The quantumness of particle decays

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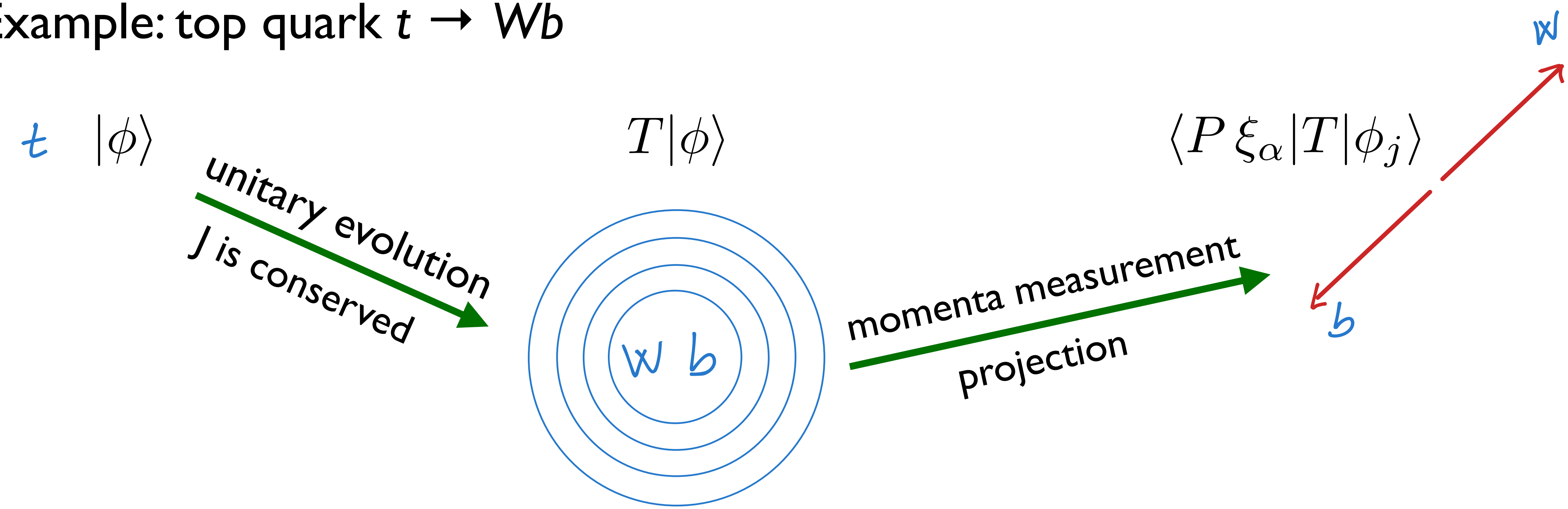
What is a decay?

Deconstructing particle decay

What does decay *mean* in a particle detector?

Example: top quark $t \to Wb$



$$J = L + S_1 + S_2$$

Detectors measure momenta in the quantum-mechanical sense.

They do not measure spin.

The measurement of momenta influences the spin state but in general it does not collapse it as a Stern-Gerlach experiment would do.

This leads to surprising quantum effects yet untested

Deconstructing particle decay

General states are described by a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

One can fully characterise the effects of a particle decay

$$A \rightarrow A_1 \, A_2 \, \ldots$$

by specifying how the post-decay operator ρ' relates to the initial one ρ

➡ we will focus on spin degrees of freedom

Deconstructing particle decay

Consider a system of two particles *A*, *B*, with spin state described by

$$\rho = \sum_{ijkl} \rho_{ij}^{kl} |\phi_i \chi_k\rangle\langle\phi_j \chi_l| \qquad |\phi_i\rangle \in \mathcal{H}_A\,, \quad |\chi_k\rangle \in \mathcal{H}_B$$

$\mathcal{H}$ are the spin spaces

Let *A* decay *A* → A_1 A_2 … with amplitudes

$$M_{\alpha j} = \langle P\,\xi_\alpha|T|\phi_j\rangle \qquad |\xi_\alpha\rangle \in \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \ldots$$

Then, the spin state of A_1 A_2 … and *B* is described by

these come from the projector

$$\rho' = \frac{1}{\sum_{\alpha k}(M\rho^{kk}M^\dagger)_{\alpha\alpha}} \sum_{\alpha\beta kl} (M\rho^{kl}M^\dagger)_{\alpha\beta} |\xi_\alpha\,\chi_k\rangle\langle\xi_\beta\,\chi_l|$$

in particular, the entanglement properties between *A* and *B* can be inherited by { the decay products of *A* } vs *B*

A post-selection experiment

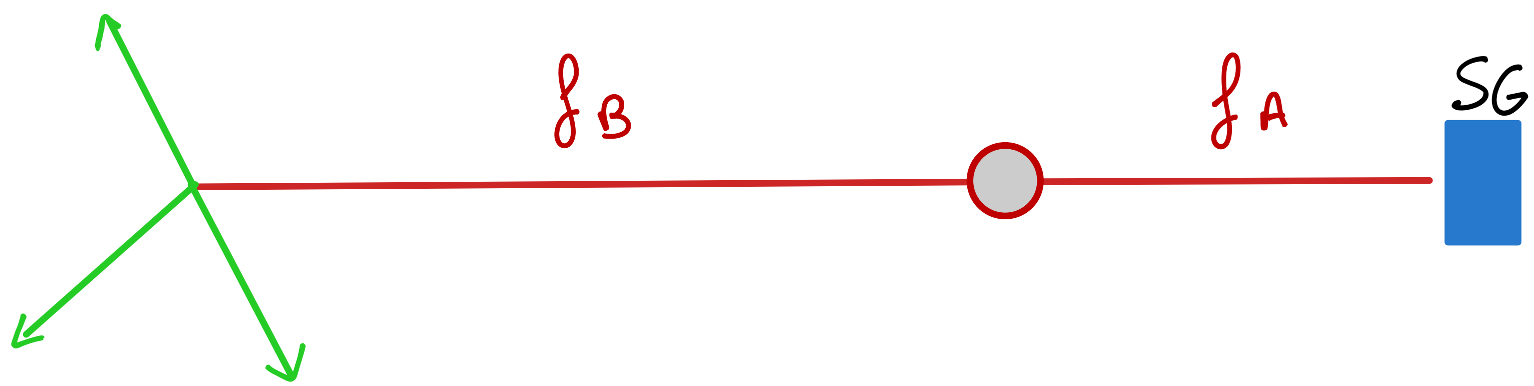
[JAAS 2308.07412]

A pre-selection experiment

Assume fermion pairs f_A f_B produced in an entangled state, say

$$\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$$

We perform a Stern-Gerlach experiment on f_A, and after that, f_B decays



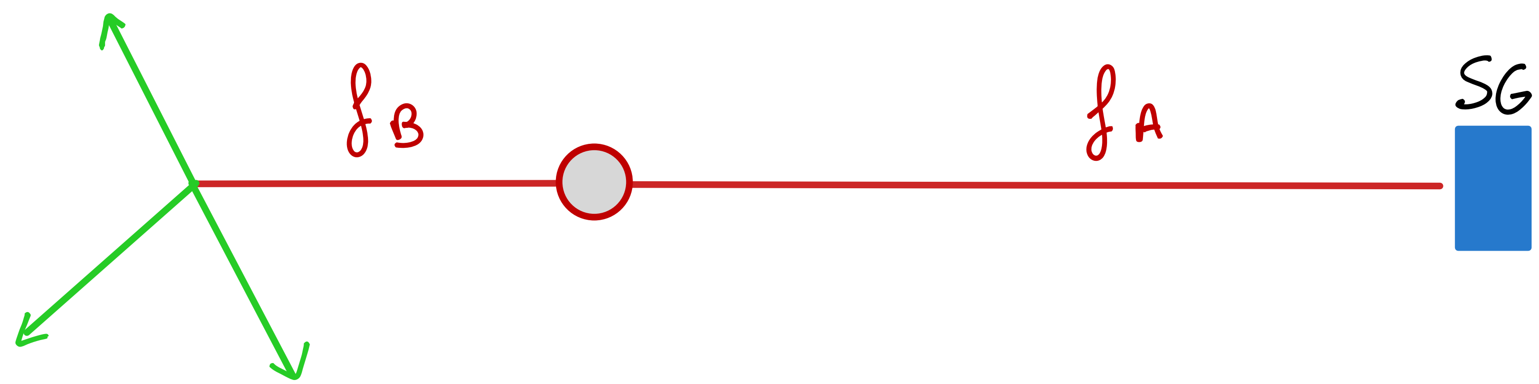
We select the subset of f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

Then, the decay distribution of those pre-selected f_B corresponds to having spin $|\downarrow\rangle$

A post-selection experiment

Remarkably, the same happens time-backwards:

f_B decays and after that, we perform a Stern-Gerlach experiment on f_A



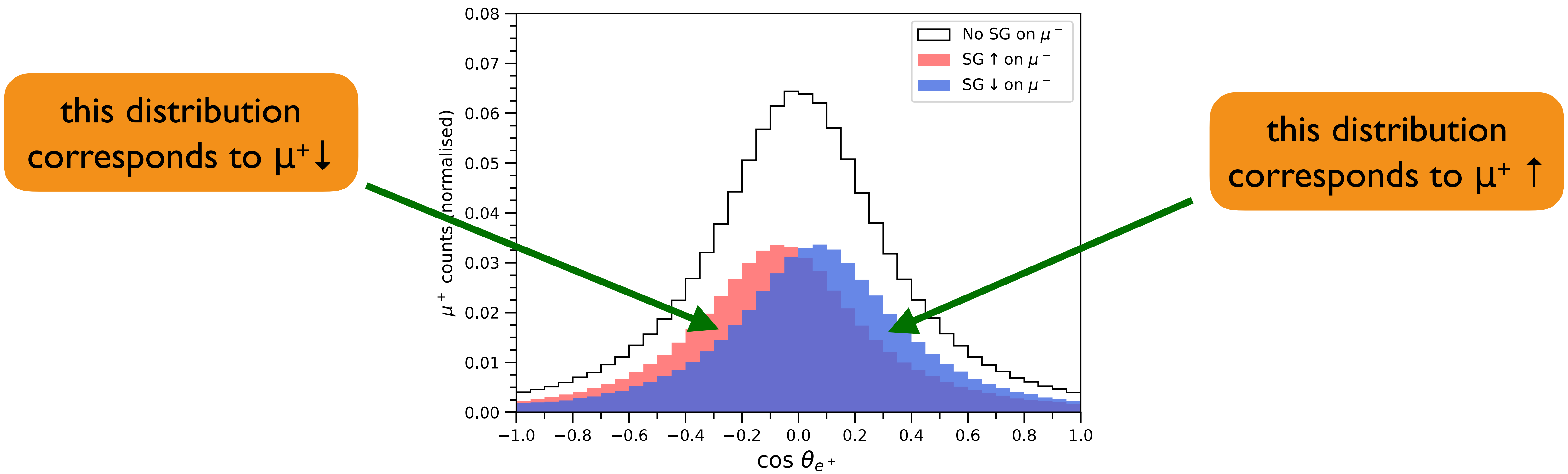
We select the subset of f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

Then, the decay distribution of those f_B that had decayed before the outcome of the SG experiment corresponds to having spin $|\downarrow\rangle$

Interlude: a post-selection experiment

This experiment can be performed with low-energy $\mu^+\mu^-$ pairs produced in Drell-Yan or from the decay of a η meson

The muon polarisation can be measured from the daughter electron

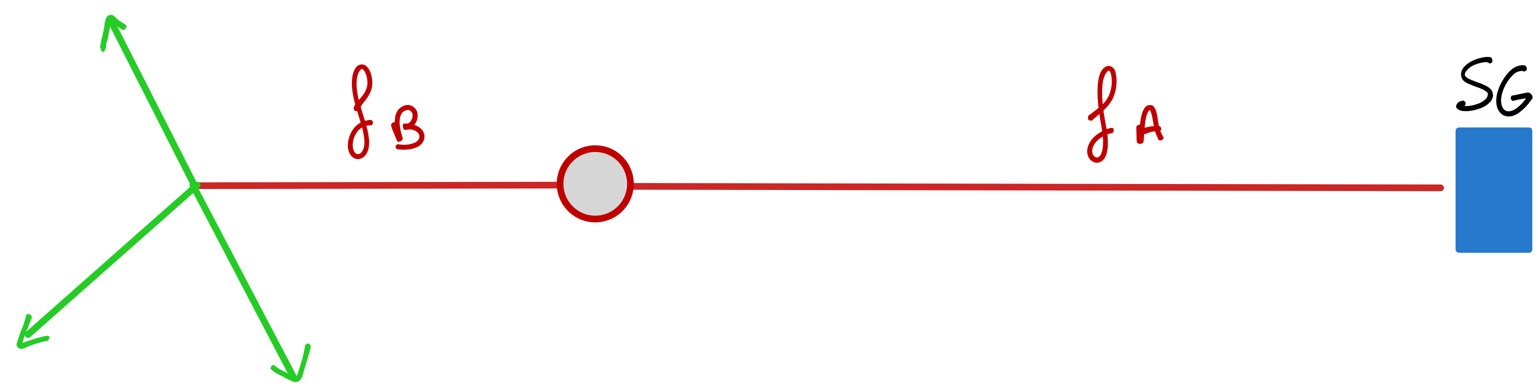


Spooky EPR action to the past? Not really. [Only PRL referees are allowed to violate causality.]

