Flavor-nondiagonal neutral Higgs Yukawa couplings revisited

Howard E. Haber

5 June 2024

Outline

1. Introduction
2. Lightning review of the 2HDM
3. 2HDM Yukawa couplings
4. Specializing to the CP-conserving 2HDM
5. The Fritzsch flavor textures
6. An ansatz for the flavor structure of the 2HDM Yukawa matrices
7. The (modified) Cheng-Sher ansatz in light of LHC Higgs data
8. Conclusions

Introduction

In the Standard Model (SM), the Higgs-fermion Yukawa coupling matrices are proportional to the corresponding diagonal fermion mass matrices.

- This is a very good feature of the SM, since experimental data reveals that flavor-changing neutral currents (FCNC) are highly suppressed.

The absence of tree-level Higgs-mediated FCNCs is not a generic feature of extended Higgs sectors.

- For example, it is standard practice introduce a symmetry of the two-Higgs doublet model (2HDM) Lagrangian to provide a natural explanation for the absence of tree-level Higgs-mediated FCNCs.

Cheng and Sher[1] advocated for a mechanism that replicated the hierarchies of the quark masses and CKM angles in the structure of the Yukawa matrices.

- As a consequence of the Cheng-Sher ansatz, the tree-level off-diagonal neutral Higgs–fermion couplings are suppressed (but not set to zero).[2]

It is sometimes asserted that the Cheng-Sher ansatz is no longer viable in light of the most recent collider data. Joseph M. Connell and I have revisited the Cheng-Sher ansatz in the context of the basis-independent approach to the 2HDM[3] (and a recent update of the Fritzsch textures for the quark mass matrices).

[1] T.P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987)

[2] The detailed phenomenology of this proposal was further investigated in a series of papers by J.L. Díaz-Cruz, R. Noriega-Papaqui, and A. Rosado, Phys. Rev. D **69**, 095002 (2004); **71**, 015014 (2005) [with follow up works by M.A. Arroyo-Ureña, J.L. Díaz-Cruz and collaborators), and in a series of papers by M. Gómez-Bock and collaborators. Additional works by J. Hernández-Sánchez, S. Moretti, and collaborators are also noteworthy.

[3] H.E. Haber and D. O'Neil, Phys. Rev. D **74**, 015018 (2006); **D83**, 055017 (2011).

Lightning review of the 2HDM

Φ_1 and Φ_2: two (complex) hypercharge-one SU(2)$_L$ doublets of scalar fields, with scalar potential $\mathcal{V}(\Phi_1, \Phi_2)$, in the Φ-basis.

The scalar field vacuum expectations values (vevs) are of the form

$$\langle \Phi_i \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ \widehat{v}_i \end{pmatrix} ,$$

where $\widehat{v}$ is a complex vector of unit norm,

$$\widehat{v} = (\widehat{v}_1, \widehat{v}_2) = (c_\beta \, , \, s_\beta e^{i\xi}),$$

$c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$, and $v = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}$.

$\mathcal{H}_1$ and $\mathcal{H}_2$: Higgs basis fields

$$\mathcal{H}_1 = (\mathcal{H}_1^+\,,\,\mathcal{H}_1^0) \equiv \widehat{v}_i^{\,*}\Phi_i\,, \qquad \mathcal{H}_2 = (\mathcal{H}_2^+\,,\,\mathcal{H}_2^0) \equiv e^{i\eta}\epsilon_{ij}\widehat{v}_i\Phi_j\,,$$

where $\epsilon_{12} = -\epsilon_{21} = 1$ and $\epsilon_{11} = \epsilon_{22} = 0$, and there is an implicit sum over repeated indices. The phase factor $e^{i\eta}$ indicates that the Higgs basis is not unique, since one can always rephase the Higgs basis field $\mathcal{H}_2$ while maintaining $\langle\mathcal{H}_2^0\rangle = 0$.

Physical neutral scalars: h_1, h_2 and h_3 obtained by diagonalizing the neutral scalar squared-mass matrix

$$R\mathcal{M}^2R^{\mathsf{T}} = \mathrm{diag}\,(m_1^2\,,\,m_2^2\,,\,m_3^2)\,,$$

where $R \equiv R_{12}R_{13}R_{23}$ is the product of three rotation matrices parametrized by θ_{12}, θ_{13} and θ_{23}, respectively.

The physical neutral mass-eigenstate scalar fields are

$$h_k = q_{k1}\big(\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_1^0 - v\big) + \frac{1}{\sqrt{2}}\big(q_{k2}^*\mathcal{H}_2^0 e^{i\theta_{23}} + \mathrm{h.c.}\big)\,,$$

where the q_{k1} and q_{k2} are exhibited in table below (where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$).

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - is_{12}s_{13}$
3	s_{13}	ic_{13}

Without loss of generality, one can set $\theta_{23} = 0$ by rephasing $\mathcal{H}_2^0$.

2HDM Yukawa couplings

In the Higgs basis,

$$
\begin{aligned}
-\mathscr{L}_Y = \sum_{i,m,m} & \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{U}})_{mn} \mathcal{H}_1^{0\dagger}\, \widehat{\bar{u}}_{mL} \widehat{u}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{U}})_{mn} \mathcal{H}_2^{0\dagger}\, \widehat{\bar{u}}_{mL} \widehat{u}_{nR} + \text{h.c.} \Big\} \\
& + \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{D}\,\dagger})_{mn} \mathcal{H}_1^{0}\, \widehat{\bar{d}}_{mL} \widehat{d}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{D}\,\dagger})_{mn} \mathcal{H}_2^{0}\, \widehat{\bar{d}}_{mL} \widehat{d}_{nR} + \text{h.c.} \Big\} \\
& + \Big\{ (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{E}\,\dagger})_{mn} \mathcal{H}_1^{0}\, \widehat{\bar{e}}_{mL} \widehat{e}_{nR} + (\widehat{\boldsymbol{\rho}}^{\boldsymbol{E}\,\dagger})_{mn} \mathcal{H}_2^{0}\, \widehat{\bar{e}}_{mL} \widehat{e}_{nR} \text{h.c.} \Big\},
\end{aligned}
$$

where $f_R \equiv \frac{1}{2}(1+\gamma_5)f$ and $f_L \equiv \frac{1}{2}(1-\gamma_5)f$ [with four-component fermion fields $f = u, d, \nu, e$]. The hatted fields correspond to the fermion interaction-eigenstate fields. Setting $\mathcal{H}_1^0 = \mathcal{H}_1^{0\,\dagger} = v/\sqrt{2}$ yields the fermion mass matrices

$$
(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{mn} = \frac{v}{\sqrt{2}} (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{U}})_{mn}\,, \qquad (\widehat{\boldsymbol{M}}_{\boldsymbol{D},\boldsymbol{E}})_{mn} = \frac{v}{\sqrt{2}} (\widehat{\boldsymbol{\kappa}}^{\boldsymbol{D},\boldsymbol{E}\,\dagger})_{mn}\,.
$$

The singular value decompositions of the quark mass matrices yield:

$$L_u^\dagger \, \widehat{\boldsymbol{M}}_{\boldsymbol{U}} \, R_u \equiv \boldsymbol{M}_{\boldsymbol{U}} = \mathrm{diag}(m_u \, , \, m_c \, , \, m_t) \, ,$$

$$L_d^\dagger \, \widehat{\boldsymbol{M}}_{\boldsymbol{D}} \, R_d \equiv \boldsymbol{M}_{\boldsymbol{D}} = \mathrm{diag}(m_d \, , \, m_s \, , \, m_b) \, ,$$

$$L_e^\dagger \, \widehat{\boldsymbol{M}}_{\boldsymbol{E}} \, R_e \equiv \boldsymbol{M}_{\boldsymbol{E}} = \mathrm{diag}(m_e \, , \, m_\mu \, , \, m_\tau) \, ,$$

where the masses m_f are real and nonnegative, and the unitary matrices L_f and R_f relate hatted interaction-eigenstate fermion fields with unhatted mass-eigenstate fields,

$$\widehat{f}_{mL} = (L_f)_{mn} f_{nL} \, , \qquad \widehat{f}_{mR} = (R_f)_{mn} f_{nR} \, .$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is defined by $\boldsymbol{K} \equiv L_u^\dagger L_d$.

The corresponding Higgs–fermion interactions involving mass-eigenstate scalar and fermion fields are given by

$$
\begin{aligned}
-\mathscr{L}_Y = \overline{U}\Big\{\frac{\boldsymbol{M_U}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q_{k2}^*\,\boldsymbol{\rho^U}P_R + q_{k2}\,\boldsymbol{\rho^{U\dagger}}P_L\Big]\Big\}Uh_k \\
+\overline{D}\Big\{\frac{\boldsymbol{M_D}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q_{k2}\,\boldsymbol{\rho^{D\dagger}}P_R + q_{k2}^*\,\boldsymbol{\rho^D}P_L\Big]\Big\}Dh_k \\
+\overline{E}\Big\{\frac{\boldsymbol{M_E}}{v}q_{k1} + \frac{1}{\sqrt{2}}\Big[q_{k2}\,\boldsymbol{\rho^{E\dagger}}P_R + q_{k2}^*\,\boldsymbol{\rho^E}P_L\Big]\Big\}Eh_k \\
+\Big\{\overline{U}\Big[\boldsymbol{K}\boldsymbol{\rho^{D\dagger}}P_R - \boldsymbol{\rho^{U\dagger}}\boldsymbol{K}P_L\Big]DH^+ + \overline{N}\boldsymbol{\rho^{E\dagger}}P_R E H^+ + \mathrm{h.c.}\Big\},
\end{aligned}
$$

where $P_R \equiv \frac{1}{2}(1+\gamma_5)$, $P_L \equiv \frac{1}{2}(1-\gamma_5)$, and the mass-eigenstate fields of the down-type quarks, the up-type quarks, the charged leptons and the neutrinos are $D = (d, s, b)^{\mathsf{T}}$, $U \equiv (u, c, t)^{\mathsf{T}}$, $E = (e, \mu, \tau)^{\mathsf{T}}$, and $N = (\nu_e, \nu_\mu, \nu_\tau)^{\mathsf{T}}$, respectively.

The physical ρ-type Yukawa couplings are

$$\boldsymbol{\rho}^{\boldsymbol{U}} \equiv L_u^\dagger \, \widehat{\boldsymbol{\rho}}^{\boldsymbol{U}} R_u \,,$$
$$\boldsymbol{\rho}^{\boldsymbol{D}\dagger} \equiv L_d^\dagger \, \widehat{\boldsymbol{\rho}}^{\boldsymbol{D}\dagger} R_d \,,$$
$$\boldsymbol{\rho}^{\boldsymbol{E}\dagger} \equiv L_e^\dagger \, \widehat{\boldsymbol{\rho}}^{\boldsymbol{E}\dagger} R_e \,,$$

which are arbitrary complex coupling matrices that are independent of the fermion mass matrices.

Note the presence of flavor-nondiagonal, CP-violating neutral Higgs–fermion interactions due to the complex nondiagonal $\boldsymbol{\rho}^{\boldsymbol{F}}$ matrices.

Specializing to the CP-conserving 2HDM

Assume that the only source of CP-violation is the unremovable phase of the CKM matrix. Then, there exists a real Higgs basis, in which the all scalar potential parameters and Higgs-fermion Yukawa couplings are real. For example,

$$\mathcal{V} \supset \tfrac{1}{2} Z_5 e^{-2i\eta} (\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + \left[Z_6 e^{-i\eta}\, \mathcal{H}_1^\dagger \mathcal{H}_1 + Z_7 e^{-i\eta}\, \mathcal{H}_2^\dagger \mathcal{H}_2 \right] \mathcal{H}_1^\dagger \mathcal{H}_2 + \text{h.c.}$$

where η can be chosen such that $Z_{5,6,7}$ are real. If $Z_6 \neq 0$ and/or $Z_7 \neq 0$, then the real Higgs basis is unique up to an overall sign, $\mathcal{H}_2 \to -\mathcal{H}_2$. For example,

$$\varepsilon \equiv \operatorname{sgn} Z_6 \,,$$

changes sign when $\mathcal{H}_2 \to -\mathcal{H}_2$.

The physical neutral Higgs bosons consist of two CP-even scalars h and H (with $m_h < m_H$) and a CP-odd scalar A, which are related to the neutral fields of the Higgs basis via

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,\mathcal{H}_1^0 - v \\ \varepsilon\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_2^0 \end{pmatrix}, \qquad A = \varepsilon\sqrt{2}\,\mathrm{Re}\,\mathcal{H}_2^0\,,$$

and[4]

$$\varepsilon c_{\beta-\alpha} = \frac{-|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}}\,.$$

h is SM-like when $\lvert c_{\beta-\alpha}\rvert \ll 1$ $0 \le \beta - \alpha \le \pi$			
k	h_k	q_{k1}	q_{k2}
1	h	$s_{\beta-\alpha}$	$\varepsilon\, c_{\beta-\alpha}$
2	$-\varepsilon\, H$	$-\varepsilon\, c_{\beta-\alpha}$	$s_{\beta-\alpha}$
3	$\varepsilon\, A$	0	i

[4] Z_1 is the coefficient of $\frac{1}{2}(\mathcal{H}_1^\dagger \mathcal{H}_1)^2$ in the scalar potential.

If $|c_{\beta-\alpha}| \ll 1$ then h is SM-like. In this regime (the so-called *Higgs alignment limit*), the following approximate expressions are satisfied:[5]

$$
\begin{aligned}
|c_{\beta-\alpha}| &\simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,, \\
m_h^2 &\simeq v^2(Z_1 + Z_6 c_{\beta-\alpha})\,, \\
m_H^2 - m_A^2 &\simeq v^2(Z_5 - Z_6 c_{\beta-\alpha})\,, \\
m_H^2 - m_{H^\pm}^2 &\simeq \tfrac{1}{2}v^2(Z_4 + Z_5 - 2Z_6 c_{\beta-\alpha})\,.
\end{aligned}
$$

In the decoupling limit, the Higgs alignment limit is achieved with $M_H \gg v$ and $Z_i \sim \mathcal{O}(1) \Longrightarrow m_H \sim m_A \sim m_{H^\pm} \gg m_h$.

[5] Z_4 is the coefficient of $(\mathcal{H}_1^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1)$ in the scalar potential.

