Renormalization group equations of a general effective field theory

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Studying a model at different energy scales

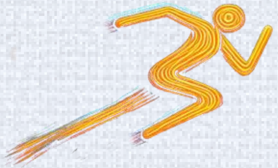
Match and run

Energy

UV theory

Integrate out heavy fields. Match the coefficients of the operators in the two theory up to the desired order

IR theory



Use the renormalization group equations (RGEs) of the lower energy theory to evolve its couplings

Studying a model at different energy scales

The running part

Conveniently, in the MS scheme, the beta functions are given by $-\mathbf{2}$ (loop order) (coefficient of the $1/\varepsilon$ term)

Lagrangian with a list of operators as Wilson coefficients

$$\mathscr{L} = \sum_i c_i \mathcal{O}_i$$

Calculate the 1/epsilon divergences

$$\mathscr{L}^{\text{div}} = \sum_j \frac{1}{16\pi^2\epsilon} c_j'(c)\mathcal{O}_j + \mathcal{O}\left(\epsilon^{-2}\right)$$

These divergences are cancelled by renormalizing the theory's couplings and fields

MS scheme

$$\mathscr{L}^{(0)} = \sum_i c_i^{(0)}\mathcal{O}_i^{(0)} = \mu^{n_i\epsilon} Z_i c_i \mathcal{O}_i \quad \text{with} \quad Z_i c_i = c_i - \frac{c_i'(c)}{16\pi^2\epsilon} + \mathcal{O}\left(\epsilon^{-2}\right)$$

loops (=1 for us)

$$16\pi^2 \beta\left(c_i\right) \equiv 16\pi^2 \mu \frac{\mathrm{d}c_i}{\mathrm{d}\mu} = n_i c_i' - \sum_j n_j c_j \frac{\partial c_i'}{\partial c_j} = -2\ell c_i'$$

The classical results

In principle, for each different model, one needs to calculate loop amplitudes to get the RGEs

However, one can avoid that with the general (2-loop) results derived in a series of papers

They considered a general renormalizable model

Jack, Osborn (1982,1983,1985)
Machacek, Vaughn (1983,1984,1985)
Luo, Wang, Xiao, hep-ph/0211440 (2003)

Martin, Vaughn, hep-ph/9311340 (1994)
Yamada, hep-ph/9401241 (1994)
(SUSY)

$$\mathscr{L}_{d\leq 4} = -\frac{1}{4}F^A_{\mu\nu}F^{B\,\mu\nu} + \frac{1}{2}D_\mu\phi_a D^\mu\phi_b + \bar{\psi}_i \mathrm{i}\not{D}\psi_j - \frac{1}{2}\left[(m_f)_{ij}\psi_i^T C\psi_j + \mathrm{h.c.}\right]$$
$$-\frac{1}{2}(m^2_\phi)_{ab}\phi_a\phi_b - \frac{1}{2}\left[Y_{ija}\psi_i^T C\psi_j\phi_a + \mathrm{h.c.}\right] - \frac{\kappa_{abc}}{3!}\phi_a\phi_b\phi_c - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d$$

Coefficient × Operator

$$D_\mu\psi_i = \partial_\mu\psi_i - \mathrm{i}g t^A_{ij}V^A_\mu\psi_j$$
$$D_\mu\phi_a = \partial_\mu\phi_a - \mathrm{i}g\theta^A_{ab}V^A_\mu\phi_b$$

t^A and θ^A are Hermitian matrices
(θ^A are also anti-symmetric)

In a particular model one must specify the shape of generic tensor coefficients shown here

In practice, this usually involves simply enforcing gauge invariance on these tensor coefficients

