

# S-wave contribution to rare $D \rightarrow \pi\pi\ell\ell$ decays in the SM and sensitivity to NP

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based on PRD109 (2024) 3 (2312.07501)*

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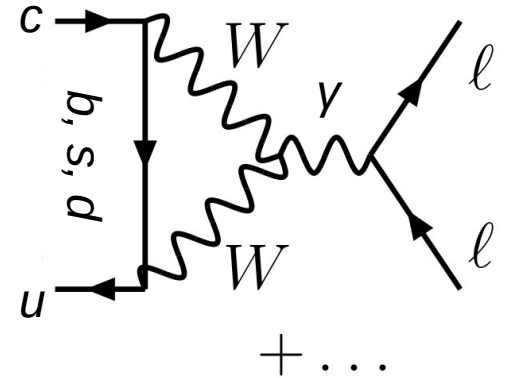
# Rare charm decays



- Flavour physics of the **up-type**: complementary, but less well known than **down-type** **strange** ( $\chi\text{PT}_3$ ) and **bottom** (HQET) sectors

- More effective GIM mechanism, CKM almost diagonal texture: **non-perturbative effects play a very important role**; QCD @ intermediate regime

[Fajfer, Prelovsek '06; Cappiello, Cata, D'Ambrosio '13; Feldmann, Muller, Seidel '17; De Boer, Hiller '18; Bharucha, Boito, Meaux '20...]



- **Large data set available**, allowing for a closer look into the SM  
[various charm-meson decays: LHCb, BESIII, CLEO, BaBar, etc.]
- Having control over the SM, move to observables measuring **SM–NP interference**: analysis of a **rich set of angular observables**

# Large available dataset

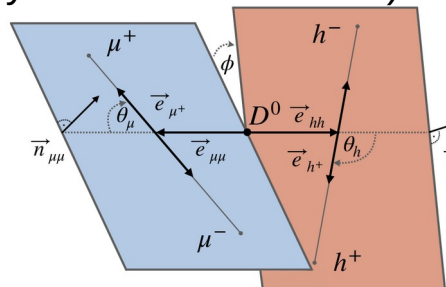
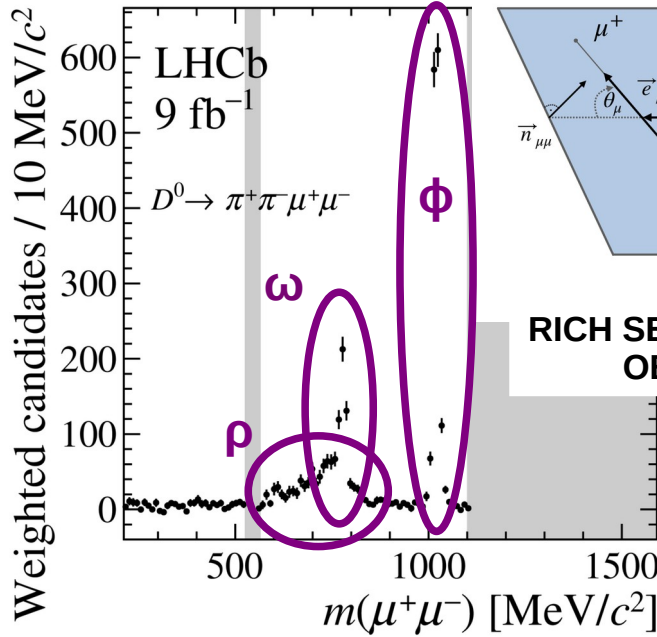
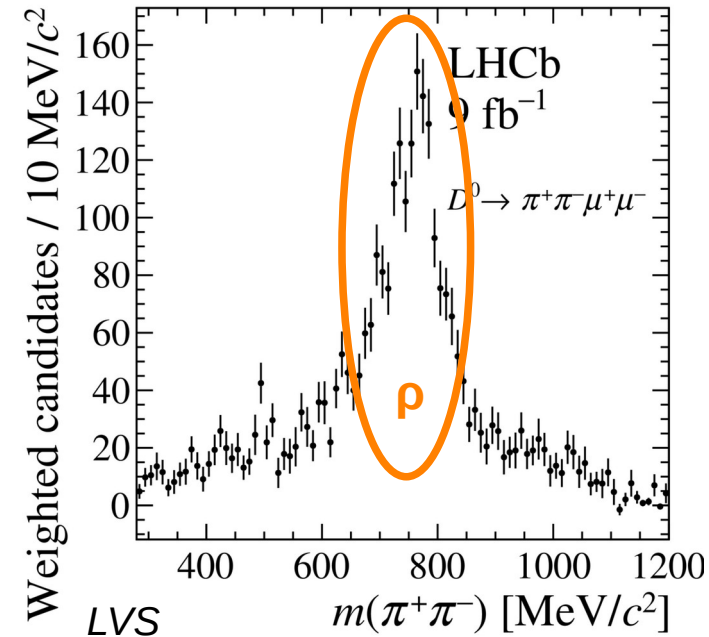


Much more is known about the **muonic** rare decay mode

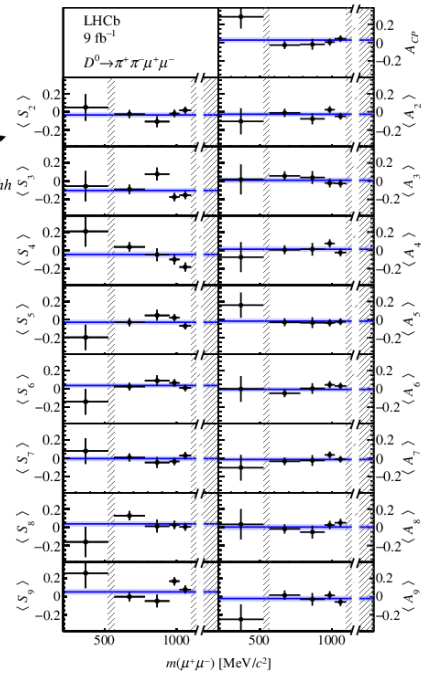
**LHCb:  $D^0 \rightarrow \mu^+\mu^-$  (1305.5059; 2212.11203);  $D^+ \rightarrow \pi^+\mu^+\mu^-$  (1304.6365; 2011.00217);**

**$D^0 \rightarrow h^+h^-\mu^+\mu^-$  (1310.2535; 1707.08377; 1806.10793; 2111.03327 - 9/fb @ 7, 8, 13 TeV); etc.**

- **Differential BRs:** clear resonant peaks in  $m(\pi\pi)$  and  $m(\mu\mu)$
- **Binned angular observables** (CP-sym. “S”, and CP-asym. “A” combinations)



**RICH SET OF ANGULAR OBSERVABLES!**



# Testing Short-Distance (SD) physics 3

- The SM effective weak interactions for  $c \rightarrow u\ell^+\ell^-$  @  $\mu \sim m_c$  are:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \underbrace{\sum_{i=1}^2 C_i(\mu) (\lambda_d Q_i^d + \lambda_s Q_i^s)}_{\text{current-current (4-quark) operators:}} - \underbrace{\lambda_b (C_7(\mu) Q_7 + C_9(\mu) Q_9 + C_{10}(\mu) Q_{10})}_{\text{GIM \& CKM: small contributions; } C_{10}: \text{ higher order in EW interactions } G_F^2} \right] + \text{h.c.}$$

current-current (4-quark) operators:

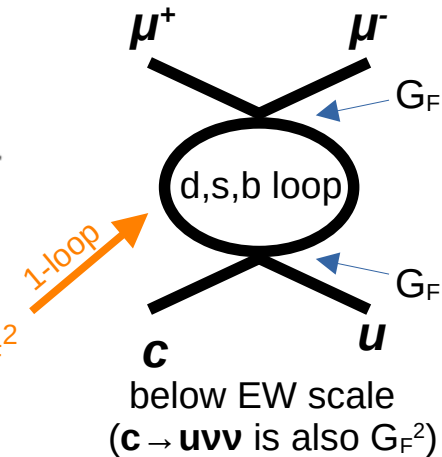
long-distance contribution,  
encoded in  $C_7^{\text{eff}}, C_9^{\text{eff}}$



GIM & CKM: small contributions;

$C_{10}$ : higher order in EW interactions  $G_F^2$

$$Q_{10} = \frac{\alpha_{em}}{2\pi} (\bar{u}\gamma_\mu(1 - \gamma_5)c)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

