Yukawas at infinite distance

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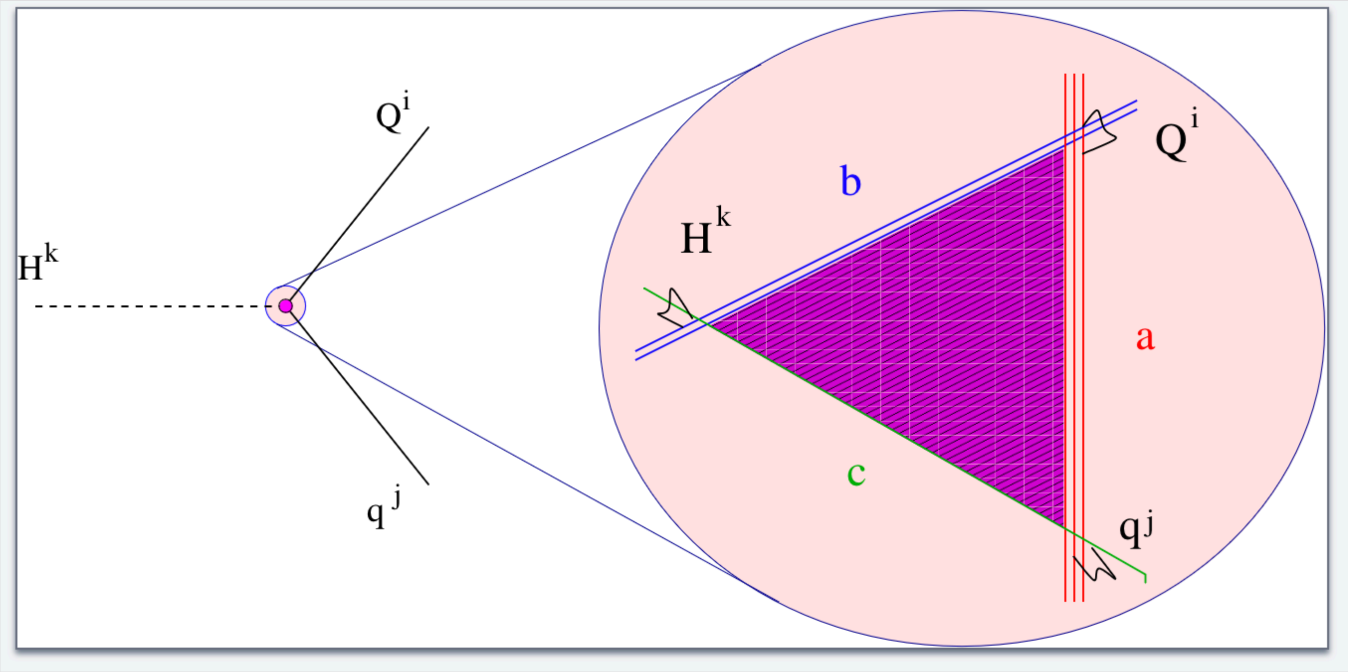


Work in collaboration with:

G.F. Casas, F. Marchesano, L.E.I.
arXiv: 2403.09775 and 2406.XXXX to appear

H. Bosch

$\sim 1490 - 1500$

Some classical Swampland lore

Vafa 2005, Palti (2019), van Beest et al. (2021), Graña, Herraez (2021)

- U(1) WGC: a charged particle exists with $m \leq qgM_p$

- Consistency under dimensional reduction in a circle: (sub)Lattice/Tower WGC $m_n = nqM_p$

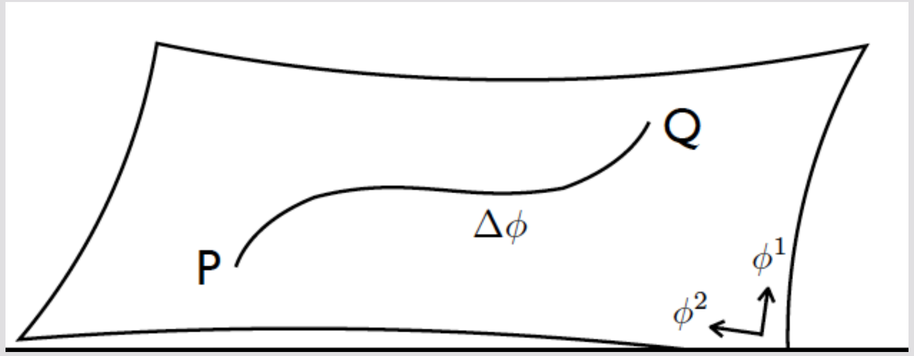
Heidenreich et al (2015,2017); Montero et al. (2016); Andriolo et al (2018)

- The limit $g \longrightarrow 0 \; is \; singular$

A tower of particles should become massless: limit at infinite distance

- Related: as a modulus goes to infinity a tower arises: $\Delta\phi \to \infty$

Ooguri, Vafa 2006



$$m(Q) \simeq m(P)e^{-\lambda\Delta\phi}$$
$$\lambda \sim 1$$

Tower: either strings or decompactification

Lee, Lerche, Weigand (2019)

The case of 4D, N=1 and Yukawas

- Not much studied. New features like 4d CHIRALITY

- Also Yukawa couplings of charged matter fields $Y_{abc}(\Phi^a\Phi^b\Phi^c)$

- Although less SUSY, results reliable in the perturbative regime

Questions:

$$Is\ Y_{abc} \longrightarrow 0\ at\ infinite\ distance?$$

- What goes wrong if at all in that limit ? Are there towers of particles becoming massless?

- If there are towers, what is their structure?

- Is there a constraint of the WGC type for Yukawas ? (no BH argument for that but…)

$$m \leq Y M_p$$

(like insisting Yukawas cannot be weaker than gravity?)

- Do towers of particles arise in the limit $Y \longrightarrow 0$?

A good reason to study the small Yukawa limit:

- Most Yukawas of the SM are small:

$$Y_e \simeq 10^{-6} \; ; \; Y_u, Y_d \simeq 10^{-5} \; ; \; Y_\mu \simeq 10^{-4}$$

- If neutrinos are Dirac, tiny Yukawa couplings are needed

$$Y_\nu \simeq 10^{-12} \sim 10^{-13}$$

- E.G.do these small Yukawas of the SM come along with some tower of states?

