Flavour-Invariant Contributions to the Strong CP Problem

based on 2402.09361

in collaboration with Ravneet Bedi, Tony Gherghetta, Christophe Grojean, Guilherme Guedes and Pham Ngoc Hoa Vuong

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HELMHOLTZ

The strong CP problem

- Can add $\theta_{QCD} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$ to the Lagrangian without breaking QCD symmetries
- Neutron EDM measurements suggest that $\theta_{\rm QCD} \leq 10^{-10}$ [nEDM collab., 2001.11966]
- Well-motivated solution is the axion relaxing $\theta_{\rm QCD}$ to 0

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- Well-motivated solution is the axion relaxing θ_{QCD} to 0
- QCD axion: potential fully determined by QCD. For instance,

$$m_a f_a = \sqrt{\frac{4 m_u m_d}{(m_u + m_d)^2}} m_\pi f_\pi$$

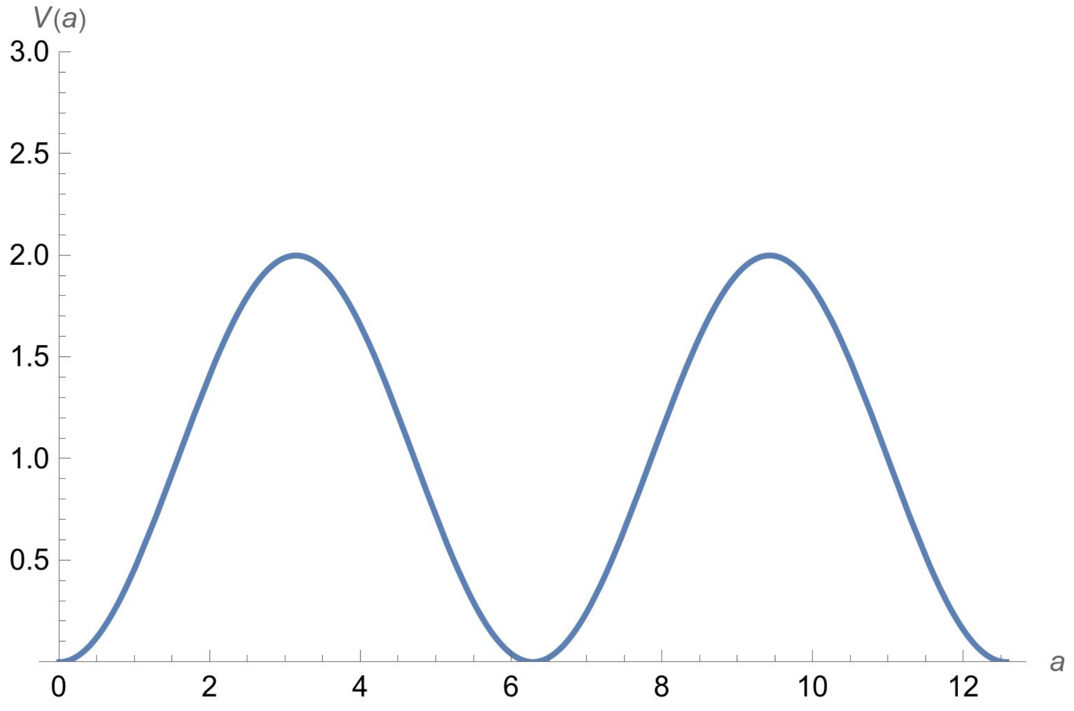
[Weinberg, 1977]

- Quality problem: How stable is the axion solution against additional contributions to its potential?

  In particular: What happens if there are **additional CP-violating parameters** in the theory?

Corrections to the axion potential

- Corrections to the axion potential can come in different forms



QCD axion

… solves strong CP problem.

$$V(a) \approx m_\pi^2 f_\pi^2 \left(1 - \cos\left(\frac{a}{f}\right)\right)$$

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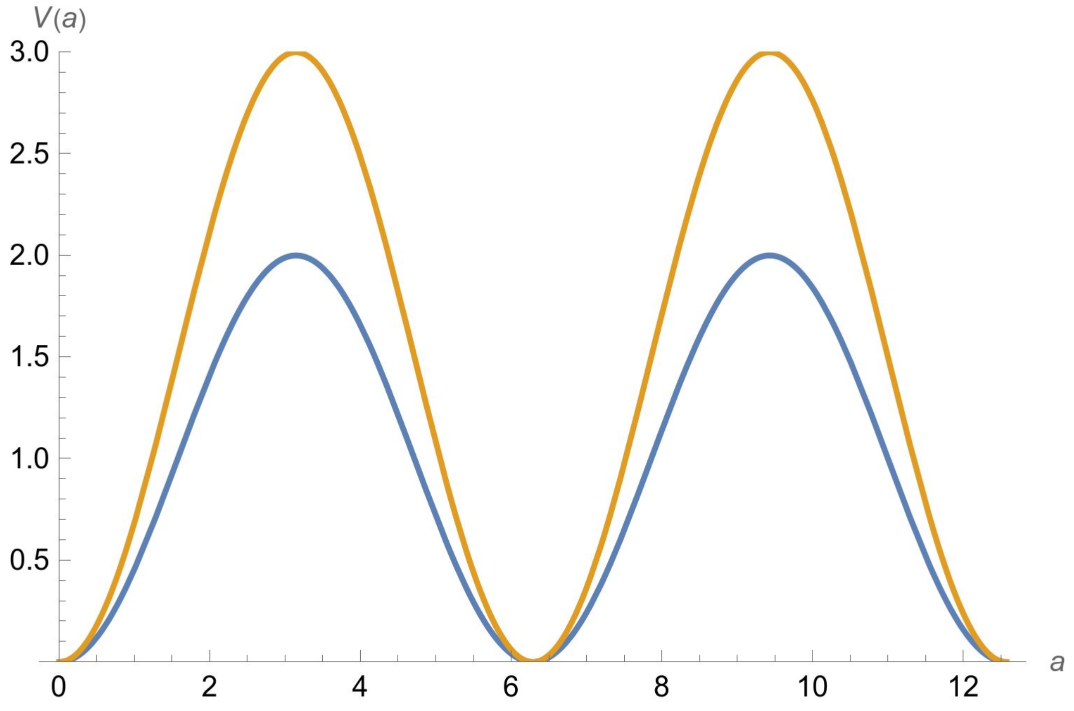

QCD axion
Aligned contribution

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… solves strong CP problem but changes axion mass.

