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A Minimal Solution to the Strong CP Problem

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Based on a work in preparation with L. J. Hall and B. Noether

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Introduction

What is the Strong CP Problem?

The QCD Lagrangian with N flavors has a large global symmetry in the limit of vanishing quark masses.

$$U(N)_V \times U(N)_A$$

Accounting for quark masses

$$m_u, m_d \ll \Lambda_{\text{QCD}} \implies U(2)_V \times U(2)_A \quad \text{expected}$$

EXPERIMENTALLY

$$U(2)_V = SU(2)_I \times U(1)_B \quad \text{observed in hadron multiplets}$$

$$U(2)_A \quad \text{broken by quark condensate} \quad \langle \bar{q} q \rangle$$

We expect four Nambu-Goldstone bosons associated with the breaking of $U(2)_A$

HOWEVER EXPERIMENTALLY WE OBSERVE ONLY THREE OF THEM

$$m_{\pi^+} \sim m_{\pi^-} \sim m_{\pi^0} \sim 0 \qquad m_\eta \gg m_{\pi^+} \; !$$

$U(1)_A$ PROBLEM

Phys.Rev.D11,3583(1975)3

SOLUTION: $U(1)_A$ is anomalous with QCD

$$\partial_\mu J_5^\mu = \frac{g_s^2 N}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \quad \text{with} \quad J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi$$

$$q \to e^{i\alpha\frac{\gamma_5}{2}} q \implies \delta S = \alpha \frac{g_s^2 N}{32\pi^2} \int d^4x G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

't Hooft showed that there are gauge configuration for which $\delta S \neq 0 \implies U(1)_A$ Is not a real symmetry of the Lagrangian

The STRONG CP PROBLEM

This result effectively adds to the QCD Lagrangian a new term which is P and T violating (CP violating)

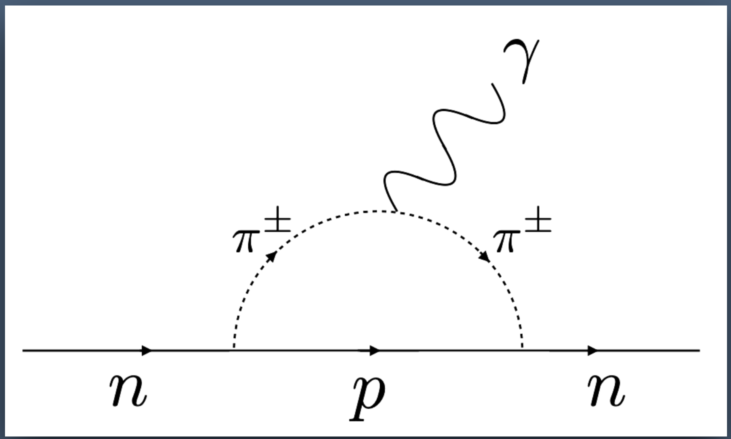
$$\mathscr{L}_{\rm QCD} \supset \theta \, \frac{g_s^2}{32\pi^2} \, G^a_{\mu\nu} \, \tilde{G}^{a\,\mu\nu}$$

θ measures how much is P (CP) violated in the strong interactions

Including weak interactions, θ is shifted by the chiral transformation needed to diagonalize the quark mass matrix M

$$\mathscr{L}_{\rm QCD} \supset \bar{\theta}\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\,\tilde{G}^{a\,\mu\nu} = \Big(\theta + \arg\det(Y_u Y_d)\Big)\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\,\tilde{G}^{a\,\mu\nu}$$

How do we measure $\bar{\theta}$?

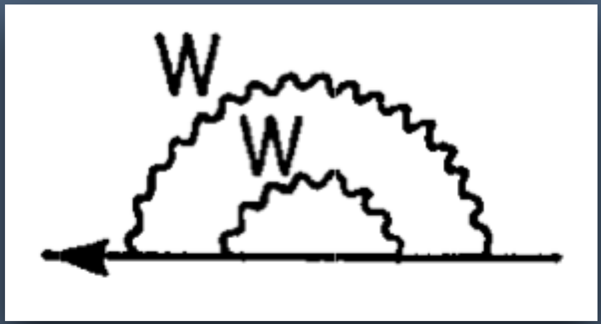


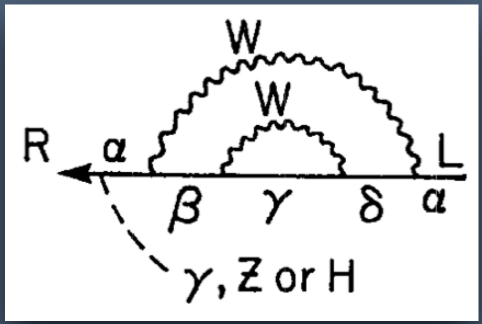
$$d_n = \frac{e\,\bar{\theta}\,m_q}{m_N^2}$$

