$U(1)$-charged Dark Matter in three-Higgs-doublet models

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In collaboration with: **P. Osland**, **M. N. Rebelo**

PLANCK 2024

June 3–7

CFTP
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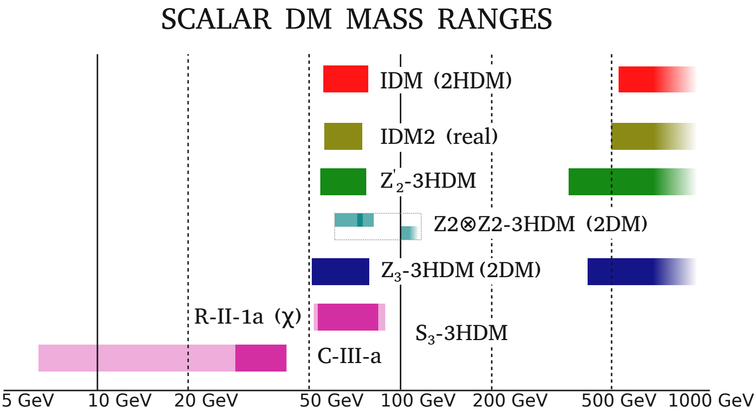

IDM: **(A. Belyaev, G. Cacciapaglia, I. P. Ivanov, F. Rojas-Abatte, M. Thomas, 2016)**,
(J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska, A. F. Zarnecki, 2018);

IDM2: **(M. Merchand, M. Sher, 2019)**;

$\mathbb{Z}_n$-3HDM: **(A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S. F. King, S. Moretti, D. Rojas, D. Sokołowska, 2014-2022)**;

S_3-3HDM: **(A. Kunčinas, O. M. Øgreid, P. Osland, M. N. Rebelo, 2021-2023)**;

CP4-3HDM: **(I. Ivanov, M. Laletin, 2018)**;

$U(1)$-Symmetric Three-Higgs-Doublet Models

Framework for $U(1)$-NHDMs **(I. Ivanov, V. Keus, E. Vdovin, 2011)**:

$$[U(1)]^N \subset U(N) \quad \mathrm{diag}\left[e^{i\alpha_1},\, e^{i\alpha_2},\, ...,\, e^{i\alpha_N}\right].$$

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$$U(1)_1 \equiv \mathrm{diag}\left(e^{-i\alpha},\, e^{i\alpha},\, 1\right) = e^{i\alpha}\mathrm{diag}\left(e^{-2i\alpha},\, 1,\, e^{-i\alpha}\right) = e^{-i\alpha}\mathrm{diag}\left(1,\, e^{2i\alpha},\, e^{i\alpha}\right),$$

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The scalar doublets are decomposed as

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Classification: **(I. de Medeiros Varzielas, I. Ivanov, 2019)** and **(N. Darvishi, A. Pilaftsis, 2019)**.

Mass-Degeneracy Patterns

Neutral $U(1)$-stabilised states result in a mass-degenerate pair.

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Mass degeneracies in the 2HDM **(H. Haber, O. M. Øgreid, P. Osland, M. N. Rebelo, 2018).**

Example of Allowed Couplings

μ_{11}^2	μ_{22}^2	μ_{33}^2	$\left(\mu_{12}^2\right)^{\mathrm{R}}$
λ_{1111}	λ_{2222}	λ_{3333}	
λ_{1122}	λ_{1133}	λ_{2233}	
λ_{1221}	λ_{1331}	λ_{2332}	

Mass terms (top row): 3+1

Quartic couplings (bottom rows): 9

$U(1) \times U(1)$

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μ_{11}^2	μ_{22}^2	μ_{33}^2	$\left(\mu_{12}^2\right)^{\mathrm{R}}$	3+1
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$U(1) \times U(1)$

$[U(1) \times U(1)] +$

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3+1 (mass terms); 9 (quartic couplings)

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3+1; 10

$= U(1) \otimes \mathbb{Z}_2$

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λ_{1111}	λ_{2222}	λ_{3333}		9
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λ_{1323}	10

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$[U(1) \times \mathbb{Z}_2] +$

μ^2_{12}	$\left(\mu^2_{23}\right)^{\mathrm{R}}$	4+1
λ_{1112}	λ_{1222}	11
λ_{1233}	λ_{1332}	

$= U(1)_2$

Considered Symmetries

$$U(1) \otimes U(1)$$

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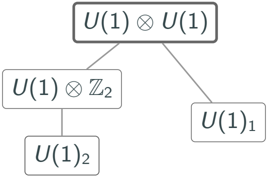

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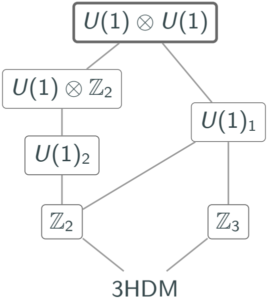