$$V(a) \approx (m_\pi^2 f_\pi^2 + \underline{C}) \left(1 - \cos\left(\frac{a}{f}\right)\right)$$

Example: **instantons**, but contribution scales as $\propto e^{-\frac{2\pi}{\alpha_s}}$

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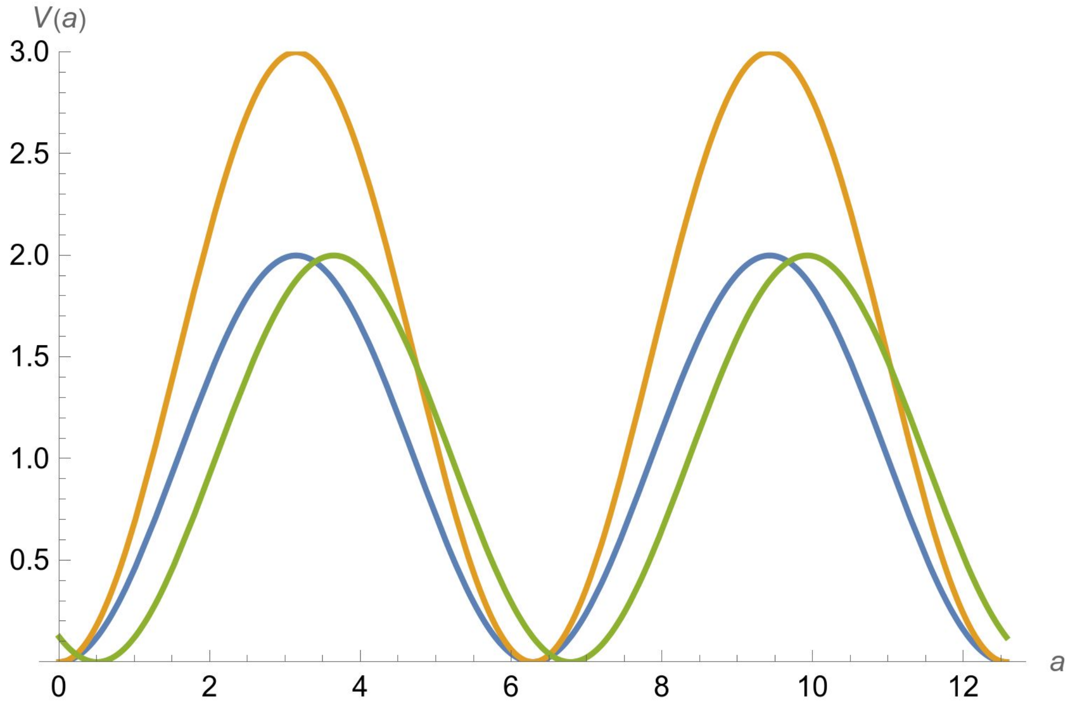


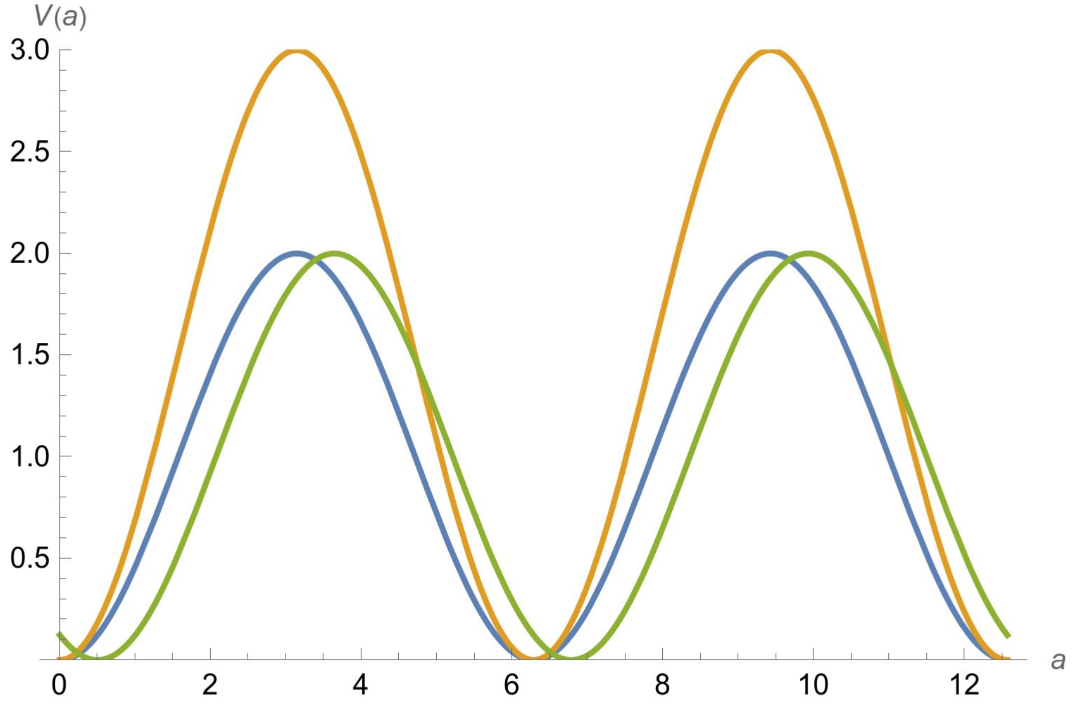
QCD axion
Aligned contribution
Misaligned contribution

… solves strong CP problem.
… solves strong CP problem but changes axion mass.
… does not solve strong CP problem and can change mass.

$$V(a) \approx (m_\pi^2 f_\pi^2 + \underline{C}) \left(1 - \cos\left(\frac{a + \delta}{f}\right)\right)$$

Corrections to the axion potential in the SM

Sources of CP violation can offset minimum of axion potential



Not a problem in SM, due to flavour suppression of CPV:

$$J_4 = \mathrm{ImTr}\left([X_u, X_d]^3\right) \qquad X_{u,d} = Y_{u,d} Y_{u,d}^\dagger$$

[Jarlskog, 1985]
[Bernabeu, Branco, Gronau, 1985]

