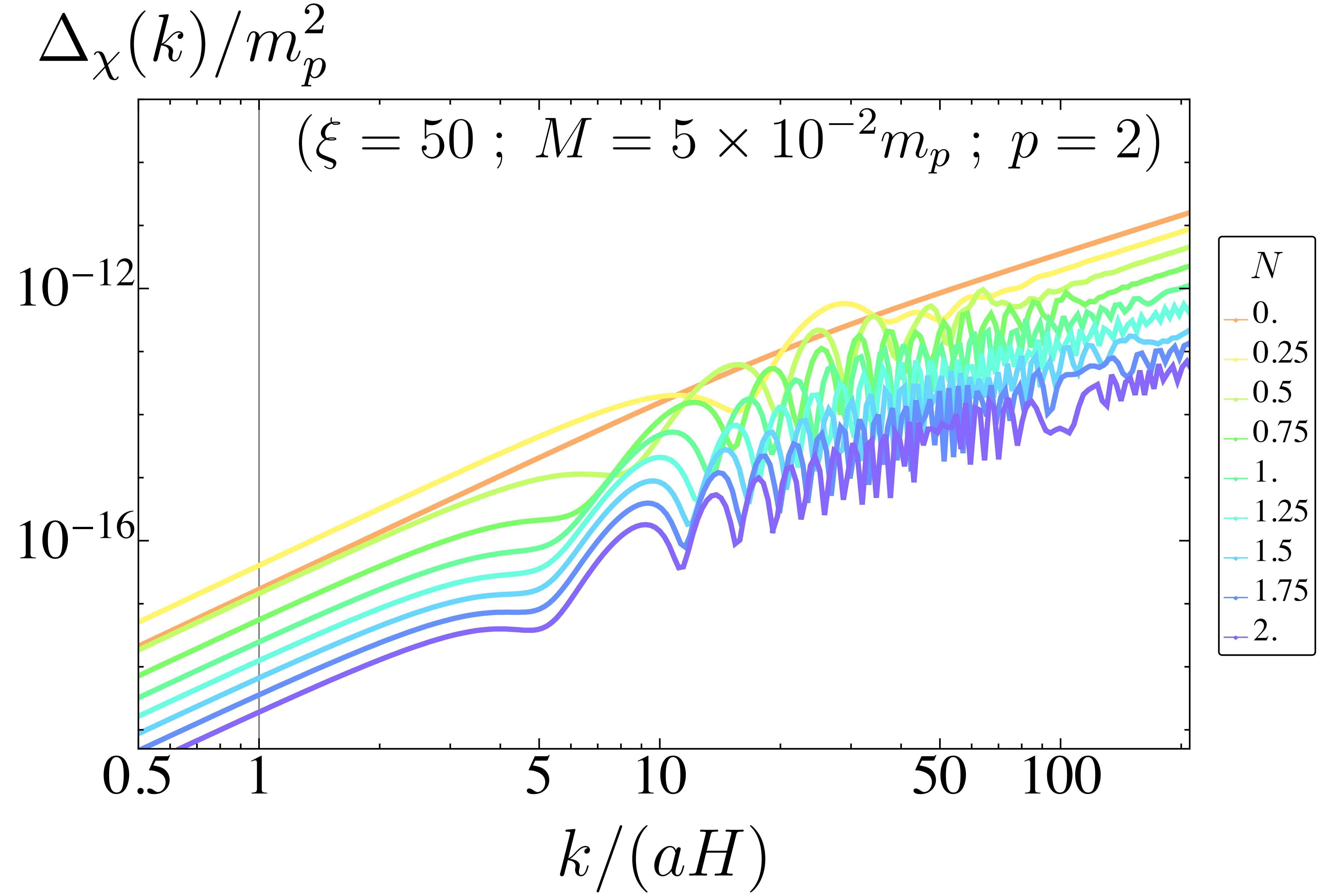
Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 2$



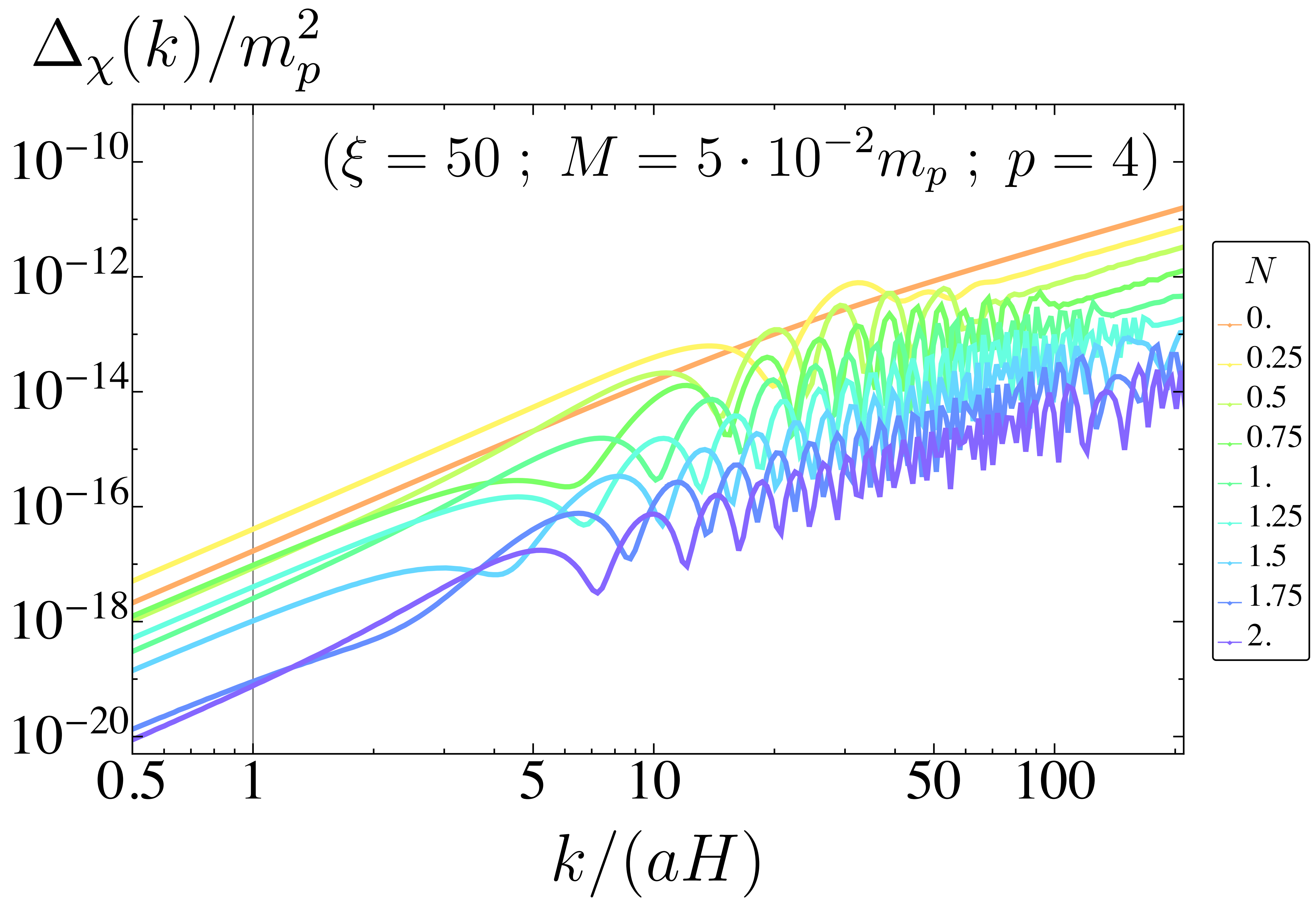

R/H^2
$(M = 5\times10^{-2}m_p\ ;\ p=2)$
10
5
0
−5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
N

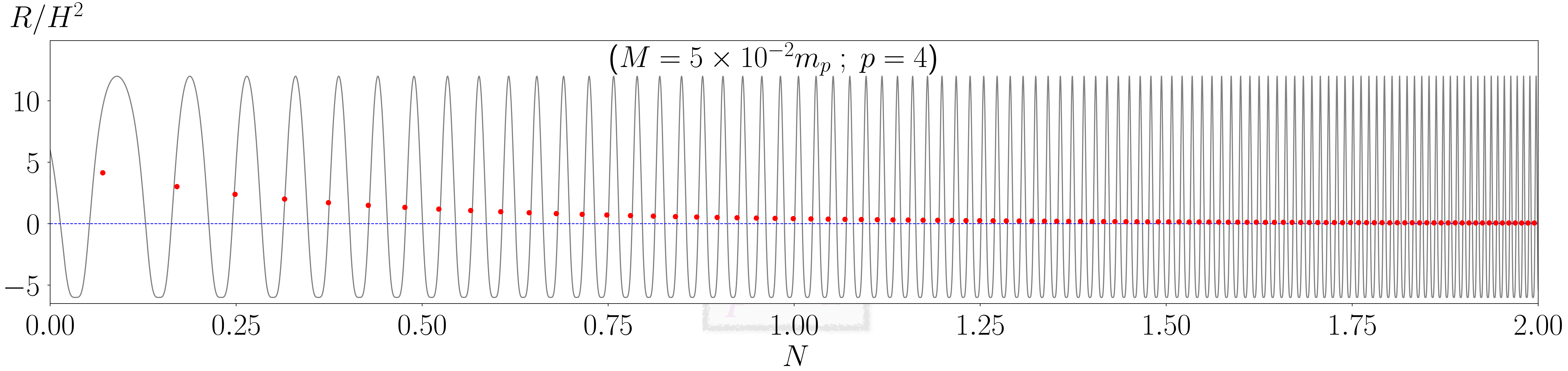

$\Delta_\chi(k)/m_p^2$
$(\xi = 50\ ;\ M = 5\times10^{-2}m_p\ ;\ p=2)$
10^{-12}
10^{-16}
0.5
1
5
10
50
100
$k/(aH)$
N
0.
0.25
0.5
0.75
1.
1.25
1.5
1.75
2.

Geometric preheating: Linear regime

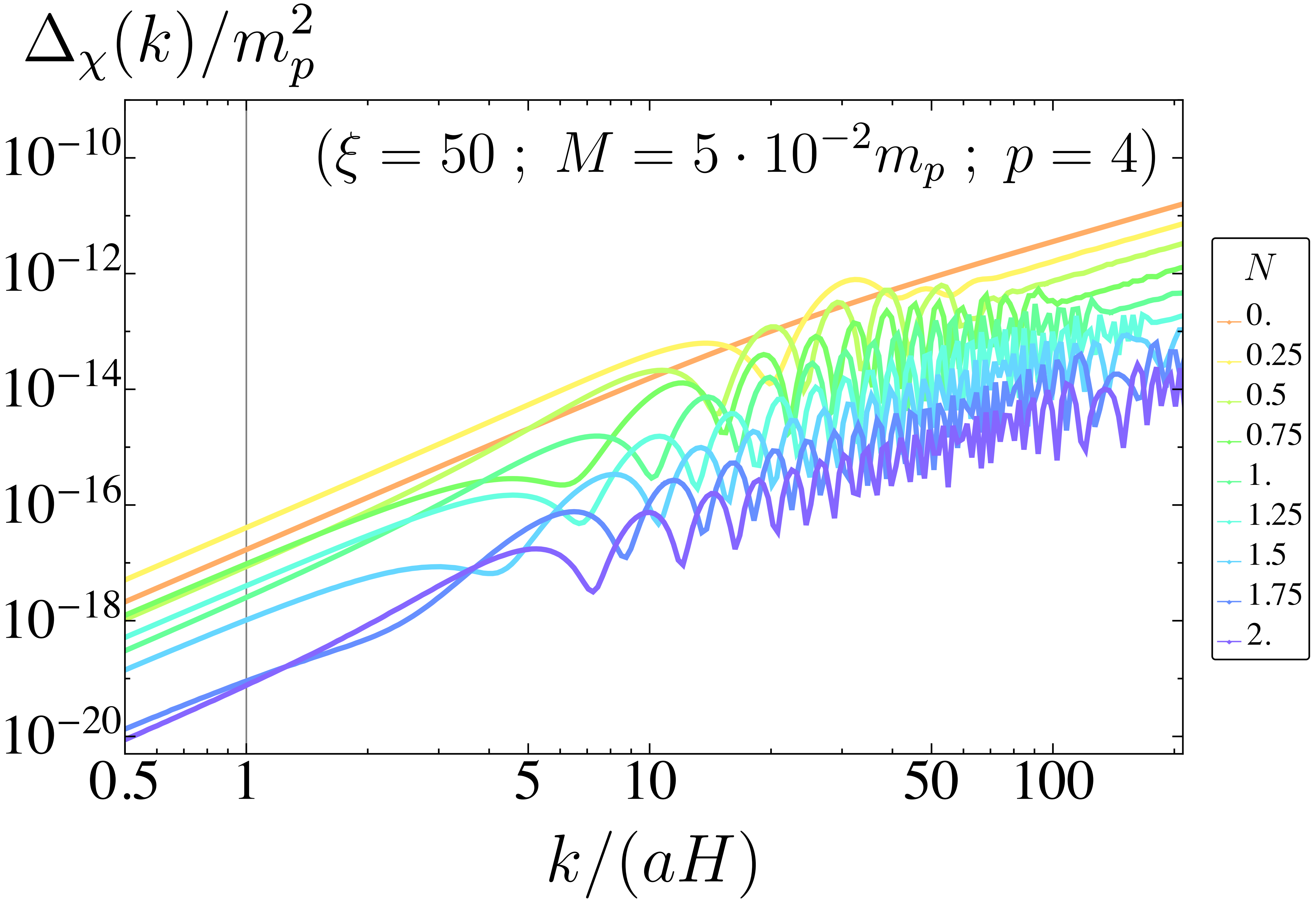
We can solve linear equation for χ_k modes

$p = 4$





R/H^2
$(M = 5 \times 10^{-2} m_p\ ;\ p = 4)$
10
5
0
−5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
N

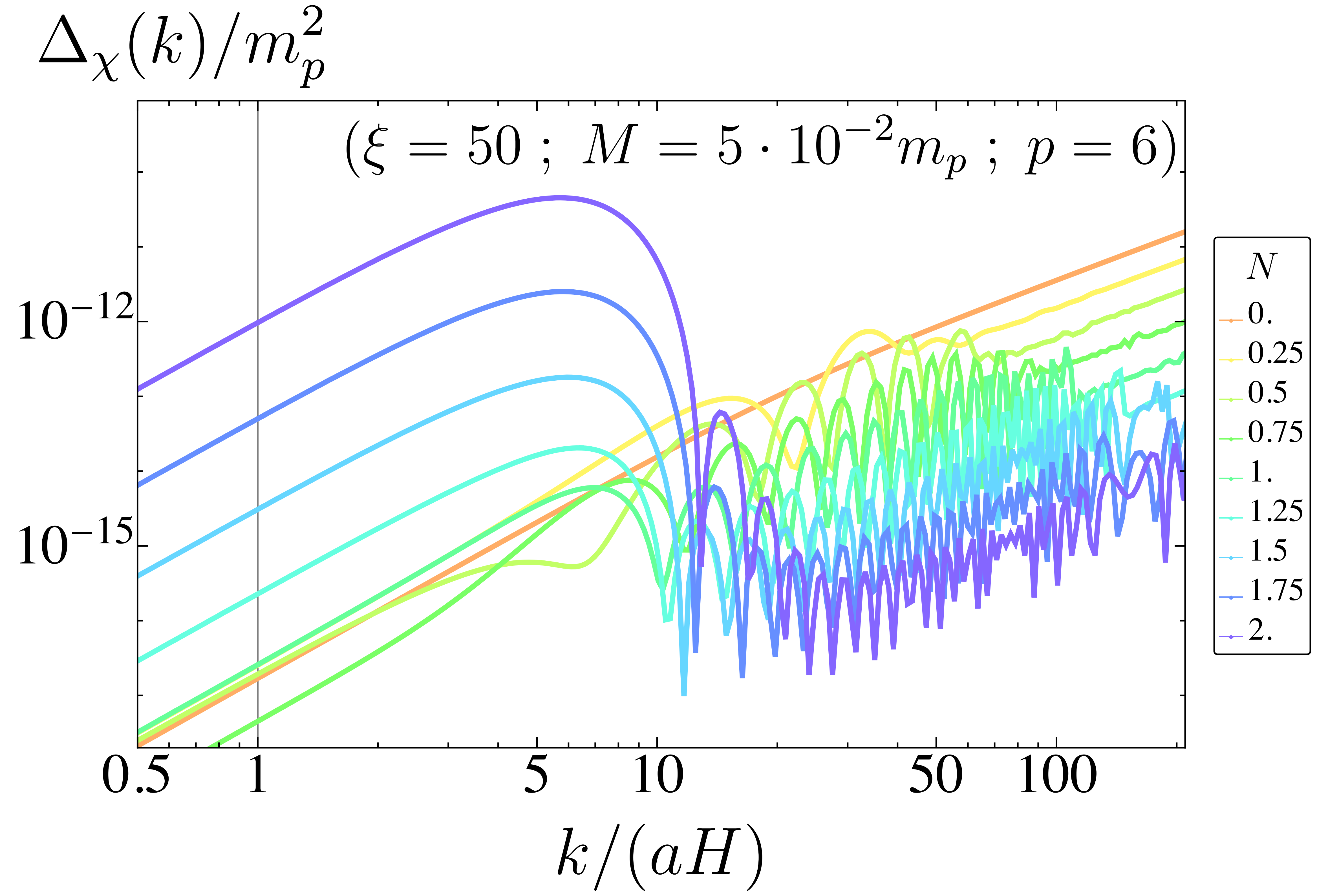


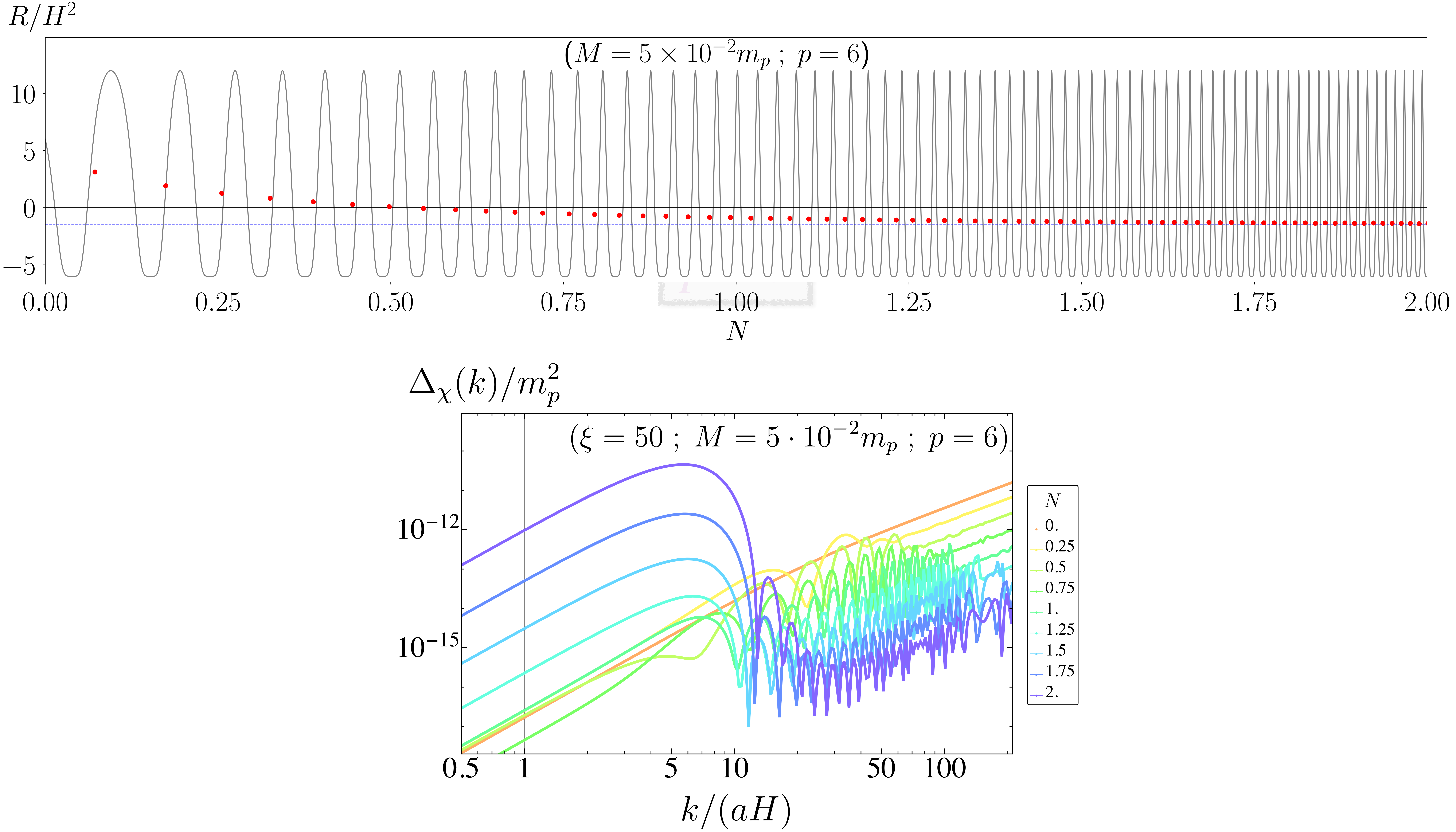
$\Delta_\chi(k)/m_p^2$
$(\xi = 50\ ;\ M = 5 \cdot 10^{-2} m_p\ ;\ p = 4)$
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
0.5
1
5
10
50
100
$k/(aH)$
N
0.
0.25
0.5
0.75
1.
1.25
1.5
1.75
2.

Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 6$





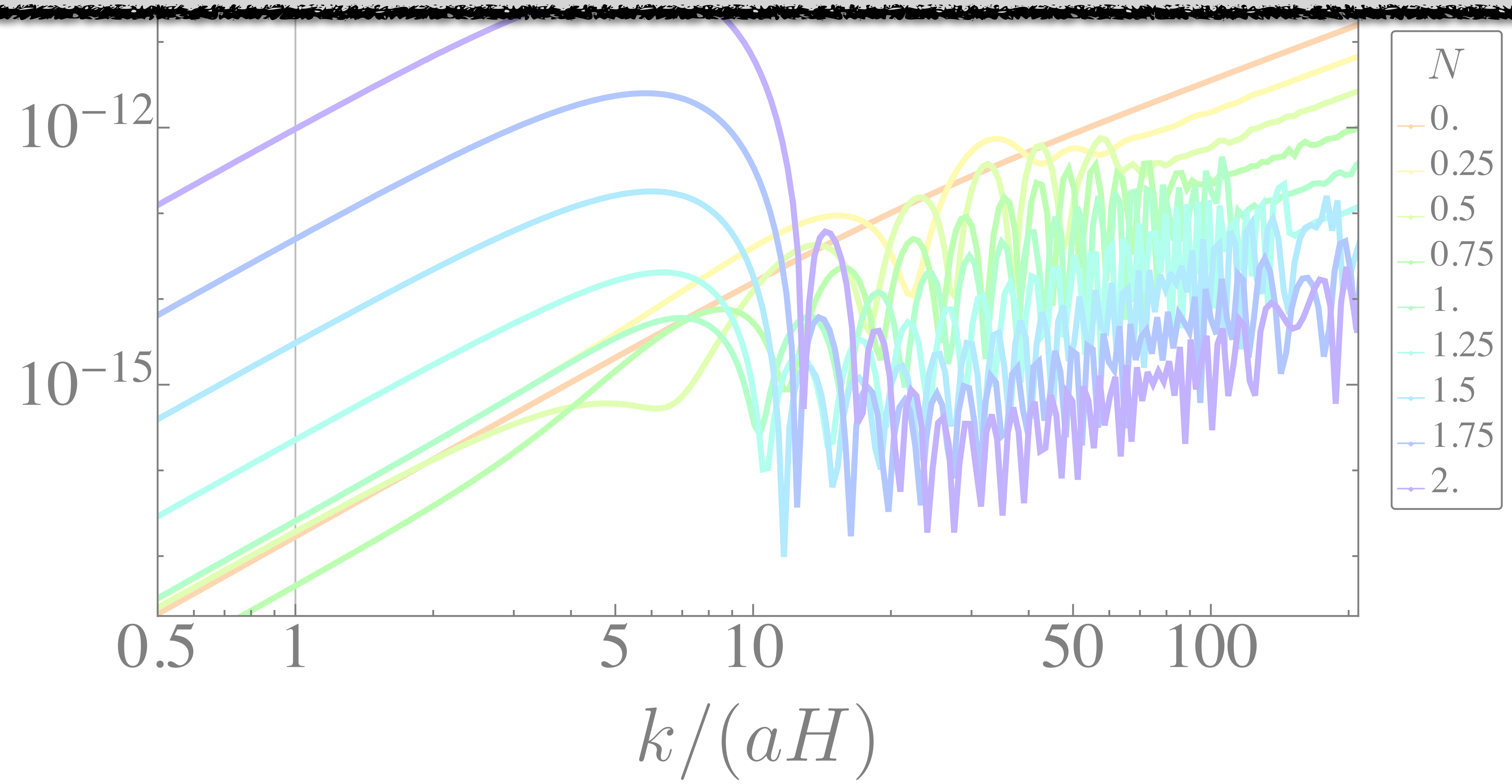
R/H^2
$(M = 5\times10^{-2}m_p\ ;\ p=6)$
10
5
0
−5
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
N
$\Delta_\chi(k)/m_p^2$
$(\xi = 50\ ;\ M = 5\cdot10^{-2}m_p\ ;\ p=6)$
10^{-12}
10^{-15}
0.5
1
5
10
50
100
$k/(aH)$
N
0.
0.25
0.5
0.75
1.
1.25
1.5
1.75
2.