The RGEs were given for these tensors

E.g.: in SM one has 45 Weyl fermions and 4 real scalars: the t^A are 45-dim; the θ^A are 4-dim. The Yukawa couplings are given by the most general Y tensor obeying

$$t^A_{ii'}Y_{i'ja} + t^A_{jj'}Y_{i'ja} + \theta^A_{aa'}Y_{ija'} = 0$$

In the SM, Y has 27 complex degrees of freedom

The classical results

Example: 2-loop beta function for the Yukawa coupling

$$
\begin{aligned}
(4\pi)^4 \boldsymbol{\beta}^a\big|_{2\text{-loop}} = {} & 2\boldsymbol{Y}^c\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^a(\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^b - \boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^c) - \boldsymbol{Y}^b[\boldsymbol{Y}_2(\mathrm{F})\boldsymbol{Y}^{\dagger a} + \boldsymbol{Y}^{\dagger a}\boldsymbol{Y}_2^{\dagger}(\mathrm{F})]\boldsymbol{Y}^b \\
& - \tfrac{1}{8}[\boldsymbol{Y}^b\boldsymbol{Y}_2(\mathrm{F})\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^a + \boldsymbol{Y}^a\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}_2^{\dagger}(\mathrm{F})\boldsymbol{Y}^b] \\
& - 4\kappa Y_2^{ac}(\mathrm{S})\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^b - \tfrac{3}{2}\kappa Y_2^{bc}(\mathrm{S})[\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^a + \boldsymbol{Y}^a\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^b] \\
& - \kappa\boldsymbol{Y}^b\,\mathrm{Tr}\,\{\tfrac{3}{2}[\boldsymbol{Y}_2(\mathrm{F})\boldsymbol{Y}^{\dagger b} + \boldsymbol{Y}^{\dagger b}\boldsymbol{Y}_2^{\dagger}(\mathrm{F})]\boldsymbol{Y}^a + 2\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^c\boldsymbol{Y}^{\dagger a}\boldsymbol{Y}^c\} \\
& - 2\lambda_{abcd}\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^d + \tfrac{1}{12}\lambda_{acde}\lambda_{bcde}\boldsymbol{Y}^b + 3g^2\{\boldsymbol{C}_2(\mathrm{F}),\ \boldsymbol{Y}^b\boldsymbol{Y}^{\dagger a}\boldsymbol{Y}^b\} \\
& + 5g^2\boldsymbol{Y}^b\{\boldsymbol{C}_2(\mathrm{F}),\ \boldsymbol{Y}^{\dagger a}\}\boldsymbol{Y}^b - \tfrac{7}{4}g^2[\boldsymbol{C}_2(\mathrm{F})\boldsymbol{Y}_2^{\dagger}(\mathrm{F})\boldsymbol{Y}^a + \boldsymbol{Y}^a\boldsymbol{Y}_2(\mathrm{F})\boldsymbol{C}_2(\mathrm{F})] \\
& - \tfrac{1}{4}g^2[\mathbf{Y}^b\mathbf{C}_2(\mathrm{F})\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^a + \boldsymbol{Y}^a\boldsymbol{Y}^{\dagger b}\boldsymbol{C}_2(\mathrm{F})\boldsymbol{Y}^b] \\
& + 6g^2[\boldsymbol{t}^A\boldsymbol{Y}^a\boldsymbol{Y}^{\dagger b}\boldsymbol{t}^A\boldsymbol{Y}^b + \boldsymbol{Y}^b\boldsymbol{t}^A\boldsymbol{Y}^{\dagger b}\boldsymbol{Y}^a\boldsymbol{t}^A] \\
& + 5\kappa g^2\boldsymbol{Y}^b\,\mathrm{Tr}\,\{\boldsymbol{C}_2(\mathrm{F}),\ \boldsymbol{Y}^a\}\boldsymbol{Y}^{\dagger b} \\
& + 6g^2[C_2^{bc}(\mathrm{S})\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger a}\boldsymbol{Y}^c - 2C_2^{ac}(\mathrm{S})\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^b] \\
& + \tfrac{9}{2}g^2[\boldsymbol{Y}^b\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^a + \boldsymbol{Y}^a\boldsymbol{Y}^{\dagger c}\boldsymbol{Y}^b] - \tfrac{3}{2}g^4\{[C_2(\mathrm{F})]^2,\ \boldsymbol{Y}^a\} \\
& + g^4[6C_2(\mathrm{S}) - \tfrac{97}{6}C_2(\mathrm{G}) + \tfrac{10}{3}\kappa S_2(\mathrm{F}) + \tfrac{11}{12}S_2(\mathrm{S})]\{\boldsymbol{C}_2(\mathrm{F}),\ \boldsymbol{Y}^a\} \\
& - g^4C_2(\mathrm{S})[\tfrac{21}{2}C_2(\mathrm{S}) - \tfrac{49}{4}C_2(\mathrm{G}) + 2\kappa S_2(\mathrm{F}) + \tfrac{1}{4}S_2(\mathrm{S})]\boldsymbol{Y}^a\,, \qquad (3.3)
\end{aligned}
$$