Corrections only appear at 4-loop and 7-loop level, corresponding to

$$\bar{\theta}_{\text{ind}} \sim 10^{-19}$$

[Ellis, Gaillard, 1979]
[Khriplovich, 1986]
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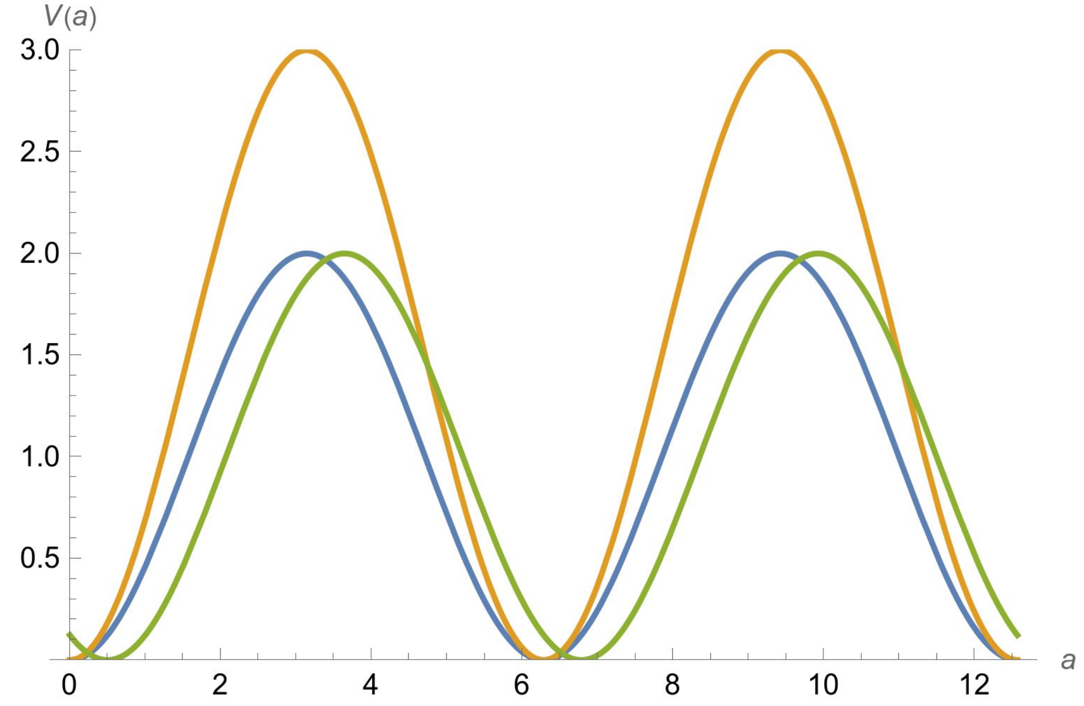

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…but what about new physics?

BSM scenario

- Generally, when only QCD explicitly breaks PQ, CPV effects suppressed by $\left(\frac{\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{CPV}}}\right)^p$

$\Longrightarrow$ **Consider small instantons by enhancing strong coupling in UV**

$\propto e^{-\frac{2\pi}{\alpha_s}}$

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[Kitano et al., 2103.08598]
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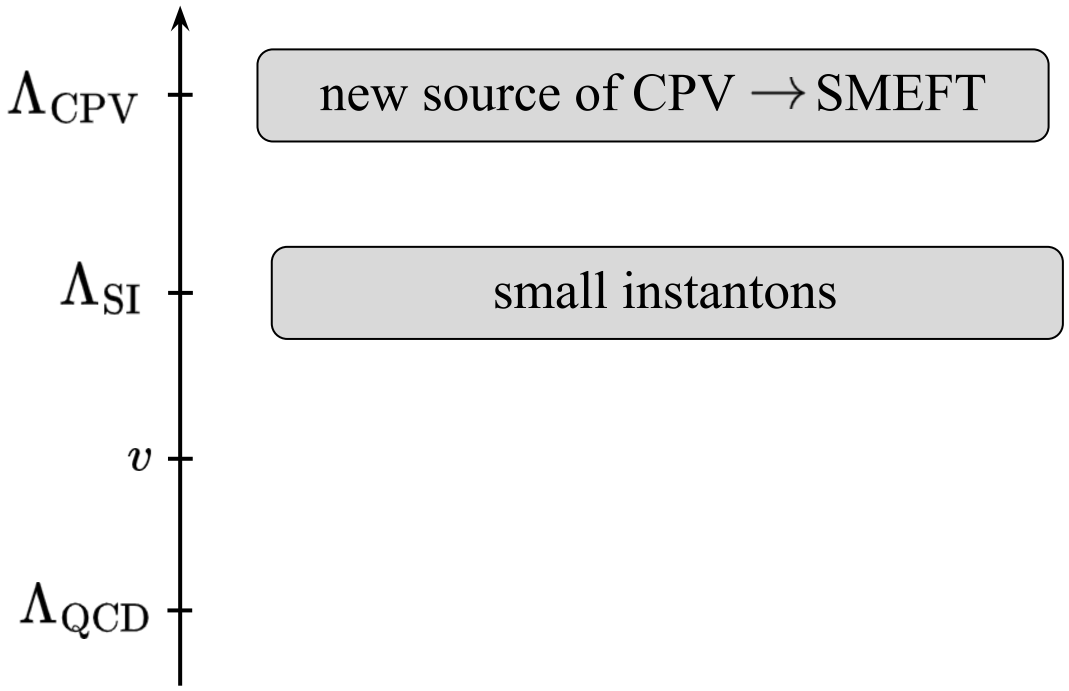

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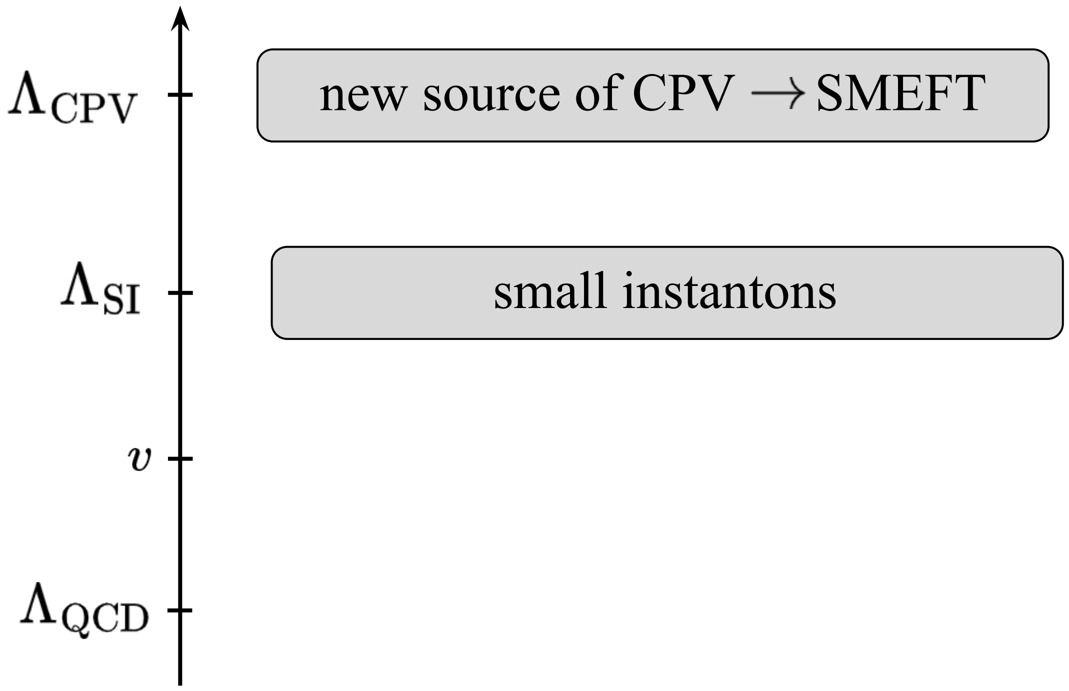

