Spontaneous CP violation and μ-τ symmetry in two-Higgs-doublet models with flavour conservation

Miguel Nebot

U. of Valencia – IFIC

Outline

1. Setup, notation
2. Motivation and the "CP conserving argument"
3. Counterexample and general aspects
4. Lepton sector examples with μ–τ symmetric PMNS
   - Dirac neutrinos
   - Type I seesaw Majorana neutrinos
   - Phenomenological considerations

Based on:
J.A. Alves, F.J. Botella, C. Miró & MN

☞ arXiv:2306.14952, EPJC83 (2023)

Setup, notation

[See plenary talks by F.J. Botella and H. Haber]

- In 2HDMs the Yukawa sector is

$$\mathscr{L}_{\rm Y} = -\overline{Q}^0_L \left(\Phi_1 Y_1^{(\rm d)} + \Phi_2 Y_2^{(\rm d)}\right) d^0_R - \overline{Q}^0_L \left(\tilde{\Phi}_1 Y_1^{(\rm u)} + \tilde{\Phi}_2 Y_2^{(\rm u)}\right) u^0_R - \overline{L}^0_L \left(\Phi_1 Y_1^{(\ell)} + \Phi_2 Y_2^{(\ell)}\right) \ell^0_R + \text{H.c.}$$

N.B. $\tilde{\Phi}_j = i\sigma_2 \Phi_j^*$ (neutrinos later)

- Going to the Higgs and fermion mass bases

$$\mathscr{L}_{\rm Y} = -\frac{\sqrt{2}}{v}\bar{Q}_L \left(H_1 \text{M}_d + H_2 \text{N}_d\right) d_R - \frac{\sqrt{2}}{v}\bar{Q}_L \left(\tilde{H}_1 \text{M}_u + \tilde{H}_2 \text{N}_u\right) u_R - \frac{\sqrt{2}}{v}\bar{L}_L \left(H_1 \text{M}_\ell + H_2 \text{N}_\ell\right) \ell_R + \text{H.c.}$$

where

- M_f are the diagonal fermion mass matrices
- N_f are the new flavour structures

Setup, notation

Higgs basis

- Expansion around vacuum appropriate for electroweak symmetry breaking

$$\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ \frac{v_j + \rho_j + i\eta_j}{\sqrt{2}} \end{pmatrix}, \quad \langle \Phi_j \rangle = \frac{e^{i\theta_j} v_j}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Higgs basis, $c_\beta \equiv \cos\beta = \frac{v_1}{v}$, $s_\beta \equiv \sin\beta = \frac{v_2}{v}$, $t_\beta \equiv \tan\beta$, $\theta = \theta_2 - \theta_1$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \mathcal{R}_\beta \begin{pmatrix} e^{-i\theta_1}\Phi_1 \\ e^{-i\theta_2}\Phi_2 \end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \; \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad v^2 = v_1^2 + v_2^2 = \frac{1}{\sqrt{2} G_F}$$

Setup, notation

- Higgs basis

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+H^0+iG^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \mathrm{H}^+ \\ \frac{R^0+iI^0}{\sqrt{2}} \end{pmatrix}$$

  - would-be Goldstone bosons G^0, $G^\pm$
  - physical charged scalar $H^\pm$
  - neutral scalars $\{H^0, R^0, I^0\}$, not the mass eigenstates

Setup, notation (quarks)

- Mass matrices M_f^0

$$\mathrm{M}_u^0 = \frac{ve^{-i\theta_1}}{\sqrt{2}}(c_\beta Y_1^{(\mathrm{u})} + e^{-i\theta} s_\beta Y_2^{(\mathrm{u})}), \quad \mathrm{M}_d^0 = \frac{ve^{i\theta_1}}{\sqrt{2}}(c_\beta Y_1^{(\mathrm{d})} + e^{i\theta} s_\beta Y_2^{(\mathrm{d})})$$

