Generalized BGL 2HDM models

(About SCPV and the flavour sector of 2HDM)

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Collaboration

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Introduction I

- Are the couplings of the Higgs to fermions like in the SM or do we have a more complex scalar sector?
- A natural scenario is Two Higgs Doublet Model (2HDM) where symmetries are needed to avoid or suppress FCNC.
- To avoid FCNC has been a very important guiding principle.
- In 2HDM a Z_2 symmetry a la Glashow-Weinberg (GW) leads to Natural Flavour Conservation (NFC) in the scalar sector.
- Beyond NFC there are 2HDM MODELS - enforced by symmetries- that give rise to Scalar Flavour Changing Neutral Couplings (SFCNC) controlled by V_{CKM} and fermion masses, realizing the Minimal Flavour Violation (MFV) idea. These are the so called BGL models (Branco, Grimus, Lavoura) that have SFCNC in the up or in the down sector, but not in both.

Introduction II

- In the framework of GW NFC Spontaneous CP violation (SCPV) cannot generate complex V_{CKM}, BGL cannot develop SCPV.
- These BGL's have been generalize to gBGL having controlled SFCNC both in the up and down sectors.
- gBGL can be formulated in the framework of Spontaneous CP violation (SCPV), generating a complex V_{CKM} and they are good laboratory to explore the relation between SCPV, SFCNC and a complex CKM.

2HDM I

- The Yukawa sector of the 2HDM

$$
\begin{aligned}
L_Y = & -\overline{Q}_L \left(\Gamma_1 \Phi_1 + \Gamma_2 \Phi_2\right) d_R - \overline{Q}_L \left(\Delta_1 \widetilde{\Phi}_1 + \Delta_2 \widetilde{\Phi}_2\right) u_R + .h.c. \\
& -\overline{L}_L \left(\Gamma_1^{(e)} \Phi_1 + \Gamma_2^{(e)} \Phi_2\right) e_R - \overline{L}_L \left(\Delta_1^{(\nu)} \widetilde{\Phi}_1 + \Delta_2^{(\nu)} \widetilde{\Phi}_2\right) \nu_R + .h.c.
\end{aligned}
$$

- With the vev's given by $\langle \Phi_i \rangle^T = e^{i\theta_i} \begin{pmatrix} 0 & v_i/\sqrt{2} \end{pmatrix}$ we define the Higgs basis by $\langle H_1 \rangle^T = \begin{pmatrix} 0 & v/\sqrt{2} \end{pmatrix}, \langle H_2 \rangle^T = \begin{pmatrix} 0 & 0 \end{pmatrix}, v^2 = v_1^2 + v_2^2$, $c_\beta = v_1/v$, $s_\beta = v_2/v$, $t_\beta = v_2/v_1$

$$
\begin{pmatrix} e^{-i\theta_1} \Phi_1 \\ e^{-i\theta_2} \Phi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}
$$

then we have

$$
H_1 = \begin{pmatrix} G^+ \\ \left(v + H^0 + iG^0\right)/\sqrt{2} \end{pmatrix} \quad ; \quad H_2 = \begin{pmatrix} H^+ \\ \left(R^0 + iA\right)/\sqrt{2} \end{pmatrix}
$$

- $G^{\pm}$ and G^0 longitudinal degrees of freedom of $W^{\pm}$ and Z^0.
- $H^{\pm}$ new charged Higgs bosons.
- A new CP odd scalar (we will have CP invariant Higgs potential).
- H^0 and R^0 CP even scalars. If they do not mix, H^0 the SM Higgs.

- The Lagrangian in the Higgs basis $(\theta = \theta_2 - \theta_1)$:

$$\begin{aligned} L_Y \;=\; & -\overline{Q}_L \frac{\sqrt{2}}{v} \left(M_d^0 H_1 + N_d^0 H_2 \right) d_R - \overline{Q}_L \frac{\sqrt{2}}{v} \left(M_u^0 \widetilde{H}_1 + N_u^0 \widetilde{H}_2 \right) u_R \\ & + h.c \end{aligned}$$

$$M_d^0 = \frac{v}{\sqrt{2}} \left(c_\beta \Gamma_1 + e^{i\theta} s_\beta \Gamma_2 \right) ; N_d^0 = \frac{v}{\sqrt{2}} \left(s_\beta \Gamma_1 - e^{i\theta} c_\beta \Gamma_2 \right)$$

$$M_u^0 = \frac{v}{\sqrt{2}} \left(c_\beta \Delta_1 + e^{-i\theta} s_\beta \Delta_2 \right) ; N_u^0 = \frac{v}{\sqrt{2}} \left(s_\beta \Delta_1 - e^{-i\theta} c_\beta \Delta_2 \right)$$