Some answers we find :

- We use Type IIA CY orientifolds with chiral matter at intersecting branes

- $Y_{abc} \longrightarrow 0 \; IS \; at \; infinite \; distance$

- $Y_{abc} \longrightarrow 0$

  Large volume: $Y_{abc} \sim \left(\frac{m_{D0}}{M_p}\right) \; decompactification \; to \; M-th$

  Large C.S. $Y_{abc} \sim \prod_a \left(\frac{m^a_{gon}}{M_p}\right)^{1/2} and \; p = 2,4,6 \; large \; dimensions$

- 'Gonions' are oscillator states localised at intersections,

$$m^2_{gon} \simeq n \, \theta \, M^2_s$$

- Gonions have all same charge (violate Lattice/Tower WGC) and not extremal

$$m_{gon} \simeq g^2_* M_p$$

- …but no violation of WGC under dimensional reduction

- For limits parametrised by a single growing c.s. field u

$$r = 1/4, 1/2, 3/4, 1$$

$$Y \sim \frac{1}{u^r} \qquad\qquad Y \sim g_*^{2r}$$

Explains what goes wrong…

Dirac Neutrinos at infinite distance:

- If neutrinos are Dirac $Y_\nu \simeq 10^{-12} \gg Y_{U,D,E}$

- Unique way to obtain tiny Yukawa : 2 large dimensions and a single gonion tower

$$Y_\nu \simeq \left(\frac{m^\nu_{gon}}{M_p}\right)^{1/2} \simeq g_* \sim \frac{1}{u^{1/2}} \sim 7 \times 10^{-13}$$

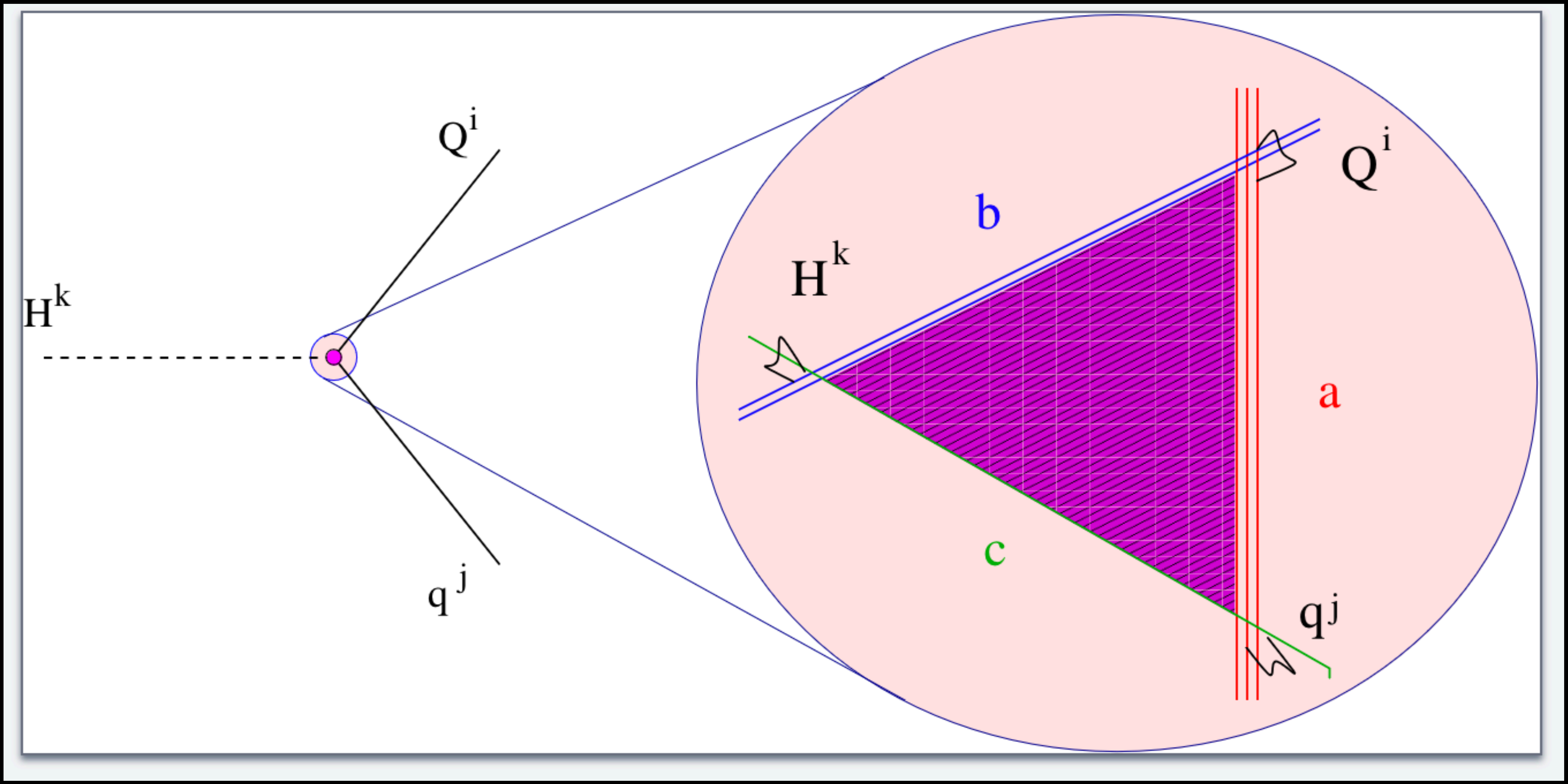
$$*A\ tower\ of\ \nu_R - like\ gonions\ at\ scale\ \sim\ 500eV$$

$$*Two\ large\ dimensions\ at\ m_{KK} \simeq\ 500\ eV$$

$$*A\ new\ gauge\ boson\ U(1)_\nu\ with\ mass\ M_V \geq\ 0.1\ eV$$

$$*String\ scale\ M_s \simeq 700\ TeV$$

Yukawas at infinite distance



4d, N=1 Type IIA CY Orientifolds as a laboratory

- CY compactification of IIA with orientifold quotient $\Omega_{ws}(-1)^{F_L}\mathcal{R}$, $\mathcal{R}(J,\Omega) = (-J,\overline{\Omega})$
- There are O(6) planes and D6-branes wrapping 3-cycles Π_α *and mirrors* Π_{α^*}

$$[\Pi_\alpha] = P_{\alpha\,J}[\Sigma_+^J] + Q_\alpha^K[\Sigma_K^-]$$

$$[\Sigma_K^-]\cdot[\Sigma_+^J] = 2\delta_K^J$$

- Closed string moduli:

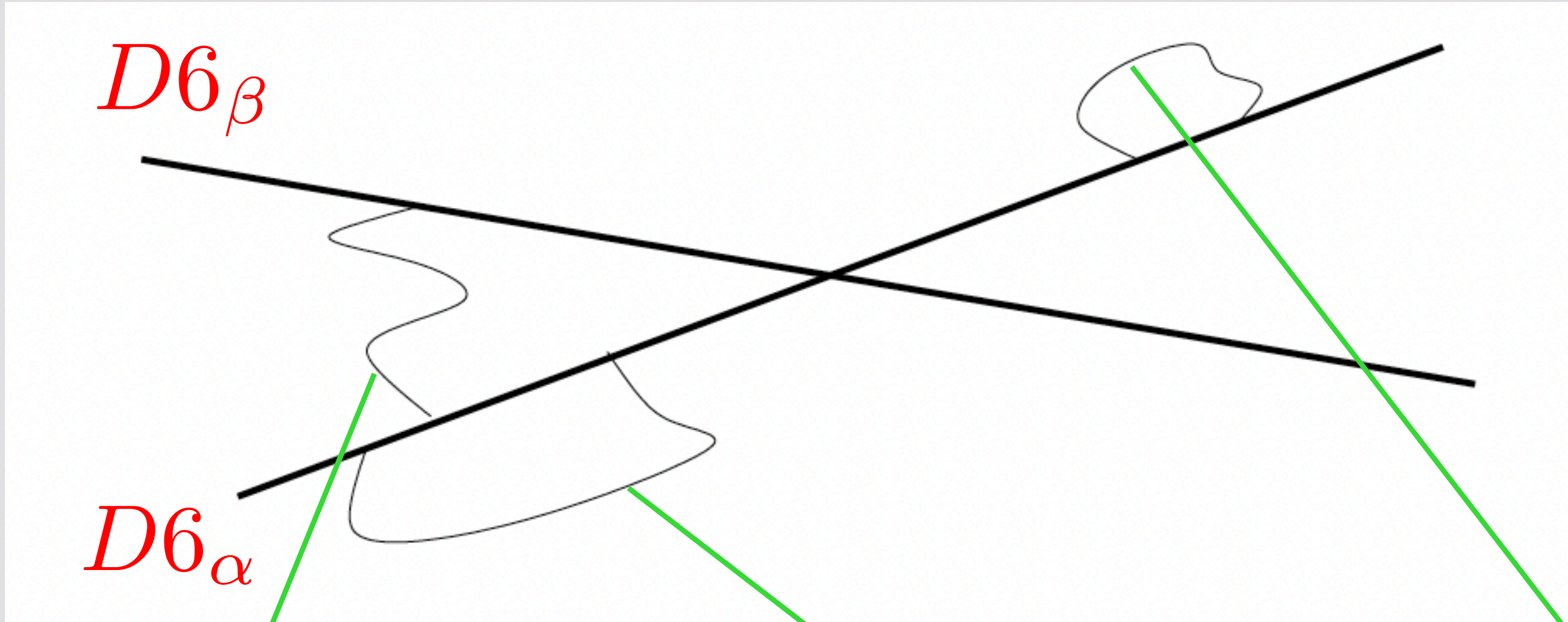
$$Kahler: \; T^a = b^a + it^a \;, where \; J_c \equiv B + iJ = (b^a + it^a)\omega_a$$

$$C.S: \; U^K = \zeta^K + iu^K \;, where \; \Omega_c \equiv C_3 + ie^{-\phi}Re\Omega = (\zeta^K + iu^K)\alpha_K$$

- Closed string moduli: (up to w.s. and D2 instantons)

$$K_K \equiv -\log\left(\mathrm{Vol}_{X_6}\right) = -\log\left(\frac{i}{48}\mathcal{K}_{abc}(T^a - \bar{T}^a)(T^b - \bar{T}^b)(T^c - \bar{T}^c)\right)$$

$$K_Q \equiv -2\log\mathcal{H} = -2\log\left(\frac{i}{8\ell_s^6}\int_{X_6} e^{-2\phi}\Omega\wedge\bar{\Omega}\right) = 4\phi_4$$



$(also\ SO, Sp)$

$U(N_\beta)$

$U(N_\alpha)$

$I_{\alpha\beta} \times (N_\alpha, \overline{N}_\beta)$

$$2\pi i f_{\alpha\alpha} = P_{\alpha K} U^K$$

$chiral\ bi-fundamentals\ at\ intersections$

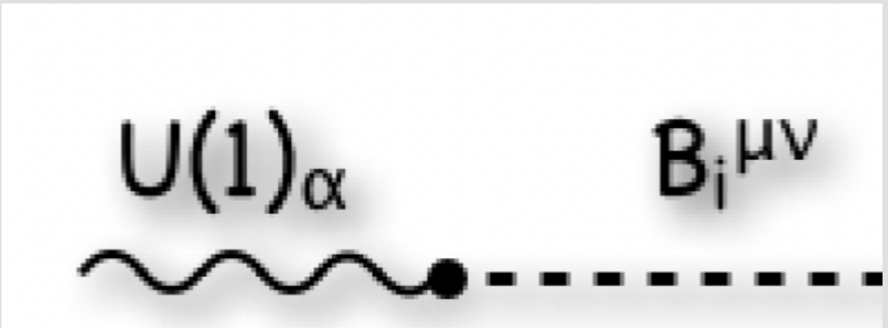
Berkooz, Douglas and Leigh, (1996)

Reviews: Blumenhagen et al(2006), Marchesano (2007), Ibañez, Uranga (2012), Marchesano, Schellekns, Weigand (2024)

$Quarks, leptons$

U(1)'s

- Some U(1)'s become massive getting Stuckelberg mass mixing with 2-forms:

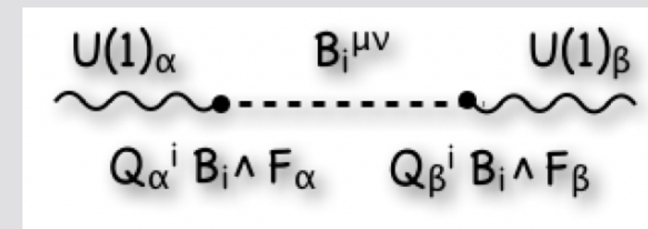


$$G_{KL}\; Q_\alpha^K (d\zeta^K \;\; A^\alpha)$$

dual to $B^K_{\mu\nu}$

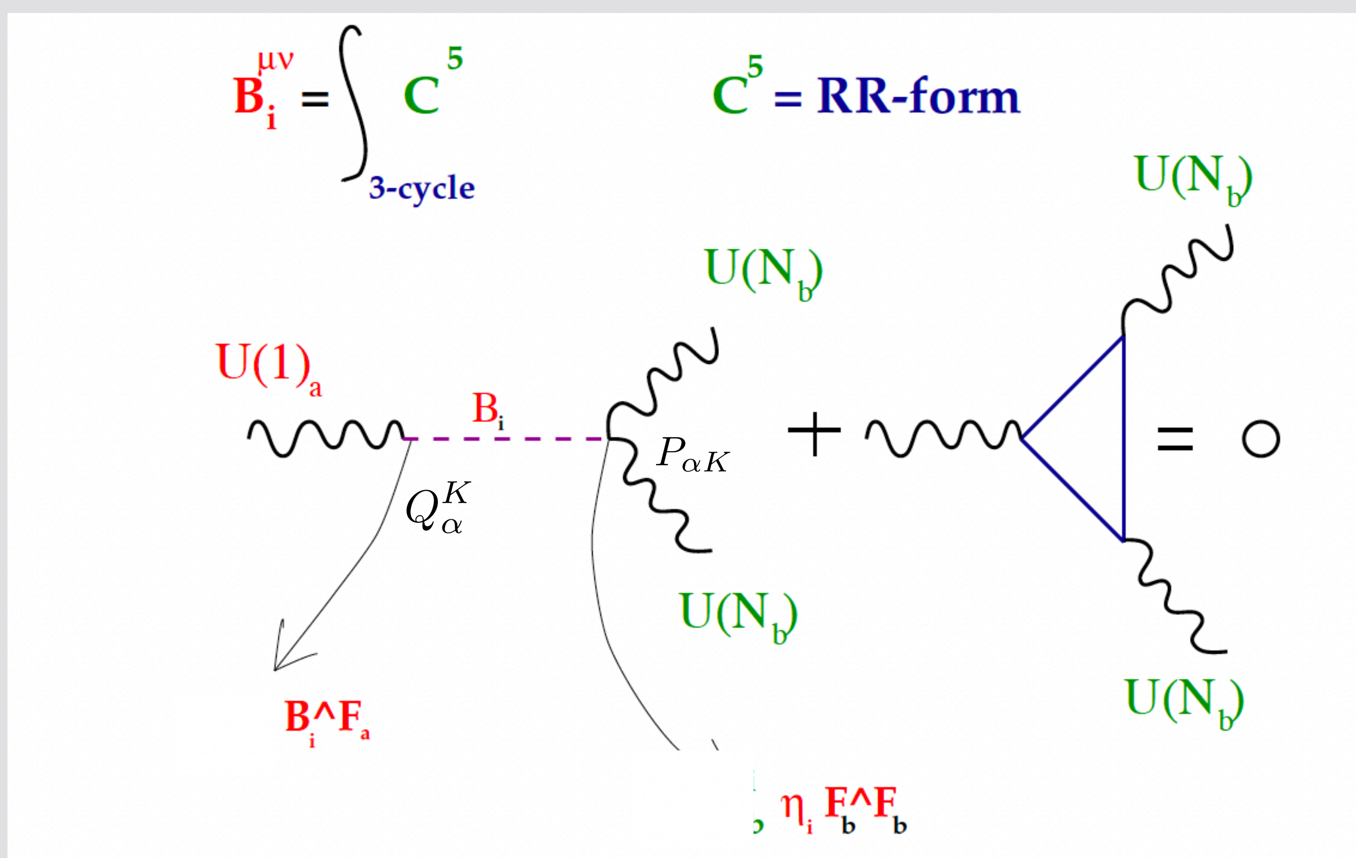
- Stuckelberg masses:

$$M^2_{\alpha\beta} = 4 G_{KL} Q^K_\alpha Q^L_\beta g_\alpha g_\beta\, M^2_{\rm P}$$



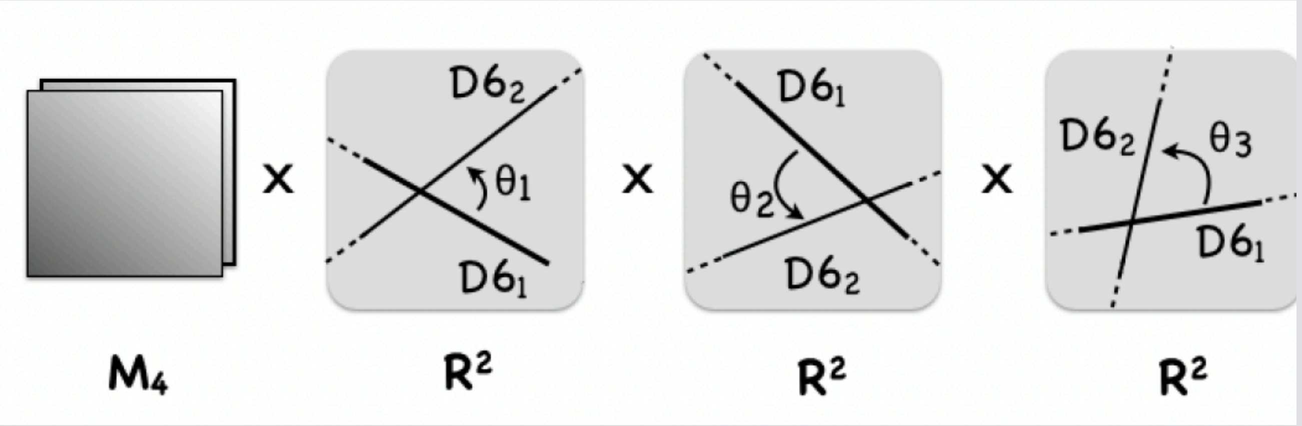
$$(may\ have\ massless\ eigenvalues, e.g.\ hypercharge\ in\ SM)$$