- N_f^0 matrices

$$\mathrm{N}_u^0 = \frac{ve^{-i\theta_1}}{\sqrt{2}}(-s_\beta Y_1^{(\mathrm{u})} + e^{-i\theta} c_\beta Y_2^{(\mathrm{u})}), \quad \mathrm{N}_d^0 = \frac{ve^{i\theta_1}}{\sqrt{2}}(-s_\beta Y_1^{(\mathrm{d})} + e^{i\theta} c_\beta Y_2^{(\mathrm{d})})$$

- Diagonalization of mass matrices

$$\mathcal{U}_{f_L}^\dagger\, \mathrm{M}_f^0 \mathrm{M}_f^{0\dagger} \mathcal{U}_{f_L} = \mathrm{diag}(m_{f_1}^2, m_{f_2}^2, m_{f_3}^2)$$
$$\mathcal{U}_{f_R}^\dagger\, \mathrm{M}_f^{0\dagger} \mathrm{M}_f^0 \mathcal{U}_{f_R} = \mathrm{diag}(m_{f_1}^2, m_{f_2}^2, m_{f_3}^2)$$
$$\mathrm{M}_f = \mathcal{U}_{f_L}^\dagger\, \mathrm{M}_f^0 \mathcal{U}_{f_R} = \mathrm{diag}(m_{f_1}, m_{f_2}, m_{f_3})$$
$$\mathrm{N}_f = \mathcal{U}_{f_L}^\dagger\, \mathrm{N}_f^0 \mathcal{U}_{f_R} = ?$$

- Mixing matrix (CKM): $V = \mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L}$

Setup, notation

We are interested in:

- CP invariant lagrangian
  - scalar potential with real coefficients
  - real Yukawa matrices
- Spontaneous CP violation $\theta \neq 0$, source of CP violating CKM
- Flavour conservation, i.e. $\mathrm{N}_f = \mathrm{diag}(n_{f_1}, n_{f_2}, n_{f_3})$?

Setup, notation

- A model with
  - CP invariant lagrangian,
  - spontaneous CP violation sourcing all CP violation,
    including a realistic CP violating CKM matrix,
  - controlled SFCNC,

MN, F.J. Botella & G.C. Branco

☞ arXiv:1808.00493, EPJC79 (2019)

If SFCNC absent, CKM is not CP violating

Motivation

The "CP conserving mixing" argument

Flavour conservation means that the matrices M_f^0 *and* N_f^0, $f = u, d$, *are simultaneously bidiagonalized. This is equivalent to* $Y_1^{(\mathrm{f})}$ *and* $Y_2^{(\mathrm{f})}$ *being bidiagonalized simultaneously.* $Y_1^{(\mathrm{f})}$ *and* $Y_2^{(\mathrm{f})}$ *are real, and thus the bidiagonalization is achieved with real orthogonal matrices,* $O_{f_L}^T Y_j^{(\mathrm{f})} O_{f_R} = \mathrm{diag}(y_{j1}^{(\mathrm{f})}, y_{j2}^{(\mathrm{f})}, y_{j3}^{(\mathrm{f})})$, $y_{jk}^{(\mathrm{f})} \in \mathbb{R}$, *implying that* $\mathrm{M}_f = O_{f_L}^T \mathrm{M}_f^0 O_{f_R}$ *and* $\mathrm{N}_f = O_{f_L}^T \mathrm{N}_f^0 O_{f_R}$ *are diagonal. Then, the CKM matrix is* $V = R_U\, O_{u_L}^T O_{d_L}\, R_D$ *with* R_U, R_D *diagonal rephasing matrices, which can be absorbed in a redefinition of the fields: the CKM matrix is thus essentially real, not CP violating.*

G.C. Branco ☞ PRL44 (1980)

Motivation

The "CP conserving mixing" argument is convincing **but** it has a loophole: even if $Y_1^{(\mathrm{q})}$ and $Y_2^{(\mathrm{q})}$ are real,