2HDM III

- The quark mass basis is obtained by bidiagonalizing M_d^0, M_u^0

$$\begin{aligned} U_L^{d\dagger} M_d^0 U_R^d &= D_d = diag\left(m_d, m_s, m_b\right) \\ U_L^{u\dagger} M_u^0 U_R^u &= D_u = diag\left(m_u, m_c, m_t\right) \end{aligned}$$

  The components of H_1 $\left(H^0, G^0\right)$ are coupled in a flavour diagonal way.
- In the mass basis the neutral components of H_2 $\left(R^0, A\right)$ generate SFCNC proportional to the arbitrary matrices

$$\begin{aligned} N_d = U_L^{d\dagger} N_d^0 U_R^d \\ N_u = U_L^{u\dagger} N_u^0 U_R^u \end{aligned}$$

2HDM IV

- The components of H_1 and H_2 in the quark mass basis interact with

$$
\begin{aligned}
\mathcal{L}_Y \;=\; & -\frac{\sqrt{2}H^+}{v}\bar{u}\left(VN_d\gamma_R - N_u^\dagger\, V\gamma_L\right)d + h.c. \\
& -\frac{H^0}{v}\left(\bar{u}M_u u + \bar{d}M_d\; d\right) - \\
& -\frac{R^0}{v}\left[\bar{u}(N_u\gamma_R + N_u^\dagger\gamma_L)u + \bar{d}(N_d\gamma_R + N_d^\dagger\gamma_L)\; d\right] \\
& +i\frac{A}{v}\left[\bar{u}(N_u\gamma_R - N_u^\dagger\gamma_L)u - \bar{d}(N_d\gamma_R - N_d^\dagger\gamma_L)\; d\right]
\end{aligned}
$$

- Where the CKM matrix is $V = U_L^{u\dagger}U_L^d$.

The BGL models I

- Branco, Grimus and Lavoura put forward models, enforced by flavour symmetries, that realize the most simple MFV expansion with controlled FCNC. For example one BGL model is enforced by the $U(1)$ flavour symmetry

$$Q_{L_3} \to e^{i\alpha} Q_{L_3} \quad ; \quad u_{R_3} \to e^{i2\alpha} u_{R_3} \quad ; \quad \Phi_2 \to e^{i\alpha} \Phi_2$$

In the quark basis where the symmetry is defined, it correspond to the model defined by the MFV expansion -$(P_3)_{ij} = \delta_{i3}\delta_{j3}$-

$$N_d^0 = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) U_L^u P_3 U_L^{u\dagger} \right] M_d^0$$
$$N_u^0 = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) U_L^u P_3 U_L^{u\dagger} \right] M_u^0$$

The BGL models II

or to the model with the following Yukawa couplings

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

This model is called a **top type model** after $u_{R_3} = t_R$.

The BGL models III

- In the quark sector we have **three up type models** and **three down type models**($u_1 = u, u_2 = c, u_3 = t$) with couplings in the mass basis:defined by the following symmetries and with the corresponding couplings

$$\begin{array}{c} Q_{L_k} \to e^{i\alpha} Q_{L_k} \\ u_{R_k} \to e^{i2\alpha} u_{R_k} \\ \Phi_2 \to e^{i\alpha}\Phi_2 \end{array} \left\{ \begin{array}{c} N_d = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) V^\dagger P_k V \right] D_d \\ N_u = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_k \right] D_u \end{array} \right.$$

$$\begin{array}{c} Q_{L_k} \to e^{i\alpha} Q_{L_k} \\ d_{R_k} \to e^{i2\alpha} d_{R_k} \\ \Phi_2 \to e^{-i\alpha}\Phi_2 \end{array} \left\{ \begin{array}{c} N_d = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_k \right] D_d \\ N_u = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) V P_k V^\dagger \right] D_u \end{array} \right.$$

  They have SFCNC in the down sector N_d or in the up sector N_u, never in both.

- A general BGL model is defined both in the quark and in the leptonic sector making a total $6 \times 6 = 36$ models.

The BGL models IV

- All BGL models are invariant under $\Phi_2 \rightarrow e^{\pm i\alpha}\Phi_2$. Therefore the Higgs potential should be the CP conserving

$$
\begin{aligned}
V \;=\; & \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}\left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) \\
& + 2\lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + 2\lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) \\
& + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2
\end{aligned}
$$

  where a soft breaking term has been introduced to avoid a Goldstone boson.