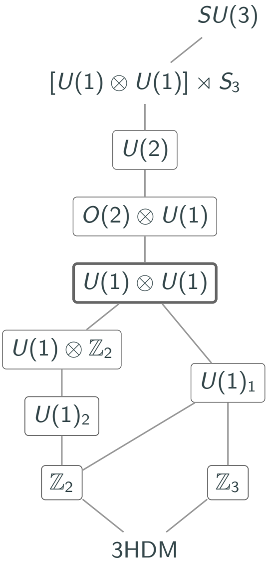
$SU(3)$
$[U(1) \otimes U(1)] \rtimes S_3$
$U(2)$
$O(2) \otimes U(1)$
$U(1) \otimes U(1)$
$U(1) \otimes \mathbb{Z}_2$
$U(1)_1$
$U(1)_2$
$\mathbb{Z}_2$
$\mathbb{Z}_3$
3HDM

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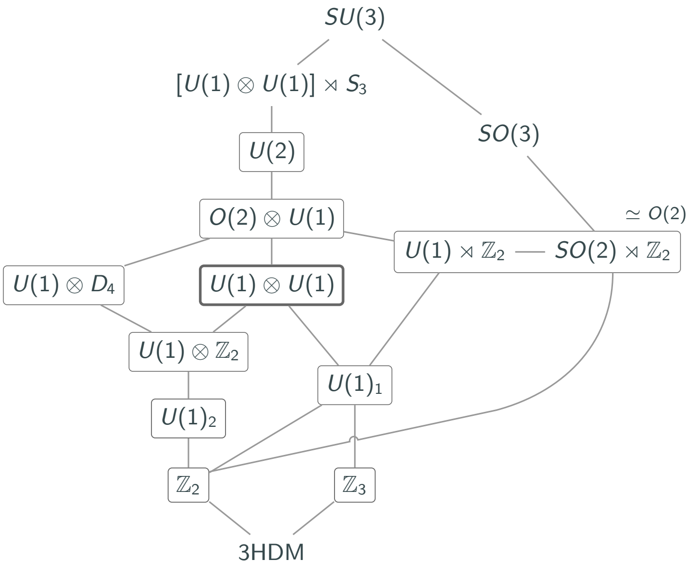

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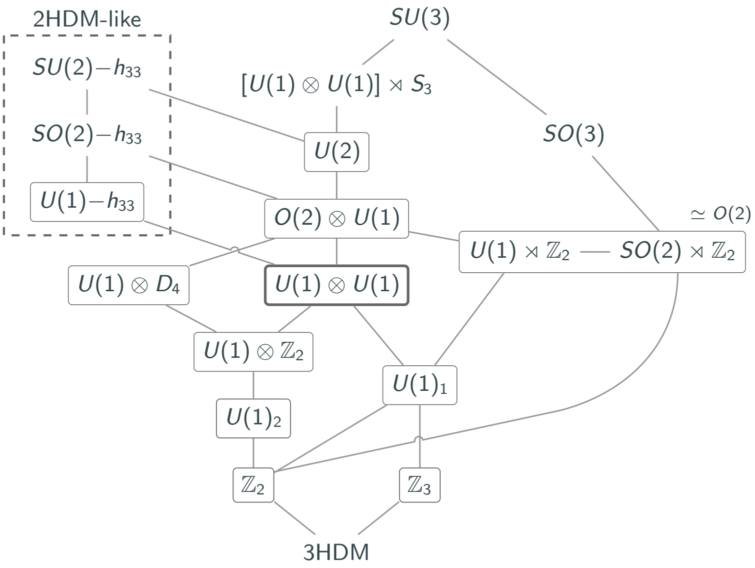

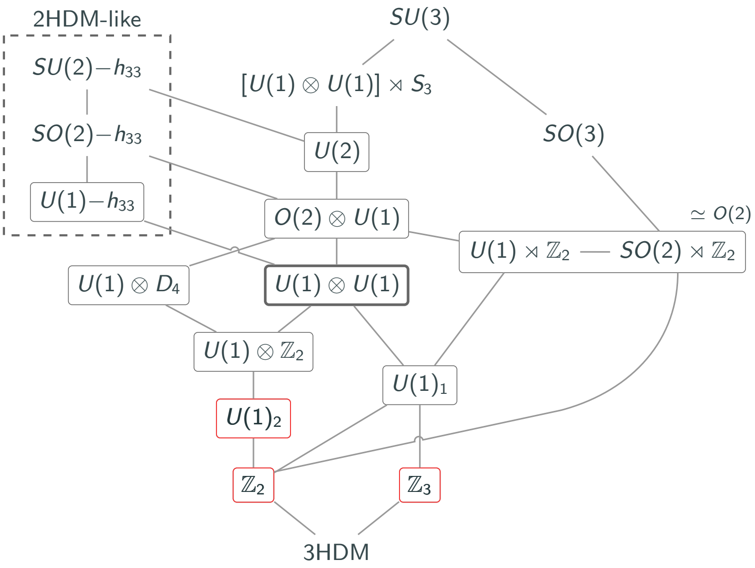
2HDM-like
$SU(2)-h_{33}$
$SO(2)-h_{33}$
$U(1)-h_{33}$
$SU(3)$
$[U(1)\otimes U(1)]\rtimes S_3$
$SO(3)$
$U(2)$
$O(2)\otimes U(1)$
$\simeq O(2)$
$U(1)\rtimes \mathbb{Z}_2$ — $SO(2)\rtimes \mathbb{Z}_2$
$U(1)\otimes D_4$
$U(1)\otimes U(1)$
$U(1)\otimes \mathbb{Z}_2$
$U(1)_1$
$U(1)_2$
$\mathbb{Z}_2$
$\mathbb{Z}_3$
3HDM