[Machacek & Vaughn Nucl. Phys. B 236 (1984) 221-232]

Deriving the Wilson Coefficient (WC) tensors

The job of getting the RGEs of a specific model is not over, but a significant amount of work was done once and for all.

There is no need to go back and calculate divergences from diagrams for every new model. One only needs to compute the specific gauge invariant Lagrangian of a model and apply the general RGEs [i.e. the work is no longer about the RG but rather about gauge invariance = group theory/linear algebra essentially.]

The model-specific work is still non-trivial and there are programs to help

Some code of Susyno included in SARAH

Some code of Susyno included in PyR@TE

SARAH
Staub (2010, 2012,...)

Susyno
R.F. (2011)

PyR@TE
Lyonnet, Schienbein, Staub, Wingerter (2014) | Lyonnet, Schienbein (2017) | Sartore, Schienbein (2021)

ARGES
Litim, Steudtner (2020)

RGBeta
Thomsen (2021)

GroupMath
R.F. (2021)

Group theory code separated and expanded

What do these tensors look like (in the SM)

fields

$$\psi = (u^c[\mathrm{R}], u^c[\mathrm{G}], u^c[\mathrm{B}], d^c[\mathrm{R}], d^c[\mathrm{G}], d^c[\mathrm{B}], Q[\mathrm{R},1], Q[\mathrm{R},2], Q[\mathrm{G},1], Q[\mathrm{G},2], Q[\mathrm{B},1], Q[\mathrm{B},2], e^c, L[1], L[2])^T$$

$$\phi = \left(H_R^+, H_R^0, H_I^+, H_I^0\right)^T \quad F_{\mu\nu} = \left(G_{\mu\nu}^1, G_{\mu\nu}^2, G_{\mu\nu}^3, G_{\mu\nu}^4, G_{\mu\nu}^5, G_{\mu\nu}^6, G_{\mu\nu}^7, G_{\mu\nu}^8, W_{\mu\nu}^1, W_{\mu\nu}^2, W_{\mu\nu}^3, B_{\mu\nu}\right)^T$$

θ^A

scalar representation matrices

SU(3)

$$\begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix} \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}}_{\mathrm{SU(3)}} \underbrace{\begin{pmatrix} 0&0&0&\frac{i}{2}\\0&0&\frac{i}{2}&0\\0&-\frac{i}{2}&0&0\\-\frac{i}{2}&0&0&0 \end{pmatrix} \begin{pmatrix} 0&-\frac{i}{2}&0&0\\\frac{i}{2}&0&0&0\\0&0&0&-\frac{i}{2}\\0&0&\frac{i}{2}&0 \end{pmatrix} \begin{pmatrix} 0&0&\frac{i}{2}&0\\0&0&0&-\frac{i}{2}\\-\frac{i}{2}&0&0&0\\0&\frac{i}{2}&0&0 \end{pmatrix}}_{\mathrm{SU(2)}} \underbrace{\begin{pmatrix} 0&0&\frac{i}{2}&0\\0&0&0&\frac{i}{2}\\-\frac{i}{2}&0&0&0\\0&-\frac{i}{2}&0&0 \end{pmatrix}}_{\mathrm{U(1)}}$$