- Therefore the physical Higgses will be

$$
\begin{pmatrix} H^0 \\ R^0 \end{pmatrix} = \begin{pmatrix} c_{\beta\alpha} & s_{\beta\alpha} \\ -s_{\beta\alpha} & c_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}
$$

  with $c_{\beta\alpha}$ small

The BGL models V

- The Yukawa couplings of the 125 GeV scalar is for all type of fermions f

$$
\begin{aligned}
L_{h\bar{f}f} &= -\overline{f_L} Y^{(f)} f_R h + h.c \\
Y^{(f)} &= \frac{1}{v}\left[s_{\beta\alpha} M_f + c_{\beta\alpha} N_f\right]
\end{aligned}
$$

- In the k-up type model u_k we have SFCNC in the down sector controlled by

$$Y_{ij}^{(d)}\left[u_k\right] = -c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right) V_{ki}^* V_{kj} \frac{m_{d_j}}{v} \quad ; \quad i \neq j$$

- In the k-down type model d_k we have SFCNC in the up sector controlled by

$$Y_{ij}^{(u)}\left[d_k\right] = -c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right) V_{ik} V_{jk}^* \frac{m_{u_j}}{v} \quad ; \quad i \neq j$$

Generalized BGL models: gBGL I

- The generalized BGL models (gBGL) are implemented through a Z_2 symmetry, where u_R and d_R are even and only one of the scalars doublets and one of the left-handed quark doublets are odd:

$$\begin{array}{rcl} Q_{L_3} \to -Q_{L_3} & , & \\ d_R \to d_R & , & \Phi_1 \to \Phi_1 \\ u_R \to u_R & , & \Phi_2 \to -\Phi_2 \end{array}$$

- Now the Yukawa textures are:

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

Generalized BGL models: gBGL II

Obviously they include both up-type and down-type BGL models. Note that the G-W NFC model is also implemented by this Z_2 symmetry. The difference is the way the left-handed fields transform under this symmetry: the principle leading to gBGL constraints the Yukawa couplings so that each line of Γ_i, Δ_j couples only to one Higgs doublet.

- This time, in the quark sector, the model is fully defined by

$$
\begin{aligned}
N_d^0 &= \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_3 \right] M_d^0 \\
N_u^0 &= \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_3 \right] M_u^0
\end{aligned}
$$

and in the mass basis

$$
\begin{aligned}
N_d &= \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) V^\dagger \left| \widehat{n}_u \right\rangle \left\langle \widehat{n}_u \right| V \right] D_d \\
N_u &= \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) \left| \widehat{n}_u \right\rangle \left\langle \widehat{n}_u \right| \right] D_u
\end{aligned}
$$

Generalized BGL models: gBGL III

the new free parameters are two angles to define the unitary vector $\left|\widehat{n}_u\right\rangle$ or $\left|\widehat{n}_d\right\rangle$ and two phases of the three complex component

$$\left|\widehat{n}_u\right\rangle = V\left|\widehat{n}_d\right\rangle$$

where

$$\widehat{n}_{[u]i} \equiv \langle\widehat{n}_u|i\rangle = (\mathcal{U}_L^u)_{3i}$$

- Note that BGL models inherit from the standard Yukawa couplings the masses and V. gBGL models additionally inherit also a unitary vector from $\mathcal{U}_L^u$,introducing four additional parameters.

Generalized BGL models: gBGL IV

- By choosing

$$\left|\widehat{n}_u\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \text{or} \quad \left|\widehat{n}_u\right\rangle = \begin{pmatrix} V_{ub} \\ V_{cb} \\ V_{tb} \end{pmatrix}$$

we are in the limit where a gBGL model arrives to the top BGL limit or to the bottom BGL limit, for example.

- The Yukawa coupling to the $125\,GeV$ Higgs

$$\begin{aligned} Y^{(q)} &= \frac{1}{v}\left[s_{\beta\alpha} D_q + c_{\beta\alpha} N_q\right] \\ N_d &= \left[t_\beta I - \left(t_\beta + t_\beta^{-1}\right)\left|\widehat{n}_d\right\rangle\left\langle\widehat{n}_d\right|\right] D_d \end{aligned}$$

with SFCNC

$$Y^{(q)} = -c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right)\left|\widehat{n}_q\right\rangle\left\langle\widehat{n}_q\right| \frac{D_q}{v}$$

Generalized BGL models: gBGL V

- All SFCNC effects are proportional to $c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right)$
- In an $i \longrightarrow j$ transition it is proportional to m_{q_i}/v
- In an $i \longrightarrow j$ transition it is proportional to $\left(|\widehat{n}_q\rangle\langle\widehat{n}_q|\right)_{ji}$ with maximal value $\left(1/\sqrt{2}\right)\left(1/\sqrt{2}\right) = 1/2$
- To be compared with the most intense case of BGL u model in the $s \longrightarrow d$ transition $\sim V_{ud}^* V_{us} \sim \lambda$