A post-selection experiment

The initial state is $\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$ and if we do a SG on f_A before f_B decays, we get up or down with equal probability.

The decay of f_B projects f_A into a state $a_+|\uparrow\rangle + a_-|\downarrow\rangle$ with a_+, a_- depending on the decay configuration. The probability to have SG up or down is not the same.



f_B decay configuration *more compatible* with spin $|\downarrow\rangle$

time

higher probability that SG on f_A gives $|\uparrow\rangle$

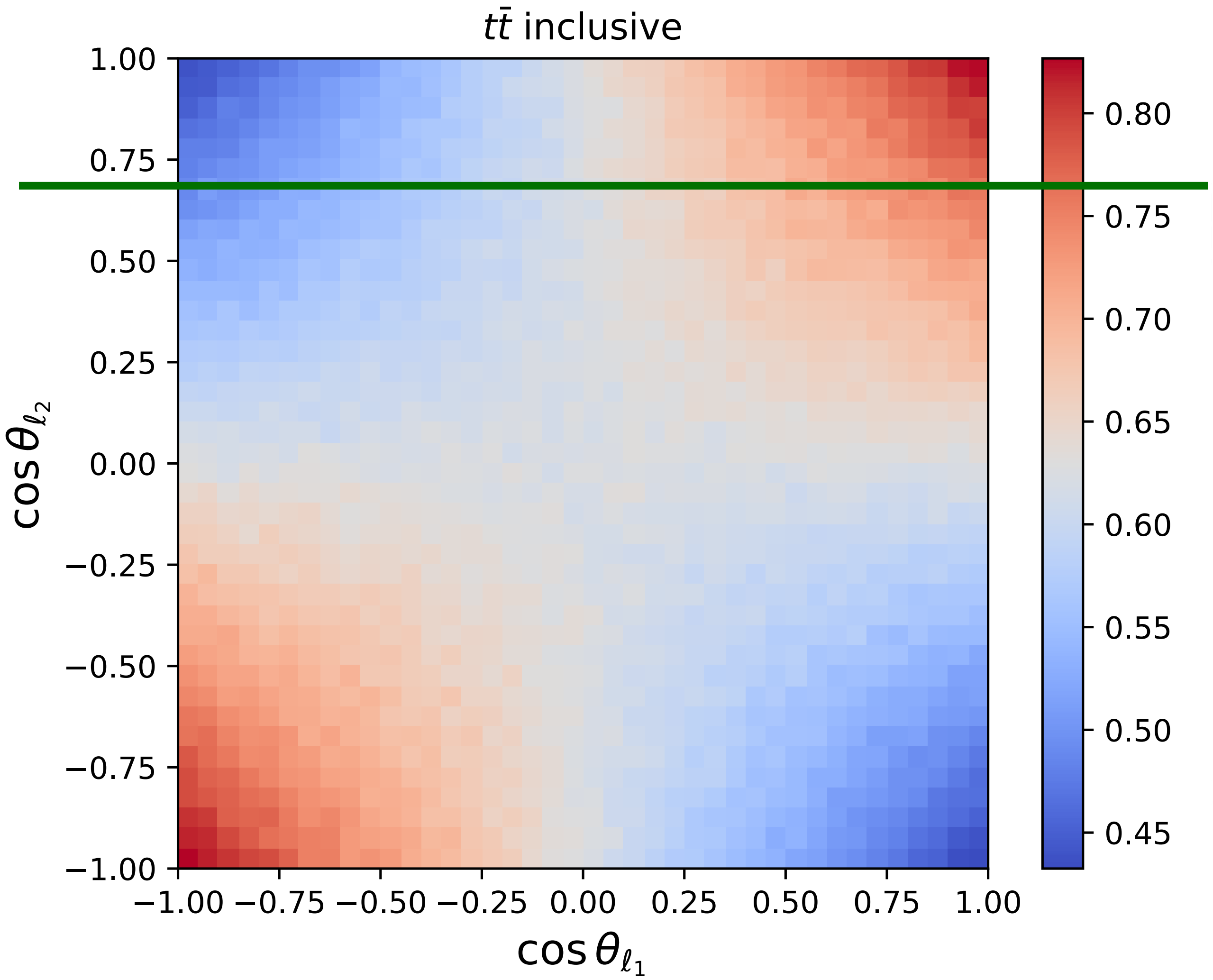
Then, when we post-select events where SG gives $|\uparrow\rangle$, we recover f_B decay distributions just as if f_B had spin $|\downarrow\rangle$ when it decayed. Beautiful and amazing.

A post-selection experiment

Is this effect genuinely quantum? [the quantumness has to be proved]

Consider for example inclusive t t-bar production, label as t_1 the quark that decays first, t_2 the one that decays last, and ℓ_1, ℓ_2 their daughter leptons.

$\theta_{\ell_1}\,, \theta_{\ell_2}$: angles between leptons and top helicity direction



A selection on the latter decay results in a selection on the first decay.

But this happens despite the t t-bar spin state state being separable.

Nothing to do with quantum entanglement!

A post-selection experiment

Definitive test: CHSH inequalities

A useful formulation of Bell-like inequalities for spin-1/2 systems is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables *A*, *A´*. Bob measures two spin observables *B*, *B´*. [Both normalised to unity]. Then, clasically:

$$|\langle AB \rangle - \langle AB' \rangle + \langle A'B \rangle + \langle A'B' \rangle| \leq 2$$

these are spin correlation observables

A post-selection experiment

For the spin-singlet state

$$A = \sigma_3\,, \quad A' = \sigma_2\,, \quad B = \frac{1}{\sqrt{2}}(\sigma_2 + \sigma_3)\,, \quad B' = \frac{1}{\sqrt{2}}(\sigma_2 - \sigma_3)$$

How is it measured?

- Bob registers μ^+ decay
- Alice chooses whether to measure spin in $\hat{z}$ or $\hat{y}$ axis for μ^-
- Expected values for Bob are calculated for each choice of Alice, e.g.

$$\langle AB \rangle = \frac{1}{2}\left[\langle B \rangle_\uparrow - \langle B \rangle_\downarrow\right]$$

$\langle B \rangle$ when Alice gets $\uparrow$

$\langle B \rangle$ when Alice gets $\downarrow$

- It turns out that $\langle AB \rangle = -\langle AB' \rangle = \langle AB' \rangle = \langle A'B' \rangle = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ $|\cdots| = 2\sqrt{2} > 2$ CHSH inequality violated

Post-decay entanglement

[JAAS 2307.06991, 2401.10998]

Post-decay entanglement

A density operator describing a composite system is separable if it can be written as

$$\rho_{\text{sep}} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator ρ^{T2} is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \longrightarrow \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

Post-decay entanglement

It is quite complicated to prove that a composite system is in a separable state [extensive work on PPT entangled states]

However, we are interested in showing that the system is entangled.

For this, one can use the counter-reciprocal of Peres-Horodecki necessary condition

ρ^{T2} non-positive $\Rightarrow$ ρ^{T2} not valid $\Rightarrow$ system entangled

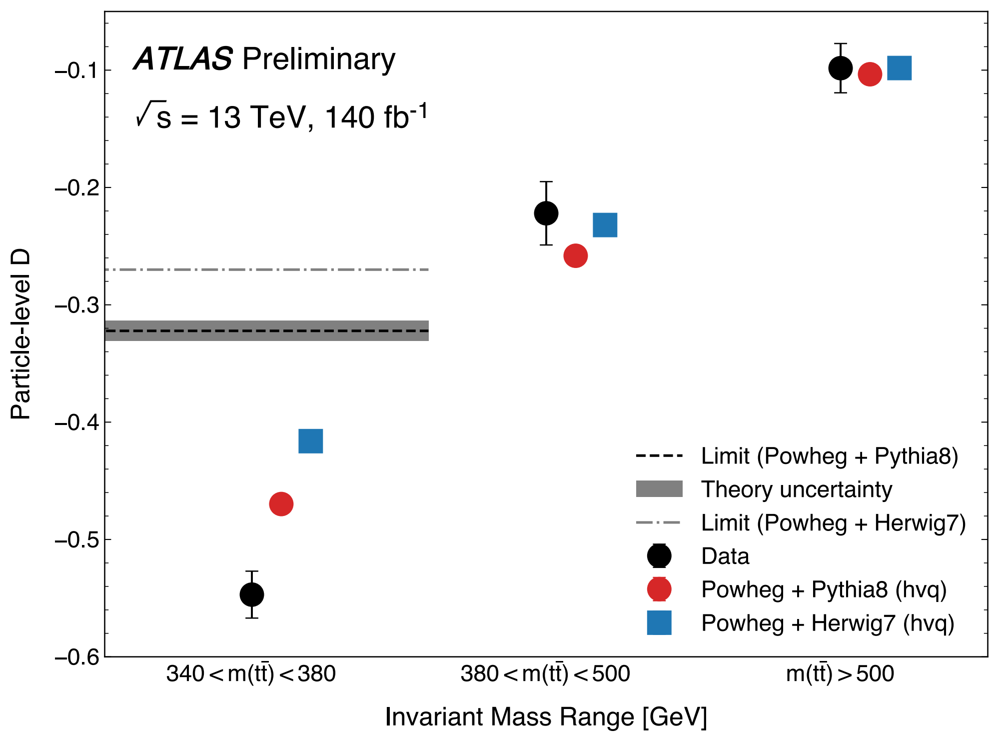
Associated to it, there is a measure of entanglement that can be used for general systems

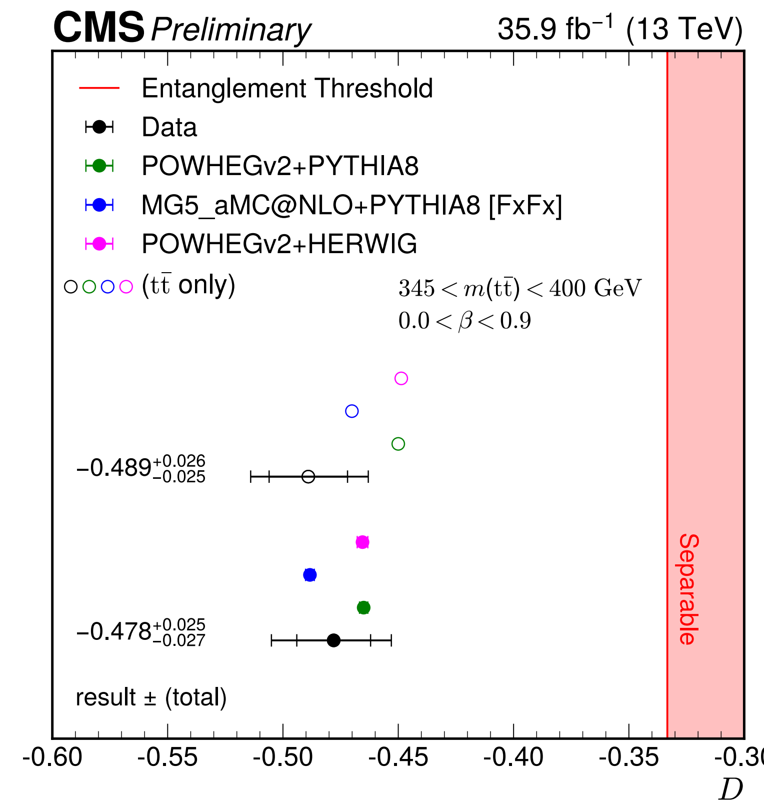
$$N(\rho) = \frac{\|\rho^{T_2}\| - 1}{2}$$

equals the sum of negative eigenvalues of ρ^{T2}

Post-decay entanglement

ATLAS and CMS have measured entanglement of *t t*-bar pairs produced near threshold at the LHC.