Geometric preheating: Linear regime

We can solve linear equation for χ_k modes

$p = 6$

p=6 shows to be the best candidate to achieve reheating for low energy scales.



Geometric (p)reheating: Non-linear regime

In linear regime χ_k modes grow unbounded

Let us go back to the action

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} m_p^2 R - \frac{1}{2} \xi R \chi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi, \chi) \right)$$

and the NMC-field EoM

$$\chi'' + (3 - \alpha)\mathcal{H}\chi' - a^{-2(1-\alpha)} \nabla^2 \chi + a^{2\alpha} \left(\xi R \chi + V_{,\chi} \right) = 0$$

can we get the effect of the χ growth on R dynamics?

Geometric (p)reheating: Non-linear regime

Let us write trace of $T_{\mu\nu}$ for the NMC sector

$$T^{\chi}_{\mu\nu} = \partial_\mu\chi\partial_\nu\chi - g^{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\partial_\rho\chi\partial_\sigma\chi + V_{\rm NMC}\right) + \xi\left(G_{\mu\nu} + g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \nabla_\mu\nabla_\nu\right)\chi^2$$

$$T_\chi \equiv g^{\mu\nu}T^{\chi}_{\mu\nu} = (6\xi - 1)(\partial^\mu\chi\partial_\mu\chi + \xi R\chi^2) + 6\xi\chi\partial_\chi V_{\rm NMC} - 4V_{\rm NMC}(\chi)$$

using trace part of Einstein's equations $m_p^2 R = -T$

$$R = \frac{(1-6\xi)\langle\partial^\mu\chi\partial_\mu\chi\rangle + 4\langle V\rangle - 6\xi\langle\chi V_{,\chi}\rangle + \langle\partial^\mu\phi\partial_\mu\phi\rangle}{m_p^2 + (6\xi - 1)\xi\langle\chi^2\rangle}$$

where we have used $T_\phi = \partial^\mu\phi\partial_\mu\phi - 4V_{\rm inf}(\phi)$

Geometric (p)reheating: Non-linear regime

We have now a full system of equations that characterize the system

$$\chi'' + (3-\alpha)\left(\frac{a'}{a}\right)\chi' - a^{-2(1-\alpha)}\nabla^2\chi = -a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right)$$

$$\phi'' + (3-\alpha)\left(\frac{a'}{a}\right)\phi' - a^{-2(1-\alpha)}\nabla^2\phi = -a^{2\alpha}V_{,\phi}$$

$$\frac{a''}{a} + (1-\alpha)\left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{6}R$$

with

$$R = \frac{\left[(6\xi-1)\left(\frac{\langle\chi'^2\rangle}{a^{2\alpha}} - \frac{\langle(\nabla\chi)^2\rangle}{a^2}\right) - 6\xi\left\langle\chi V_{,\chi}\right\rangle + 4\langle V\rangle - \frac{\langle\phi'^2\rangle}{a^{2\alpha}} + \frac{\langle(\nabla\phi)^2\rangle}{a^2}\right]}{m_p^2 + (6\xi-1)\xi\langle\chi^2\rangle}$$

Geometric (p)reheating: Non-linear regime

We have now a full system of equations that characterize the system

$$\chi'' + (3-\alpha)\left(\frac{a'}{a}\right)\chi' - a^{-2(1-\alpha)}\nabla^2\chi = -\,a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right)$$

$$\phi'' + (3-\alpha)\left(\frac{a'}{a}\right)\phi' - a^{-2(1-\alpha)}\nabla^2\phi = -\,a^{2\alpha}V_{,\phi}$$

$$\frac{a''}{a} + (1 - \ldots$$

$$R = \frac{\left[(6\xi - 1)\left(\frac{\langle\chi'^2\rangle}{a^{2\alpha}} - \frac{\langle(\nabla\chi)^2\rangle}{a^2}\right) \ldots\right.}{m_p^2 \ldots}$$

With the Hubble rate given by

$$\mathscr{H}^2 \equiv \left(\frac{a'}{a}\right)^2 = \frac{a^{2\alpha}}{3m_p^2}[E_\phi + E_\chi]$$

where

$$E_\phi = \frac{1}{2a^{2\alpha}}\left\langle\phi'^2\right\rangle + \frac{1}{2a^2}\left\langle(\nabla\phi)^2\right\rangle + \left\langle V_{\rm inf}(\phi)\right\rangle$$

$$E_\chi = \frac{1}{2a^{2\alpha}}\left\langle\chi'^2\right\rangle + \frac{1}{2a^2}\left\langle(\nabla\chi)^2\right\rangle + \left\langle V_{\rm NMC}(\chi)\right\rangle + \frac{3\xi}{a^{2\alpha}}\mathscr{H}^2\left\langle\chi^2\right\rangle + \frac{6\xi}{a^{2\alpha}}\mathscr{H}\left\langle\chi\chi'\right\rangle$$

Figueroa, Florio, Opferkuch, Stefanek '23

Geometric (p)reheating: Non-linear regime

We have now a full system of equations that characterize the system

$$\chi'' + (3-\alpha)\left(\frac{a'}{a}\right)\chi' - a^{-2(1-\alpha)}\nabla^2\chi = -\,a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right)$$

$$\phi'' + (3-\alpha)\left(\frac{a'}{a}\right)\phi' - a^{-2(1-\alpha)}\nabla^2\phi = -\,a^{2\alpha}V_{,\phi}$$

Full non-linear system impossible to solve analytically

$$R = \frac{\left[(6\xi-1)\left(\frac{\cdots}{a^{2\alpha}} - \frac{\cdots}{a^2}\right) + 6\xi\left\langle\chi V_{,\chi}\right\rangle + 4\left\langle V\right\rangle - \frac{\cdots}{a^{2\alpha}} + \frac{\cdots}{a^2}\right]}{m_p^2 + (6\xi-1)\xi\langle\chi^2\rangle}$$

where

$$E_\phi = \frac{1}{2a^{2\alpha}}\left\langle\phi'^2\right\rangle + \frac{1}{2a^2}\left\langle(\nabla\phi)^2\right\rangle + \left\langle V_{\rm inf}(\phi)\right\rangle$$

$$E_\chi = \frac{1}{2a^{2\alpha}}\left\langle\chi'^2\right\rangle + \frac{1}{2a^2}\left\langle(\nabla\chi)^2\right\rangle + \left\langle V_{\rm NMC}(\chi)\right\rangle + \frac{3\xi}{a^{2\alpha}}\mathscr{H}^2\left\langle\chi^2\right\rangle + \frac{6\xi}{a^{2\alpha}}\mathscr{H}\left\langle\chi\chi'\right\rangle$$

Geometric (p)reheating: Non-linear regime

We have now a full system of equations that characterize the system

$$\chi'' + (3-\alpha)\left(\frac{a'}{a}\right)\chi' - a^{-2(1-\alpha)}\nabla^2\chi = -a^{2\alpha}\left(\xi R\chi + V_{,\chi}\right)$$

$R = \frac{\left[(6\xi - \dots\right.}{\dots}$

Full non-linear system impossible to solve analytically

↓

Lattice simulations

CosmoLattice

$E_\chi = \frac{\dots}{2a^{2\alpha}}\langle\chi'^2\rangle + \frac{\dots}{2a^2}\langle(\nabla\chi)^2\rangle + \langle V_{\rm NMC}(\chi)\rangle + \frac{\dots}{a^{2\alpha}}\mathcal{H}^2\langle\chi^2\rangle + \frac{6\xi}{a^{2\alpha}}\mathcal{H}\langle\chi\chi'\rangle$

Figueroa, Florio, Torrenti, Valkenburg ’21, ’23

Geometric (p)reheating:

Let us have a common language

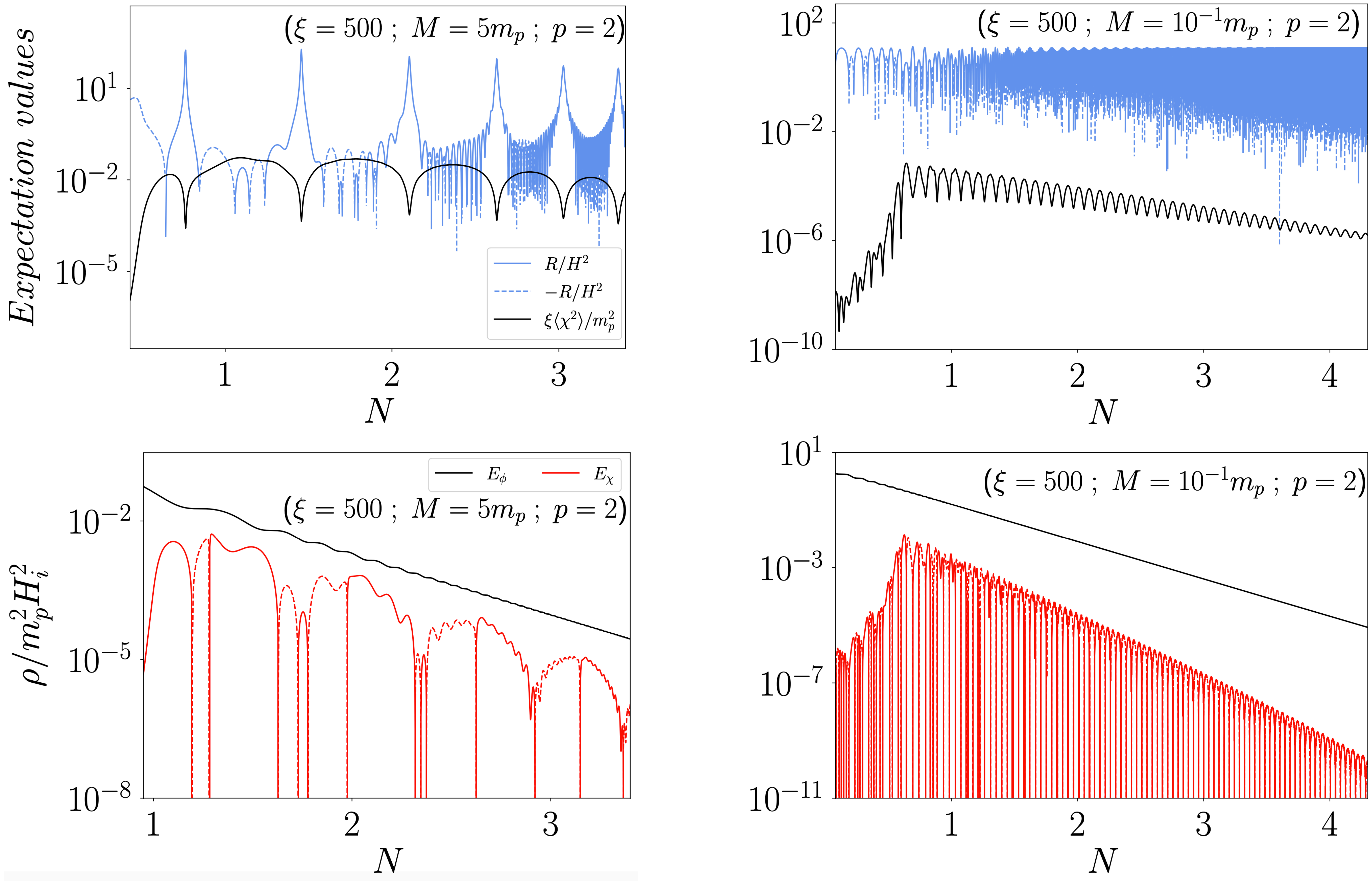
Preheating:

Any non-perturbative transfer of energy from inflaton to any other sector

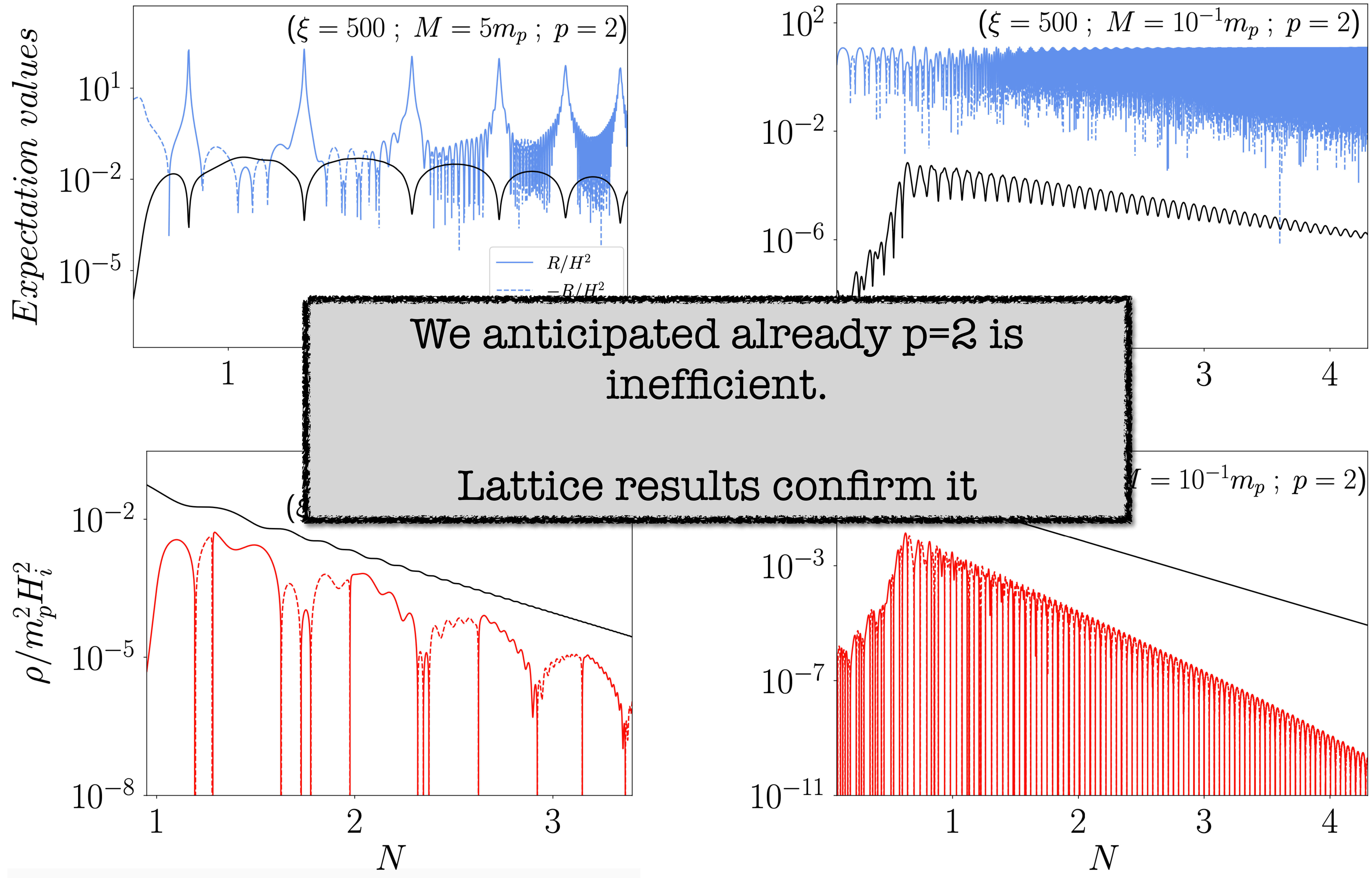
Reheating:

All energy is stored in the daughter fields, and Radiation Domination has been achieved

Geometric (p)reheating: Lattice results p=2



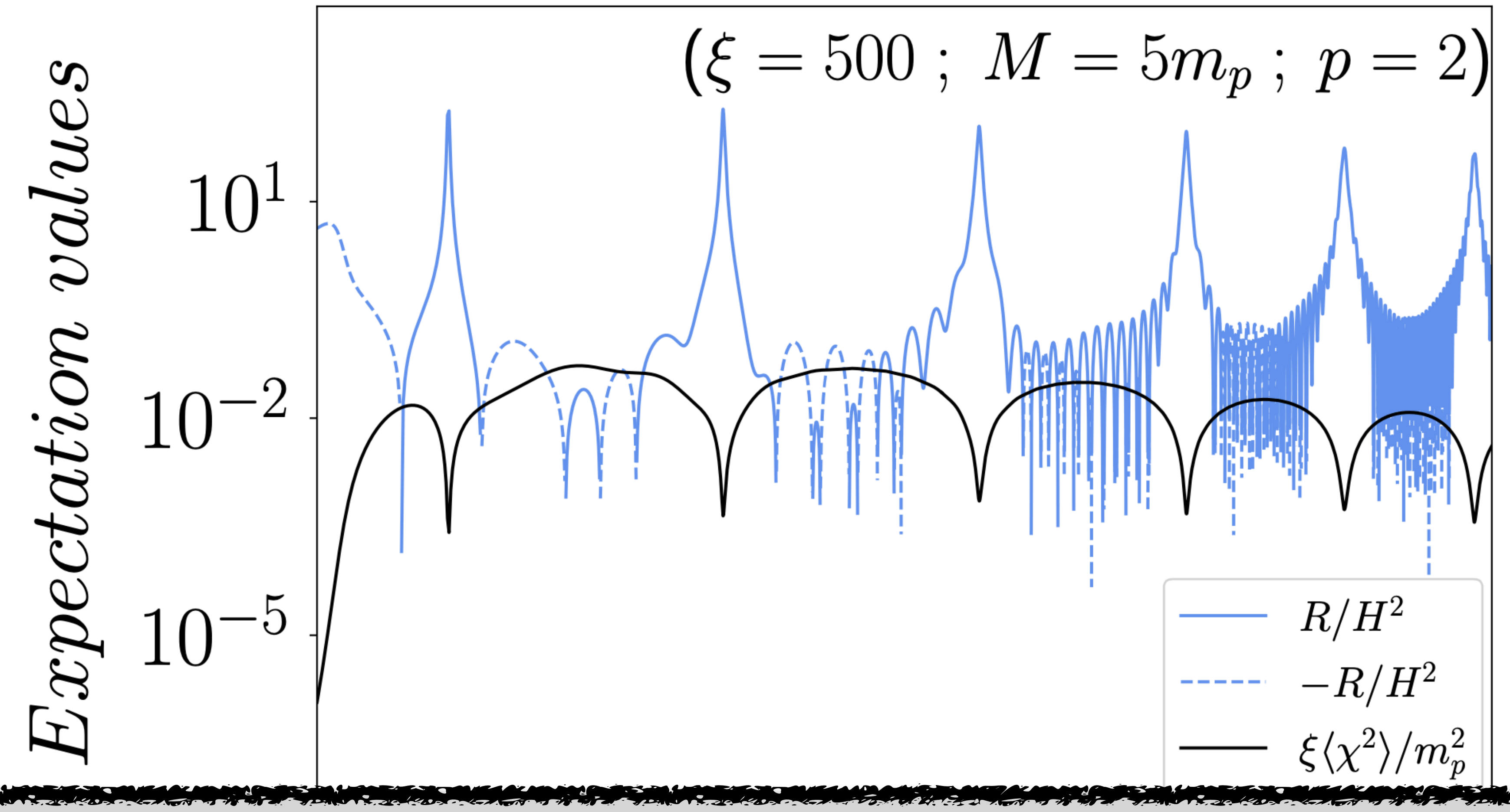
Geometric (p)reheating: Lattice results p=2

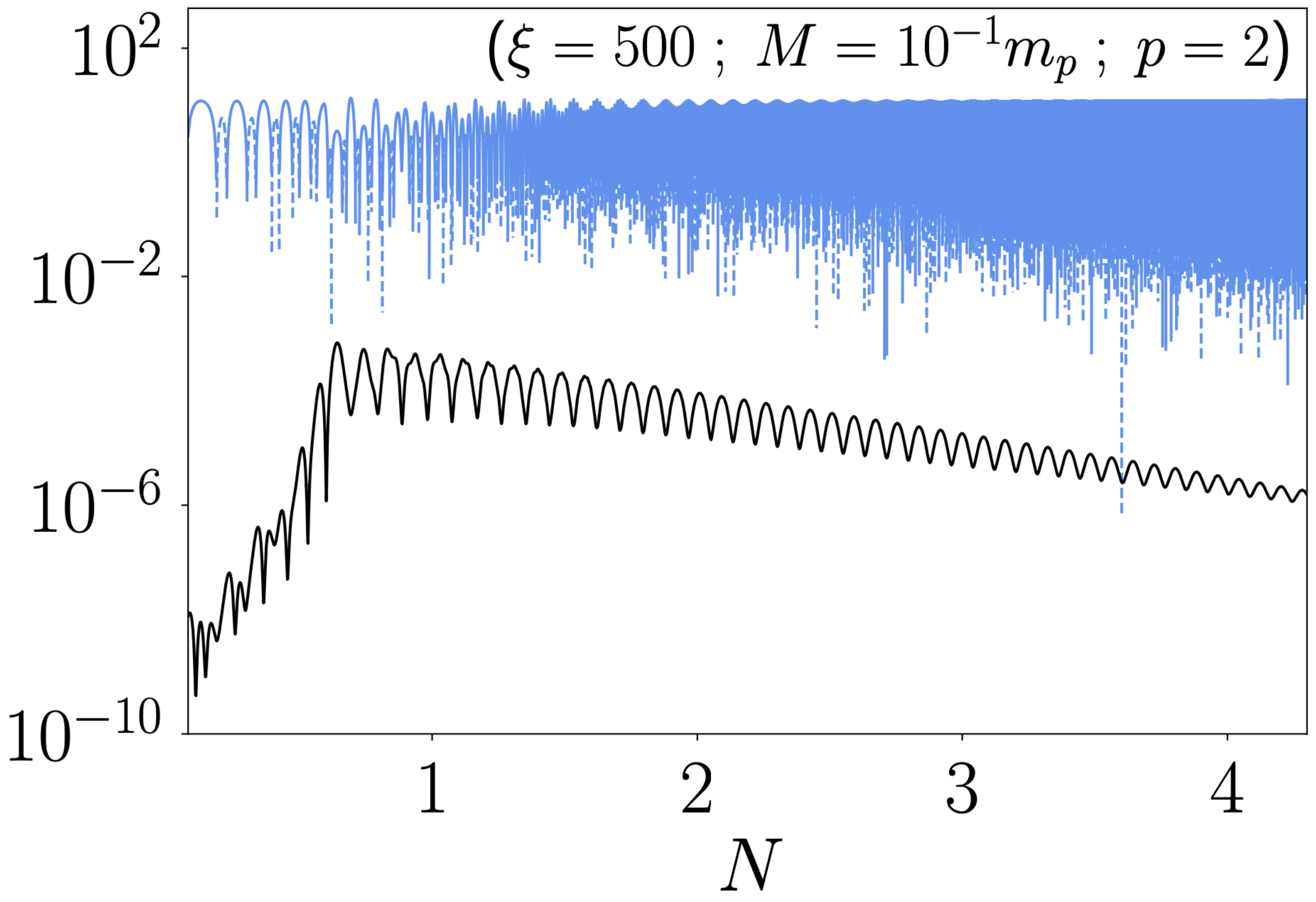


We anticipated already p=2 is inefficient.