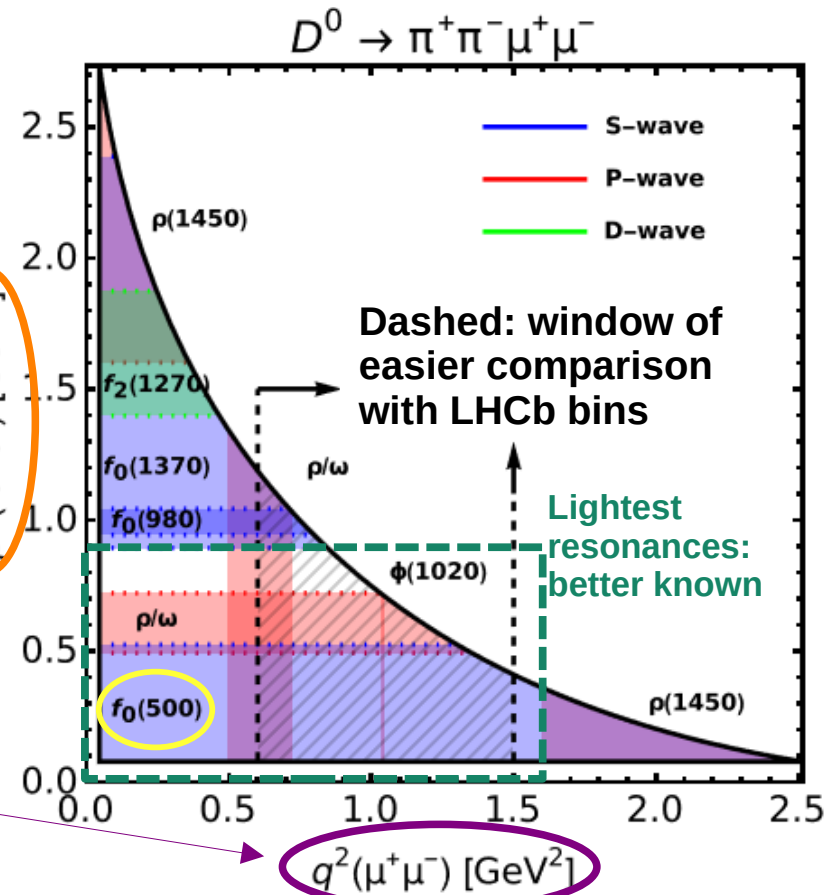
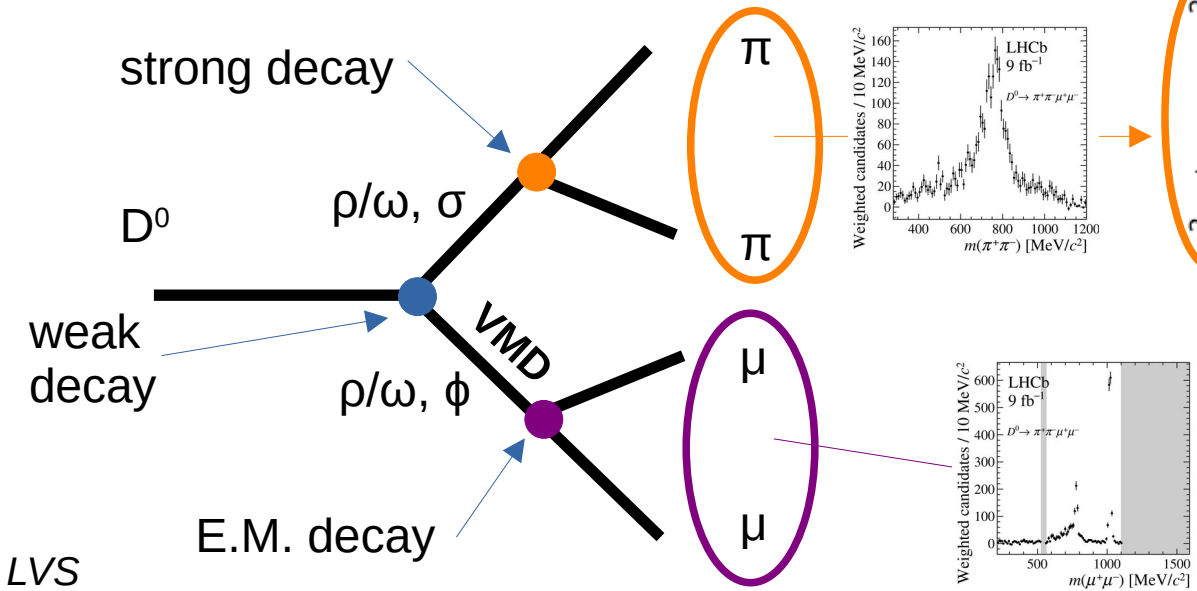


- SM null tests**, e.g., NP in  $C_{10}$ : interference with SM Long-Distance (LD) **enhances sensitivity to NP**, i.e.,  $(C_9^{\text{eff}})^* \times C_{10}^{\text{NP}}$  [De Boer, Hiller '18]
- Tests of SD require good enough description of the LD part

# Available phase space

4

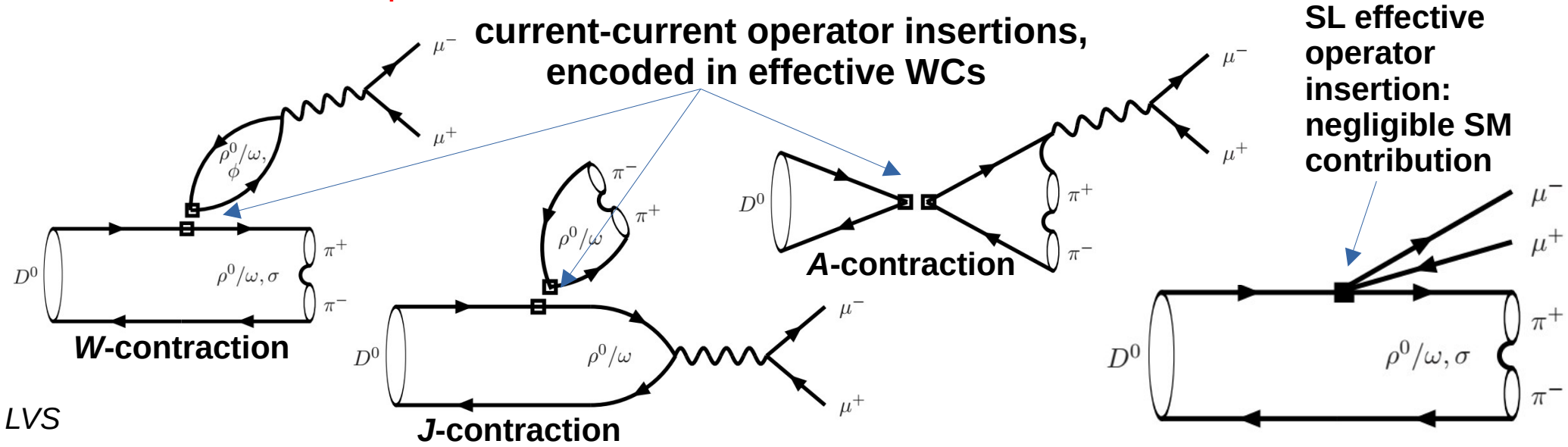
- Phase space heavily populated with resonances (cf. B sector)
- Quasi-two body (Q2B) decays
- Focus: “high-energy window”, thus avoiding tower of heavier S-, P-, D-resonances



# Factorization model

5

- Required **non-perturbative inputs**: **decay constants** (from  $\rho^0, \omega, \phi \rightarrow e^+e^-$ ), **form factors** (BESIII SL  $D^+ \rightarrow \pi^+\pi^-e^+\nu_e$ ), **line-shapes** ( $\rho^0/\omega \rightarrow \pi^+\pi^-$ : Gounaris-Sakurai;  $\phi, \omega \rightarrow \mu^+\mu^-$ : Breit-Wigner;  $\sigma$ : Bugg)
- Beyond naive factorization: free  $O(1)$  normalization coefs, constant complex phases among intermediate resonances (no clear need for dynamics in these parameters)
- We **fit these free parameters from LHCb data**