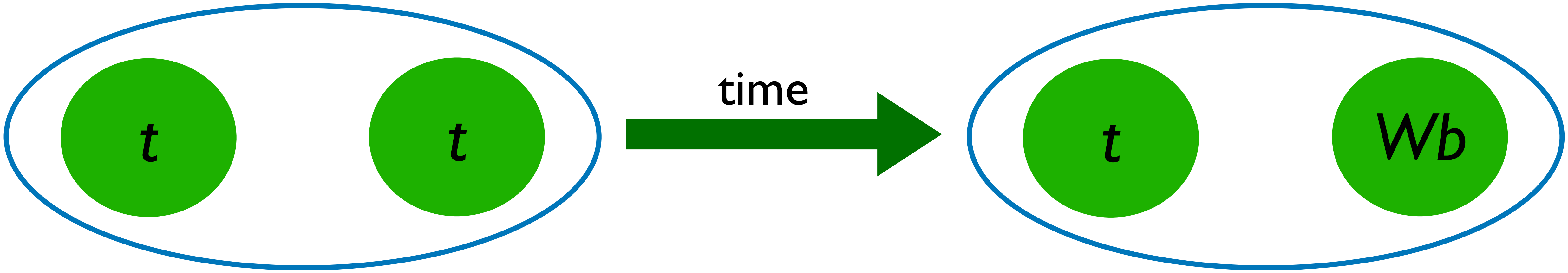
This is very nice, and these are the highest-energy measurements ever. But they are not *essentially different* from entanglement measurements between electron pairs.

Post-decay entanglement can be measured in top pair production too.

And this is a quantum effect that has never been tested

Post-decay entanglement

When *t* *t*-bar are entangled and *t*-bar decays into W^- *b*-bar, *t* is entangled with the W^- *b*-bar pair



Potential problem:

The *b* spin is, in principle, not measurable.

When we have several entangled particles and trace over [unobserved] degrees of freedom, entanglement may be lost.

but *b*-bar has RH helicity up to small mass effects, trace maintains entanglement

Post-decay entanglement

Threshold region $m_{tt} \leq 390$ GeV, $\beta \leq 0.9$, beamline basis z = (0,0,1)

θ 👉 angle between W^{-} momentum in *t*-bar rest frame and $\hat{z}$ axis *or any fixed axis*

entanglement measure

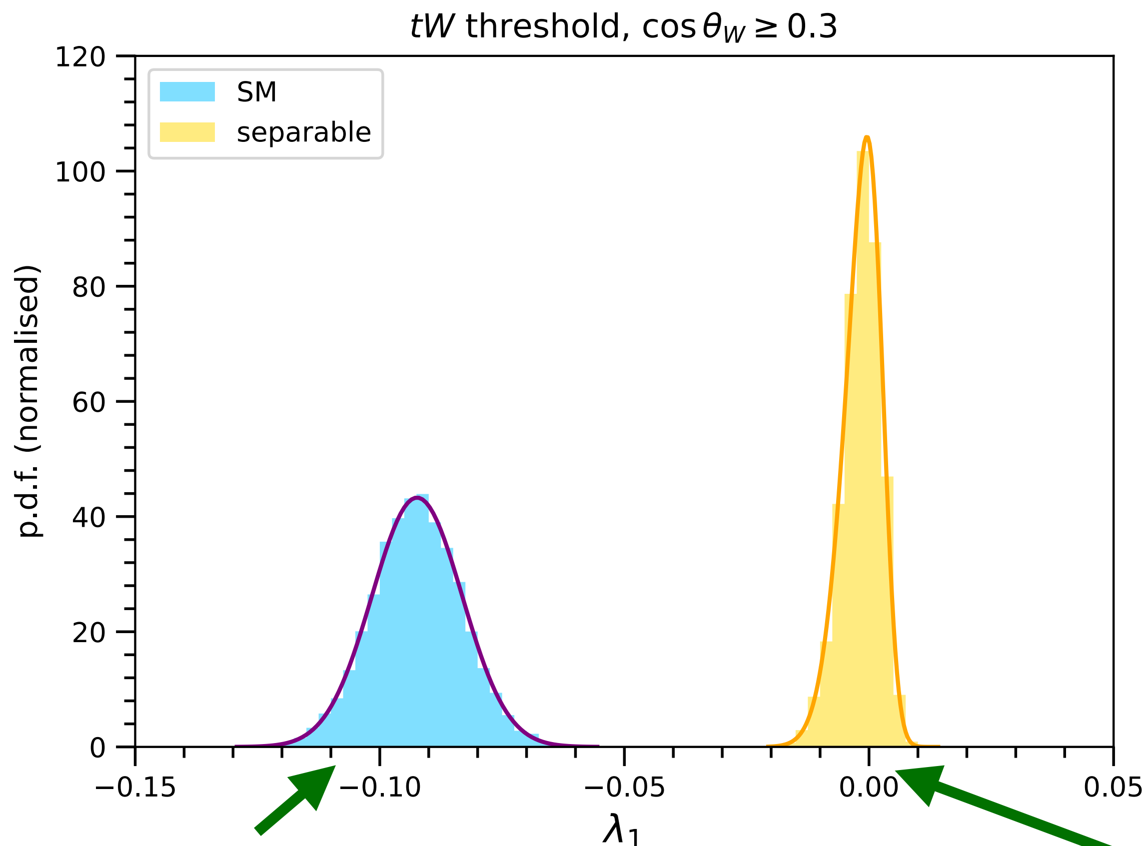
phase space region	N(ρ)
θ = 0	0.13
cos θ > 0.9	0.12
cos θ > 0.5	0.10
cos θ > 0	0.07
all θ	0

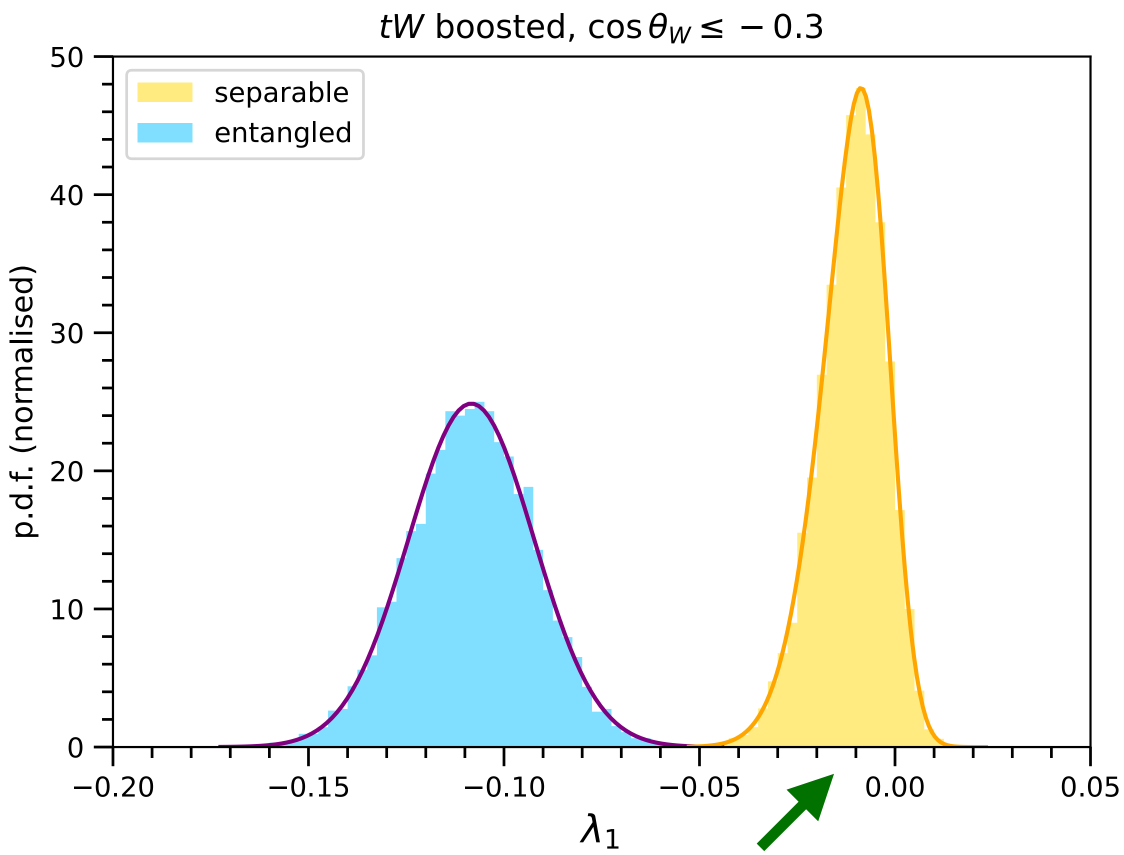
The amount of entanglement is the same in any direction but the quantum state is not, so integration washes out entanglement

Post-decay entanglement

Entanglement indicator:
lowest eigenvalue λ_l of the ρ^{T2} matrix for *tW*

$\lambda_l < 0 \Leftrightarrow$ Entanglement





stat uncertainty

Bias: even if $\lambda_l > 0$, in a small sample we may find it negative

Run 2	Significance [stat + 10% sys + bias]
Threshold	7.0 σ
Boosted	5.0 σ

these numbers can possibly be improved by combining several regions...

Entanglement autodistillation

[JAAS & Casas 2401.06854]

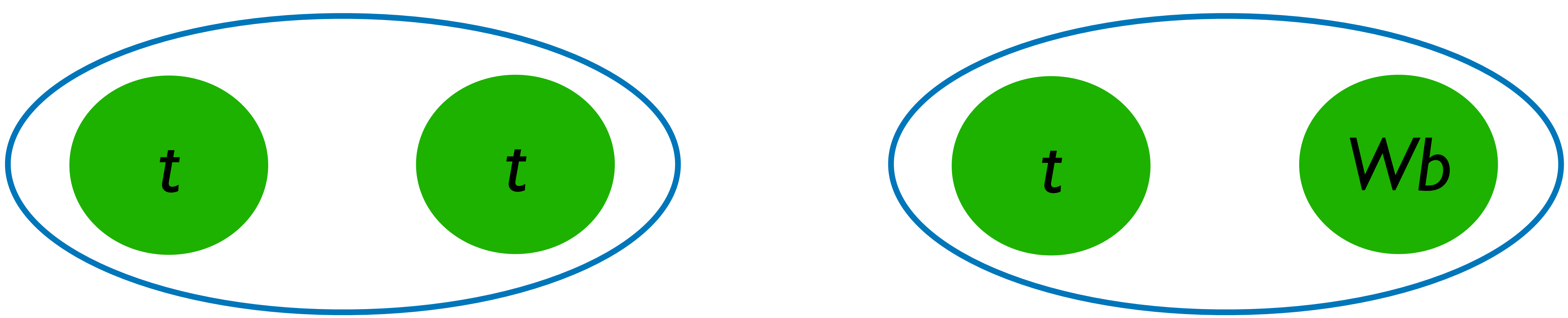
Entanglement autodistillation

Entanglement decreases by measurements [collapse], interaction with environment [decoherence] …

Methods are known [distillation] to manipulate a sub-system and, if lucky, increase entanglement

Most remarkably, the decay can increase entanglement spontaneously.