Lattice results confirm it

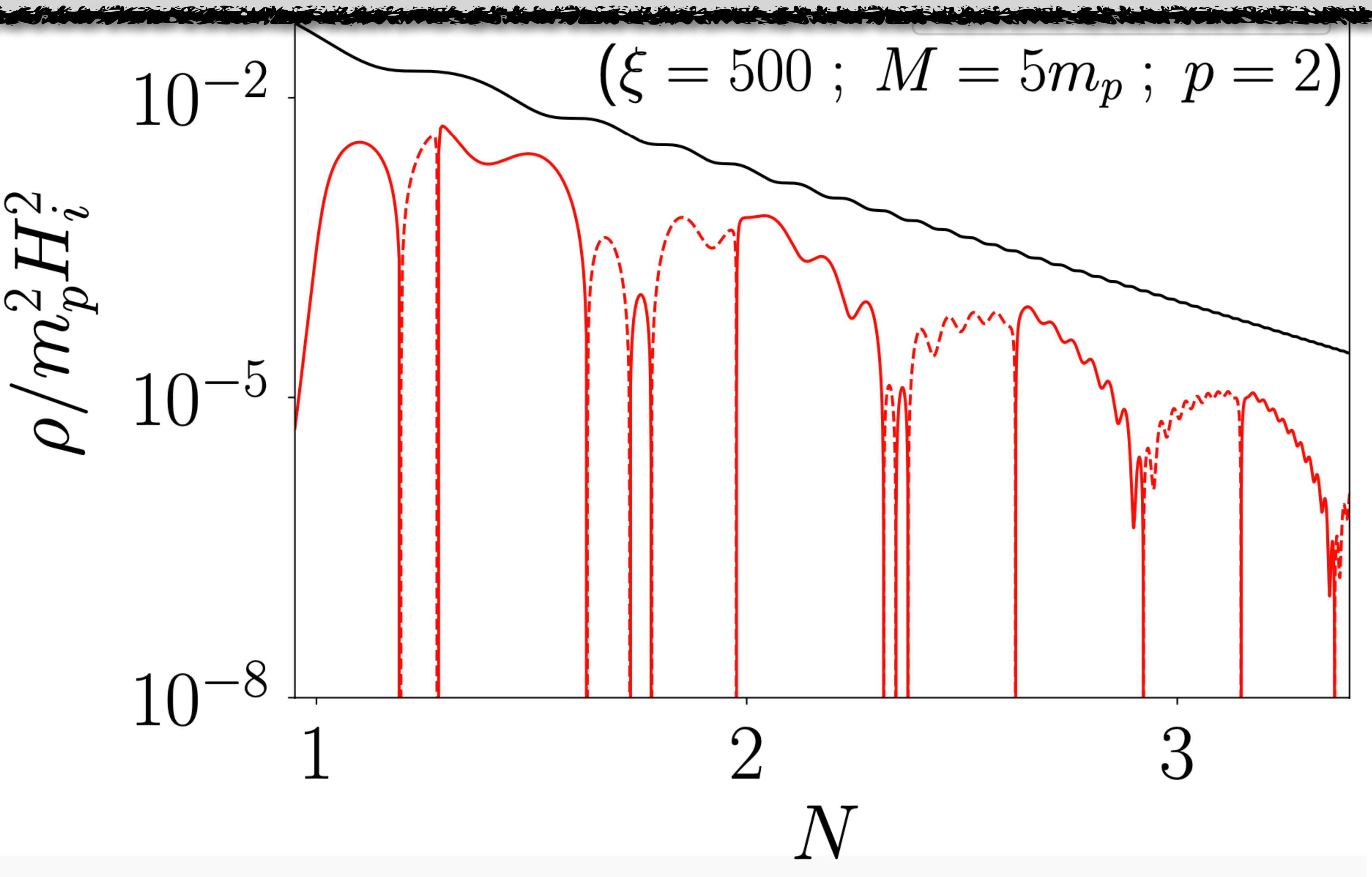
Geometric (p)reheating: Lattice results p=2

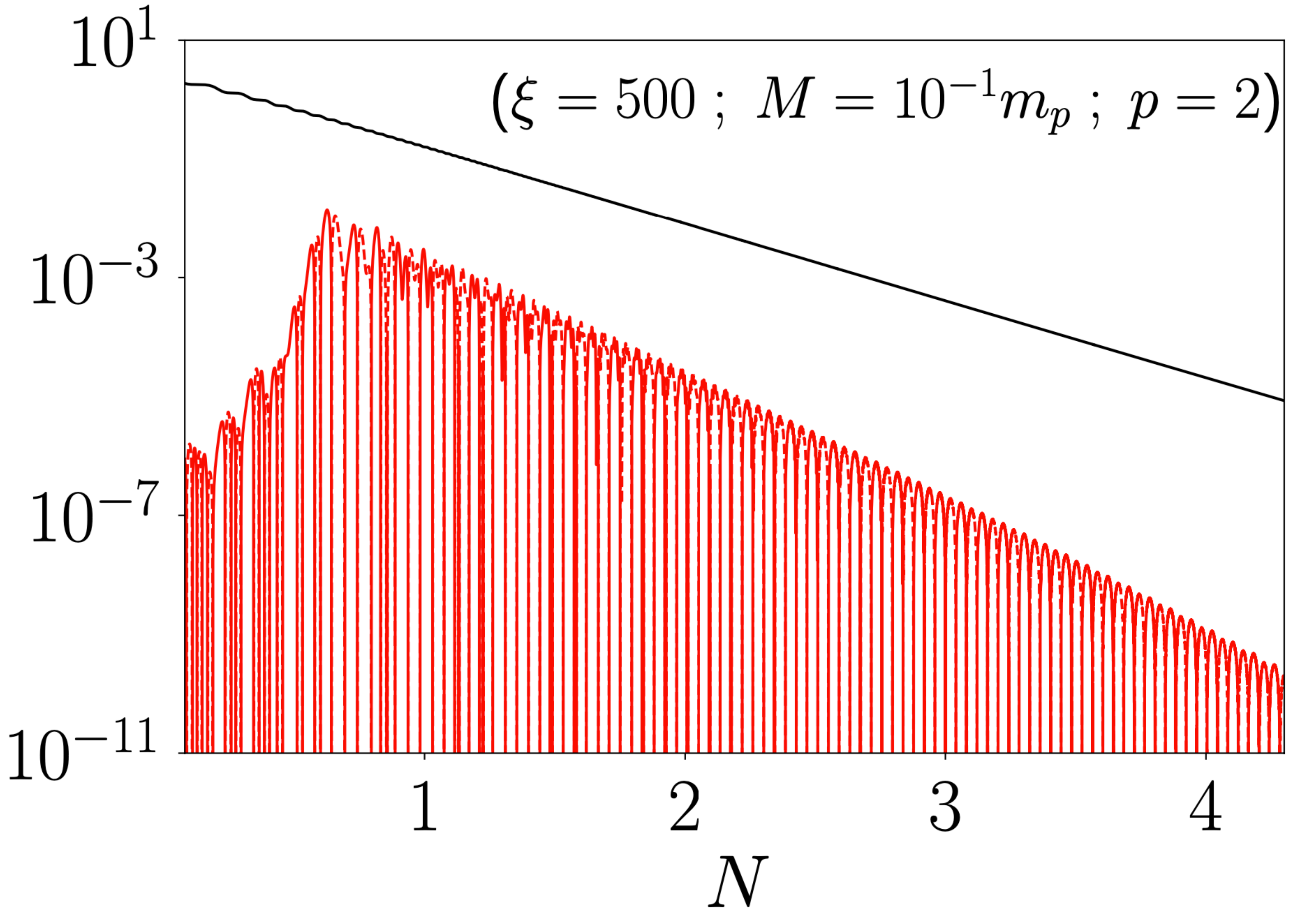




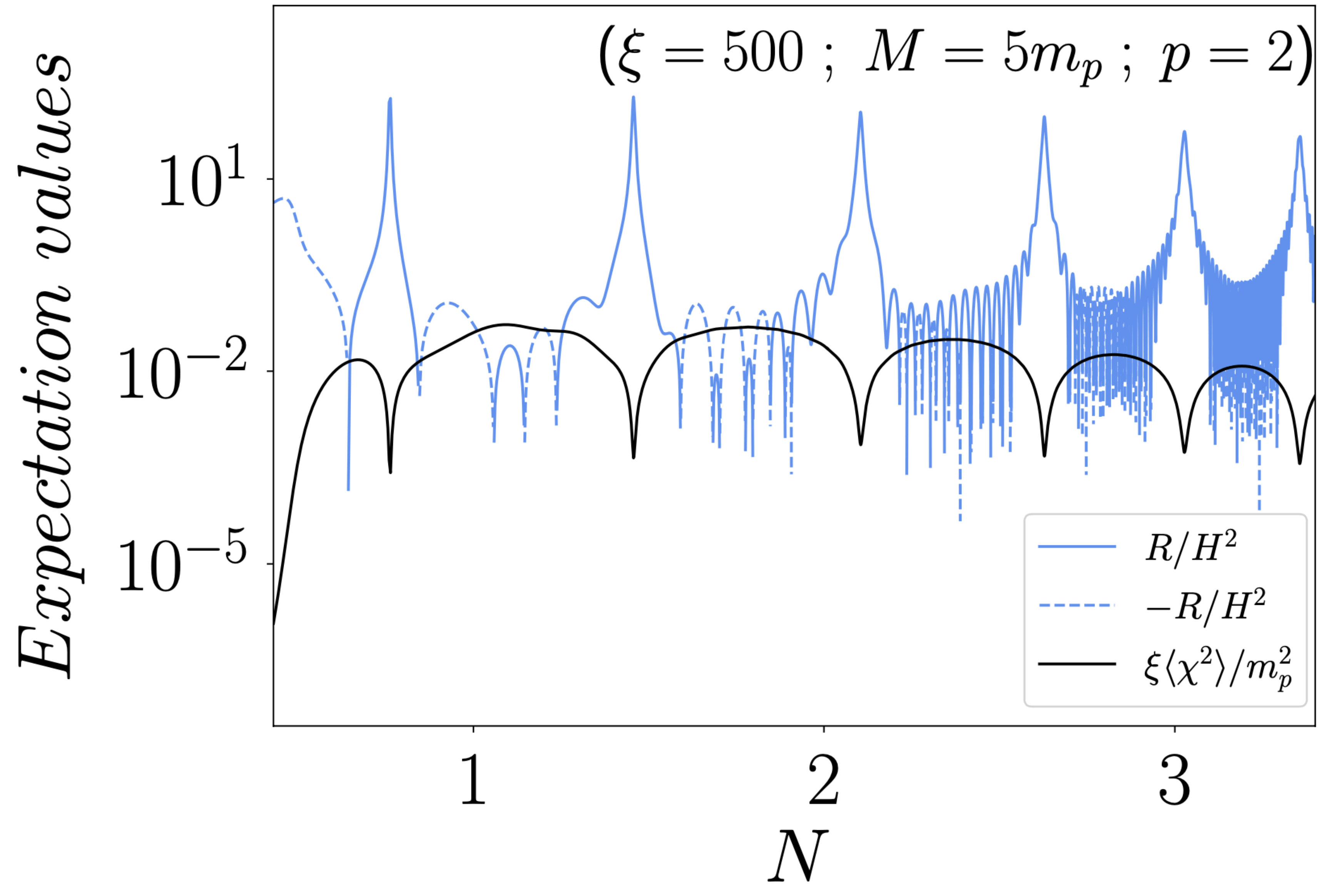
For high M scale and largest ξ

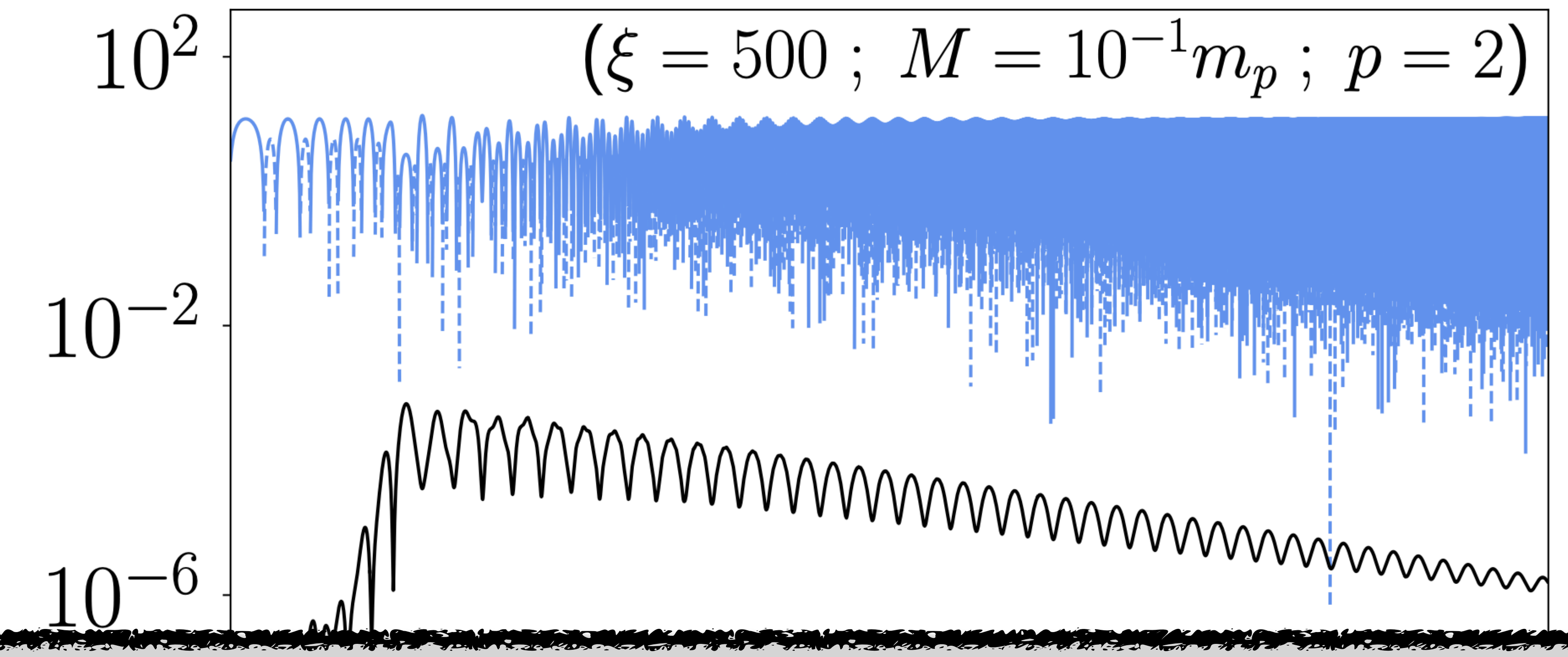
$E_\chi < E_\phi$





Geometric (p)reheating: Lattice results p=2

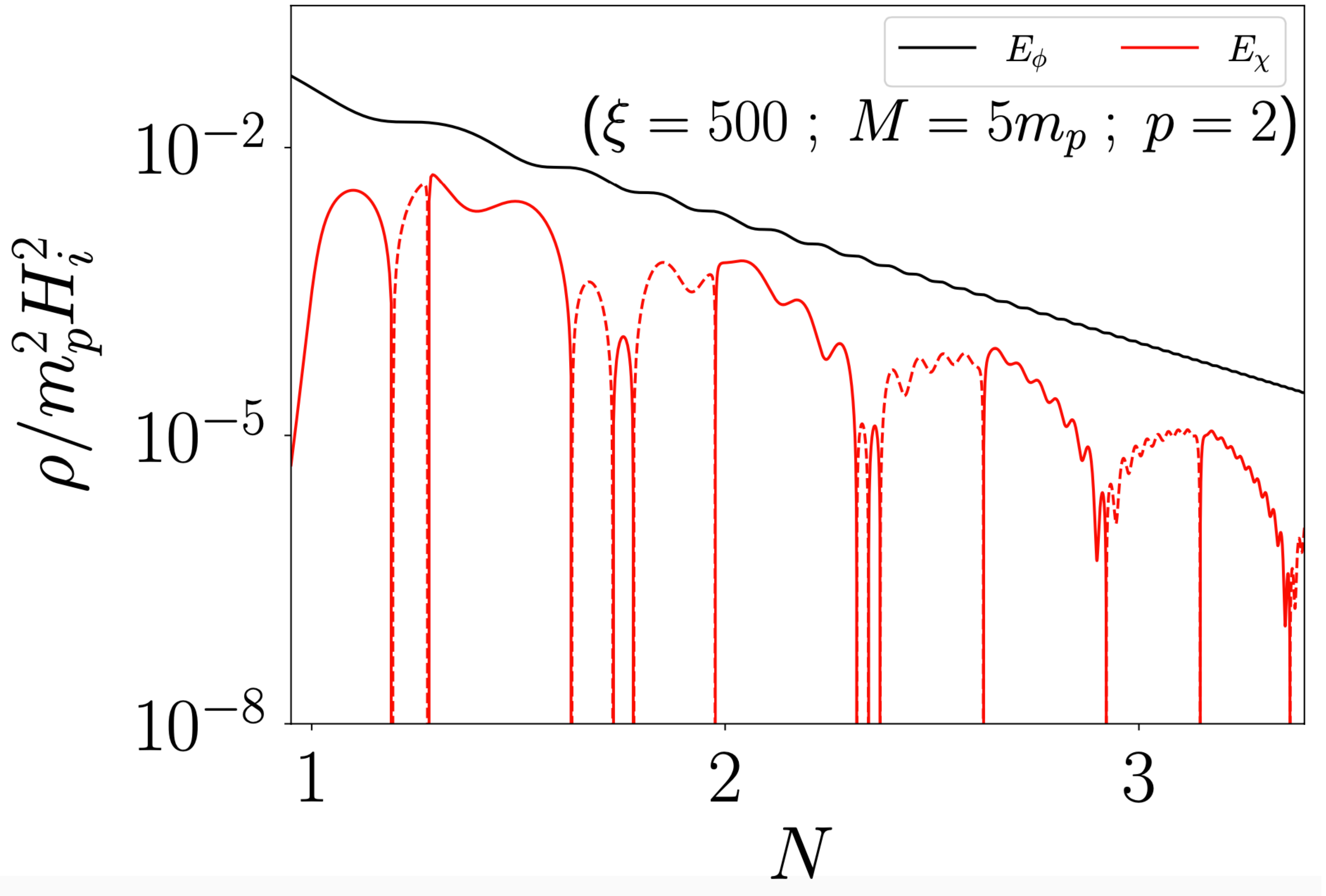


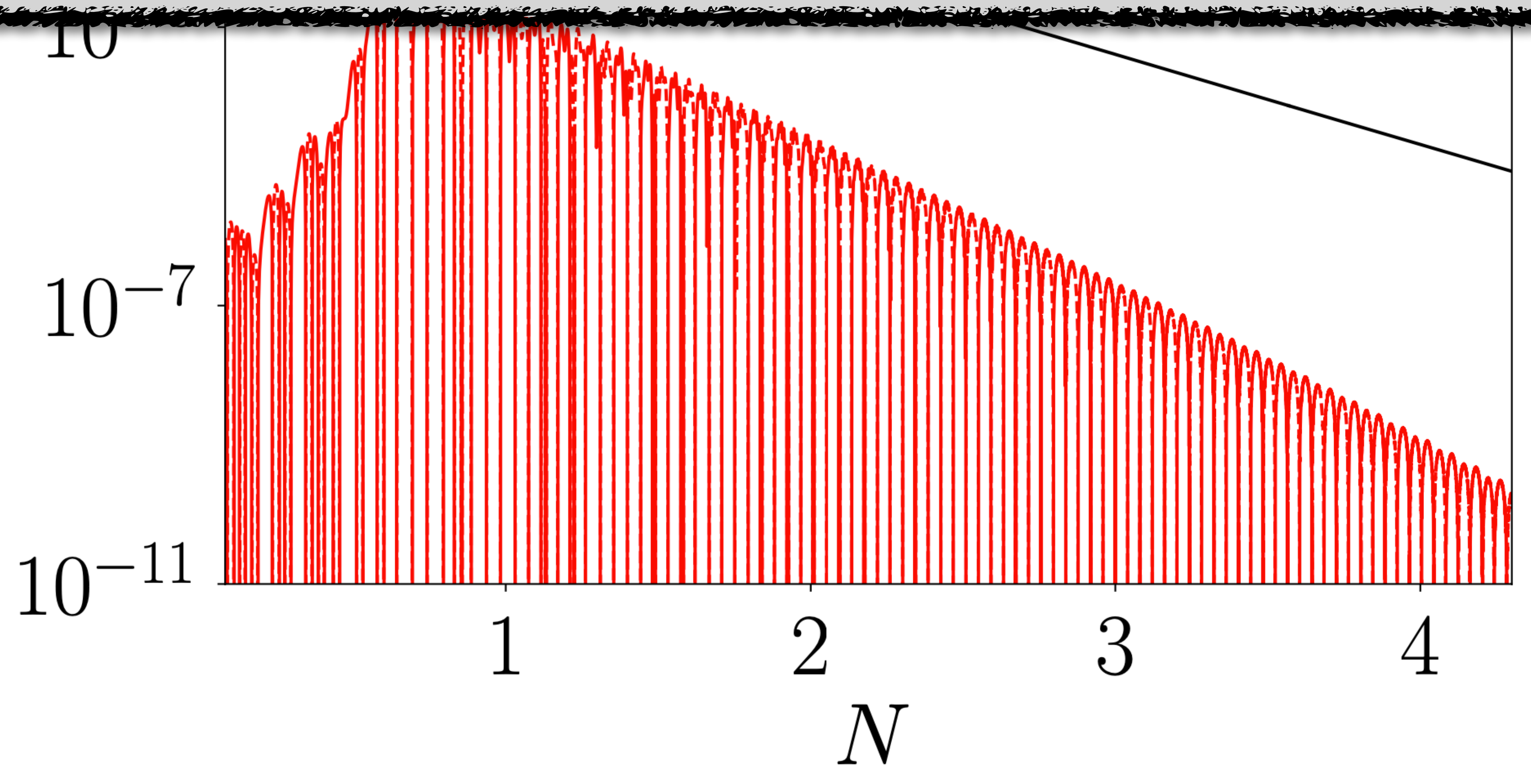


For low M scale and largest ξ

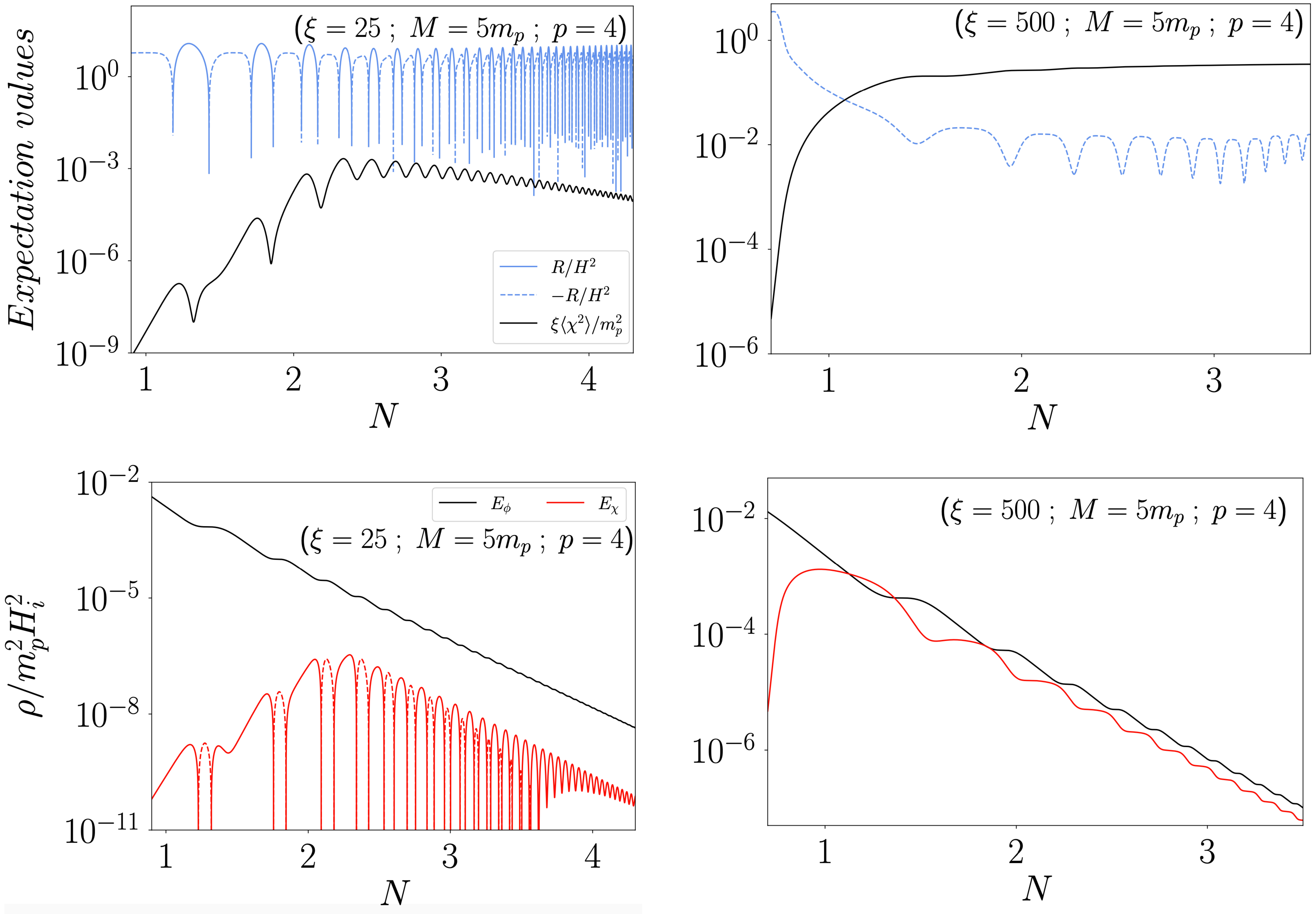
$E_\chi \ll E_\phi$

χ dilutes faster (as radiation) than ϕ (as matter)