$U(1) \otimes U(1)$-3HDM:

$$V_0 = \sum_i \mu_{ii}^2 h_{ii} + \sum_i \lambda_{iiii} h_{ii}^2 + \sum_{i<j} \lambda_{iijj} h_{ii} h_{jj} + \sum_{i<j} \lambda_{ijji} h_{ij} h_{ji}.$$

Vacuum	SSB	V	Mixing of the neutral states	Comments
$(\hat{v}_1,\, \hat{v}_2,\, 0)$	✓	V_0	$\{\eta_1,\, \eta_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$, $m_{\chi_1} = m_{\chi_2} = 0$
$(\hat{v}_1,\, \hat{v}_2,\, 0)$	✓	$V_0 + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\, \eta_2\}-\{\chi_1,\, \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v,\, 0,\, 0)$	×	V_0	diagonal	$m_{\eta_2} = m_{\chi_2}$, $m_{\eta_3} = m_{\chi_3}$

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$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	V_0	$\{\eta_1,\ \eta_2\}-\{\eta_3\}-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$, $m_{\chi_1} = m_{\chi_2} = 0$
$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	$V_0 + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2\}-\{\chi_1,\ \chi_2\}-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
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$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	$V_0 + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2\}-\{\chi_1,\ \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
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$(\hat{v}_1,\, \hat{v}_2,\, 0)$	✓	V_0	$\{\eta_1,\, \eta_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$, $m_{\chi_1} = m_{\chi_2} = 0$
$(\hat{v}_1,\, \hat{v}_2,\, 0)$	✓	$V_0 + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\, \eta_2\}-\{\chi_1,\, \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v,\, 0,\, 0)$	×	V_0	diagonal	$m_{\eta_2} = m_{\chi_2}$, $m_{\eta_3} = m_{\chi_3}$

Implementations within $U(1)_1$-3HDM

Vacuum	SSB	V	Mixing of the neutral states	Comments
$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	$V_{U(1)_1}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\chi_1} = m_{\chi_2} = 0$
$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	$V_{U(1)_1} + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	-
$(0,\ \hat{v}_2,\ \hat{v}_3)$	✓	V_0	$\{\eta_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_1} = m_{\chi_1}$ $m_{\chi_2} = m_{\chi_3} = 0$
$(v,\ 0,\ 0)$	×	$V_{U(1)_1}$	diagonal	$m_{\eta_2} = m_{\chi_2}$, $m_{\eta_3} = m_{\chi_3}$
$(0,\ 0,\ v)$	×	$V_{U(1)_1}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	Two pairs of mass-degenerate states

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$(\hat{v}_1,\ \hat{v}_2,\ 0)$	✓	$V_{U(1)_1} + \{(\mu_{12}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	-
$(0,\ \hat{v}_2,\ \hat{v}_3)$	✓	V_0	$\{\eta_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_1\}-\{\chi_2\}-\{\chi_3\}$	$m_{\eta_1} = m_{\chi_1}$ $m_{\chi_2} = m_{\chi_3} = 0$
$(v,\ 0,\ 0)$	×	$V_{U(1)_1}$	diagonal	$m_{\eta_2} = m_{\chi_2}$, $m_{\eta_3} = m_{\chi_3}$
$(0,\ 0,\ v)$	×	$V_{U(1)_1}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	Two pairs of mass-degenerate states

Implementations within $U(1)_2$-3HDM

Vacuum	SSB	V	Mixing of the neutral states	Comments
$(v_1,\, v_2,\, 0)$	$\times$	$V_{U(1)_2} \in \mathbb{R}$	$\{\eta_1,\, \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\, \chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v_1,\, v_2,\, 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\, v_2,\, v_3)$	✓	$V_{U(1)_2}$	$\{\eta_1,\, \eta_2,\, \eta_3,\, \chi_1\}$ $-\{\chi_2\}-\{\chi_3\}$	$m_{\chi_2} = m_{\chi_3} = 0$ DM might be unstable
$(0,\, v_2,\, v_3)$	✓	$V_{U(1)\otimes\mathbb{Z}_2} + \{\lambda_{1112}\}$	$\{\eta_1,\, \chi_1\}-\{\eta_2,\, \eta_3\}$ $-\{\chi_2\}-\{\chi_3\}$	No underlying symmetry $m_{\chi_2} = m_{\chi_3} = 0$, DM might be unstable
$(0,\, \hat{v}_2,\, \hat{v}_3)$	✓	$V_{U(1)_2} + \{(\mu_{23}^2)^{\mathrm{R}}\}$	$\{\eta_1,\, \eta_2,\, \eta_3,\, \chi_1\}$ $-\{\chi_2,\, \chi_3\}$	DM might be unstable
$(0,\, \hat{v}_2,\, \hat{v}_3)$	✓	$V_{U(1)\otimes\mathbb{Z}_2} + \{(\mu_{23}^2)^{\mathrm{R}},\, \lambda_{1112}\}$	$\{\eta_1,\, \chi_1\}-\{\eta_2,\, \eta_3\}$ $-\{\chi_2,\, \chi_3\}$	No underlying symmetry DM might be unstable
$(v,\, 0,\, 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\, \eta_2,\, \chi_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\, 0,\, v)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	Two pairs of mass-degenerate neutral states