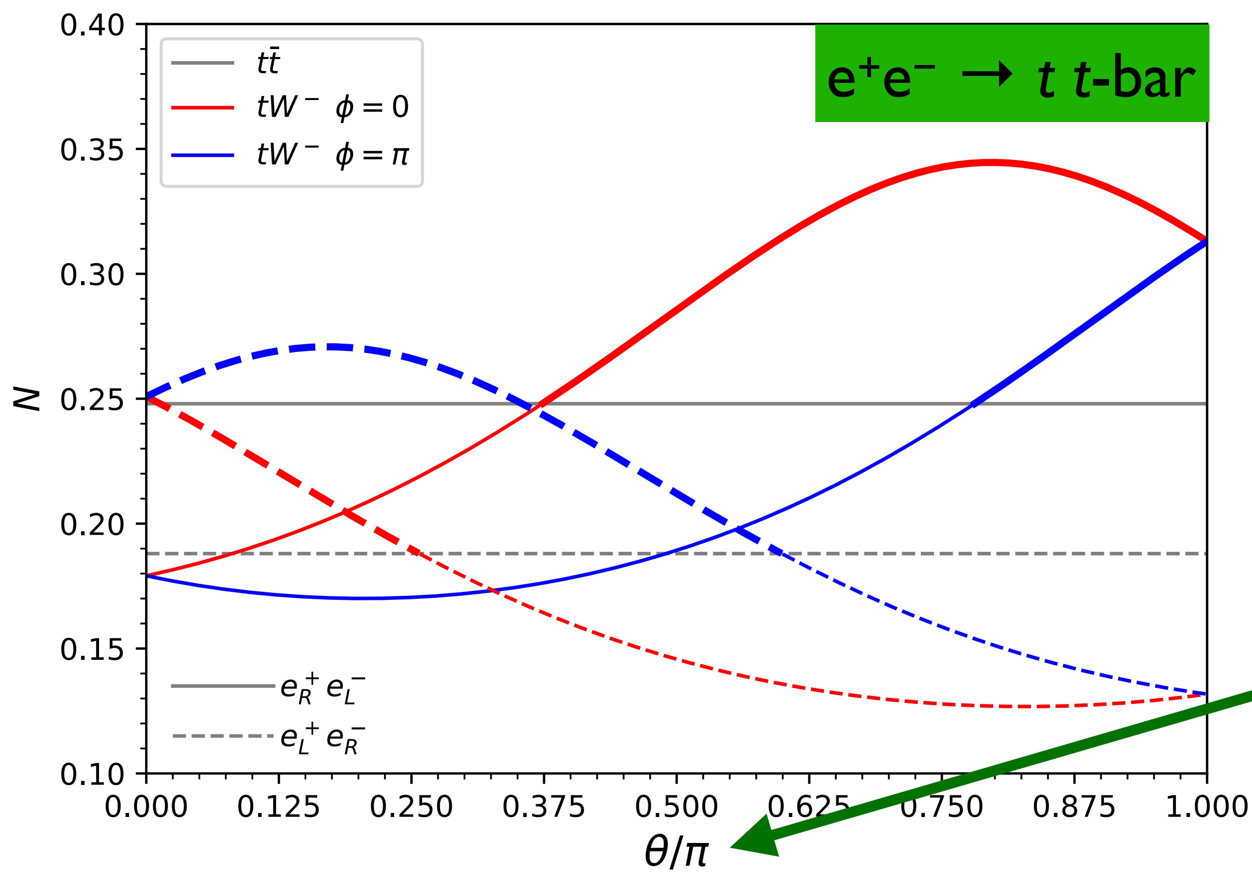
N (before) $<$ N (after)



Entanglement autodistillation

Since the *b* spins are, in principle, not measurable, we can use the *t*-*W* entanglement as a proxy to probe the entanglement increase.

And this could be observed in e+ e− colliders [needs that tops are polarised]



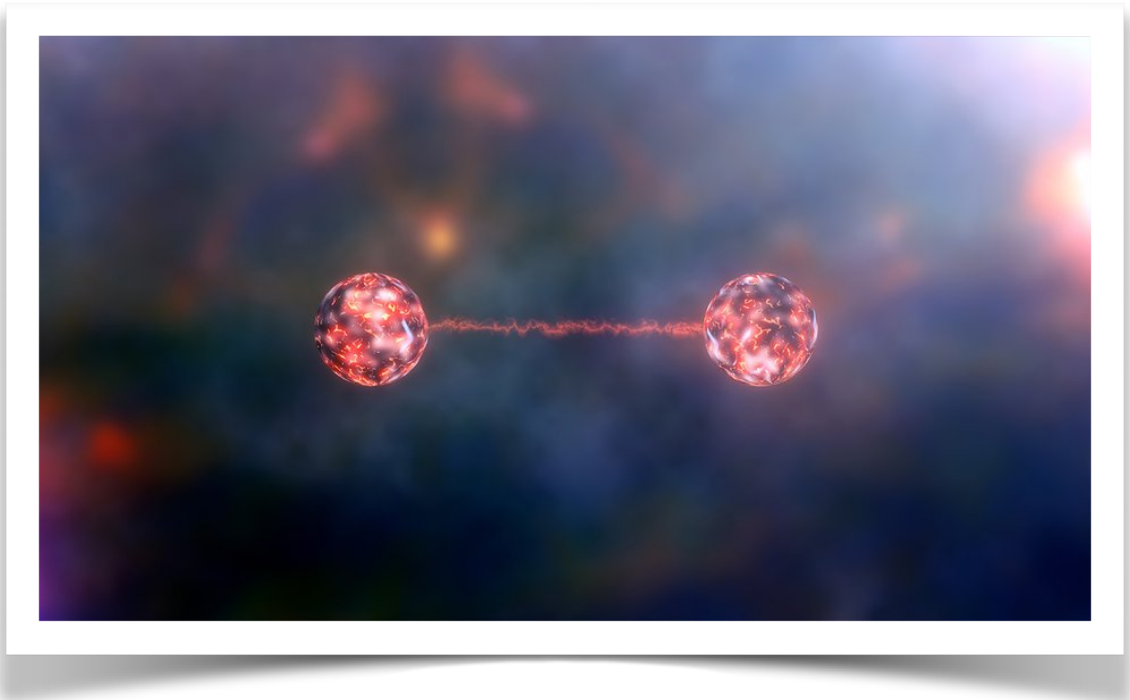
Unique quantum effect that requires large luminosity to be observed

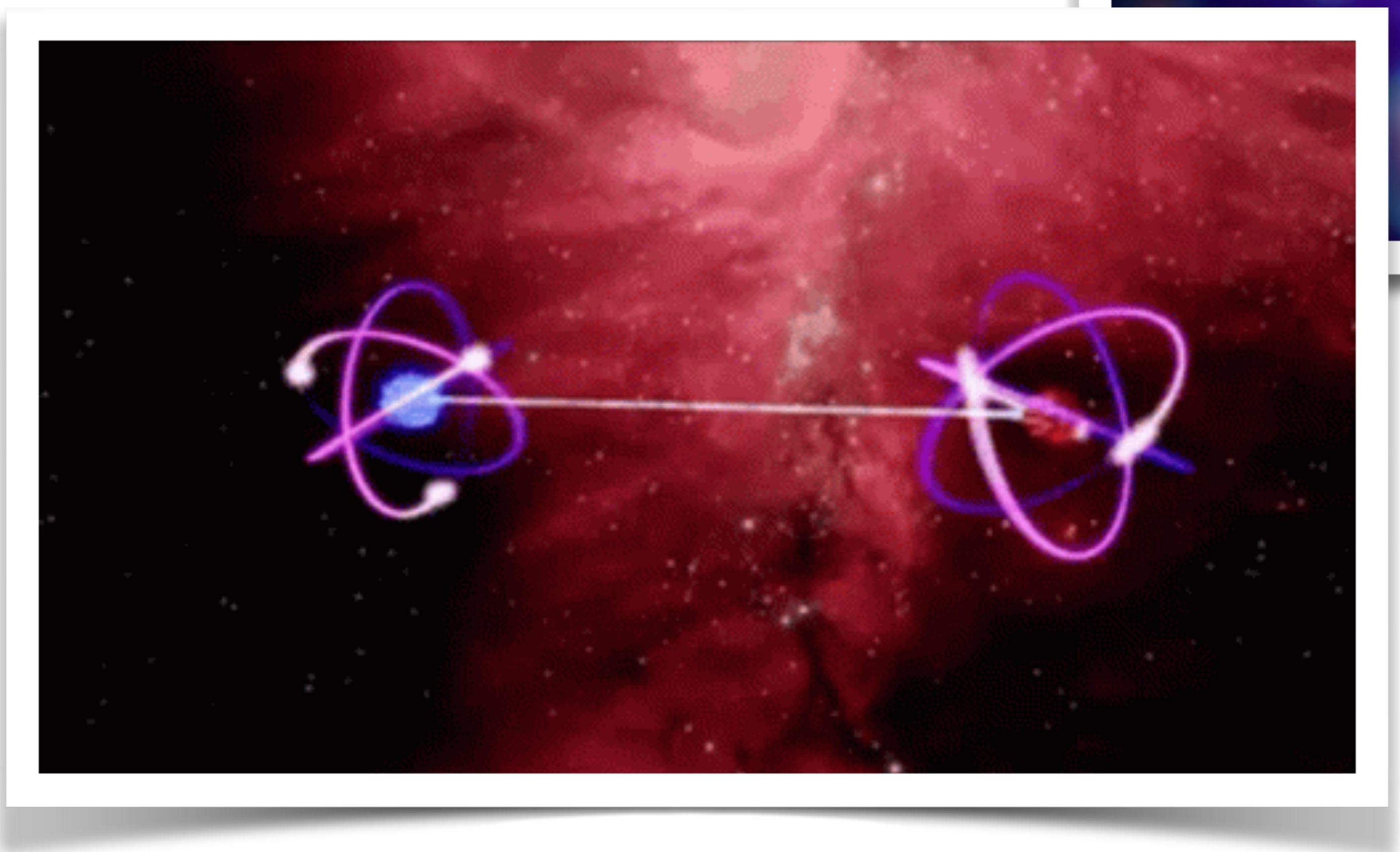
polar angle between *W* momentum in top rest frame and top direction in c.m. frame

To take away

- Particle decay and subsequent momenta projection is a very special kind of "measurement" in QM sense
- Unique QM effects yet untested:
  - post-selection
  - autodistillation
- Post-decay entanglement never tested either, 5σ sensitivity is possible at LHC with Run 2 data

Quantum entanglement in the media



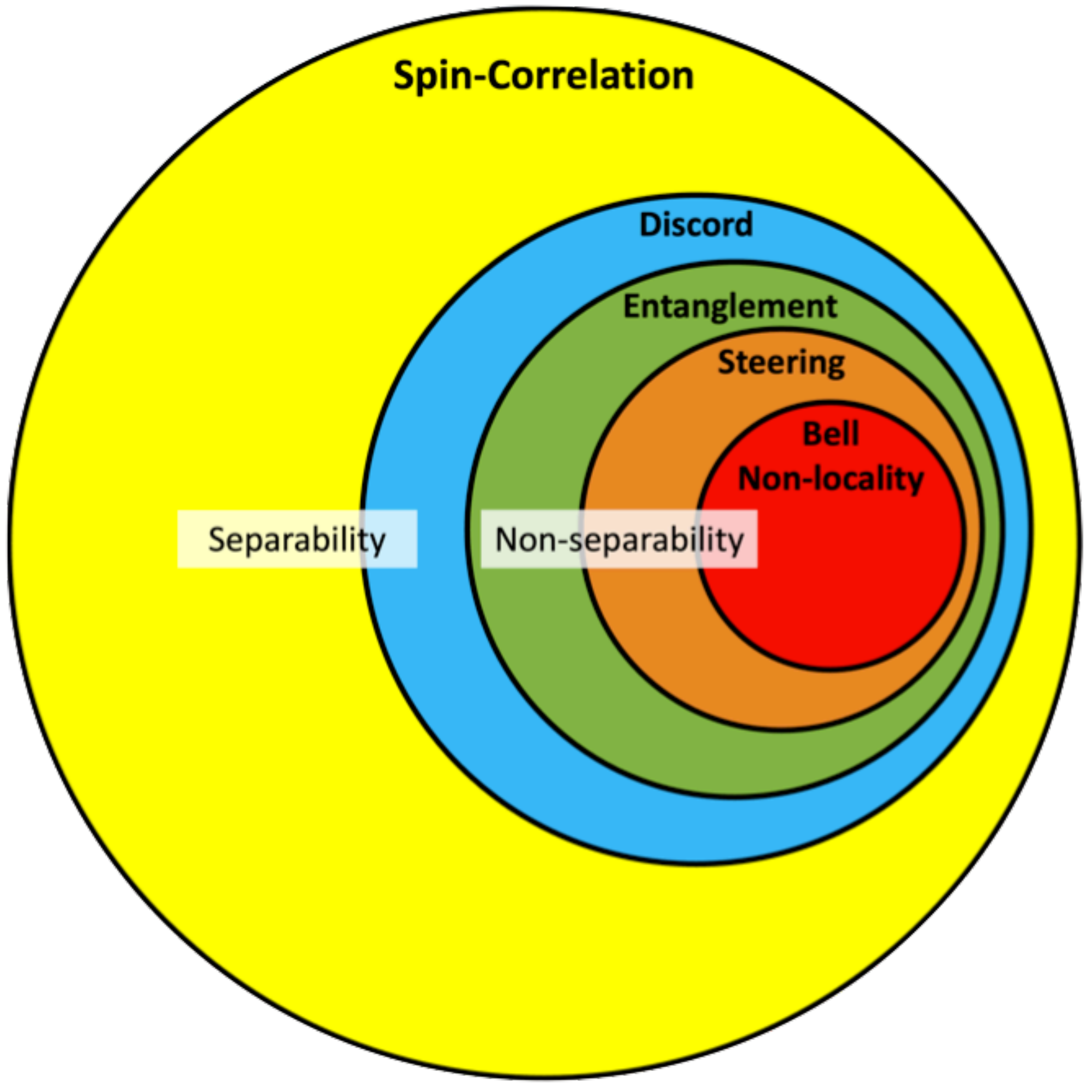


🤔 does not exactly look like a spooky action at a distance ...

End

What?

