Lepton family numbers of neutrinos at low energies and leptogenesis

Takuya Morozumi (Hiroshima U.)

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Work based on the collaboration with
Nicholas James Benoit (Hiroshima U.)
Saki Kawano (Higashi-Hiroshima),Yuta Kawamura(Mie), Yusuke Shimizu (Niigata Kaishi prof.U)
Kei Yamamoto (Hiroshima Institute of Technology)

Ref. Arxiv.2212.00142 "Determination of Majorana type-phases from the time evolution of lepton numbers" and the under-revision project

Abstract

- The lepton family numbers of Majorana neutrinos are known to be sensitive to the Majorana type phases which is a rephaing invariant of lepton mixing matrix for Majorana neutrinos.

- In this talk, we wish to discuss the lepton family numbers at low energies in the light of the seesaw model with two right-handed neutrinos with electron and muon families.

Contents

1. Lepton mixing matrix (PMNS) and Unitarity triangles, effective Majorana mass and Majorana type phases
2. Time evolution of lepton numbers and Majorana type phases
3. The link to lepton family asymmetries of leptogensis for the seesaw model with two right-handed neutrinos

1-1.effective Majorana mass , Lepton Mixing Matrix, Majorana phases

Charged lepton mass basis

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu \nu_{L\alpha}W^- - \frac{1}{2}\overline{(\nu_{L\alpha})^c} m_{\nu\alpha\beta}\nu_{L\beta} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

$$\alpha = e,\ \mu, \tau :\ \begin{pmatrix}\nu_{L\alpha}\\ l_{L\alpha}\end{pmatrix} = \left\{\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix}, \begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}, \begin{pmatrix}\nu_{L\tau}\\ l_{L\tau}\end{pmatrix}\right\}, \quad m_{\nu\alpha\beta} = \begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} & m_{\nu e\tau} \\ m_{\nu e\mu} & m_{\nu\mu\mu} & m_{\nu\mu\tau} \\ m_{\nu e\tau} & m_{\nu\mu\tau} & m_{\nu\tau\tau}\end{pmatrix}$$

Neutrinos' mass basis

$$\nu_{L\alpha} = U_{\alpha i}\nu_{Li} \quad i = 1, 2, 3$$

$$\mathrm{U}^T_{i\alpha} m_{\nu\alpha\beta}\, \mathrm{U}_{\beta j} = m_i \delta_{ij}$$

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu U_{\alpha i}\nu_{Li}W^- - \frac{1}{2}\overline{(\nu_{Li})^c}\, m_i \nu_{Li} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

$$l_\alpha \to e^{i\theta_\alpha}\, l_\alpha,\ \nu_i \not\to e^{i\theta_i}\, \nu_i$$

(Rephasing allowed only for charged lepton)

$$\mathrm{U}_{\alpha i}(\text{6 phases} + \text{3angles}) \to e^{i(\not\theta_i - \theta_\alpha)}\ \mathrm{U}_{\alpha i}(\text{3phases} + \text{3angles})$$

[Arxiv.2212.00142, N. J. Benoit, S. Kawano, Y. Kawamura, T. Morozumi, Y. Shimizu, K.Yamamoto]

Parametrization for PMNS matrix for Majorana Neutrinos and Majorana phases and Kobayashi Maskawa phase

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Doi, Kotani, Nishiura, Okuda,Takasugi, PLB. 1981

1. α_{21}, α_{31} are called Majorana phases. They are physical when all the three neutrinos are massive.

2. δ is a Kobayashi Maskawa phase, which also is present for Dirac neutrinos.

1-2. Unitarity triangles and Majorana type phases

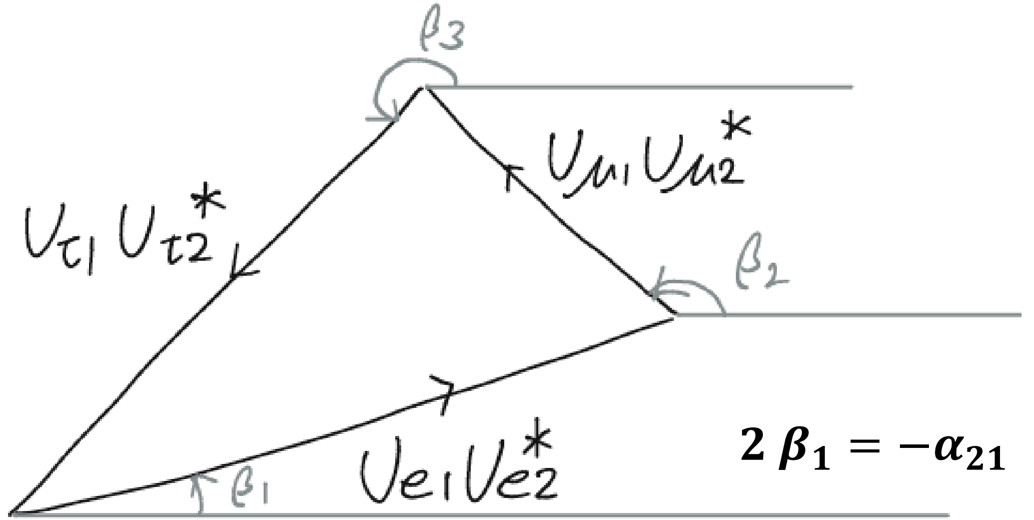
1. Unitarity triangles: orthogonality conditions of different rows(columns)

$$U_{ei}\,U_{ej}^{*} + U_{\mu i}\,U_{\mu j}^{*} + U_{\tau i}\,U_{\tau j}^{*} = 0 \quad i \neq j$$