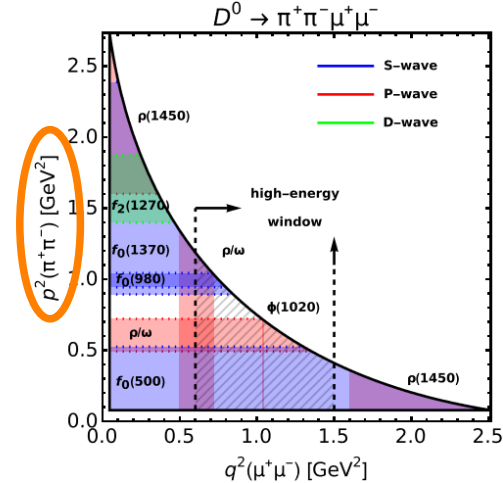
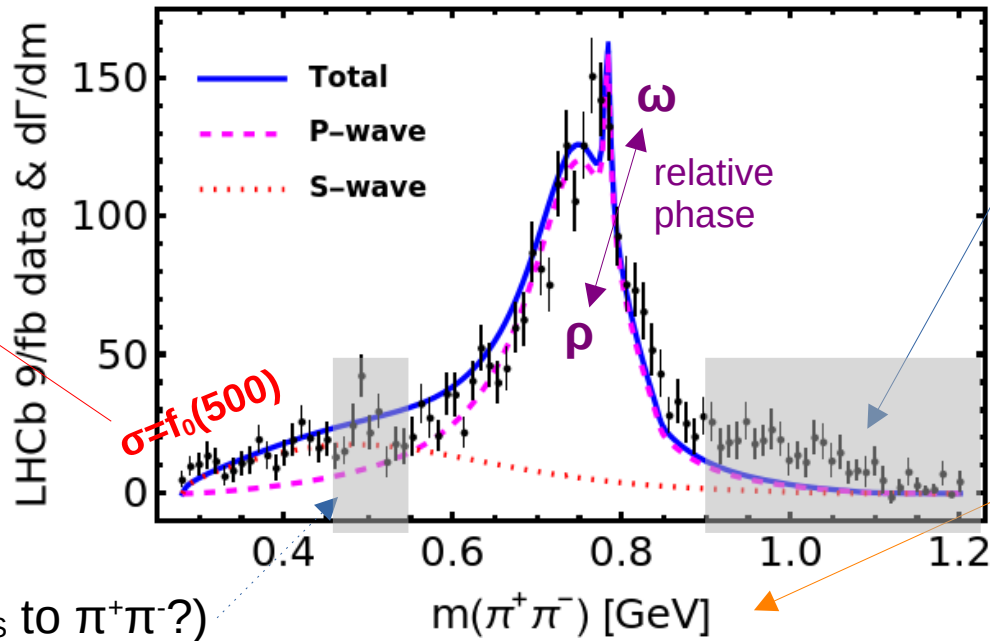
# Fits to differential BRs



- S-wave:  $f_0(500)$  is clearly seen in present data, **despite** not interfering with the dominant P-wave in the BR
- Consistent with BESIII SL decay:  $D^+ \rightarrow \pi^+\pi^-\ell^+\nu_\ell$

Other scalar contributions [ $\pi\pi \rightarrow KK \rightarrow \pi\pi$  (inelastic rescattering effect)], and P- and D-waves; also, isospin-2, and Bremsstrahlung

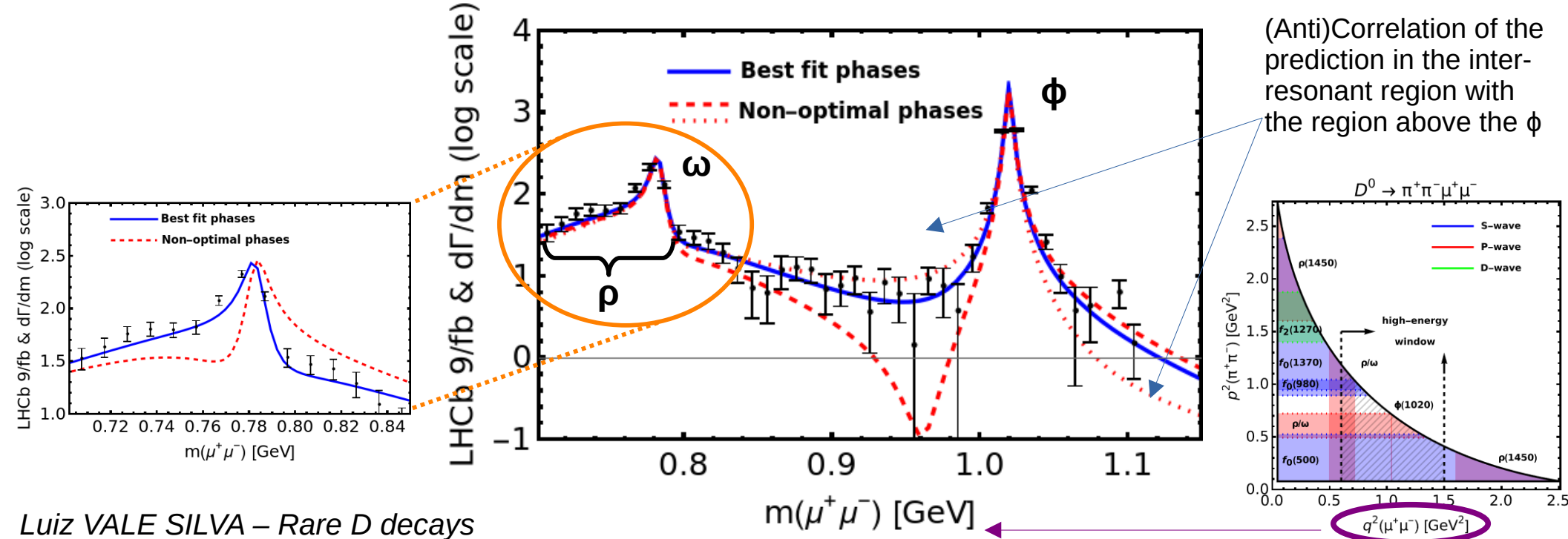
At the level of ~20% of total  $\Gamma$



# Fits to differential BRs



- Relative strong-phases among resonances: important impact on differential BR
- Such phase differences can be probed by present data





# Angular observables

$$\langle I_i \rangle_- \equiv \left[ \int_0^{+1} d \cos \theta_\pi - \int_{-1}^0 d \cos \theta_\pi \right] I_i, \quad \langle I_i \rangle_+ \equiv \int_{-1}^{+1} d \cos \theta_\pi I_i$$

- LHCb measured  $|S|^2+|P|^2$  (i.e., o) & P-wave only (i.e., x); **straightforward to extend their analysis to include S- and P-waves interference** (i.e., ✓)
- SM predictions, use previous strong-phase differences (“S” stands for CP-symmetric,  $I_i^\dagger \equiv \mathbf{S}_i$ ,  $i=1, \dots, 9$ ):

- $\mathbf{S}_2, \mathbf{S}_3, \mathbf{S}_4 \sim -10\%$  ( $\mathbf{S}_1$  is related to  $\Gamma$  and  $\mathbf{S}_2$ )
- $\mathbf{S}_5, \mathbf{S}_6, \mathbf{S}_7 = 0$  (null tests of the SM)
- $\mathbf{S}_7, \mathbf{S}_8, \mathbf{S}_9 \sim 0$  (imaginary part among P-wave contributions)

- exp vs. theo: **similar pattern seen in LHCb data**, but large exp and theo uncertainties of O(few)% prevent better tests of the SM

$\int \langle I_i \rangle_- / \Gamma^r$		$\int \langle I_i \rangle_+ / \Gamma^r$	
$i$	S-wave	$i$	S-wave
1	✓	1†	o
2	✓	2†	o
4†	x	3†	x
5†	x	4	✓
7†	x	5	✓
8†	x	6†	x
		7	✓
		8	✓
		9†	x

“+” or “-”: ways of integrating over  $\cos\theta_\pi$

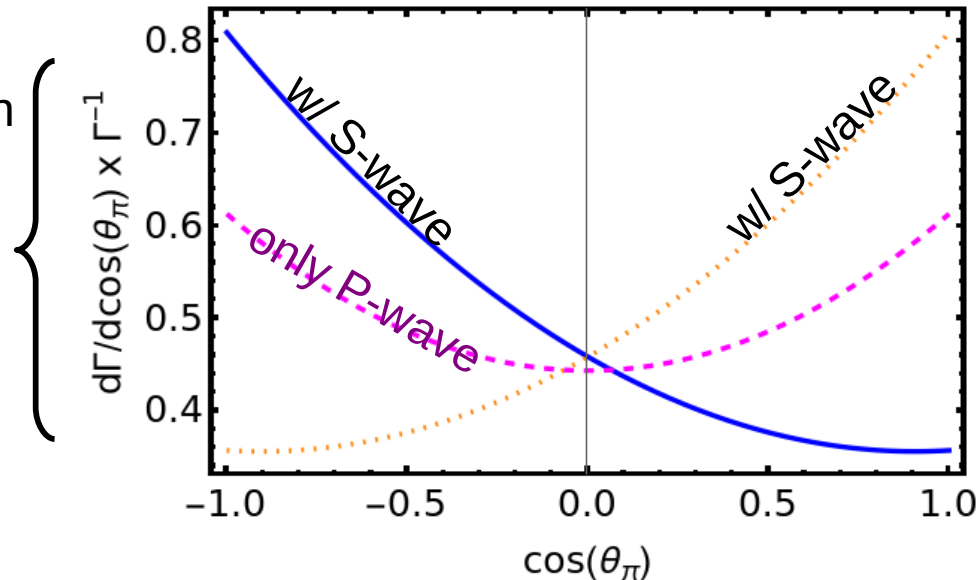
- o:  $|S|^2+|P|^2$
- ✓: S\*P interference
- x: only P-wave
- †: LHCb 2111.03327