There are many levels of quantum correlations



Captured from Yoav Afik talks

- Spin correlation: statistical correlation between spins, classical
- Discord: quantum correlations yet in separable states
- Entanglement: subsystems are not separable
- Steering: measurement in one subsystem influences the other
- Bell non-locality: correlation cannot be described by local hidden variables

more stringent tests

What?

Example: top pair production

- q_L q_L-bar → t t-bar at threshold gives a spin configuration $|\leftarrow\rangle \otimes |\leftarrow\rangle$
  that is obviously separable [in the q direction]

- q_R q_R-bar → t t-bar at threshold gives a spin configuration $|\rightarrow\rangle \otimes |\rightarrow\rangle$
  that is separable too [in the q direction]

⇒ q q-bar → t t-bar gives 50% of each [density operator], separable.
We do have a *classical* spin correlation

- g g → t t-bar at threshold gives $\frac{1}{\sqrt{2}}\left(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle\right)$

  This one is entangled [actually, it is maximally entangled, violates Bell inequalities, etc.]

What?

The CHSH inequalities involve spin correlations. Therefore, for a particle of spin 1/2, they involve the C_{ij} spin-correlation coefficients [already measured for top pair production]

It can be shown that the maximum of the l.h.s.

$$|\langle AB \rangle - \langle AB' \rangle + \langle A'B \rangle + \langle A'B' \rangle|$$

is given by

$$2\sqrt{\lambda_1 + \lambda_2}$$

where λ_1 and λ_2 are the two largest eigenvalues of the positive definite matrix $C^T C$

Horodecki, Horodecki, Horodecki, ' 95

What?

Simpler but equally effective: Take judicious choice of [non-commuting] spin observables

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(2S_i + 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(-2S_i + 2S_j)$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \to |C_{ii} + C_{jj}|$$

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \to |C_{ii} - C_{jj}|$$

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(-2S_i - 2S_j)$$
$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(2S_i - 2S_j)$$

CHSH violation is probed by testing if $|C_{ii} \pm C_{jj}| > \sqrt{2}$

These estimators are optimal when off-diagonal C_{ij} vanish

What?

For spin-1 systems there is an inequality that is stronger than CHSH. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B] CGLMP PRL ‘02

$$\begin{aligned} I_3 =& P\left(A_1 = B_1\right) + P\left(B_1 = A_2 + 1\right) + P\left(A_2 = B_2\right) + P\left(B_2 = A_1\right) \\ & - \left[P\left(A_1 = B_1 - 1\right) + P\left(B_1 = A_2\right) + P\left(A_2 = B_2 - 1\right) + P\left(B_2 = A_1 - 1\right)\right] \leq 2 \end{aligned}$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|+-\rangle - |00\rangle + |-+\rangle\right)$$

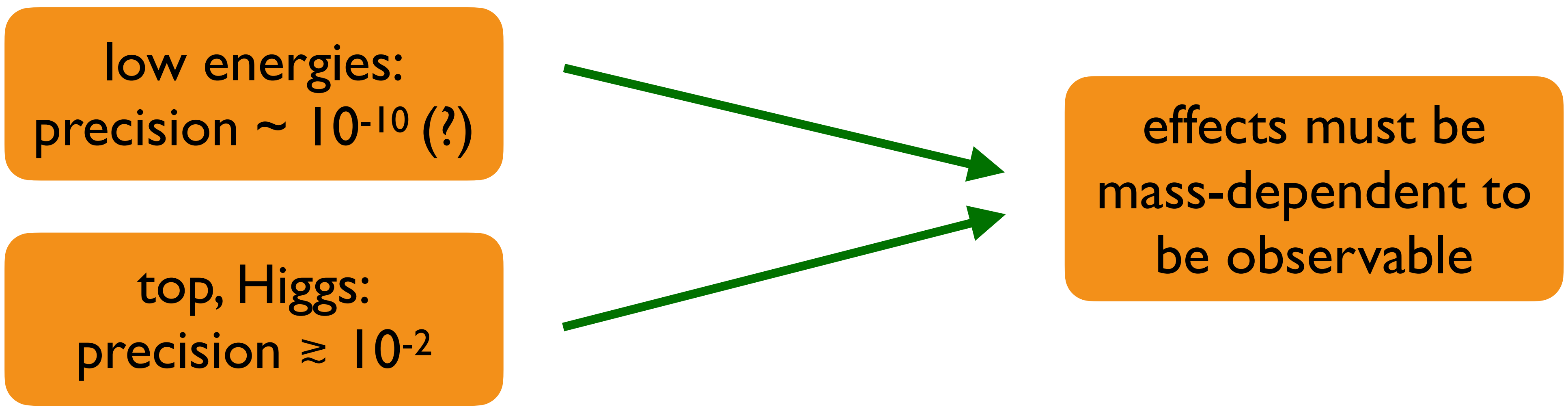
However, it is not optimal for the mixed spin state of the *VV* pair resulting from *H* decay

$$\rho = \int d\beta \ \mathcal{P}(\beta)|\psi_\beta\rangle\langle\psi_\beta| \qquad |\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}}\left(|+-\rangle - \beta|00\rangle + |-+\rangle\right)$$

Why?

Q: Should we see any breaking of QM at the LHC?

A: it is not clear that we should see any effect at LHC even if QM has to be corrected (e.g. with non-linear terms)



… and it remains to be shown that effects should precisely be seen in entanglement measurements!

Why?

Entanglement observables involve spin correlations, which are sensitive to new physics.

we can parameterise deviations from SM in terms of dim-6 operators, which provide a definite framework for comparisons

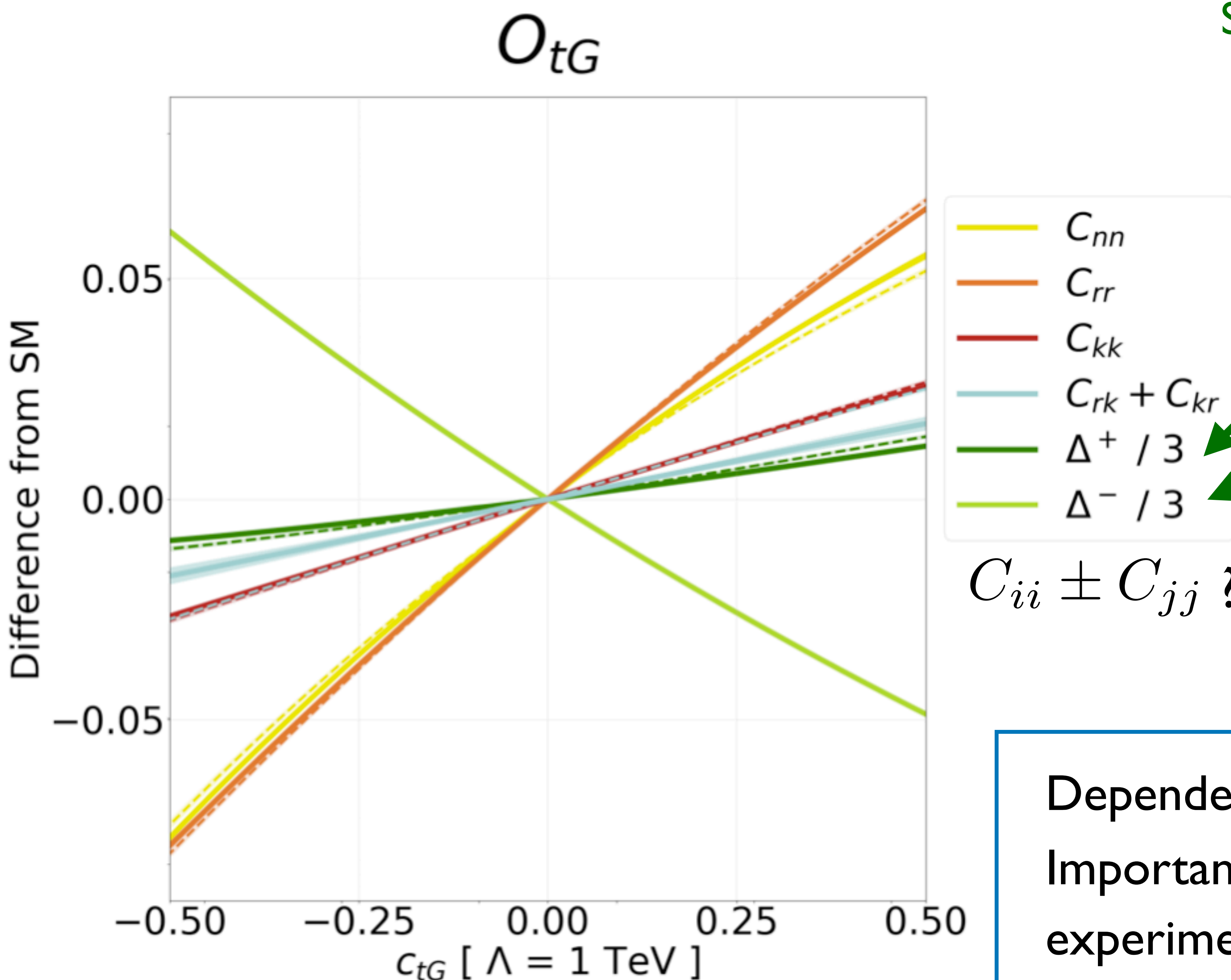
Spin correlations are measured with angular distributions, with a relation that may be modified by new physics

we can also introduce dim-6 operators for the decay of top, W, Z, but typically there are better ways to constrain them

Why?

t t-bar example: top chromomagnetic dipole operator

Severi, Vryonidou, 2210.09330



$C_{ii} \pm C_{jj}$???

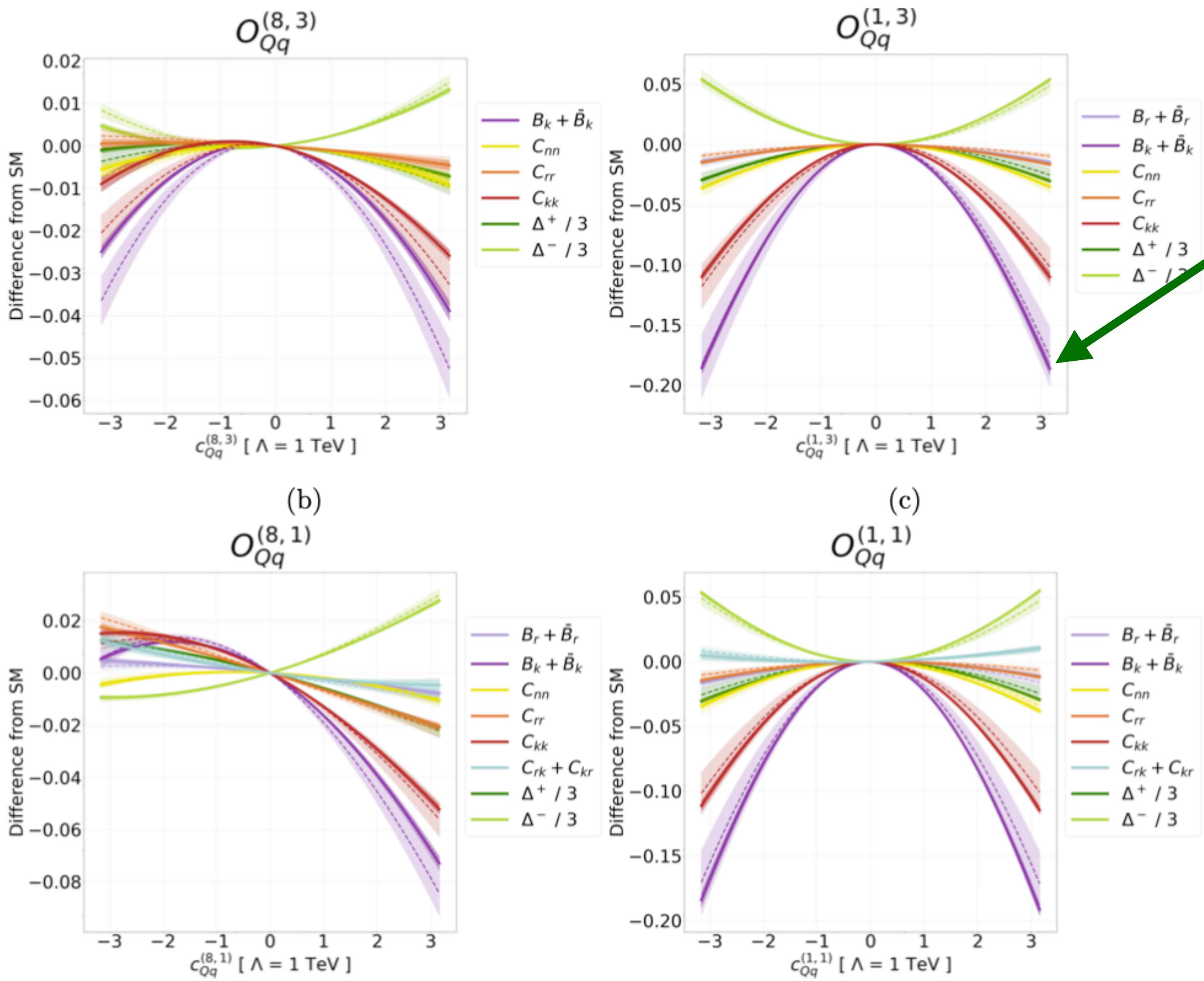
this is a new observable

this is essentially the old good D

Dependence on c is the first step. Important missing piece: expected experimental error bars for these quantities

Why?

t t-bar example: some four-fermion operators



polarisation in helicity direction

Polarisation seems to outperform the rest of observables [note that experimental uncertainties are likely smaller] but this statement is basis-dependent (!)