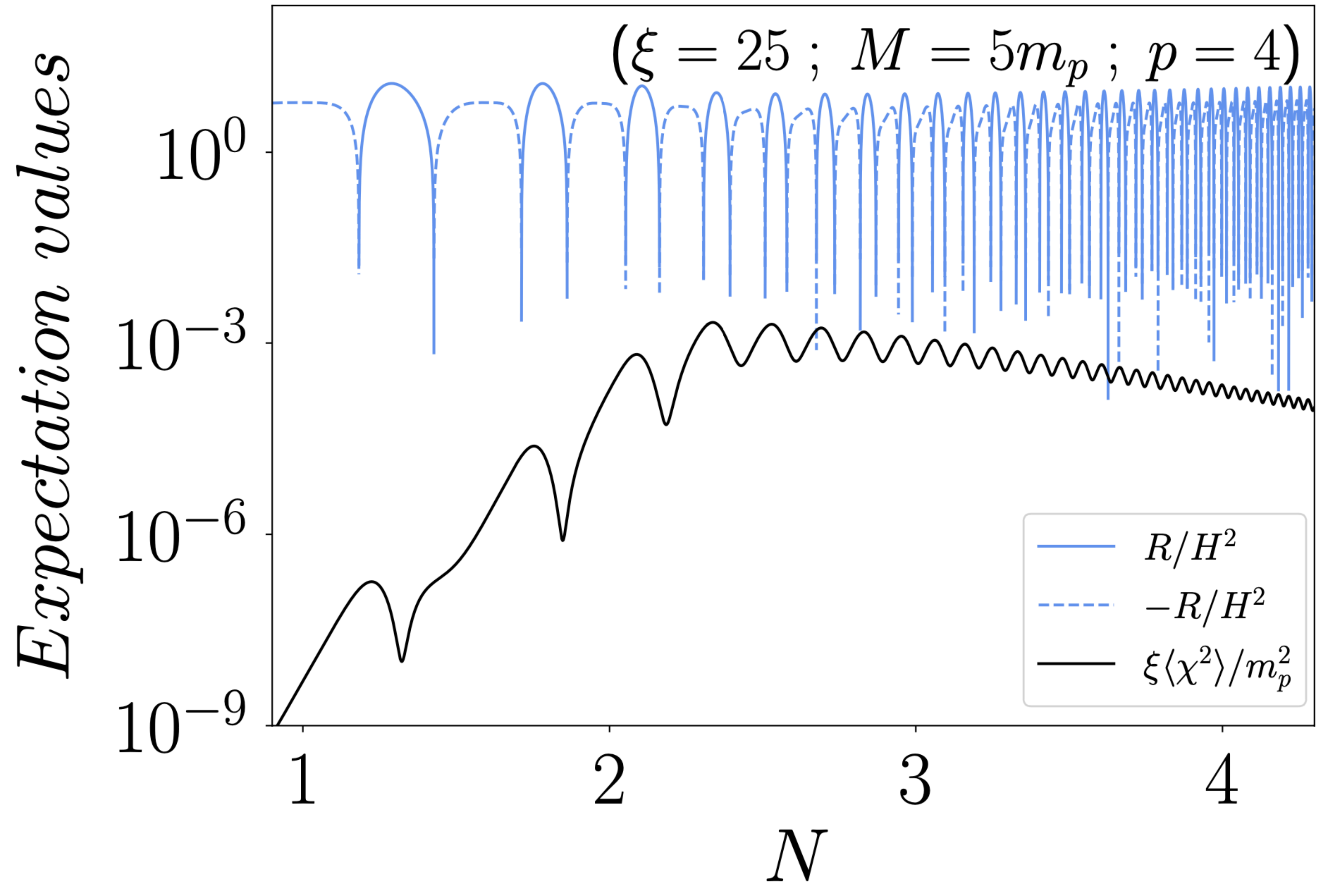


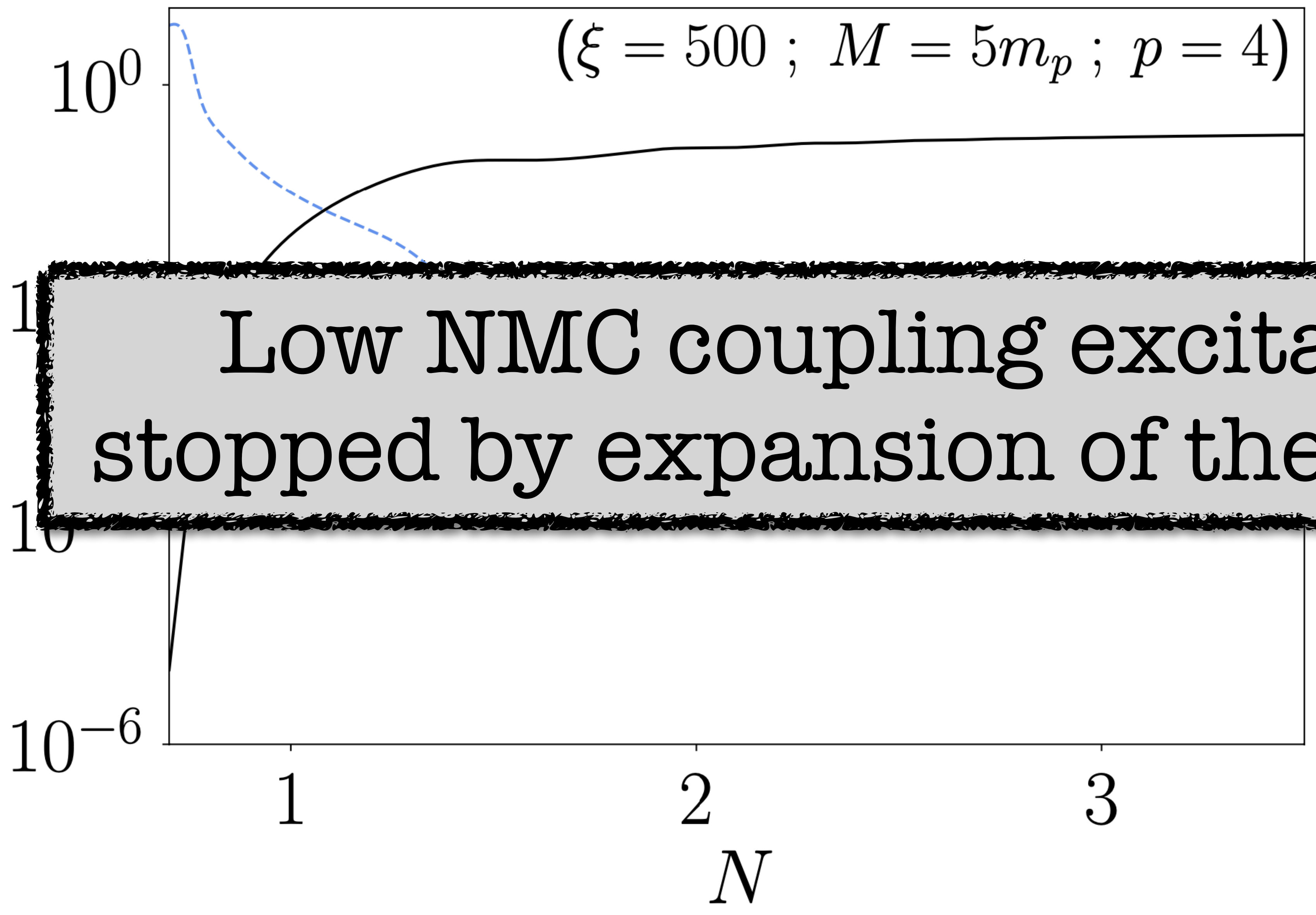


Geometric (p)reheating: Lattice results p=4

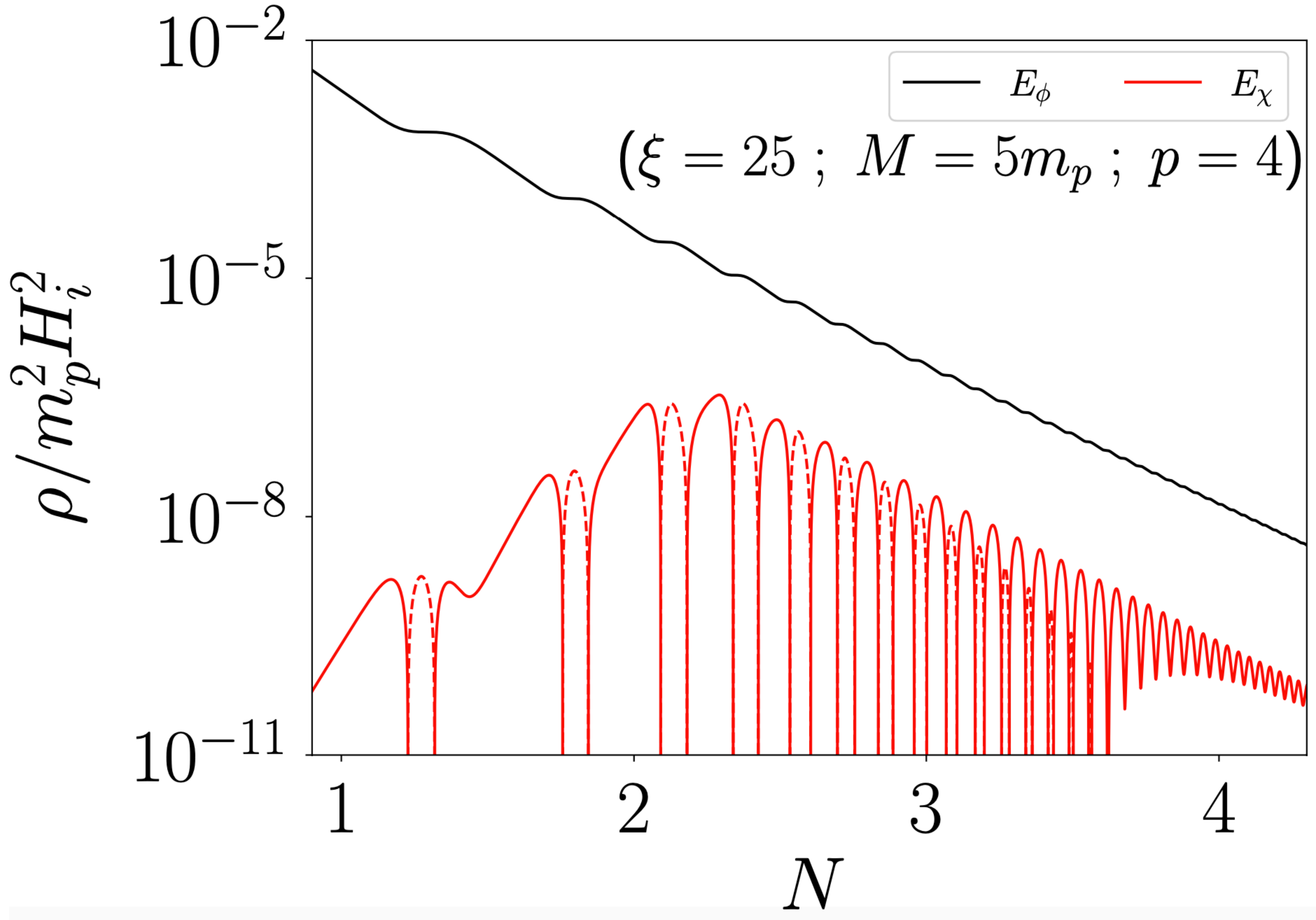


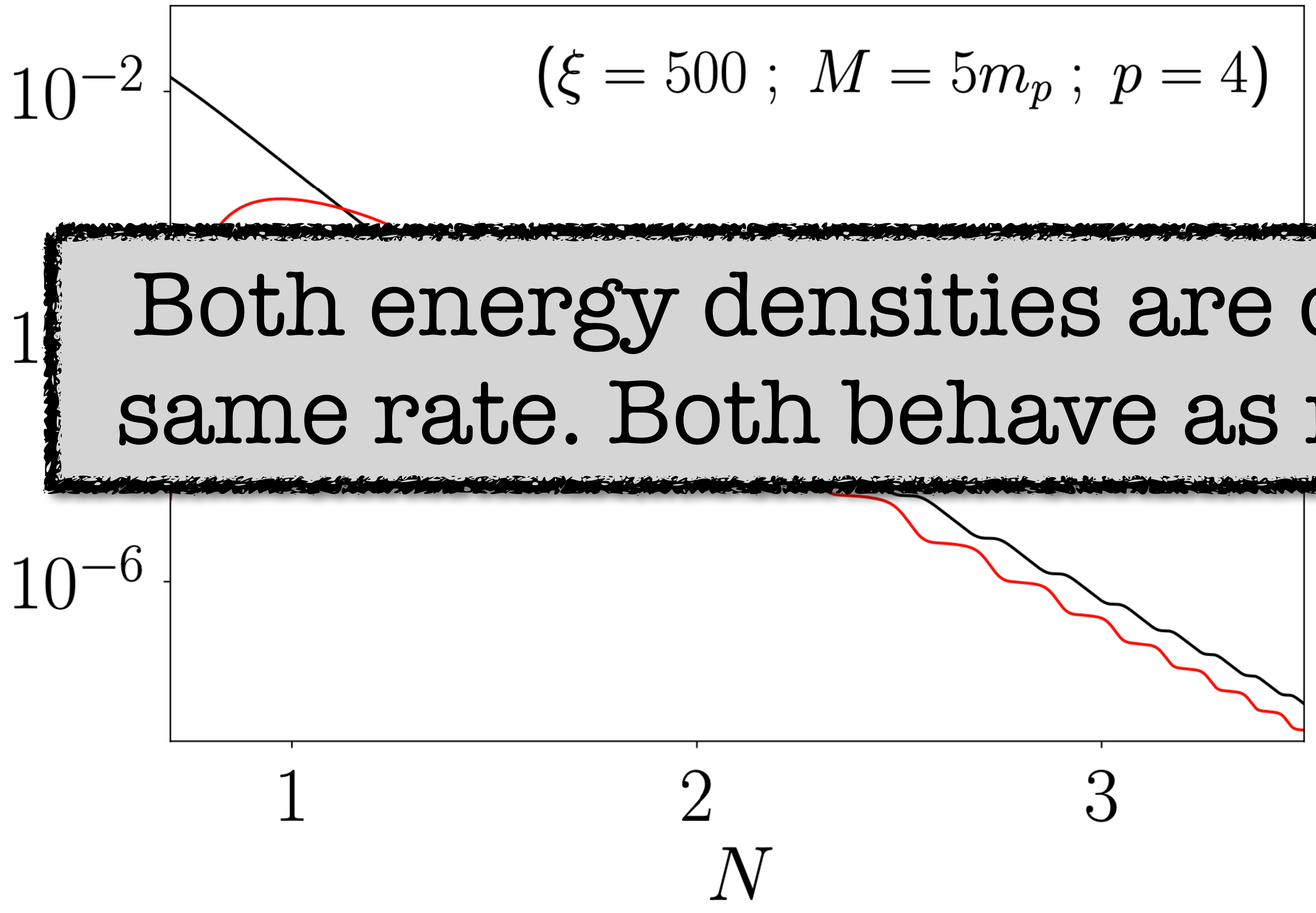
Geometric (p)reheating: Lattice results p=4





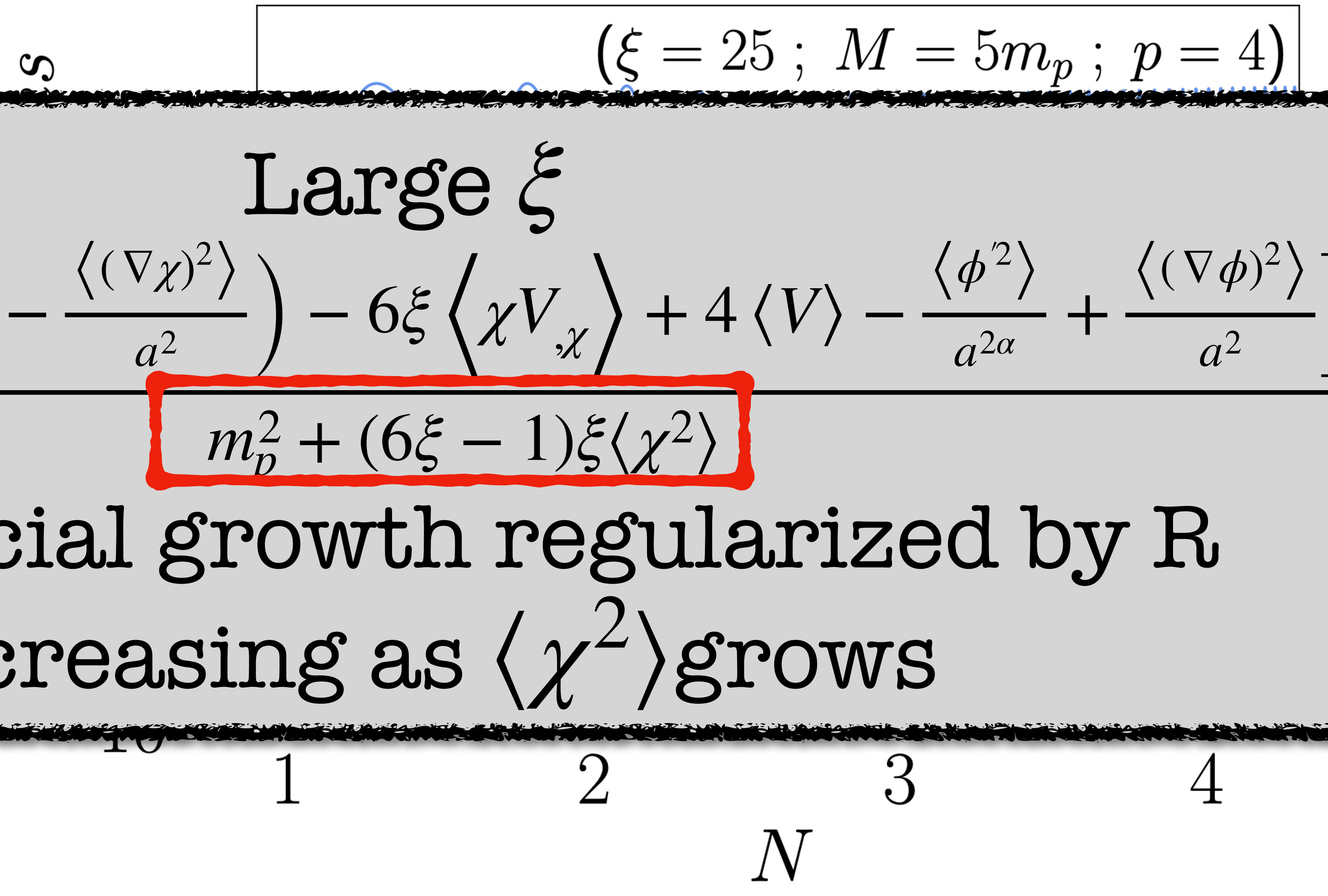
Low NMC coupling excitation is stopped by expansion of the universe





Both energy densities are diluted at same rate. Both behave as radiation.

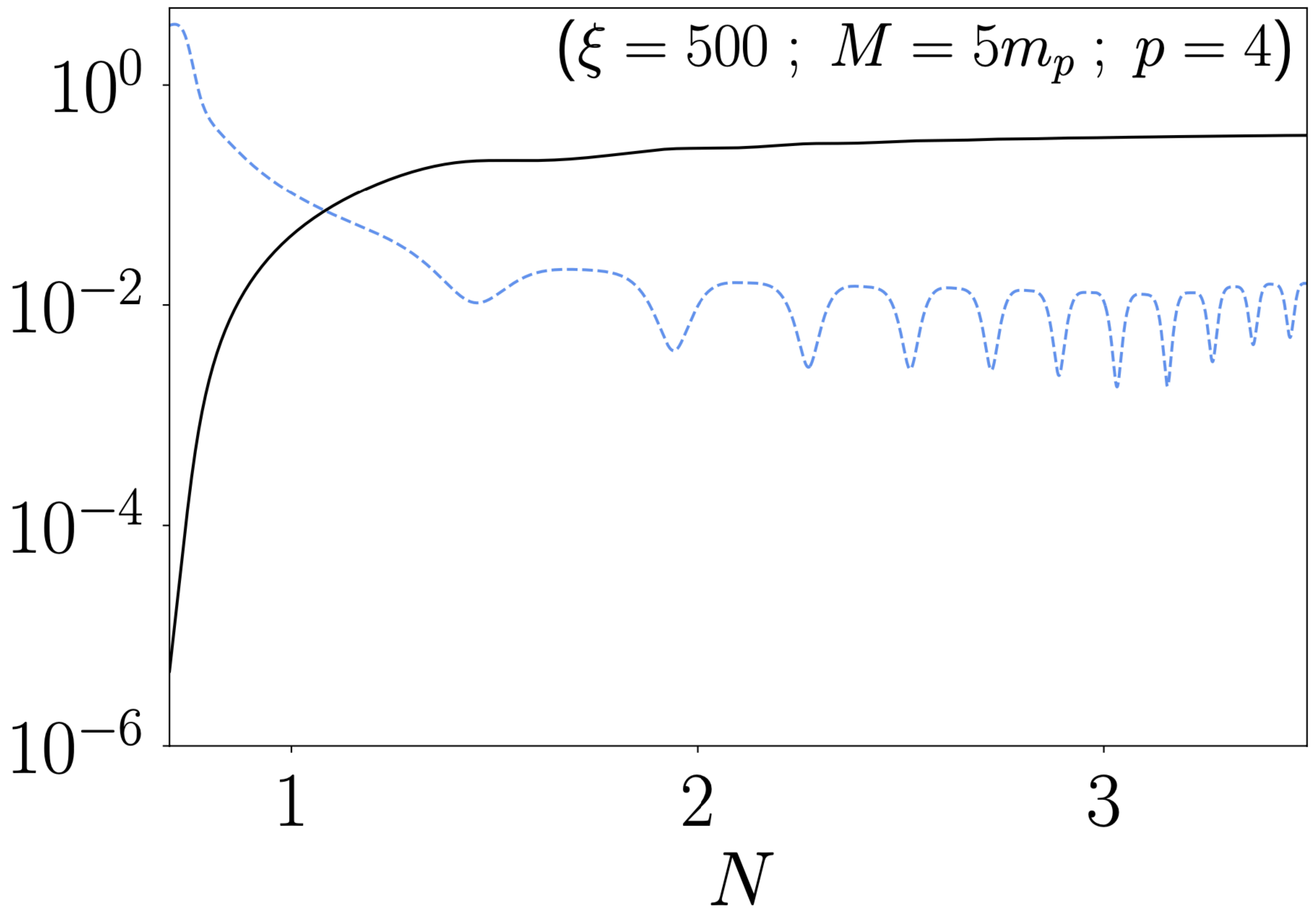
Geometric (p)reheating: Lattice results p=4