2. "Majorana type" phases [**Branco, Rebelo, PRD.79, 013001(2009)**]

From quadratic rephasing invariants [J. Nieves and P. B. Pal, Phys. Rev. D36(1987)] of lepton mixing matrix elements, they define:

$$arg(U_{e1}\,U_{e2}^{*}) = \beta_1\ , arg(U_{\mu 1}\,U_{\mu 2}^{*}) = \beta_2, arg(U_{\tau 1}\,U_{\tau 2}^{*}) = \beta_3$$



$$2\,\beta_1 = -\alpha_{21}$$

Fig. : Saki Kawano, Master thesis, Hiroshima Univ.

orthogonality condition

$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U_{e2}^{0} e^{i\frac{\alpha_{21}}{2}} & U_{e3}^{0} e^{i\frac{\alpha_{31}}{2}} \\ U_{\mu 1} & U_{\mu 2}^{0} e^{i\frac{\alpha_{21}}{2}} & U_{\mu 3}^{0} e^{i\frac{\alpha_{31}}{2}} \\ U_{\tau 1} & U_{\tau 2}^{0} e^{i\frac{\alpha_{21}}{2}} & U_{\tau 3}^{0} e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$\Delta(\beta_i + \beta_j) = -\Delta\alpha_{21}:\ \Delta\beta_i = \Delta\beta_j$$

$\beta_i - \beta_j$ are independent of Majorana phases

Majorana type phases determine the rotation angles of each side from a real axis on the complex plane

Branco and Rebelo defined 6 independent arguments of each side of the three Unitarity triangles,

$$\sum_{\alpha=e}^{\tau} U_{\alpha i} U_{\alpha j}^{*}=0$$

$U_{\alpha i} U_{\alpha j}^{*}$

$\alpha \backslash (i,j)$	$(1,2)$	$(2,3)$	$(3,1)$
e	$U_{e1}U_{e2}^*$	$U_{e2}U_{e3}^*$	$U_{e3}U_{e1}^*$
μ	$U_{\mu1}U_{\mu2}^*$	$U_{\mu2}U_{\mu3}^*$	$U_{\mu3}U_{\mu1}^*$
τ	$U_{\tau1}U_{\tau2}^*$	$U_{\tau2}U_{\tau3}^*$	$U_{\tau3}U_{\tau1}^*$

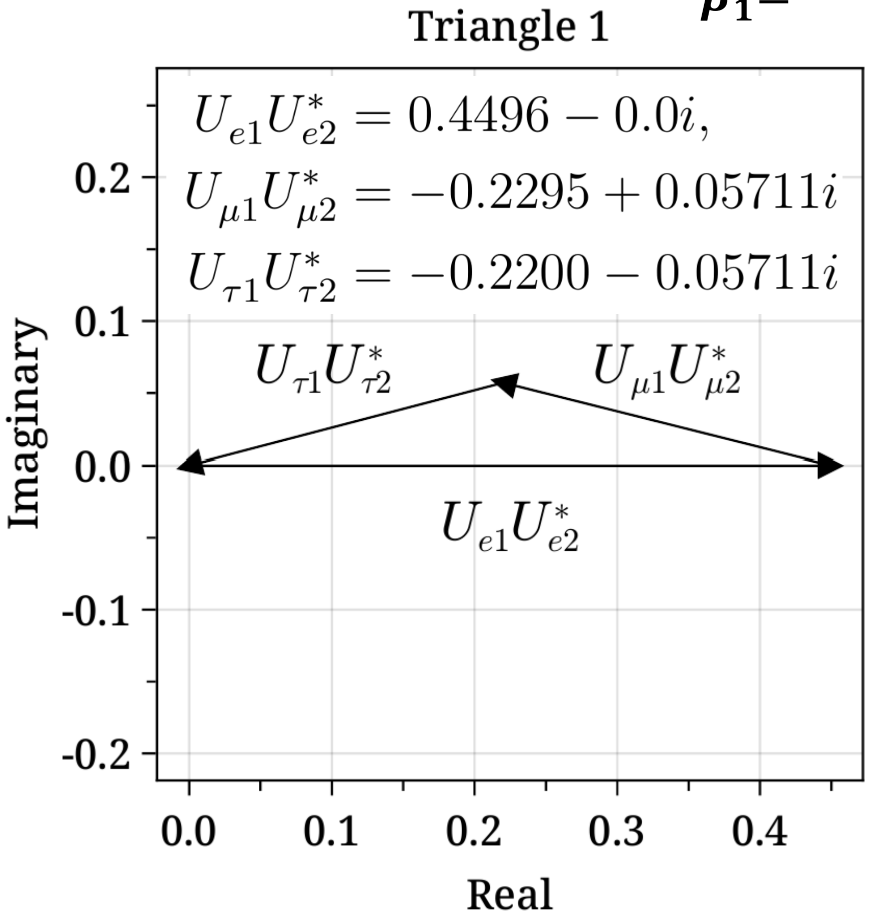
$arg\left(U_{\alpha i} U_{\alpha j}^{*}\right)$

$\alpha \backslash (i,j)$	$(1,2)$	$(2,3)$	$(3,1)$
e	β_1	$\gamma_1-\beta_1$	$-\gamma_1$
μ	β_2	$\gamma_2-\beta_2$	$-\gamma_2$
τ	β_3	$\gamma_3-\beta_3$	$-\gamma_3$