What do these tensors look like (in the SM)

fields

$$\psi = \left(u^c[\mathrm{R}], u^c[\mathrm{G}], u^c[\mathrm{B}], d^c[\mathrm{R}], d^c[\mathrm{G}], d^c[\mathrm{B}], Q[\mathrm{R},1], Q[\mathrm{R},2], Q[\mathrm{G},1], Q[\mathrm{G},2], Q[\mathrm{B},1], Q[\mathrm{B},2], e^c, L[1], L[2]\right)^T$$

$$\phi = \left(H_R^+, H_R^0, H_I^+, H_I^0\right)^T \quad F_{\mu\nu} = \left(G_{\mu\nu}^1, G_{\mu\nu}^2, G_{\mu\nu}^3, G_{\mu\nu}^4, G_{\mu\nu}^5, G_{\mu\nu}^6, G_{\mu\nu}^7, G_{\mu\nu}^8, W_{\mu\nu}^1, W_{\mu\nu}^2, W_{\mu\nu}^3, B_{\mu\nu}\right)^T$$

t^A

fermion representation Matrices (1 flavor)

<u>No surprise:</u>

These are **block diagonal matrices** because the fermions are in a reducible representation of the gauge group

$$\begin{pmatrix}
0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \ldots,
\begin{pmatrix}
-\frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}
\end{pmatrix}$$

A=1 [first SU(3) generator]

A=12 [U(1) generator]

What do these tensors look like (in the SM)

fields

$$\psi = (u^c[\mathrm{R}], u^c[\mathrm{G}], u^c[\mathrm{B}], d^c[\mathrm{R}], d^c[\mathrm{G}], d^c[\mathrm{B}], Q[\mathrm{R},1], Q[\mathrm{R},2], Q[\mathrm{G},1], Q[\mathrm{G},2], Q[\mathrm{B},1], Q[\mathrm{B},2], e^c, L[1], L[2])^T$$

$$\phi = \left(H_R^+, H_R^0, H_I^+, H_I^0\right)^T \quad F_{\mu\nu} = \left(G^1_{\mu\nu}, G^2_{\mu\nu}, G^3_{\mu\nu}, G^4_{\mu\nu}, G^5_{\mu\nu}, G^6_{\mu\nu}, G^7_{\mu\nu}, G^8_{\mu\nu}, W^1_{\mu\nu}, W^2_{\mu\nu}, W^3_{\mu\nu}, B_{\mu\nu}\right)^T$$

Y_{ija}

Yukawa couplings

$$-\frac{1}{2}\left[Y_{ija}\psi_i^T C\psi_j\phi_a + \mathrm{h.c.}\right]$$

Show here is

$$\frac{1}{\sqrt{2}} y^*_{ij1}$$

i.e. the interactions of H_R^+

u^cQ $\quad$ d^cQ $\quad$ e^cL

0	0	0	0	0	0	0	-Yu[f2, f1]	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-Yu[f2, f1]	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-Yu[f2, f1]	0	0	0
0	0	0	0	0	0	Yd[f2, f1]	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	Yd[f2, f1]	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	Yd[f2, f1]	0	0	0	0
0	0	0	Yd[f1, f2]	0	0	0	0	0	0	0	0	0	0	0
-Yu[f1, f2]	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	Yd[f1, f2]	0	0	0	0	0	0	0	0	0	0
0	-Yu[f1, f2]	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	Yd[f1, f2]	0	0	0	0	0	0	0	0	0
0	0	-Yu[f1, f2]	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	Ye[f2, f1]	0
0	0	0	0	0	0	0	0	0	0	0	0	Ye[f1, f2]	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Flavor is unexpanded (f1,f2 indices); otherwise, Y would be a 45x45x4 tensor

Our goal

To develop and extend the same formalism to non-renormalization operators of an EFT

step 1 — **Write down a basis of operators for a <u>general EFT</u> up to dimension 6 (for now)**

step 2 — **Derive the 1-loop (for now) RGEs for the Wilson coefficients**

← Validate the results by comparing with the known RGEs of specific models (e.g. SMEFT, LEFT, ...)