CP-conserving neutral Higgs-fermion Yukawa couplings:

$$
\begin{aligned}
-\mathscr{L}_Y = \overline{U}\Bigg\{ & \left[\frac{\boldsymbol{M_U}}{v} s_{\beta-\alpha} + \frac{1}{\sqrt{2}}\varepsilon c_{\beta-\alpha}\Big(\boldsymbol{\rho^U} P_R + [\boldsymbol{\rho^U}]^\top P_L\Big)\right] h \\
& + \varepsilon\left[\frac{\boldsymbol{M_U}}{v}\varepsilon c_{\beta-\alpha} - \frac{1}{\sqrt{2}} s_{\beta-\alpha}\Big(\boldsymbol{\rho^U} P_R + [\boldsymbol{\rho^U}]^\top P_L\Big)\right] H - \frac{i}{\sqrt{2}}\,\varepsilon\Big(\boldsymbol{\rho^U} P_R - [\boldsymbol{\rho^U}]^\top P_L\Big) A \Bigg\} U \\
+ \sum_{F=D,E} \overline{F}\Bigg\{ & \left[\frac{\boldsymbol{M_F}}{v} s_{\beta-\alpha} + \frac{1}{\sqrt{2}}\varepsilon c_{\beta-\alpha}\Big([\boldsymbol{\rho^F}]^\top P_R + \boldsymbol{\rho^F} P_L\Big)\right] h \\
& + \varepsilon\left[\frac{\boldsymbol{M_F}}{v}\varepsilon c_{\beta-\alpha} - \frac{1}{\sqrt{2}} s_{\beta-\alpha}\Big([\boldsymbol{\rho^F}]^\top P_R + \boldsymbol{\rho^F} P_L\Big)\right] H + \frac{i}{\sqrt{2}}\,\varepsilon\Big([\boldsymbol{\rho^F}]^\top P_R - \boldsymbol{\rho^F} P_L\Big) A \Bigg\} F\,.
\end{aligned}
$$

Note that: (i) $\varepsilon c_{\beta-\alpha} = -|c_{\beta-\alpha}|$; (ii) no $\tan\beta$ dependence.

<u>Remark</u>: In the Higgs alignment limit, where $c_{\beta-\alpha} \to 0$, the Yukawa couplings of h coincide with those of the SM (and are hence flavor-diagonal). In contrast, H and A generically possess flavor-nondiagonal couplings in the Higgs alignment limit.

Flavor textures

A long-standing program initiated by H. Fritzsch[6] provides a phenomenological explanation of the quark mixing hierarchy based on a correlation with the quark mass hierarchy. Du and Xing subsequently proposed that[7]

$$\widehat{\boldsymbol{M}}_{\boldsymbol{F}} = \begin{pmatrix} 0 & C_F & 0 \\ C_F^* & \widetilde{B}_F & B_F \\ 0 & B_F^* & A_F \end{pmatrix} ,$$

where A_F, $\widetilde{B}_F \in \mathbb{R}$ (with no loss of generality, one can take $A_F > 0$). In particular, by choosing $A_F \gg |B_F|$, $|\widetilde{B}_F|$, C_F, one can reproduce the hierarchy of quark masses and CKM angles.

[6] H. Fritzsch, "Calculating the Cabibbo angle," Phys. Lett. B **70** (1977) 436.

[7] D.-s. Du and Z.-z. Xing, Phys. Rev. D **48**, 2349 (1993).

This proposed form is called the four-zero texture of hermitian quark mass matrices, since there are a total of four independent zeros[8] in $\widetilde{M}_F$ for $F = U, D$.[9] A previous proposal in which $\widetilde{B}_F = 0$ (the six-zero texture) is no longer consistent with data.

Writing $B_F = |B_F| e^{i\phi_{B_F}}$ and defining $C_F = |C_F| e^{i\phi_{C_F}}$ and $P_F \equiv \mathrm{diag}\big(1\,,\, e^{-i\phi_{C_F}}\,,\, e^{-i\left(\phi_{B_F}+\phi_{C_F}\right)}\big)$, it is convenient to define:

$$\overline{\boldsymbol{M}}_{\boldsymbol{F}} = P_F^\dagger\, \widehat{\boldsymbol{M}}_{\boldsymbol{F}}\, P_F = \begin{pmatrix} 0 & |C_F| & 0 \\ |C_F| & \widetilde{B}_F & |B_F| \\ 0 & |B_F| & A_F \end{pmatrix} .$$

[8] Due to the assumption of hermiticity, a pair of off-diagonal zeros is counted as one texture zero.

[9] The assertion that $\widehat{\boldsymbol{M}}_{\boldsymbol{U}}$ and $\widehat{\boldsymbol{M}}_{\boldsymbol{D}}$ are hermitian matrices with $(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{11} = (\widehat{\boldsymbol{M}}_{\boldsymbol{D}})_{11} = (\widehat{\boldsymbol{M}}_{\boldsymbol{D}})_{13} = 0$ does not require an extra set of assumptions, since these conditions can always be achieved by an appropriately chosen weak-basis transformation [e.g., see G.C. Branco, D. Emmanuel-Costa, and R. González Felipe, Phys. Lett. B **477** (2000) 147 and **670** (2009) 340 (with H. Serôdio)]. The additional constraint of $(\widehat{\boldsymbol{M}}_{\boldsymbol{U}})_{13} = 0$ is chosen to provide a good fit to the CKM matrix elements as a function of the quark masses.

Since $\overline{\boldsymbol{M}}_{\boldsymbol{F}}$ is a a real symmetric matrix, its eigenvalues (denoted by λ_i^F) are real numbers, denoted by λ_i^F $(i = 1, 2, 3)$

$$\lambda^3 - \lambda^2(A_F + \widetilde{B}_F) - \lambda(|C_F|^2 + |B_F|^2 - \widetilde{B}_F A_F) + |C_F|^2 A_F$$
$$= (\lambda - \lambda_1^F)(\lambda - \lambda_2^F)(\lambda - \lambda_3^F)\,.$$

in a convention where $|\lambda_1^F| < |\lambda_2^F| < |\lambda_3^F|$. The λ_i^F are related to the coefficients of the characteristic equation above,

$$\widetilde{B}_F = \lambda_1^F + \lambda_2^F + \lambda_3^F - A_F\,,$$
$$|B_F| = \sqrt{\frac{(A_F - \lambda_1^F)(A_F - \lambda_2^F)(\lambda_3^F - A_F)}{A_F}}\,,$$
$$|C_F| = \sqrt{\frac{-\lambda_1^F \lambda_2^F \lambda_3^F}{A_F}}\,.$$

Under the assumption that $A_F \gg |B_F|$, $|\widetilde{B}_F|$, C_F,

$$\lambda^F_{1,2} \simeq \frac{1}{2}\left[\widetilde{B}_F - \frac{|B_F|^2}{A_F} \pm \sqrt{\left(\widetilde{B}_F - \frac{|B_F|^2}{A_F}\right)^2 + 4|C_F|^2}\right], \qquad \lambda^F_3 \simeq A_F + \frac{|B_F|^2}{A_F},$$

where the maximal eigenvalue is denoted by λ^F_3 and terms of $\mathcal{O}(1/A_F^2)$ have been dropped. Since $A_F > 0$, it follows that $\lambda^F_1\lambda^F_2 < 0$ and $\lambda^F_3 > A_F$. It is convenient to adopt a convention where $|\lambda^F_1| < |\lambda^F_2| < \lambda^F_3$, with $\eta_F \equiv \operatorname{sgn}\lambda_2$. In particular,

$$\eta_F = \begin{cases} +1\,, & \text{if } \lambda^F_1 < 0 \text{ and } \lambda^F_2 > 0 \quad \Longrightarrow \quad |B_F|^2 < A_F\widetilde{B}_F \\ -1\,, & \text{if } \lambda^F_1 > 0 \text{ and } \lambda^F_2 < 0 \quad \Longrightarrow \quad |B_F|^2 > A_F\widetilde{B}_F \end{cases}$$

We now introduce the matrix $H_F = \operatorname{diag}\big(-\eta_F\,,\,\eta_F\,,\,1\big)$.

Hence,

$$Q_F^{\mathsf{T}} P_F^{\dagger} \widehat{\boldsymbol{M}}_{\boldsymbol{F}} P_F Q_F H_F = \mathrm{diag}\big(m_1^F\,,\, m_2^F\,,\, m_3^F\big)\,,$$

where $m_i^F \equiv (-\eta_F \lambda_1^F\,,\, \eta_F \lambda_2^F\,,\, \lambda_3^F)$ and

$$Q_F = \begin{pmatrix} \sqrt{\dfrac{\lambda_2^F \lambda_3^F \left(A_F - \lambda_1^F\right)}{A_F \left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_1^F\right)}} & \eta_F \sqrt{\dfrac{\lambda_1^F \lambda_3^F \left(\lambda_2^F - A_F\right)}{A_F \left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} & \sqrt{\dfrac{\lambda_1^F \lambda_2^F \left(A_F - \lambda_3^F\right)}{A_F \left(\lambda_3^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} \\ -\eta_F \sqrt{\dfrac{\lambda_1^F \left(\lambda_1^F - A_F\right)}{\left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_1^F\right)}} & \sqrt{\dfrac{\lambda_2^F \left(A_F - \lambda_2^F\right)}{\left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} & \sqrt{\dfrac{\lambda_3^F \left(\lambda_3^F - A_F\right)}{\left(\lambda_3^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} \\ \eta_F \sqrt{\dfrac{\lambda_1^F \left(A_F - \lambda_2^F\right)\left(A_F - \lambda_3^F\right)}{A_F \left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_1^F\right)}} & -\sqrt{\dfrac{\lambda_2^F \left(A_F - \lambda_1^F\right)\left(\lambda_3^F - A_F\right)}{A_F \left(\lambda_2^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} & \sqrt{\dfrac{\lambda_3^F \left(A_F - \lambda_1^F\right)\left(A_F - \lambda_2^F\right)}{A_F \left(\lambda_3^F - \lambda_1^F\right)\left(\lambda_3^F - \lambda_2^F\right)}} \end{pmatrix}.$$

That is, the singular value decomposition of the quark mass matrices can be achieved using

$$L_f = P_F Q_F\,, \qquad R_f = L_f H_F\,, \qquad \text{for } f = u, d.$$

A detailed analysis by H. Fritzsch, Z.-z. Xing and D. Zhang, Nucl. Phys. B **974** (2022) 115634, yields a very good fit to the CKM mixing angles and CP-violating phase by setting $A_U = A_D$. For example, in the case of $\eta_U = \eta_D = 1$, Fritzsch et al. obtain:

$$\widehat{\boldsymbol{M}}_{\boldsymbol{U}} \simeq m_t \begin{pmatrix} 0 & 0.00018 & 0 \\ 0.00018 & 0.18924 & 0.38787 \\ 0 & 0.38787 & 0.81444 \end{pmatrix},$$

$$\widehat{\boldsymbol{M}}_{\boldsymbol{D}} \simeq m_b \begin{pmatrix} 0 & 0.00465 & 0 \\ 0.00465 & 0.20335 & 0.38448 \\ 0 & 0.38448 & 0.81444 \end{pmatrix}.$$

with $\arg C_U - \arg C_D = 0.53216\pi$ and $\arg B_U - \arg B_D = 1.0313\pi$.

We shall extend the ansatz of Fritzsch et al. by setting

$$A_E = A_U = A_D\,.$$

An ansatz for the flavor structure of $\widehat{\boldsymbol{\rho}}^{\boldsymbol{F}}$

The ρ-type Yukawa coupling matrices in the fermion mass-eigenstate basis are given by

$$\boldsymbol{\rho}^{\boldsymbol{F}} = Q_F^{\mathsf{T}} P_F^{\dagger} \widehat{\boldsymbol{\rho}}^{\boldsymbol{F}} P_F Q_F H_F \,.$$

For simplicity we take $\boldsymbol{\rho}^{\boldsymbol{F}}$ by adopting the following ansatz:

$$P_F^{\dagger} \widehat{\boldsymbol{\rho}}^{\boldsymbol{F}} P_F = \frac{\sqrt{2}}{v} \begin{pmatrix} 0 & c_F|C_F| & 0 \\ c_F|C_F| & \widetilde{b}_F \widetilde{B}_F & b_F|B_F| \\ 0 & b_F|B_F| & a_F A_F \end{pmatrix} .$$

The a_F, b_F, $\widetilde{b}_F$ and c_F are real $\mathcal{O}(1)$ parameters (of either sign).

Note that if $a_F = b_F = \widetilde{b}_F = c_F$ then $\boldsymbol{\rho}^{\boldsymbol{F}} = a_F \boldsymbol{\kappa}^{\boldsymbol{F}}$, which corresponds to the flavor-aligned 2HDM. By taking a_F, b_F, $\widetilde{b}_F$ and c_F unequal, we inject the hierarchical structure of the fermion mass matrices into the $\boldsymbol{\rho}^{\boldsymbol{F}}$, as originally proposed by T.P. Cheng and M. Sher.[10]

We assume that $A \sim \mathcal{O}(m_3)$ and $m_1 \ll m_2 \ll m_3$ (dropping the superscript F for convenience). To obtain accurate approximate expressions for the ρ_{ij}, the size of $m_3 - A$ is critical. Suppose that $m_3 - A \sim \mathcal{O}(m_2)$. In this case, we can write

$$A = \overline{\alpha} m_3 \,, \qquad\qquad m_3 - A = \overline{\beta} m_2 \,,$$

where $\overline{\alpha}$ and $\overline{\beta}$ are positive $\mathcal{O}(1)$ parameters.[11]

[10] The original Cheng-Sher ansatz was based on the six-zero texture scheme where $\widetilde{B}_F = 0$.

[11] These parameters should not be confused with α and β, which appear in $c_{\beta-\alpha}$.

We then obtain:

$$\rho_{11} \simeq \frac{\sqrt{2}\,\eta\, m_1}{v}\left[\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b}) + \eta\big(2c-(1-\overline{\alpha})(2b-a)-\overline{\alpha}\widetilde{b}\big)\right],$$

$$\rho_{12} = -\rho_{21} \simeq \frac{\sqrt{2m_1m_2}}{v}\left[\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b}) + \eta\big(c-(1-\overline{\alpha})b-\overline{\alpha}\widetilde{b}\big)\right],$$

$$\rho_{13} = -\eta\rho_{31} \simeq \frac{\sqrt{2m_1m_3}\,\sqrt{\overline{\alpha}\overline{\beta}}}{v}\left[\overline{\alpha}(a-b) + (1-\overline{\alpha})(b-\widetilde{b})\right],$$

$$\rho_{22} \simeq \frac{\sqrt{2}\,\eta\, m_2}{v}\left[-\overline{\alpha}\overline{\beta}(2b-a-\widetilde{b}) + \eta\big((2\overline{\alpha}-1)\widetilde{b}+2(1-\overline{\alpha})b\big)\right],$$

$$\rho_{23} = \eta\rho_{32} \simeq \frac{\sqrt{2m_2m_3}\,\sqrt{\overline{\alpha}\overline{\beta}}}{v}\left[\overline{\alpha}(b-a) - (1-\overline{\alpha})(b-\widetilde{b})\right],$$

$$\rho_{33} \simeq \frac{\sqrt{2}m_3}{v}\left[\overline{\alpha}^2 a + 2\overline{\alpha}(1-\overline{\alpha})b + (1-\overline{\alpha})^2\widetilde{b}\right],$$

where terms of $\mathcal{O}(m_1/m_{2,3})$ and $\mathcal{O}(m_2/m_3)$ have been dropped.[12]

[12] Note that in an approximation where $m_2 \ll m_3$ and $\overline{\beta} \sim \mathcal{O}(1)$, one can also drop all terms that are proportional to $1-\overline{\alpha} = \overline{\beta}m_2/m_3$.