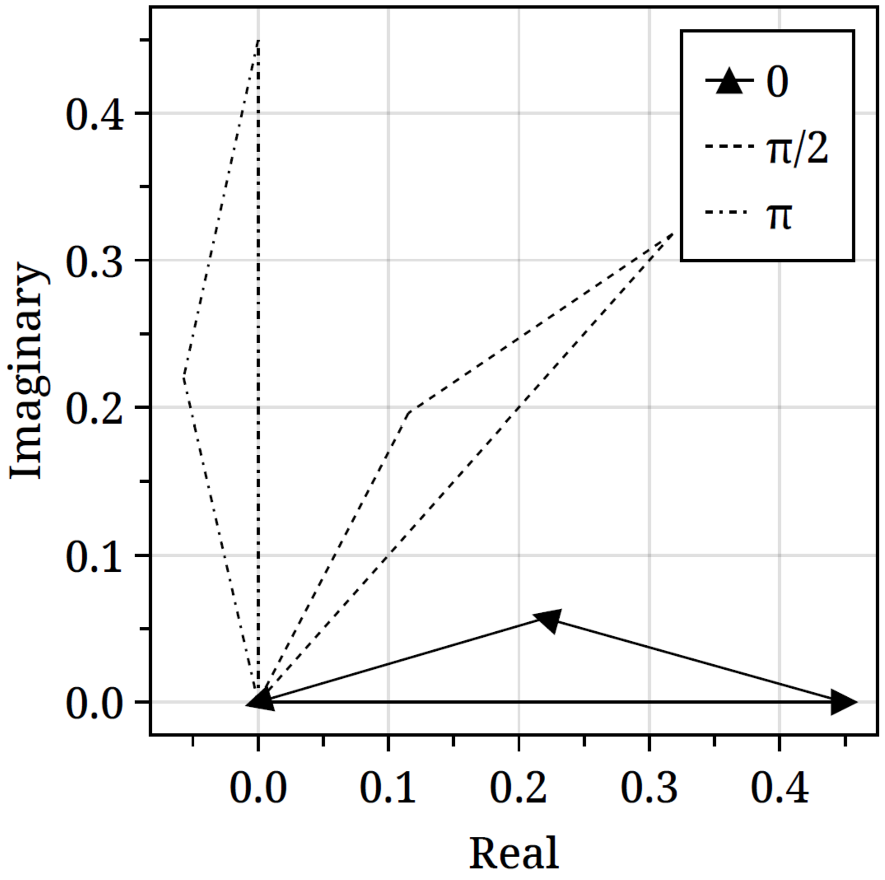
$$2\,\beta_1=-\alpha_{21}, \quad 2\gamma_1=2\delta-\alpha_{31}$$

Figures from [Arxiv.2212.00142, N. J. Benoit, S. Kawano, Y. Kawamura, T. Morozumi, Y. Shimizu, K.Yamamoto]

$\boldsymbol{\beta_1 = -\frac{\alpha_{21}}{2} = 0}$



$\beta_1 = 0, \frac{\pi}{4}, \frac{\pi}{2}$



$$\boldsymbol{arg(U_{e1}\,U_{e2}^*) = \beta_1}$$
$$\boldsymbol{arg(U_{\mu1}\,U_{\mu2}^*) = \beta_2}$$
$$\boldsymbol{arg(U_{\tau1}\,U_{\tau2}^*) = \beta_3}$$

For the left figure, we use:

Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, JHEP 09, 178 (2020) doi:10.1007/JHEP09(2020)178 [arXiv:2007.14792 [hep-ph]], NuFIT 5.1 (2021) www.nu-fit.org.

[Arxiv.2212.00142, N. J. Benoit, S. Kawano, Y. Kawamura, T. Morozumi, Y. Shimizu, K.Yamamoto
"Determination of Majorana type-phases from the time evolution of lepton numbers"]

2-1.The time evolution of lepton family number and Majorana type phases

$$L_\alpha(t) = \int \frac{d^3p}{(2\pi)^3 2|p|} [a_\alpha^\dagger(p,t) a_\alpha(p,t) - b_\alpha^+(p,t) b_\alpha(p,t)]$$

[A.S. Adam, N.J. Benoit, Y.Kawamura, Y. Matsuo, T. Morozumi, Y. Shimizu, Y. Tokunaga, PTEP 2021, 5 (2021) , See also A.S. Adam, N.J. Benoit, Y.Kawamura, Y. Matsuo, T. Morozumi, N. Toyota, Y.Shimizu, PRD2023]

$$|\boldsymbol{\sigma}(\boldsymbol{q})\rangle = \boldsymbol{a}_{\boldsymbol{\sigma}}^+(\boldsymbol{q})|\boldsymbol{0}> \quad \boldsymbol{E}_{\boldsymbol{i}}(\boldsymbol{q}) = \sqrt{\boldsymbol{q}^2 + \boldsymbol{m}_{\boldsymbol{i}}^2}$$

