Quarks at the Modular S_4 Cusp

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POPH PROGRAMA OPERACIONAL POTENCIAL HUMANO

Goal

Modular Invariance

Natural Hierarchies

Predictive Models
w/ Natural Hierarchies
for Quarks

Modular Framework

What are modular symmetries?

- Modular Group

$$\Gamma \equiv SL(2,\mathbb{Z}) \equiv \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad a,b,c,d \in \mathbb{Z}, ad-bc=1 \right\}$$

- Principal Congruence Subgroups

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z}), \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod}\, N) \right\}$$

- Quotient Group

$$\Gamma'_N \equiv \Gamma/\Gamma(N) \simeq SL(2,\mathbb{Z}_N)$$

Modular Group

- 3 generators
$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

- Group Relations
$$S^2 = R, \qquad (ST)^3 = \mathbb{1}, \qquad R^2 = \mathbb{1}, \qquad RT = TR.$$

- Transformation
$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \, : \, \tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

- Generating Transformations
$$S : \tau \to -\frac{1}{\tau}, \qquad T : \tau \to \tau + 1, \qquad R : \tau \to \tau$$

Why do we care?

- Applications to Flavour

$$\Gamma_2 \simeq S_3 \qquad \Gamma'_2 \simeq S_3$$

$$\Gamma_3 \simeq A_4 \qquad \Gamma'_3 \simeq T'$$

$$\Gamma_4 \simeq S_4 \qquad \Gamma'_4 \simeq S'_4$$

$$\Gamma_5 \simeq A_5 \qquad \Gamma'_5 \simeq A'_5$$

Why do we care?

- If (super)fields transform trivially under Γ(N)

$$\varphi_i \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{ij}(\gamma) \varphi_j$$

Representation Matrix of Γ_N

Why do we care?

- If (super)fields transform trivially under Γ(N)

$$\varphi_i \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{ij}(\gamma)\varphi_j$$

Automorphy Factor

Why do we care?

- If (super)fields transform trivially under Γ(N)

$$\varphi_i \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{ij}(\gamma) \varphi_j$$

Select Γ_N

- Dealer's choice: $\varphi_i \sim \left(\mathbf{r}_i , k_i\right)$

Why do we care?

- If (super)fields transform trivially under Γ(N)

$$\varphi_i \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{ij}(\gamma)\varphi_j$$

Modular

- Dealer's choice: $\varphi_i \sim (\mathbf{r}_i, k_i)$

Modular Framework

- Superpotential

$$W(\tau, \psi_I) = \sum \left(Y_{I_1 \ldots I_n}(\tau)\, \psi_{I_1} \cdots \psi_{I_n} \right)_{\mathbf{1}}$$

Modular Framework

- Modular Forms (Yukawas):
  - "Weighted-representations" (-*ish)*
  - Representations (and weights) constrained by Γ_N
  - Specific functions of τ
  - $$Y_{\mathbf{r}}^{(k)}(\tau) = \left(\bigotimes_{n=1}^{k} Y_{\mathbf{r_1}}^{(1)}(\tau) \right)_{\mathbf{r}}$$