In particular, taking $\overline{\beta} \sim \mathcal{O}(1)$ yields the Cheng-Sher ansatz

$$\rho_{ij} = k_{ij} \frac{\sqrt{m_i m_j}}{v}, \qquad \text{where } k_{ij} \sim \mathcal{O}(1).$$

However, in light of the analysis by Fritzsch et al. previously cited, $A_F/m_f = 0.81444$, which yields

$$\overline{\beta} \simeq 0.18556\, m_3/m_2\,.$$

Using $\overline{\mathrm{MS}}$ quark masses evaluated at m_Z and the lepton masses yield: $m_t/m_c \simeq 271$, $m_b/m_s \simeq 53.4$, and $m_\tau/m_\mu = 16.81$. Hence,

$$\overline{\beta}_U \simeq 50\,, \qquad \overline{\beta}_D \simeq 10\,, \qquad \overline{\beta}_E = 3.12\,.$$

That is, k_{11}, k_{12}, k_{21}, and k_{22} are enhanced by an $\mathcal{O}(\overline{\beta})$ factor, while k_{13}, k_{31}, k_{23}, and k_{32} are enhanced by an $\mathcal{O}(\overline{\beta}^{1/2})$ factor.

The (modified) Cheng-Sher ansatz in light of LHC Higgs data

In the absence of FCNC phenomena mediated by the scalars of the 2HDM, one can ascertain an upper limit for $|c_{\beta-\alpha}|$ and lower limits for the masses of H, A, and $H^{\pm}$, assuming the ansatz for the flavor structure of $\widehat{\rho}^F$ adopted above.

Preliminary results for our analysis are shown below, where we have fixed $m_h = 125$ GeV and $m_H \sim m_A \sim m_{H^{\pm}} \sim 800$ GeV.[13]

[13] With these masses, one-loop FCNC phenomena mediated by $H^{\pm}$ (such as $b \to s + \gamma$) yield only small corrections to the corresponding contributions mediated by $W^{\pm}$ and cannot be ruled out by current experimental data.

Constraints imposed on our parameter scans

We scan over the $\mathcal{O}(1)$ parameters that define the $\boldsymbol{\rho}^{\boldsymbol{F}}$ and $|c_{\beta-\alpha}|$ [the latter determines the parameter $|Z_6|$) subject to the following constraints:

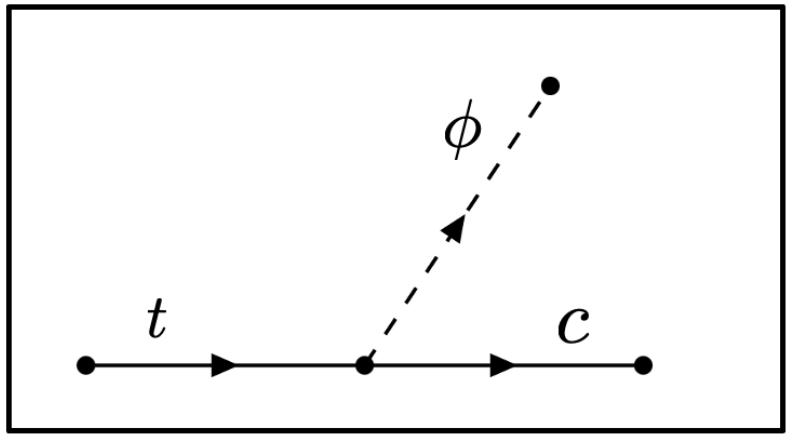
- The scalar potential is bounded from below.
- Tree-level unitarity and perturbativity.
- precision electroweak constraints on S, T, and U.
- precision LHC Higgs data (h BRs and cross sections)

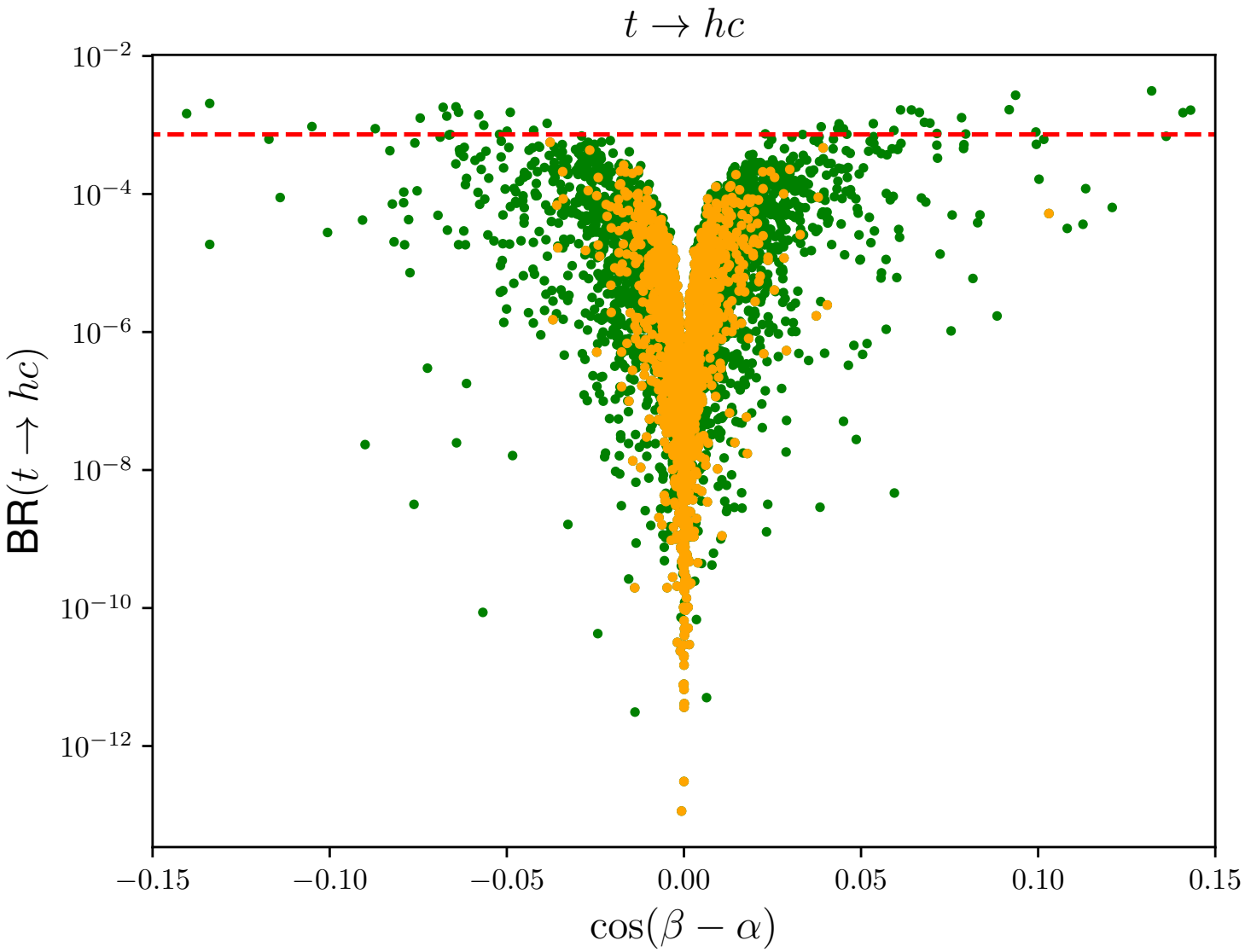
Constraints are checked using the public codes `2HDMC` and `HiggsTools`. We exclude points with $\Delta\chi^2 \gtrsim 6$ as provided by `HiggsSignals` (corresponding to a 95% CL exclusion limit for the joint estimation of two parameters).

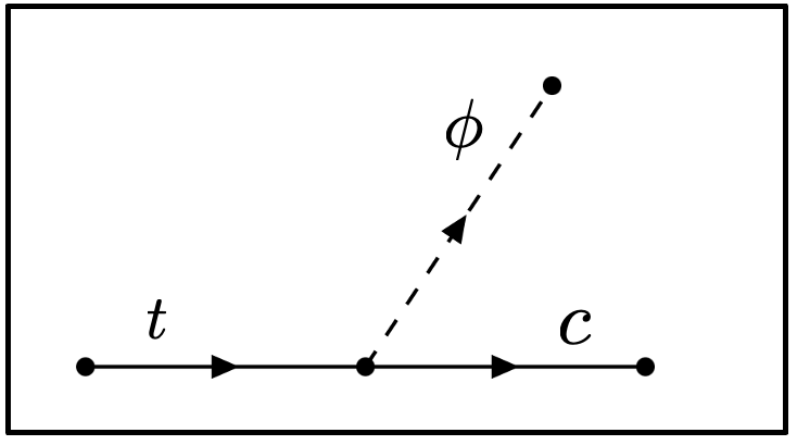
Flavor-changing processes mediated by neutral scalars

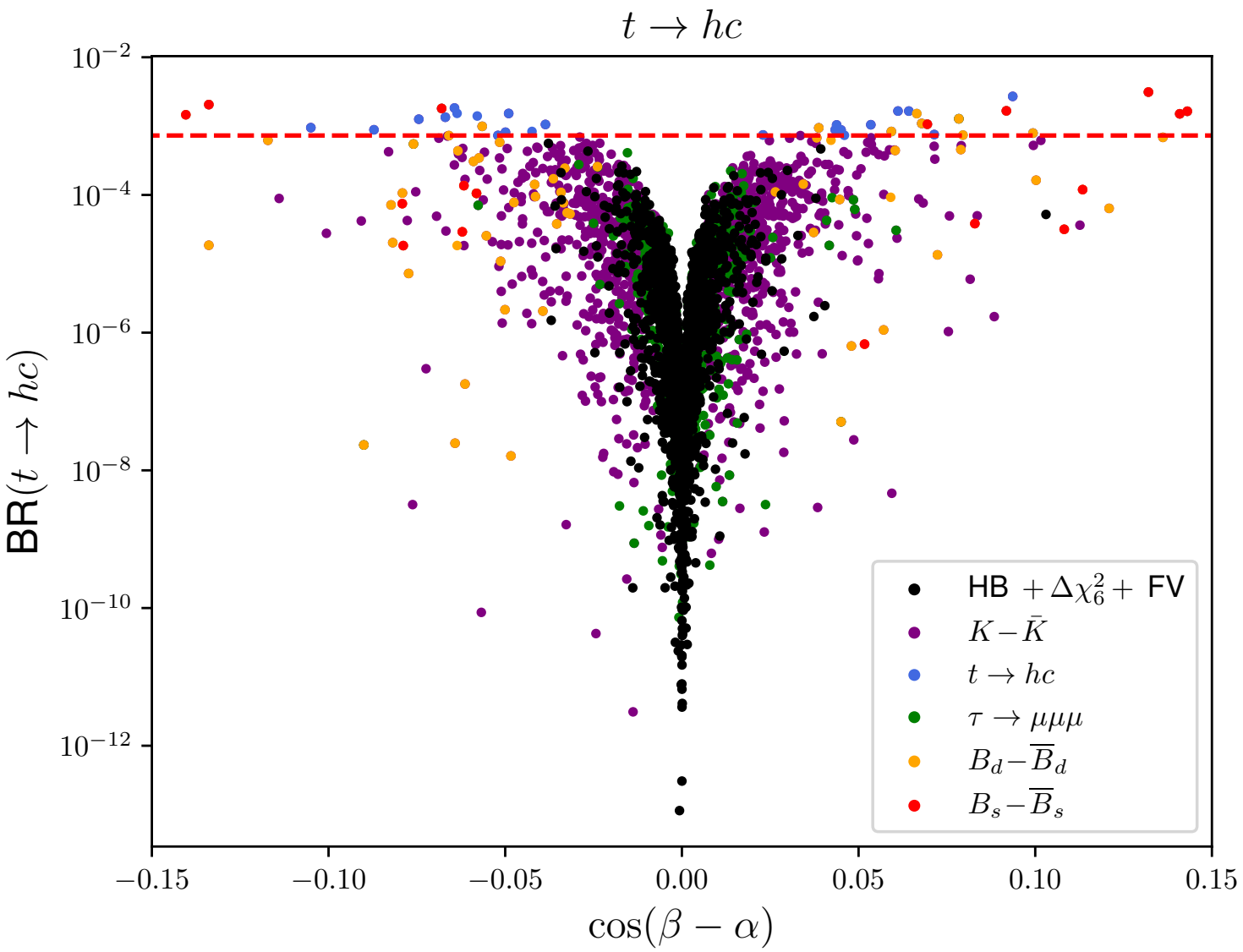
1. $t \to hc$
2. $h \to \tau^\pm \mu^\mp$, $h \to \tau^\pm e^\mp$, $h \to \mu^\pm e^\mp$
3. $\tau^\pm \to \mu^\pm \gamma$, $\tau^\pm \to e^\pm \gamma$, $\mu^\pm \to e^\pm \gamma$
4. $\tau^- \to \mu^- \mu^+ \mu^-$, $\mu^- e^+ e^-$, $e^- \mu^+ \mu^-$, $\mu^- \to e^- e^+ e^-$
5. K^0–$\bar{K}^0$ mixing
6. $P^0_{s,d}$–$\bar{P}_{s,d}$ mixing ($P = B$, D)
7. $B^0_{s,d} \to \mu^+ \mu^-$, $\tau^+ \tau^-$
8. $b \to s\mu^+ \mu^-$, $s\tau^+ \tau^-$

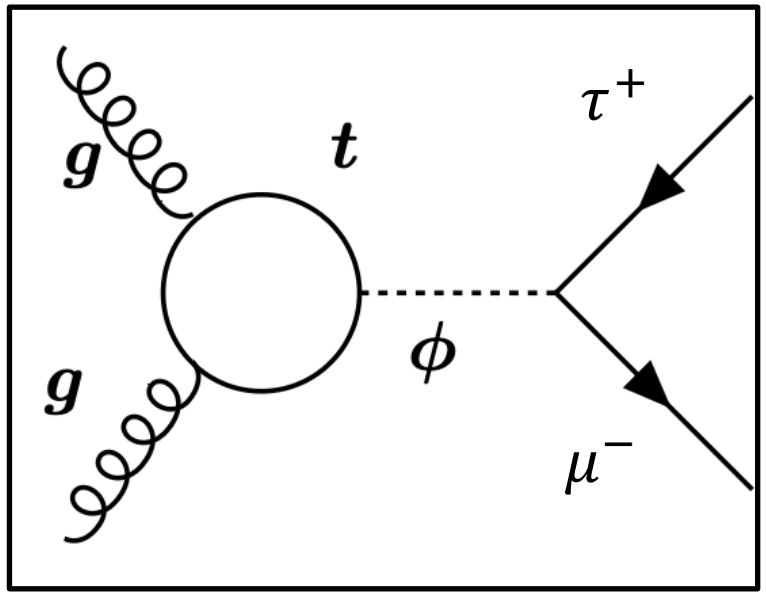
All plots shown are preliminary.