- **depends on Majorana phases and the combinations of $\beta_i + \beta_j / \gamma_i + \gamma_j$**
- $(\boldsymbol{\alpha}, \boldsymbol{\sigma}, \mathrm{i}, \mathrm{j}) \to (\mathrm{e}, \mu, 2,1)$: $\boldsymbol{arg}(\boldsymbol{U}_{\alpha j} \boldsymbol{U}_{\alpha i}^*) + \boldsymbol{arg}(\boldsymbol{U}_{\sigma j} \boldsymbol{U}_{\sigma i}^*) \to \beta_1 + \beta_2$

$$\begin{aligned}
&\langle\sigma|\, L_\alpha^M(t)\, |\sigma\rangle \\
&= \sum_{i,j}^{3} \Bigg[\mathrm{Re}(U_{\alpha i}^* U_{\sigma i} U_{\alpha j} U_{\sigma j}^*) \left(\cos\{E_i(\mathbf{q})t\} \cos\{E_j(\mathbf{q})t\} + \frac{\mathbf{q}^2}{E_i(\mathbf{q}) E_j(\mathbf{q})} \sin\{E_i(\mathbf{q})t\} \sin\{E_j(\mathbf{q})t\} \right) \\
&\quad - \mathrm{Im}(U_{\alpha i}^* U_{\sigma i} U_{\alpha j} U_{\sigma j}^*) \left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})} \sin\{E_i(\mathbf{q})t\} \cos\{E_j(\mathbf{q})t\} - \frac{|\mathbf{q}|}{E_j(\mathbf{q})} \cos\{E_i(\mathbf{q})t\} \sin\{E_j(\mathbf{q})t\} \right) \\
&\quad - \mathrm{Re}(U_{\alpha i}^* U_{\sigma i}^* U_{\alpha j} U_{\sigma j}) \frac{m_i}{E_i(\mathbf{q})} \frac{m_j}{E_j(\mathbf{q})} \sin\{E_i(\mathbf{q})t\} \sin\{E_j(\mathbf{q})t\} \Bigg] \qquad (17)
\end{aligned}$$

- **depends Kobayashi Maskawa phase and the combinations of $\beta_i - \beta_j / \gamma_i - \gamma_j$**
- $(\boldsymbol{\alpha}, \boldsymbol{\sigma}, \mathrm{i}, \mathrm{j}) \to (\mathrm{e}, \mu, 2,1)$: $\boldsymbol{arg}(\boldsymbol{U}_{\alpha j} \boldsymbol{U}_{\alpha i}^*) - \boldsymbol{arg}(\boldsymbol{U}_{\sigma j} \boldsymbol{U}_{\sigma i}^*) \to \beta_1 - \beta_2$

2-2: The extraction of Majorana phases from the second time derivatives of Lepton family numbers

To see the connection to leptogenesis later, we show the results with two active neutrinos family here.

- $\begin{pmatrix} \nu_{L\alpha} \\ l_{L\alpha} \end{pmatrix} = \left\{ \begin{pmatrix} \nu_{Le} \\ l_{Le} \end{pmatrix}, \begin{pmatrix} \nu_{L\mu} \\ l_{L\mu} \end{pmatrix} \right\}$, $m_{\nu\alpha\beta} = \begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} \\ m_{\nu e\mu} & m_{\nu\mu\mu} \end{pmatrix}$

- $U_{\alpha i} = \begin{pmatrix} c_{12} & -s_{12} e^{i\frac{\alpha_{21}}{2}} \\ s_{12} & c_{12} e^{i\frac{\alpha_{21}}{2}} \end{pmatrix}$ $s_{12} = \sin\theta_{12}$ (we take $\theta_{12} = -\frac{\pi}{4}$)

- $\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.42 \times 10^{-5} (eV^2)$

- The remaining parameters are the lightest neutrino mass m_1 and a Majorana phase $\alpha_{21} = -2\beta_1$. $(\beta_2 = \beta_1 + \pi)$

Time evolution of lepton number (2 families Case)

$$\langle\nu_e|\,L_e(t)\,|\nu_e\rangle = c_{12}^4\left(1-\frac{2m_1^2\sin^2(E_1t)}{E_1^2}\right)+s_{12}^4\left(1-\frac{2m_2^2\sin^2(E_2t)}{E_2^2}\right)$$
$$+s_{12}^2c_{12}^2\left\{\left(1+\frac{q^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1-E_2)t\}\right.$$
$$\left.+\left(1-\frac{q^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1+E_2)t\}\right\}$$