Implementations within $U(1)_2$-3HDM

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$(v_1,\ v_2,\ 0)$	$\times$	$V_{U(1)_2} \in \mathbb{R}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v_1,\ v_2,\ 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_1,\ \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\ v_2,\ v_3)$	$\checkmark$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \eta_3,\ \chi_1\}$ $-\{\chi_2\}-\{\chi_3\}$	$m_{\chi_2} = m_{\chi_3} = 0$ DM might be unstable
$(0,\ v_2,\ v_3)$	$\checkmark$	$V_{U(1)\otimes\mathbb{Z}_2} + \{\lambda_{1112}\}$	$\{\eta_1,\ \chi_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_2\}-\{\chi_3\}$	No underlying symmetry $m_{\chi_2} = m_{\chi_3} = 0$, DM might be unstable
$(0,\ \hat{v}_2,\ \hat{v}_3)$	$\checkmark$	$V_{U(1)_2} + \{(\mu_{23}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2,\ \eta_3,\ \chi_1\}$ $-\{\chi_2,\ \chi_3\}$	DM might be unstable
$(0,\ \hat{v}_2,\ \hat{v}_3)$	$\checkmark$	$V_{U(1)\otimes\mathbb{Z}_2} + \{(\mu_{23}^2)^{\mathrm{R}},\ \lambda_{1112}\}$	$\{\eta_1,\ \chi_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_2,\ \chi_3\}$	No underlying symmetry DM might be unstable
$(v,\ 0,\ 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\ 0,\ v)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_1,\ \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	Two pairs of mass-degenerate neutral states

Implementations within $U(1)_2$-3HDM

Vacuum	SSB	V	Mixing of the neutral states	Comments
$(v_1,\ v_2,\ 0)$	$\times$	$V_{U(1)_2} \in \mathbb{R}$	$\{\eta_1,\ \eta_2\}-\{\eta_3\}$ $-\{\chi_1,\ \chi_2\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(v_1,\ v_2,\ 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_1,\ \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\ v_2,\ v_3)$	✓	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \eta_3,\ \chi_1\}$ $-\{\chi_2\}-\{\chi_3\}$	$m_{\chi_2} = m_{\chi_3} = 0$ DM might be unstable
$(0,\ v_2,\ v_3)$	✓	$V_{U(1)\otimes\mathbb{Z}_2} + \{\lambda_{1112}\}$	$\{\eta_1,\ \chi_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_2\}-\{\chi_3\}$	No underlying symmetry $m_{\chi_2} = m_{\chi_3} = 0$, DM might be unstable
$(0,\ \hat{v}_2,\ \hat{v}_3)$	✓	$V_{U(1)_2} + \{(\mu_{23}^2)^{\mathrm{R}}\}$	$\{\eta_1,\ \eta_2,\ \eta_3,\ \chi_1\}$ $-\{\chi_2,\ \chi_3\}$	DM might be unstable
$(0,\ \hat{v}_2,\ \hat{v}_3)$	✓	$V_{U(1)\otimes\mathbb{Z}_2} + \{(\mu_{23}^2)^{\mathrm{R}},\ \lambda_{1112}\}$	$\{\eta_1,\ \chi_1\}-\{\eta_2,\ \eta_3\}$ $-\{\chi_2,\ \chi_3\}$	No underlying symmetry DM might be unstable
$(v,\ 0,\ 0)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_2\}-\{\eta_3\}$ $-\{\chi_1\}-\{\chi_3\}$	$m_{\eta_3} = m_{\chi_3}$
$(0,\ 0,\ v)$	$\times$	$V_{U(1)_2}$	$\{\eta_1,\ \eta_2,\ \chi_1,\ \chi_2\}$ $-\{\eta_3\}-\{\chi_3\}$	Two pairs of mass-degenerate neutral states

Cases *without* spontaneous symmetry breaking:

• $U(1) \otimes U(1)$	$(v, 0, 0)$		
• $U(1)_1$	$(v, 0, 0)$	$(0, 0, v)$	
• $U(1) \otimes \mathbb{Z}_2$	$(v, 0, 0)$	$(0, 0, v)$	
• $U(1)_2$	$(v, 0, 0)$	$(0, 0, v)$	$(v_1, v_2, 0)$
• $O(2) \otimes U(1)$		$(0, 0, v)$	
• $O(2)$		$(0, 0, v)$	
• $U(1) \otimes D_4$		$(0, 0, v)$	
• $U(2)$		$(0, 0, v)$	
• $U(1)-h_{33}$	$(v, 0, 0)$	$(0, 0, v)$	
• $SO(2)-h_{33}$		$(0, 0, v)$	
• $SU(2)-h_{33}$		$(0, 0, v)$	

$[U(1) \otimes U(1)] \rtimes S_3$, $SO(3)$, $SU(3)$ result in spontaneous symmetry breaking.