# Angular observables

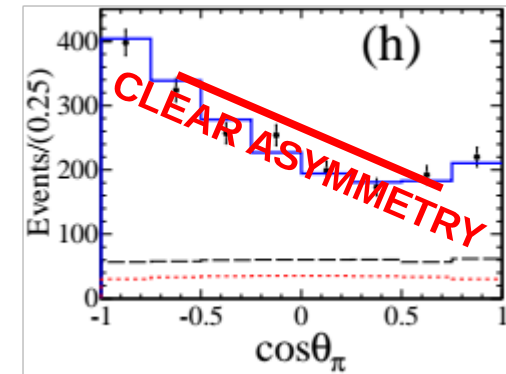


- Probe S- and P-waves interference also with distinct differential quantities

Observable depends on an S- and P-waves relative phase not probed by  $d\Gamma/dq^2$ , but by the previous S\*P observables



BESIII (1809.06496)  
SL:  $D^+ \rightarrow \pi^+ \pi^- e^+ \nu_e$



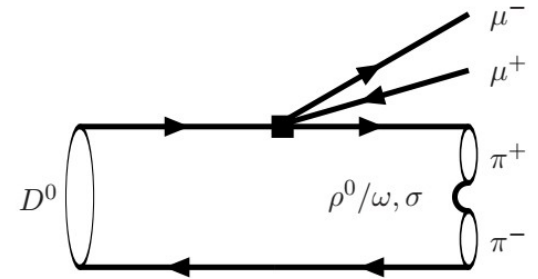
Also, BaBar (1012.1810)  
SL:  $D^+ \rightarrow K^- \pi^+ e^+ \nu_e$

# Null tests: SM-NP interference

- NP can introduce contributions to **semi-leptonic contact interactions**, e.g.:  $|V_{ub} V_{cb}^* C_{10}^{NP}| < 0.43$  @ 95% CL (from  $D^0 \rightarrow \mu^+ \mu^-$  LHCb, 2212.11203)

[similar bound from  $pp \rightarrow \mu^+ \mu^-$ , Fuentes-M., Greljo, Camalich, Ruiz-A. '20]

- P-wave only:  $S_5, S_6$  can reach **O(few)%**
- **Claiming NP requires exhaustive tests**; similar **O(few)%** reach in analogous S- and P-waves interference observables
- **Not possible to conclude yet about novel bounds on NP**, given bounds from other decay processes & presence of extra strong-phases in the theo prediction & experimental precision



# Conclusions



- Long-distance is dominant in rare SM modes: must consider resonances for a meaningful phenomenological description
- $D^0 \rightarrow \pi^+\pi^-\mu^+\mu^-$ : impact of **present data (new LHCb binned analysis)** on the charm sector
- **Improved SM description**: first quantitative assessment of the S-wave
  - Significant ingredient of the non-perturbative dynamics
  - **Straightforward LHCb measurements will further probe the S-wave**
  - **S-wave provides novel null tests of the SM**

**Thanks!**

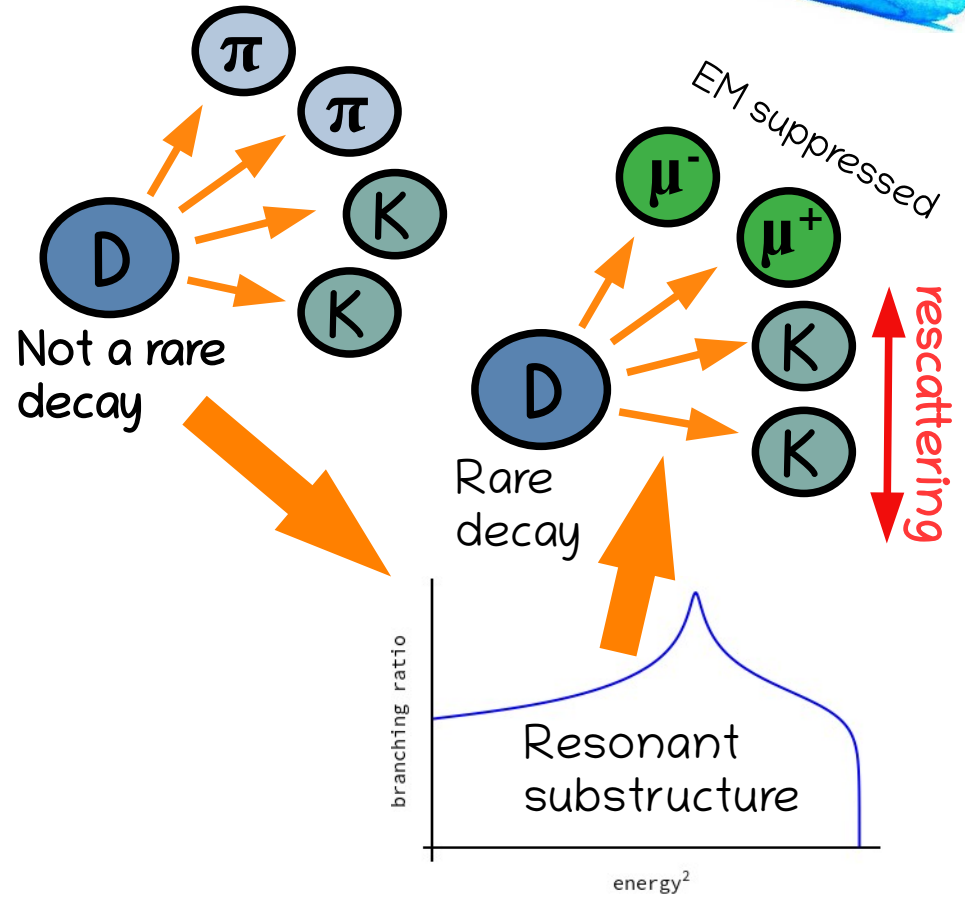
# BACK UP

Cover painting: **Joaquín Sorolla**,  
*María en la playa de Zarautz*

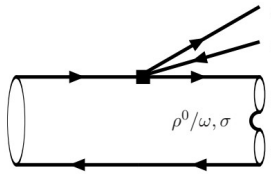
# Amplitude Analyses (AAs)



- $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$  (CLEO 1703.08505; BESIII 2312.02524),  $D^0 \rightarrow K^+K^-\pi^+\pi^-$  (LHCb 1811.08304)
- $D^0 \rightarrow f_0(500)\rho(770)^0$  distinguished
- $D^0 \rightarrow f_0(500)\phi(1020)$  suppressed
- Cascade topologies  $D^0 \rightarrow \pi^-a_1(1260)^+$ ,  $D^0 \rightarrow K^-K_1(1270)^+$  ( $\mu^+\mu^-$ -peak at  $\phi(1020)$  or  $\rho(770)^0$ ) may give relevant contributions
- At the moment, only a qualitative use is made of AAs in the present analysis
- D to  $hh\ell\ell$  5-dimensional AA: extraction of possible NP contamination?