With these results, there will be no need to ever do physics again (calculate amplitudes of diagrams) to compute RGEs for a specific EFT

Only some algebra is needed in order to compute the the Wilson coefficient tensors (Y_{ija} , λ_{abcd}, etc)

Dimension 5 Green basis

$$\mathscr{L}_5^{\text{phys}} = \left[\frac{1}{2}\left(a_{\psi F}^{(5)}\right)_{Aij}\psi_i^T C\sigma^{\mu\nu}\psi_j F_{\mu\nu}^A + \frac{1}{4}\left(a_{\psi\phi^2}^{(5)}\right)_{ijab}\psi_i^T C\psi_j\phi_a\phi_b + \text{h.c.}\right]$$
$$+\frac{1}{2}\left(a_{\phi F}^{(5)}\right)_{ABa}F^{A\,\mu\nu}F_{\mu\nu}^B\phi_a + \frac{1}{2}\left(a_{\phi\widetilde{F}}^{(5)}\right)_{ABa}F^{A\,\mu\nu}\widetilde{F}_{\mu\nu}^B\phi_a + \frac{1}{5!}\left(a_{\phi}^{(5)}\right)_{abcde}\phi_a\phi_b\phi_c\phi_d\phi_e$$
$$\mathscr{L}_5^{\text{red}} = \frac{1}{2}\left(r_{\phi\Box}^{(5)}\right)_{abc}(D_\mu D^\mu\phi_a)\phi_b\phi_c + \left[\frac{1}{2}\left(r_{\psi}^{(5)}\right)_{ij}(D_\mu\psi_i)^T C D^\mu\psi_j + \left(r_{\psi\phi}^{(5)}\right)_{ija}\bar{\psi}_i \mathrm{i}\not{D}\psi_j\phi_a + \text{h.c.}\right]$$

The Wilson coefficients have important symmetries (in some cases non-trivial)

$$\left(a_{\psi F}^{(5)}\right)_{ij} = -\left(a_{\psi F}^{(5)}\right)_{ji} \qquad \left(a_{\psi\phi^2}^{(5)}\right)_{ijab} = \left(a_{\psi\phi^2}^{(5)}\right)_{jiab} = \left(a_{\psi\phi^2}^{(5)}\right)_{ijba}$$
$$\left(a_{\phi F}^{(5)}\right)_{ABa} = \left(a_{\phi F}^{(5)}\right)_{BAa} \qquad \left(a_{\phi\widetilde{F}}^{(5)}\right)_{ABa} = \left(a_{\phi\widetilde{F}}^{(5)}\right)_{BAa} \qquad \left(a_{\phi}^{(5)}\right)_{abcde} = \text{fully symmetric}$$
$$\left(r_{\psi}^{(5)}\right)_{ij} = \left(r_{\psi}^{(5)}\right)_{ji} \qquad \left(r_{\phi\Box}^{(5)}\right)_{abc} = \left(r_{\phi\Box}^{(5)}\right)_{acb}$$

Integration-by-parts (IBPs) equations of motion (EOM) redundancies may affect only parts of these tensors (e.g. they can remove the symmetric part of some WC and leave untouched the anti-symmetric)

Dimension 6 Green basis

$$\mathscr{L}_6^{\text{phys}} = \frac{1}{3!}\left(a_{3F}^{(6)}\right)_{ABC}(F^A)_\mu^{\ \nu}(F^B)_\nu^{\ \rho}(F^C)_\rho^{\ \mu} + \frac{1}{3!}\left(a_{3\widetilde{F}}^{(6)}\right)_{ABC}(F^A)_\mu^{\ \nu}(F^B)_\nu^{\ \rho}(\widetilde{F}^C)_\rho^{\ \mu}$$
$$+ \frac{1}{4}\left(a_{\phi F}^{(6)}\right)_{ABab}F_{\mu\nu}^A F^{B\,\mu\nu}\phi_a\phi_b + \frac{1}{4}\left(a_{\phi\widetilde{F}}^{(6)}\right)_{ABab}F_{\mu\nu}^A \widetilde{F}^{B\,\mu\nu}\phi_a\phi_b$$
$$+ \frac{1}{4}\left(a_{\phi D}^{(6)}\right)_{abcd}(D_\mu\phi_a)(D^\mu\phi_b)\phi_c\phi_d + \frac{1}{6!}\left(a_{\phi}^{(6)}\right)_{abcdef}\phi_a\phi_b\phi_c\phi_d\phi_e\phi_f$$
$$+ \frac{1}{2}\left(a_{\phi\psi}^{(6)}\right)_{ijab}\bar{\psi}_i\gamma^\mu\psi_j[\phi_a D_\mu\phi_b - \phi_b D_\mu\phi_a] + \frac{1}{4}\left(a_{\bar{\psi}\psi}^{(6)}\right)_{ijkl}(\bar{\psi}_i\gamma^\mu\psi_j)(\bar{\psi}_k\gamma_\mu\psi_l)$$
$$+ \left[\frac{1}{2}\left(a_{\psi F}^{(6)}\right)_{Aija}F_{\mu\nu}^A\psi_i^T C\sigma^{\mu\nu}\psi_j\phi_a + \frac{1}{2!3!}\left(a_{\psi\phi}^{(6)}\right)_{ijabc}\psi_i^T C\psi_j\phi_a\phi_b\phi_c\right.$$
$$\left. + \frac{1}{4!}\left(a_{\psi\psi}^{(6)}\right)_{ijkl}(\psi_i^T C\psi_j)(\psi_k^T C\psi_l) + \text{h.c.}\right]$$