- they can have complex eigenvalues and in that case they are not necessarily bidiagonalised simultaneously with real orthogonal matrices

Counterexample

G. Ecker, W. Grimus & H. Neufeld, ☞ PLB194 (1987)

Complex conjugate eigenvalues and mixing moduli relations

M. Gronau, A. Kfir, G. Ecker, W. Grimus & H. Neufeld, ☞ PRD37 (1988)

Counterexample

- Model with 2HDM and 4 generations "out of the blue"
  G. Ecker, W. Grimus & H. Neufeld, ☞ PLB194 (1987)
  Yukawa matrices

$$Y_j^{(\mathrm{d})} = \mathrm{diag}(y_{j1}^{(\mathrm{d})}, y_{j2}^{(\mathrm{d})}, y_{j3}^{(\mathrm{d})}, y_{j4}^{(\mathrm{d})}), \quad Y_j^{(\mathrm{u})} = O^T \begin{pmatrix} y_{j1}^{(\mathrm{u})} & 0 & 0 & 0 \\ 0 & y_{j2}^{(\mathrm{u})} & 0 & 0 \\ 0 & 0 & a_j & b_j \\ 0 & 0 & -b_j & a_j \end{pmatrix}$$

with real $Y_j^{(\mathrm{d})}$, $Y_j^{(\mathrm{u})}$ and O orthogonal

- Crucial ingredient: the blocks

$$B_j = \begin{pmatrix} a_j & b_j \\ -b_j & a_j \end{pmatrix}$$

N.B. $b_1/a_1 \neq b_2/a_2$, otherwise degenerate mass eigenstates

Counterexample

Special blocks

- they obey (no sum over j)

$$B_j B_j^T = B_j^T B_j = (a_j^2 + b_j^2)\mathbf{1}_2$$

- B_j has two *complex conjugate* eigenvalues $a_j \pm i b_j$ while $B_j B_j^T$ has two *degenerate* eigenvalues $a_j^2 + b_j^2$
- The simultaneous real orthogonal bidiagonalization of both $Y_1^{(\mathrm{u})}$ and $Y_2^{(\mathrm{u})}$ *fails*
- However

$$U^\dagger B_j U = \begin{pmatrix} a_j + i b_j & 0 \\ 0 & a_j - i b_j \end{pmatrix}, \quad \text{with} \quad U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

Counterexample

- $Y_1^{(\mathrm{u})}$ and $Y_2^{(\mathrm{u})}$ are simultaneously diagonalized *unitarily*

$$U_{u_L}^\dagger Y_j^{(\mathrm{u})} U_{u_R} = \mathrm{diag}(y_{1j}^{(\mathrm{u})}, y_{2j}^{(\mathrm{u})}, a_j + ib_j, a_j - ib_j)$$

$$U_{u_L} = O^T U_{[34]}\,,\ U_{u_R} = U_{[34]}, \quad U_{[34]} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

- The resulting CKM matrix, up to rephasings, is

$$V = U_{34}^\dagger O$$

- It follows that $V_{3j} = V_{4j}^* = (O_{3j} - iO_{4j})/\sqrt{2}$, i.e. the rephasing invariant relation

$$|V_{3j}| = |V_{4j}|, \quad j = 1, 2, 3, 4$$

Counterexample

No real orthogonal simultaneous bidiagonalisation

- Notice that

$$B_j = \begin{pmatrix} a_j & b_j \\ -b_j & a_j \end{pmatrix} = \lambda_j \begin{pmatrix} \cos\alpha_j & \sin\alpha_j \\ -\sin\alpha_j & \cos\alpha_j \end{pmatrix}$$

with $\lambda_j = \sqrt{a_j^2 + b_j^2}$, $\frac{a_j}{\lambda_j} = \cos\alpha_j$, $\frac{b_j}{\lambda_j} = \sin\alpha_j$