How do UV contributions look like compared to **SM contributions suppressed by Jarlskog invariant?**

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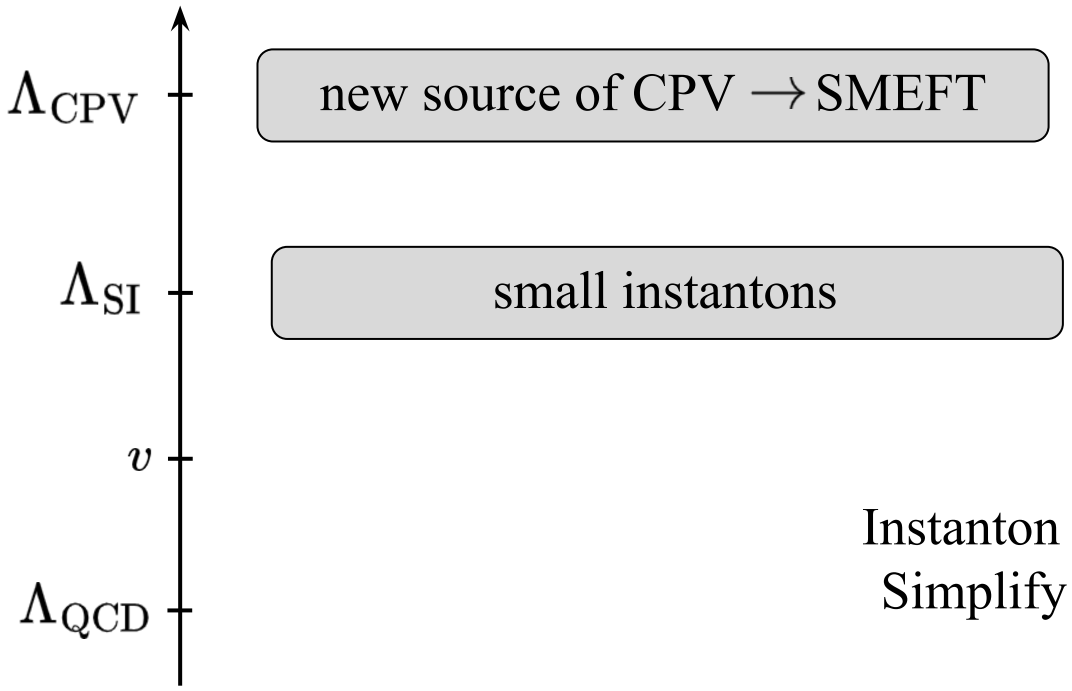

Instanton computations are **difficult!**
Simplify using **flavour invariants.**

How do UV contributions look like compared to **SM contributions suppressed by Jarlskog invariant?**

CP-violation in the SMEFT

- Use the **SMEFT** to describe new sources of **CP violation** beyond the SM

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{\mathcal{L}^{(6)}}{\Lambda^2} + \dots$$

- At dimension 6: 1149 CP-odd couplings in the SMEFT. How to study them systematically?

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$\Longrightarrow$ **Flavour invariants!**

- Construct generalised Jarlskog invariants. Example: $\mathcal{O}_{uH} = |H|^2 \bar{Q}\tilde{H}u$

[Bonnefoy et al.,2112.03889]
[Bonnefoy et al.,2302.07288]

- Wilson coefficient has 9 complex parameters

- Can be captured by flavour invariants of the form

$$\text{ImTr}\left(X_u^a X_d^b X_u^c X_d^d C_{uH} Y_u^\dagger\right) \qquad X_{u,d} = Y_{u,d} Y_{u,d}^\dagger$$

Can show: **only 705 dimension-6 SMEFT phases** can be written in **flavour-invariant** way (possibly less flavour suppressed than Jarlskog invariant).

More flavour invariants?

- Allowing for non-perturbative effects, Jarlskog invariant is not the only CP-odd invariant in the SM.
- We have another spurion now: θ_{QCD}

	$U(3)_Q$	$U(3)_u$	$U(3)_d$	$U(3)_L$	$U(3)_e$
$e^{i\theta_{\mathrm{QCD}}}$	$\mathbf{1}_{+6}$	$\mathbf{1}_{-3}$	$\mathbf{1}_{-3}$	$\mathbf{1}_{0}$	$\mathbf{1}_{0}$
Y_u	$\mathbf{3}_{+1}$	$\bar{\mathbf{3}}_{-1}$	$\mathbf{1}_{0}$	$\mathbf{1}_{0}$	$\mathbf{1}_{0}$
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- Can build more invariants, e.g. $J_\theta = \mathrm{Im}(e^{-i\theta_{\mathrm{QCD}}} \det Y_u \det Y_d)$
- Are there also more non-redundant EFT invariants? No!
- But invariants featuring θ_{QCD} are more suitable to describe SMEFT CPV in non-perturbative computations. E.g.,

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}} \epsilon^{IJK} \epsilon^{ijk} Y_{u,Ii} Y_{u,Jj} C_{uH,Kk} \det Y_d) \qquad \mathcal{O}_{uH} = |H|^2 \bar{Q}\tilde{H}u$$

Small Instanton Contribution

- Small instantons generate axion potential of the form

$$V(a) = \chi_6(0)\frac{a}{f_a} + \chi(0)\frac{a^2}{f_a^2} \qquad \Longrightarrow \left\langle \frac{a}{f_a} \right\rangle \equiv \theta_{\text{ind}} = -\frac{\chi_6(0)}{\chi(0)}$$