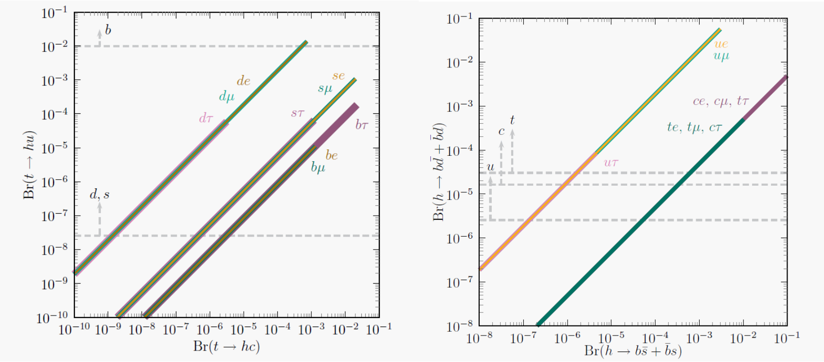
- From meson mixing we have the following naive constraints

	$D^0 - \overline{D}^0$	$K^0 - \overline{K}^0$	$B^0 - \overline{B}^0$	$B_s^0 - \overline{B}_s^0$
$\left\|c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right)\right\| \leq$	0.02	0.04	0.003	0.007

there are many regions of the model parameter space where $\left|c_{\beta\alpha}\left(t_\beta + t_\beta^{-1}\right)\right|$ can get its maximum value of order one.

Generalized BGL models: gBGL VI

- Some results for SFCNC are given here:



- Some results for the BAU are

$$I_{12} = \mathrm{Im}\ Tr\left[\left(M_u^0 M_u^{0\dagger}\right)\left(M_d^0 M_d^{0\dagger}\right)\left(M_u^0 M_u^{0\dagger}\right)^2\left(M_d^0 M_d^{0\dagger}\right)^2\right] \sim m_t^4 m_c^2 m_b^4 m_s^2 J$$

$$\begin{aligned} I_4\left(\widehat{n}_d\right) &= \mathrm{Im}\ Tr\left[N_d^0 M_d^{0\dagger} M_u^0 M_u^{0\dagger}\right] \\ &= \frac{i}{2}\left(t_\beta + t_\beta^{-1}\right)\sum_{i,j,k}\left(m_{d_i}^2 - m_{d_j}^2\right) m_{u_k}^2 \left(\left|\widehat{n}_d\right\rangle\left\langle\widehat{n}_d\right|\right)_{ij} V_{ki} V_{kj}^* \\ &\sim \left(t_\beta + t_\beta^{-1}\right) m_t^2 m_b^2\, \mathrm{Im}\left[\left(\left|\widehat{n}_d\right\rangle\left\langle\widehat{n}_d\right|\right)_{32} V_{tb} V_{ts}^*\right] \end{aligned}$$

enhancements of $10^{12} - 10^{13}$ in the CP contribution to the BAU are completely safe.

2HDM and SCPV I

- The first model of spontaneous CP violation (SCPV) proposed by T.D. Lee in 1973 a two Higgs doublets model (2HDM), with vacuum expectation values with a relative phase which violates T and CP.
- But SCPV and NFC generate real CKM (Branco)
- BGL models cannot have SCPV because the Z_4 symmetry is a too strong constraint to have SCPV.
- Keeping Z_2 softly broken allows for SCPV in the Higgs potential and the particular realization of the Z_2 -that defines gBGL models-

$$\begin{array}{rcl} \Phi_1 \longrightarrow \Phi_1 & ; & \Phi_2 \longrightarrow -\Phi_2 \\ Q_{L_{1,2}} \longrightarrow Q_{L_{1,2}} & ; & Q_{L_3} \longrightarrow -Q_{L_3} \\ u_R \longrightarrow u_R & ; & d_R \longrightarrow d_R \end{array}$$

do not meet NFC criteria and as we will see they generate a complex CKM

2HDM and SCPV II

- The Higgs potential is the standard for 2HDM with a Z_2 symmetry, that with a soft breaking term opens the possibility to have CP violation from the vacuum

$$
\begin{aligned}
V &= \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 - \mu_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\
&+ \left[\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 + h.c. \right] + 2\lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\
&+ 2\lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2
\end{aligned}
$$

$$
\cos\theta = \frac{\mu_{12}^2}{2\lambda_5 v_1 v_2}, \quad \theta \neq 0, \pm\frac{\pi}{2}, \pm\pi
$$

Generation of CP violating CKM and PMNS matrices I

- CP invariance of the Lagrangian

$$\Gamma_i^{(f)} = \Gamma_i^{(f)*}$$

- From the structure of the mass matrices

$$M_d^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(d)} c_\beta + \Gamma_2^{(d)} s_\beta e^{i\theta}\right); \quad M_u^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(u)} c_\beta + \Gamma_2^{(u)} s_\beta e^{-i\theta}\right)$$
$$M_l^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(l)} c_\beta + \Gamma_2^{(l)} s_\beta e^{i\theta}\right); \quad M_\nu^0 = \frac{v}{\sqrt{2}}\left(\Gamma_1^{(\nu)} c_\beta + \Gamma_2^{(\nu)} s_\beta e^{-i\theta}\right)$$