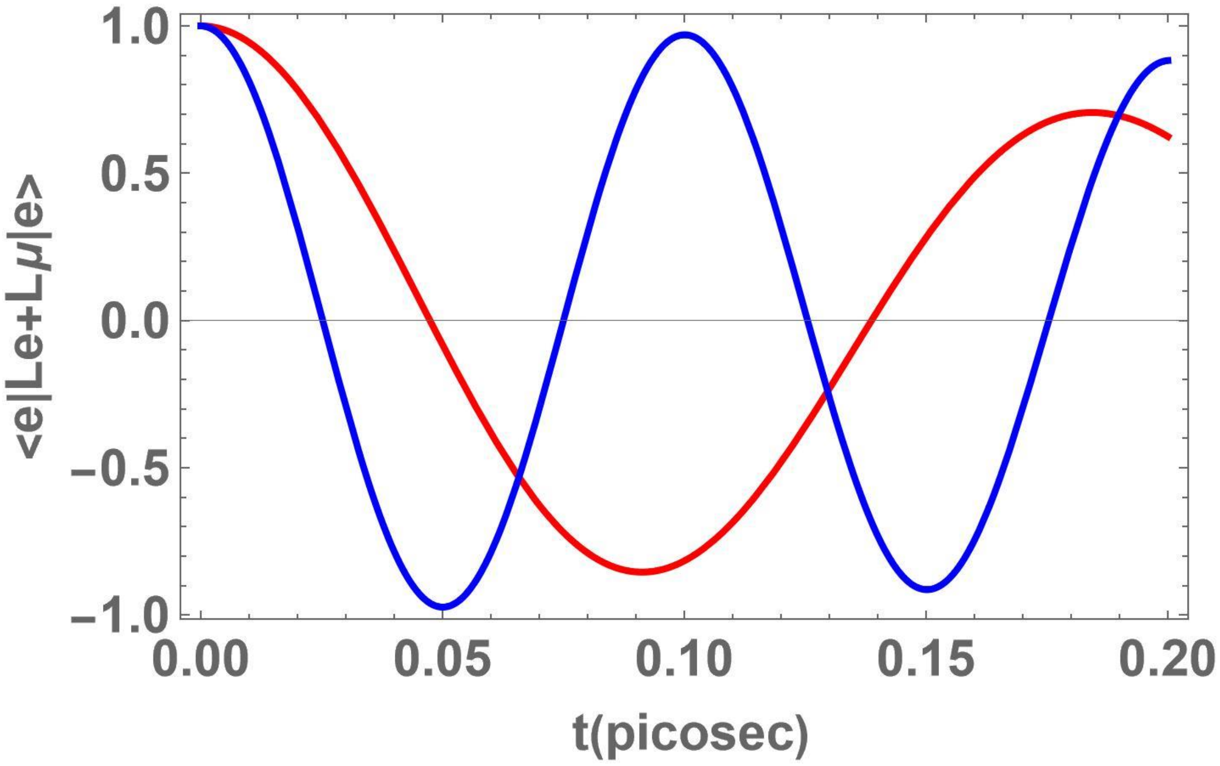
$$\langle\nu_e|L_\mu(t)|\nu_e\rangle = c_{12}^2s_{12}^2\left(\left(1-\frac{2m_1^2\sin^2(E_1t)}{E_1^2}\right)+\left(1-\frac{2m_2^2\sin^2(E_2t)}{E_2^2}\right)\right)$$
$$-s_{12}^2c_{12}^2\left\{\left(1+\frac{q^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1-E_2)t\}\right.$$
$$\left.+\left(1-\frac{q^2-m_1m_2\cos(\alpha_{21})}{E_1E_2}\right)\cos\{(E_1+E_2)t\}\right\}.$$

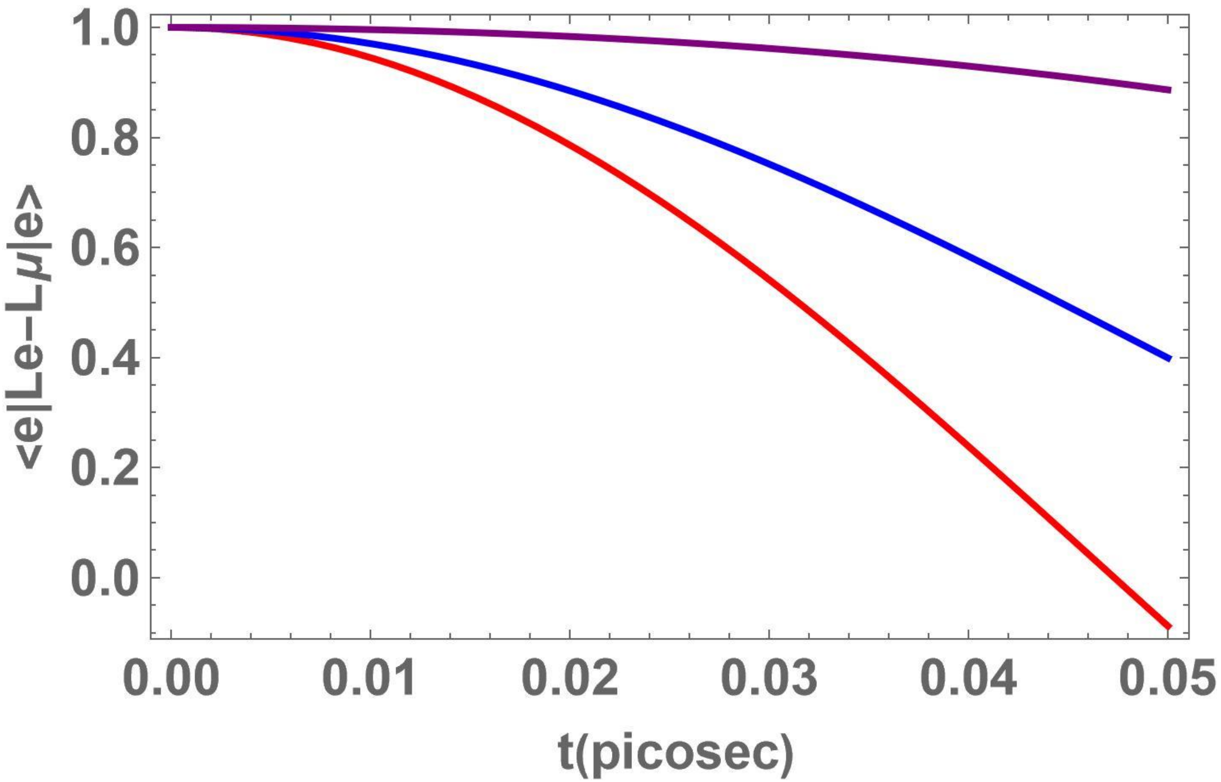
Total Lepton Number

$$\langle e|L_e(t)+L_\mu(t)|e\rangle$$
$$=\langle\nu_e|L_e(t)|\nu_e\rangle+\langle\nu_e|L_\mu(t)|\nu_e\rangle$$

electron-muon number

$$\langle e|L_{e-\mu}(t)|e\rangle$$
$$=\langle\nu_e|L_e(t)|\nu_e\rangle-\langle\nu_e|L_\mu(t)|\nu_e\rangle$$