B_1 and B_2 not proportional $\Leftrightarrow$ $\alpha_2 \neq \alpha_1\,[\pi]$

- $SO(2,\mathbb{R})$

$$O_2(\alpha) \equiv \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$[O_2(\alpha)]^{-1} = [O_2(\alpha)]^T = O_2(-\alpha), \quad O_2(\alpha_a)O_2(\alpha_b) = O_2(\alpha_a + \alpha_b)$$

- Orthogonal bidiagonalisation

$$O_2(-\alpha_L) B_j B_j^T O_2(\alpha_L) = O_2(-\alpha_R) B_j B_j^T O_2(\alpha_R) = \lambda_j^2 \mathbf{1}_2$$

it looks like we have full freedom to choose α_L and α_R

Counterexample

No real orthogonal simultaneous bidiagonalisation

- It looks like we have full freedom to choose α_L and α_R ... but

$$O_2(-\alpha_L)\, B_1\, O_2(\alpha_R) = \lambda_1\, O_2(-\alpha_L + \alpha_1 + \alpha_R)$$

$$O_2(-\alpha_L)\, B_2\, O_2(\alpha_R) = \lambda_2\, O_2(-\alpha_L + \alpha_2 + \alpha_R)$$

B_1 bidiagonalised iff $-\alpha_L + \alpha_1 + \alpha_R = 0[\pi]$,
but then $-\alpha_L + \alpha_2 + \alpha_R \neq 0[\pi]$ i.e. B_2 not bidiagonalised

General analysis

Important points in

$$U^\dagger \begin{pmatrix} \cos\alpha_j & \sin\alpha_j \\ -\sin\alpha_j & \cos\alpha_j \end{pmatrix} U = \begin{pmatrix} e^{i\alpha_j} & 0 \\ 0 & e^{-i\alpha_j} \end{pmatrix}, \quad \text{with} \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$$

- eigenvectors $\vec{v}_+ = \frac{1}{\sqrt{2}} \left(\begin{smallmatrix} 1 \\ i \end{smallmatrix} \right)$, $\vec{v}_- = (\vec{v}_+)^*$, corresponding to eigenvalues $e^{\pm i\alpha}$, are orthonormal
  $\Rightarrow$ unitary diagonalisation exists
- eigenvectors "independent" of the eigenvalues
  (unique diagonalisation for all $SO(2,\mathbb{R})$ matrices)

General analysis

With three generations in mind, notice that $O(3,\mathbb{R})$ matrices

- have two complex conjugate eigenvalues $e^{\pm i\alpha}$ and one real eigenvalue ± 1
- orthonormal eigenvectors ($\Rightarrow$ unitary diagonalisation exists), two are complex conjugate of each other
- the eigenvectors "do not depend" on the eigenvalues
  $\Rightarrow$ different $O(3,\mathbb{R})$ matrices can be diagonalised simultaneously

Model with Dirac neutrinos

- Yukawa lagrangian

$$-\mathscr{L}_{\mathrm{Y}\ell} = \overline{L}_L^0 \left(Y_1^{(\nu)} \tilde{\Phi}_1 + Y_2^{(\nu)} \tilde{\Phi}_2 \right) \nu_R^0 + \overline{L}_L^0 \left(Y_1^{(\ell)} \Phi_1 + Y_2^{(\ell)} \Phi_2 \right) \ell_R^0 + \mathrm{H.c.}$$

with

$$Y_j^{(\nu)} = \mathrm{diag}(y_{j1}^{(\nu)}, y_{j2}^{(\nu)}, y_{j3}^{(\nu)})$$

$$Y_j^{(\ell)} = O^T \lambda_j \begin{pmatrix} y_{j1}^{(\ell)}/\lambda_j & 0 & 0 \\ 0 & \cos\varphi_j & \sin\varphi_j \\ 0 & -\sin\varphi_j & \cos\varphi_j \end{pmatrix}$$