$$|\,d_n\,| \leq 3 \times 10^{-26}\,\mathrm{e}\cdot\mathrm{cm}^2 \implies \bar{\theta} \lesssim 10^{-10}$$

Radiative Corrections in the SM

Nucl.Phys.B150.1979





$$\sum_j V_{ij} V_{kj}^*$$

These sum to 0

First non-zero finite imaginary contributions.

$$\Delta\bar{\theta} = \left(\frac{\alpha}{\pi}\right)^2 s_1^2 s_2 s_3 \sin\delta \frac{m_s^2 m_c^2}{m_W^4} \sim 10^{-16}$$

CP Solutions

CP: $$\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} \rightarrow -\theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

$$\theta = -\theta = 0$$

$$Y_u, Y_d \in \mathscr{R} \implies \begin{cases} \arg\det(Y_u Y_d) = 0 \\ \delta_{\text{CKM}} = 0 \end{cases}$$

What is the minimal extension of the SM that allows to implement a CP solution to the strong CP problem?

- We want to impose CP in the UV
- We need to break it to get the CKM phase
- We need to ensure that after breaking $\arg\det(Y_u Y_d) = 0$

We found that two ingredients aree needed: an extension of the scalar sector which softly breaks CP and a family symmetry!

N Higgs

$$\mu_{\alpha\beta} \in \mathscr{C} \text{ for } \alpha \neq \beta$$

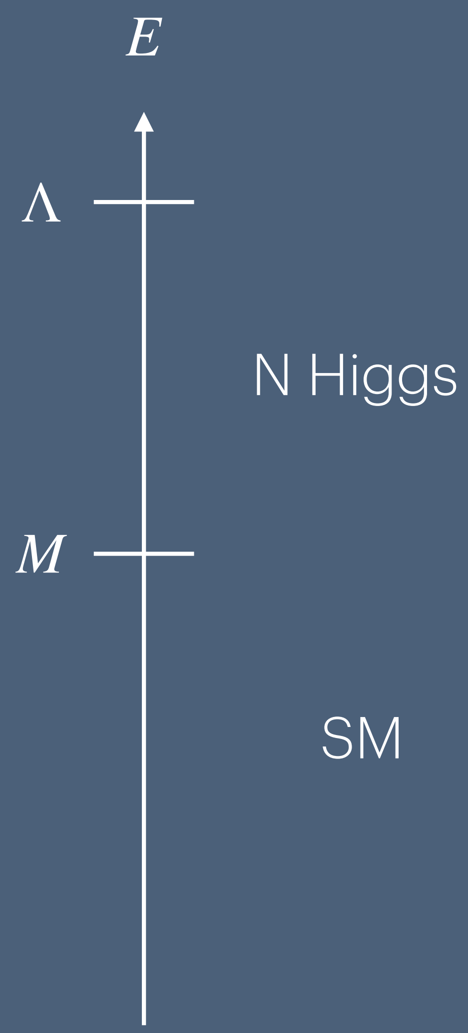
$$V = M_\alpha^2 \phi_\alpha^\dagger \phi_\alpha + \mu_{\alpha\beta}^2 \phi_\alpha^\dagger \phi_\beta + \lambda_{\alpha\beta}\, \phi_\alpha^\dagger \phi_\alpha \phi_\beta^\dagger \phi_\beta + \lambda_{\alpha\beta\gamma\delta}\, \phi_\alpha^\dagger \phi_\beta \phi_\gamma^\dagger \phi_\delta + h\,.\,c\,.$$

In our picture all Higgs but one are heavy. Let rotate to the mass basis, moving all phases in the rotation matrix U, such that

$$\phi_\alpha = U_{\alpha\beta}\, h_\beta \qquad \text{with only} \qquad \langle h_1 \rangle \neq 0$$