Why?

H → *ZZ* example: test anomalous *HZZ* interaction Fabbrichesi et al. 2304.02403

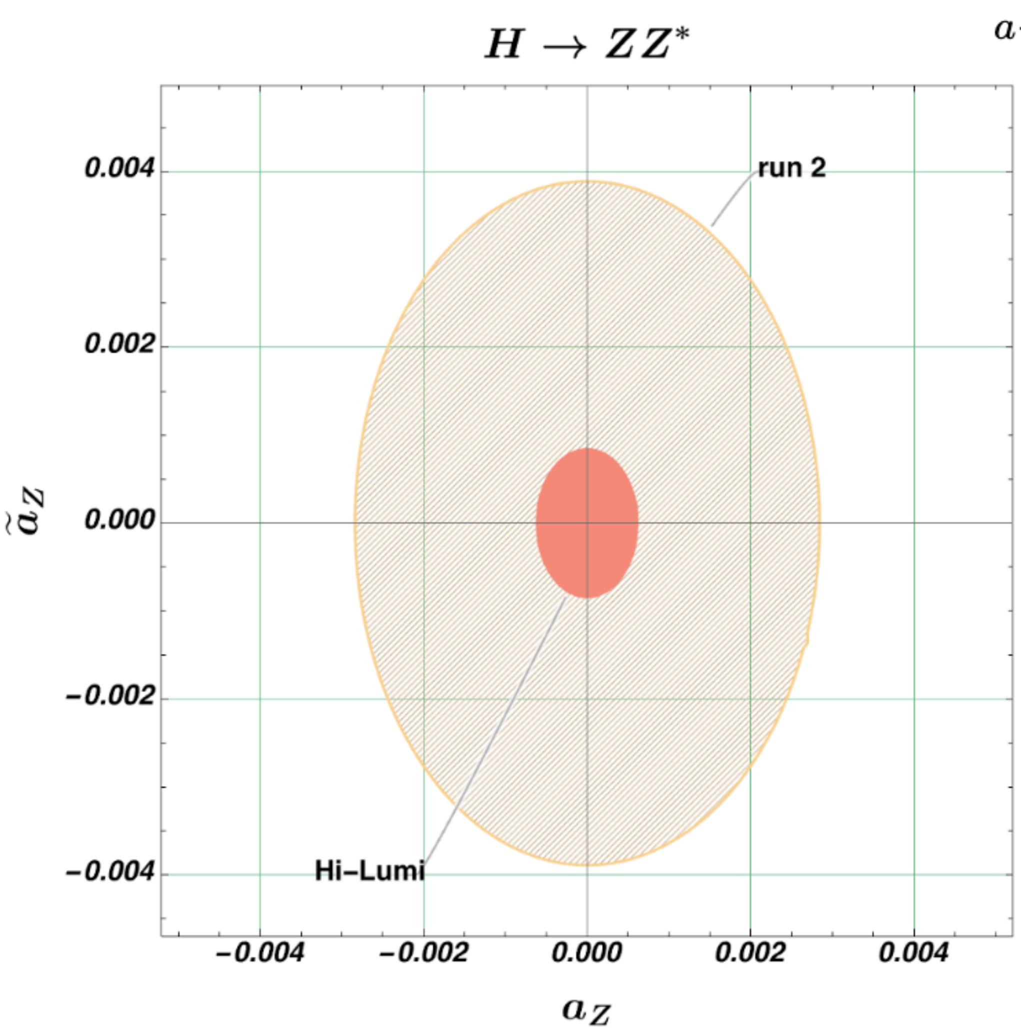
entanglement witness

$$\mathscr{C}_2 = 2\max\Big[0,\ -\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,,$$
$$-\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]$$

nothing to do with entanglement

$$\mathscr{C}_{\rm odd} = \frac{1}{2}\sum_{\substack{a,b\\a<b}} \left|h_{ab} - h_{ba}\right|$$

parameters of *ZZ* density matrix



- Why not using *ZZ* density matrix elements instead ofx $\mathscr{C}_2$?
- Why use $\mathscr{C}_{\rm odd}$ and not dedicated triple-product observables?
- Same applies to EW diboson production Aoude et al. 2307.09675

How?

If we want to study quantum information stuff with the spin of elementary particles, we have to measure it. All of it! $\longrightarrow$ density operator

As we all know, top quarks, *W/Z* bosons, … even τ leptons decay before one can pass them through a Stern-Gerlach experiment to measure spin.

But: the spin leaves its imprint in angular distributions.

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_3 \end{pmatrix}$$

density operator for spin 1/2

take daughter particle as spin analyser

constant

momentum direction of daughter particle

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1 + \alpha\vec{P}\cdot\hat{n})$$

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

How?

Top pair: two spin-1/2 particles, simplest example of quantum correlation

$$\rho = \frac{1}{4}\left(1\otimes 1 + \sum_i B_i^+ \sigma_i \otimes 1 + \sum_i B_i^- 1 \otimes \sigma_i + \sum_{ij} C_{ik}\sigma_i \otimes \sigma_j\right)$$

normalisation

$$\hat{n}_a = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$$
$$\hat{n}_b = (\sin\theta_b \cos\varphi_b, \sin\theta_b \sin\varphi_b, \cos\theta_b)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\left[1 + \alpha_a \vec{B}^+ \cdot \hat{n}_a + \alpha_b \vec{B}^- \cdot \hat{n}_b + \alpha_a\alpha_b \hat{n}_a^T C \hat{n}_b\right]$$

3 coefficients corresponding to top polarisation

3 coefficients corresponding to anti-top polarisation

9 spin correlations

Measured by ATLAS and CMS since some time

How?

For two qubits [e.g. spin-1/2 fermions] sufficient entanglement conditions are

$$|C_{11}+C_{22}| > 1 + C_{33} \qquad \text{or} \qquad |C_{11}-C_{22}| > 1 - C_{33}$$

Afik, Nova 2003.02280
Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

And Bell-like inequalities are violated if

$$|C_{ii}+C_{jj}| > \sqrt{2} \qquad \text{or} \qquad |C_{ii}-C_{jj}| > \sqrt{2}$$

Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

For $H \to VV$ [spin 1, extra symmetry] sufficient entanglement conditions are

$$C_{212-1} \neq 0 \qquad \text{or} \qquad C_{222-2} \neq 0$$

JAAS, Bernal, Casas, Moreno 2209.13441

And [optimised] sufficient condition for violation of Bell-like inequalities

$$I_3 = \frac{1}{36}\left[(18+16\sqrt{3}) - \sqrt{2}(9-8\sqrt{3})A^1_{20} - 8(3+2\sqrt{3})C_{212-1} + 6C_{222-2}\right] > 2$$

JAAS, Bernal, Casas, Moreno 2209.13441

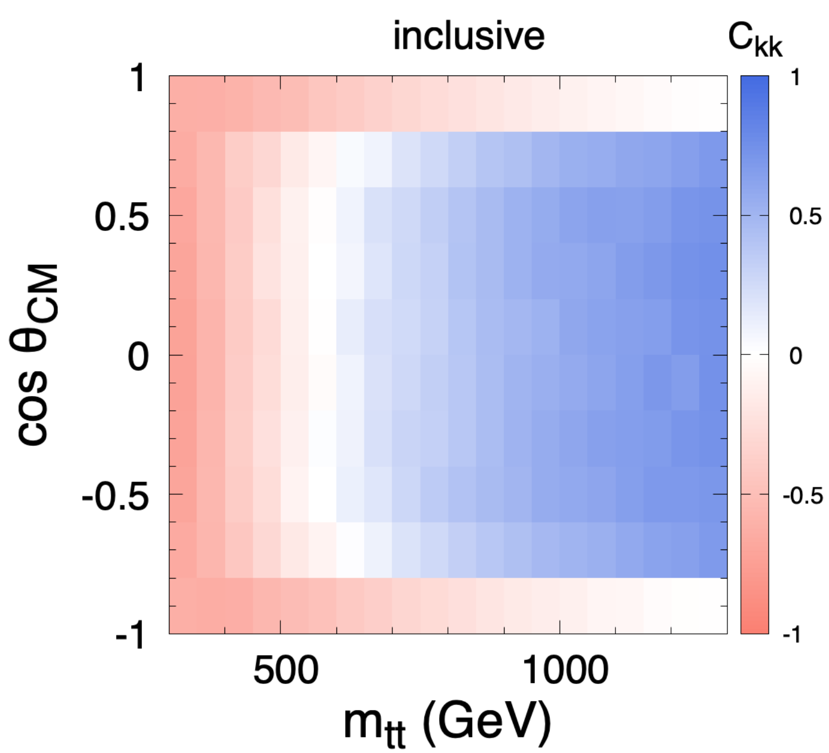
For different dimensions, fall back into Peres-Horodecki criterion [backup]

Top pair entanglement

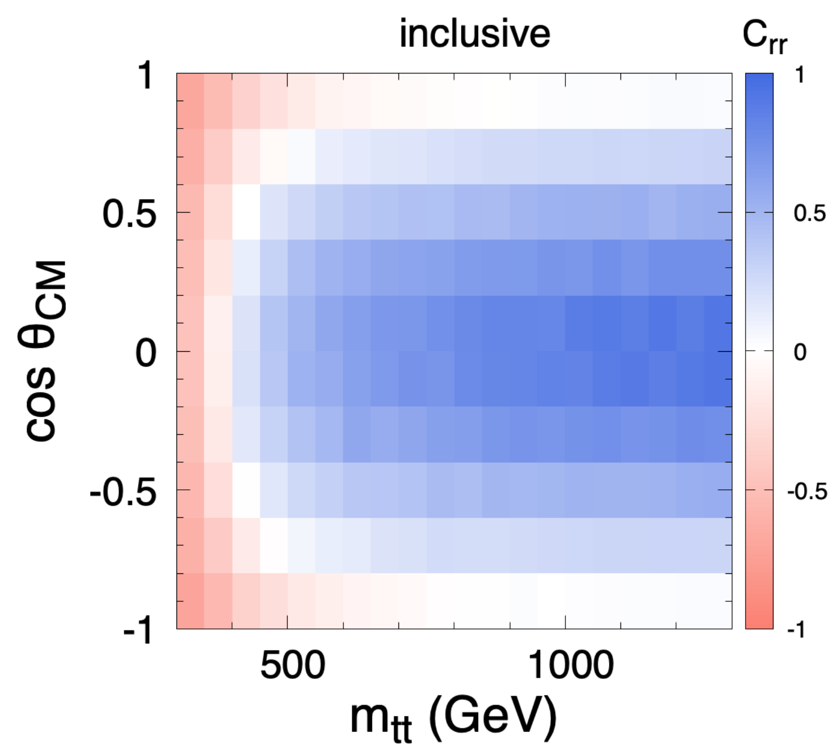
There is a dependence of the C_{ij} coefficients on the kinematics.