- Green-Schwarz cancel some U(1) pure and mixed anomalies



Gonions: flavour towers at intersections

- At an intersection local geometry in a CY specified by 3 angles

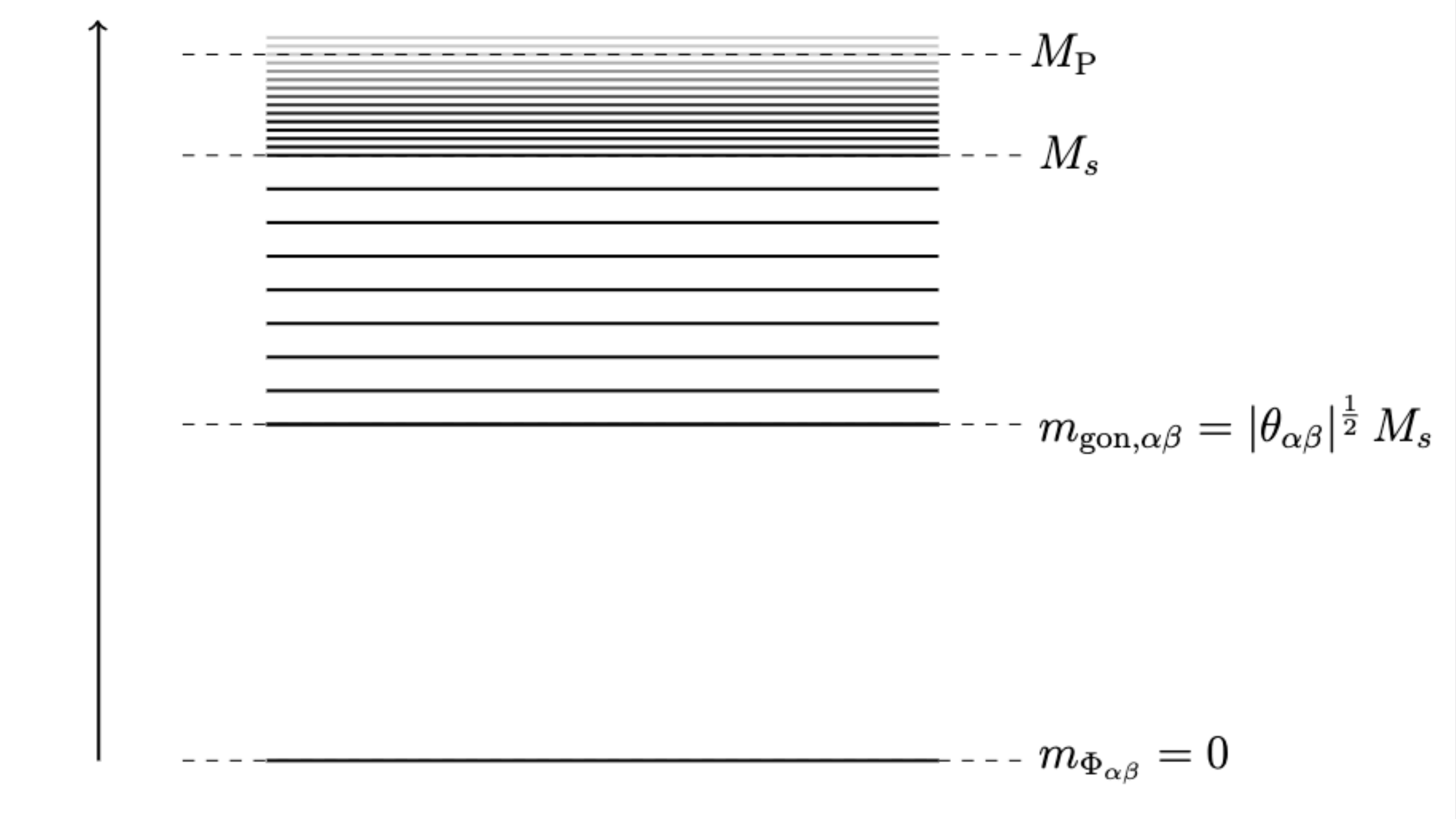


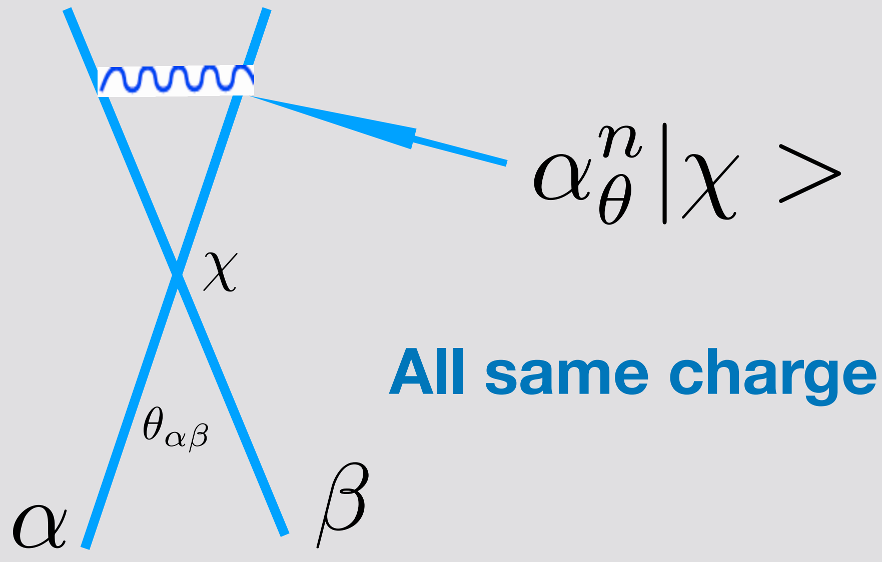
Berkooz, Douglas and Leigh, (1996)

- SUSY preserved at intersection: $\theta_1 + \theta_2 + \theta_3 \;\; \in 2\mathbf{Z}$

Aldazabal et al (2001)

- In addition to massless chiral bifundamental, a tower of bi-fundamentals: 'Gonions'





All same charge

$$m^2_{n;\alpha\beta} \simeq n\;|\theta_{\alpha\beta}|\;M_s^2$$

'p =2 tower'

(They are dual to Landau levels in the dual given by magnetized D9 branes)

• Gonion masses may be understood as coming from FI-terms associated to U(1)'s of the branes:

$$V_D \;=\; \frac{1}{2}\sum_{\alpha} g_\alpha^2 \left(\xi_\alpha \;+\; \sum_i q_\alpha^i \Phi_i \overline{\Phi}_{\bar{i}}\right)^2$$

$$Q_\alpha^K B^K \wedge F_\alpha$$

$$\downarrow$$

$$\xi_\alpha \;=\; \frac{Q_\alpha^K}{u^K} M_p^2$$

$$scalar \;\; at \; an \; (\alpha\beta) \; intersection:$$

$$m_{\alpha\beta}^2 \;=\; q_\alpha^i g_\alpha^2 \xi_\alpha \;+\; q_\beta^i g_\beta^2 \xi_\beta$$

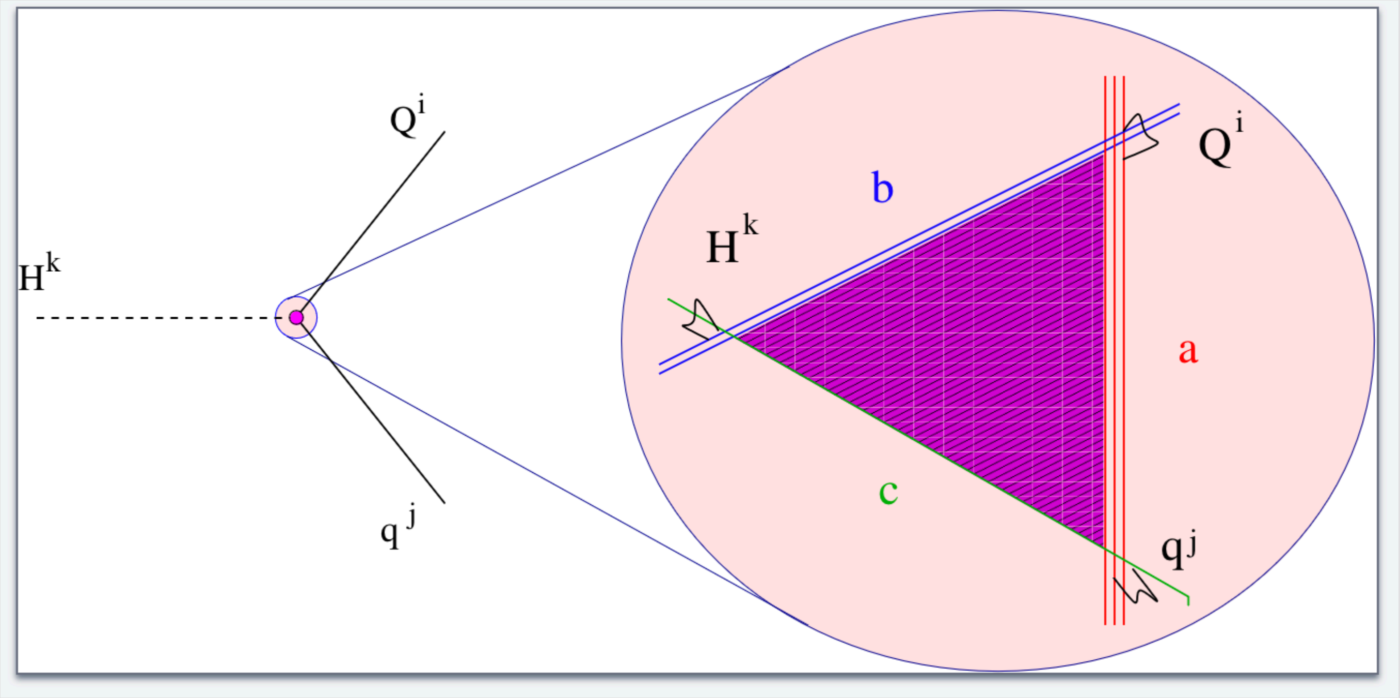
• Vanishes in a SUSY configuration in which a complex scalar partner of the chiral fermion becomes massless

$$m_{\alpha\beta}^2 \simeq (\theta_{\alpha\beta}^1 + \theta_{\alpha\beta}^2 + \theta_{\alpha\beta}^3) M_s^2 \;=\; 0$$