The Dark Matter Candidates

Model	Vacuum	Symmetry	h_{SM}
$m_{\eta_3} = m_{\chi_3}$			
I-a	$(v,\, 0,\, 0)$	$U(1) \otimes \mathbb{Z}_2$	η_1
I-b	$(v,\, 0,\, 0)$	$U(1)_2$	$\{\eta_1,\, \eta_2,\, \chi_2\}$
I-c	$(v,\, 0,\, 0)$	$U(1){-}h_{33}$	η_1
I-d	$(\hat{v}_1,\, \hat{v}_2,\, 0)$	$U(1)_2$	$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$
I-e	$(\hat{v}_1 e^{i\sigma},\, \hat{v}_2,\, 0)$	$U(1)_2$	$\{\eta_1,\, \eta_2,\, \chi_1,\, \chi_2\}$
$m_{\eta_i} = m_{\chi_i}$, for $\langle h_i \rangle = 0$			
II-a	$(v,\, 0,\, 0)$	$U(1) \otimes U(1)$	η_1
II-b	$(v,\, 0,\, 0)$	$U(1)_1$	η_1
II-c	$(0,\, 0,\, v)$	$U(1) \otimes \mathbb{Z}_2$	η_3
$m_{\{\eta_1,\eta_2\}} = m_{\{\chi_1,\chi_2\}}$, η_i and χ_i mix separately			
II-d	$(0,\, 0,\, v)$	$U(1)_1$	η_3
II-e	$(0,\, 0,\, v)$	$O(2)$	η_3
$m_{\{\eta_1,\eta_2,\chi_1,\chi_2\}}$, all states mix together			
II-f	$(0,\, 0,\, v)$	$U(1)_2$	η_3
$m_{\eta_1} = m_{\eta_2} = m_{\chi_1} = m_{\chi_2}$			
II-g	$(0,\, 0,\, v)$	$O(2) \otimes U(1)$	η_3
II-h	$(0,\, 0,\, v)$	$U(1) \otimes D_4$	η_3
II-i	$(0,\, 0,\, v)$	$U(2)$	η_3

The Dark Matter Candidates

Symmetry	$(v, 0, 0)$	$(0, 0, v)$	$(v_1, v_2, 0)$
$U(1) \otimes U(1)$	$7 + 32\ (5 + 10)$		
$U(1)_1$	$15 + 40\ (8 + 13)$	$9 + 53\ (6 + 24)$	
$U(1) \otimes \mathbb{Z}_2$	$7 + 32\ (6 + 11)$	$7 + 38\ (5 + 16)$	
$U(1)_2$	$24 + 55\ (20 + 40)$	$12 + 73\ (8 + 36)$	$\mathbb{R} : 16 + 41\ (13 + 24)$ $\mathbb{C} : 24 + 55\ (20 + 40)$
$O(2) \otimes U(1)$		$7 + 32\ (3 + 7)$	
$O(2)$		$7 + 39\ (4 + 12)$	
$U(1) \otimes D_4$		$7 + 34\ (3 + 11)$	
$U(2)$		$7 + 32\ (3 + 6)$	
$U(1)-h_{33}$	$4 + 16\ (3 + 5)$	$1 + 26\ (1 + 6)$	
$SO(2)-h_{33}$		$1 + 26\ (1 + 5)$	
$SU(2)-h_{33}$		$1 + 26\ (1 + 4)$	

Dark Matter in $U(1) \otimes U(1)$-3HDM

$$
\begin{aligned}
V_{\mathrm{IDM}} &= \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
&\quad + \lambda_{1212} \left(h_{12}^2 + h_{21}^2 \right). \\
V_{U(1)\otimes U(1)} &= \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
&\quad + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} \\
&\quad + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}.
\end{aligned}
$$

$$\begin{aligned}
V_{\mathrm{IDM}} &= \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
&\quad + \lambda_{1212} \left(h_{12}^2 + h_{21}^2 \right). \\
V_{U(1)\otimes U(1)} &= \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\
&\quad + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} \\
&\quad + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}.
\end{aligned}$$

Masses:

$$\begin{aligned}
m_{h_i^+}^2 &= \mu_{ii}^2 + \frac{1}{2}\lambda_{11ii} v^2, \ \text{for } i = \{2, 3\}, \\
m_{\eta_i}^2 &= m_{\chi_i}^2 \equiv m_{H_i}^2 = \mu_{ii}^2 + \frac{1}{2}\left(\lambda_{11ii} + \lambda_{1ii1}\right) v^2, \ \text{for } i = \{2,\ 3\}.
\end{aligned}$$