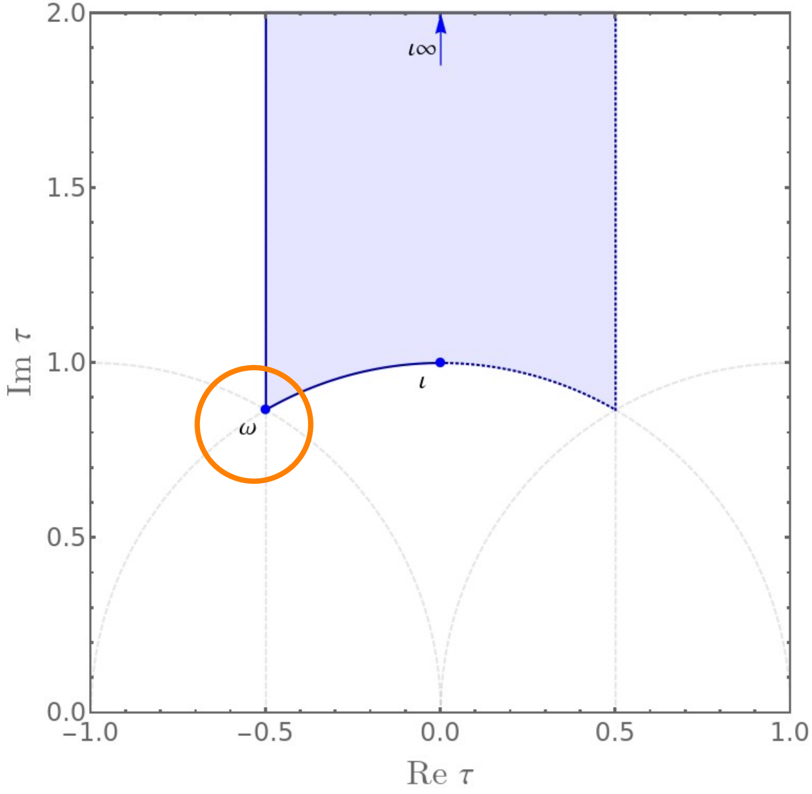
Residual Symmetries

Fundamental Domain

- $$\tau = i\infty : \quad T \circ i\infty = i\infty + 1 = i\infty$$
  $$\mathbb{Z}_N^T \times \mathbb{Z}_2^R \qquad T^N = \mathbb{1} \quad \text{and} \quad R^2 = \mathbb{1}$$

- $$\tau = i : \qquad S \circ i = \frac{-1}{i} = i$$
  $$\mathbb{Z}_4^S \qquad S^2 = R \quad \text{and} \quad R^2 = \mathbb{1}$$

- $$\tau = \omega : \qquad ST \circ \omega = \frac{-1}{-\omega^2} = \omega$$
  $$\mathbb{Z}_3^{ST} \times \mathbb{Z}_2^R \qquad (ST)^3 = \mathbb{1} \quad \text{and} \quad R^2 = \mathbb{1}$$



Natural Hierarchies

Subgroup Decomposition @ Cusp

- Field Transformations

$$\varphi_i \xrightarrow{\gamma} (c\tau + d)^{-k} \rho_{ij}(\gamma) \varphi_j$$

$$ST = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\rho_{\mathbf{3}}(ST) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\varphi \to \omega^k \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \varphi = \begin{pmatrix} \omega^k & 0 & 0 \\ 0 & \omega^{k+1} & 0 \\ 0 & 0 & \omega^{k+2} \end{pmatrix} \varphi$$

$$\mathbf{3} \rightsquigarrow \mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$$

Mass Matrices: Power Structure @ cusp

- $\varphi_i M(\tau)_{ij} \varphi_j^c \longrightarrow M(\tau) \xrightarrow{\gamma} M(\gamma\tau) = (c\tau + d)^{k_Y} \rho^* M(\tau) \rho^{c\dagger}$

$$
\begin{aligned}
\tilde{\rho}_i &= w^k \rho_i \,, \\
u &= \frac{\tau - \omega}{\tau - \omega^2} \,, \quad \epsilon = |u| \\
\widetilde{M}_{ij}(u) &= (1-u)^{-k_Y} M_{ij}(u) \,,
\end{aligned}
$$

$$\omega^{2n} \widetilde{M}_{ij}^{(n)}(0) = \left(\tilde{\rho}_i \tilde{\rho}_j^c\right)^* \widetilde{M}_{ij}^{(n)}(0)$$

$$\tilde{\rho}_i^c \tilde{\rho}_j = \omega^l \Rightarrow \tilde{M}_{ij} \sim M_{ij} \sim \mathcal{O}(\epsilon^l)$$

See M. Tanimoto's talk

Natural Hierarchies: An Example

S_4' irrep $\mathbf{r}$	$\mathbf{1}, \mathbf{1}', \hat{\mathbf{1}}, \hat{\mathbf{1}}'$	$\mathbf{2}, \hat{\mathbf{2}}$	$\mathbf{3}, \mathbf{3}', \hat{\mathbf{3}}, \hat{\mathbf{3}}'$
$\mathbb{Z}_3$ decomposition	$\mathbf{1}_k$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$

$$Q \sim (\mathbf{2}, 0) \oplus (\mathbf{1}, 2) \rightsquigarrow \mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{1}_2$$

$$q^c \sim (\mathbf{2}, 4) \oplus (\mathbf{1}, 2) \rightsquigarrow \mathbf{1}_2 \oplus \mathbf{1}_0 \oplus \mathbf{1}_2$$

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

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$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