• still the FI induced scalar mass fixes the scale of the gonion towers, schematically

$$m_{gon}^2 \;\simeq\; g_*^2 \xi_{FI}^* \;\simeq\; g_*^2 \frac{Q_*^u}{u} M_p^2$$

Yukawas in toroidal and CY orientifolds



Aldazabal et al(2001), Cremades et al(2003), Cvetic et al (2003), Lust et al (2004))

$$E.g.\ \ T^2 \times T^2 \times T^2\ orientifolds\ (or\ Abelian\ orbifolds)$$

$$Y_{ijk} = e^{\phi_4/2} \prod_{r=1}^{3} (\mathrm{Im}\, T^r)^{-1/2} \left[\Theta^{(r)}\right]^{1/4} W_{ijk}^{(r)}$$

$(ignoring\ open\ string\ moduli)$

$$\Theta^{(r)} = 2\pi \frac{\Gamma(1-|\chi^r_{ab}|)}{\Gamma(|\chi^r_{ab}|)} \frac{\Gamma(1-|\chi^r_{bc}|)}{\Gamma(|\chi^r_{bc}|)} \frac{\Gamma(1-|\chi^r_{ca}|)}{\Gamma(|\chi^r_{ca}|)}$$

$|\chi^r_{\alpha\beta}| = |\theta^r_{\alpha\beta}|$ or $1 - |\theta^r_{\alpha\beta}|$;

- Essentially depends only on local geometry

$$Recall\ \Gamma(\theta) \simeq \frac{1}{\theta}$$

- Depends only on local geometry: Expect structure valid for general CY:

$$Y_{ijk} = \frac{e^{\phi_4/2}}{Vol_X^{1/2}} \Theta_{ijk}^{1/4} \, W_{ijk}$$

- N=1 supergravity: canonically normalized Yukawas given by

$$Y_{ijk} = e^{K/2} \left(K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}}\right)^{-1/2} W_{ijk}$$

  - Gives information about Kahler metric of chiral matter fields:

$$K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}} = e^{3\phi_4} \Theta_{ijk}^{-1/2}$$

Kinetic terms of chiral fields

- Thus for the toroidal case, recalling

$$\Theta^{(r)} = 2\pi \frac{\Gamma(1-|\chi^r_{ab}|)}{\Gamma(|\chi^r_{ab}|)} \frac{\Gamma(1-|\chi^r_{bc}|)}{\Gamma(|\chi^r_{bc}|)} \frac{\Gamma(1-|\chi^r_{ca}|)}{\Gamma(|\chi^r_{ca}|)}$$

$$K_{i\bar{i}} = e^{\phi_4}(2\pi)^{-1/2} \prod_{r=1}^{3} \left(\frac{\Gamma(|\chi^r_i|)}{\Gamma(1-|\chi^r_i|)} \right)^{1/2}$$

$$|\chi^r_{\alpha\beta}| = |\theta^r_{\alpha\beta}| \text{ or } 1-|\theta^r_{\alpha\beta}|$$

- For small angles, recalling $\theta^r \simeq (m^r_{gon}/M_s)^2$

$$K_{i\bar{i}} \simeq \frac{e^{\phi_4}}{(\theta^{min}_{\alpha\beta})^{1/2}} \simeq e^{2\phi_4} \frac{M_p}{m^{min}_{gon,\alpha\beta}}$$

- Will give rise to a singular behaviour as $m^{min}_{gon,\alpha\beta} \to 0$

$The\ Y \longrightarrow\ 0\ limit\ and\ infinite\ distance$

$$Y_{ijk} \quad = \quad \frac{W_{ijk}}{Vol_X^{1/2}} \quad e^{\phi_4/2}\Theta_{ijk}^{1/4}$$

- Infinite distance and Kahler moduli (fixed c.s.)

$$Y_{ijk} \simeq \ A \ \frac{W_{ijk}}{Vol_X^{1/2}} \ \longrightarrow \ 0 \qquad \Rightarrow \qquad Vol_X \ \longrightarrow \ \infty$$

$(W_{ijk} \rightarrow 0\ typically\ requires\ non-generic\ fine-tuning)$

- These limits are at infinite distance: SDC a tower of particles should become massles: the D0's

$$m_{D0}^2 \ \simeq \ \frac{M_p^2}{Vol_X} \ \longrightarrow \qquad |Y| \ \simeq \ \frac{m_{D0}}{M_p}$$

- So this limit is the M-theory limit

A tower of particles should become massless

Small Yukawas and gonion masses

- Infinite distance and complex structure (fixed Kahler moduli)

$$Y_{ijk} \simeq B \; e^{\phi_4/2} \Theta_{ijk}^{1/4}$$

- With e.g. at least one small angle per complex plane and SUSY (No N=2 planes)

$$Y_{ijk} \sim e^{K_Q/2} \left(K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}} \right)^{-1/2}$$

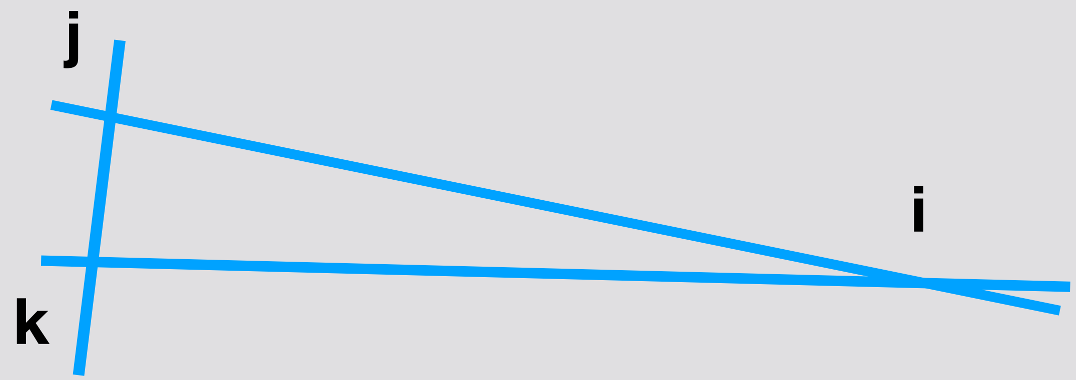
$$Y_{ijk} \sim e^{-\phi_4} \left(\frac{m_{gon,i}}{M_p} \right)^{1/2} \left(\frac{m_{gon,j}}{M_p} \right)^{1/2} \left(\frac{m_{gon,k}}{M_p} \right)^{1/2}$$

$$Y_{ijk} \rightarrow 0$$

- At least one tower of light charged gonions !