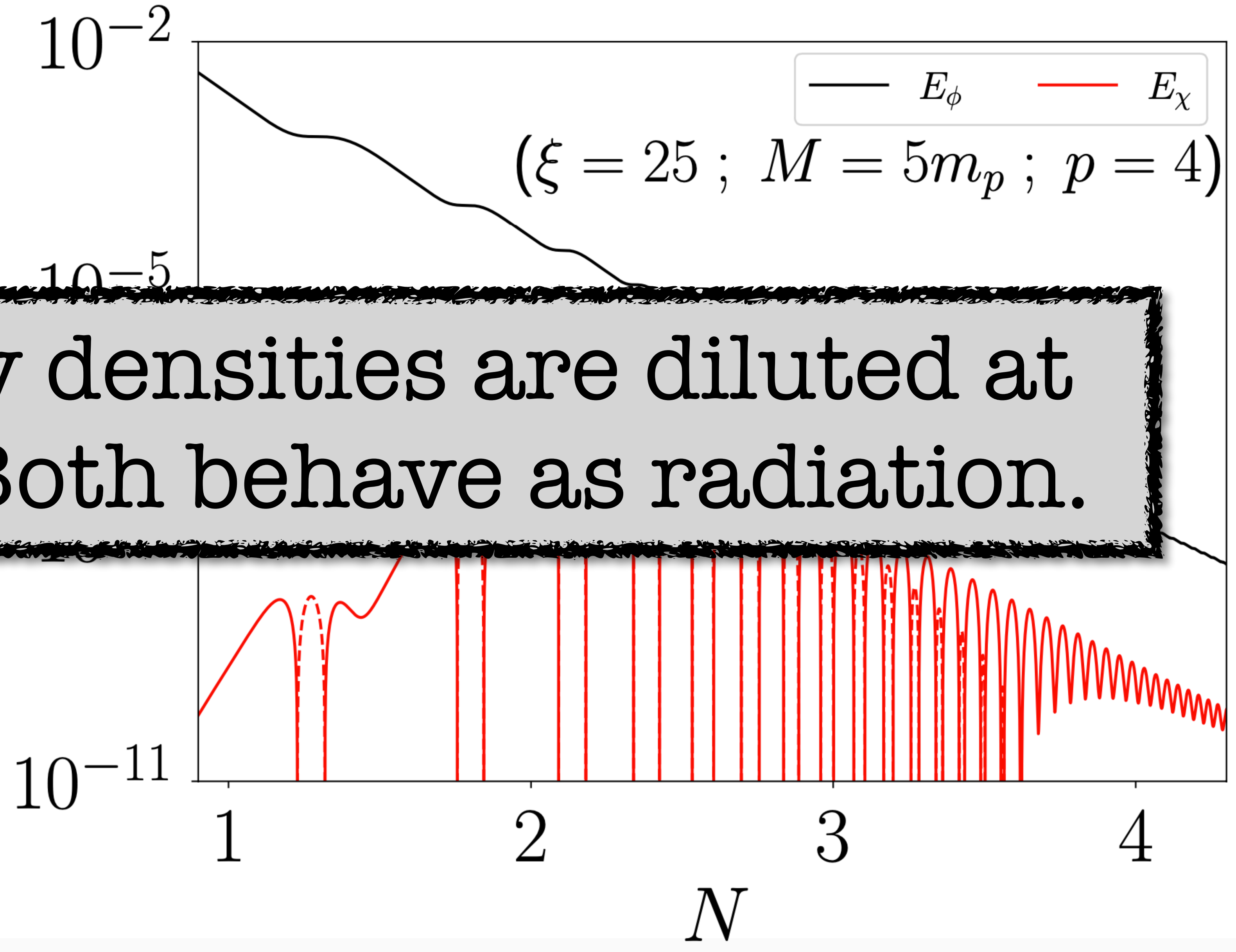


Large ξ

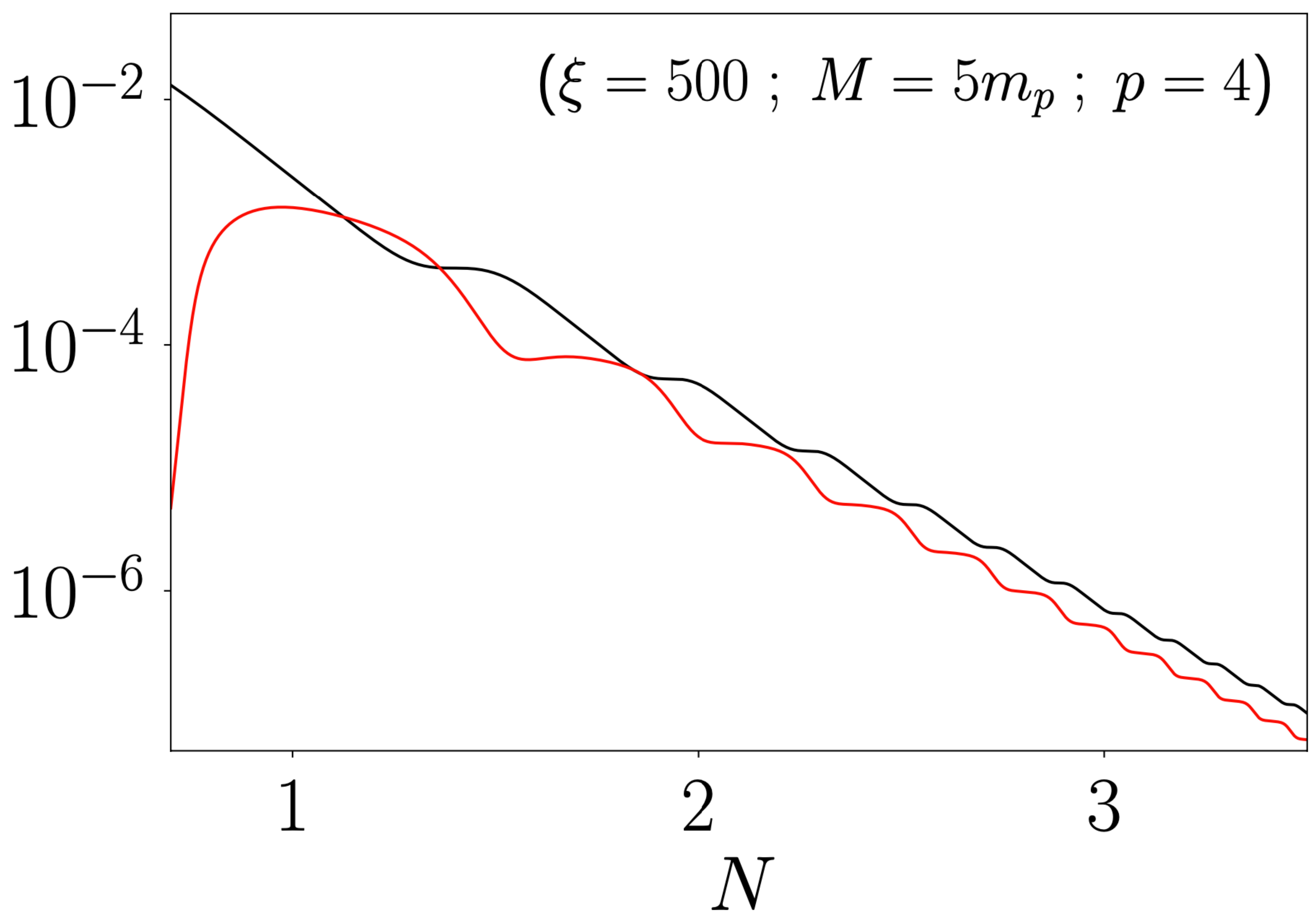
$$R = \frac{\left[(6\xi - 1)\left(\frac{\langle \chi'^2 \rangle}{a^{2\alpha}} - \frac{\langle (\nabla\chi)^2 \rangle}{a^2}\right) - 6\xi\left\langle \chi V_{,\chi}\right\rangle + 4\langle V \rangle - \frac{\langle \phi'^2 \rangle}{a^{2\alpha}} + \frac{\langle (\nabla\phi)^2 \rangle}{a^2}\right]}{m_p^2 + (6\xi - 1)\xi\langle \chi^2 \rangle}$$

Exponencial growth regularized by R decreasing as $\langle \chi^2 \rangle$grows

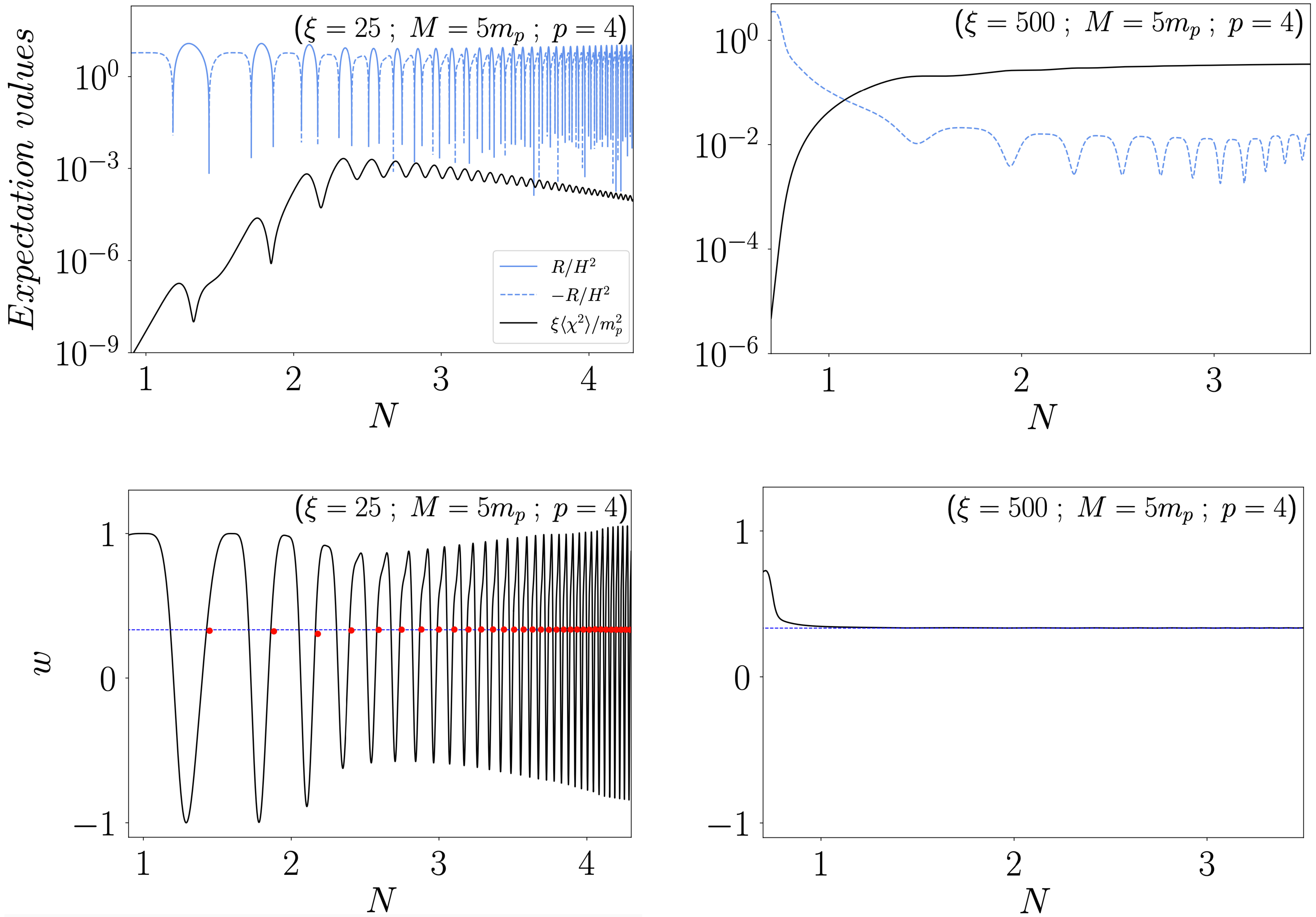




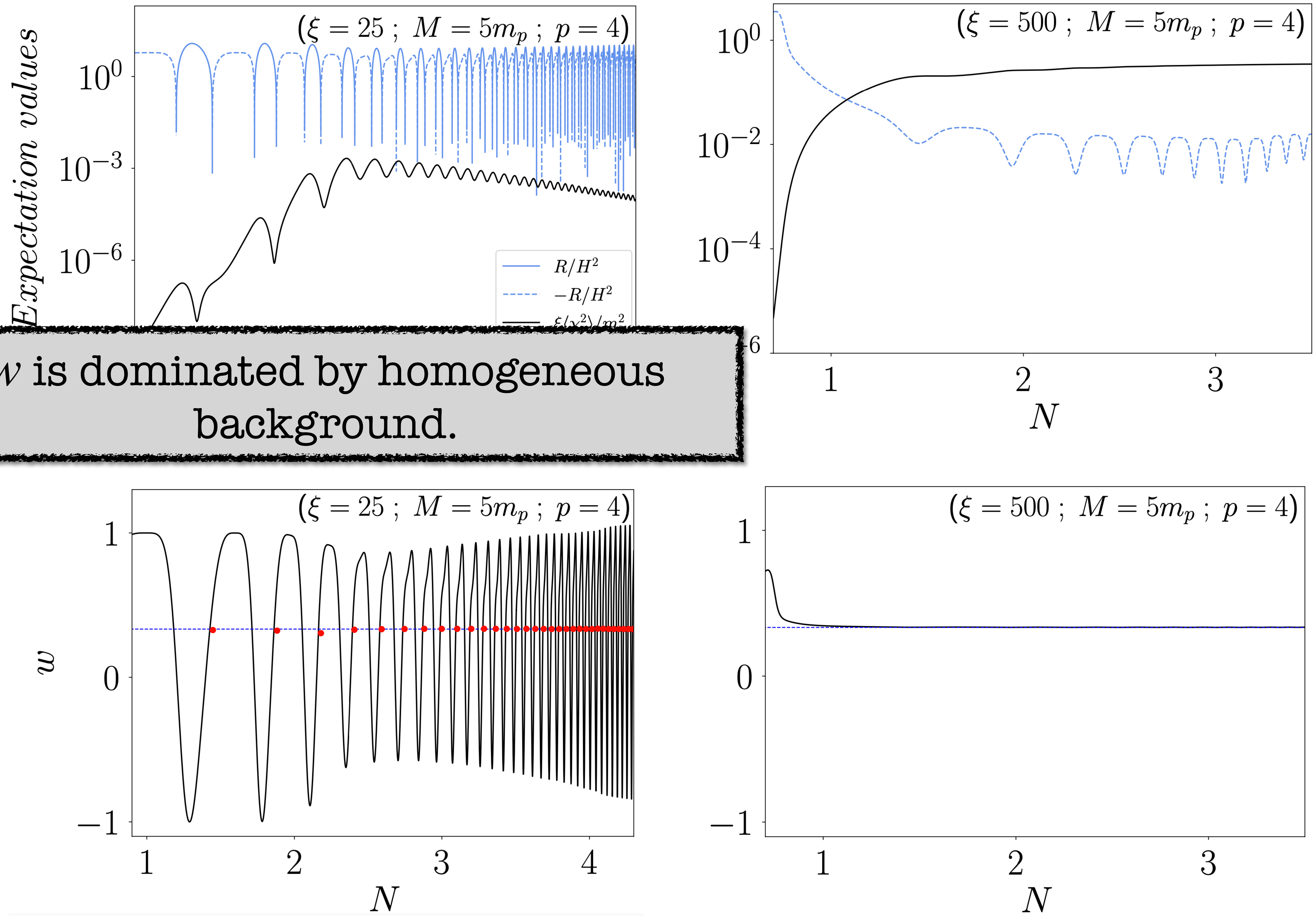
Both energy densities are diluted at same rate. Both behave as radiation.



Geometric (p)reheating: Lattice results p=4

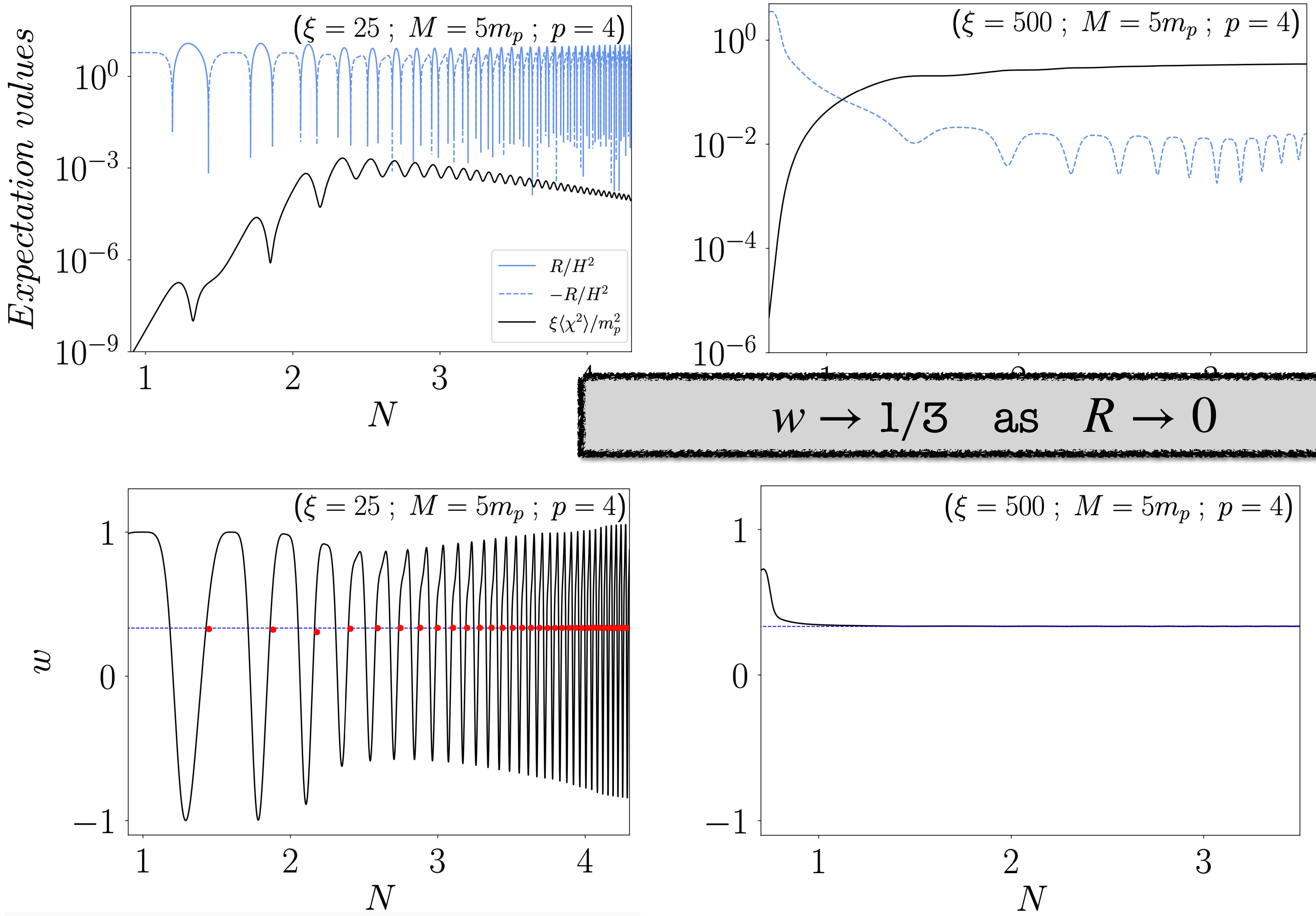


Geometric (p)reheating: Lattice results p=4



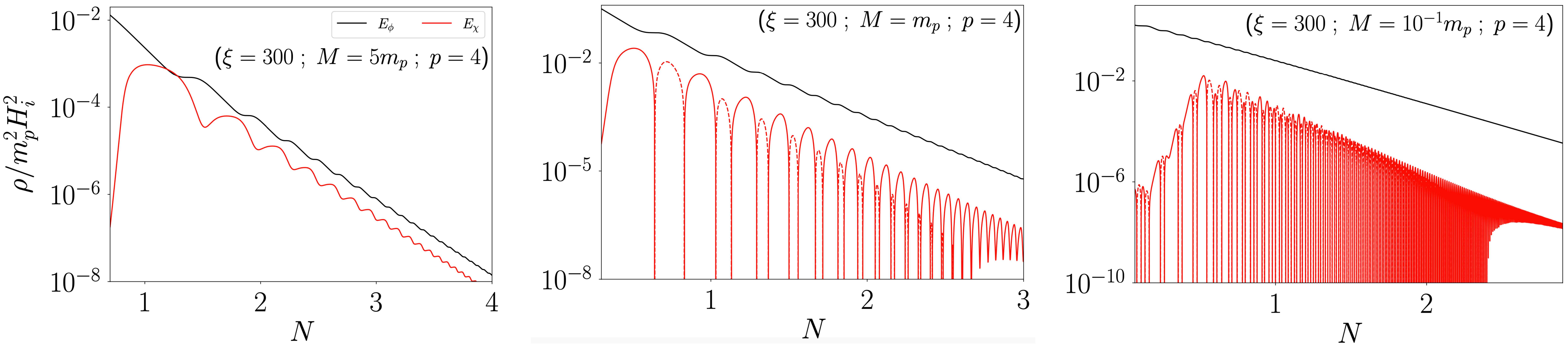
w is dominated by homogeneous background.

Geometric (p)reheating: Lattice results p=4



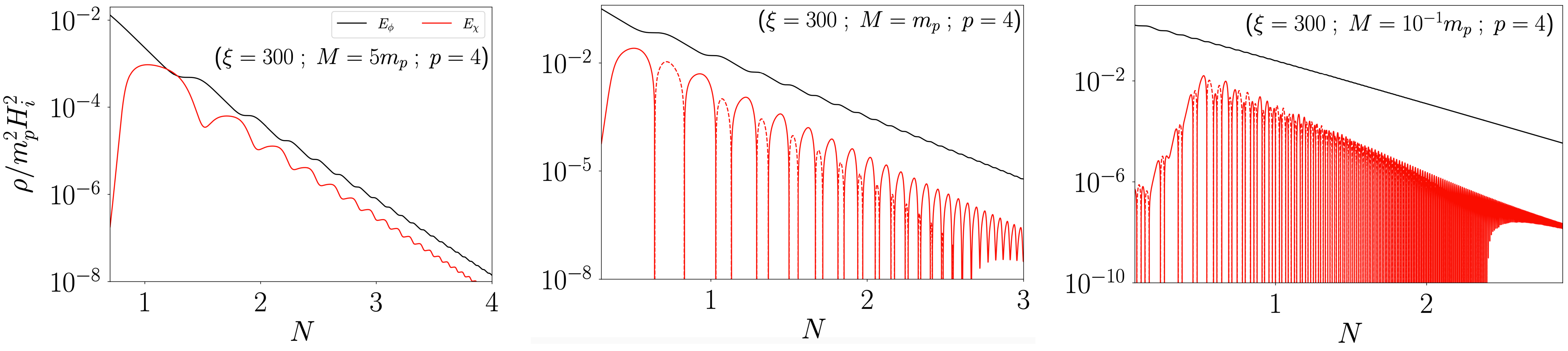
$w \to 1/3$ as $R \to 0$

Geometric (p)reheating: Lattice results p=4



As we decrease M preheating becomes less efficient.

Geometric (p)reheating: Lattice results p=4



Once growth is blocked both energy densities scale equally (as radiation)

Geometric (p)reheating: Lattice results p=4

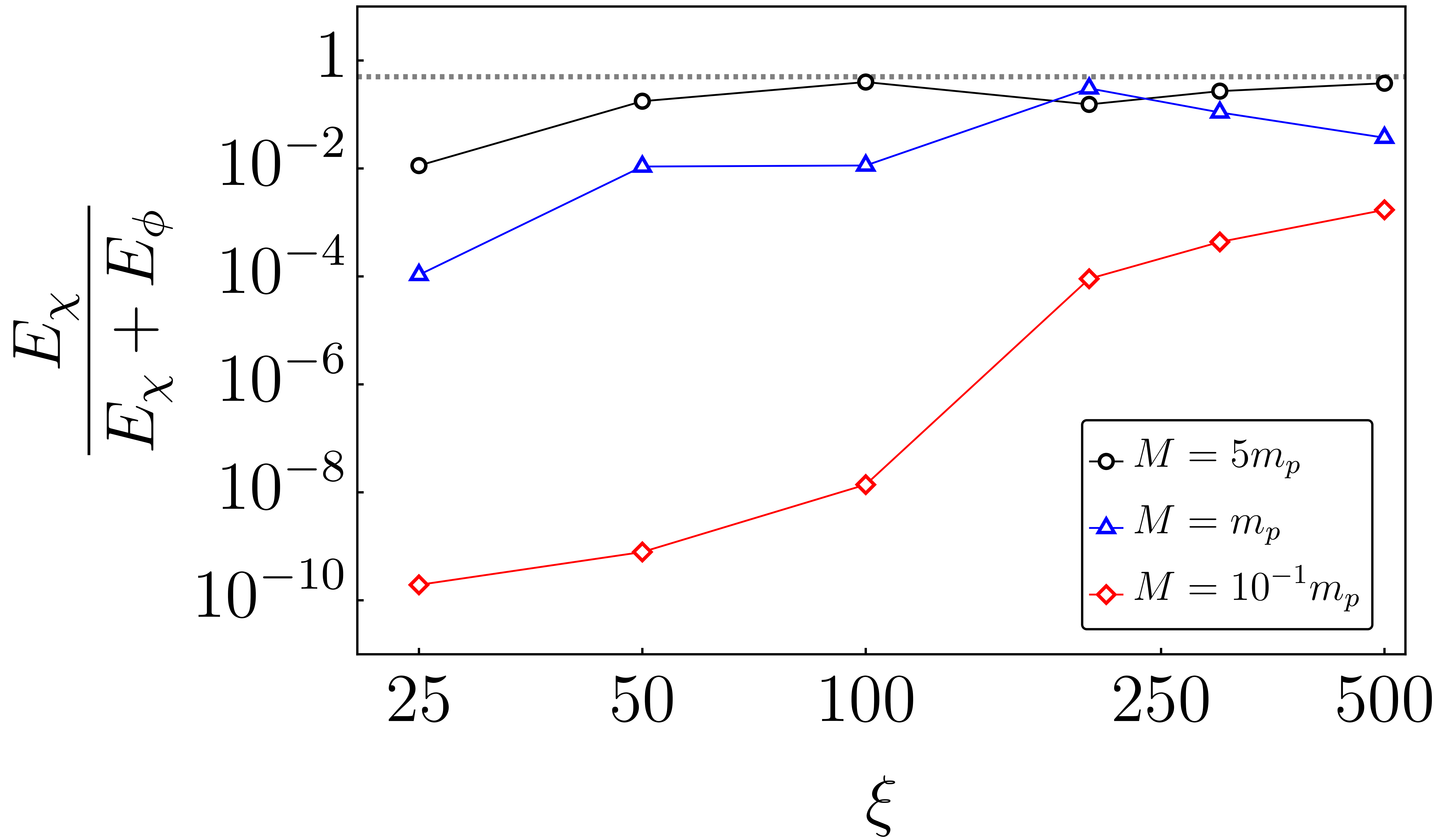
Geometric (p)reheating: Lattice results p=4

As we decrease M preheating becomes less efficient.