Natural Hierarchies: Caveats

S_4' irrep $\mathbf{r}$	$\mathbf{1}, \mathbf{1}', \hat{\mathbf{1}}, \hat{\mathbf{1}}'$	$\mathbf{2}, \hat{\mathbf{2}}$	$\mathbf{3}, \mathbf{3}', \hat{\mathbf{3}}, \hat{\mathbf{3}}'$
$\mathbb{Z}_3$ decomposition	$\mathbf{1}_k$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$

$$Q \sim \big(\mathbf{2}, 0\big) \oplus \big(\mathbf{1}, 2\big) \rightsquigarrow \mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{1}_2$$

$$q^c \sim \big(\mathbf{2}, 4\big) \oplus \big(\mathbf{1}, 2\big) \rightsquigarrow \mathbf{1}_2 \oplus \mathbf{1}_0 \oplus \mathbf{1}_2$$

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

Natural Hierarchies: Normalisations

- Zero in the Symmetric Point
- Governed by the Same Coefficient
  (can absorb ϵ …)

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

c

How to Normalise The Modular Forms?

Natural Hierarchies: Normalisations

- Zero in the Symmetric Point
- Governed by the Same Coefficient
  (can absorb ϵ …)

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

How to Normalise The Modular Forms?

No idea… but

Natural Hierarchies: Normalisations

- Relative Hierarchies are unavoidable

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline \epsilon & \epsilon^2 & \epsilon \end{array}\right)$$

- Euclidean Norm

Choice as good as any … ?

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline 1 & \epsilon & 1 \end{array}\right)$$

Natural Hierarchies: representations

- > 3 sub-blocks ⇨ > 1 $\mathcal{O}(1)$ masses

$$M \sim \left(\begin{array}{cc|c} 1 & \epsilon & 1 \\ \epsilon & \epsilon^2 & \epsilon \\ \hline 1 & \epsilon & 1 \end{array}\right)$$

- Fully Hierarchical ⇨ < 4 sub-blocks ⇨

$$Q \sim \mathbf{3}^*$$

$$q^c \sim \mathbf{3}^*$$

$$\text{both} \sim \mathbf{3}^*$$

Natural Hierarchies: representations

S_4' irrep $\mathbf{r}$	$\mathbf{1}, \mathbf{1}', \hat{\mathbf{1}}, \hat{\mathbf{1}}'$	$\mathbf{2}, \hat{\mathbf{2}}$	$\mathbf{3}, \mathbf{3}', \hat{\mathbf{3}}, \hat{\mathbf{3}}'$
$\mathbb{Z}_3$ decomposition	$\mathbf{1}_k$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$

- $\left(\mathbf{3}^*,\ k\right) \rightsquigarrow \mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$
- Only one large mass ⇨ $\text{partner} \rightsquigarrow \mathbf{1}_{k'} \oplus \mathbf{1}_{k'} \oplus \mathbf{1}_{k'}$

- No Triplets
- No doublets

Natural Hierarchies: representations

S_4' irrep $\mathbf{r}$	$\mathbf{1}, \mathbf{1}', \hat{\mathbf{1}}, \hat{\mathbf{1}}'$	$\mathbf{2}, \hat{\mathbf{2}}$	$\mathbf{3}, \mathbf{3}', \hat{\mathbf{3}}, \hat{\mathbf{3}}'$
$\mathbb{Z}_3$ decomposition	$\mathbf{1}_k$	$\mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$	$\mathbf{1}_k \oplus \mathbf{1}_{k+1} \oplus \mathbf{1}_{k+2}$

- Only one large mass

$$Q \sim \left(\mathbf{3}^*,\ k\right) \Rightarrow q^c \sim \bigoplus_\beta \left(\mathbf{1}_\beta,\ k_\beta\right) \qquad k_\beta - k_{\beta'} = 0 \pmod{3}$$

- Mass Matrix $M = f\left(Y_{\mathbf{3}^*}^{k_Y}, Y_{\mathbf{3}^*}^{k_Y+3}, Y_{\mathbf{3}^*}^{k_Y+6}, \ldots\right)$

Minimal Mass Matrices

Counting Possibilities

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
$k=1$	0	1
$k=4$	1	1
$k=7$	2	2
$k=10$	2	3

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
$k=2$	0	1
$k=5$	1	2
$k=8$	2	2
$k=11$	3	3

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
$k=3$	1	1
$k=6$	1	2
$k=9$	2	3
$k=12$	3	3

- Predictive: at most 4 Yukawas
  (4x2 + τ = 10 = 6 masses + 3 angles + δ)

Counting Possibilities: 3 parameter matrices

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
$k=1$	0	1
$k=4$	1	1
$k=7$	2	2
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$k=3$	1	1
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$k=12$	3	3