$$\mathrm{M}_\nu^0 = \frac{ve^{-i\theta_1}}{\sqrt{2}} \left(c_\beta Y_1^{(\nu)} + s_\beta e^{-i\theta} Y_1^{(\nu)} \right), \quad \mathrm{M}_\ell^0 = \frac{ve^{i\theta_1}}{\sqrt{2}} \left(c_\beta Y_1^{(\ell)} + s_\beta e^{i\theta} Y_2^{(\ell)} \right)$$

$$\mathrm{N}_\nu^0 = \frac{ve^{-i\theta_1}}{\sqrt{2}} \left(-s_\beta Y_1^{(\nu)} + c_\beta e^{-i\theta} Y_2^{(\nu)} \right), \quad \mathrm{N}_\ell^0 = \frac{ve^{i\theta_1}}{\sqrt{2}} \left(-s_\beta Y_1^{(\ell)} + c_\beta e^{i\theta} Y_2^{(\ell)} \right)$$

Model with Dirac neutrinos

- Simultaneous diagonalisation of $\{Y_1^{(\ell)}, Y_2^{(\ell)}\} \Leftrightarrow \{\mathrm{M}_\ell^0, \mathrm{N}_\ell^0\}$

$$U_{\ell_L}^\dagger \, \mathrm{M}_\ell^0 \, U_{\ell_R} = \mathrm{M}_\ell = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$U_{\ell_L} = O^T \, U_{[23]} \qquad U_{\ell_R} = U_{[23]} R_{\ell_R} \qquad U_{[23]} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{pmatrix}$$

(N.B. rephasings R_{ℓ_R})

- M_ν^0 and N_ν^0 diagonal
- PMNS matrix (up to rephasings)

$$U = U_{23}^\dagger O$$

and then

$$U_{2j} = U_{3j}^* = \frac{1}{\sqrt{2}}(O_{2j} - iO_{3j}) \;\Rightarrow\; |U_{2j}| = |U_{3j}|, \; j = 1, 2, 3$$

Model with Dirac neutrinos

- With

$$|U_{2j}| = |U_{3j}|,\ j = 1, 2, 3$$

the PMNS matrix has "$\mu - \tau$ symmetry"
(in standard PDG parameterisation $\theta_{23} = \pi/4$, $\delta = \pm\pi/2$)

- Freedom left in O_{1j} is sufficient to have a realistic PMNS

Model with Dirac neutrinos

Comments

- CP violation in PMNS is in the end independent of the value of the vacuum SCPV phase (!)
- This choice (generations 2 and 3 of charged leptons) is the only viable implementation of the idea, no other rows or columns could work
- $m_\mu \neq m_\tau \leftrightarrow c_\beta s_\beta \sin\theta\, \lambda_1 \lambda_2 \sin(\varphi_2 - \varphi_1) \neq 0$

$$\frac{2}{v^2} m_\mu^2 = c_\beta^2 \lambda_1 + s_\beta^2 \lambda_2 + 2 c_\beta s_\beta \lambda_1 \lambda_2 \cos(\theta + \varphi_2 - \varphi_1)$$

$$\frac{2}{v^2} m_\tau^2 = c_\beta^2 \lambda_1 + s_\beta^2 \lambda_2 + 2 c_\beta s_\beta \lambda_1 \lambda_2 \cos(\theta - \varphi_2 + \varphi_1)$$

- Diagonalisation of the neutrino mass matrix has had almost no role ($\Rightarrow$ easy to do Majorana neutrinos with type I seesaw)
- Not symmetry based

  (simplest scenario not stable under one loop RGE)