$$\mathscr{L}_6^{\text{red}} = \frac{1}{2!}\left(r_{2F}^{(6)}\right)_{AB}(D_\mu F^{A\,\mu\nu})(D^\rho F_{\rho\nu}^B) + \frac{1}{2!}\left(r_{FD\phi}^{(6)}\right)_{Aab}(D_\nu F^{A,\mu\nu})\left[(D_\mu\phi_a)\,\phi_b - (a\leftrightarrow b)\right]$$
$$+ \frac{1}{2!}\left(r_{D\phi}^{(6)}\right)_{ab}(D_\mu D^\mu\phi_a)\,(D_\nu D^\nu\phi_b) + \frac{1}{3!}\left(r_{\phi D}^{(6)}\right)_{abcd}(D_\mu D^\mu\phi_a)\,\phi_b\phi_c\phi_d$$
$$+ \ldots$$

These tensors also have flavor symmetries

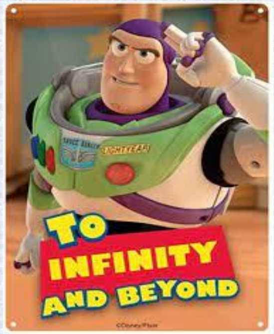
Deriving the 1-loop RGEs

Matchmakereft
Carmona, Lazopoulos, Olgoso, Santiago, 2112.10787

With José Santiago and Pablo Olgoso, we are in the process of **computing the 1-loop RGEs of a general EFT up to dimension 6 using Matchmakereft**

Side comment(s)

But one can go beyond RGEs with this approach



Matching

In the same spirit, why not calculate the matching for a general light+heavy set of fields? (Diagrammatic *vs* functional *vs* 'do the matching once and for all' method?)

Generate operators

Maybe one can do the same generating operators of an EFT: start with the general basis of and from there just deal with gauge invariance on a model-by-model basis

Example

$\frac{1}{\varepsilon}$ pole of $\left(a^{(5)}_{\psi\phi^2}\right)_{ijab}$

Preliminary (subject to checks!)

Complicated expressions

These complicated expressions can sometimes be simplified a bit using gauge invariance conditions. E.g.:

$$t^A_{ii'}\left(a^{(5)}_{\psi\phi^2}\right)_{i'jab} + t^A_{jj'}\left(a^{(5)}_{\psi\phi^2}\right)_{ij'ab} + \theta^A_{aa'}\left(a^{(5)}_{\psi\phi^2}\right)_{ija'b} + \theta^A_{bb'}\left(a^{(5)}_{\psi\phi^2}\right)_{ijab'} = 0$$

More importantly, one must deal with dummy indices and tensors with symmetries

We have developed our own code for this

See also xAct, Cadabra

```
T[i1, i2] × T[i1, i3] + 2 α T[j1, i2] × T[j1, i3]
T[i1, i2] × T[i1, i3] + 2 α T[j1, i2] × T[j1, i3]
```

The two terms are the same, up to a factor, but because of the dummy indices the expression is no simplified

```
CanonicalForm[T[i1, i2] × T[i1, i3] + 2 α T[j1, i2] × T[j1, i3]]
(1 + 2 α) T[i1, i2] × T[i1, i3]
```