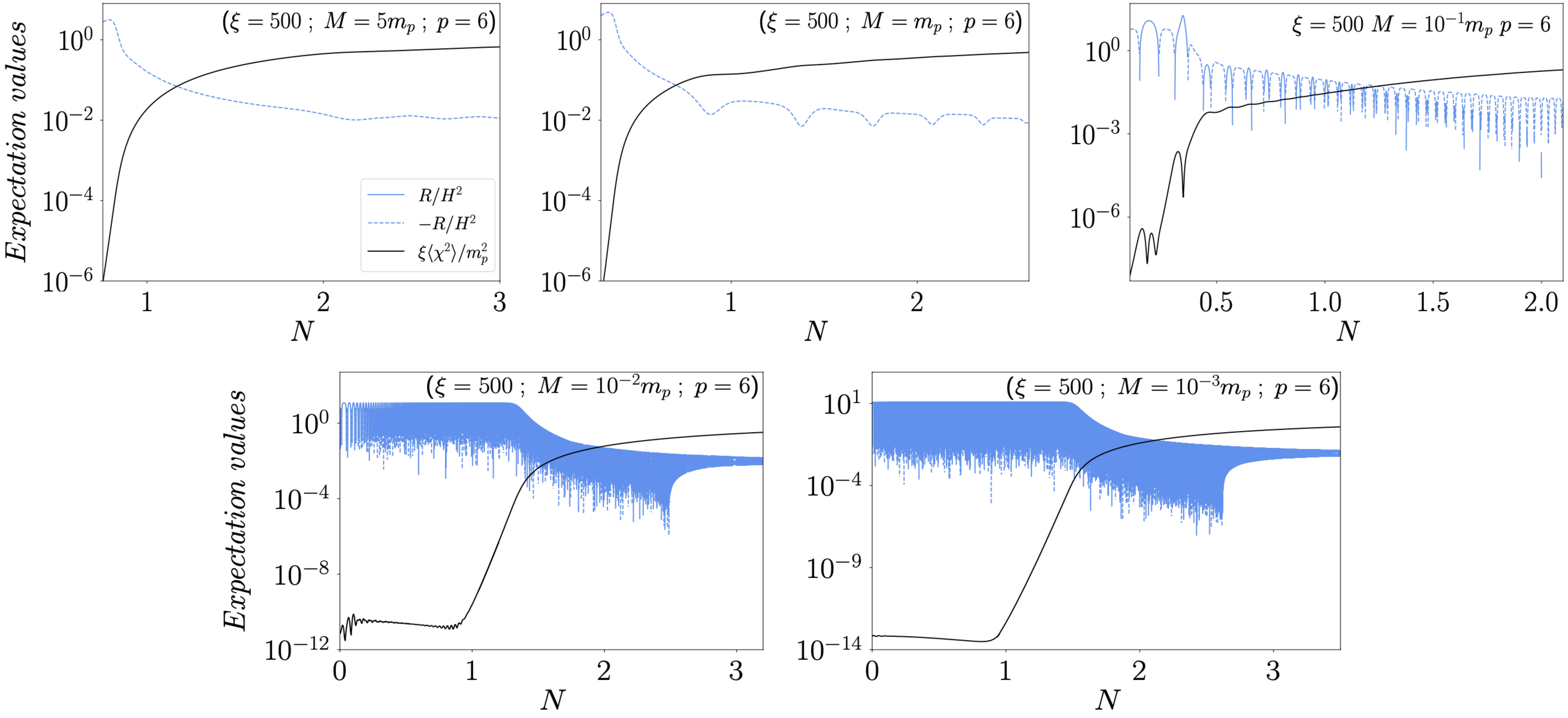
When $E_\chi < E_\phi$ EoS restores to homogeneous oscillations.

Geometric (p)reheating: Lattice results p=4

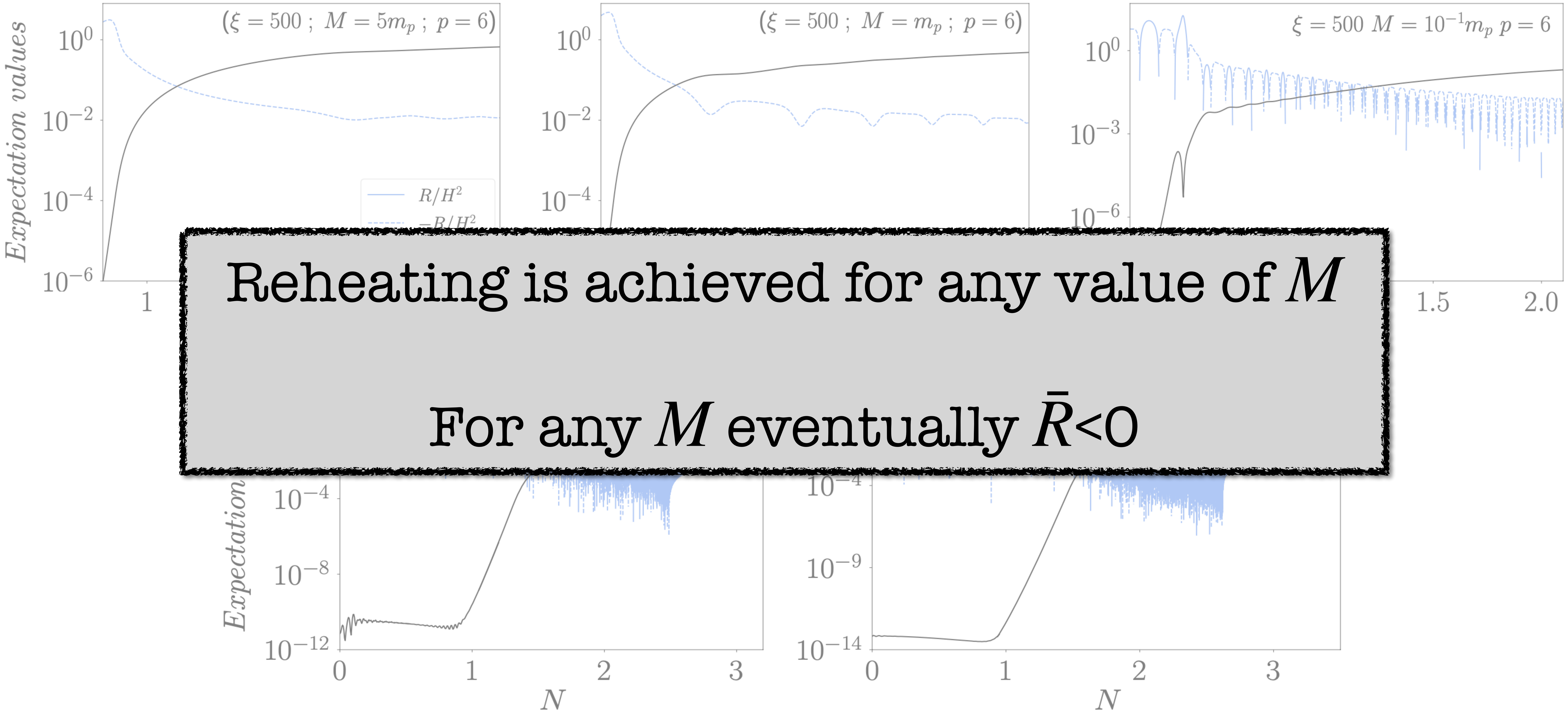
As we decrease M preheating becomes less efficient.



Geometric (p)reheating: Lattice results p=6



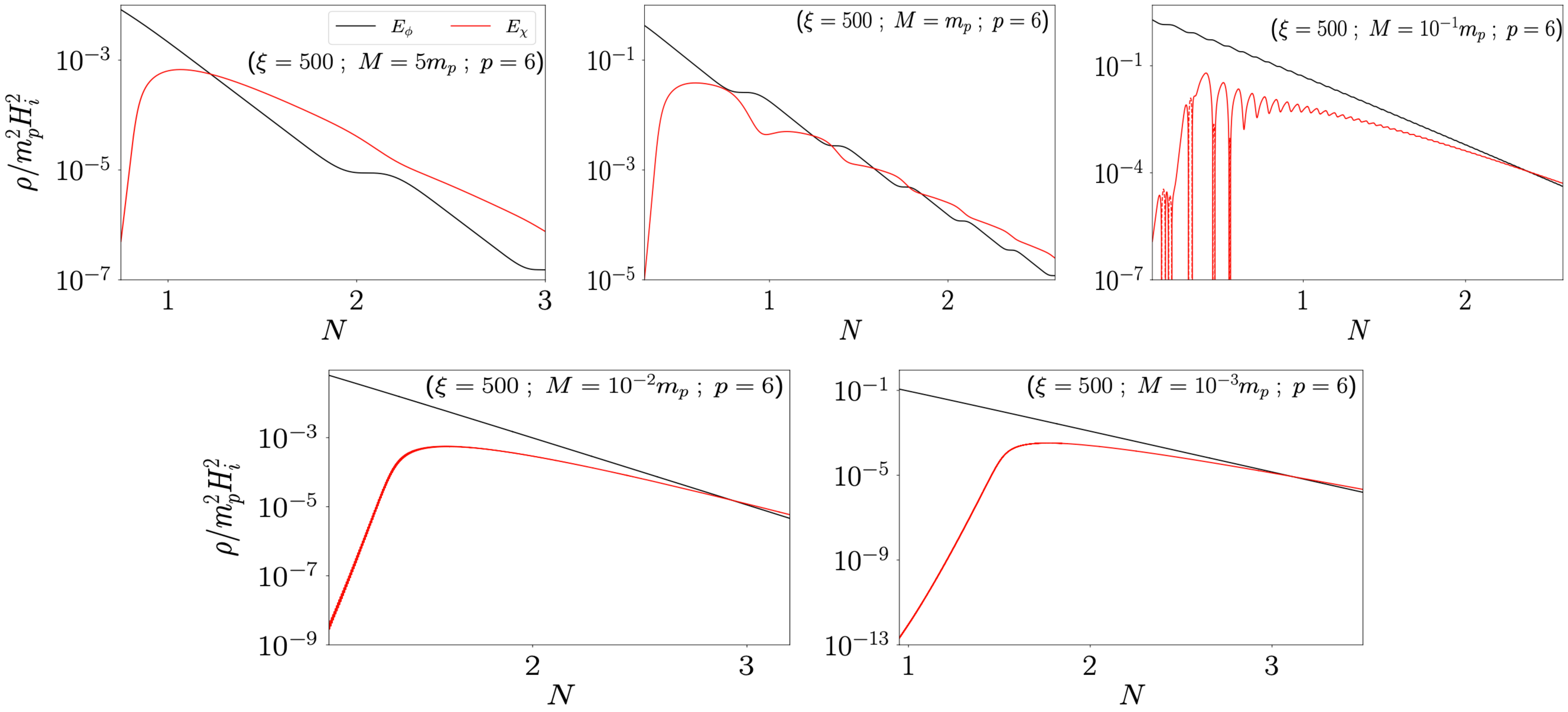
Geometric (p)reheating: Lattice results p=6



Reheating is achieved for any value of M

For any M eventually $\bar{R}$<0

Geometric (p)reheating: Lattice results p=6



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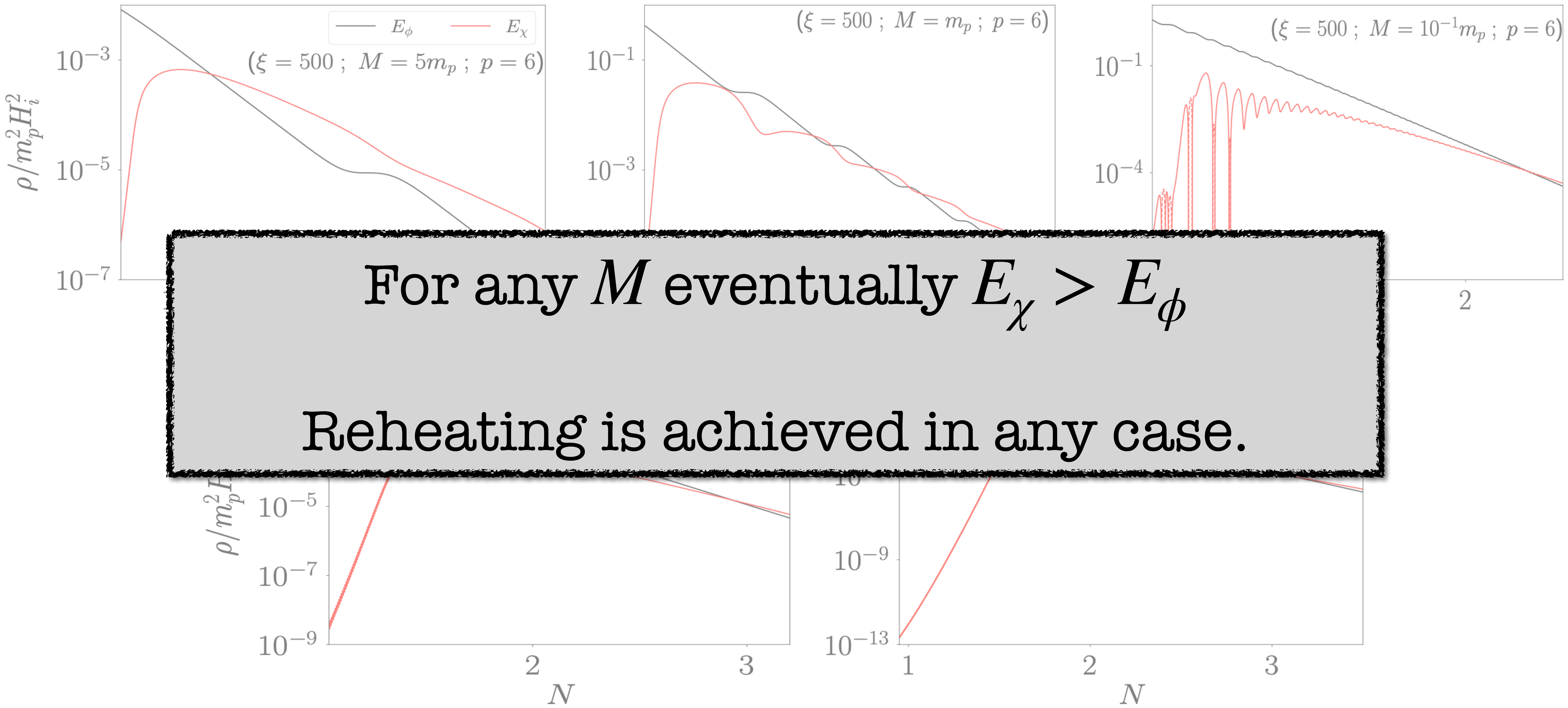

For any M eventually $E_\chi > E_\phi$

Reheating is achieved in any case.

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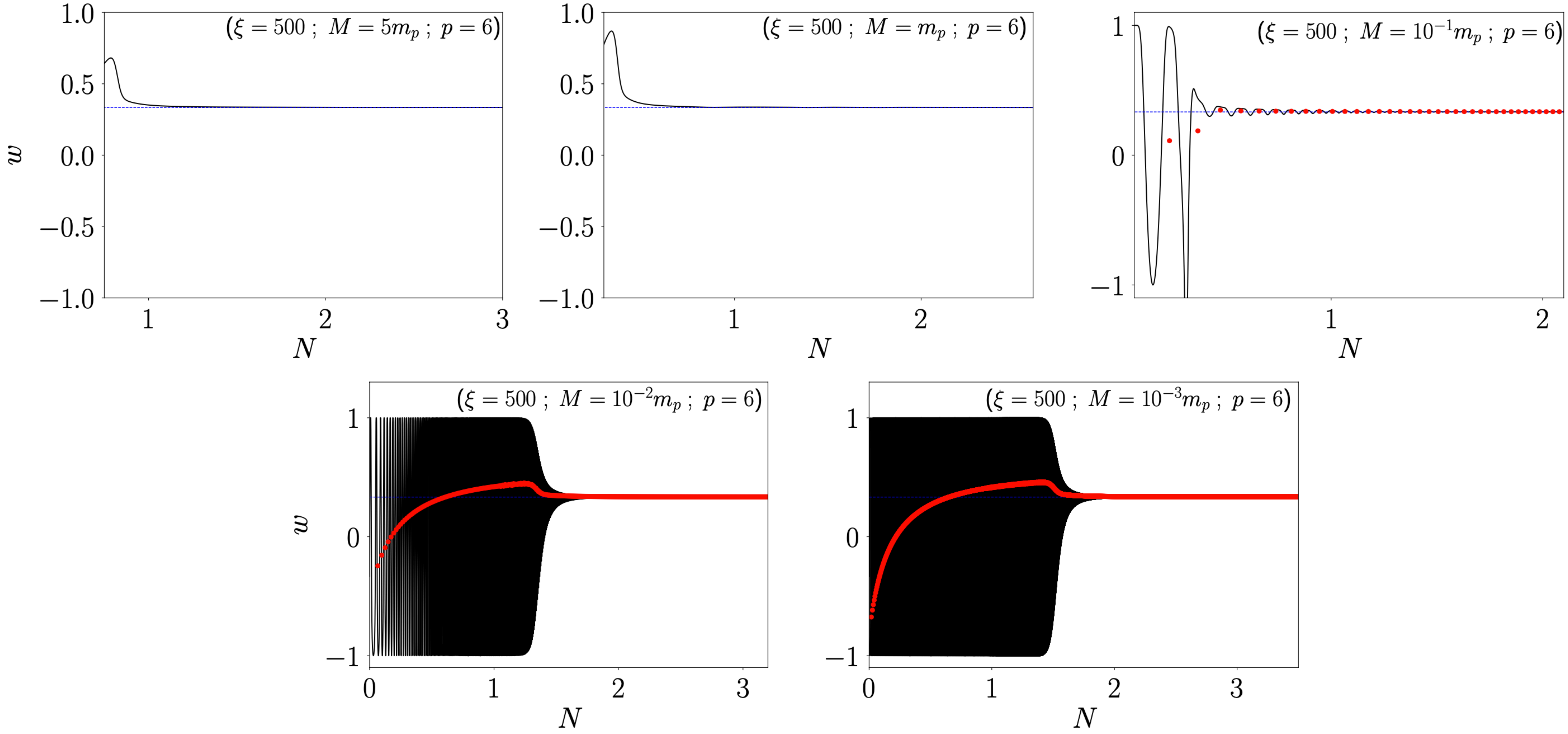

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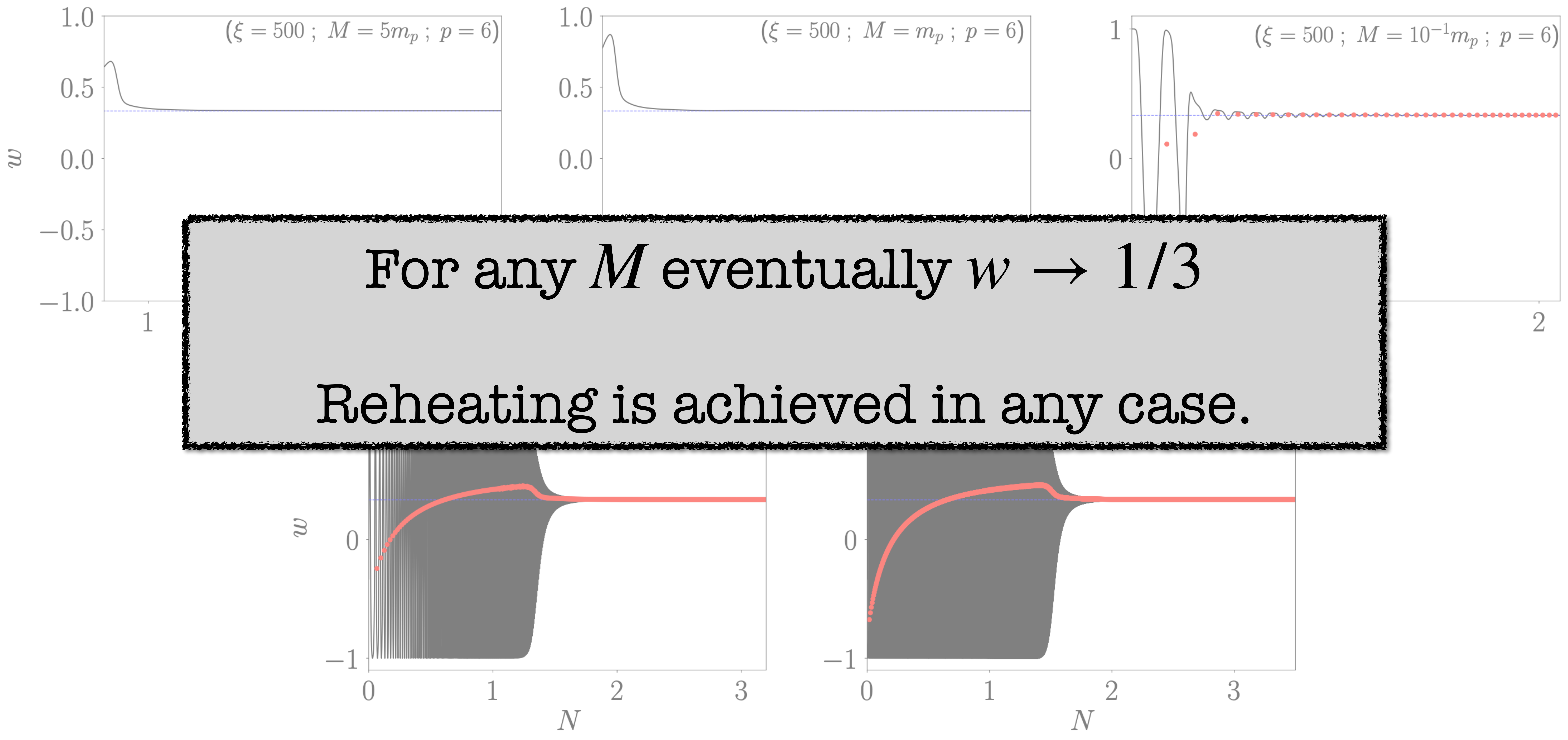
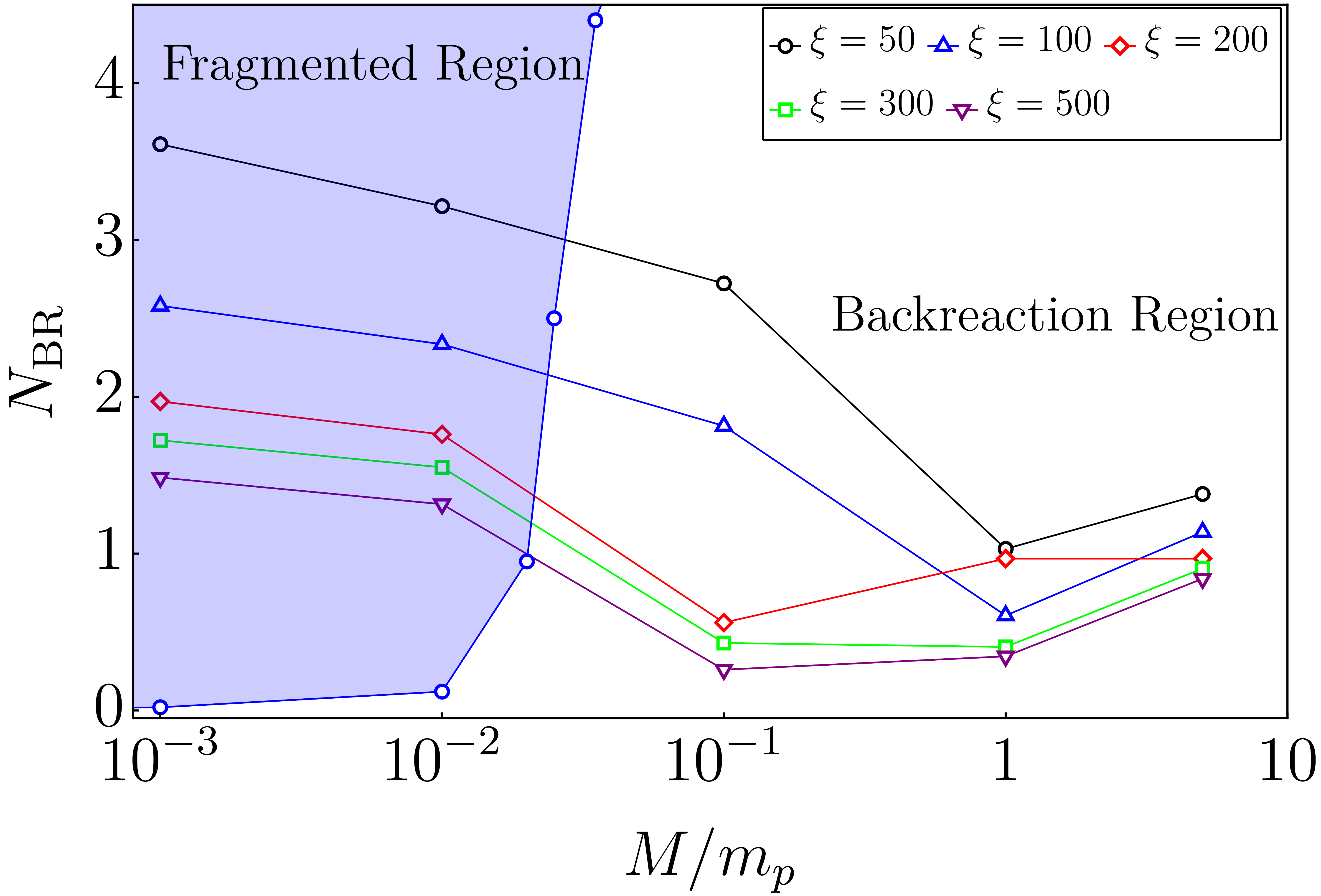


For any M eventually $w \to 1/3$

Reheating is achieved in any case.