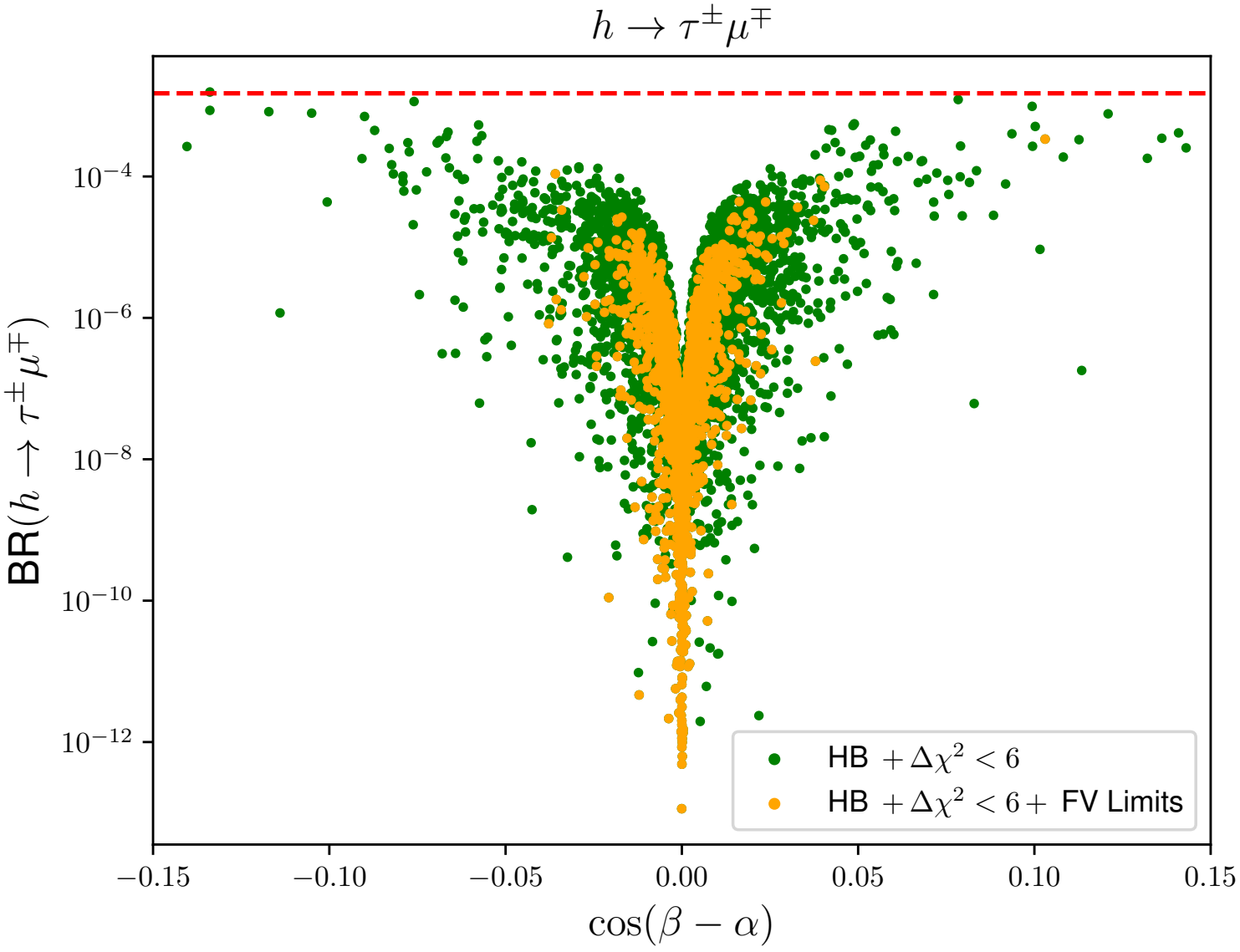

ϕ
t
c

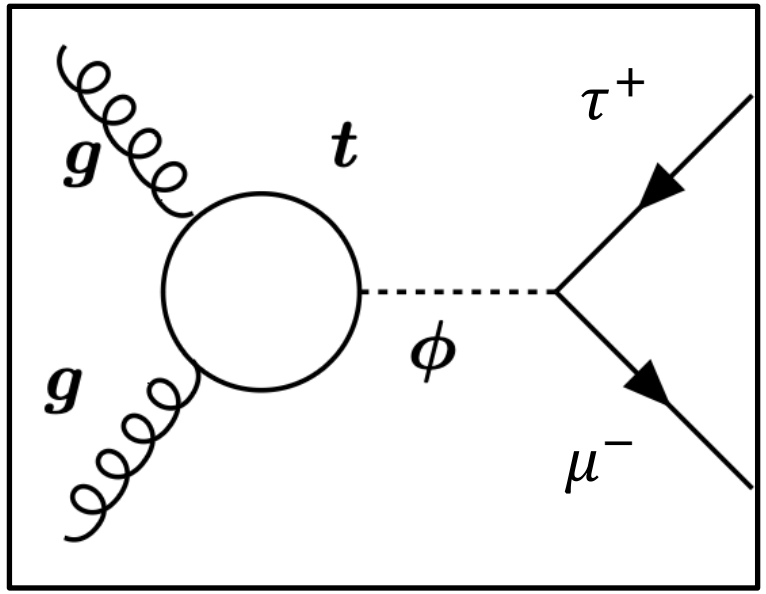

$t \to hc$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
$\mathrm{BR}(t \to hc)$
-0.15
-0.10
-0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$

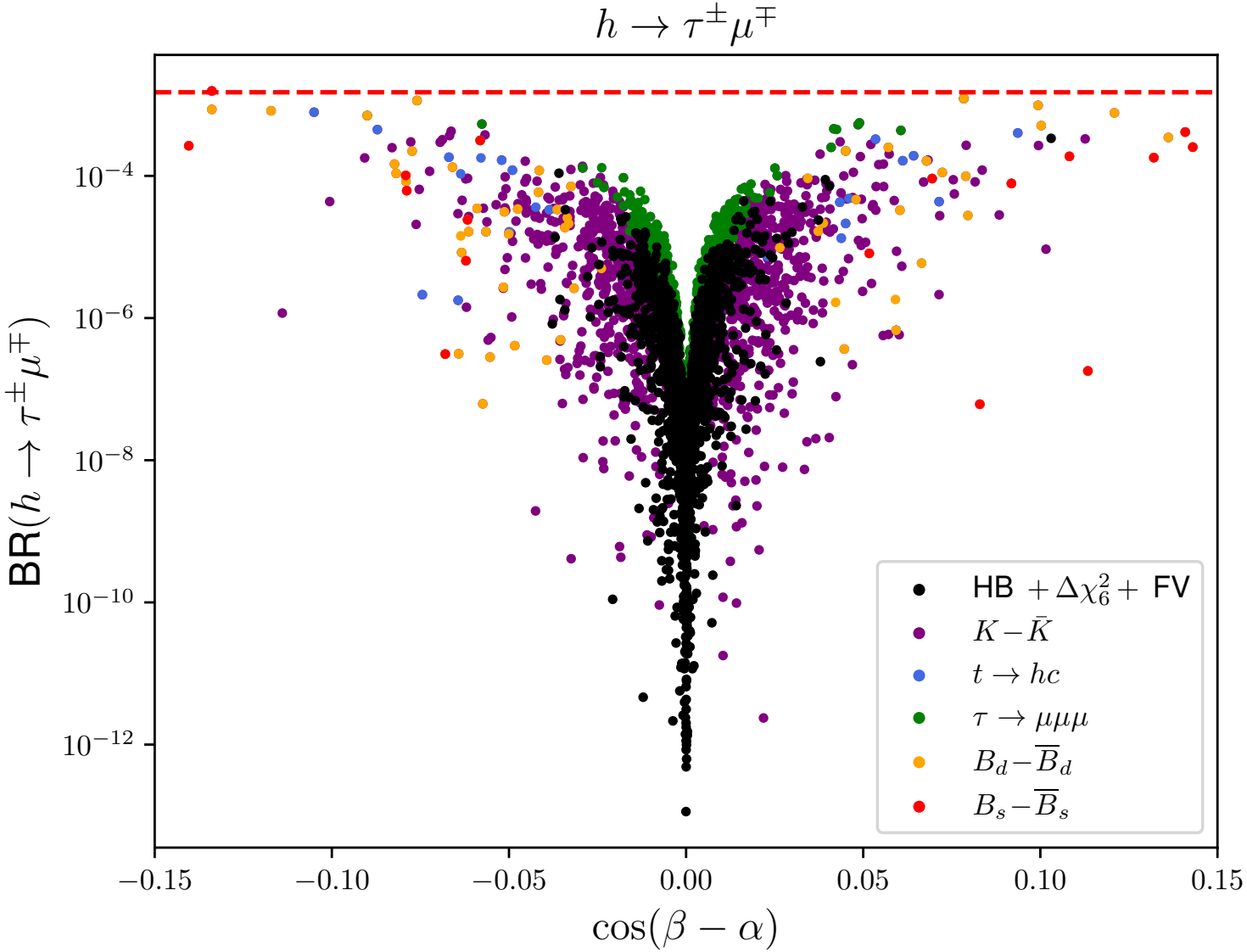

ϕ
t
c

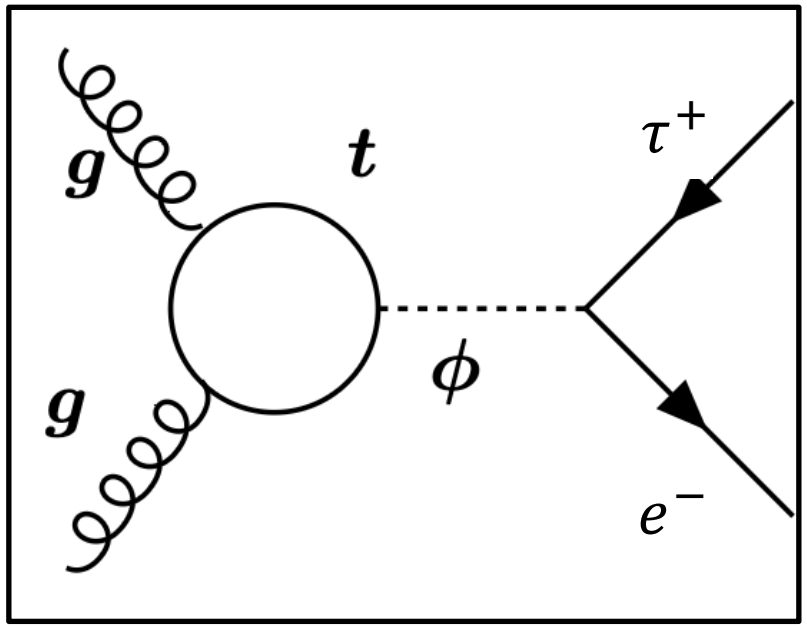

$t \to hc$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
BR($t \to hc$)
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$

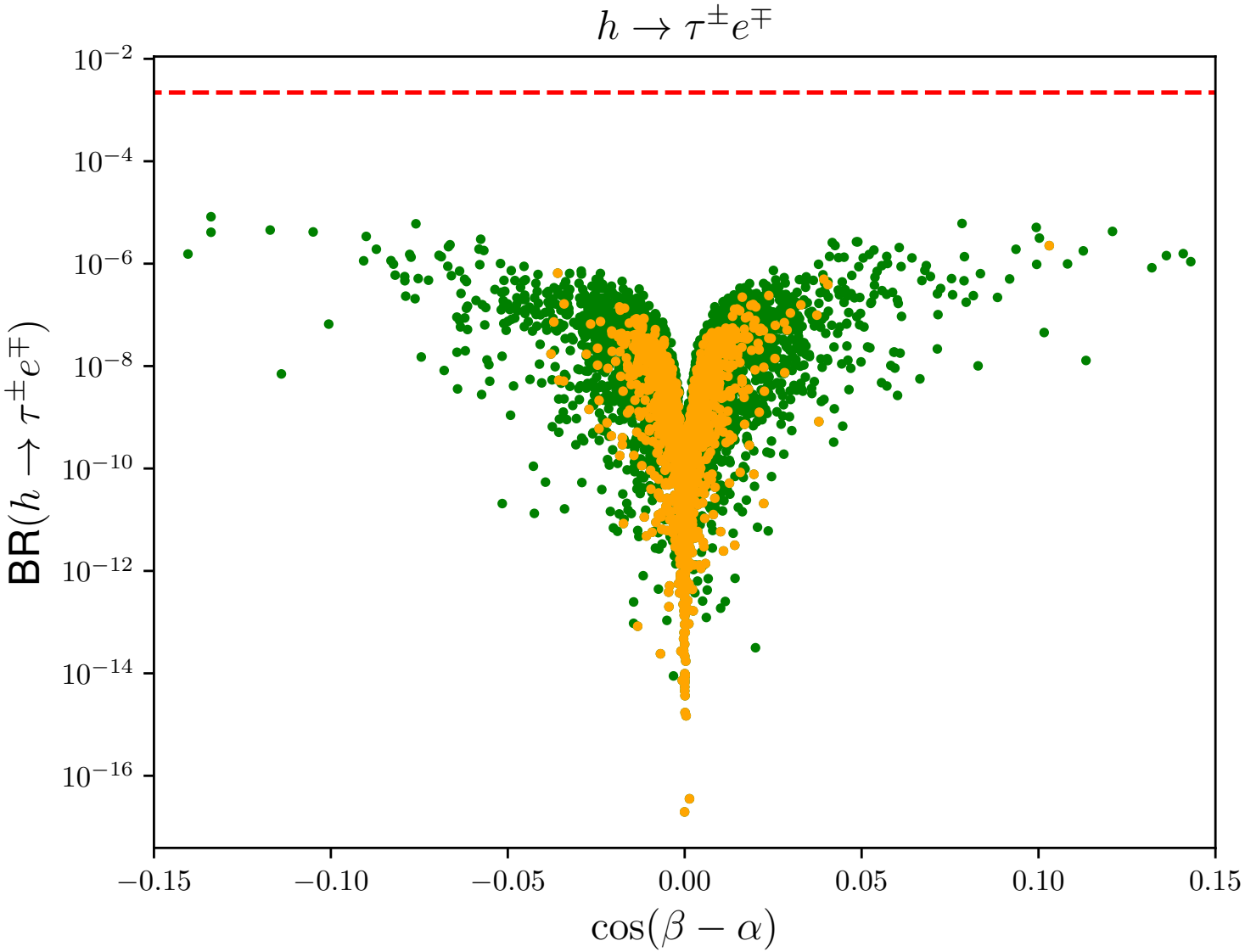

g
t
τ+
g
ϕ
μ−

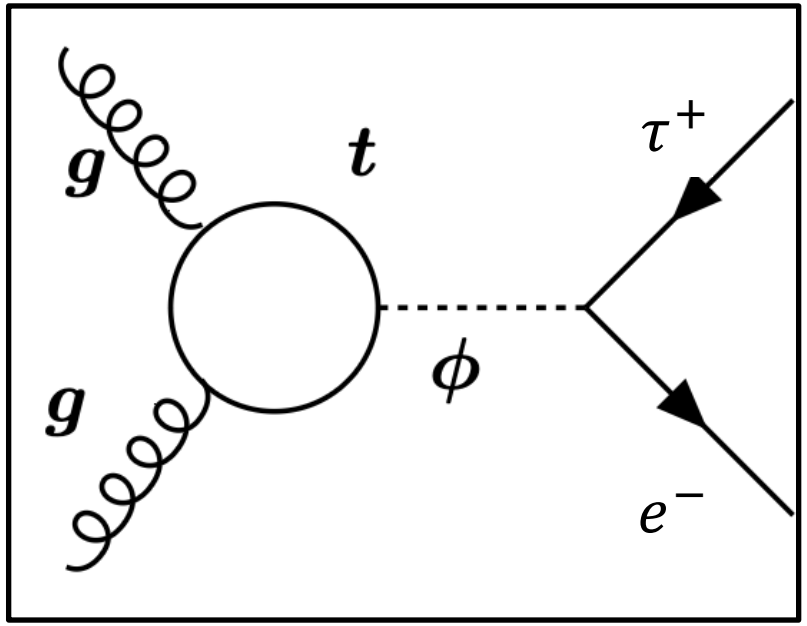

$h \to \tau^{\pm}\mu^{\mp}$
$\mathrm{BR}(h \to \tau^{\pm}\mu^{\mp})$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+\Delta\chi^2 < 6$
HB $+\Delta\chi^2 < 6 +$ FV Limits