- it is evident that

$$M_f^0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma_f} \end{pmatrix} \widehat{M}_f^0 \equiv \Phi_3\left(\sigma_f\right) \widehat{M}_f^0$$

with $\widehat{M}_f^0$ arbitrary real mass matrices and

$$\sigma_f = \pm\theta \; \begin{matrix} f=d,e \\ f=u,\nu \end{matrix}$$

Generation of CP violating CKM and PMNS matrices II

- Obviously

$$\begin{aligned} M_f^0 M_f^{0\dagger} &= \Phi_3(\sigma_f)\,\widehat{M}_f^0\,\widehat{M}_f^{0\,T}\,\Phi_3(-\sigma_f) \\ M_f^{0\dagger} M_f^0 &= \widehat{M}_f^{0\,T}\,\widehat{M}_f^0 \end{aligned}$$

therefore $M_f^0 M_f^{0\dagger}$ will be diagonalized by

$$\mathcal{U}_{f_L}^\dagger M_f^0 M_f^{0\dagger} \mathcal{U}_{f_L} = \begin{pmatrix} m_{f_1}^2 & 0 & 0 \\ 0 & m_{f_2}^2 & 0 \\ 0 & 0 & m_{f_3}^2 \end{pmatrix} \; ; \mathcal{U}_{f_L} = \Phi_3(\sigma_f)\,\mathcal{O}_{f_L}$$

where $\mathcal{O}_{f_L}$ are real orthogonal matrices

$$\mathcal{O}_{f_L}^T \widehat{M}_f^0 \widehat{M}_f^{0\,T} \mathcal{O}_{f_L} = \begin{pmatrix} m_{f_1}^2 & 0 & 0 \\ 0 & m_{f_2}^2 & 0 \\ 0 & 0 & m_{f_3}^2 \end{pmatrix}$$

$$M_f = \mathcal{U}^\dagger_{f_L} M^0_f \mathcal{O}_{f_R} = \begin{pmatrix} m_{f_1} & 0 & 0 \\ 0 & m_{f_2} & 0 \\ 0 & 0 & m_{f_3} \end{pmatrix}$$

- Therefore because $V_{CKM} = \mathcal{U}^\dagger_{u_L} \mathcal{U}_{d_L}$ and $U_{PMNS} = \mathcal{U}^\dagger_{e_L} \mathcal{U}_{\nu_L}$ we will have

$$\boxed{V \equiv V_{CKM} = \mathcal{O}^T_{u_L} \Phi_3 \left(2\theta\right) \mathcal{O}_{d_L}} \; ; \; \boxed{U \equiv U_{PMNS} = \mathcal{O}^T_{e_L} \Phi_3 \left(-2\theta\right) \mathcal{O}_{\nu_L}}$$

  Since $\mathcal{O}_{f_L}$ are arbitrary real rotations, it is evident that there is enough freedom to obtain arbitrary V and U, except that any CP violating observable in the quark sector and any CP violating observable in the lepton sector, must vanish with $\theta \to 0$.

- **An obvious way of testing these models will be to relate the CP violating phases in V and U, δ_{CKM} and δ_{PMNS} respectively. ¿Could we predict δ_{PMNS} from the quark data (δ_{CKM}) ? After all both are generated by θ the vacuum phase.**

CP violation in CKM and PMNS and SFCNC I

- Our SFCNC are controlled by

$$N_f = \mathcal{U}_{f_L}^\dagger N_f^0 \mathcal{O}_{f_R} = \left[t_\beta I - \left(t_\beta + t_\beta^{-1} \right) P_3^f \right] D_f$$

where we have introduced the projectors

$$\begin{aligned} \left(P_3^f \right)_{ij} &= \left(\mathcal{U}_{f_L}^\dagger P_3 \mathcal{U}_{f_L} \right)_{ij} = \left(\mathcal{O}_{f_L}^T P_3 \mathcal{O}_{f_L} \right)_{ij} \\ &= (\mathcal{O}_{f_L})_{3i} (\mathcal{O}_{f_L})_{3j} \equiv \widehat{r}_{[f]i} \widehat{r}_{[f]j} \end{aligned}$$

our $\widehat{n}_{[f]i}$ now become real unitary vectors incorporating **two additional parameters** in total to the CKM or PMNS sectors, instead of four.