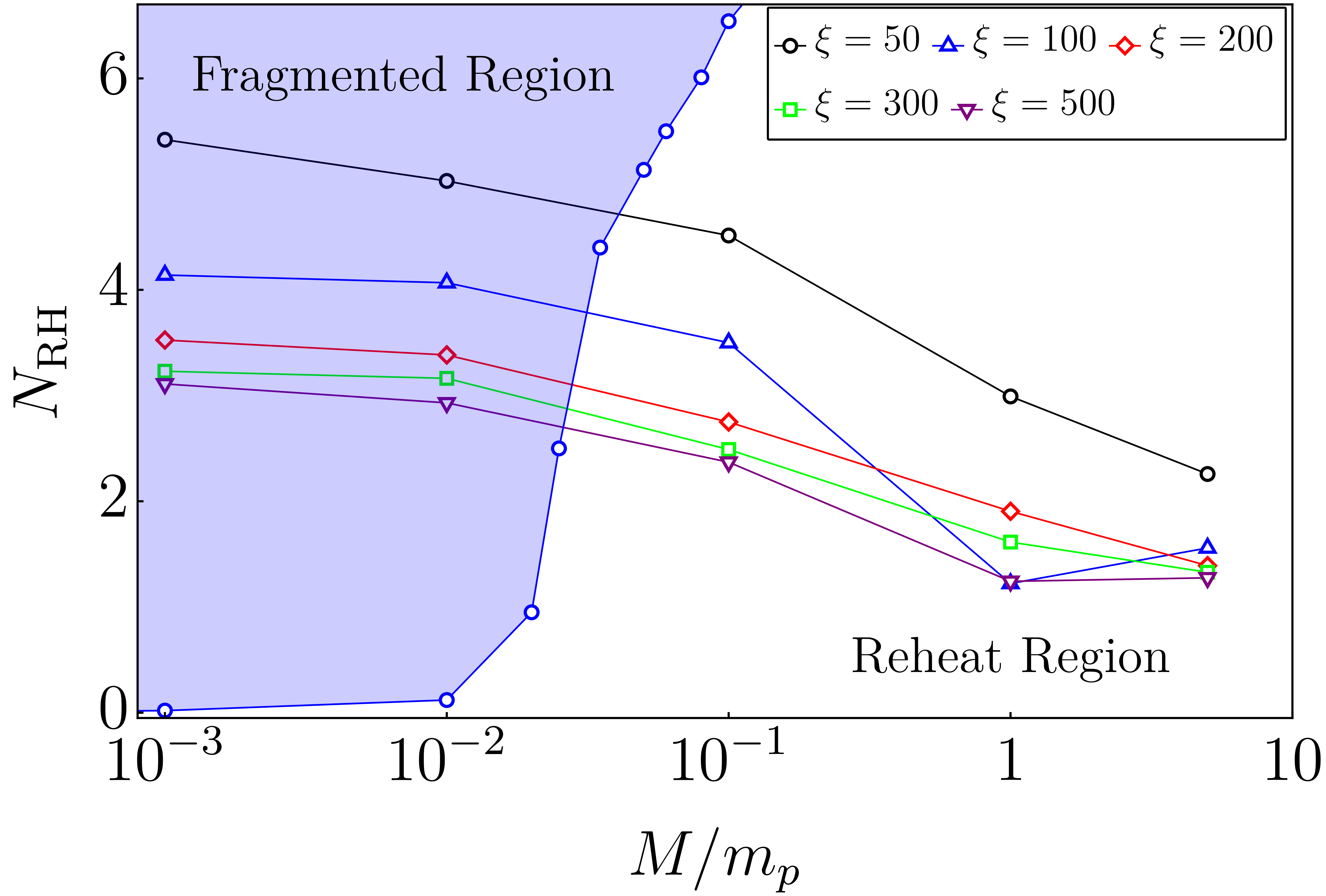
Geometric (p)reheating: Lattice results p=6

N_{BR} ≡ when w (EoS) envelope falls bellow 90% its Max value



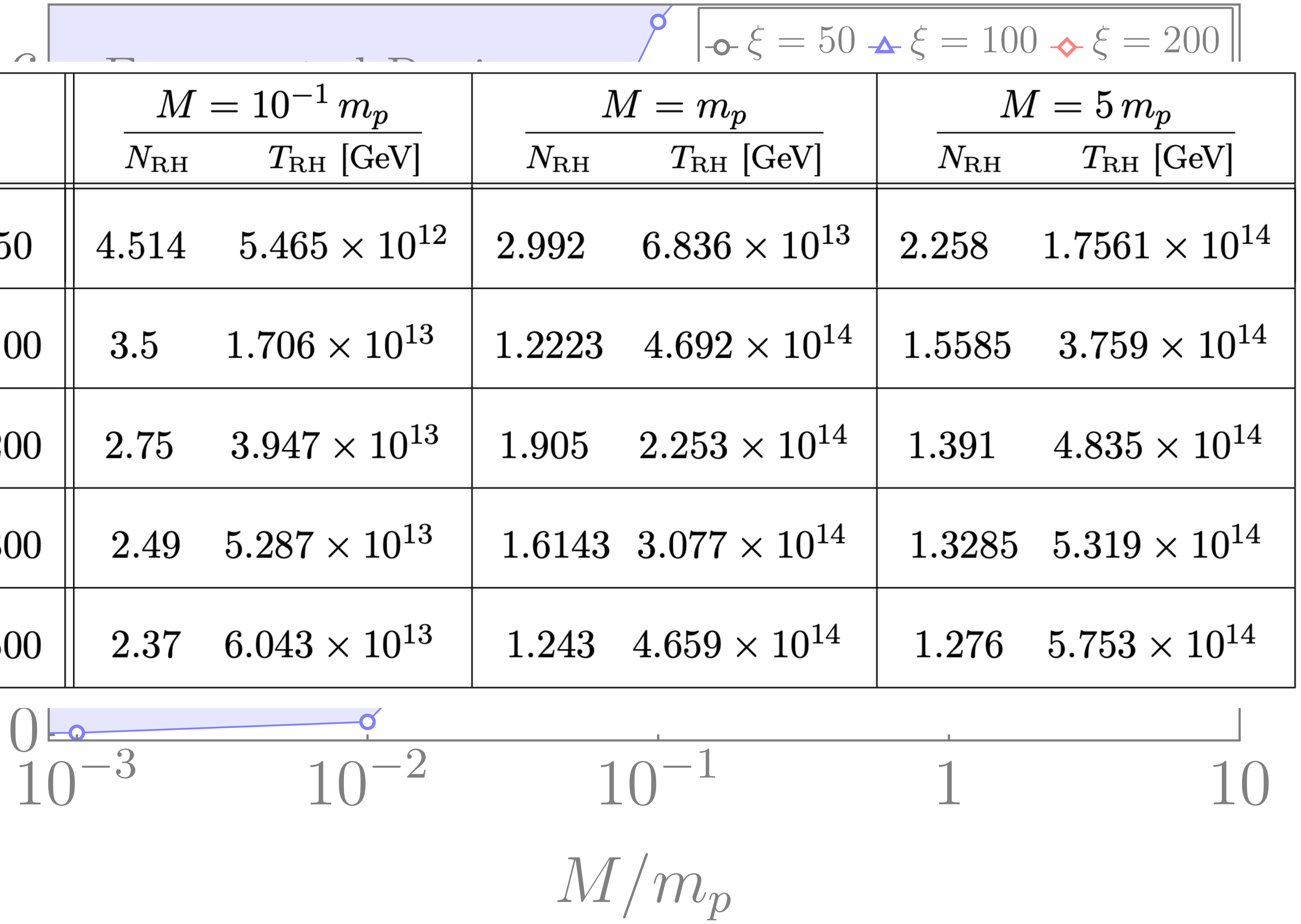
Geometric (p)reheating: Lattice results p=6

$N_{\mathrm{RH}} \equiv$ moment when $E_\chi = E_\phi$



Geometric (p)reheating: Lattice results p=6

We summarize $N_{\rm RH}$ and $T_{\rm RH}$ for allowed cases



	$M = 10^{-1}\, m_p$		$M = m_p$		$M = 5\, m_p$	
	$N_{\rm RH}$	$T_{\rm RH}$ [GeV]	$N_{\rm RH}$	$T_{\rm RH}$ [GeV]	$N_{\rm RH}$	$T_{\rm RH}$ [GeV]
$\xi = 50$	4.514	5.465×10^{12}	2.992	6.836×10^{13}	2.258	1.7561×10^{14}
$\xi = 100$	3.5	1.706×10^{13}	1.2223	4.692×10^{14}	1.5585	3.759×10^{14}
$\xi = 200$	2.75	3.947×10^{13}	1.905	2.253×10^{14}	1.391	4.835×10^{14}
$\xi = 300$	2.49	5.287×10^{13}	1.6143	3.077×10^{14}	1.3285	5.319×10^{14}
$\xi = 500$	2.37	6.043×10^{13}	1.243	4.659×10^{14}	1.276	5.753×10^{14}

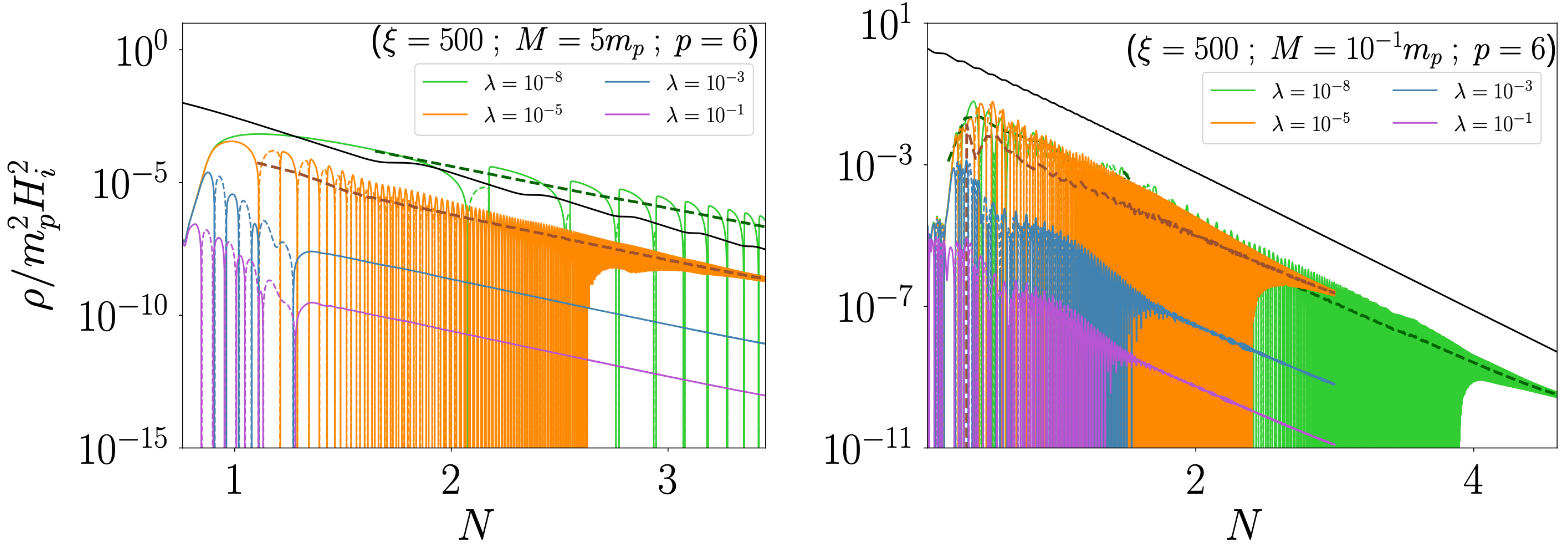
Geometric (p)reheating: Lattice results p=6 and $V(\chi) \equiv \frac{1}{4}\lambda\chi^4$

We have $m_\chi = 3\lambda\langle\chi\rangle^2 + \xi R$

Tachyonic excitation ends for $m_\chi = 0$

$$\chi_{\rm br} \equiv \sqrt{\langle\chi^2\rangle}\Big|_{m^2_{\rm tot}=0} \simeq \sqrt{\frac{\xi}{3\lambda}}\sqrt{\overline{R}}$$

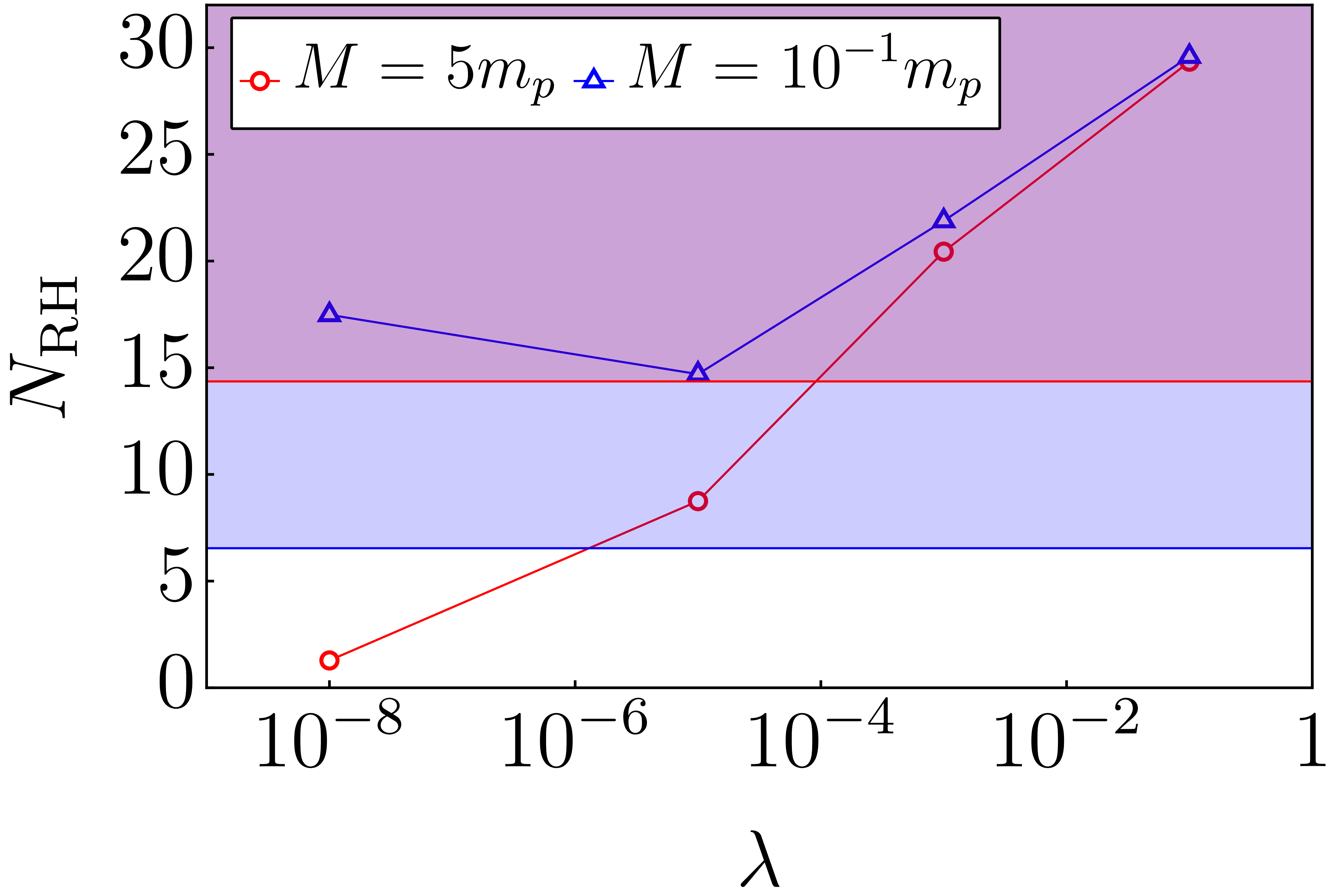
Geometric (p)reheating: Lattice results p=6 and $V(\chi) \equiv \frac{1}{4}\lambda\chi^4$



Geometric (p)reheating: Lattice results p=6 and

$$V(\chi) \equiv \frac{1}{4}\lambda\chi^4$$

$N_{\rm RH} \equiv$ moment when $E_\chi = E_\phi$



	$M = 5\,m_p$	
	$N_{\rm RH}$	$T_{\rm RH}$ [GeV]
$\lambda = 10^{-8}$	1.281	6.511×10^{14}
$\lambda = 10^{-5}$	8.747	3.382×10^{11}

Conclusions

α-attractor models allow for low energy inflation description.

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Inflaton oscillations are deeply affected by low M scenarios

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- Enhances Fragmentation

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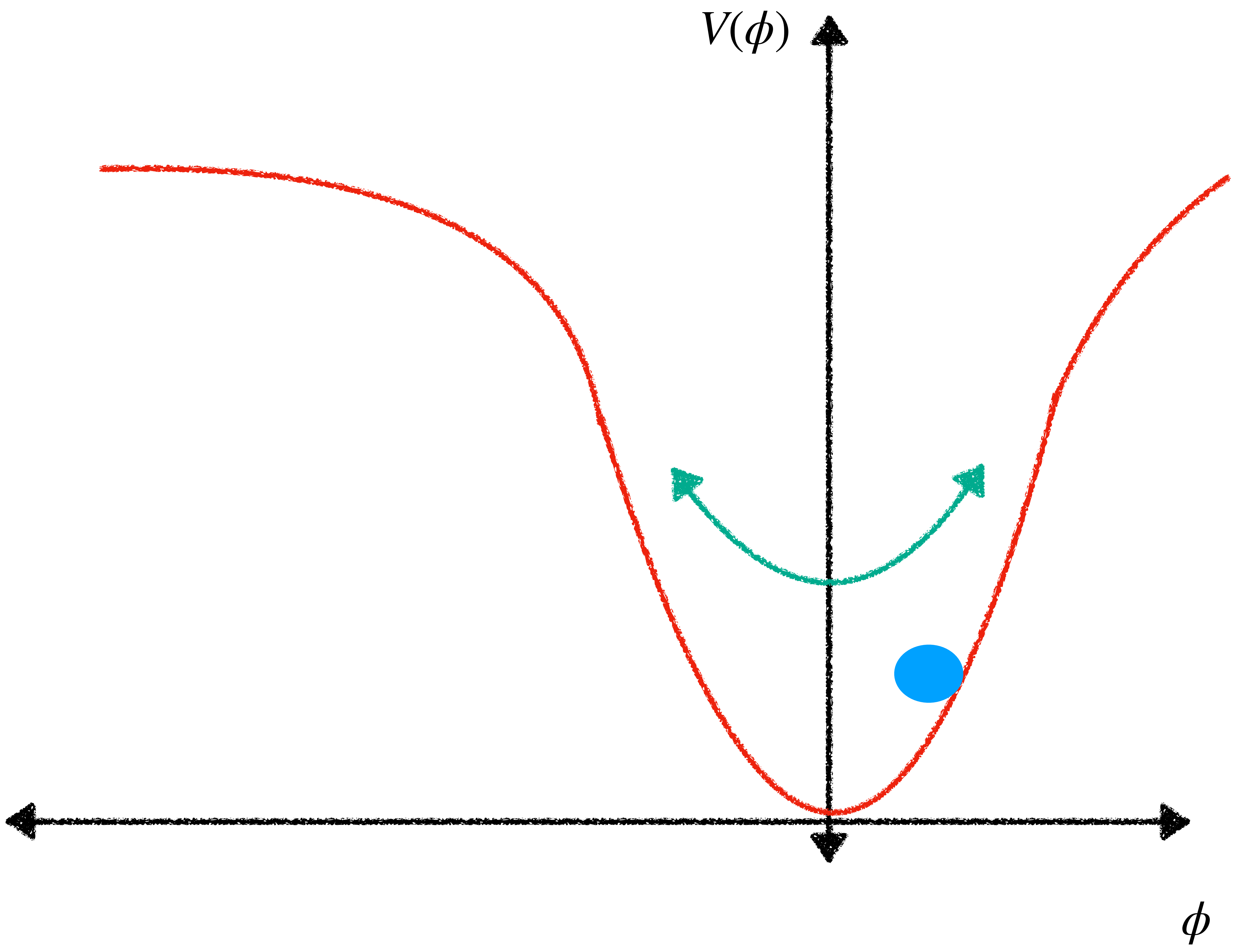
- χ - self interaction prevents preheating for almost any λ and M

Inflaton fragmentation

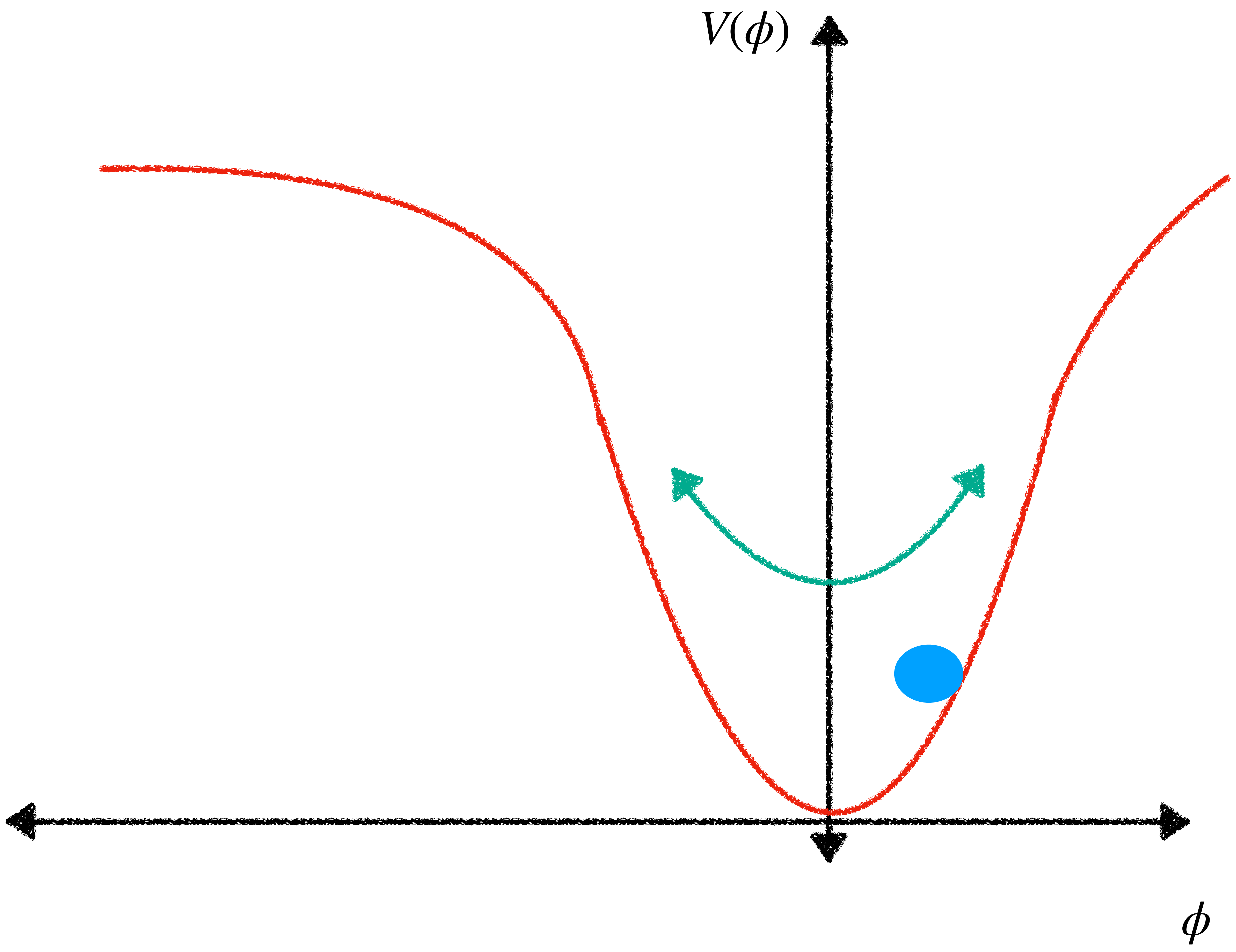


Inflaton inhomogeneous perturbations $\delta\phi(\mathbf{x}, t)$ couple to the background

$$\delta\ddot{\phi}_k + \left[\kappa^2 + (p-1)\,|\,\phi\,|^{p-2}\right]\delta\phi_k = 0$$