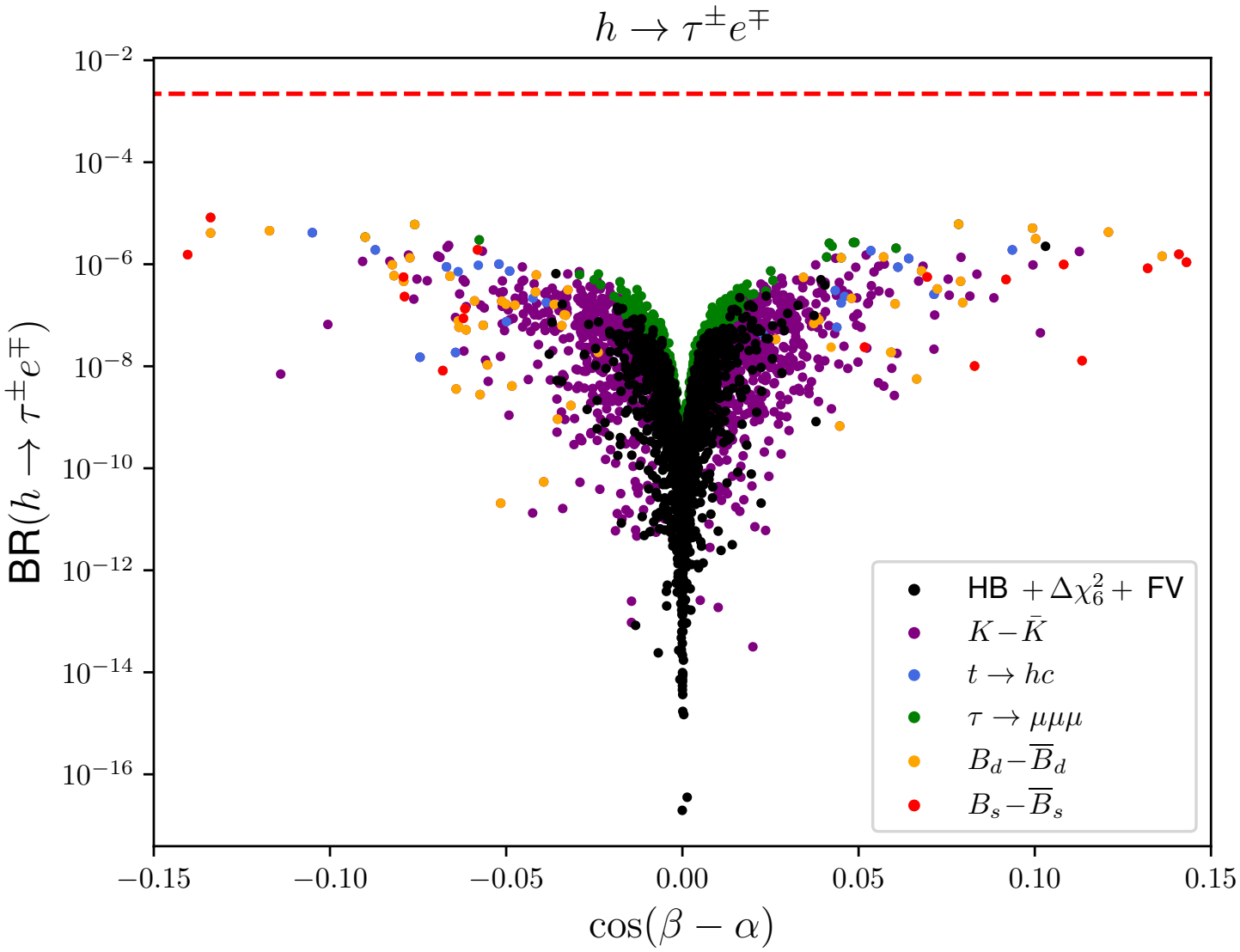

g
t
τ+
g
ϕ
μ−

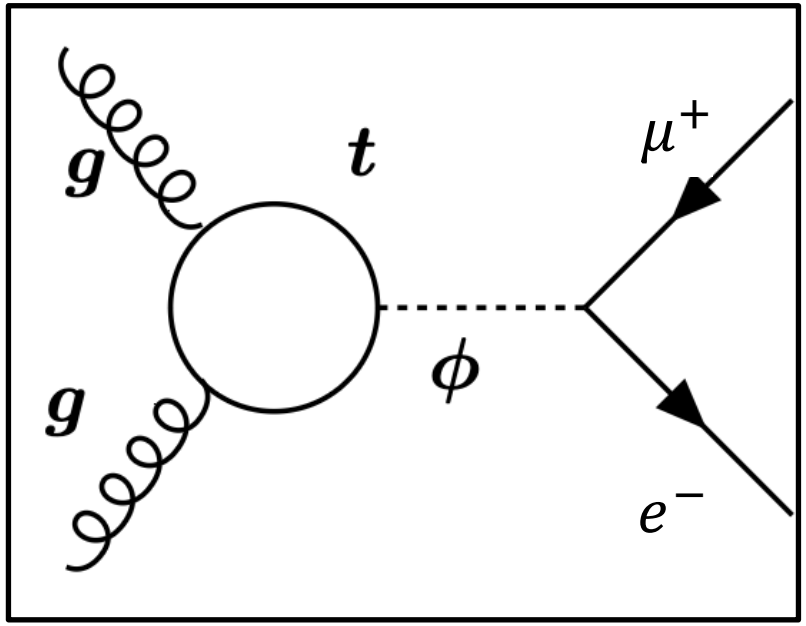

$h \to \tau^{\pm}\mu^{\mp}$
$\mathrm{BR}(h \to \tau^{\pm}\mu^{\mp})$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$

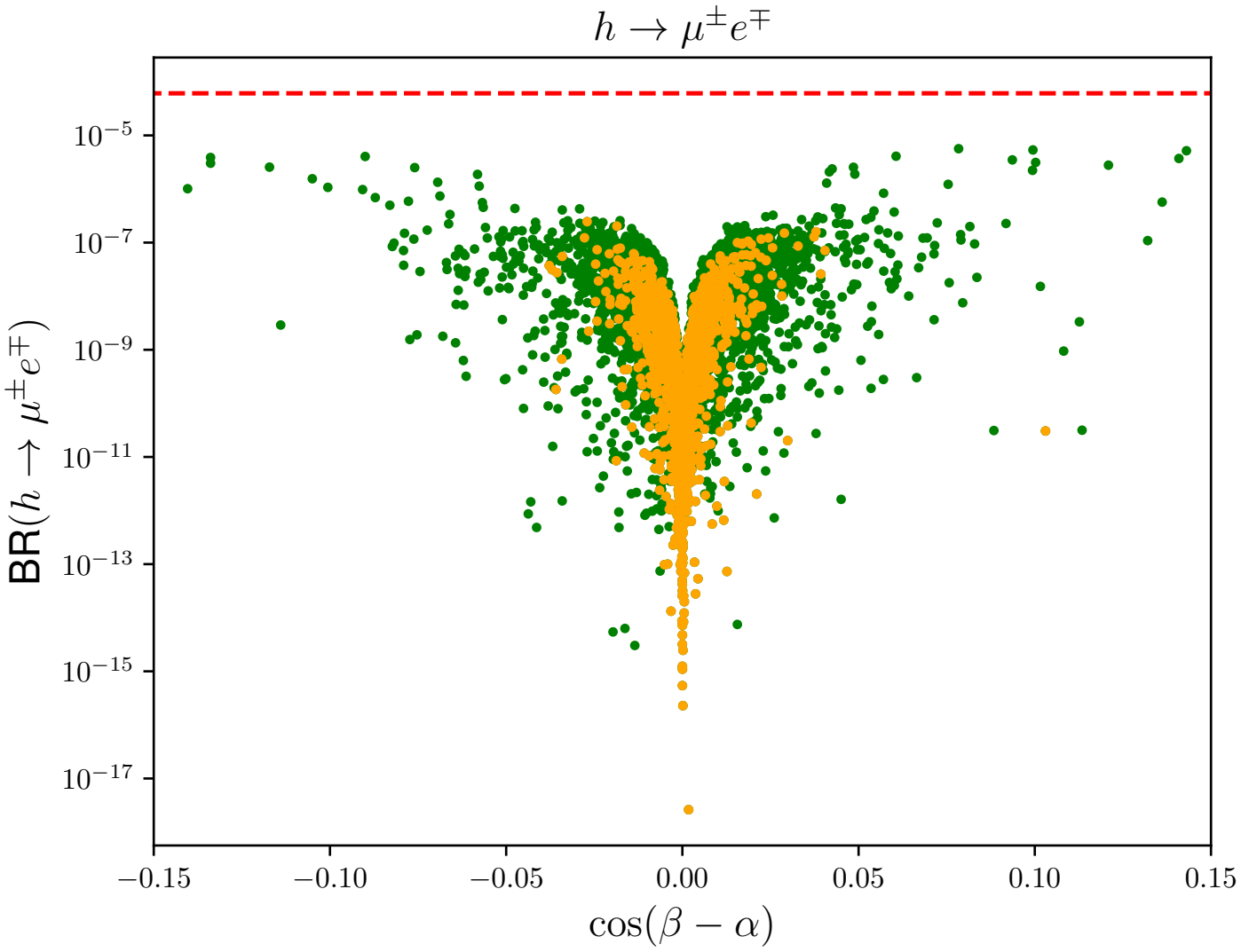

g
t
τ+
g
ϕ
e−

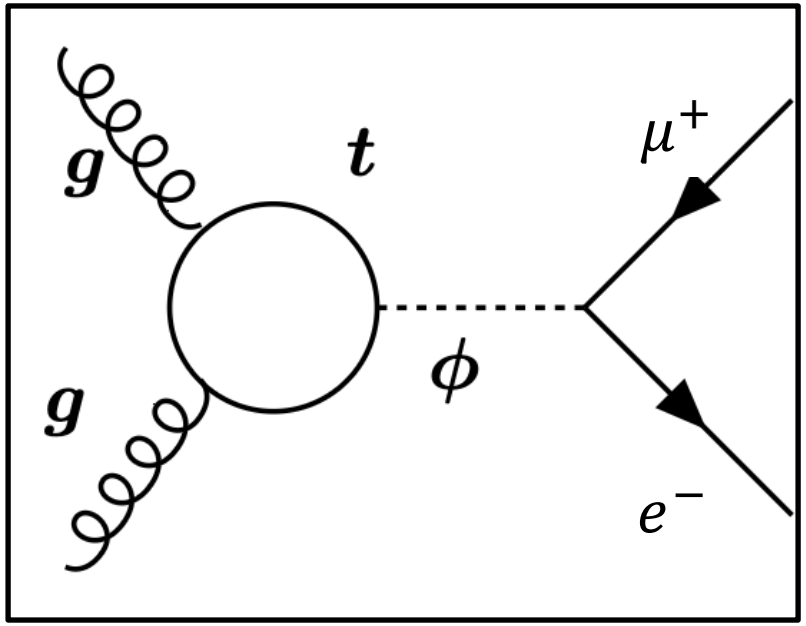

$h \to \tau^{\pm} e^{\mp}$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
$\mathrm{BR}(h \to \tau^{\pm} e^{\mp})$
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$

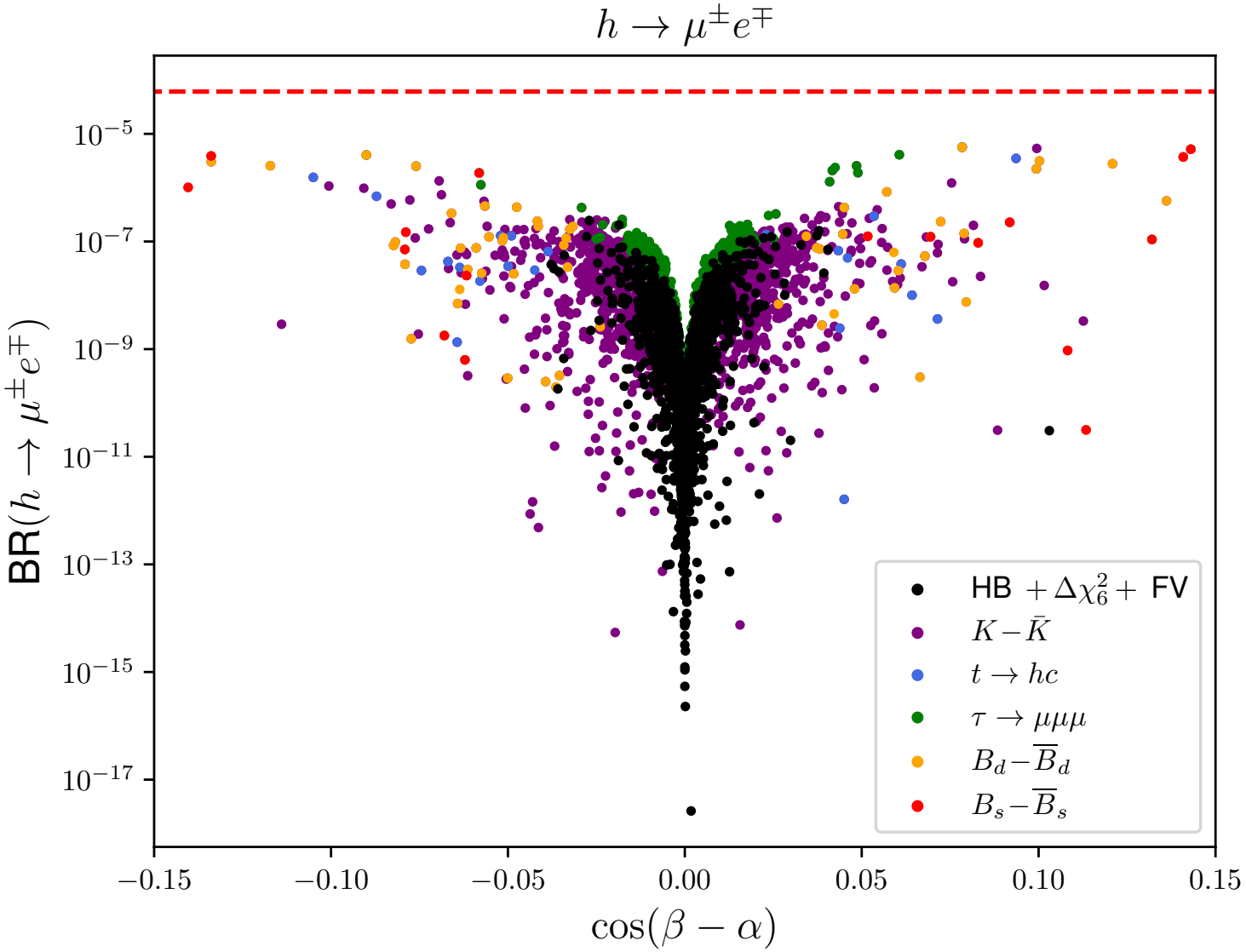

g
t
τ+
g
ϕ
e−


$h \to \tau^{\pm} e^{\mp}$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
$\mathrm{BR}(h \to \tau^{\pm} e^{\mp})$
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+ \Delta\chi^2_6 +$ FV
$K - \bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d - \overline{B}_d$
$B_s - \overline{B}_s$