Dark Matter in $U(1) \otimes U(1)$-3HDM

$$V_{\mathrm{IDM}} = \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} + \lambda_{1212}\left(h_{12}^2 + h_{21}^2\right).$$

$$V_{U(1)\otimes U(1)} = \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}.$$

Masses:

$$m_{h_i^+}^2 = \mu_{ii}^2 + \frac{1}{2}\lambda_{11ii} v^2, \text{ for } i = \{2,3\},$$

$$m_{\eta_i}^2 = m_{\chi_i}^2 \equiv m_{H_i}^2 = \mu_{ii}^2 + \frac{1}{2}\left(\lambda_{11ii} + \lambda_{1ii1}\right) v^2, \text{ for } i = \{2,\,3\}.$$

Z interactions: $\mathcal{L}_{VHH} = -\frac{g}{2\cos\theta_W} Z^\mu \eta_i \overleftrightarrow{\partial}_\mu \chi_i$.

Dark Matter in $U(1) \otimes U(1)$-3HDM

$$V_{\mathrm{IDM}} = \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} + \lambda_{1212} \left(h_{12}^2 + h_{21}^2 \right).$$

$$V_{U(1)\otimes U(1)} = \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}.$$

Masses:

$$m_{h_i^+}^2 = \mu_{ii}^2 + \frac{1}{2}\lambda_{11ii} v^2, \text{ for } i = \{2, 3\},$$

$$m_{\eta_i}^2 = m_{\chi_i}^2 \equiv m_{H_i}^2 = \mu_{ii}^2 + \frac{1}{2}\left(\lambda_{11ii} + \lambda_{1ii1}\right) v^2, \text{ for } i = \{2, 3\}.$$

Z interactions: $\mathcal{L}_{VHH} = -\frac{g}{2\cos\theta_W} Z^\mu \eta_i \overleftrightarrow{\partial}_\mu \chi_i$.

Higgs portal: $g(\eta_i^2 h) = g(\chi_i^2 h) = v\left(\lambda_{11ii} + \lambda_{1ii1}\right)$.

Dark Matter in $U(1) \otimes U(1)$-3HDM

$$\begin{aligned} V_{\mathrm{IDM}} &= \mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\ &\quad + \lambda_{1212}\left(h_{12}^2 + h_{21}^2\right). \\ V_{U(1)\otimes U(1)} &= \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{22}^2 h_{22} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{2222} h_{22}^2 + \lambda_{1122} h_{11} h_{22} + \lambda_{1221} h_{12} h_{21} \\ &\quad + \frac{1}{2}\mu_{11}^2 h_{11} + \mu_{33}^2 h_{33} + \frac{1}{2}\lambda_{1111} h_{11}^2 + \lambda_{3333} h_{33}^2 + \lambda_{1133} h_{11} h_{33} + \lambda_{1331} h_{13} h_{31} \\ &\quad + \lambda_{2233} h_{22} h_{33} + \lambda_{2332} h_{23} h_{32}. \end{aligned}$$

Masses:

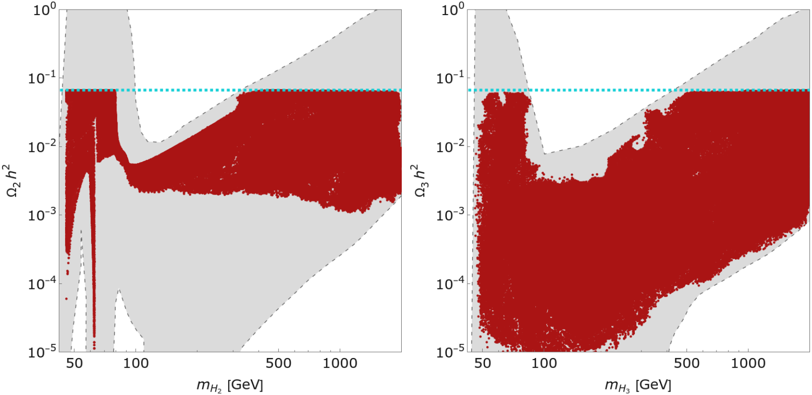
$$\begin{aligned} m_{h_i^+}^2 &= \mu_{ii}^2 + \frac{1}{2}\lambda_{11ii} v^2, \text{ for } i = \{2,3\}, \\ m_{\eta_i}^2 &= m_{\chi_i}^2 \equiv m_{H_i}^2 = \mu_{ii}^2 + \frac{1}{2}\left(\lambda_{11ii} + \lambda_{1ii1}\right) v^2, \text{ for } i = \{2,\, 3\}. \end{aligned}$$

Z interactions: $\mathcal{L}_{VHH} = -\frac{g}{2\cos\theta_W} Z^\mu \eta_i \overleftrightarrow{\partial}_\mu \chi_i$.