Red : m_1=**0.01**(eV) **Blue** : m_1=**0.02**(eV)

Red **:** $\boldsymbol{\alpha_{21}=0}$ **Blue** : $\boldsymbol{\alpha_{21}=\frac{\pi}{2}}$ **Purple**: $\boldsymbol{\alpha_{21}=\pi}$

$$\frac{d^2}{\mathrm{dt}^2}\langle\nu_e|L(t)|\nu_e\rangle\Big|_{t=0} = -4(m_1^2c_{12}^2 + m_2^2s_{12}^2)$$

$$\frac{d^2}{\mathrm{dt}^2}\langle\nu_e|L_{e-\mu}(t)|\nu_e\rangle\Big|_{t=0} = -4|m_{ee}|^2$$

- strong dependence on the lightest neutrino mass m_1^2
- strong dependence on the Majorana phase α_{21} through $|m_{ee}|$component of the effective Majorana mass matrix .

(Similar to the neutrino-less double β decay)

$$|m_{ee}|^2 = |m_1c_{12}^2 + m_2s_{12}^2e^{-i\alpha_{21}}|^2 = m_1^2c_{12}^4 + m_2^2s_{12}^4 + 2m_1m_2c_{12}^2s_{12}^2\cos(\alpha_{21})$$

3. The link to lepton family asymmetries of leptogensis for the seesaw model with two right-handed neutrinos (under revision project)

So far we study the relation between the low energy parameters such as the lightest neutrino mass and Majorana type phase and low energy physical observables of lepton family numbers. The next question is:

- If the effective Majorana mass matrix is generated from the seesaw model, what can we learn about the high energy phenomena such as leptogenesis (Fukugita, Yanagida, PLB174(1986)?

- Conversely, if the seasaw's parameters are specified, what would be the predictions of the low energy observables ?

- We try to answer to this question for the case with two active neutrino and two gauge singlet neutrinos, (2,2) model.

Two Steps to show the answer

3-1. The effective neutrino mass matrix and low energy parameters
3-2. The seesaw and the effective mass matrix at low energies

3-1. The effective Majorana mass matrix and low energy parameters

- **This two by two effective Majorana matrix has four parameters after changing weak basis:**
- $\left\{\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix},\begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}\right\}\rightarrow\left\{e^{-i\frac{\theta_{ee}}{2}}\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix},e^{-i\frac{\theta_{\mu\mu}}{2}}\begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}\right\}$ $\theta_{\alpha\beta}=arg(m_{\alpha\beta})$

$$\begin{pmatrix}m_{ee} & m_{e\mu}\\ m_{e\mu} & m_{\mu\mu}\end{pmatrix}\rightarrow\begin{pmatrix}|m_{ee}| & |m_{e\mu}|e^{i\theta'_{e\mu}}\\ |m_{e\mu}|e^{i\theta'_{e\mu}} & |m_{\mu\mu}|\end{pmatrix},\theta'_{e\mu}=\theta_{e\mu}-\frac{\theta_{ee}+\theta_{\mu\mu}}{2},$$

$$\begin{pmatrix} |m_{ee}| & |m_{e\mu}|e^{i\theta'_{e\mu}} \\ |m_{e\mu}|e^{i\theta'_{e\mu}} & |m_{\mu\mu}| \end{pmatrix}$$

- The matrix contains the same number of parameters as the mass basis, i.e., a mixing angle($\boldsymbol{\theta_{12}}$): a Majorana phase $\boldsymbol{\alpha_{21}}$ and two mass eigenvalues (m_1, m_2).
- $U_{\alpha i} = \begin{pmatrix} c_{12} & -s_{12}e^{i\frac{\alpha_{21}}{2}} \\ s_{12} & c_{12}e^{i\frac{\alpha_{21}}{2}} \end{pmatrix}$, $s_{12} = \sin\theta_{12}$
- Mass basis parameters with moduli $|m_{\alpha\beta}|$ and a phase $\theta'_{e\mu}$

$$\sin 2\theta_{12} = -\frac{2|m_{e\mu}|\sqrt{(|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}m_{\mu\mu}|\cos(2\theta'_{e\mu}))}}{\Delta m^2_{21}} < 0$$

$$\Delta m^2_{21} = \sqrt{(|m_{ee}|^2 - |m_{\mu\mu}|^2)^2 + 4|m_{e\mu}|^2(|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}m_{\mu\mu}|\cos(2\theta'_{e\mu}))}$$

$$m_2^2 + m_1^2 = |m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{e\mu}|^2$$

$$\alpha_{21} = \tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu} + \Delta m^2_{21}}\right] - \tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu} - \Delta m^2_{21}}\right]$$