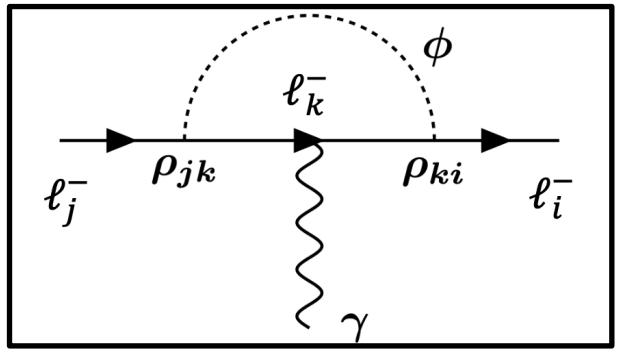

g
t
μ+
g
ϕ
e−


$h \to \mu^{\pm} e^{\mp}$
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
$\mathrm{BR}(h \to \mu^{\pm} e^{\mp})$
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$

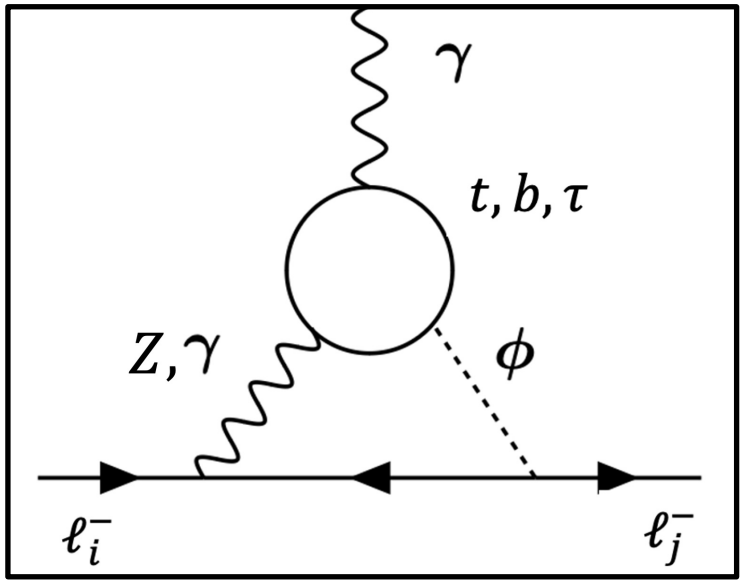

g
t
μ+
g
ϕ
e−

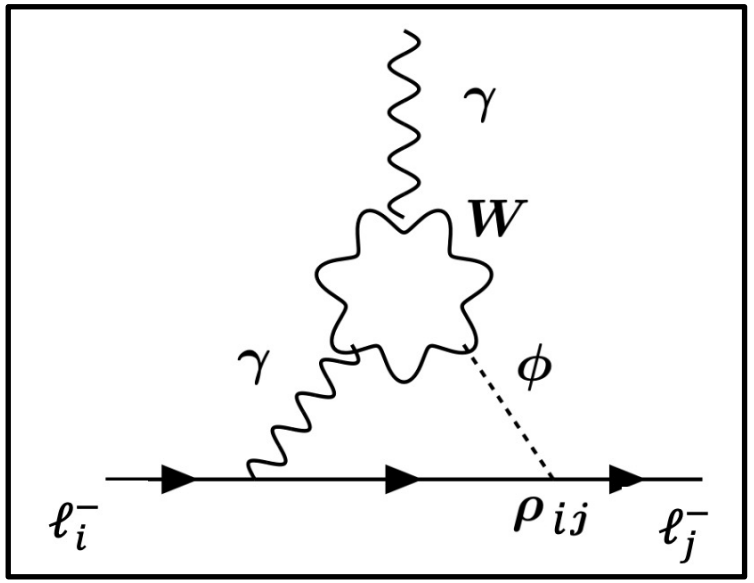

$h \to \mu^{\pm} e^{\mp}$
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
$\mathrm{BR}(h \to \mu^{\pm} e^{\mp})$
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+ \Delta\chi^2_6 +$ FV
$K - \bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d - \overline{B}_d$
$B_s - \overline{B}_s$