Seesaw I model with Majorana neutrinos

- Add 3 ν_R with

$$\mathscr{L}_{\nu,\text{Maj}} = -\frac{1}{2}\left[\overline{(\nu_R^0)^c}\, M_R\, \nu_R^0 + \overline{\nu_R^0}\, M_R\, (\nu_R^0)^c\right]$$

with $M_R = \text{diag}(M_{R1}, M_{R2}, M_{R3})$, $M_{Rj} \in \mathbb{R}$

- Neutrino mass terms

$$\mathscr{L}_{\nu,\text{Mass}} = -\frac{1}{2}\begin{pmatrix}\overline{(\nu_L^0)^c} & \overline{\nu_R^0}\end{pmatrix} \mathscr{M} \begin{pmatrix}\nu_L^0 \\ (\nu_R^0)^c\end{pmatrix} + \text{H.c.}, \quad \mathscr{M} = \begin{pmatrix} 0 & \text{M}_\nu^{0*} \\ \text{M}_\nu^{0\dagger} & M_R\end{pmatrix}$$

- Diagonalization of $\mathscr{M}$ reduced to 3 textbook 2×2 seesaws

$$\begin{pmatrix} 0 & \mu_j \\ \mu_j & M_{Rj}\end{pmatrix}, \quad \mu_j = \frac{v e^{i\theta_1}}{\sqrt{2}}(c_\beta y_{1j}^{(\nu)} + e^{i\theta} s_\beta y_{2j}^{(\nu)})$$

$\mu_j \in \mathbb{C}$, $|\mu_j| \ll M_{Rj}$

Seesaw I model with Majorana neutrinos

That is

$$\mathcal{U}^T \mathscr{M} \mathcal{U} = \begin{pmatrix} m_{\text{light}} & 0 \\ 0 & m_{\text{heavy}} \end{pmatrix}, \quad \mathcal{U} = \begin{pmatrix} C & S \\ -S^* & C \end{pmatrix} \begin{pmatrix} R_\nu & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \text{diag}(\cos\alpha_1, \cos\alpha_2, \cos\alpha_3)$$

$$S = \text{diag}(\sin\alpha_1, \sin\alpha_2, \sin\alpha_3)$$

$$R_\nu = i\,\text{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

$$\tan 2\alpha_j = 2\frac{|\mu_j|}{M_{Rj}} \ll 1, \quad \beta_j = -\arg(\mu_j)$$

$$[m_{\text{light}}]_{jk} = \delta_{jk}\,|\mu_j|\,\tan\alpha_j \simeq \delta_{jk}\frac{|\mu_j|^2}{M_{Rj}}$$

$$[m_{\text{heavy}}]_{jk} = \delta_{jk}\,M_{Rj}\,\frac{\tan 2\alpha_j}{2\tan\alpha_j} \simeq \delta_{jk}\,M_{Rj}$$

Seesaw I model with Majorana neutrinos

- Resulting 3×6 PMNS matrix

$$U = \mathcal{U}_{[23]}^{\dagger} \, O \, \begin{pmatrix} C R_\nu & S \end{pmatrix}$$

- As in the Dirac scenario
  - no SFCNC for charged leptons and light neutrinos
  - PMNS with μ-τ symmetry
- light-heavy neutrino SFCNCs

Phenomenological concerns

- Benefit from phenomenological studies in $(g-2)_{e,\mu}$-oriented work
  F.J. Botella, F. Cornet-Gómez, C. Miró & MN

  ☞ arXiv:2302.05471, JPhysG51 (2024)
  ☞ arXiv:2205.01115, EPJC82 (2022)
  ☞ arXiv:2006.01934, PRD102 (2020)

  No problem with experimental constraints

  EW precision, Higgs signal strengths, universality constraints in semileptonic decays with π's and K's, $b \to s\gamma$, meson mixings, ...
- Special attention to the electron EDM: one loop contributions OK, two loop Barr-Zee not automatically safe

Conclusions

- In 2HDMs (or nHDM) one can reconcile flavour conservation and a spontaneous origin of CP violation, with CP violating fermion mixings
- Caveat: OK for leptons but not OK for quarks
- Essentially only one implementation is phenomenologically viable (simple illustration with Dirac neutrinos and type I seesaw Majorana neutrinos)
- This implementation implies a PMNS with μ–τ symmetry

Muito obrigado!