With some code, this can be issue can be addressed (not just in this simple example but rather in full generality)

```
CanonicalForm[T[i1, i2] × T[i1, i3] + 2 α T[j1, i2] × T[i3, j1], {T[a, b] + T[b, a]}]
(1 - 2 α) T[i1, i2] × T[i1, i3]
```

Tensor may have some symmetries. In this example T[a,b]+T[b,a]=0 (the tensor is anti-symmetric).
We have been working on a code to deal with arbitrarily complicated symmetries (including the so-called multi-term ones)

Complicated symmetries

There are many symmetries to consider. Here are the ones of dimension 6 operators:

$\left(a^{(6)}_{3F}\right)_{ABC}$ = fully anti-symmetric, $\in \mathbb{R}$

$\left(a^{(6)}_{3\widetilde{F}}\right)_{ABC}$ = fully anti-symmetric, $\in \mathbb{R}$

$$\left(a^{(6)}_{\phi\psi}\right)_{ijab} = -\left(a^{(6)}_{\phi\psi}\right)_{ijba} = \left[\left(a^{(6)}_{\phi\psi}\right)_{jiab}\right]^*$$

$$\left(a^{(6)}_{\bar{\psi}\psi}\right)_{ijkl} = \left(a^{(6)}_{\bar{\psi}\psi}\right)_{kjil} = \left(a^{(6)}_{\bar{\psi}\psi}\right)_{ilkj} = \left[\left(a^{(6)}_{\bar{\psi}\psi}\right)_{jilk}\right]^*$$

$$\left(a^{(6)}_{\phi D}\right)_{abcd} = \left(a^{(6)}_{\phi D}\right)_{bacd} = \left(a^{(6)}_{\phi D}\right)_{abdc} = \left(a^{(6)}_{\phi D}\right)_{cdab} \text{ and}$$

$$\left(a^{(6)}_{\phi D}\right)_{abcd} + \left(a^{(6)}_{\phi D}\right)_{adbc} + \left(a^{(6)}_{\phi D}\right)_{acdb} = 0,\ \left(a^{(6)}_{\phi D}\right)_{abcd} \in \mathbb{R}$$

$$\left(a^{(6)}_{\phi F}\right)_{ABab} = \left(a^{(6)}_{\phi F}\right)_{BAab} = \left(a^{(6)}_{\phi F}\right)_{ABba} \in \mathbb{R}$$

$$\left(a^{(6)}_{\phi \widetilde{F}}\right)_{ABab} = \left(a^{(6)}_{\phi \widetilde{F}}\right)_{BAab} = \left(a^{(6)}_{\phi \widetilde{F}}\right)_{ABba} \in \mathbb{R}$$

$\left(a^{(6)}_{\phi}\right)_{abcdef}$ = fully symmetric $\in \mathbb{R}$

$$\left(a^{(6)}_{\psi\psi}\right)_{ijkl} = \left(a^{(6)}_{\psi\psi}\right)_{jikl} = \left(a^{(6)}_{\psi\psi}\right)_{ijlk} = \left(a^{(6)}_{\psi\psi}\right)_{klij} \text{ and}$$

$$\left(a^{(6)}_{\psi\psi}\right)_{ijkl} + \left(a^{(6)}_{\psi\psi}\right)_{iljk} + \left(a^{(6)}_{\psi\psi}\right)_{iklj} = 0$$

$$\left(a^{(6)}_{\psi F}\right)_{Aija} = -\left(a^{(6)}_{\psi F}\right)_{Ajia}$$

$\left(a^{(6)}_{\psi\phi}\right)_{ijabc}$ = fully symmetric in (i,j) and also (a,b,c)

$$\left(r^{(6)}_{\psi D}\right)_{ij} = \left(r^{(6)}_{\psi D}\right)_{ji}$$

$$\left(r^{(6)}_{D\phi}\right)_{ab} = \left(r^{(6)}_{D\phi}\right)_{ba} \in \mathbb{R}$$