N Higgs

EFT picture



N Higgs

$$\mathscr{L}_Y = x_{ij}^{\alpha}\, q_i\, \bar{u}_j\, \phi_\alpha + \tilde{x}_{ij}^{\alpha}\, q_i\, \bar{d}_j\, \phi_\alpha^{\dagger}$$

The quark Yukawa matrices are now complex

$$Y_u = x^{\alpha}\, U_{\alpha 1}\,, \qquad Y_d = \tilde{x}^{\alpha}\, U_{\alpha 1}^{*}$$

It remains to find a family symmetry that ensures

$$\arg\det(Y_u Y_d) = 0$$

$$\delta_{\mathrm{CKM}} \neq 0$$

Examples

N = 2

	ϕ_1	ϕ_2	q_1	q_2	q_3	u_1	u_2	u_3	d_1	d_2	d_3
$\mathbb{Z}_3$	1	α^2	α^2	α	1	α^2	1	1	α^2	α^2	1

$$\mathscr{L}_Y = \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} x_{11}U_{11} & 0 & 0 \\ 0 & x_{22}U_{21} & x_{23}U_{21} \\ x_{31}U_{21} & x_{32}U_{11} & x_{33}U_{11} \end{pmatrix} \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} h_1$$

$$+ \begin{pmatrix} q_1 & q_2 & q_3 \end{pmatrix} \begin{pmatrix} \tilde{x}_{11}U^*_{11} & \tilde{x}_{12}U^*_{11} & \tilde{x}_{13}U^*_{21} \\ \tilde{x}_{21}U^*_{21} & \tilde{x}_{22}U^*_{21} & 0 \\ 0 & 0 & \tilde{x}_{33}U^*_{11} \end{pmatrix} \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} h^\dagger$$

Examples

N = 3

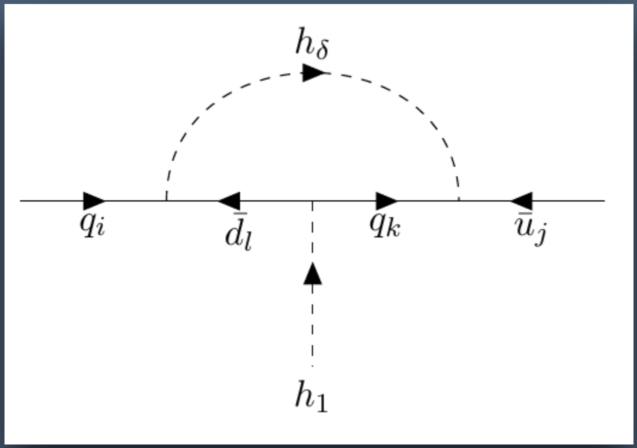
	ϕ_1	ϕ_2	ϕ_3	q_1	q_2	q_3	u_1	u_2	u_3	d_1	d_2	d_3
$U(1)_{\rm F}$	0	2	-1	0	1	-1	-2	1	-1	2	1	-1

$$\mathscr{L}_Y = (q_1 \quad q_2 \quad q_3)\begin{pmatrix} 0 & 0 & x_{13}U_{31} \\ 0 & x_{22}U_{11} & 0 \\ x_{31}U_{31} & x_{32}U_{21} & x_{33}U_{11} \end{pmatrix}\begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{pmatrix} h_1$$

$$+(q_1 \quad q_2 \quad q_3)\begin{pmatrix} 0 & \tilde{x}_{12}U^*_{31} & 0 \\ \tilde{x}_{21}U^*_{31} & \tilde{x}_{22}U^*_{11} & \tilde{x}_{23}U^*_{21} \\ 0 & 0 & \tilde{x}_{33}U^*_{11} \end{pmatrix}\begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \end{pmatrix} h^\dagger$$

Radiative Corrections

$$Y_u = x^\alpha U_{\alpha 1} \quad +$$



Using the unitarity of the U matrix one can cast the result as

$$\delta Y_u = \frac{1}{16\pi^2}\tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\gamma \, U_{\beta 1}\left[U^*_{\alpha 1} U_{\gamma 1} \log\left(\frac{\mu^2}{m_h^2}\right) + (\delta_{\alpha\gamma} - U^*_{\alpha 1} U_{\gamma 1}) \log\left(\frac{\mu^2}{M_2^2}\right)\right.$$

$$\left. + \sum_{\delta=3}^{N} U^*_{\alpha\delta} U_{\gamma\delta} \log\left(\frac{M_2^2}{M_\delta^2}\right)\right] + \frac{1}{16\pi^2}\tilde{x}^\alpha \tilde{x}^{\dagger\beta} x^\gamma \, U_{\beta 1}\left[\frac{1}{\epsilon} + 1\right].$$

Conclusions

- I presented a class of theories where the strong CP problem is solved with:
  - a softly broken CP symmetry
  - an extension of the scalar sector
  - a family symmetry

- For N=2 only SM corrections. For N>2 additional corrections proportional to the log of the ratio of the heavy scalars.