# BESIII: D to $\pi\pi e^+\nu_e$ [1809.06496]

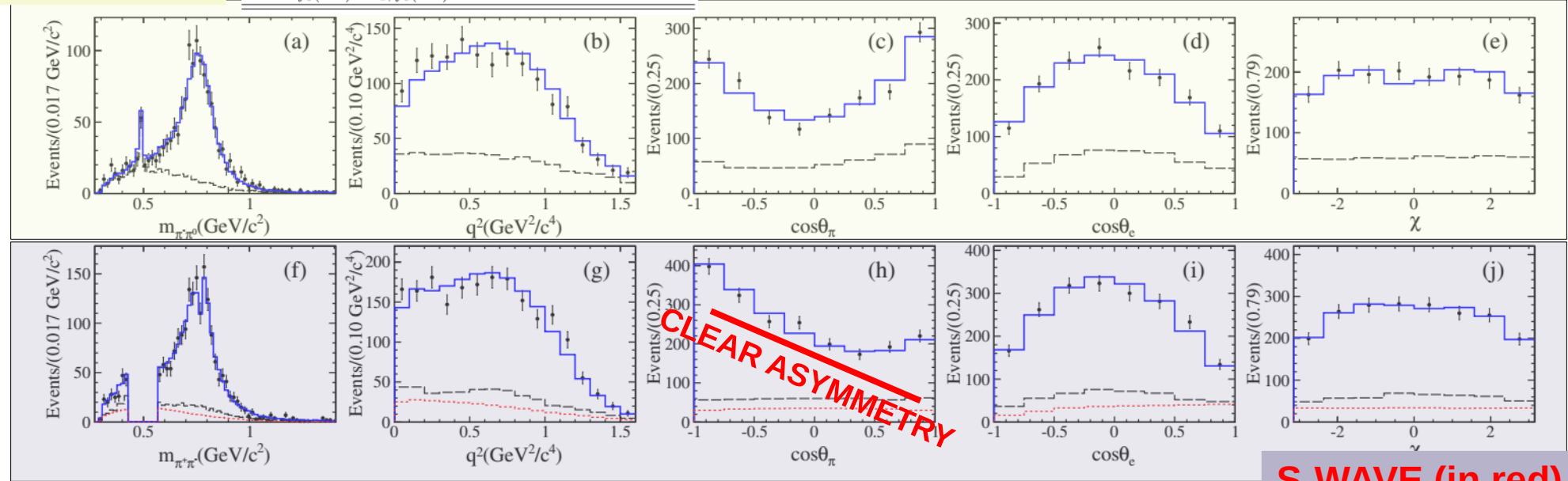


Signal mode	this analysis ( $\times 10^{-3}$ )
$D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e$	$1.445 \pm 0.058 \pm 0.039$
$D^0 \rightarrow \rho^- e^+ \nu_e$	$1.445 \pm 0.058 \pm 0.039$
$D^+ \rightarrow \pi^- \pi^+ e^+ \nu_e$	$2.449 \pm 0.074 \pm 0.073$
$D^+ \rightarrow \rho^0 e^+ \nu_e$	$1.860 \pm 0.070 \pm 0.061$
$D^+ \rightarrow \omega e^+ \nu_e$	$2.05 \pm 0.66 \pm 0.30$
$D^+ \rightarrow f_0(500) e^+ \nu_e, f_0(500) \rightarrow \pi^+ \pi^-$	$0.630 \pm 0.043 \pm 0.032$
$D^+ \rightarrow f_0(980) e^+ \nu_e, f_0(980) \rightarrow \pi^+ \pi^-$	$< 0.028$

S-wave at the level of 25%!

Also, BESIII SL decays:  $D^+ \rightarrow \pi^+ \pi^- \mu^+ \nu_\mu$  [2401.13225]

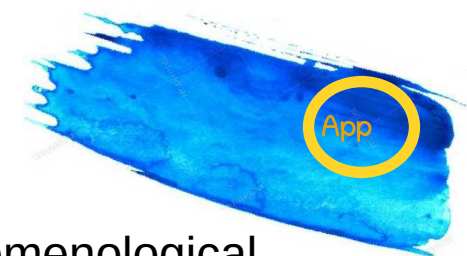
**NO S-WAVE**



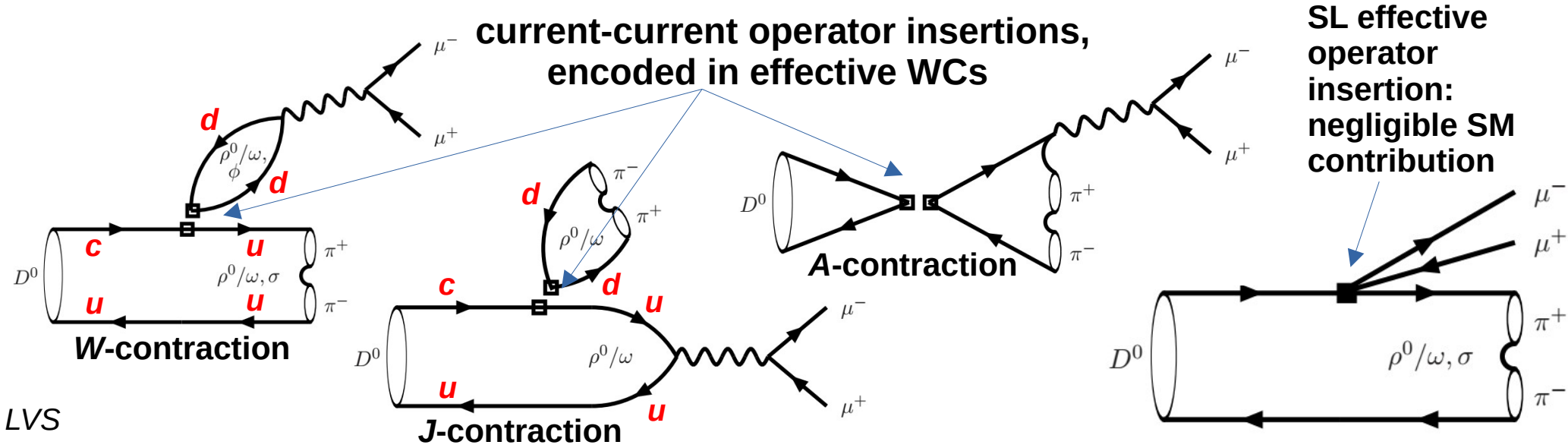
$D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e$   
 $D^+ \rightarrow \pi^- \pi^+ e^+ \nu_e$

FIG. 2. Projections of the data and simultaneous PWA fit onto the five kinematic variables for  $D^0 \rightarrow \pi^- \pi^0 e^+ \nu_e$  (top) and  $D^+ \rightarrow \pi^- \pi^+ e^+ \nu_e$  (bottom) channels. The dots with error bars are data, the solid lines are the fits, the dashed lines show the MC simulated backgrounds, and the short-dashed lines in (f)–(j) show the component of  $D^+ \rightarrow f_0(500) e^+ \nu_e$ .

# Factorization model



- More crude than QCD factorization ( $1/m_c$ ,  $\alpha_s$ ), but allows a good phenomenological description of the binned data
- **Distinct contributions:**  $W$ -,  $J$ - and  $A$ -contractions; SM short-distance negligible
  - $A$ -contraction: suppressed in naive factorization by light quark masses [Bauer, Stech, Wirbel '87]
  - $J$ -contraction in  $B^+$  to  $K^{(*)+}\ell^+\ell^-$ : light flavours are CKM suppressed  $V_{ub}^*V_{us}/(V_{cb}^*V_{cs})$
  - Cappiello, Cata, D'Ambrosio '13: Bremsstrahlung, @ low- $m(\mu^+\mu^-)$





# P-wave suppressions in $S_{2,3}$

App

$q^2$ -bin $r$	$\Gamma^r$ (SM)	$\frac{\Gamma^r}{\Gamma^r} [\%]$	$\int \langle I_2 \rangle_+^r \times 100$	$\frac{\int \langle I_2 \rangle_+^r}{\int \langle I_2 \rangle_+^r} [\%]$	$\int \langle I_3 \rangle_+^r \times 100$	$\int \langle I_4 \rangle_-^r \times 100$
$r(\rho: \text{sup})$	[0.64, 0.87]	[23, 43]	[-16, -8.5]	[59, 78]	[-7.2, -4.7]	[8.3, 13]
$r(\phi: \text{inf})$	[1.6, 1.9]	[0.3, 8]	[-11, -6.2]	[3, 45]	[-30, -26]	[36, 41]
$r(\phi: \text{sup})$	[1.2, 1.3]	[0.8, 10]	[-8.7, -4.3]	[8, 53]	[-22, -19]	[26, 29]

$$\langle I_1 \rangle_+ = \frac{1}{8} \left[ 2|\mathcal{F}_S|^2 \rho_{1,S}^- + \frac{2}{3} |\mathcal{F}_P|^2 \rho_{1,P}^- + 2|\mathcal{F}_\parallel|^2 \rho_{1,P}^- + 2|\mathcal{F}_\perp|^2 \rho_{1,P}^+ \right]$$

$$\xrightarrow{\text{SM}} + \frac{1}{8} \left\{ 2|\mathcal{F}_S|^2 |C_9^{\text{eff}:S}|^2 + \frac{2}{3} (|\mathcal{F}_P|^2 + 2(|\mathcal{F}_\parallel|^2 + |\mathcal{F}_\perp|^2)) |C_9^{\text{eff}:P}|^2 \right\}, \quad (64)$$

P-wave unsuppressed

$$\langle I_2 \rangle_+ = -\frac{1}{8} \left[ 2|\mathcal{F}_S|^2 \rho_{1,S}^- + \frac{2}{3} \{ |\mathcal{F}_P|^2 \rho_{1,P}^- - |\mathcal{F}_\parallel|^2 \rho_{1,P}^- - |\mathcal{F}_\perp|^2 \rho_{1,P}^+ \} \right]$$