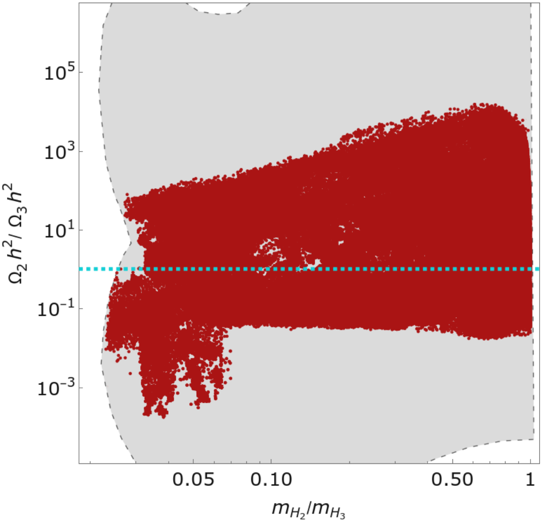
Higgs portal: $g(\eta_i^2 h) = g(\chi_i^2 h) = v\left(\lambda_{11ii} + \lambda_{1ii1}\right)$.

There are 4 (2) DM candidates, $\Omega_{\mathrm{DM}} h^2 = \Omega_2 h^2 + \Omega_3 h^2$.

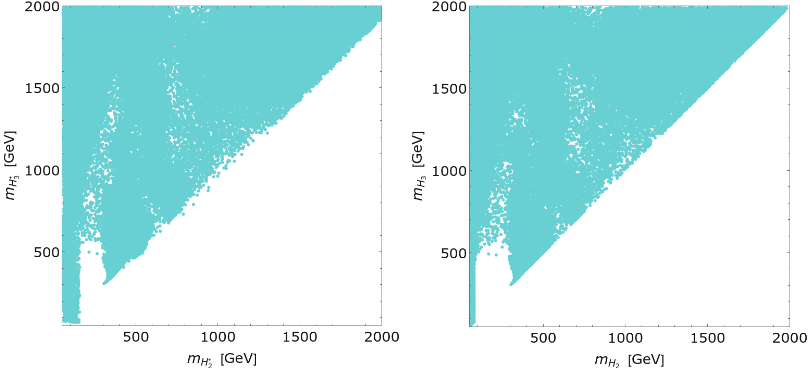
Dark Matter in $U(1) \otimes U(1)$-3HDM

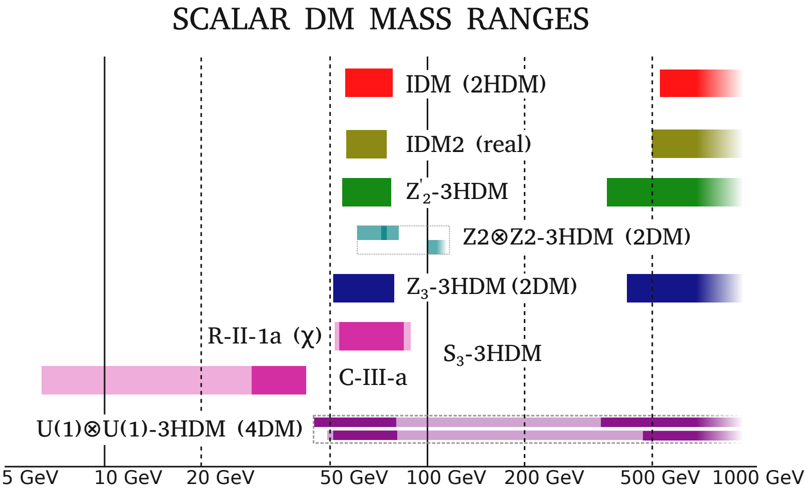


- No points when $m_{H_2} \in [100; 300]$ GeV and $m_{H_3} \in [100; 450]$ GeV;
- In $[50; 80] \cup [300; 2000]$ GeV both mass scales and their contributions to the total relic density are comparable;
- Regions where Ωh^2 is dominated by a single mass scale, $m_{H_2} = [45; 80] \cup [330; 2000]$ GeV and $m_{H_3} = [52; 80] \cup [470; 2000]$ GeV;


10^5
10^3
10^1
10^{-1}
10^{-3}
$\Omega_2 h^2 / \Omega_3 h^2$
0.05
0.10
0.50
1
m_{H_2}/m_{H_3}

Dark Matter in $U(1) \otimes U(1)$-3HDM




SCALAR DM MASS RANGES
IDM (2HDM)
IDM2 (real)
Z'2-3HDM
Z2⊗Z2-3HDM (2DM)
Z3-3HDM (2DM)
R-II-1a (χ)
S3-3HDM
C-III-a
U(1)⊗U(1)-3HDM (4DM)
5 GeV
10 GeV
20 GeV
50 GeV
100 GeV
200 GeV
500 GeV
1000 GeV

Conclusions

- We classified and identified models, embedding $U(1)$-stabilised DM in 3HDMs. These models contain mass-degenerate pairs of DM candidates, due to the unbroken $U(1)$. Such classification and identification of models is useful for model builders interested in three-Higgs-doublet models stabilised by continuous symmetries;
- We performed a numerical scan of the $U(1) \otimes U(1)$-3HDM. Within the model there is a multi-component DM sector, with two different mass scales. We found that there are possible solutions throughout a broad DM mass range, 45–2000 GeV;

Work supported by the Fundação para a Ciência e a Tecnologia (FCT, Portugal) PhD fellowship with reference UI/BD/150735/2020 as well as through the FCT projects CERN/FIS-PAR/0002/2021, UIDB/00777/2020, UIDP/00777/2020, CERN/FIS-PAR/0008/2019, PTDC/FIS-PAR/29436/2017.