- Massless Quark (det = 0)

$$Y_{\hat{\mathbf{3}}}^{(1)} \propto Y_{\mathbf{3}}^{(4)}$$

Counting Possibilities: 3 parameter matrices

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$k=1$	0	1
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$k=3$	1	1
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$k=9$	2	3
$k=12$	3	3

- Massless Quark (det = 0)

$$Y_{\hat{\mathbf{3}}}^{(3)} \propto Y_{\mathbf{3}}^{(6)}$$

Counting Possibilities: 4 parameter matrices

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
$k=1$	0	1
$k=4$	1	1
$k=7$	2	2
$k=10$	2	3

	$\mathbf{3}\ (\hat{\mathbf{3}}')$	$\mathbf{3}'\ (\hat{\mathbf{3}})$
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$k=3$	1	1
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$k=9$	2	3
$k=12$	3	3

	M_1	M_2	M_3	M_4	M_5	M_6
$(\mathbf{r}_1, k_1)$	$(\hat{\mathbf{3}}, 1)$	$(\mathbf{3}, 4)$	$(\mathbf{3}', 2)$	$(\mathbf{3}', 2)$	$(\hat{\mathbf{3}}', 3)$	$(\hat{\mathbf{3}}', 3)$
$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$	$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$	$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$	$(\mathbf{3}', 6)$

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$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$	$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$	$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$	$(\mathbf{3}', 6)$

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$k=12$	3	3

$$Y^{(1)}_{\hat{\mathbf{3}}} \propto Y^{(4)}_{\mathbf{3}}$$

	M_1 $\sim$	M_2	M_3	M_4	M_5 $\sim$	M_6
$(\mathbf{r}_1, k_1)$	$(\hat{\mathbf{3}}, 1)$	$(\mathbf{3}, 4)$	$(\mathbf{3}', 2)$	$(\mathbf{3}', 2)$	$(\hat{\mathbf{3}}', 3)$	$(\hat{\mathbf{3}}', 3)$
$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$	$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$	$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$	$(\mathbf{3}', 6)$

$$Y^{(3)}_{\hat{\mathbf{3}}} \propto Y^{(6)}_{\mathbf{3}}$$

From Matrices to *Models*

Assignments, Transposition, and gCP

	M_1	$\sim$	M_2	M_3	M_4	M_5	$\sim$	M_6
$(\mathbf{r}_1, k_1)$	$(\hat{\mathbf{3}}, 1)$		$(\mathbf{3}, 4)$	$(\mathbf{3}', 2)$	$(\mathbf{3}', 2)$	$(\hat{\mathbf{3}}', 3)$		$(\hat{\mathbf{3}}', 3)$
$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$		$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$		$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$		$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$		$(\mathbf{3}', 6)$

Are all combinations Possible?

Assignments, Transposition, and gCP

	M_1	$\sim$ M_2	M_3	M_4	M_5	$\sim$ M_6
$(\mathbf{r}_1, k_1)$	$(\hat{\mathbf{3}}, 1)$	$(\mathbf{3}, 4)$	$(\mathbf{3}', 2)$	$(\mathbf{3}', 2)$	$(\hat{\mathbf{3}}', 3)$	$(\hat{\mathbf{3}}', 3)$
$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$	$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$	$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$	$(\mathbf{3}', 6)$

Are all combinations Possible?

$Q \sim \mathbf{3}^*$ → 7 field weights to 6 Yukawa weights → Yes! ✓

$q^c \sim \mathbf{3}^*$ → 5 field weights to 6 Yukawa weights → No :(

Assignments, Transposition, and gCP

$Q \sim \mathbf{3}^*$

	M_1	$\sim$ M_2	M_3	M_4	M_5	$\sim$ M_6
$(\mathbf{r}_1, k_1)$	$(\hat{\mathbf{3}}, 1)$	$(\mathbf{3}, 4)$	$(\mathbf{3}', 2)$	$(\mathbf{3}', 2)$	$(\hat{\mathbf{3}}', 3)$	$(\hat{\mathbf{3}}', 3)$
$(\mathbf{r}_2, k_2)$	$(\mathbf{3}', 4)$	$(\mathbf{3}', 4)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}', 5)$	$(\hat{\mathbf{3}}, 3)$	$(\mathbf{3}, 6)$
$(\mathbf{r}_3, k_3)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}', 7)$	$(\hat{\mathbf{3}}, 5)$	$(\mathbf{3}, 8)$	$(\mathbf{3}', 6)$	$(\mathbf{3}', 6)$

