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Reduction of Couplings in the 2HDM

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Motivation

The ad hoc Yukawa and Higgs sectors of the Standard Model induce ~ 20 free parameters. How can they be related to the gauge sector in a more *fundamental* level?

The straightforward way to induce relations among parameters is to add **more symmetries**.

→ i.e. GUTs.

Another approach is to look for **renormalization group invariant (RGI)** relations among couplings at the GUT scale that hold up to the Planck scale.

→ **less free** parameters → **more predictive** theories

Reduction of Couplings Basics

An RGI expression among couplings

$$\mathcal{F}(g_1, \dots, g_A) = 0$$

must satisfy the pde

$$\mu \frac{d\mathcal{F}}{d\mu} = \sum_{\alpha=1}^A \beta_{\alpha} \frac{\partial \mathcal{F}}{\partial g_{\alpha}} = 0$$

There are $(A - 1)$ independent \mathcal{F} s and finding them is equivalent to solve the ode

$$\beta_g \left(\frac{dg_{\alpha}}{dg} \right) = \beta_{\alpha}, \quad \alpha = 1, \dots, A - 1$$

where g is considered the *primary coupling*.

The above equations are called *reduction equations (RE)*.

Zimmermann (1985)

However, the general solutions of the REs have integration constants.

→ We just traded an integration constant for each coupling → we have not reduced the freedom of the parameter space.

→ Assume **power series solutions** to the REs (which preserve perturbative renormalizability):

$$g_a = \sum_n \rho_a^{(n)} g^{2n(+1)}$$

- Examining in one-loop sufficient for uniqueness to all loops

Oehme, Sibold, Zimmermann (1984); (1985)

For some models this **complete reduction** can prove to be too restrictive → use fewer \mathcal{F} s as RGI constraints (**partial reduction**).

RoC Applications

- Standard Model

→ $m_t \sim 173 \text{ GeV}$

→ $m_h \sim 125 \text{ GeV}$

Kubo, Sibold, Zimmermann (1984); (1985)

- Non-Supersymmetric SM Extensions

- Two Higgs Doublet Models

- SM + Vector-like Quarks

- SM + Asymptotically Safe Gravity

- Supersymmetric SM Extensions

- Reduced MSSM

Mondragon, Tracas, Zoupanos (2014)

- Finite Unified Theories

- Reduced Minimal $N = 1 \text{ SU}(5)$

Kubo, Mondragon, Zoupanos (1994)

- All-loop Finite $N = 1 \text{ SU}(5)$

Heinemeyer, Mondragon, Zoupanos (2008)

- Two-loop Finite $N = 1 \text{ SU}(3)^3$

Ma, Mondragon, Zoupanos (2004)

Heinemeyer, Mondragon, GP, Tracas, Zoupanos (2020)

Heinemeyer, Kalinowski, Kotlarski, Mondragon, GP, Tracas, Zoupanos (2021)

The 2HDM

$$\begin{aligned}
 V_h = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) \\
 & + \frac{1}{2} \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2} \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\
 & + \left[\frac{1}{2} \lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + \lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \text{h.c.} \right]
 \end{aligned}$$

→ We choose all parameters to be real

→ We choose to work with the **Type-II** scenario ($u_R^j \rightarrow \Phi_2$, $d_R^j, e_R^j \rightarrow \Phi_1$):

- $\lambda_6 = \lambda_7$ to avoid tree-level FCNCs

and we need

- $\lambda_4 < 0$ to conserve the electric charge and
- $\lambda_1 > 0$, $\lambda_2 > 0$ and $\sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0$ for the potential to be bounded from below.

A first Attempt

- Reduction on **dimensionless** parameters, m_{11}^2 , m_{22}^2 and m_{12}^2 remain free, will be fixed to give m_A
- Partial 1-loop reduction on the $g_3 - y_t - \lambda_i$ space, g_3 is the **primary** coupling
- g_2 , g_1 *switched off*, will be added as corrections to the reduction process

$$\beta_3 \equiv \mathcal{D}g_3 = -7g_3^3, \quad \beta_2 \equiv \mathcal{D}g_2 = -3g_2^3, \quad \beta_1 \equiv \mathcal{D}g_1 = 7g_1^3$$

$$\beta_t = \beta_{t_0} + \beta_{t_c} = \left(\frac{9}{2}y_t^2 - 8g_3^2\right)y_t + \left(-\frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right)y_t$$

$$\beta_{\lambda_i} = \beta_{\lambda_{i0}} + \beta_{\lambda_{ic}}$$

$$\beta_{\lambda_1} = 12\lambda_1^2 + 4\lambda_3^3 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_6^2 + \frac{3}{4}(3g_2^4 + g_1^4 + 2g_2^2g_1^2) - 3\lambda_1(3g_2^2 + g_1^2)$$

$$\beta_{\lambda_2} = 12\lambda_2^2 + 4\lambda_3^3 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2 + 24\lambda_7^2 + 12\lambda_2\lambda_7^2 - 12\lambda_7^4 + \frac{3}{4}(3g_2^4 + g_1^4 + 2g_2^2g_1^2) - \dots$$

$$\beta_{\lambda_3} = \dots$$

where $\mathcal{D} = 16\pi^2\mu(d/d\mu)$, $\lambda_t = y_t \sin\beta$ and y_b , y_τ are considered negligible