Thank you!

Backup

- $O(3,\mathbb{R})$ matrices
- μ–τ symmetry with $O(3,\mathbb{R})$ matrices

$O(3,\mathbb{R})$ matrices

- $O \in SO(3,\mathbb{R})$ are of the form $O = \exp(\alpha A)$ with real $\alpha \in [0; 2\pi[$ and A a normalized antisymmetric matrix

$$A = \begin{pmatrix} 0 & \hat{n}_3 & -\hat{n}_2 \\ -\hat{n}_3 & 0 & \hat{n}_1 \\ \hat{n}_2 & -\hat{n}_1 & 0 \end{pmatrix}, \quad \hat{n}_j \in \mathbb{R}, \quad \hat{n}_1^2 + \hat{n}_2^2 + \hat{n}_3^2 = 1$$

- A has eigenvalues $\lambda = 0, \pm i$; the normalized eigenvectors are

$$\lambda = 0,\ \vec{v}_0^T = (\hat{n}_1, \hat{n}_2, \hat{n}_3) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),$$

$$\lambda = i,\ \vec{v}_+^T = \frac{1}{\sqrt{2}}(-\cos\theta\cos\varphi + i\sin\varphi, -\cos\theta\sin\varphi - i\cos\varphi, \sin\theta),$$

$$\lambda = -i,\ \vec{v}_- = (\vec{v}_+)^*,$$

$O(3,\mathbb{R})$ matrices

- Diagonalisation

$$U^\dagger A U = \mathrm{diag}(0, i, -i), \qquad U = \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{v}_0 & \vec{v}_+ & \vec{v}_- \\ \downarrow & \downarrow & \downarrow \end{pmatrix}$$

- $O = \exp(\alpha A)$ has eigenvalues $\{1, e^{i\alpha}, e^{-i\alpha}\}$ and the same eigenvectors

$$U^\dagger O U = \mathrm{diag}(1, e^{i\alpha}, e^{-i\alpha})$$

- Geometrically O represents a rotation in $\mathbb{R}^3$ of angle α around the axis $(\hat{n}_1, \hat{n}_2, \hat{n}_3)$
- The eigenvectors of O do not depend on $\alpha \Rightarrow O_1 = \exp(\alpha_1 A)$ and $O_2 = \exp(\alpha_2 A)$ with $\alpha_1 \neq \alpha_2$ can be unitarily diagonalised simultaneously.

μ–τ symmetry with $O(3,\mathbb{R})$ matrices

PMNS matrix $U =$ unitary$^\dagger\times$orthogonal with $\vec{r}_k$ real

$$\begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{r}_0 & \vec{v}_+ & (\vec{v}_+)^* \\ \downarrow & \downarrow & \downarrow \end{pmatrix}^\dagger \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ \vec{r}_1 & \vec{r}_2 & \vec{r}_3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} = \begin{pmatrix} \vec{r}_0\cdot\vec{r}_1 & \vec{r}_0\cdot\vec{r}_2 & \vec{r}_0\cdot\vec{r}_3 \\ (\vec{v}_+)^*\cdot\vec{r}_1 & (\vec{v}_+)^*\cdot\vec{r}_2 & (\vec{v}_+)^*\cdot\vec{r}_3 \\ \vec{v}_+\cdot\vec{r}_1 & \vec{v}_+\cdot\vec{r}_2 & \vec{v}_+\cdot\vec{r}_3 \end{pmatrix}$$

and $U_{2j} = (U_{3j})^* = (\vec{v}_+)^*\cdot\vec{r}_j$, $j = 1,2,3$, that is, again, μ–τ symmetry:

$$|U_{2j}| = |U_{3j}|,\ \ j = 1,2,3$$