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- Coefficients in the potential can be computed from following correlators

$$\chi(0) = -i \lim_{k \to 0} \int d^4x\, e^{ik\cdot x} \left\langle 0 \left| \frac{1}{32\pi^2} G\tilde{G}(0), \frac{1}{32\pi^2} G\tilde{G}(x) \right| 0 \right\rangle$$

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- Compute these in instanton background

$$A_\mu^a(x) = \frac{2\eta^a_{\mu\nu}(x-x_0)^\nu}{(x-x_0)^2+\rho^2} \qquad\qquad Q = \frac{1}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$$

$Q = 1$ BPST solution

Adding the SM degrees of freedom

- We want to calculate contributions of SMEFT operators → have to add fermions and Higgs
- Calculate the correlation function in topological susceptibility from path integral

$$\langle 0\,|\mathcal{O}|0\rangle = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}h\, e^{-S_E[A]} e^{iS_0[\psi,\bar{\psi}]} e^{iS_0[h]} e^{iS_{int}[\psi,\bar{\psi},h]}\mathcal{O}$$

$$S_E[A] = \frac{8\pi^2}{g^2}|Q| + iQ\theta$$

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- Zero modes of $SU(3)$ charged fermions take special form in instanton background

$$i\gamma_\mu D^\mu\psi^{(n)} = \lambda_n\psi^{(n)}, \quad \lambda_0 = 0$$

$$\psi^{(0)}(x) = \begin{pmatrix}\chi_L\\ \chi_R\end{pmatrix} = \frac{1}{\pi}\frac{\rho}{\left[(x-x_0)^2+\rho^2\right]^{3/2}}\begin{pmatrix}0\\1\end{pmatrix}\varphi$$

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- Can split of zero modes from the non-zero modes to simplify calculation ['t Hooft, 1976]

$$\psi = \psi^{(0)}\xi_\psi + \psi' \qquad \mathcal{D}\psi\mathcal{D}\bar{\psi} = ||\psi^{(0)}||^{-1}d\xi_\psi||\bar{\psi}^{(0)}||^{-1}d\bar{\xi}_\psi\,\mathcal{D}\psi'\mathcal{D}\bar{\psi}'$$

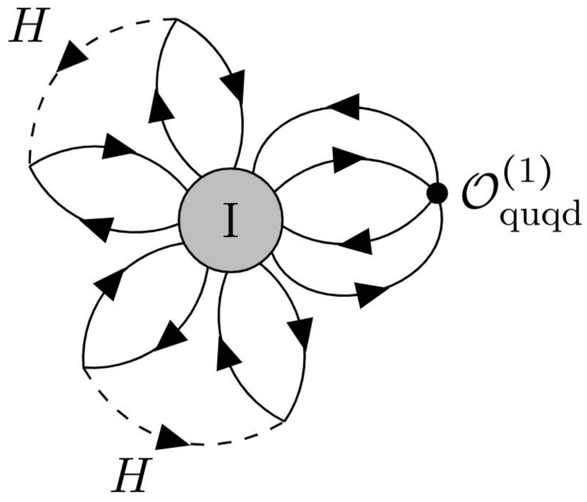
simple Grassmann integration

- First, ignore non-zero mode interactions and integrate over free action of non-zero modes

Identifying the flavor invariants

- As an example, let's take a 4-fermion operator, like $\mathcal{O}^{(1)}_{quqd} = (\bar{Q}^i u)\epsilon_{ij}(\bar{Q}^j d)$
- After splitting of the zero modes we have something that schematically looks like

$$\chi_6(0) \sim (\text{bosonic}) \times \int d\xi_{\psi_1} d\bar{\xi}_{\psi_1} d\xi_{\psi_2} d\bar{\xi}_{\psi_2}\, e^{i\int_x \bar{\xi}_1 \bar{\psi}_1^{(0)} Y_1 h \psi_1^{(0)} \xi_1 + \bar{\xi}_2 \bar{\psi}_2^{(0)} Y_2 h \psi_2^{(0)} \xi_2 + \text{h.c.}} \quad C\bar{\xi}_1 \psi_1^{(0)} \xi_1 \bar{\psi}_1^{(0)} \bar{\xi}_2 \bar{\psi}_2^{(0)} \psi_2^{(0)} \xi_2$$



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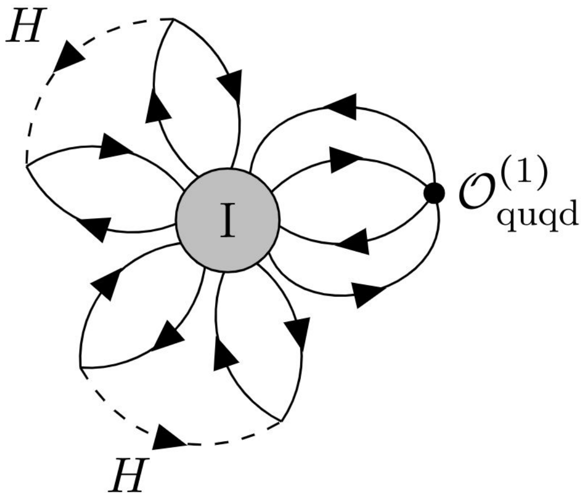
- This almost looks like the well-known Grassmann integration identity

$$\int d^n\alpha d^n\beta\, e^{\alpha A \beta} = \det A$$

- Need generalisation with operator insertion, for 3 generations

$$\int d^3\xi_1 d^3\xi_2\, e^{\xi_1 A \xi_2} \xi_1 B \xi_2 = \frac{1}{2}\epsilon^{i_1 i_2 i_3}\epsilon^{j_1 j_2 j_3} A_{i_1 j_1} A_{i_2 j_2} B_{i_3 j_3}$$



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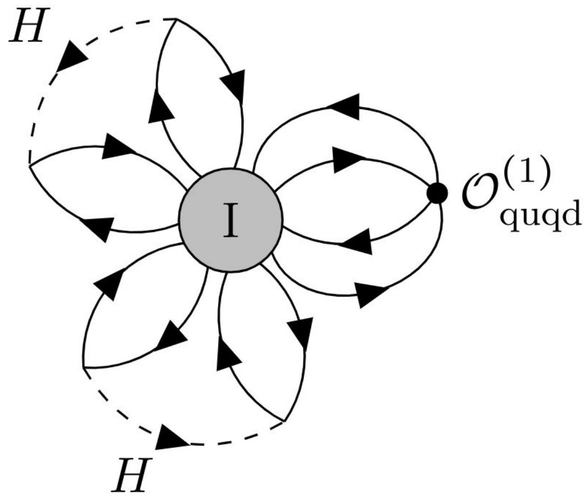