$q^c \sim \mathbf{3}^*$

$$(M_1^T, M_4^T)\,, \quad (M_2^T, M_5^T)\,, \quad (M_3^T, M_6^T)\,,$$
$$(M_4^T, M_1^T)\,, \quad (M_5^T, M_2^T)\,, \quad (M_6^T, M_3^T)\,,$$
$$(M_i^T, M_i^T) \text{ with } i = 1, \ldots, 6$$

of Parameters

$$M_q \sim \begin{pmatrix} | & | & | \\ \alpha_1 Y_1 & \alpha_2 Y_2 & \alpha_3 Y_3 + \alpha_4 Y_4 \\ | & | & | \end{pmatrix} \text{ or its transpose}$$

gCP

(4 real) x 2 + τ

$Q \sim \mathbf{3}^*$

(4 real) x 2 + τ
+
(1 phase) x 2

$q^c \sim \mathbf{3}^*$

(4 real) x 2 + τ
+
(2 phase) x 2

Numerical Results

Can we find the elusive predictive Quark model?

Foreshadowing: no

Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w/ δ)	10	✗	✗
pheno phase	11	✓	—
special τ	10“+2”	✓	—
2 τ	8+4	✓	✓

Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w			
pheno p			
specia			
2 τ			

M_u \ M_d	$M_{1,2}$	M_3	M_4	$M_{5,6}$
$M_{1,2}$	0.0	9+	9+	9+
M_3	9+	0.0	0.0	9+
M_4	9+	0.0	0.0	9+
$M_{5,6}$	9+	9+	9+	0.0

(a) Models with $Q \sim \mathbf{3}^*$.

M_u \ M_d	$M^T_{1,2}$	M^T_3	M^T_4	$M^T_{5,6}$
$M^T_{1,2}$	0.0	–	1.5	1.5
M^T_3	–	1.0	–	1.0
M^T_4	0.0	–	1.0	–
$M^T_{5,6}$	0.0	1.5	–	1.4

(b) Models with $q^c \sim \mathbf{3}^*$.

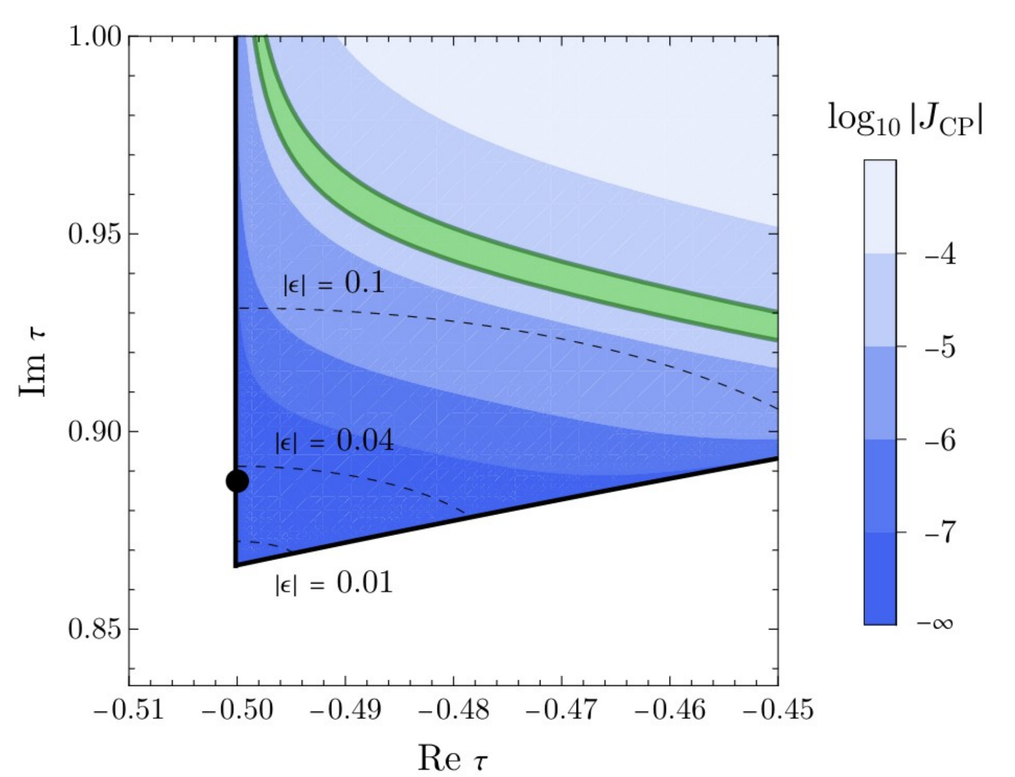
Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w/ δ)	10	✗	✗
pheno phase	11	✓	—
special τ	10“+2”	✓	—
2 τ	8+4	✓	✓