CP violation in CKM and PMNS and SFCNC II

- The only way to avoid SFCNC in P_3^f is to set one component $\widehat{r}_{[f]k} = 1$ and the others $\widehat{r}_{[f]j} = 0$ $j \neq k$

$$\left(P_3^f\right)_{ij} = \delta_{ik}\delta_{jk} \equiv (P_k)_{ij}$$

- If we kill SFCNC in any sector of the quarks and in any sector of the leptons V_{CKM} and U_{PMNS} are CP conserving

$$\begin{aligned} U &= \mathcal{O}_{e_L}^T \Phi_3(-2\theta)\, \mathcal{O}_{\nu_L} = \mathcal{O}_{e_L}^T \left[I + \left(e^{-i2\theta} - 1\right) P_3\right] \mathcal{O}_{\nu_L} \\ &= \mathcal{O}_{e_L}^T \mathcal{O}_{\nu_L} \mathcal{O}_{\nu_L}^T \left[I + \left(e^{-i2\theta} - 1\right) P_3\right] \mathcal{O}_{\nu_L} \\ &= \mathcal{O}_{e_L}^T \mathcal{O}_{\nu_L} \left[I + \left(e^{-i2\theta} - 1\right) P_3^{\nu}\right] \end{aligned}$$

that is CP conserving: a **real rotation times a diagonal of phases.** Similar in the other cases.

CP violation in CKM and PMNS and SFCNC III

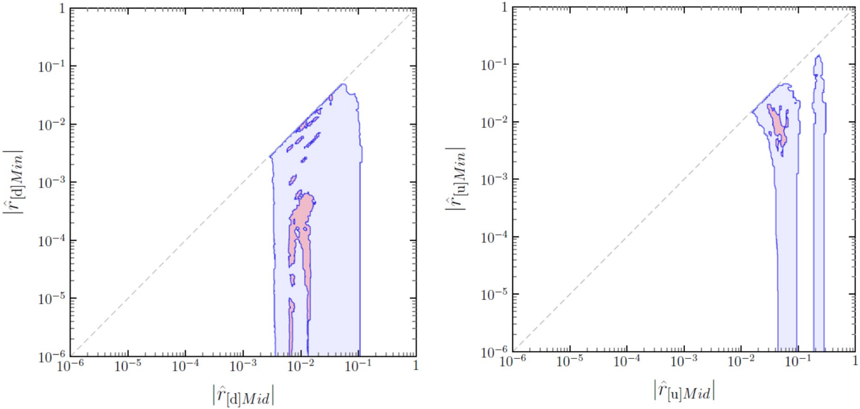
- **Therefore, in this model, to have CP violation in the CKM matrix, there must be tree level SFCNC both in the up and in the down quark sectors.**
- **In order to have a non-vanishing CP violating phase in the PMNS matrix, there must be tree level SFCNC both in the neutrino and in the charged lepton sectors.**

The relation between the CKM and PMNS phases I

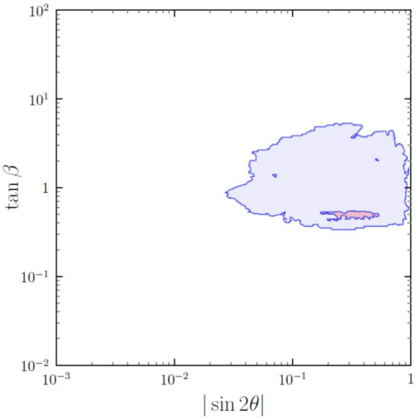
- In the quark sector now we have 6 parameters: five angles from $\mathcal{O}_{u_L}$ and $\mathcal{O}_{d_L}$ and θ. To be extracted from V_{CKM} and information from processes mediated by SFCNC. The same happens in the lepton sector, therefore one could predict δ_{PMNS} with enough information from the leptonic sector if previously we have θ from the quark information.
- The analysis of the quark sector shows that the model is viable after surmounting flavour constraints, Higgs constraints, electroweak constraints and overall that, as we have shown, SFCNC cannot be eliminated to produce a correct δ_{CKM}.
- Surprisingly, we have still a lot of room in the SFCNC and consequently in the value of θ. Therefore, generalizing the full analysis to include the leptonic sector does not look the more promising way to begin with, specially if we are trying to show how it works the connection among δ_{CKM} and δ_{PMNS} in this kind of models.

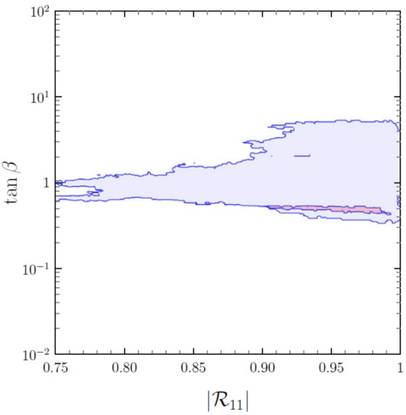
The relation between the CKM and PMNS phases II