Use the helicity basis to parameterise C_{ij}:

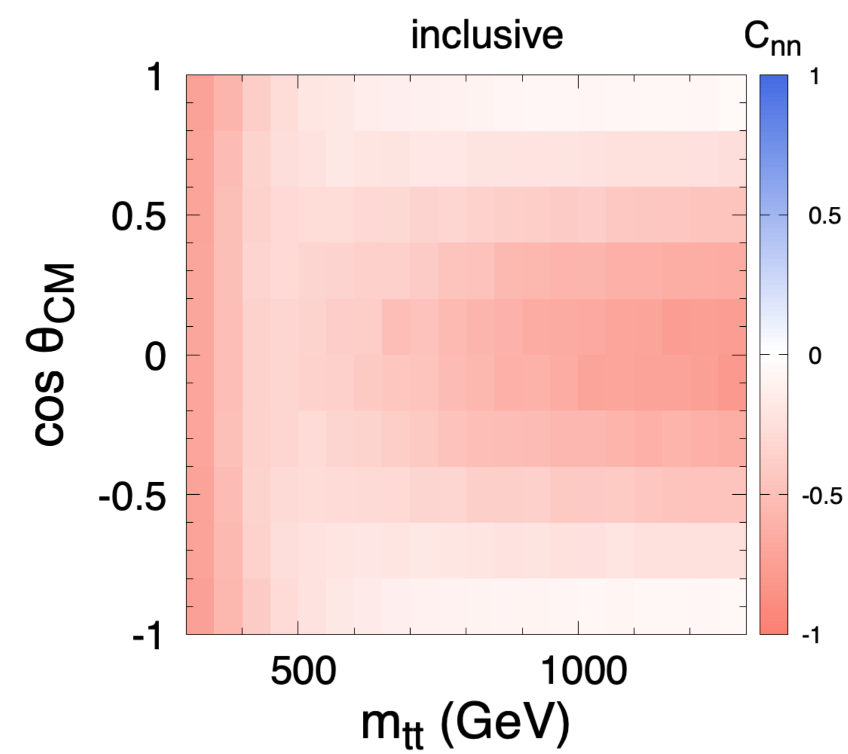
K: top helicity



R: ⊥ in production plane



N: ⊥ to K and N



Most convenient entanglement criterion: $|C_{11} + C_{22}| > 1 + C_{33}$

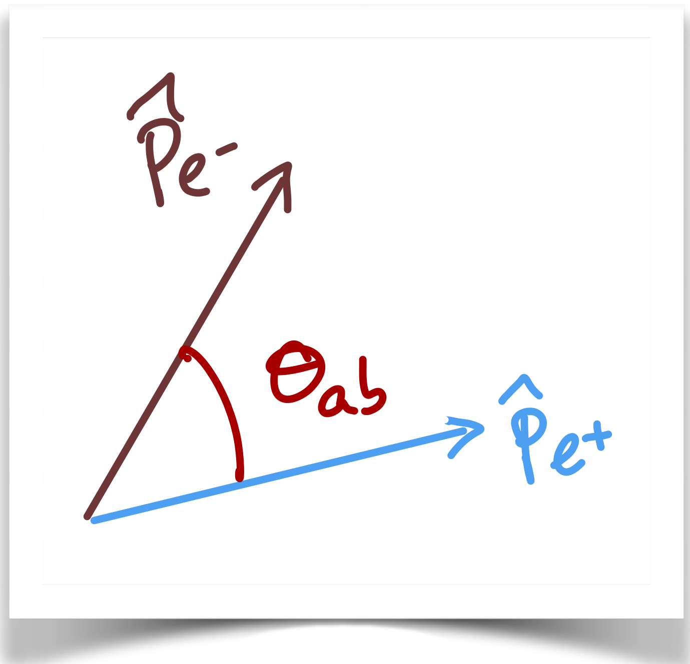
with 3 → N because $C_{nn} < 0$

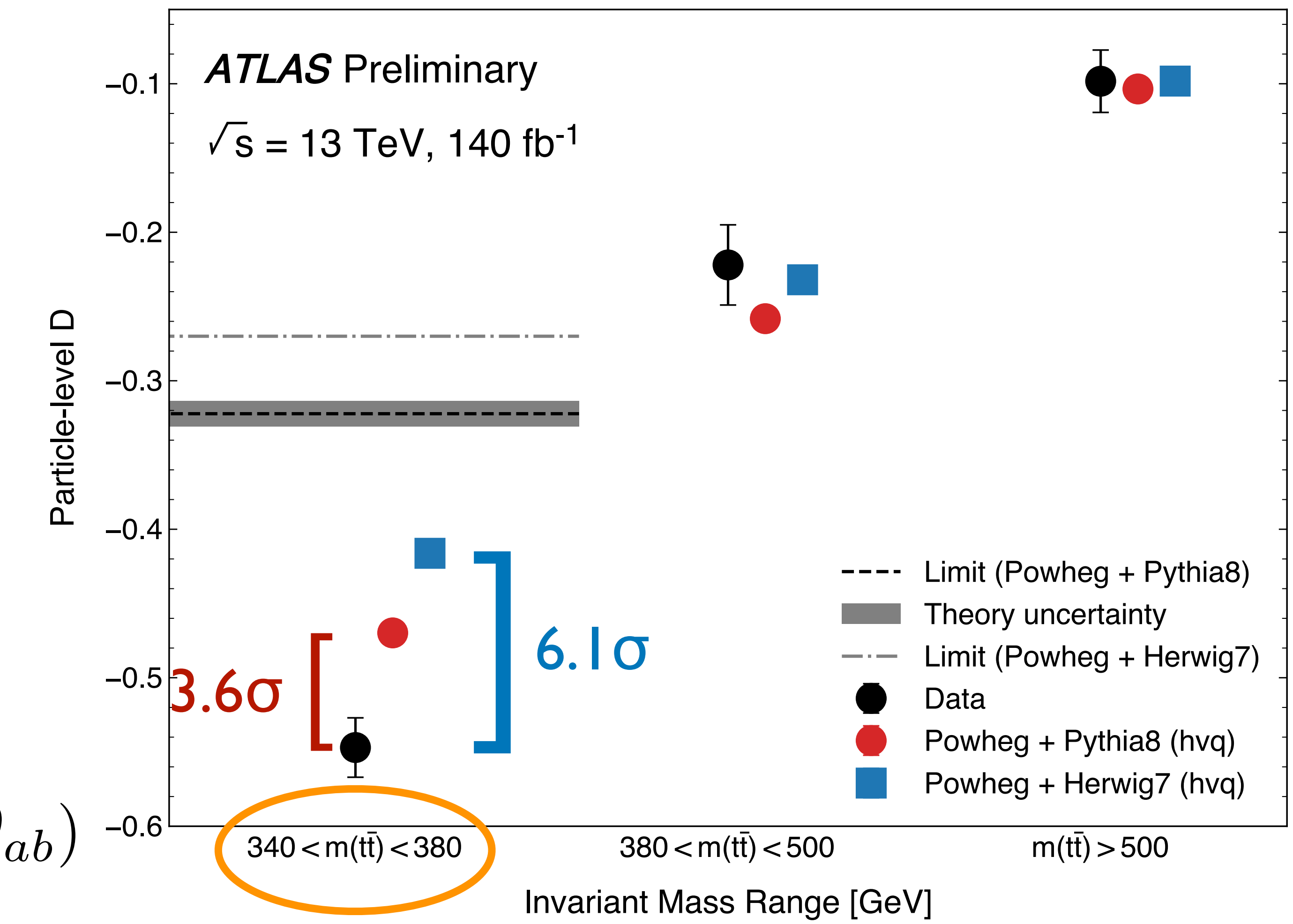
Near threshold: $|C_{kk} + C_{rr}| = - C_{kk} - C_{rr}$

Boosted central: $|C_{kk} + C_{rr}| = C_{kk} + C_{rr}$

Top pair entanglement

ATLAS has performed a measurement at threshold using the *D* observable, related to the angle between the two leptons





$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{ab}} = \frac{1}{2}\left(1 + \alpha_a\alpha_b D\cos\theta_{ab}\right)$$

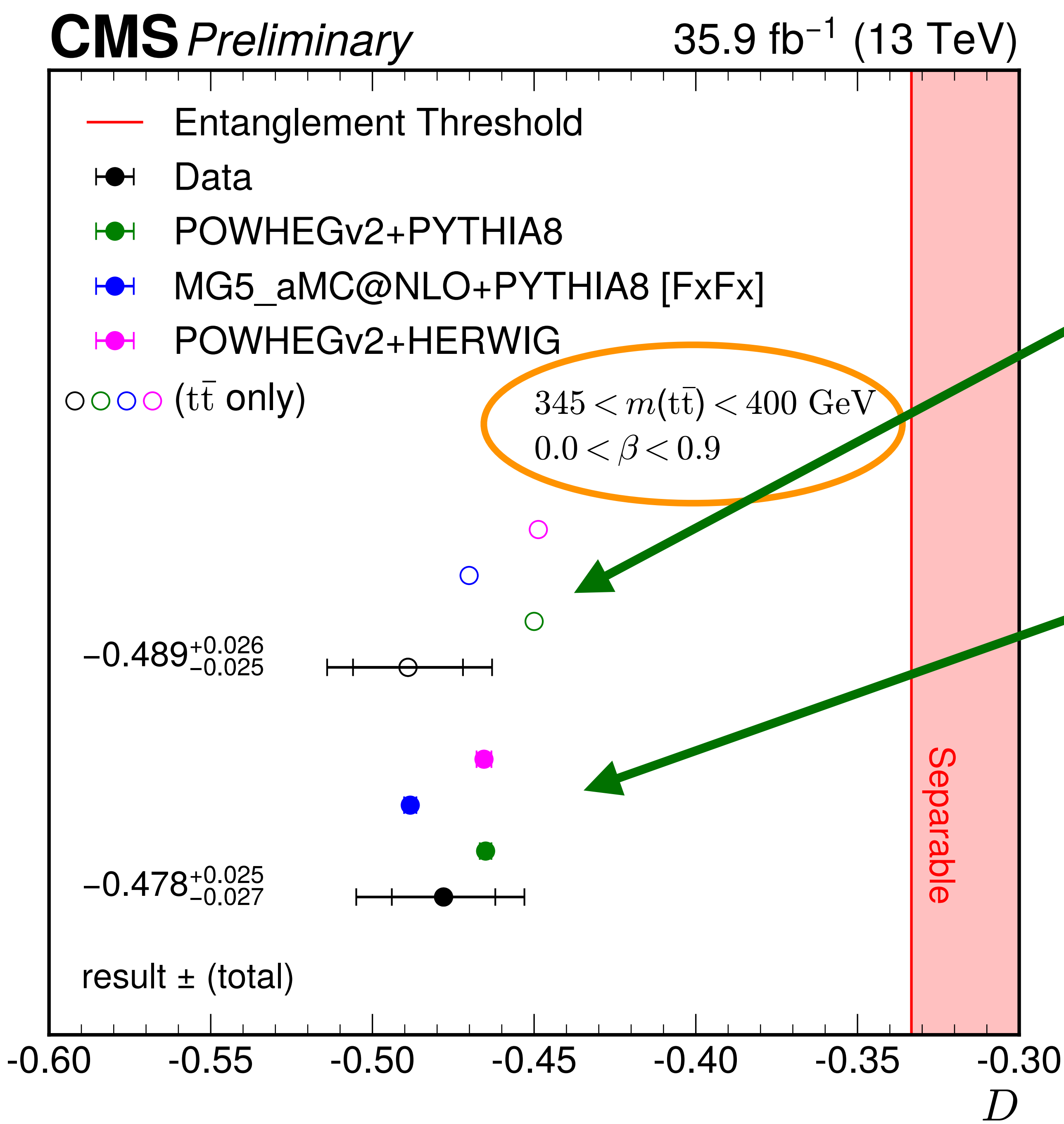
$$D = \frac{1}{3}(C_{11} + C_{22} + C_{33})$$

Entanglement test near threshold: $-3D - 1 > 0$

Bottom line: we know there are spin correlations since a decade, but entanglement is a stronger condition

Top pair entanglement

CMS has measured entanglement using the same observable, in a slightly different kinematical region



t t-bar NLO

t t-bar NLO ...
+ toponium!

σ = 6.43 pb
m = 343 GeV
Γ = 7 GeV

Fuks et al. 2102.11281

Overall agreement much better than ATLAS, even without toponium

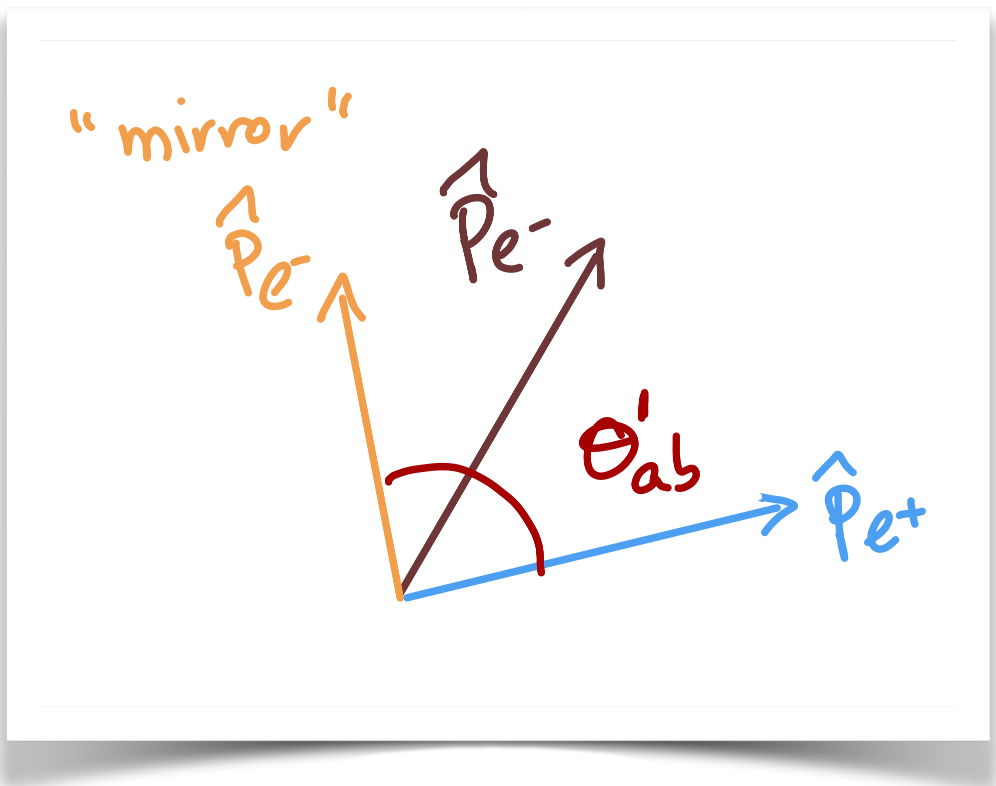
Top pair entanglement

What about the boosted central region?