- A few comments:
  - the solution to the strong CP problem is in the scalar sector (possible connections with the electroweak hierarchy problem)
  - the μ parameters can induce a hierarchy in the quark sector (possible connections with the flavor puzzle)
  - potentially interesting connections between CKM and PMNS phase

Backup

Nelson Barr Mechanism

Add vector-like quarks q, and scalars η, which spontaneously break CP with a complex vev.

$$\mathscr{L} = \mu \bar{q} q + \lambda \eta \bar{q} d + Y_d \bar{Q} d H$$

$$M_d = \begin{pmatrix} \mu & B \\ 0 & m_d \end{pmatrix}$$

Not so simple…

- Generally more than 1 η to have CKM phase
- η must not couple to $\bar{Q}d$
- μ < B for sizable CKM phase
- Dangerous higher dimensional operators and radiative corrections

Radiative Corrections

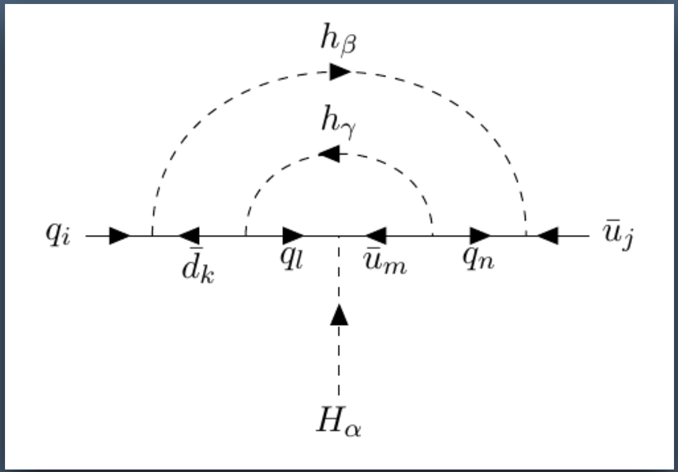
Work in Progress

The argument can be repeated at any order.
For N=2 it means that there are no contributions other than the SM ones.

For instance, at 2-loops*:

$$Y_u = x^\alpha U_{\alpha 1}$$

$$(\delta Y_u)_{2-\text{loops}} = A_1\, \tilde{x}^\beta \tilde{x}^{\dagger\,\gamma} x^\alpha x^{\dagger\gamma} x^\beta\, U_{\alpha 1} + A_2 \log\left(\frac{M_2^2}{m_h^2}\right)$$

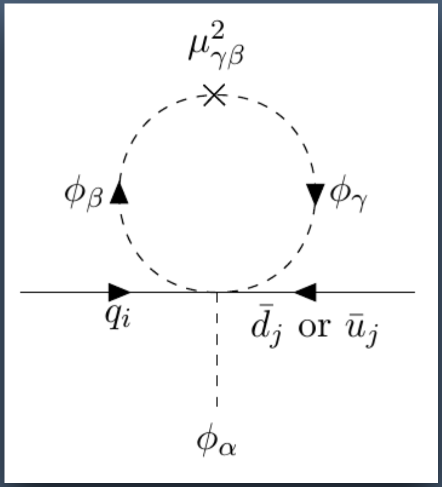


* similar results are obtained for each of the other 6 diagrams at 2 loops.

Higher Dimensional Operators

At dim 6 there are dangerous operators

$$\frac{C_{ijk}}{\Lambda^2} Q\, \bar{u}\, \phi_\alpha \phi_\beta \phi_\gamma^\dagger$$



$$\delta\bar{\theta} = \frac{1}{8\pi^2} \log\left(\frac{\Lambda}{M}\right) \sum_{\beta\gamma} \frac{\mu^2_{\beta\gamma}}{\Lambda^2} f(U_{\beta\gamma})$$