Reducing the parameters w.r.t. g_3 , the power series solutions are:

$$y_t = p_t g_3 \quad , \quad \lambda_i = p_i g_3^2$$

Substituting the solutions into the REs:

$$\beta_3 \frac{dy_t}{dg_3} = \beta_{t_0} \quad , \quad \beta_3 \frac{d\lambda_i}{dg_3} = \beta_{\lambda_{i0}}$$

we get sets of p_i, p_t that depend on $\sin \beta$ and are RGI.

$$\rightarrow m_t \sim \sin \beta \text{ 105 GeV}$$

Switching on g_2 and g_1 , we have solutions of the form:

$$y_t = p_t g_3 + q_t g_2 + r_t g_1 \quad , \quad \lambda_i = p_i g_3^2 + q_i g_2^2 + r_i g_1^2$$

where p_t, p_i are known from the above procedure and now the full REs will be

$$\beta_3 \frac{dy_t}{dg_3} = \beta_t \quad , \quad \beta_3 \frac{d\lambda_i}{dg_3} = \beta_{\lambda_i}$$

Solving them requires the conditions

$$D(q_a g_2) \sim 0 \quad , \quad D(r_a g_1) \sim 0 \quad , \quad a = t, 1, \dots, 7$$

They hold for $\mu \geq 10^7 \text{ GeV}$ and are not RGI \rightarrow X

Realistic Approach - Reduction at a Boundary Scale

Main Idea:

- Solve the REs at a **specific scale** M_{bdry}
- Above M_{bdry} a **covering theory** is assumed that makes the solutions RGI
- Use solutions as **BCs** to run the usual 2HDM RGEs from M_{bdry} to M_{EW}

About the boundary scale:

- Solutions wrt g_2, g_1 demand $M_{bdry} \geq 10^7 \text{ GeV}$
- g_2, g_1 are treated as corrections, should not be comparable to g_3 at the boundary scale. This demands $M_{bdry} < 10^8 \text{ GeV}$

$$\rightarrow M_{bdry} \sim 10^7 \text{ GeV}$$

This is the scale that **New Physics** appear.

We can now reduce our system using only the following **input**:

$$g_i(M_{EW}) \quad , \quad m_A$$

First we focus on the $g_3 - y_t$ reduction, as it can be performed independently from the λ_i 's and we switch off $g_{1,2}$. We find

$$y_t = 0.471 g_3$$

Now, switching on the two remaining gauge couplings, we solve the full REs at M_{bdry} , using their respective values $g_{1,2}(M_{bdry})$:

$$y_t = 0.471 g_3 - 0.119 g_2 + 1.228 g_1$$

Choosing the appropriate value for $\tan \beta$:

$$\tan \beta = 2.2 \pm 0.5$$

we obtain a pole top mass that satisfies the most recent experimental limits:

$$m_t = (172.69 \pm 0.30) \text{ GeV}$$



allowing for a 1 GeV theoretical uncertainty due to higher order contributions and the absence of y_b, y_τ

Now we can repeat the procedure for the full system $g_3 - y_t - \lambda_i$.

We choose $m_{11}^2, m_{22}^2, m_{12}^2$ values appropriately, in order for the CP odd Higgs scalar mass to be:

$$m_A = 800 \text{ GeV}$$

Out of all the possible reduction solutions, we look for those that satisfy the light Higgs mass experimental limits:

$$m_h^{\text{exp}} = (125.25 \pm 0.17) \text{ GeV}$$

where we estimate our theoretical calculations to have a 5 GeV uncertainty due to threshold corrections and higher order contributions

There are 4 viable sets of solutions



$$m_h^{(1)} = 127.28 \text{ GeV} \quad , \quad m_h^{(2)} = 120.87 \text{ GeV}$$

$$m_h^{(3)} = 121.20 \text{ GeV} \quad , \quad m_h^{(4)} = 122.81 \text{ GeV}$$

#	p_t	p_1	p_2	p_3	p_4	p_5	p_6	p_7
SET1	0.471	-1.377	-1.167	0	0	0	0	0
SET2	0.471	-1.377	-1.167	0	0	0	0	0
SET3	0.471	-1.109	-0.773	-0.955	0	0	0	0
SET4	0.471	-1.109	-0.773	-0.955	0	0	0	0
#	q_t	q_1	q_2	q_3	q_4	q_5	q_6	q_7
SET1	-0.119	4.606	4.198	-0.087	-0.060	-0.060	0	0
SET2	-0.119	4.598	4.189	0.124	-0.595	0	0	0
SET3	-0.119	3.652	2.819	3.317	0	0	0	0
SET4	-0.119	3.652	2.819	3.317	0	0	0	0
#	r_t	r_1	r_2	r_3	r_4	r_5	r_6	r_7
SET1	1.228	10.022	0.498	-1.033	-3.275	0	0	0
SET2	1.228	10.197	0.240	-0.415	-1.490	-1.490	0	0
SET3	1.228	7.929	1.245	5.518	-9.425	0	0	0
SET4	1.228	-3.196	0.312	8.017	-8.394	0	0	0

Conclusions & Outlook

- Partial RoC on the **Type-II 2HDM** at a **boundary scale** M_{bdry}
- **Input** parameters: $g_i(M_{EW}), m_A$
- **Top quark mass** obtained within experimental limits
- Light **Higgs boson mass** obtained within experimental limits
- Prediction for **$\tan \beta \sim 2.2$**
- Predicted scale of **New Physics**: $M_{bdry} \sim 10^7 \text{ GeV}$

Next:

- **2-loop** analysis
 - 2HDM with **complex** parameters
 - realistic description of **spontaneous CP violation** with minimal input
 - natural selection of one of the six **symmetries** of the Higgs potential
 - apply on **3HDM** - more predictive reduction