$$\ell_1^{\pm} \to \ell_2^{\pm} + \gamma$$



One-loop diagrams

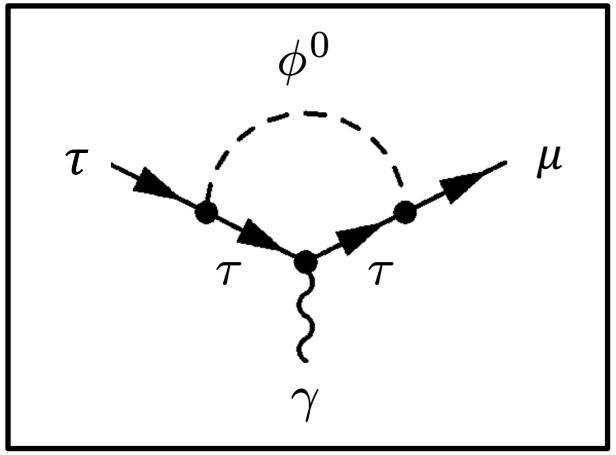


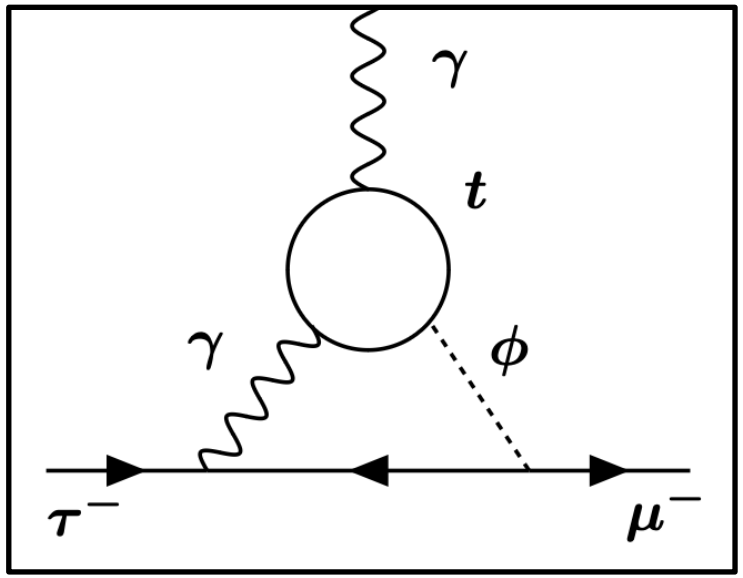


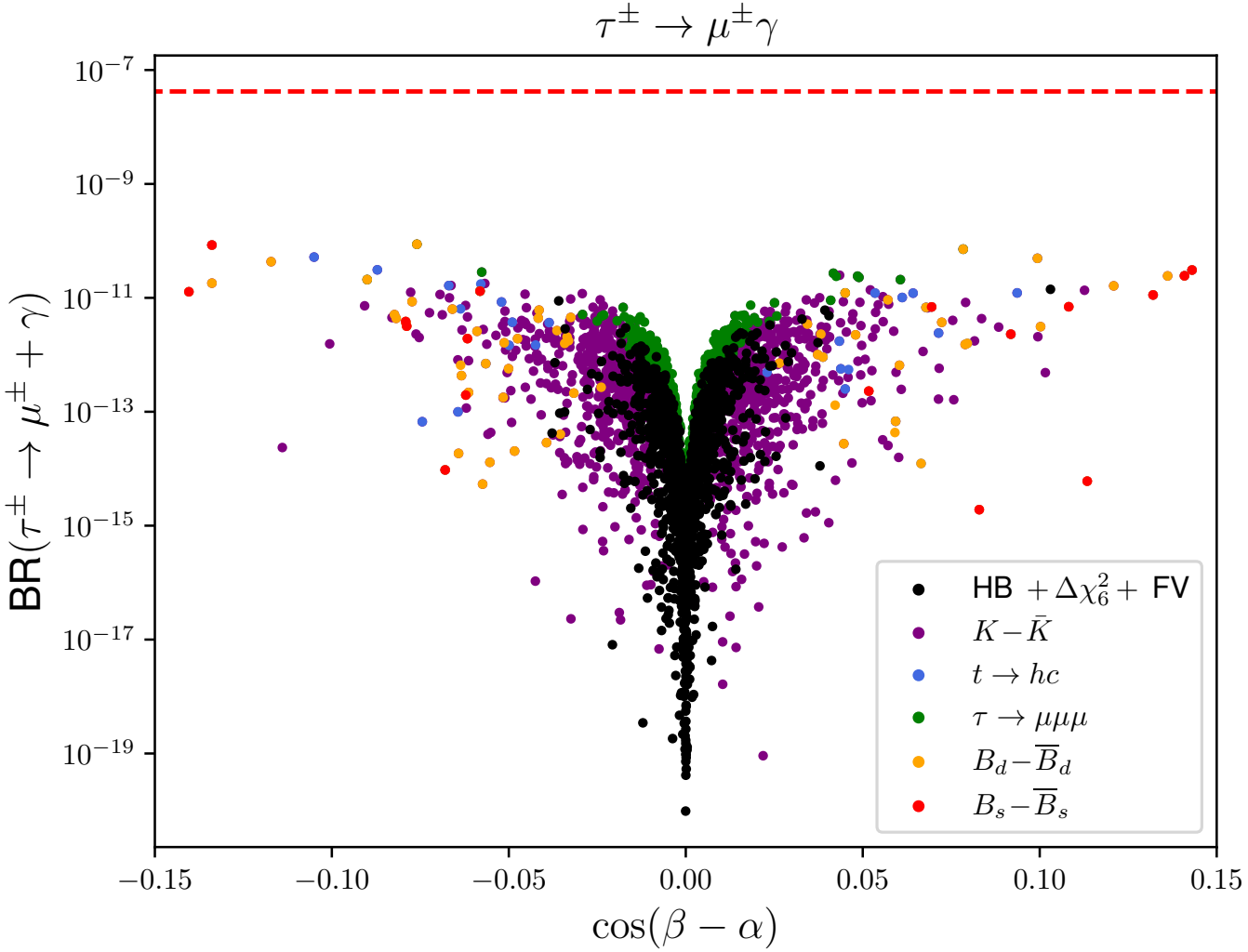
Two-loop Barr-Zee diagrams dominate

Heaviest loops dominate

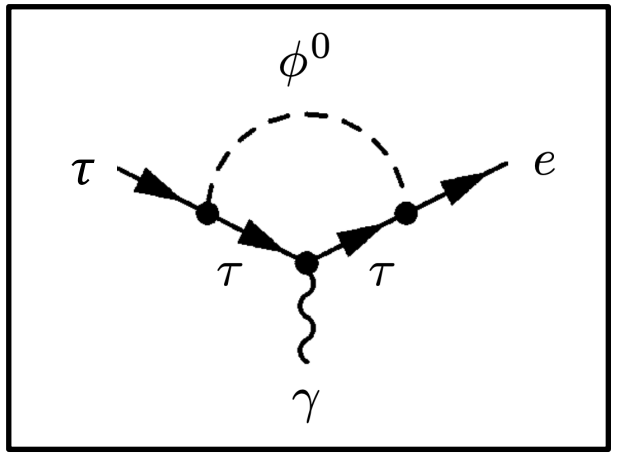
Two-loop diagrams dominate

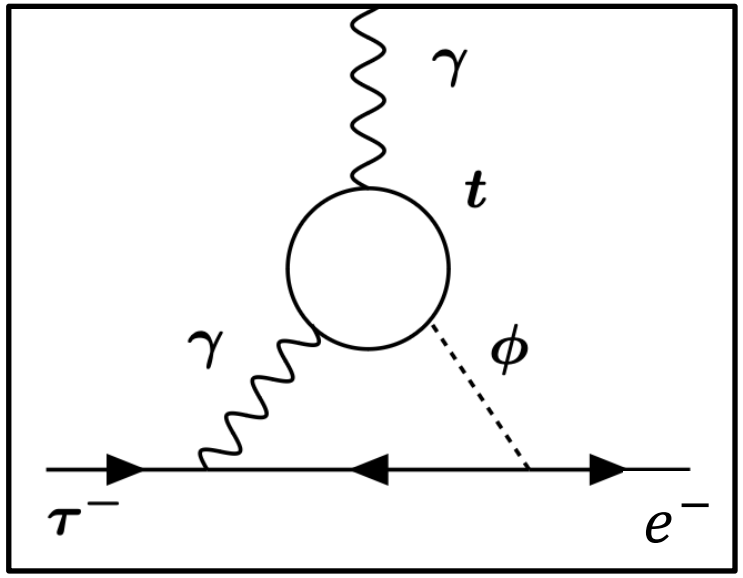


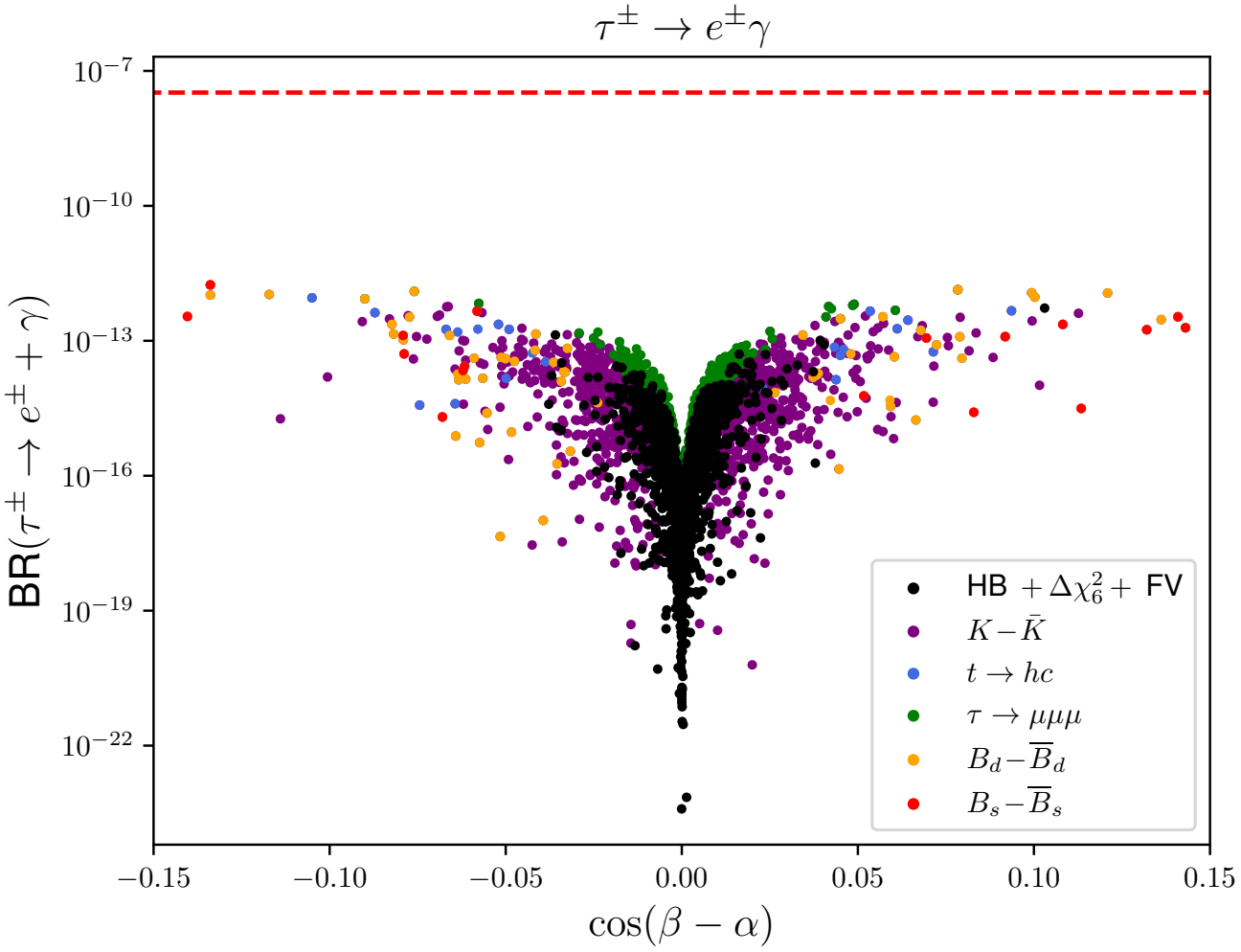




Two-loop diagrams dominate

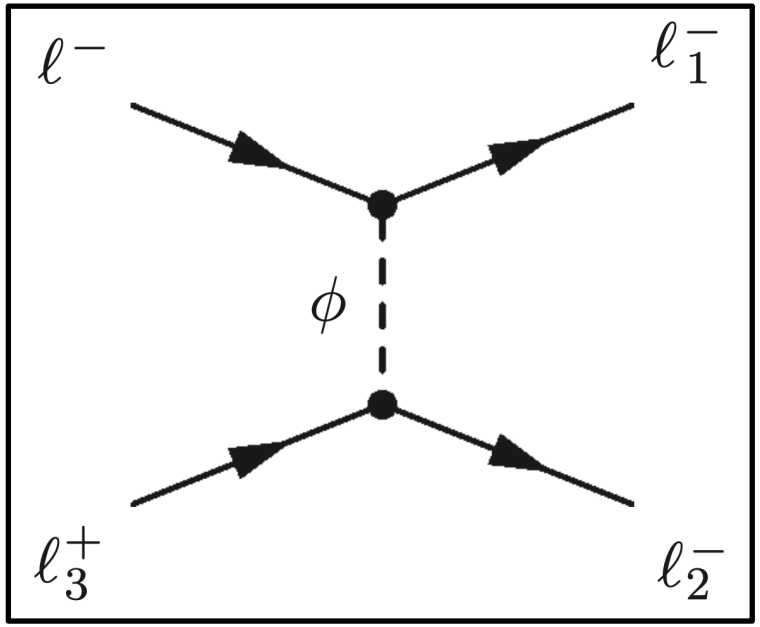


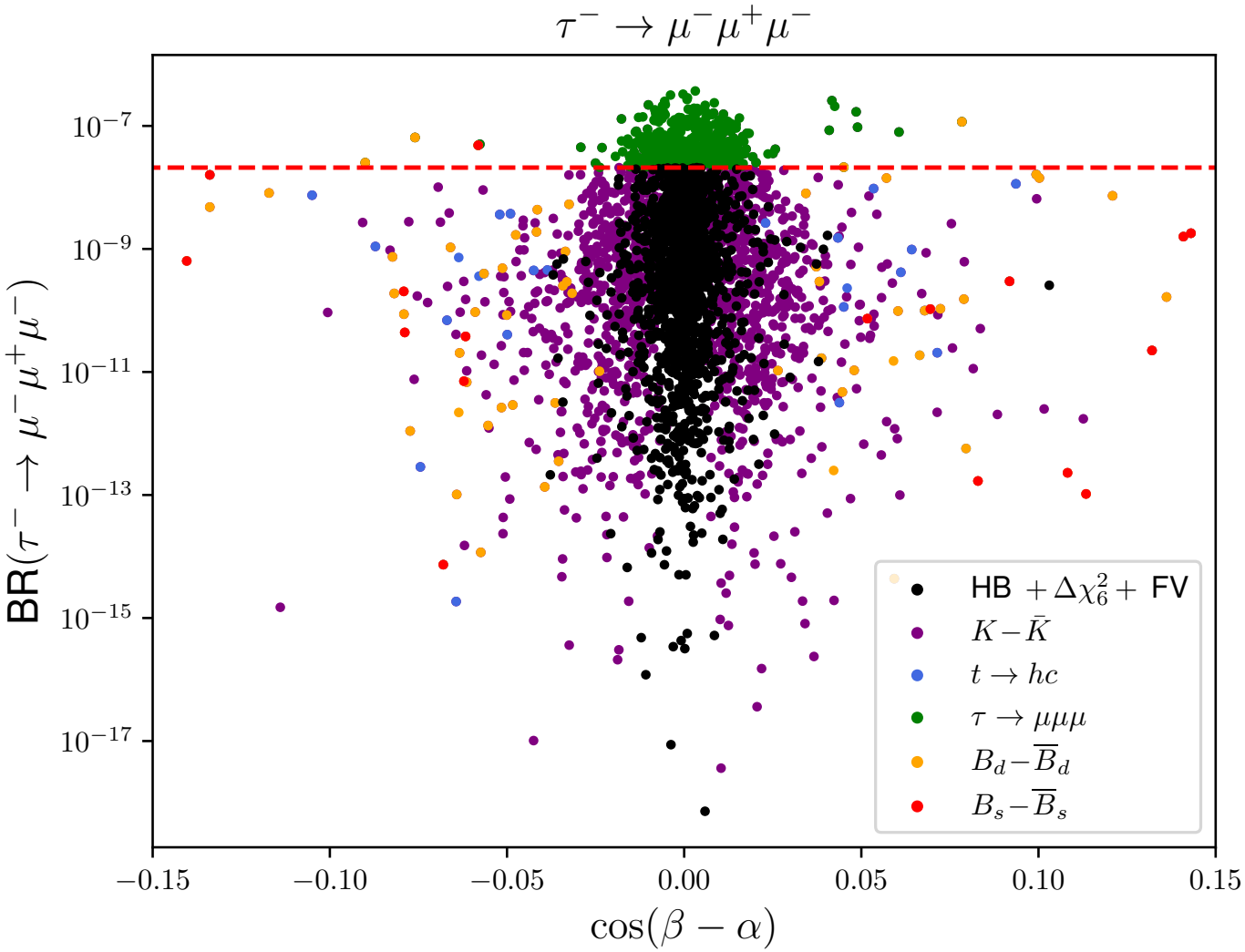




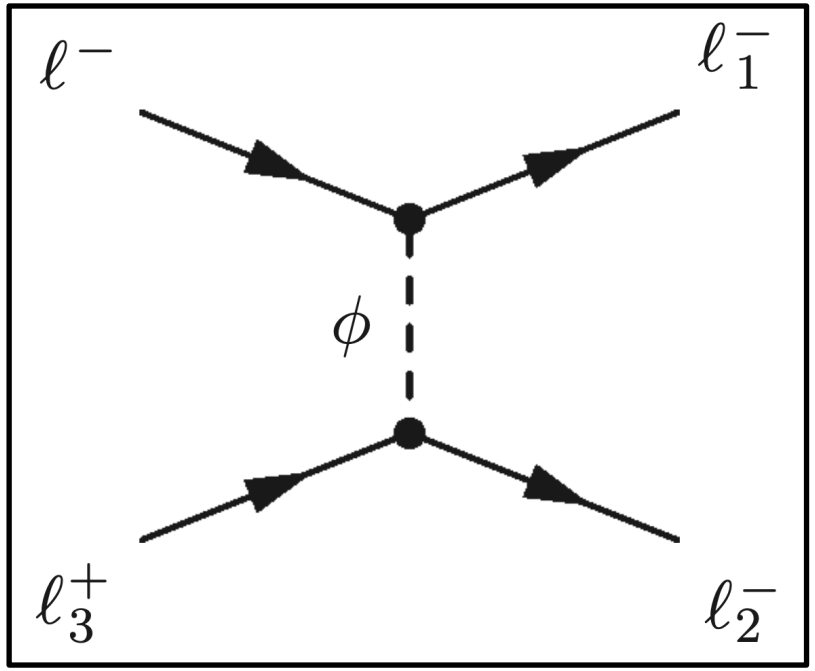
$\tau^- \to \mu^- + \mu^+ + \mu^-$

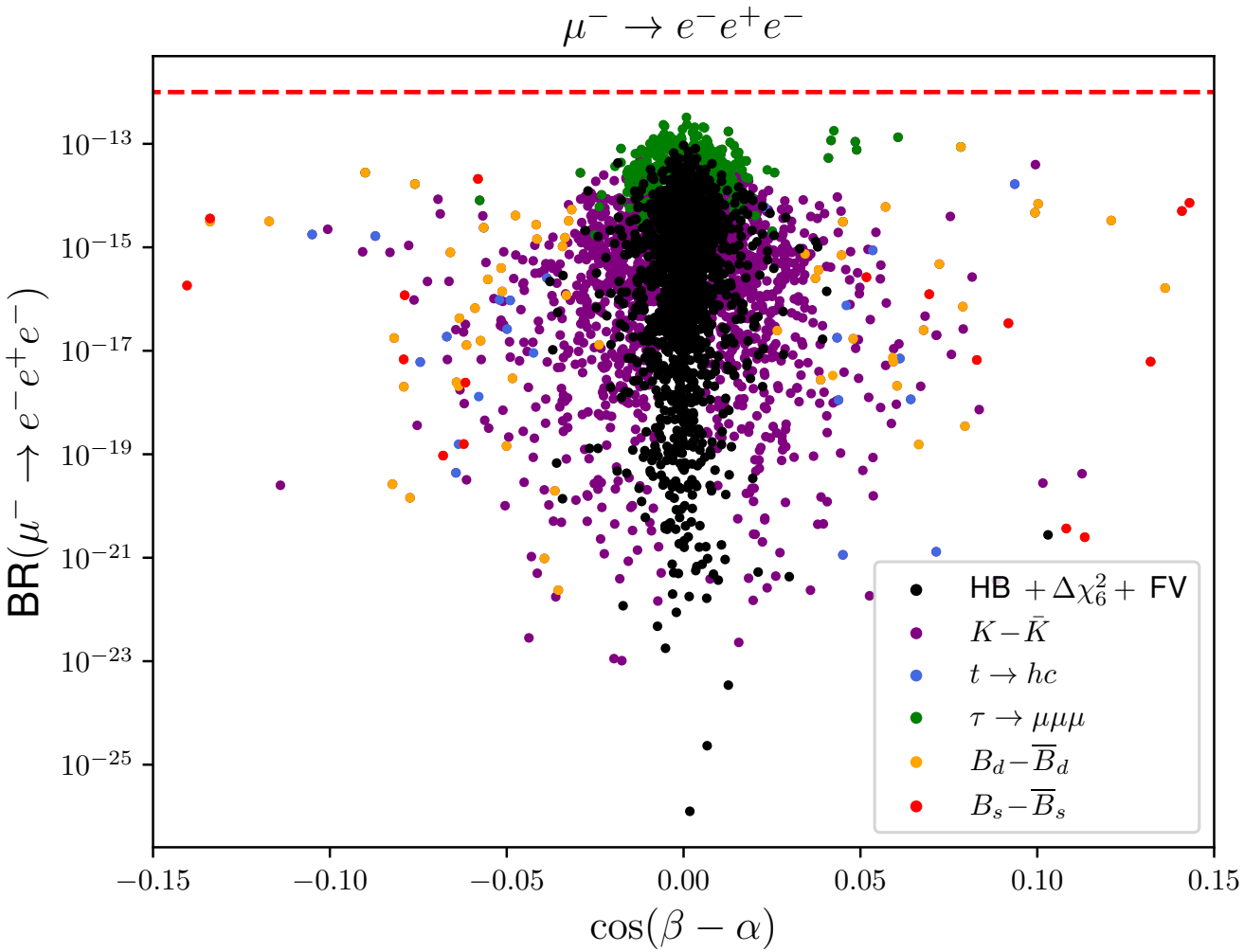
(Other 3 lepton final states all have very small BR's)





$\mu^- \to e^- + e^+ + e^-$


ℓ^-
ℓ_1^-
ϕ
ℓ_3^+
ℓ_2^-


$\mu^- \to e^- e^+ e^-$
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
$\mathrm{BR}(\mu^- \to e^- e^+ e^-)$
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
$\cos(\beta - \alpha)$
HB $+\Delta\chi^2_6+$ FV
$K-\bar{K}$
$t \to hc$
$\tau \to \mu\mu\mu$
$B_d-\overline{B}_d$
$B_s-\overline{B}_s$

Higgs-mediated Neutral meson mixing

Higgs mediated contributions to neutral meson mixing ($B_{d,s} - \bar{B}_{d,s}$, $K - \bar{K}$ and $D - \bar{D}$ mixing) arise in our model. Integrating out the three neutral Higgs bosons, we obtain the following dimension six effective Lagrangian describing B_s meson mixing

$$\mathscr{L}_{\text{eff}} = C_2 \left(\bar{b}_R s_L\right)^2 + \tilde{C}_2 \left(\bar{b}_L s_R\right)^2 + C_4 \left(\bar{b}_R s_L\right)\left(\bar{b}_L s_R\right) + \text{ h.c.}$$

with Wilson coefficients,

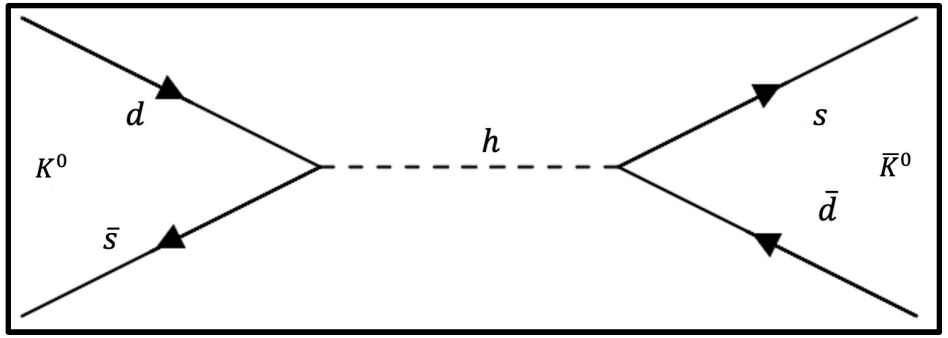
$$C_2 = \frac{\left(\rho_{32}^D\right)^2}{4}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} - \frac{1}{m_A^2}\right),$$