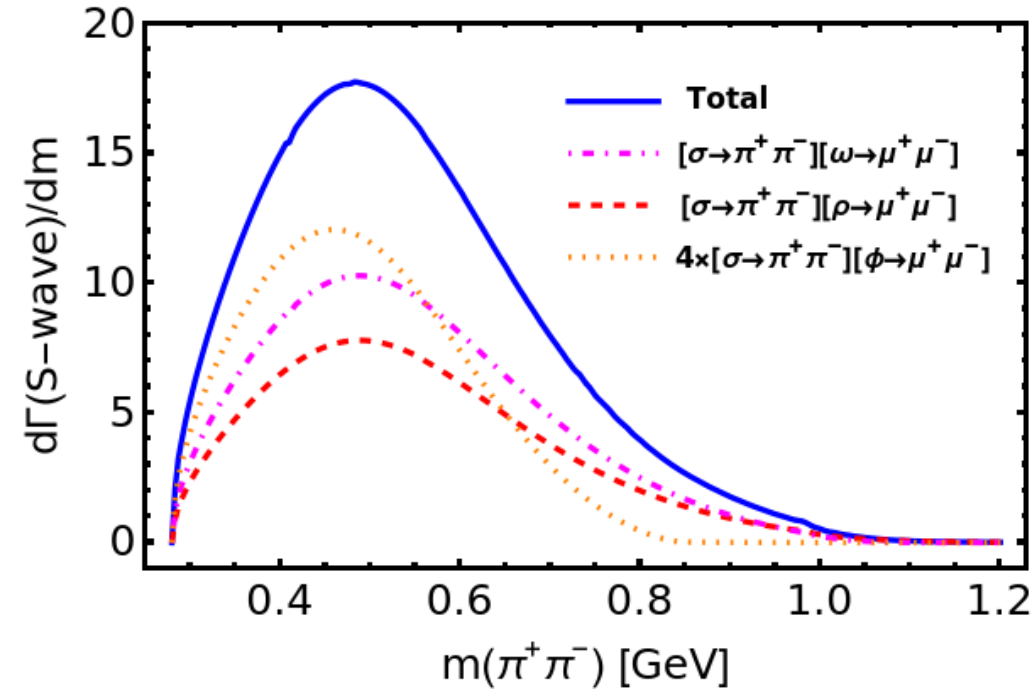
$$\xrightarrow{\text{SM}} -\frac{1}{8} \left\{ 2|\mathcal{F}_S|^2 |C_9^{\text{eff}:S}|^2 + \frac{2}{3} (|\mathcal{F}_P|^2 - |\mathcal{F}_\parallel|^2 - |\mathcal{F}_\perp|^2) |C_9^{\text{eff}:P}|^2 \right\}, \quad (65)$$

$$\langle I_3 \rangle_+ = \frac{1}{6} [|\mathcal{F}_\perp|^2 \rho_{1,P}^+ - |\mathcal{F}_\parallel|^2 \rho_{1,P}^-]$$

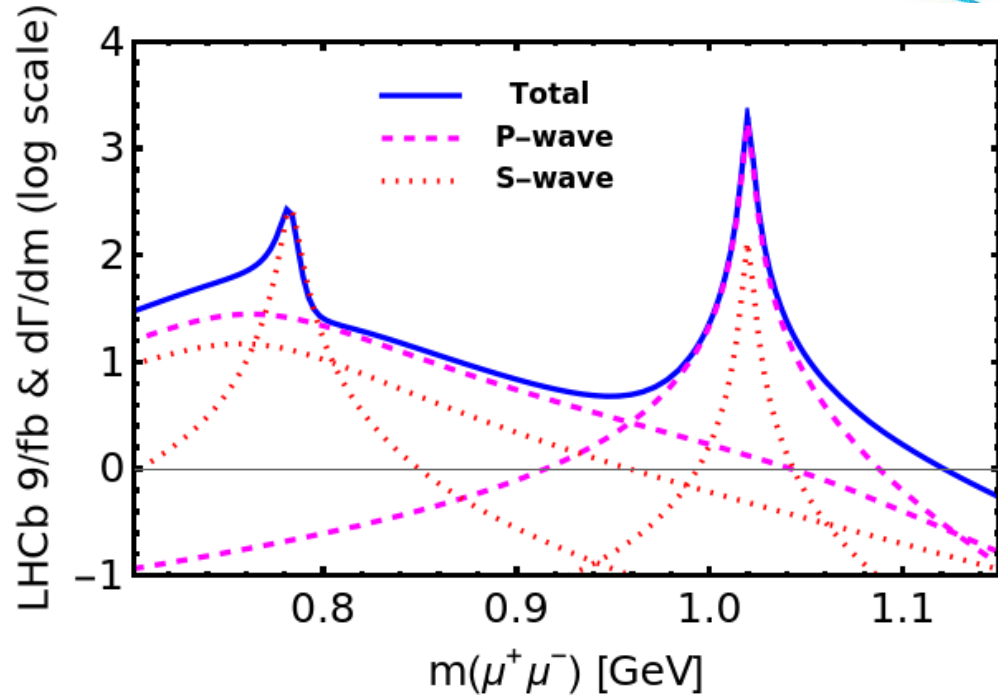
$$\xrightarrow{\text{SM}} \frac{1}{6} (|\mathcal{F}_\perp|^2 - |\mathcal{F}_\parallel|^2) |C_9^{\text{eff}:P}|^2, \quad (66)$$

P-wave suppression!

# Resonant components



$\phi$  from S-wave: distinct  $m(\pi^+\pi^-)$  dependence (see 2D plot displaying resonances); it helps in constraining its size



Naive factorization:  $\omega$  from P-wave suppressed (simpler (BW) description of  $\rho$  (from P-wave) and  $\omega$  (from S-wave))

# Parameters extracted from the fit

- 6 norm. parameters (B's,  $a_\omega$ ,  $a_S(0)$ ),  
3 strong phase differences ( $\phi_\omega$ ,  $\Delta_{1,4}$ )
- Overall normalization from **LHCb BR** [1707.08377]
- Expected from **factorization**
- **Suppression** also seen in the hadronic decay mode  $D^0 \rightarrow K^+K^-\pi^+\pi^-$
- In the ballpark of **BESIII SL**
- Large impact in  **$q^2$  distribution**

( $A_1(0)$ : FF normalization)

$$0.8 \lesssim A_1(0) B_{\rho^0} \lesssim 1.2$$

$$0.8 \lesssim B_\phi / B_{\rho^0} \lesssim 0.9,$$

$$0.9 \lesssim B_\omega^{(S)} / B_{\rho^0}^{(S)} \lesssim 1.1,$$

$$0.05 \lesssim B_\phi^{(S)} / B_{\rho^0}^{(S)} \lesssim 0.27.$$

$$0.001 \lesssim a_\omega \lesssim 0.005,$$

$$1.1 \pi \lesssim \phi_\omega \lesssim 1.7 \pi,$$

$$39 \text{ GeV} \lesssim \frac{a_S(0)}{A_1(0)} \lesssim 62 \text{ GeV}$$

$$0.5 \pi \lesssim \Delta_1 \lesssim 0.9 \pi$$

$$0.2 \pi \lesssim \Delta_4 \lesssim 0.5 \pi$$

# Angular observables



The angular distribution of  $D^0 \rightarrow h^+ h^- \mu^+ \mu^-$  ( $h = \pi, K$ ) decays can be written as 8

$$\frac{d^5\Gamma}{dq^2 dp^2 d\vec{\Omega}} = \frac{1}{2\pi} \left[ \sum_{i=1}^9 c_i(\theta_\mu, \phi) \underbrace{I_i(q^2, p^2, \cos\theta_h)} \right], \quad (5)$$

with the angular basis,  $c_i$ , defined as WCs, hadronic inputs

$$\begin{aligned} c_1 &= 1, \quad c_2 = \cos 2\theta_\mu, \quad c_3 = \sin^2 \theta_\mu \cos 2\phi, \quad c_4 = \sin 2\theta_\mu \cos \phi, \quad c_5 = \sin \theta_\mu \cos \phi, \\ c_6 &= \cos \theta_\mu, \quad c_7 = \sin \theta_\mu \sin \phi, \quad c_8 = \sin 2\theta_\mu \sin \phi, \quad c_9 = \sin^2 \theta_\mu \sin 2\phi. \end{aligned} \quad (6)$$