The relevant quantity to test is $C_{kk} + C_{rr} - C_{nn}$ and there was no specific observable for this combination [one can however measure C's and sum]

We can design a new observable

JAAS, Casas,2205.00542



Use the mirror image of ℓ^- momentum, reflected in the K-R plane

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta'_{ab}} = \frac{1}{2}(1 + \alpha_a\alpha_b D_3 \cos\theta'_{ab})$$

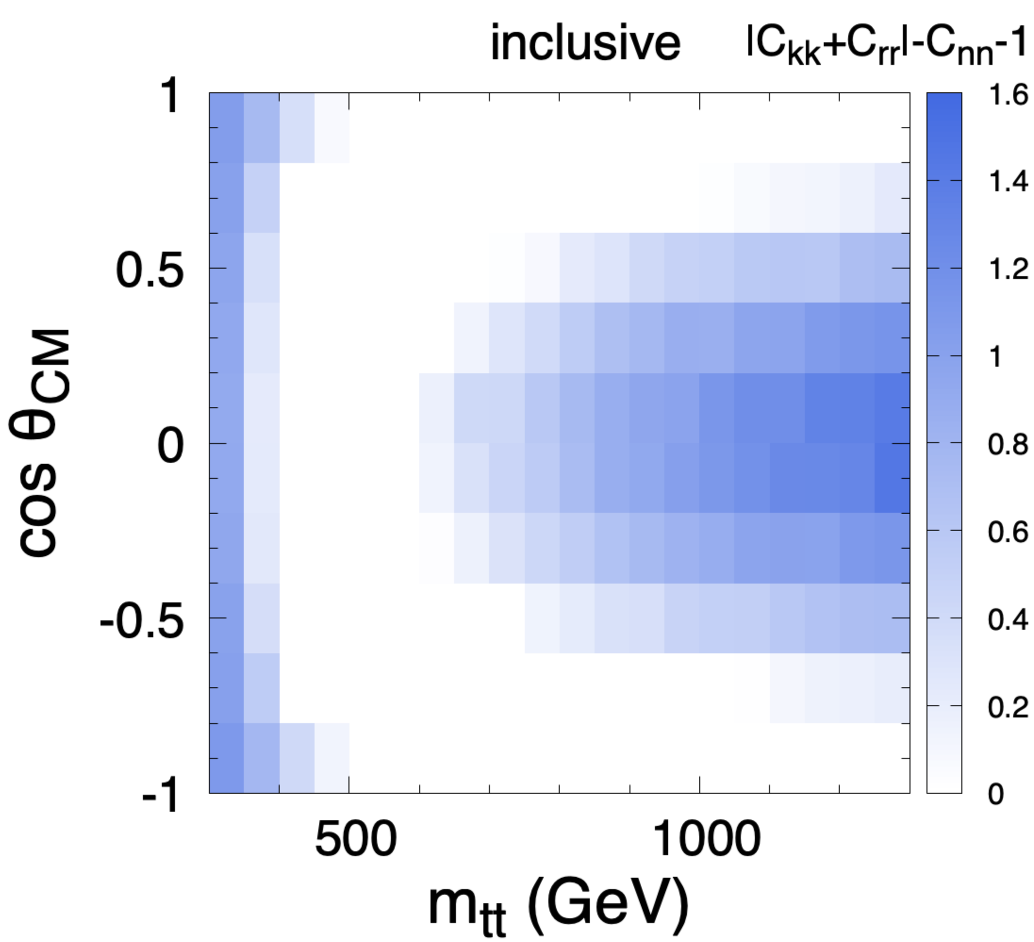
$$D_3 = \frac{1}{3}(C_{11} + C_{22} - C_{33})$$

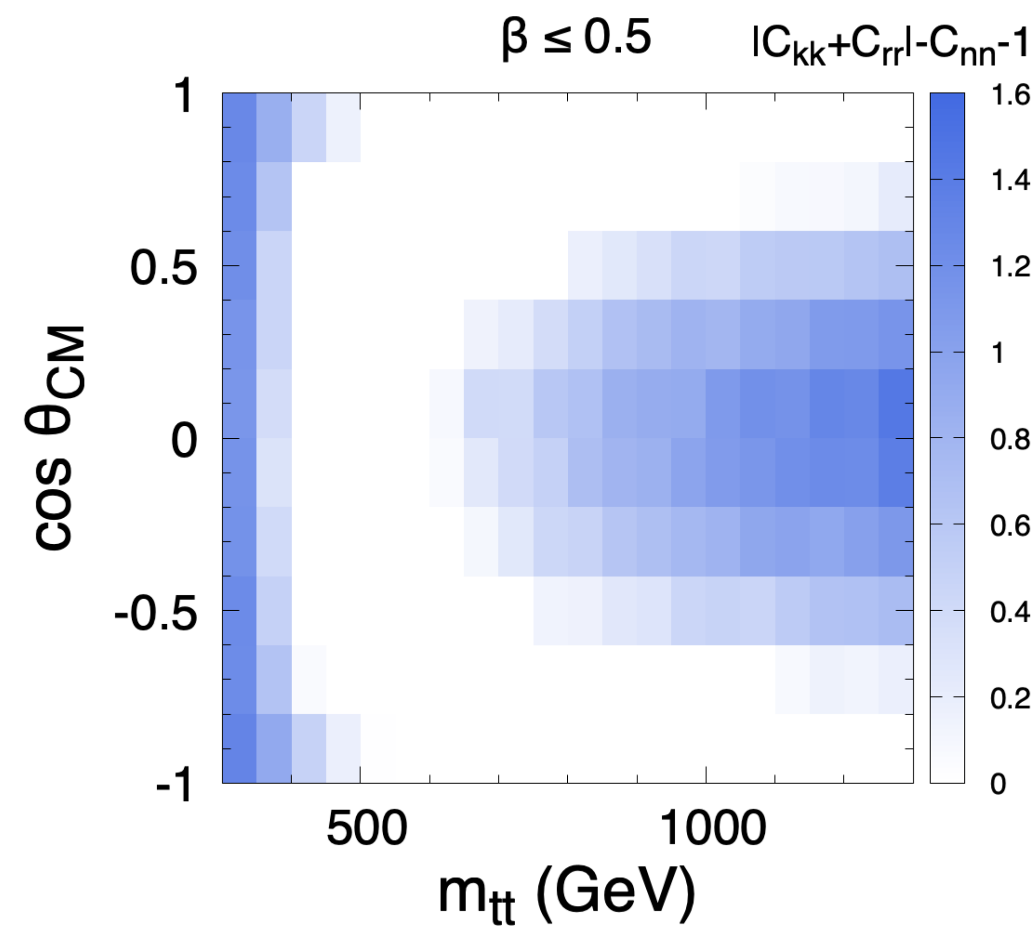
$\boxed{\text{Entanglement test for boosted region: } 3D_3 - 1 > 0}$

Top pair entanglement

Improvement: consider events that are more central: upper cut on *t t*-bar velocity β in LAB frame

JAAS, Casas,2205.00542





- opposite contributions from *qq* and *gg* sub-processes
- the upper cut reduces the *qq* fraction
- can relax upper cut on m_{tt}, reducing systematics

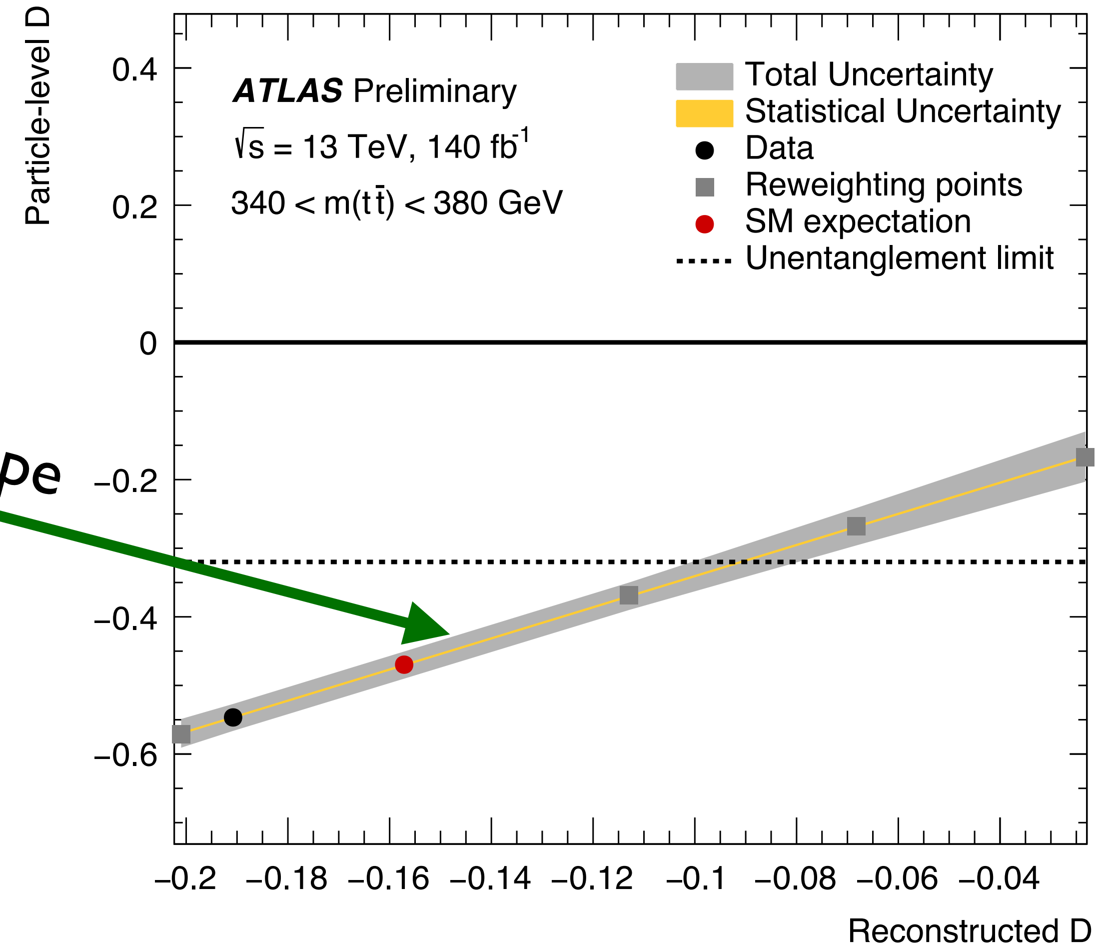
Top pair entanglement

Caveat in ATLAS measurement: calibration of D from reconstructed to particle level [3x correction]

ATLAS: unphysical reweighting of cos ϕ distribution

$$w = \frac{1 - D_\Omega(m_{t\bar{t}}) \cdot \mathcal{X} \cdot \cos\varphi}{1 - D_\Omega(m_{t\bar{t}}) \cdot \cos\varphi}$$

$\mathcal{X} \in [0.4, 1.2]$ 👉 slope



Physical alternative: mixture samples, e.g.

- gg sample ~ spin singlet
- separable sample

	D = -0.73 (LO)		D = 0.33 (LO)
f	gg → t t-bar	+ (1-f)	qq → t t-bar
f	gg → t t-bar	+ (1-f)	pp → t_R t_L-bar
f	gg → t t-bar	+ (1-f)	pp → t_L t_R-bar

Is calibration model dependent?
Do other choices give the same slope?

now this is the
end