$$\left(r^{(6)}_{2F}\right)_{AB} = \left(r^{(6)}_{2F}\right)_{BA} \in \mathbb{R}$$

$$\left(r^{(6)}_{DF\psi}\right)_{Aij} = \left[\left(r^{(6)}_{DF\psi}\right)_{Aji}\right]^*$$

$$\left(r^{(6)}_{F\psi}\right)_{Aij} = \left[\left(r^{(6)}_{F\psi}\right)_{Aji}\right]^*$$

$$\left(r^{(6)}_{\widetilde{F}\psi}\right)_{Aij} = \left[\left(r^{(6)}_{\widetilde{F}\psi}\right)_{Aji}\right]^*$$

$$\left(r^{(6)}_{FD\phi}\right)_{Aab} = -\left(r^{(6)}_{FD\phi}\right)_{Aba} \in \mathbb{R}$$

$$\left(r^{(6)}_{\phi\psi x}\right)_{ijab} = \left(r^{(6)}_{\phi\psi x}\right)_{ijba} = \left[\left(r^{(6)}_{\phi\psi x}\right)_{jiab}\right]^* \text{ for } x = 1, 2$$

$\left(r^{(6)}_{\phi D}\right)_{abcd}$ = fully symmetric in $(b,c,d) \in \mathbb{R}$

$$\left(r^{(6)}_{\psi\phi D1}\right)_{ija} = \left(r^{(6)}_{\psi\phi D1}\right)_{jia}$$

$$\left(r^{(6)}_{\psi\phi D2}\right)_{ija} = \left(r^{(6)}_{\psi\phi D2}\right)_{jia}$$

$\left(r^{(6)}_{\psi\phi D3}\right)_{ija}$ = no restrictions

Some of them are quite complicated

⊞ symmetry

Complicated symmetries

Let me give another (more complicated) example, involving the Weyl tensor ⊞ symmetry as well

Example taken from Cadabra's website [https://cadabra.science/notebooks/tensor_monomials.html]

$$W_{pqrs}W_{ptru}W_{tvqw}W_{uvsw} - W_{pqrs}W_{pqtu}W_{rvtw}W_{svuw} = W_{mnab}W_{npbc}W_{mscd}W_{spda} - \frac{1}{4}W_{mnab}W_{psba}W_{mpcd}W_{nsdc}$$

```
symmetriesWeyl = {Weyl[w, f2, f3, f4] + Weyl[f2, w, f3, f4], Weyl[f1, f2, f3, f4] + Weyl[f1, f2, f4, f3],
   Weyl[f1, f2, f3, f4] + Weyl[f1, f3, f4, f2] + Weyl[f1, f4, f2, f3]};

exprNullWeyl = Weyl[p, q, r, s] × Weyl[p, t, r, u] × Weyl[t, v, q, w] × Weyl[u, v, s, w] -
   Weyl[p, q, r, s] × Weyl[p, q, t, u] × Weyl[r, v, t, w] × Weyl[s, v, u, w] -
   Weyl[m, n, a, b] × Weyl[n, p, b, c] × Weyl[m, s, c, d] × Weyl[s, p, d, a] +
   α Weyl[m, n, a, b] × Weyl[p, s, b, a] × Weyl[m, p, c, d] × Weyl[n, s, d, c];
CanonicalForm[exprNullWeyl, symmetriesWeyl]
```

$\frac{1}{8}(-2+8\alpha)$ Weyl[m, n, a, b] × Weyl[m, p, c, d] × Weyl[n, s, d, c] × Weyl[p, s, b, a]

When $\alpha = 1/4$ we get 0

Aside from finishing touches, the code is done.
It is very flexible in how the user provides the input.

Not specific to RGEs. Might be useful to other.
So, it might be made public.

To conclude ...

RGEs for a general EFT

Suggestion: let us decouple the task of calculating the RGEs for non-renormalizable models from the task of computing amplitudes

This kind of work (listing diagrams, computing divergent part of loop amplitudes, reverting to a physical set of operators) can be done once and for all for a general EFT where the gauge group and field content are left generic

The resulting equations can then be used to compute the RGEs of specific EFTs without the need to compute loops ever again

This is work in progress...

Thank you