3-2. Effective Majorana mass matrix in a seesaw

- We assume that $\boldsymbol{m}_{\alpha\beta}$ generated by the type I seesaw with two heavy right-handed neutrinos N_1 and N_2 with mass M_1 and M_2 .
- With Dirac-Yukawa mass matrix $m_{D\alpha i} = \begin{pmatrix} \boldsymbol{\nu_{Le}} \\ \boldsymbol{\nu_{L\mu}} \end{pmatrix} \overset{\quad N_1 \qquad N_2}{\begin{bmatrix} m_{De1} & m_{De2} \\ m_{D\mu 1} & m_{D\mu 2} \end{bmatrix}}$,
- $\boldsymbol{m}_{\alpha\beta} = -\sum_{\boldsymbol{i=1}}^{\boldsymbol{2}} m_{D\alpha i} \frac{1}{M_i} m_{D\beta i} = - \sum_{\boldsymbol{i=1}}^{\boldsymbol{2}} u_{\alpha i} X_i\, u_{\beta i},\quad : X_i \equiv \frac{{m_{Di}}^2}{M_i}$
- $\begin{bmatrix} m_{De1} & m_{De2} \\ m_{D\mu 1} & m_{D\mu 2} \end{bmatrix} = \begin{bmatrix} u_{e1} & u_{e2} \\ u_{\mu 1} & u_{\mu 2} \end{bmatrix} \begin{bmatrix} m_{D1} & 0 \\ 0 & m_{D2} \end{bmatrix}$: $m_{D\alpha i} = u_{\alpha i} m_{Di}$, $\sum_{\boldsymbol{\alpha=e}}^{\mu} |u_{\alpha i}|^{\boldsymbol{2}} = 1$
- With a proper weak basis, one can choose $\begin{bmatrix} u_{e1} & u_{e2} \\ u_{\mu 1} & u_{\mu 2} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 e^{i\varphi_e} \\ S_1 & S_2 e^{i\varphi_\mu} \end{bmatrix}$

$\boldsymbol{m_{\alpha\beta}}$ in the seesaw model and Majorana phase

- **Three elements of $\boldsymbol{m_{\alpha\beta}}$ written with 6 parameters:** $X_i \equiv \frac{{m_{Di}}^2}{M_i}$, $\begin{bmatrix} u_{e1} & u_{e2} \\ u_{\mu 1} & u_{\mu 2} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 e^{i\varphi_e} \\ S_1 & S_2 e^{i\varphi_\mu} \end{bmatrix}$

  $\boldsymbol{X_1\ X_2\ S_1\ S_2\ \varphi_1 \varphi_2}$**(two CP phases)**

$$\boldsymbol{m_{ee} = |m_{ee}| e^{i\theta_{ee}} = -C_1^2 X_1 - C_2^2 e^{2i\varphi_e} X_2}$$

$$\boldsymbol{m_{\mu\mu} = |m_{\mu\mu}| e^{i\theta_{\mu\mu}} = -S_1^2 X_1 - S_2^2 e^{2i\varphi_\mu} X_2}$$

$$\boldsymbol{m_{e\mu} = |m_{e\mu}| e^{i\theta_{e\mu}} = -C_1 S_1 X_1 - C_2 S_2 e^{i(\varphi_e + \varphi_\mu)} X_2}$$

A phase combination of effective mass, $\boldsymbol{\theta'_{e\mu} = \theta_{e\mu} - \frac{\theta_{ee} + \theta_{\mu\mu}}{2}}$, relevant for Majorana phase

$$\theta'_{e\mu} = tan^{-1} \frac{sin(\varphi_e + \varphi_\mu)}{\frac{C_1 S_1 x_1}{C_2 S_2 x_2} + cos\,(\varphi_e + \varphi_\mu)} - \frac{1}{2} \left(tan^{-1} \frac{sin(2\varphi_e)}{\frac{C_1^2 x_1}{C_2^2 x_2} + cos\,(2\varphi_e)} + tan^{-1} \frac{\sin(2\varphi_\mu)}{\frac{S_1^2 x_1}{S_2^2 x_2} + \cos(2\varphi_\mu)} \right)$$

$$\alpha_{21} = \tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu} + \Delta m_{21}^2}\right] - \tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2 + |m_{\mu\mu}|^2 + 2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu} - \Delta m_{21}^2}\right]$$

[PTP111(2004), T. Endoh, T. Morozumi, .Z.H. Xiong]

Lepton family asymmetries from the decays of the heavy Majorana neutrinos

$$\epsilon_\alpha^k = \frac{\Gamma[N_k \to l_\alpha \phi] - \Gamma[N_k \to \bar{l_\alpha} \bar{\phi}]}{\Gamma[N_k \to l_\alpha \phi] + \Gamma[N_k \to \bar{l_\alpha} \bar{\phi}]}$$

$$x_{k'k} = \frac{M_{k'}^2}{M_k^2}$$

$$\epsilon_\alpha^k = \frac{1}{4\pi v^2}\left[I(x_{k'k})\,\mathrm{Im}\frac{(m_D^\dagger m_D)_{\mathrm{kk'}}(m_D)^*_{\alpha k}(m_D)_{\alpha k'}}{|(m_D)_{\alpha k}|^2} + \frac{1}{1-x_{k'k}}\mathrm{Im}\frac{(m_D^\dagger m_D)_{k'k}(m_D)^*_{\alpha k}(m_D)_{\alpha k'}}{|(m_D)_{\alpha k}|^2}\right]$$

$$\epsilon_\alpha^k = \frac{(m_{\mathrm{Dk'}})^2}{4\pi v^2}\left[I(x_{k'k})\frac{\sum_\beta \mathrm{Im}\,(u^*_{\beta k}u_{\beta k'}u^*_{\alpha k}u_{\alpha k'})}{|u^*_{\alpha k}|^2} + \frac{1}{1-x_{k'k}}\mathrm{Im}\frac{\sum_\beta \mathrm{Im}\,(u^*_{\beta k'}u_{\beta k}u^*_{\alpha k}u_{\alpha k'})}{|u^*_{\alpha k}|^2}\right](k' \neq k)$$