- The problem of CP-violation

Cases considered

	# dofs	$Q\sim$
gCp (w/o δ)	10	✓
gCp (w/ δ)	10	✗
pheno phase	11	✓
special τ	10“+2”	✓
2 τ	8+4	✓



- The problem of CP-violation

Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w/ δ)	10	✗	✗
pheno phase	11	✓	—
special τ	10"+2"	✓	—
2 τ	8+4	✓	✓

- *Pheno phase:* lift gCP only in one sector (not consistently)

Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w/ δ)	10	✗	✗
pheno phase	11	✓	—
special τ	10"+2"	✓	—
2 τ	8+4	✓	✓

- Special τ : no gCP but top-down values for τ

$$\tau \simeq \mp 0.484 + 0.884\, i,\ \mp 0.492 + 0.875\, i,\ \mp 0.495 + 0.872\, i,\ \ldots ,$$

$$\text{corresponding to } |\epsilon(\tau)| \simeq 0.04,\ 0.02,\ 0.01,\ \ldots ,$$

Cases considered

gCp (w/o δ)
gCp (w/ δ)
pheno phase
special τ
2 τ

M_u \ M_d	$M_{1,2}$	M_3	M_4	$M_{5,6}$
$M_{1,2}$	0.1	9+	9+	9+
M_3	9+	2.1	2.1	9+
M_4	9+	2.1	2.1	9+
$M_{5,6}$	9+	9+	9+	2.1

(a) $\tau = \mp 0.484 + 0.884\, i$.

M_u \ M_d	$M_{1,2}$	M_3	M_4	$M_{5,6}$
$M_{1,2}$	5+	9+	9+	9+
M_3	9+	5+	5+	9+
M_4	9+	5+	5+	9+
$M_{5,6}$	9+	9+	9+	5+

(b) $\tau = \mp 0.492 + 0.875\, i$.

$$\tau \simeq \mp 0.484 + 0.884\, i,\ \mp 0.492 + 0.875\, i,\ \mp 0.495 + 0.872\, i,\ \ldots,$$

corresponding to $|\epsilon(\tau)| \simeq 0.04,\ 0.02,\ 0.01,\ \ldots,$

Cases considered

	# dofs	$Q \sim \mathbf{3}^*$	$q^c \sim \mathbf{3}^*$
gCp (w/o δ)	10	✓	✓
gCp (w/ δ)	10	✗	✗
pheno phase	11	✓	—
special τ	10“+2”	✓	—
2 τ	8+4	✓	✓

- 2 τ : gCP but different τ for each sector (not consistent)

Are the Hierarchies *actually* from the proximity to the cusp?

Almost?

A Closer Look @ Natural Hierarchies

	gCP (masses)	gCP (all)	pheno phase	no gCP	two moduli
$\mathrm{Re}\,\tau_1$	-0.4772	-0.4823	-0.4992	-0.4978	-0.4969
$\mathrm{Im}\,\tau_1$	0.8861	0.8784	0.8852	0.8850	0.8692
$\mathrm{Re}\,\tau_2$	–	–	–	–	-0.4939
$\mathrm{Im}\,\tau_2$	–	–	–	–	0.8856
$\lvert\epsilon_1\rvert$	0.0486	0.0348	0.0306	0.0306	0.0072
$\lvert\epsilon_2\rvert$	–	–	–	–	0.0328
$(m_c/m_t)\,/\,\lvert\epsilon\rvert$	0.055	0.052	0.088	0.088	0.371
$(m_u/m_t)\,/\,\lvert\epsilon\rvert^2$	0.002	0.003	0.006	0.006	0.105
$(m_s/m_b)\,/\,\lvert\epsilon\rvert$	0.282	0.400	0.448	0.448	0.418
$(m_d/m_b)\,/\,\lvert\epsilon\rvert^2$	0.293	0.582	0.739	0.739	0.643
$N\sigma$ (masses)	0.0	3.4	0.0	0.0	0.0
$N\sigma$ (angles)	51.8	5.5	0.0	0.0	0.0
$N\sigma$ (δ_{CP})	11.2	0.0	0.0	0.0	0.0
$N\sigma$ (total)	52.9	6.4	0.0	0.0	0.0

Thank You