See also Castellano, Herraez, Ibañez (2023)

- Simple example with a dominant small angle:



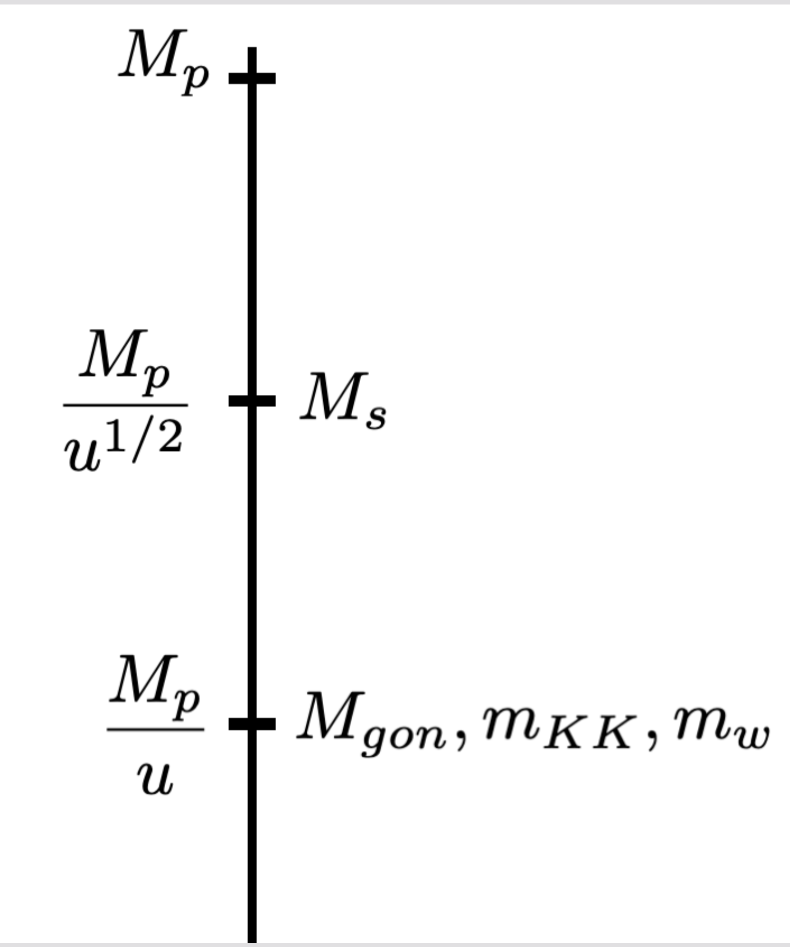
$$Y_{ijk} \simeq e^{-\phi_4} \left(\frac{m_{gon,i}}{M_p}\right)^{1/2} \left(\frac{M_s}{M_p}\right)^{1/2} \left(\frac{M_s}{M_p}\right)^{1/2}$$

$(recall\ M_s = e^{\phi_4} M_p)$

$$Y_{ijk} \simeq \left(\frac{m_{gon,i}}{M_p}\right)^{1/2}$$

- Small Yukawas imply a tower of light particles with same charge as the massless field

- In EFT gonion masses come from FI-term of U(1) gauge group felt at the intersection



$$\theta \sim 1/u$$

$$m_{gon}^2 \simeq g_*^2 \xi_{FI}^* \simeq g_*^2 \frac{Q_*^u}{u} M_p^2 \simeq \frac{Q_*^u}{u^2} M_p^2$$

$$Re f_* \simeq \frac{1}{g_*^2} \simeq u$$

g_* is the gauge coupling of the U(1) (sub)group under which the leading gonions transform

General asymptotic behaviour of Yukawas

- We have considered the infinite c.s. limit in several settings/examples:

  - 'STU' Type IIA orientifold models models, dual to magnetized Type I and SO(32) models with U(1) bundles *Blumenhagen, Honecker, Weigand (2005)*

  - 'EFT String Limits of ref. *Lanza, Marchesano, Martucci, Valenzuela (2021)*

  - Specific toroidal Type IIA orientifolds (e.g. Pati-Salam-like) *Cremades, Ibañez, Marchesano (2002)*

- For limits parametrized by a single growing c.s. field u :

$$\boxed{Y_* \sim \frac{1}{u^r}}$$

$$r = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$$

(Recall Y~ $e^{\phi_4/2}\Theta_{ijk}^{1/4}$)

- Reminds us of recent results in heterotic Yukawa couplings in CY's with U(1) bundles

- Thus in limits parametrized by a single growing c.s. field u in general one will have

$$Y_* \sim g_*^{2r}$$

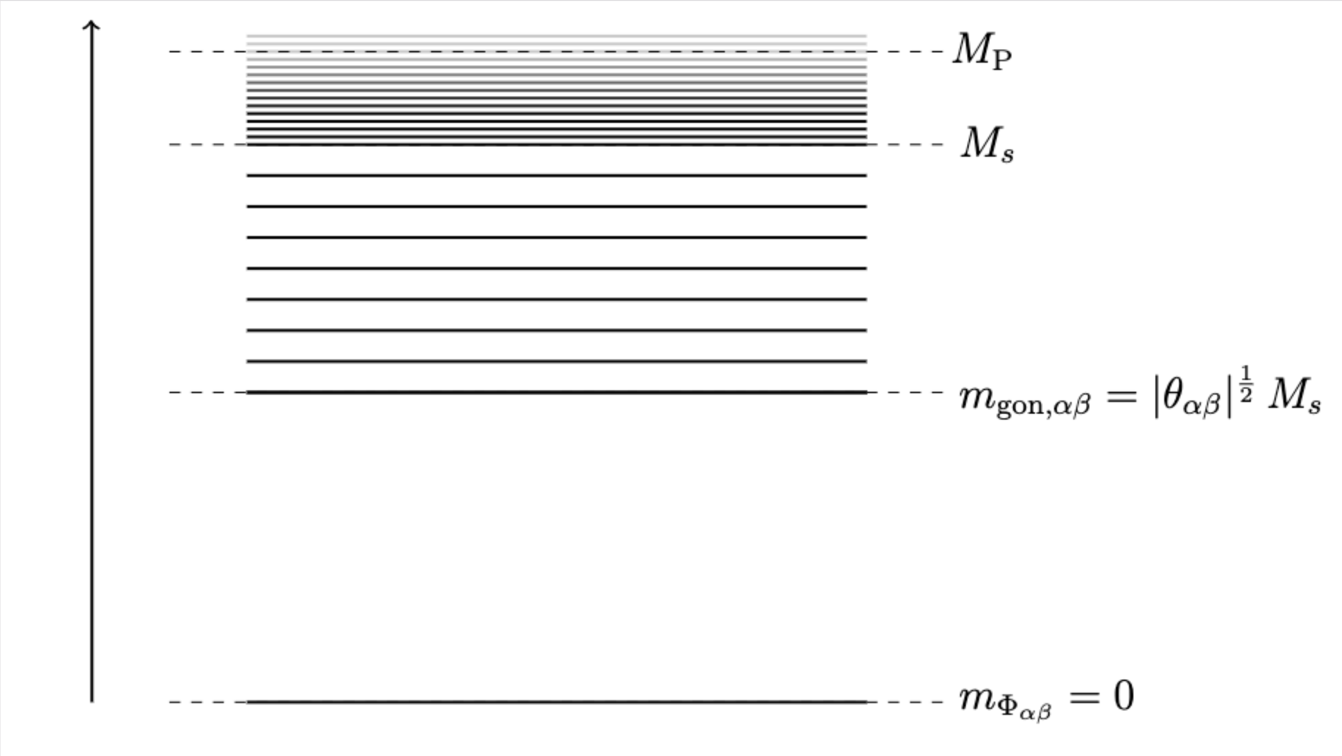
g_* is the gauge coupling under which the leading gonions transform