- Identifying A with Yukawa couplings, B with operator and adding anti-instanton, we find

$$\chi_6(0) \sim \text{Im}\left[e^{-i\theta}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} Y_{u,i_1 j_1} Y_{u,i_2 j_2} C^{(1)}_{quqd,mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{d,k_1 l_1} Y_{d,k_2 l_2}\right] \times \begin{pmatrix}\text{int. over zero mode} \\ \text{coordinates and Higgses}\end{pmatrix}$$

Now, start from invariants…

$$\mathcal{O}^{(1)}_{lequ} = (\bar{L}^i e)\epsilon_{ij}(\bar{Q}^j u)$$

$$\text{Im}\left[e^{-i\theta} C^{(1)}_{lequ,ijkl} Y^{\dagger}_{e,ji} \epsilon^{k_1 k_2 k} \epsilon^{l_1 l_2 l} Y_{u,k_1 l_1} Y_{u,k_2 l_2} \text{det} Y_d\right]$$

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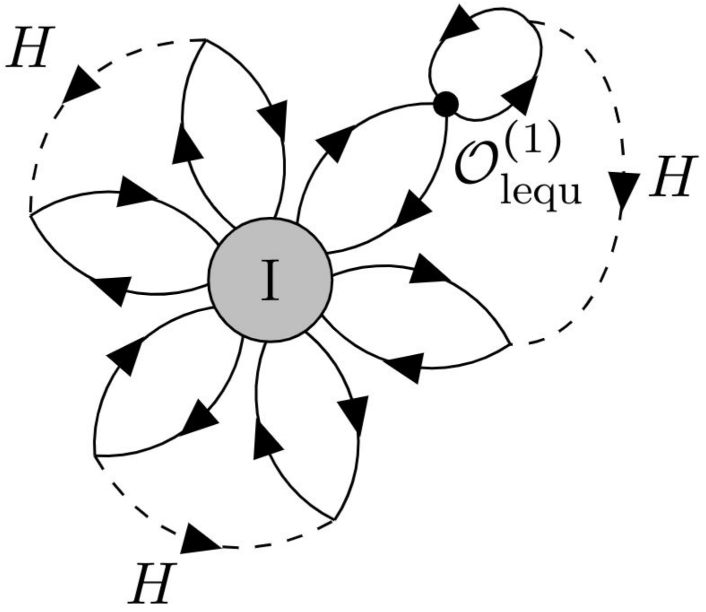
Treat non-zero modes ψ' perturbatively:

$$\bar{\psi}YP_Rh\psi = \bar{\psi}^{(0)}YP_Rh\psi^{(0)} + \bar{\psi}^{(0)}YP_Rh\psi' + \bar{\psi}'YP_Rh\psi^{(0)} + \bar{\psi}'YP_Rh\psi'$$

$$e^{iS_{int}[\psi',\bar{\psi}']} = 1 + iS_{int}[\psi',\bar{\psi}'] - \frac{S_{int}[\psi',\bar{\psi}']^2}{2} + \dots$$

$$\int \mathcal{D}\psi'\mathcal{D}\bar{\psi}'\, e^{iS_0[\psi',\bar{\psi}']}\psi'_i(x_1)\bar{\psi}'_j(x_2) = \Delta_F(x_1 - x_2)\delta_{ij}$$



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$$\mathcal{O}^{(1)}_{quqd} = (\bar{Q}^i u)\epsilon_{ij}(\bar{Q}^j d)$$

$$\text{Im}\left[e^{-i\theta}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} Y_{u,i_1 j_1} Y_{u,i_2 j_2} C^{(1)}_{quqd,mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{d,k_1 l_1} Y_{d,k_2 l_2}\right]$$

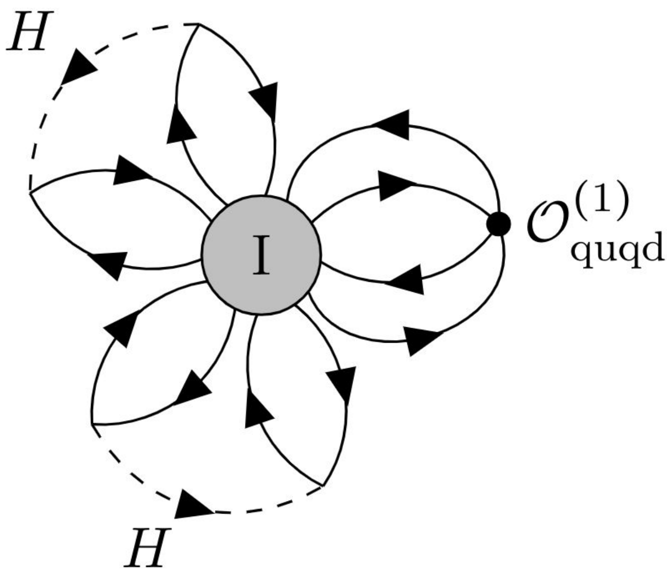
$$\text{Im}\left[e^{-i\theta}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} (X_u Y_u)_{i_1 j_1} Y_{u,i_2 j_2} C^{(1)}_{quqd,mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{d,k_1 l_1} Y_{d,k_2 l_2}\right]$$

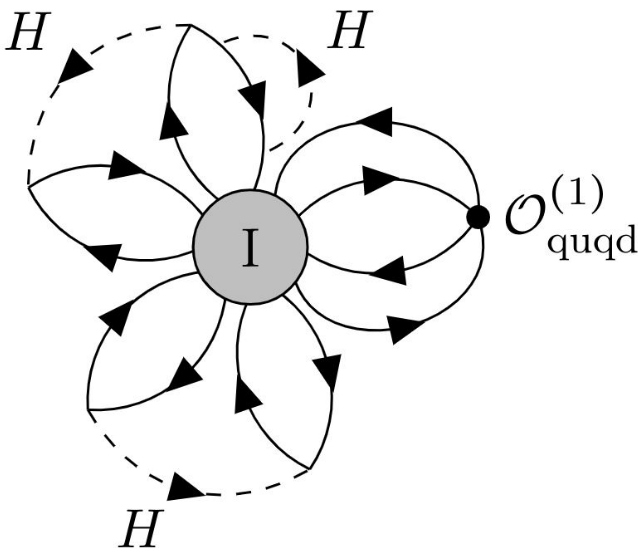
Now, start from invariants…

$$\mathcal{O}_{quqd}^{(1)} = (\bar{Q}^i u)\epsilon_{ij}(\bar{Q}^j d)$$

$$\mathrm{Im}\left[e^{-i\theta}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} Y_{u,i_1 j_1} Y_{u,i_2 j_2} C^{(1)}_{quqd,mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{d,k_1 l_1} Y_{d,k_2 l_2}\right]$$

$$\mathrm{Im}\left[e^{-i\theta}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} (X_u Y_u)_{i_1 j_1} Y_{u,i_2 j_2} C^{(1)}_{quqd,mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{d,k_1 l_1} Y_{d,k_2 l_2}\right]$$





Conclusions

- Axion solution to the strong CP problem depends on quality of PQ symmetry
- Additional contributions to axion potential can spoil resolution of strong CP problem
- We studied small instanton contributions to the axion potential and showed that they are proportional to SMEFT CPV invariants
- We can also use the invariants to study the contributions of all remaining CP-odd operators

Thank you!