FCT Fundação para a Ciência e a Tecnologia

COMPETE
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

QREN QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL

Appendix: $O(2) \otimes U(1)$

Invariant under:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ e^{i\theta_2} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}.$$

$$V = \mu_{11}^2(h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^3 + \lambda_{1122} h_{11} h_{22}$$
$$+ \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) + \lambda_{1221} h_{12} h_{21} + \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}).$$

$S_3 \otimes \mathbb{Z}_2 \simeq D_6$. However, D_n, with $n \geq 5$ result in identical V.

Perform a basis change:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\frac{\pi}{4}} & -e^{i\frac{\pi}{4}} & 0 \\ e^{-i\frac{\pi}{4}} & -e^{i\frac{\pi}{4}} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}.$$

$$V = \mu_{11}^2(h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_a(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^3 + \lambda_b h_{12} h_{21}$$
$$+ \lambda_{1331}(h_{13} h_{31} + h_{23} h_{32}) + \lambda_c h_{11} h_{22} + \lambda_{1133}(h_{11} h_{33} + h_{22} h_{33}) - \frac{1}{2}\Lambda(h_{12}^2 + h_{21}^2).$$

Invariance under $g = \begin{pmatrix} 0 & e^{i\alpha} & 0 \\ -e^{-i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

Appendix: $[U(1) \otimes U(1)] \rtimes S_3$

Invariant under:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ 0 & 0 & e^{i\theta_2} \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix} .$$

$$\begin{aligned} V &= \mu_{11}^2 (h_{11} + h_{22} + h_{33}) + \lambda_{1111} (h_{11}^2 + h_{22}^2 + h_{33}^2) \\ &\quad + \lambda_{1122} (h_{11} h_{22} + h_{22} h_{33} + h_{33} h_{11}) + \lambda_{1221} (h_{12} h_{21} + h_{23} h_{32} + h_{31} h_{13}) \\ &= \mu_{11}^2 \sum_i h_{ii} + \lambda_{1111} \sum_i h_{ii}^2 + \lambda_{1122} \sum_{i<j} h_{ii} h_{jj} + \lambda_{1221} \sum_{i<j} h_{ij} h_{ji} . \end{aligned}$$

$$g = \begin{pmatrix} 0 & e^{i\theta_1} & 0 \\ 0 & 0 & e^{i\theta_2} \\ e^{-i(\theta_1+\theta_2)} & 0 & 0 \end{pmatrix} .$$

Appendix: $U(1) \otimes D_4$

$$V^{\mathrm{ph}}_{U(1)\otimes Z_2} = \lambda_{1212} h_{12}^2 + \mathrm{h.c.},$$

$$\begin{aligned} V = {} & \mu_{11}^2 (h_{11} + h_{22}) + \mu_{33}^2 h_{33} + \lambda_{1111}(h_{11}^2 + h_{22}^2) + \lambda_{3333} h_{33}^2 + \lambda_{1122} h_{11} h_{22} \\ & + \lambda_{1133}(h_{11}h_{33} + h_{22}h_{33}) + \lambda_{1221} h_{12} h_{21} + \lambda_{1331}(h_{13}h_{31} + h_{23}h_{32}) \\ & + \lambda_{1212}(h_{12}^2 + h_{21}^2). \end{aligned}$$

Invariance under $\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 0 & -e^{i\alpha} & 0 \\ e^{i\alpha} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}$
suggests that $\{\lambda_{1112}\,(h_{11}h_{12} - h_{21}h_{22}) + \mathrm{h.c.}\}$ is allowed. Change of basis
$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\frac{i\pi}{4}} & e^{\frac{i\pi}{4}} & 0 \\ -e^{-\frac{i\pi}{4}} & e^{-\frac{i\pi}{4}} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} h_1' \\ h_2' \\ h_3' \end{pmatrix}$ yields $\lambda_{1212} \in \mathbb{C}$.

As pointed out by I. Ivanov, this should be a quotient group.

Appendix: $U(1) \otimes U(1)$ Scan

- All of the additional scalars can be as heavy as 2 TeV;
- The DM candidates are associated with the neutral states of the h_2 and h_3 doublets, $m_{H_2} \leq m_{H_3}$;
- LEP constraints. We allow for $m_{H_i^\pm} \geq 70$ GeV and assume $m_{H_i} > \frac{1}{2} m_Z$;
- Perturbativity, Unitarity (8π), Stability conditions;
- Electroweak precision observables S and T;
- $\mathrm{Br}\,(h \to \mathrm{invisible}) \leq 0.1$;
- LHC searches implemented in HiggsTools;
- DM relic density is evaluated as $\Omega_{\mathrm{DM}} h^2 = 2\Omega_2 h^2 + 2\Omega_3 h^2$;
- Direct DM searches based on XENONnT and LUX-ZEPLIN;
- Indirect DM searches.