- $\widehat{r}_{[d]}$ and $\widehat{r}_{[u]}$



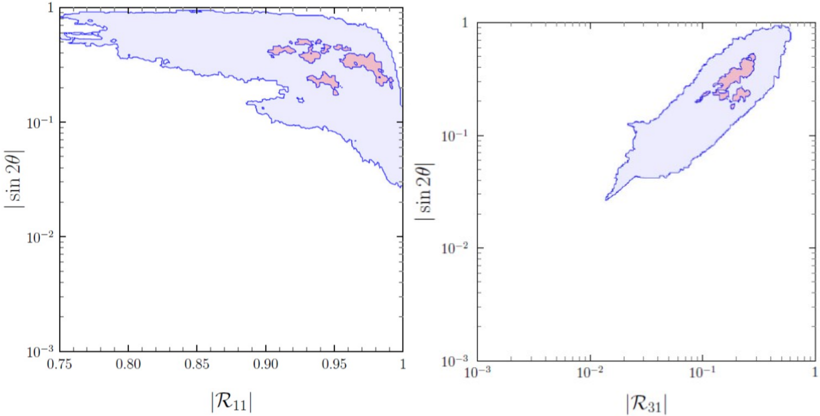
- $\tan\beta$ vs $|\sin 2\theta|$ and $|\mathcal{R}_{11}|$





The relation between the CKM and PMNS phases IV

- $|\sin 2\theta|$ vs $|\mathcal{R}_{11}|$ and $|\mathcal{R}_{31}|$



Simplified models (Quark sector) I

- The idea is to restrict the model by making simplifying assumptions about the SFCNC sector, guided by - and therefore compatible with - the experimental data.
- We cannot assume the absence of SFCNC. The way of eliminating as much as possible SFCNC is to impose a zero in the vector $\widehat{r}_{[u]}$ and a zero in the vector $\widehat{r}_{[d]}$:

$\widehat{r}_{[u]}$	$(0, \times, \times)$	$(\times, 0, \times)$	$(\times, \times, 0)$
$\widehat{r}_{[d]}$	$(0, \times, \times)$	$(\times, 0, \times)$	$(\times, \times, 0)$

only one type of SFCNC in each sector ($d_i \leftrightarrow d_j$ and $u_l \leftrightarrow u_m$)

- These models incorporate the **MFV** ansatz, only four parameters as in CKM.
- In fact the still allowed SFCNC, in each sector, will be fixed by one of the 3 mixing angles of the V_{CKM} matrix.

Simplified models (Quark sector) II

- We have 9 models in the quark sector and 9 models in the lepton sector. **81 models has been analyzed**. **Only one survives the experimental data**.
- In the quark sector the surviving model has only $t \rightleftarrows c$ and $d \rightleftarrows b$ SFCNC and parametrized as

$$\widehat{r}_{[u]} = \left(0, -\sin p_2^u, \cos p_2^u\right), \widehat{r}_{[d]} = \left(-\sin p_2^d, 0, \cos p_2^d\right)$$

$$V = R_{23}^T\left(p_2^u\right)\Phi_3\left(2\theta\right)R_{12}^T\left(p_1^u\right)R_{13}\left(p_2^d\right)$$

- The result of the fit of V to the experimental data (to the known V_{CKM}) gives

2θ	p_1^u	p_2^u	p_2^d
$1.077\left(^{+0.039}_{-0.031}\right)$	$0.22694\,(52)$	$4.235\,(59)\cdot 10^{-2}$	$3.774\,(98)\cdot 10^{-3}$

Simplified models (Quark sector) III

- In addition our fit fixes SFCNC with

$$\widehat{r}_{[u]} = (0, -0.0423, 0.9991)$$
$$\widehat{r}_{[d]} = (-0.0038, 0, 0.9999)$$

A non-trivial result is that these values are within the allowed regions of the previous figures. The precise effects in specific processes depend on other parameters like t_β and $\mathcal{R}_{11}$ that is the corresponding element of the Higgs mixing matrix, in particular it is the mixing among the $125 GeV$ Higgs and the Higgs with SM Higgs couplings. From the previous figures and taking from the θ value $|\sin(2\theta)| = 0.88$ we get

$$\mathcal{R}_{11} \in (0.82, 0.90)\,, \quad t_\beta \in (0.5, 1.8)$$

Simplified models (Quark sector) IV

- The most relevant prediction of this model in the SFCNC concerns the transition $t \to ch$

$$Br\left(t \to ch\right) = 0.1306\left(1 - \mathcal{R}_{11}^2\right)\left(t_\beta + t_\beta^{-1}\right)^2 r_{[u]2}^2 r_{[u]3}^2$$

$$2.7 \times 10^{-4} \leq Br\left(t \to ch\right) \leq 4.3 \times 10^{-4}$$

ATLAS already arrived to $Br\left(t \to ch\right) \leq 4.3 \times 10^{-4}$. In the $d \rightleftarrows b$ SFCNC we get $B_r\left(h \to b\bar{d} + d\bar{b}\right) \sim 10^{-6}$.