Backup

Trace basis vs. determinant-like basis

Example: $\mathcal{O}_{uH} = |H|^2 \bar{Q}\tilde{H}u$

$$\mathrm{ImTr}\left(C_{uH}Y_u^\dagger\right)$$

$$\mathrm{ImTr}\left(X_u C_{uH}Y_u^\dagger\right)$$

$$\mathrm{ImTr}\left(X_d C_{uH}Y_u^\dagger\right)$$

$$\mathrm{ImTr}\left(X_u X_d C_{uH}Y_u^\dagger\right)$$

$$\vdots$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}C_{uH,Kk}\det Y_d)$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}(X_u C_{uH})_{Kk}\det Y_d)$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}(X_d C_{uH})_{Kk}\det Y_d)$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}(X_u X_d C_{uH})_{Kk}\det Y_d)$$

$$\vdots$$

Trace basis vs. determinant-like basis

Example: $\mathcal{O}_{uH} = |H|^2 \bar{Q}\tilde{H}u$

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$$\mathrm{ImTr}\left(X_u C_{uH}Y_u^\dagger\right)$$

$$\mathrm{ImTr}\left(X_d C_{uH}Y_u^\dagger\right)$$

$$\mathrm{ImTr}\left(X_u X_d C_{uH}Y_u^\dagger\right)$$

$$\vdots$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}C_{uH,Kk}\det Y_d)$$

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$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}(X_d C_{uH})_{Kk}\det Y_d)$$

$$\mathrm{Im}(e^{-\mathrm{i}\theta_{QCD}}\epsilon^{IJK}\epsilon^{ijk}Y_{u,Ii}Y_{u,Jj}(X_u X_d C_{uH})_{Kk}\det Y_d)$$

$$\vdots$$

The two bases can be related:

$$\mathcal{I}_{abcd}(C_{uH}) = 2\left(J_\theta R_{(a-1)bcd}(C_{uH}Y_u^\dagger) + K_\theta L_{(a-1)bcd}(C_{uH}Y_u^\dagger)\right)$$

Det-like CP-odd inv.

SM CP-odd θ-inv.

Trace CP-even inv.

SM CP-even θ-inv.

Trace CP-odd inv.

CPV in the SM – Example: electron EDM

[Pospelov, Ritz, 1311.5537]
[Smith, Touati, 1707.06805]

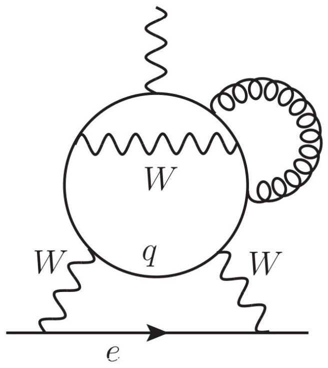
- How can we use flavor invariants? Estimate size of electron EDM in SM.

- We know: - EDM breaks chiral symmetry $\Longrightarrow d_e \propto m_e$

  - EDM breaks CP $\Longrightarrow d_e \propto J_4$

$$\mathcal{L}_{dip} = -\frac{i}{2} d_e \bar{e} \sigma^{\mu\nu} F_{\mu\nu} \gamma_5 e$$

- Draw diagram that can generate J_4



$$d_e \sim e \frac{m_e}{m_W^2} \frac{\alpha_W^3 \alpha_S}{(4\pi)^4} J_4 \frac{v^{12}}{m_W^{12}} \approx 10^{-48} e \cdot \mathrm{cm}$$

[Roussy et al., 2212.11841]

Experiment:
$|d_e| < 4.1 \times 10^{-30}\, e\,\mathrm{cm}$

- EDMs are strong experimental probes $\Longrightarrow$ Repeat analysis for new physics?

Details of instanton calculation

$$\chi^{(1)}_{\text{quqd}}(0)^{\text{1-inst.}} = -i \lim_{k\to 0} \int d^4x\, e^{ikx} \left\langle 0 \left| T\left\{ \frac{1}{32\pi^2} G\widetilde{G}(x), \frac{C^{(1)}_{\text{quqd}}}{\Lambda^2_{\not{CP}}} \mathcal{O}^{(1)}_{\text{quqd}}(0) \right\} \right| 0 \right\rangle ,$$

$$= e^{-i\theta_{\text{QCD}}} \int d^4x_0 \int \frac{d\rho}{\rho^5} d_N(\rho) \int \mathcal{D}H\mathcal{D}H^\dagger\, e^{-S_0[H,H^\dagger]} \int \prod_{f=1}^{3} \left(\rho^2\, d\xi^{(0)}_{u_f} d\xi^{(0)}_{d_f} d^2\bar{\xi}^{(0)}_{Q_f} \right)$$

$$\times\, e^{\int d^4x (\bar{Q} Y_{\text{u}} \widetilde{H} u + \bar{Q} Y_{\text{d}} H d + \text{h.c.})(x)} \frac{1}{32\pi^2} \int d^4x\, G\widetilde{G}(x) \left(\frac{C^{(1)}_{\text{quqd}}}{\Lambda^2_{\not{CP}}} \bar{Q}u\bar{Q}d(0) + \text{h.c.} \right) ,$$