Lepton family asymmetries from $\boldsymbol{N_1}$ decay.

$$\epsilon_e^1 = \frac{(m_{D2})^2 C_2^2}{4\pi v^2}\left[I(x_{21})\left(\frac{S_1}{C_1}\frac{S_2}{C_2}\sin(\varphi_\mu+\varphi_e)+\sin(2\varphi_e)\right)+\frac{1}{1-x_{21}}\frac{S_1}{C_1}\frac{S_2}{C_2}\sin(\varphi_e-\varphi_\mu))\right]$$

$$\epsilon_\mu^1 = \frac{(m_{D2})^2 S_2^2}{4\pi v^2}\left[I(x_{21})\left(\frac{C_1}{S_1}\frac{C_2}{S_2}\sin(\varphi_\mu+\varphi_e)+\sin(2\varphi_\mu)\right)-\frac{1}{1-x_{21}}\frac{C_1 C_2}{S_1 S_2}\sin(\varphi_e-\varphi_\mu))\right]$$

$$\theta'_{e\mu} = tan^{-1}\frac{sin(\varphi_e+\varphi_\mu)}{\frac{C_1 S_1 x_1}{C_2 S_2 x_2}+cos\,(\varphi_e+\varphi_\mu)} - \frac{1}{2}\left(tan^{-1}\frac{sin(2\varphi_e)}{\frac{C_1^2 x_1}{C_2^2 x_2}+cos\,(2\varphi_e)} + tan^{-1}\frac{\sin(2\varphi_\mu)}{\frac{S_1^2 x_1}{S_2^2 x_2}+\cos(2\varphi_\mu)}\right)$$

$$\alpha_{21} = \tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2+|m_{\mu\mu}|^2+2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu}+\Delta m_{21}^2}\right]-\tan^{-1}\left[\frac{2|m_{ee}||m_{\mu\mu}|\sin 2\theta'_{e\mu}}{|m_{ee}|^2+|m_{\mu\mu}|^2+2|m_{ee}||m_{\mu\mu}|\cos 2\theta'_{e\mu}-\Delta m_{21}^2}\right]$$

Since there are two independent CP violating phases in the seesaw while the effective description at low energies includes only one phase, it is impossible to constrain both phases from the knowledge of the single CP violating phase at low energy.

Some special examples that the direct connection between low and high energy CPV is possible

$$\theta'_{e\mu} = \tan^{-1}\frac{\sin(\varphi_e+\varphi_\mu)}{\frac{C_1S_1X_1}{C_2S_2X_2}+\cos(\varphi_e+\varphi_\mu)} - \frac{1}{2}\left(\tan^{-1}\frac{\sin(2\varphi_e)}{\frac{C_1^2X_1}{C_2^2X_2}+\cos(2\varphi_e)} + \tan^{-1}\frac{\sin(2\varphi_\mu)}{\frac{S_1^2X_1}{S_2^2X_2}+\cos(2\varphi_\mu)}\right)$$

Let us consider the following cases $(C_2=0(A), S_2=0(B))$

$$\begin{pmatrix} C_1 & C_2e^{i\varphi_e} \\ S_1 & S_2e^{i\varphi_\mu} \end{pmatrix} \to \begin{pmatrix} C_1 & 0 \\ S_1 & e^{i\varphi_\mu} \end{pmatrix}(A), \begin{pmatrix} C_1 & e^{i\varphi_e} \\ S_1 & 0 \end{pmatrix}(B)$$

$$\begin{pmatrix} \epsilon_e^1 \\ \epsilon_\mu^1 \end{pmatrix} \to \begin{pmatrix} 0 \\ \frac{(m_{D2})^2}{4\pi v^2}I(x_{21})\sin(2\varphi_\mu) \end{pmatrix}(A), \begin{pmatrix} \frac{(m_{D2})^2}{4\pi v^2}I(x_{21})\sin(2\varphi_e) \\ 0 \end{pmatrix}(B)$$

$$\theta'_{e\mu} \to -\frac{1}{2}\tan^{-1}\frac{\sin(2\varphi_\mu)}{\frac{S_1^2X_1}{X_2}+\cos(2\varphi_\mu)}(A), -\frac{1}{2}\tan^{-1}\frac{\sin(2\varphi_e)}{\frac{C_1^2X_1}{X_2}+\cos(2\varphi_e)}(B)$$

Conclusion

- The absolute phases of each side of the Unitarity triangles for the active Majorana neutrinos have physical meaning.
- It is also shown that the single Majorana phase in two generation model can be determined via the time derivatives of lepton family numbers.
- In the light of the seesaw model with two heavy and two light neutrinos, we derive the relation between the single phase at low energy and the parameters of the seesaw model including two CPV phases at high energy.
- When leptogensis is dominated by a single family asymmetry, the CPV at high energy and low energy phase are directly related. (Fujihara, Kaneko, Kang,Kimura, Morozumi,Tanimoto, PRD72(2005))