Simplified models (Lepton sector) I

- In the lepton sector the only model allowed experimentally is:

$$\widehat{r}_{[\nu]} = (-\sin p_2^\nu, \cos p_2^\nu, 0) \;\; ; \widehat{r}_{[e]} = (-\sin p_2^e, 0, \cos p_2^e)$$

$$U = R_{13}^T(p_2^e)\,\Phi_3(-2\theta)\,R_{12}^T(p_1^e)\,P_{23}R_{12}(p_2^\nu)$$

  The PMNS matrix is fully fixed by three mixing angles and the *CP* violating phase θ already fixed by the quark sector.

- Now we can fit U to the experimental information on PMNS encoded in $\left\{\theta_{12}^l, \theta_{13}^l, \theta_{23}^l\right\}$. In this fit we fix the quark fit result $2\theta = 1.077^{+0.039}_{-0.031}$. Although different PMNS analyses show some sensitivity to the phase δ_l, we do not include that information. The fit gives the following two solutions:

Solution 1:	$p_1^e = 0.7496$,	$p_2^e = 1.3541$,	$p_2^\nu = 0.8974$
Solution 2:	$p_1^e = 2.3889$,	$p_2^e = 1.3541$,	$p_2^\nu = 1.0542$

Simplified models (Lepton sector) II

SFCNC are controlled in both cases by

$$\hat{r}_{[e]} = (-0.9765, 0, 0.2156)$$

- Most important, the solutions differ in the values of the (unique) CP violating imaginary part of the Jarlskog invariant quartet

$$J_{PMNS} = \mathrm{Im}\left(U_{e1}\, U_{\mu 2}\, U^*_{e2}\, U^*_{\mu 1}\right)$$

and the phase $\delta_{PMNS} = \delta_l$,

Case	J_{PMNS}	$\delta_{PMNS} = \delta_l$	$\Delta\chi^2_{NO}\,(\delta_{PMNS})$	$\Delta\chi^2_{IO}\,(\delta_{PMNS})$
Solu 1	-0.0316	$1.629\pi\ (293^o)$	5	0
Solu 2	0.0282	$0.679\pi\ (126^o)$	13	> 20

$\Delta\chi^2_{NO}\,(\delta_{PMNS})$ and $\Delta\chi^2_{IO}\,(\delta_{PMNS})$ show the values that correspond to δ_{PMNS} attending to the $\Delta\chi^2$ profiles for δ_l obtained for normal and inverted neutrino mass ordering.

Simplified models (Lepton sector) III

- **Using the information on CP violation in the quark sector, we have been able to predict the phase in PMNS using the connection that SCPV provides in this model; in particular, Solution 1 has** $\delta_{PMNS} = 1.629\pi$, **which is in good agreement with the most likely values in PMNS analyses**.
- With r_e and r_τ we have a definite prediction for $h \to e\overline{\tau} + \tau\overline{e}$, through the equation

$$B_r\left(h \to e\tau\right) = \left(1 - \mathcal{R}_{11}^2\right)\left(t_\beta + t_\beta^{-1}\right)^2 r_e^2 r_\tau^2 \left(\frac{\Gamma_{SM}\left(h\right)}{\Gamma\left(h\right)}\right)$$

Taking the allowed regions of $\mathcal{R}_{11}^2$ and t_β, we have the sharp prediction

$$3 \times 10^{-3} < \left(\frac{\Gamma\left(h\right)}{\Gamma_{SM}\left(h\right)}\right) B_r\left(h \to e\tau\right) < 5 \times 10^{-3}$$

ATLAS has arrived to $B_r\left(h \to e\tau\right) < 2 \times 10^{-3}$, **CMS before.**

Simplified models (Lepton sector) IV

- Of course, those experimental bounds points to the exclussion of these simplified models. But probably and much more important they implied that we have new weapons, new sectors where to look for the consequences of SCPV. In particular it looks like that its the moment to look either for the full gBGL 2HDM with SCPV or a less restricted version.
- This relation among SCPV and SFCNC in order to generate a complex CKM present in the NFC framework and now in this gBGL model has trigger also to look for the generality of this connection, already study long time ago by Ecker et al. and Gronau et al. outside NFC.
- Surprisingly we have founded in the lepton sector that the requirement of absence of SFCNC and SCPV and generating a CP violating PMNS mixing matrix gives rise to mu-tau symmetric PMNS matrix. But this is the subject of Miguel Nebot talk.

Conclusions I

- (g)BGL two Higgs doublet models are relevant models beyond NFC very useful to study the 2HDM sector
- They offer important contributions to flavour changing Higgs participated processes and to the BAU
- gBGL are an important frame to study the connection between SCPV and SFCNC.
- The experimental Higgs flavour changing data is starting to give important information on these models in such a way that we expect soon important information on the SCPV-SFCNC connection