Details of instanton calculation

$$\chi^{(1)}_{\text{quqd}}(0)^{1-\text{inst.}} = e^{-i\theta_{\text{QCD}}} \int d^4x_0 \int \frac{d\rho}{\rho^5} d_N(\rho) \int \mathcal{D}H\mathcal{D}H^\dagger\, e^{-S_0[H,H^\dagger]} \prod_{f=1}^{3} \left(\rho^2\, d\xi^{(0)}_{u_f} d\xi^{(0)}_{d_f} d^2\bar{\xi}^{(0)}_{Q_f}\right)$$
$$\times \int d^4x_1 d^4x_2 d^4x_3 d^4x_4\, \frac{1}{4!}\Bigg[\sum_{\substack{\text{perm. over}\\\text{fermion fields}}} \bar{\xi}^{(0)}_{Q^I_{i_1}} (\bar{\psi}^{(0)} Y_{\text{u},i_1j_1} \widetilde{H}^I P_R \psi^{(0)})(x_1)\xi^{(0)}_{u_{j_1}}$$
$$\times \bar{\xi}^{(0)}_{Q^J_{i_2}} (\bar{\psi}^{(0)} Y_{\text{u},i_2j_2} \widetilde{H}^J P_R \psi^{(0)})(x_2)\xi^{(0)}_{u_{j_2}} \quad \bar{\xi}^{(0)}_{Q^K_{k_1}} (\bar{\psi}^{(0)} Y_{\text{d},k_1l_1} H^K P_R \psi^{(0)})(x_3)\xi^{(0)}_{d_{l_1}} \tag{3.14}$$
$$\times \bar{\xi}^{(0)}_{Q^L_{k_2}} (\bar{\psi}^{(0)} Y_{\text{d},k_2l_2} H^L P_R \psi^{(0)})(x_4)\xi^{(0)}_{d_{l_2}} \int d^4x \frac{G\widetilde{G}(x)}{32\pi^2} \Bigg(\frac{C^{(1)}_{\text{quqd},mnop}}{\Lambda^2_{\not{CP}}} \bar{\xi}^{(0)}_{Q^M_m} (\bar{\psi}^{(0)} P_R \psi^{(0)}) \xi^{(0)}_{u_n}\, \epsilon_{MN}$$
$$\times \bar{\xi}^{(0)}_{Q^N_o} (\bar{\psi}^{(0)} P_R \psi^{(0)}) \xi^{(0)}_{d_p}\Bigg)(0)\Bigg],$$

Details of instanton calculation

$$\chi^{(1)}_{\text{quqd}}(0)^{1-\text{inst.}} = \frac{1}{4\Lambda^2_{\not{CP}}}\Big[e^{-i\theta_{\text{QCD}}}\epsilon^{i_1i_2m}\epsilon^{j_1j_2n}Y_{\text{u},i_1j_1}Y_{\text{u},i_2j_2}C^{(1)}_{\text{quqd},mnop}\epsilon^{k_1k_2o}\epsilon^{l_1l_2p}Y_{\text{d},k_1l_1}Y_{\text{d},k_2l_2}$$

$$+e^{-i\theta_{\text{QCD}}}\epsilon^{i_1i_2m}\epsilon^{j_1j_2n}Y_{\text{u},i_1j_1}Y_{\text{u},i_2j_2}C^{(1)}_{\text{quqd},onmp}\epsilon^{k_1k_2o}\epsilon^{l_1l_2p}Y_{\text{d},k_1l_1}Y_{\text{d},k_2l_2}\Big]\int d^4x_0\int\frac{d\rho}{\rho^5}d_N(\rho)\rho^6$$

$$\times\underbrace{\int\mathcal{D}H\mathcal{D}H^\dagger\,e^{-S_0[H,H^\dagger]}\Big[\int d^4x_1d^4x_2(\bar{\psi}^{(0)}H^\dagger_I\epsilon^{IJ}P_R\psi^{(0)})(x_1)\,(\bar{\psi}^{(0)}\epsilon_{JK}H^KP_R\psi^{(0)})(x_2)\Big]^2}_{=2!\,\left[\int d^4x_1d^4x_2(\bar{\psi}^{(0)}P_R\psi^{(0)})(x_1)\Delta_H(x_1-x_2)\epsilon_{IJ}\epsilon^{JI}(\bar{\psi}^{(0)}P_R\psi^{(0)})(x_2)\right]^2\equiv 2!\,\mathcal{I}^2}$$

$$\times\Big(\epsilon_{MN}\epsilon^{MN}\bar{\psi}^{(0)}P_R\psi^{(0)}\,\bar{\psi}^{(0)}P_R\psi^{(0)}\Big)(0)\int d^4x\frac{G\widetilde{G}(x)}{32\pi^2}\,.$$

$$A^{(1)}_{\text{quqd}} = e^{-i\theta_{\text{QCD}}}\epsilon^{i_1i_2m}\epsilon^{j_1j_2n}Y_{\text{u},i_1j_1}Y_{\text{u},i_2j_2}C^{(1)}_{\text{quqd},mnop}\epsilon^{k_1k_2o}\epsilon^{l_1l_2p}Y_{\text{d},k_1l_1}Y_{\text{d},k_2l_2}$$

$$B^{(1)}_{\text{quqd}} = e^{-i\theta_{\text{QCD}}}\epsilon^{i_1i_2m}\epsilon^{j_1j_2n}Y_{\text{u},i_1j_1}Y_{\text{u},i_2j_2}C^{(1)}_{\text{quqd},onmp}\epsilon^{k_1k_2o}\epsilon^{l_1l_2p}Y_{\text{d},k_1l_1}Y_{\text{d},k_2l_2}$$

Details of instanton calculation

Final result:

$$\chi^{(1)}_{\text{quqd}}(0) = \frac{2i}{\Lambda^2_{\not{CP}}} \text{Im}\left(A^{(1)}_{\text{quqd}} + B^{(1)}_{\text{quqd}}\right) \int d^4x_0 \int \frac{d\rho}{\rho^5} d_N(\rho)\rho^6 \mathcal{I}^2 \left[\frac{4\rho^4}{\pi^4}\frac{1}{(x_0^2+\rho^2)^6}\right]$$

$$A^{(1)}_{\text{quqd}} = e^{-i\theta_{\text{QCD}}}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} Y_{\text{u},i_1 j_1} Y_{\text{u},i_2 j_2} C^{(1)}_{\text{quqd},mnop}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{\text{d},k_1 l_1} Y_{\text{d},k_2 l_2}$$
$$B^{(1)}_{\text{quqd}} = e^{-i\theta_{\text{QCD}}}\epsilon^{i_1 i_2 m}\epsilon^{j_1 j_2 n} Y_{\text{u},i_1 j_1} Y_{\text{u},i_2 j_2} C^{(1)}_{\text{quqd},onmp}\epsilon^{k_1 k_2 o}\epsilon^{l_1 l_2 p} Y_{\text{d},k_1 l_1} Y_{\text{d},k_2 l_2}$$