Lozanov, Amin '17
Antusch, Figueroa, Marschall, Torrenti '20

Inflaton fragmentation



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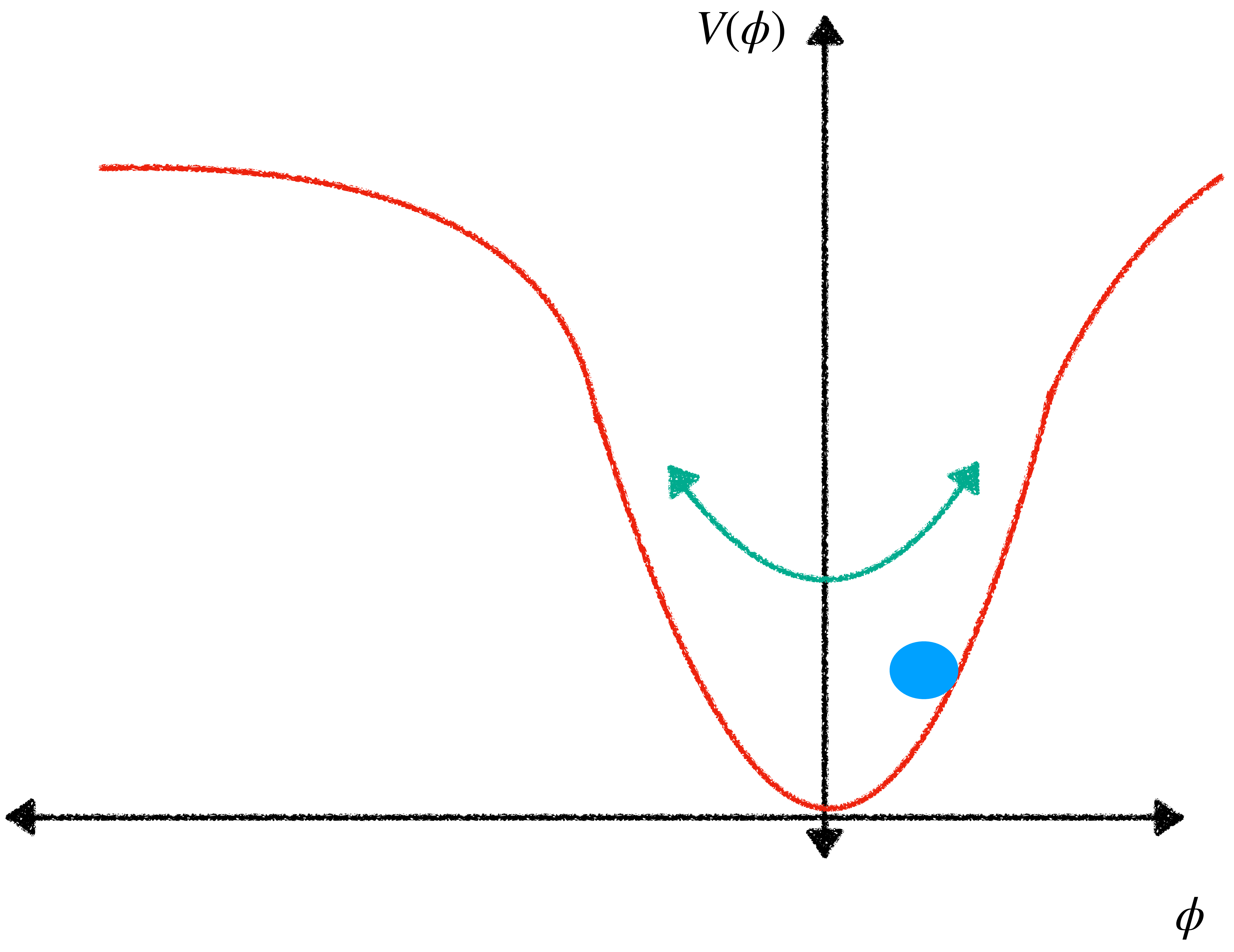
For $M \gtrsim m_p$
Inhomogeneities grow due to parametric resonance

Leading to the fragmentation of the homogeneous background

Fragmentation $\implies$ $w = 1/3$

Inflaton fragmentation



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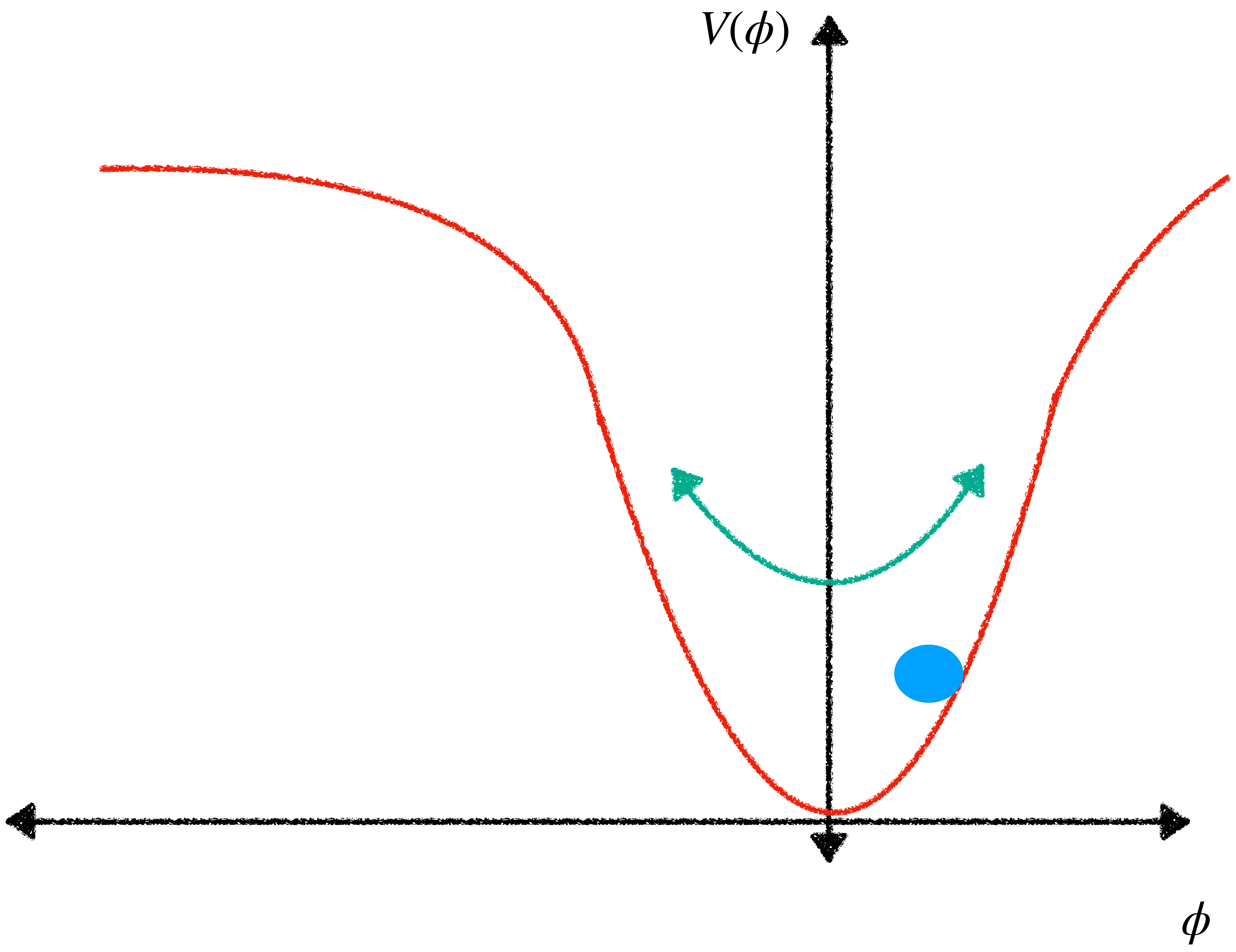
For $M \gtrsim m_p$
Inhomogeneities grow due to parametric resonance

Inefficient effect
Fragmentation time scale very long $N_{\rm frag} \sim \mathcal{O}(1-10)$

Inflaton fragmentation



Inflaton inhomogeneous perturbations $\delta\phi(\mathbf{x}, t)$ couple to the background

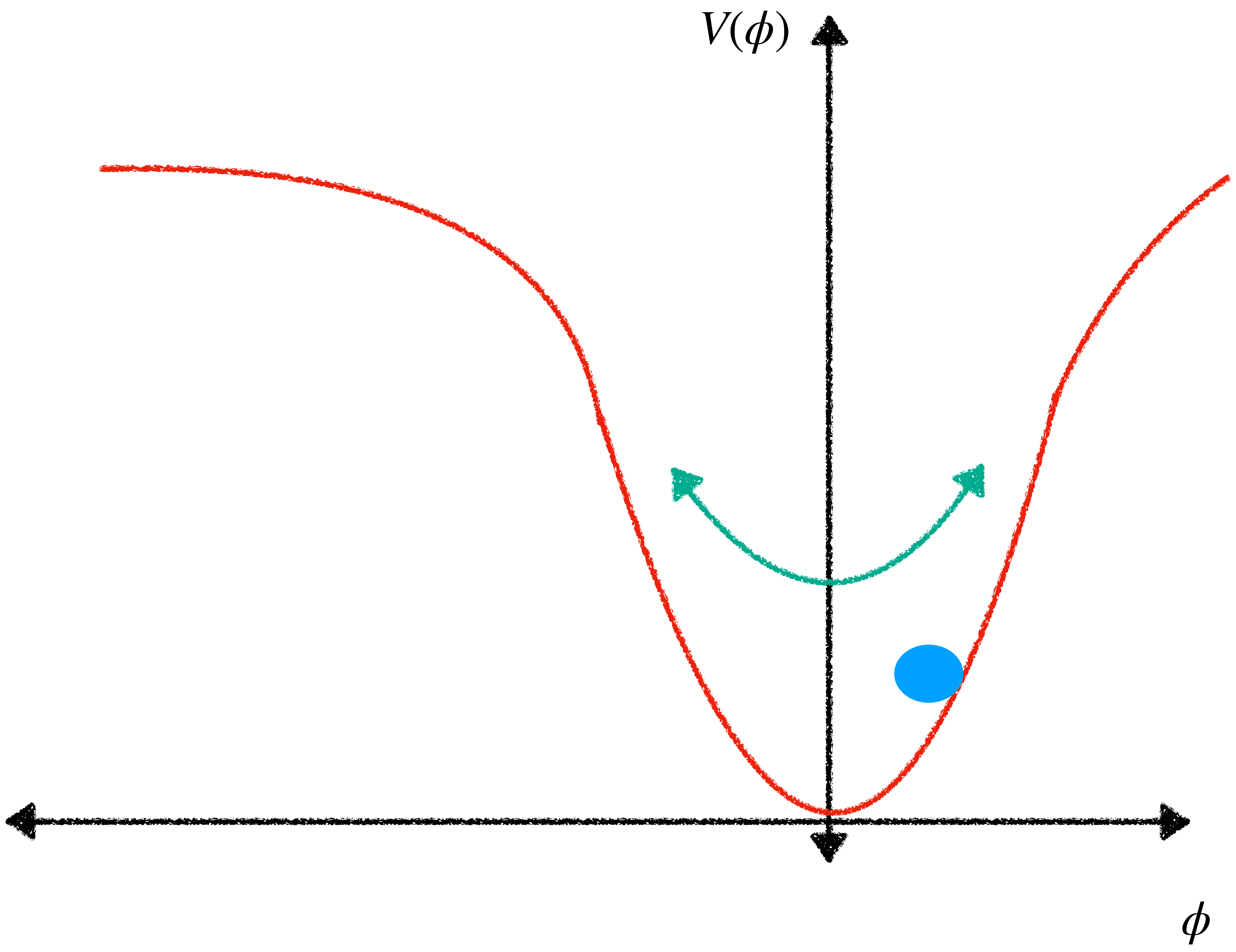
$$\delta\ddot{\phi}_k + \left[\kappa^2 + \frac{\partial^2 V(\phi)}{\partial \phi^2}\bigg|_{\phi_{\rm end}}\right]\delta\phi_k = 0$$

For $M \ll m_p$ initial effective mass of modes is negative

$$\frac{\partial^2 V(\phi)}{\partial \phi^2}\bigg|_{\phi_{\rm end}} < 0$$

Fragmentation $\implies$ $w = 1/3$

Inflaton fragmentation



Fragmentation $\implies$ $w = 1/3$

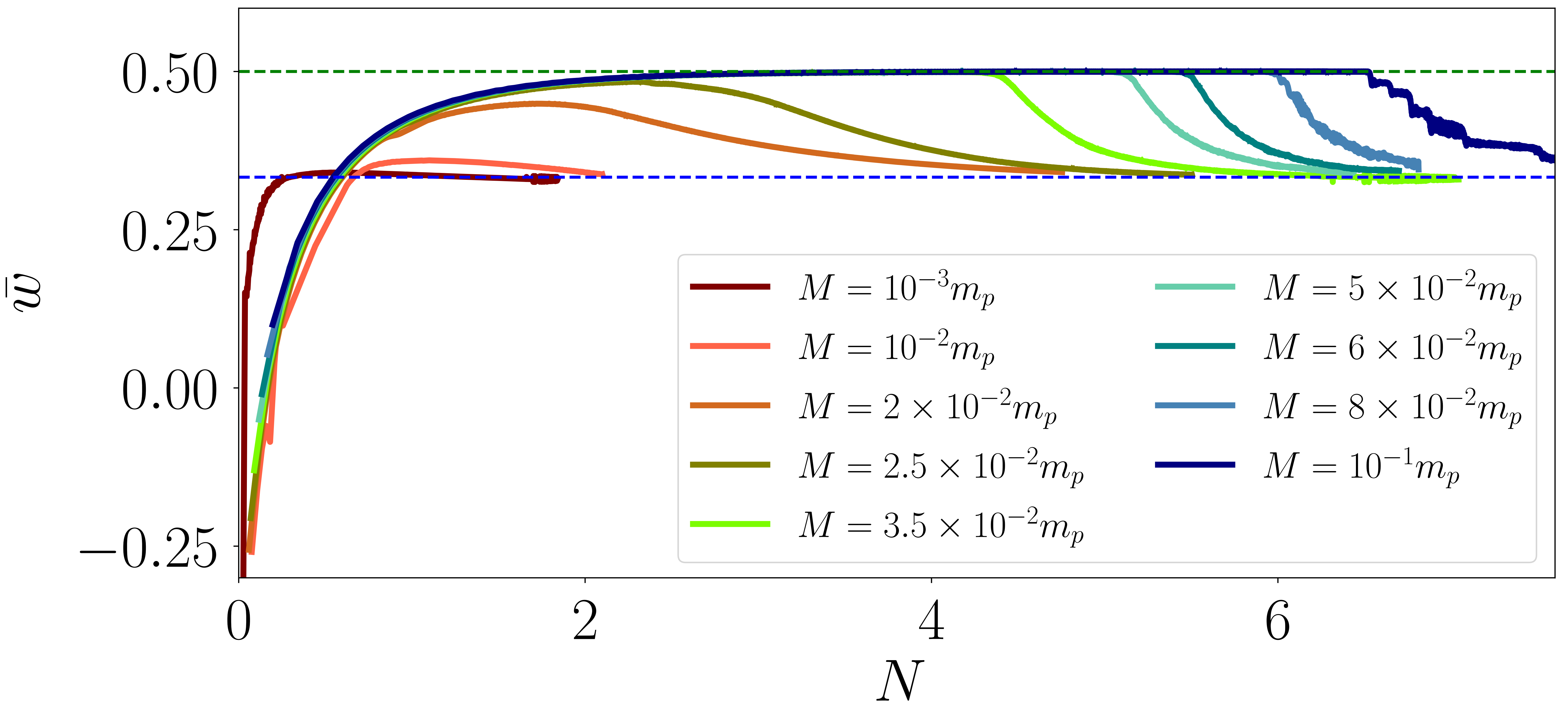
Inflaton inhomogeneous perturbations $\delta\phi(\mathbf{x}, t)$ couple to the background

$$\delta\ddot{\phi}_k + \left[\kappa^2 + \left.\frac{\partial^2 V(\phi)}{\partial \phi^2}\right|_{\phi_{\rm end}}\right] \delta\phi_k = 0$$

A very large range of modes $\kappa \lesssim (M/m_p)^{-1}$ grow exponentially (tachyonic excitation)

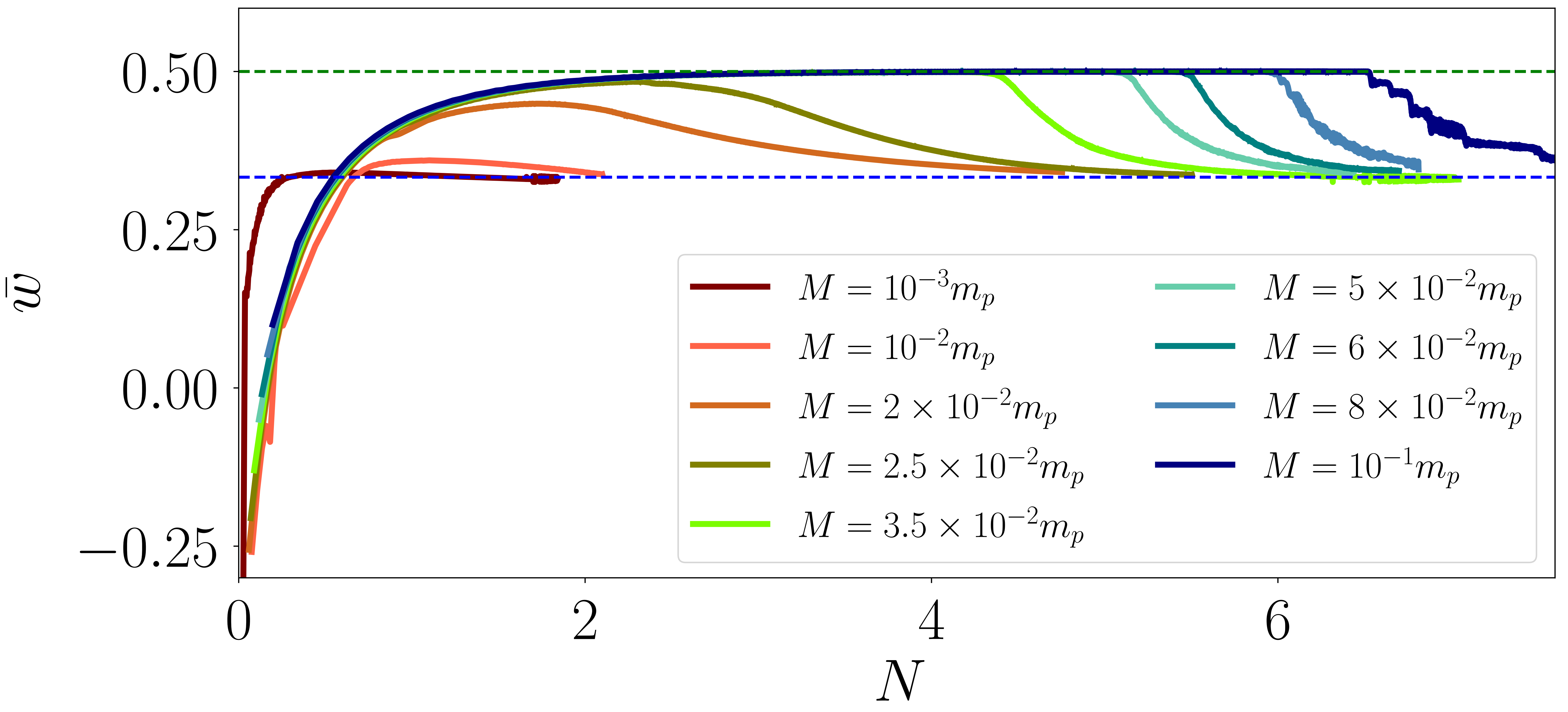
Efficient effect
Fragmentation time scale very short $N_{\rm frag} \sim \mathcal{O}(10^{-2})$

Inflaton fragmentation



Lattice simulations for $p = 6$

Inflaton fragmentation



Lattice simulations for $p = 6$

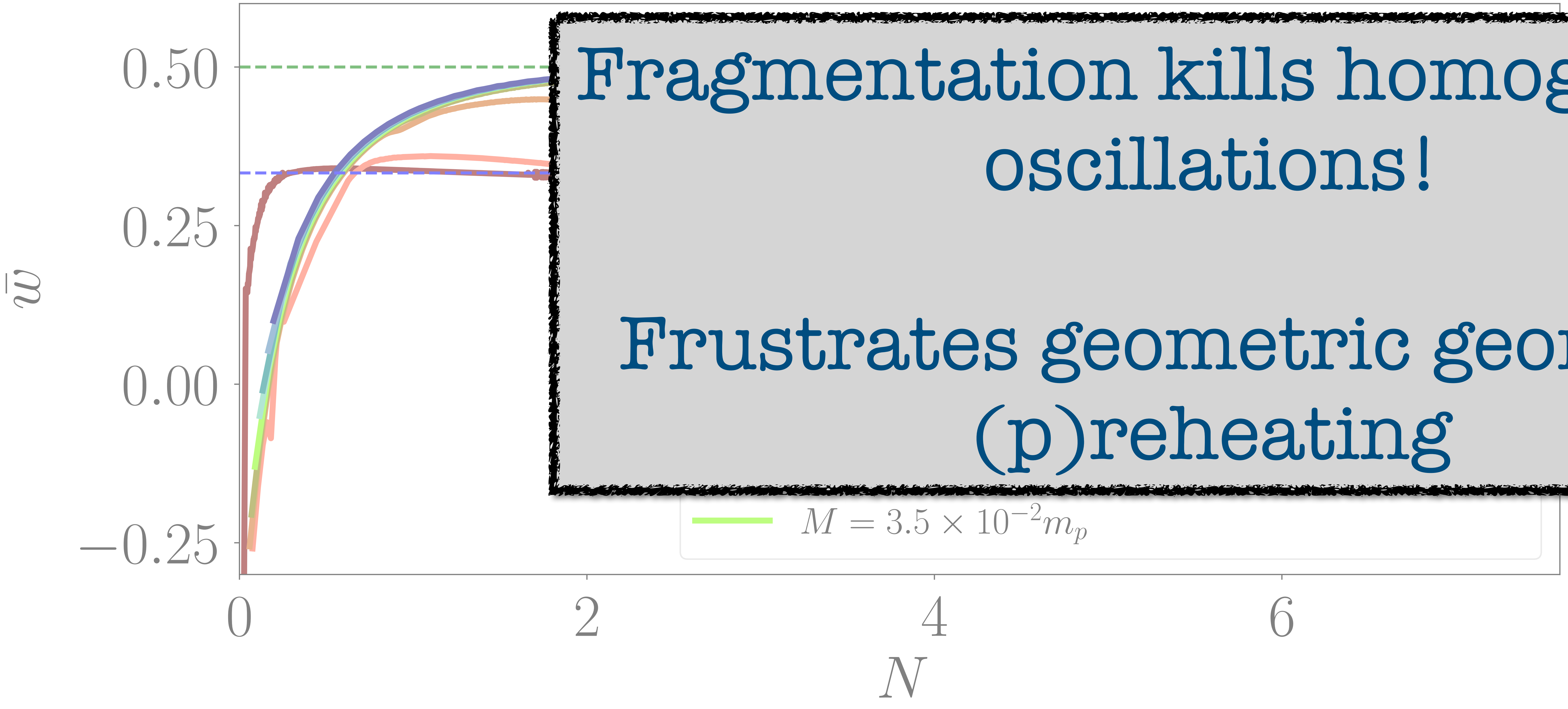
For $M \gtrsim 10^{-2} m_p$ competition between tachyonic and parametric resonance

Long $N_{\rm frag}$

For $M \lesssim 10^{-2} m_p$ dominated tachyonic resonance

Short $N_{\rm frag}$

Inflaton fragmentation



Fragmentation kills homogeneous oscillations!

Frustrates geometric geometric (p)reheating

Lattice simulations for $p = 6$

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…een tachyonic and
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