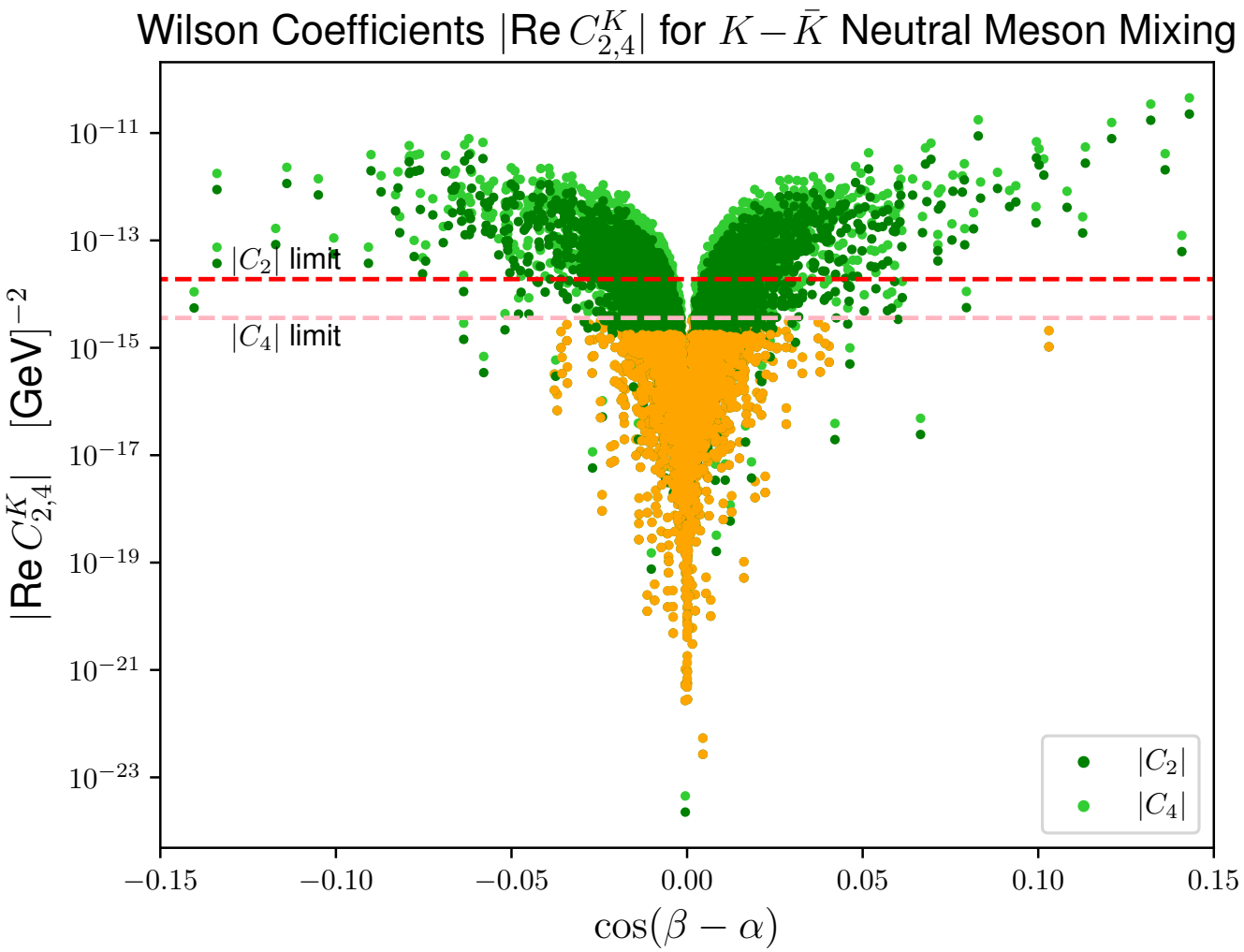
$$\tilde{C}_2 = \frac{\left(\rho_{23}^{D*}\right)^2}{4}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} - \frac{1}{m_A^2}\right),$$

$$C_4 = \frac{\left(\rho_{32}^D\right)\left(\rho_{23}^{D*}\right)}{2}\left(\frac{\sin^2(\beta-\alpha)}{m_H^2} + \frac{\cos^2(\beta-\alpha)}{m_h^2} + \frac{1}{m_A^2}\right),$$

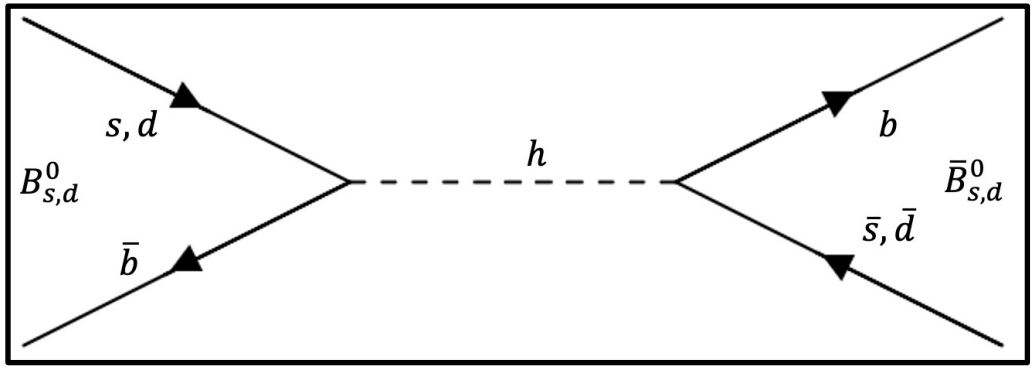
and corresponding Wilson coefficients for B_d, K, and D mixing.

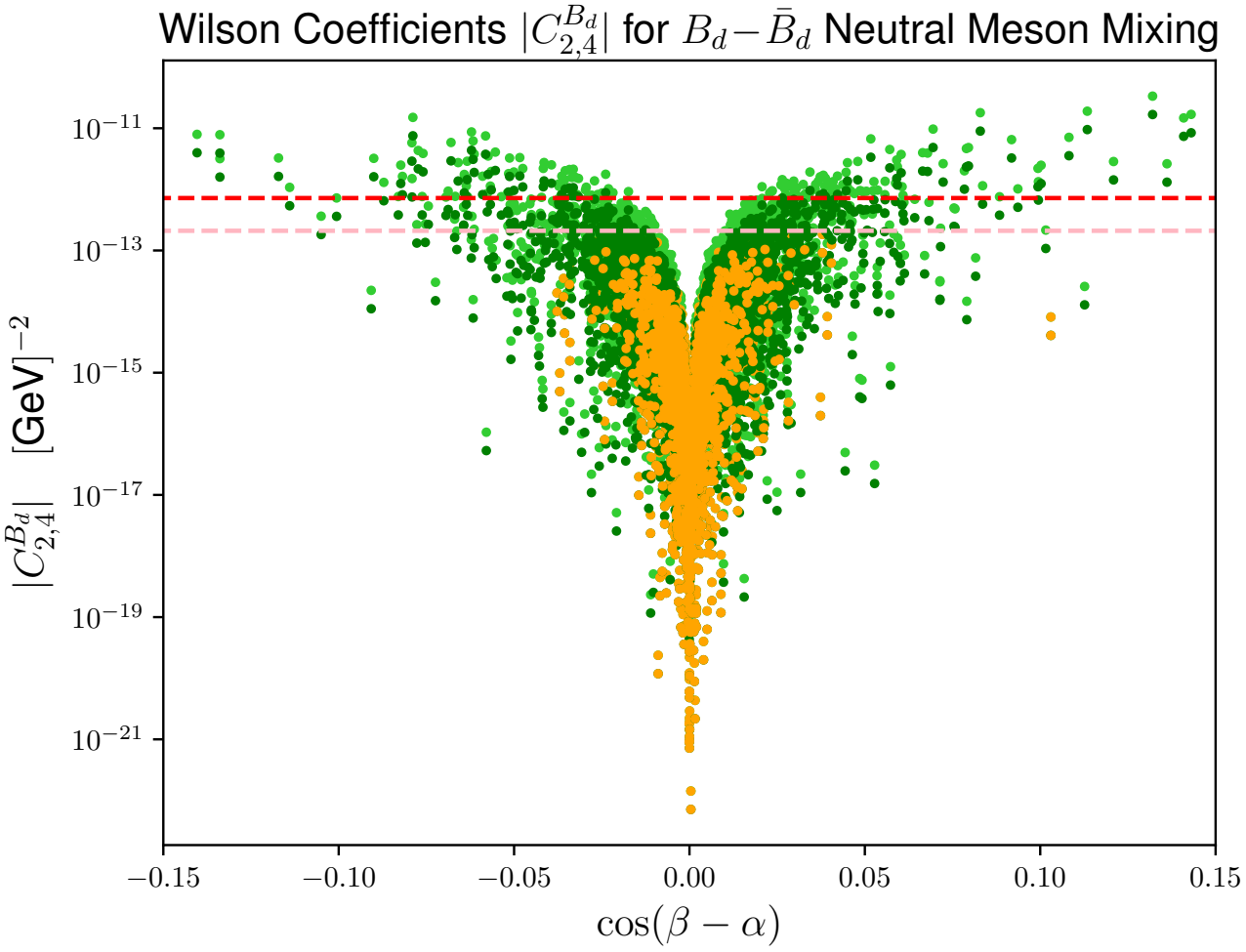
Kaon Mixing $K^0 - \bar{K}^0$



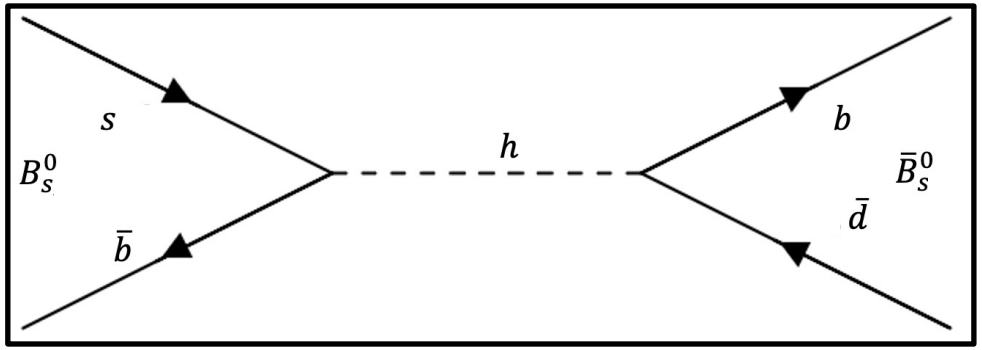


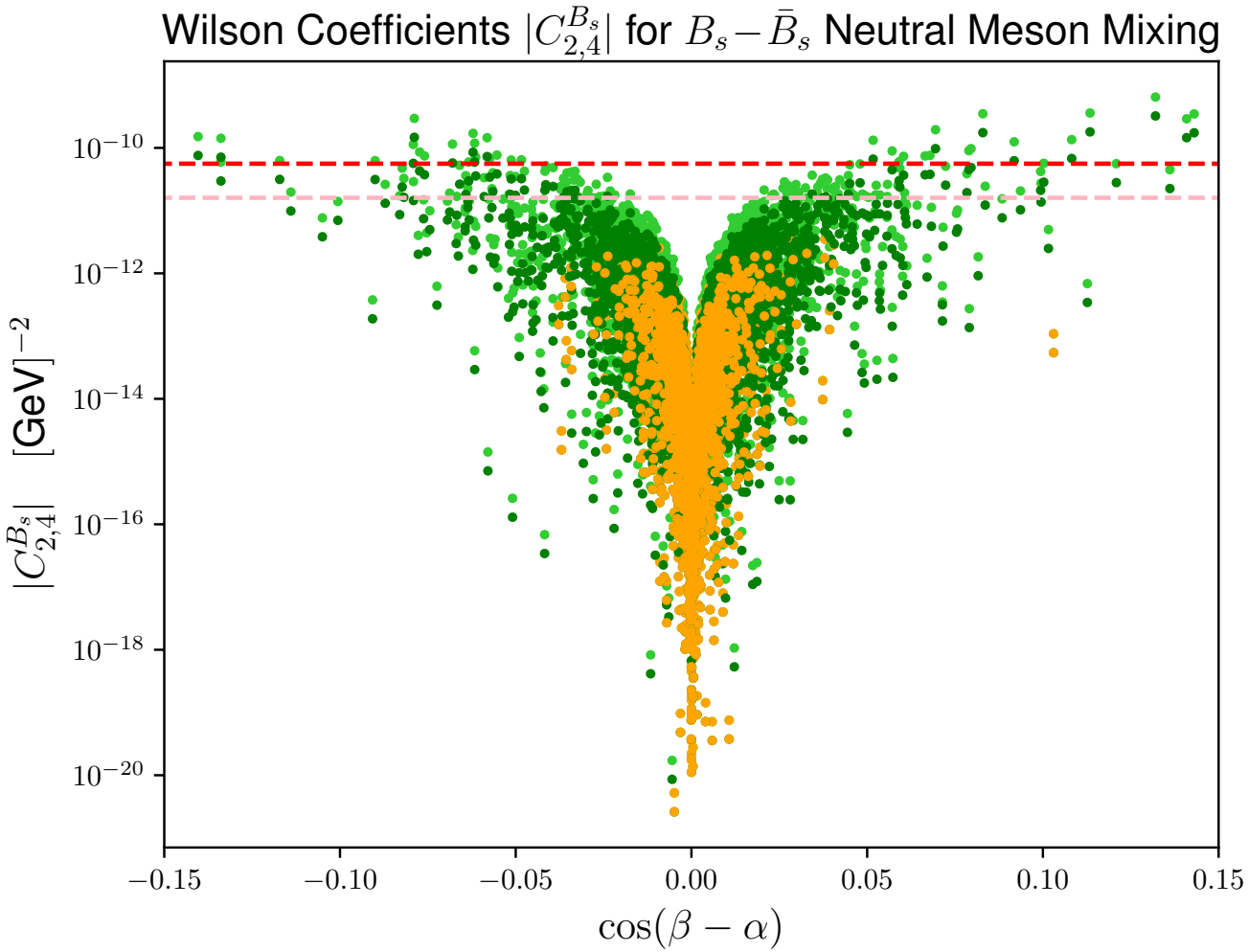
Neutral B Meson Mixing: $B_d^0 - \bar{B}_d^0$





Neutral Strange B Meson Mixing: $B_s^0 - \bar{B}_s^0$





Conclusions and Future work

- The viability of the Cheng-Sher ansatz for off-diagonal neutral Higgs–fermion Yukawa couplings should be examined...
  - in a formalism where the unphysical parameter $\tan\beta$ never appears.
  - by making use of the most recent analysis of the CKM parameters based on the Fritzsch textures for the up and down quark mass matrices.
- Phenomenological implications of (the less suppressed) flavor off-diagonal decays of the heavy Higgs scalars should be investigated.
- Extend the analysis to allow for CP-violating phases in the ρ-type Yukawa matrices and scalar potential.
- The Fritzsch and Cheng-Sher textures are not RG-stable. Thus, it would be useful to construct UV completions of the 2HDM that could provide an (approximate) explanation for the Yukawa matrix textures used here.