The normalised and integrated observables  $\langle I_i \rangle$  are defined as

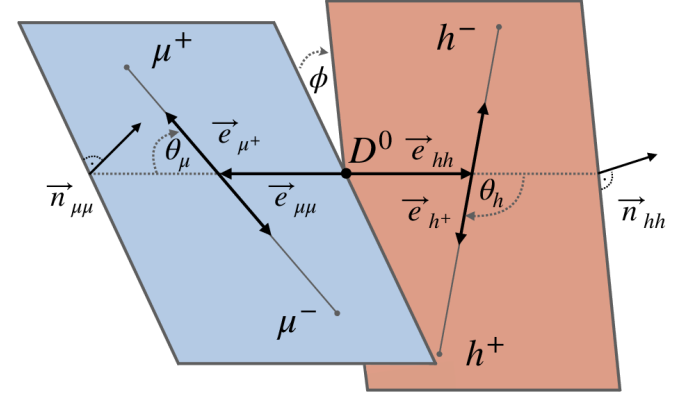
$$\begin{aligned} \langle I_{2,3,6,9} \rangle &= \frac{1}{\Gamma} \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \int_{p_{\min}^2}^{p_{\max}^2} dp^2 \int_{-1}^{+1} d\cos\theta_h I_{2,3,6,9}, \\ \langle I_{4,5,7,8} \rangle &= \frac{1}{\Gamma} \int_{q_{\min}^2}^{q_{\max}^2} dq^2 \int_{p_{\min}^2}^{p_{\max}^2} dp^2 \left[ \int_0^{+1} d\cos\theta_h - \int_{-1}^0 d\cos\theta_h \right] I_{4,5,7,8}. \end{aligned} \quad (10)$$

The observables reported in the Letter are the  $CP$  averages,  $\langle S_i \rangle$ , and asymmetries,  $\langle A_i \rangle$ , defined as

$$\begin{aligned} \langle S_i \rangle &= \frac{1}{2} [\langle I_i \rangle + (-)\langle \bar{I}_i \rangle], \\ \langle A_i \rangle &= \frac{1}{2} [\langle I_i \rangle - (+)\langle \bar{I}_i \rangle], \end{aligned} \quad (11)$$

for the  $CP$ -even ( $CP$ -odd) coefficients  $\langle I_{2,3,4,7} \rangle$  ( $\langle I_{5,6,8,9} \rangle$ ).

See LHCb (2111.03327);  
De Boer, Hiller '18



$$\cos \theta_\mu = \vec{e}_{\mu\mu} \cdot \vec{e}_{\mu^+},$$

$$\cos \theta_h = \vec{e}_{hh} \cdot \vec{e}_{h^+}.$$

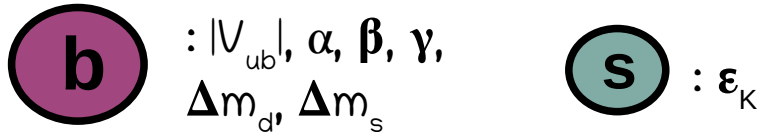
$$\cos \phi = \vec{n}_{\mu\mu} \cdot \vec{n}_{hh},$$

$$\sin \phi = [\vec{n}_{\mu\mu} \times \vec{n}_{hh}] \cdot \vec{e}_{hh},$$

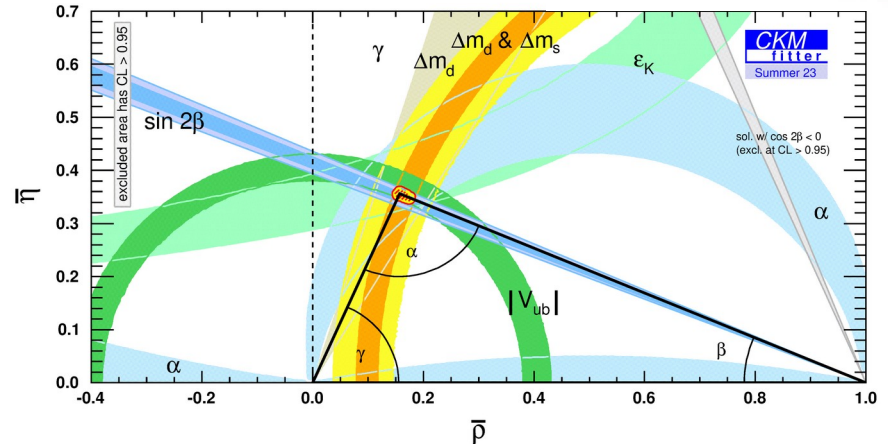
# CP violation in the charm sector



- CKM: a single CP-odd phase must be responsible for **CPV phenomena** in all quark flavour sectors of the SM



[CKMfitter Collaboration: Charles, Deschamps, Descotes-G., Monteil, Orloff, Qian, Tisserand, Trabelsi, Urquijo, LVS]



- Direct CP violation discovered by LHCb (2019) in  $D^0 \rightarrow h^+h^-$
- Unclear yet whether this can be explained within the SM

[Khodjamirian, Petrov '17; Li, Lu, Yu '19; Soni '19; Cheng, Chiang '19; Pich, Solomonidi, LVS '23; Lenz, Piscopo, Rusov '23; ...]

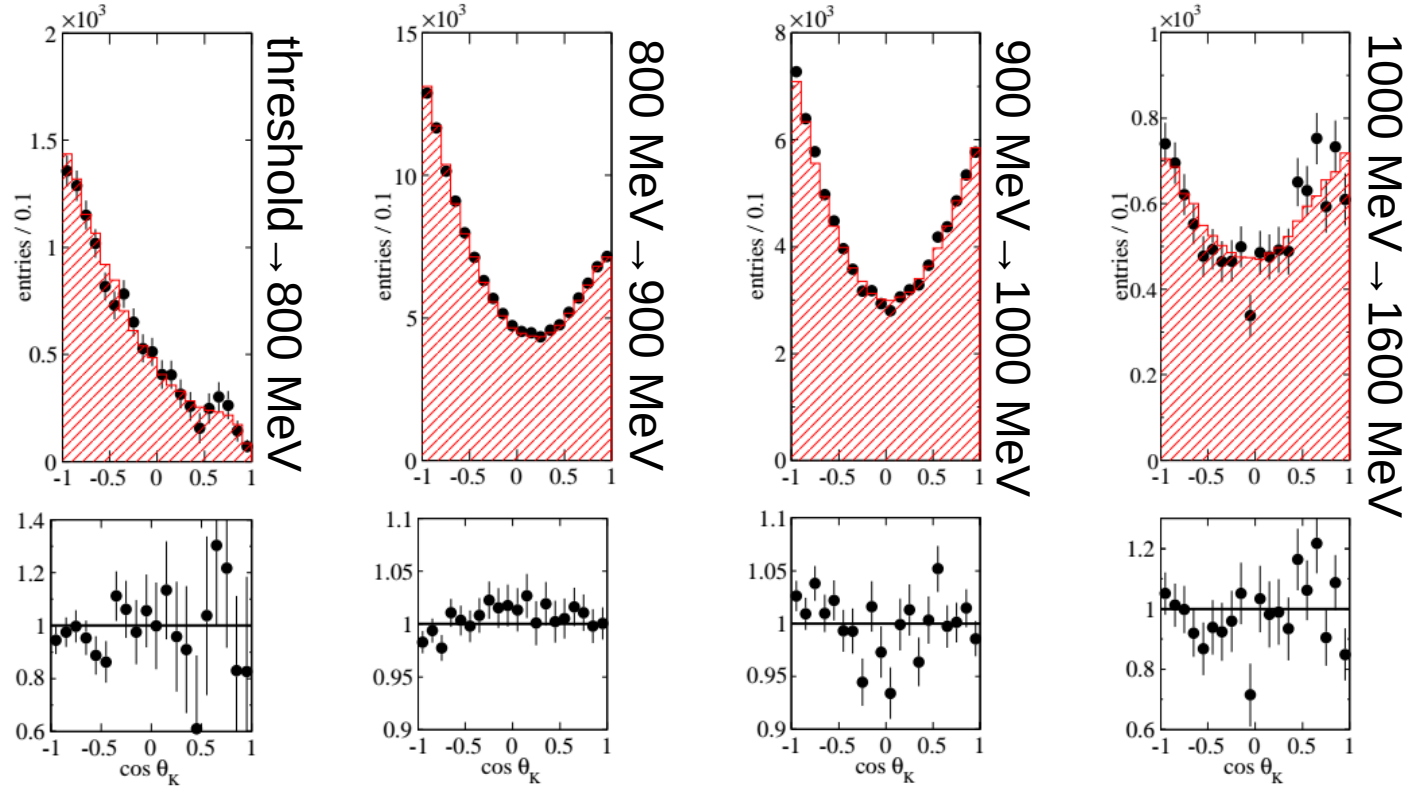
- Rare charm-meson decays consistent with no CP violation:

-  $A_1, \dots, A_9 \sim 0$  (small CP violation)

# BaBar SL decays: $D^+$ to $K^-\pi^+ e^+\nu_e$ [1012.1810]



Different  $K\pi$ -energy slices



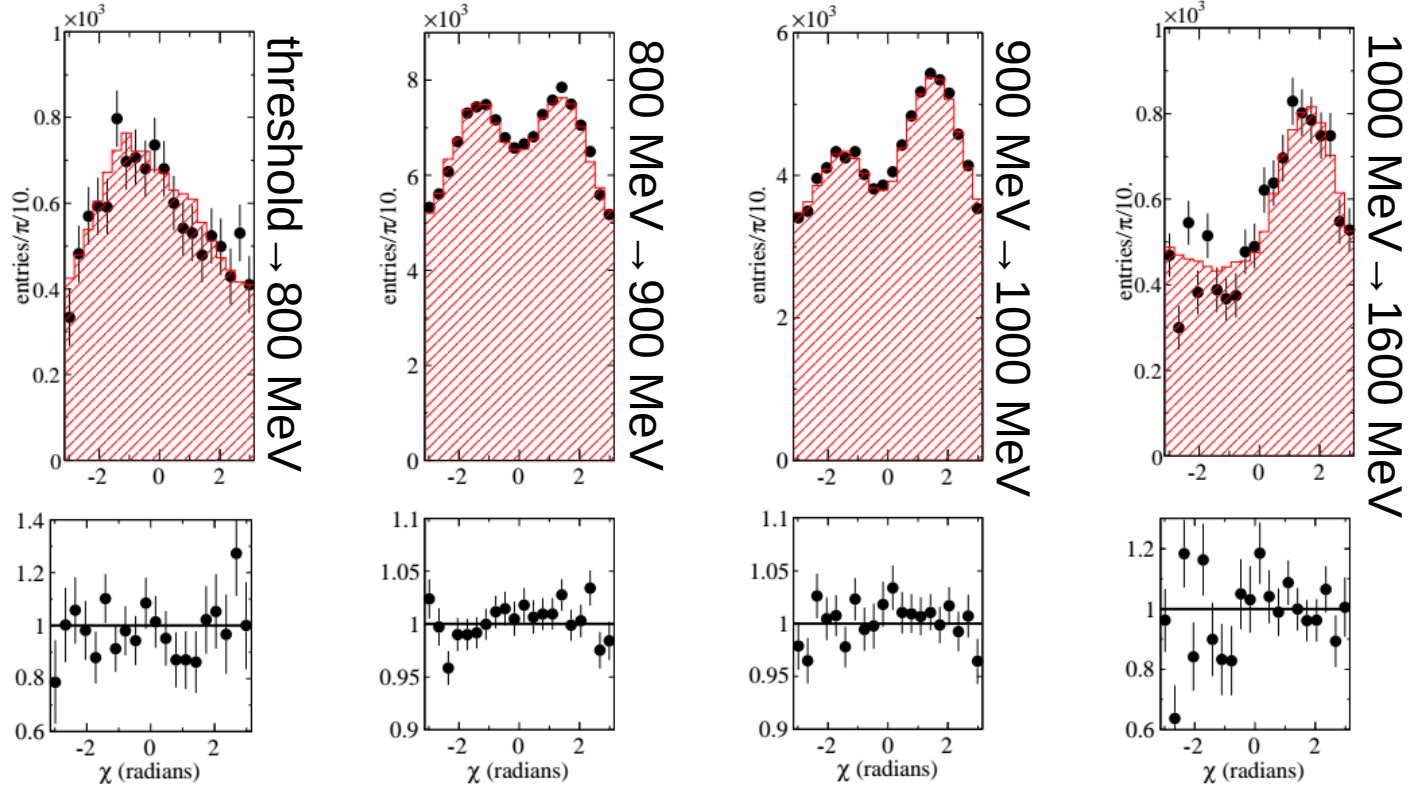
S- and P-waves interference produces  $\cos(\theta_K)$  term; “P-wave only” gives a  $\cos^2(\theta_K)$  term

S-wave from  $K_0^*(800)=\kappa$  and  $K_0^*(1430)$

# BaBar SL decays: $D^+$ to $K^-\pi^+ e^+\nu_e$ [1012.1810]



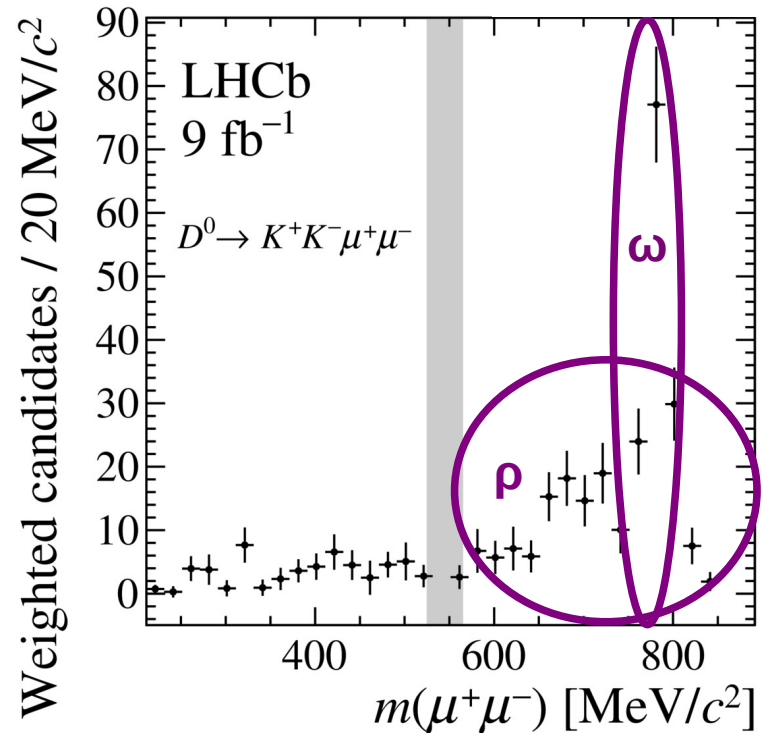
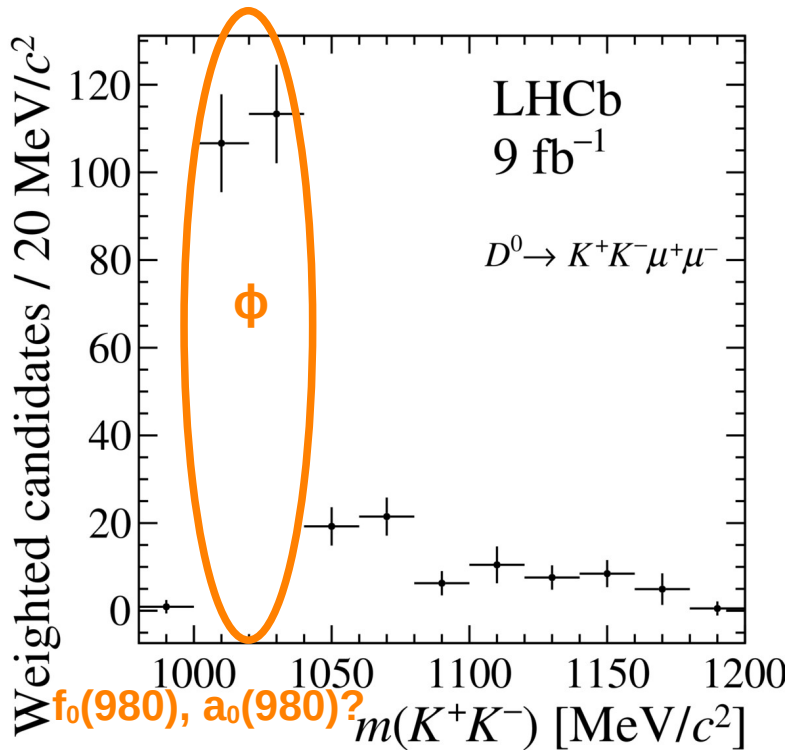
Different  $K\pi$ -energy slices



S- and P-waves interference produces  $\sin(\chi)$  term, and also  $\cos(\chi)$  term; “P-wave only” gives a  $\cos(2\chi)$  term

S-wave from  $K_0^*(800)=\kappa$  and  $K_0^*(1430)$

# Related channel: $D^0 \rightarrow K^+ K^- \mu^+ \mu^-$



**Threshold effects complicate the description of  $f_0(980)$ ,  $a_0(980)$**

Dedicated analysis still needed; having  $f_0(980)$ ,  $a_0(980)$  and  $\phi$  close may produce an interesting S- and P-waves interference effect



# Outlook



- SL (hadronic) modes: quantitative (qualitative) information
- Currently only looking at a fraction of the allowed phase space
- Long-term goal (**dreaming out loud**): **more intensive data-driven approach**
  - (i) data on **semi-leptonic decay modes**
    - $D$  to  $\pi\pi\ell\nu_\ell$
  - (ii) data on **alternative rare decay modes**, including **radiative** ones
    - $D$  to  $KK\ell\ell$ ,  $D$  to  $hhy$ , etc.
  - (iii) data on **purely hadronic decay modes**
    - $D$  to  $\pi\pi\pi\pi$ ,  $D$  to  $\pi\pi KK$ , etc.
  - (iv) data on **rescattering of final states**
    - $\pi\pi$  to  $KK$