- Thus in a vanishing Yukawa coupling limit :

$$Y_* \to 0 \quad \longrightarrow \quad g_* \to 0$$

- This explains why this limit is singular: the gauge group would survive as a global symmetry, which is forbidden in QG

More about the gonion towers



$$m^2_{n;\alpha\beta} \simeq n\ |\theta_{\alpha\beta}|\ M_s^2$$

'p =2 tower'

All same charge: not BPS

- Thus e.g. in the simple class of models with a single leading gonion tower (along with two large dimensions) one has

$$m_{\rm gon} \sim g_* M_s \sim g_*^2 M_{\rm P}\ , \qquad Y_* \sim g_*\ , \qquad g_* \sim e^{\phi_4}\ .$$

$$The\ WGC\ condition\ m\ \leq\ \sqrt{2} g M_p\ verified\ with\ room\ to\ spare$$

- The (sub)Lattice/Tower WGC not realised here (all gonions have same charge)

Neutrinos at infinite distance?

$$If\ neutrinos\ are\ Dirac\ tiny\ Yukawas\ \sim\ 10^{-13}\ needed....$$

Neutrinos and String Theory

$$\mathcal{L}_\nu = K_{ii}^{\nu_R}(\nu_R^i \not\partial \nu_R^i) + M_R^i(\nu_R^i \nu_R^i) + W_{ij}(\nu_R^i \nu_L^i)\overline{H} + \ldots$$

- In presence of ν_R, essentially two ways to get tiny neutrino masses with $|W_{ij}| \simeq \mathcal{O}(1)$
- See-saw mechanism:

$$K_{ii}^{\nu_R} \sim 1 \qquad M_R \gg |H| \longrightarrow M_\nu^{Maj} \simeq \frac{|H|^2}{M_R}$$

Possible, but not easy, from stringy (charged) instantons (E2 in IIA orientifolds)

L.I., A.Uranga (2006); Blumenhagen et al (2006)

- Dirac neutrino mass:

$$K_{ii}^{\nu_R} \gg 1 \qquad M_R^i = 0 \longrightarrow M_\nu^{Dirac} \simeq |K_{ii}^{-1}||H|$$

We follow here this second path: turns out to be quite predictive !!

(Also some swampland arguments select Dirac neutrinos to avoid 3D AdS SM vacua)

Somewhat similar in spirit to Arkani-Hamed et al (1998)

A U(5)xU(1)xU(1) intersecting brane 'quiver' example

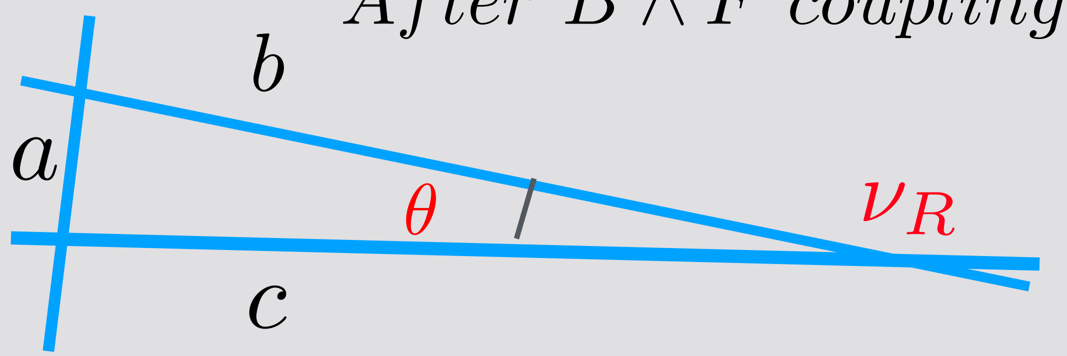
- Anastasopoulos,Leontaris, Kiritsis, Schellekens hep-th/1010.5188

Intersection	$SU(5)$	$U(1)_5$	$U(1)_b$	$U(1)_c$	Inter.
(aa^*)	10	2	0	0	+3
(ab)	$\bar{5}$	-1	1	0	-3
(ac)	$5_u + \bar{5}_d$	± 1	0	∓ 1	± 1
(bc) ν_R	1	0	-1	1	-3

- Structure may be obtained
- in CFT Type II orientifolds
- a la Schellekens et al.

(SM-like quivers also possible !!) **(F.G. Casas, L.E.I., F.Marchesano, 2406.XXXXX)**

$$After\ B \wedge F\ couplings\ U(1)_5 \times U(1)_b \times U(1)_c\ massive$$



$$Neutrino\ Yukawa:\ Y_\nu^{ij} (\bar{5}^i 5_{\overline{H}}) \nu_R^j$$

$$Gauge\ group\ felt\ by\ \nu_R's$$

$$Q_\nu \equiv Q_b - Q_c$$

- Direct application of the limit with a single gonion tower, parametrized by a single growing c.s. field u with axion coupling to $U(1)_\nu$

• Consider a c.s. field direction with a single large modulus U, coupling to the U(1) group of generator Q_ν through a B^F term:

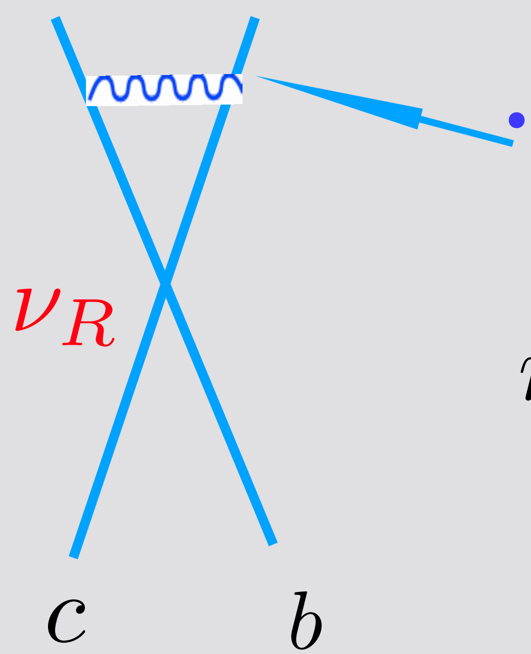
$$u \equiv Im\,U \longrightarrow \infty$$

$$S_{BF}^{(U)} = Q_U^\nu B_U \wedge F_\nu$$

$$a_U \longrightarrow a_U + Q_U^\nu \Lambda_\nu$$

• Cancelation of $U(1)_\nu^3$ through a GS mechanism requires a kinetic term

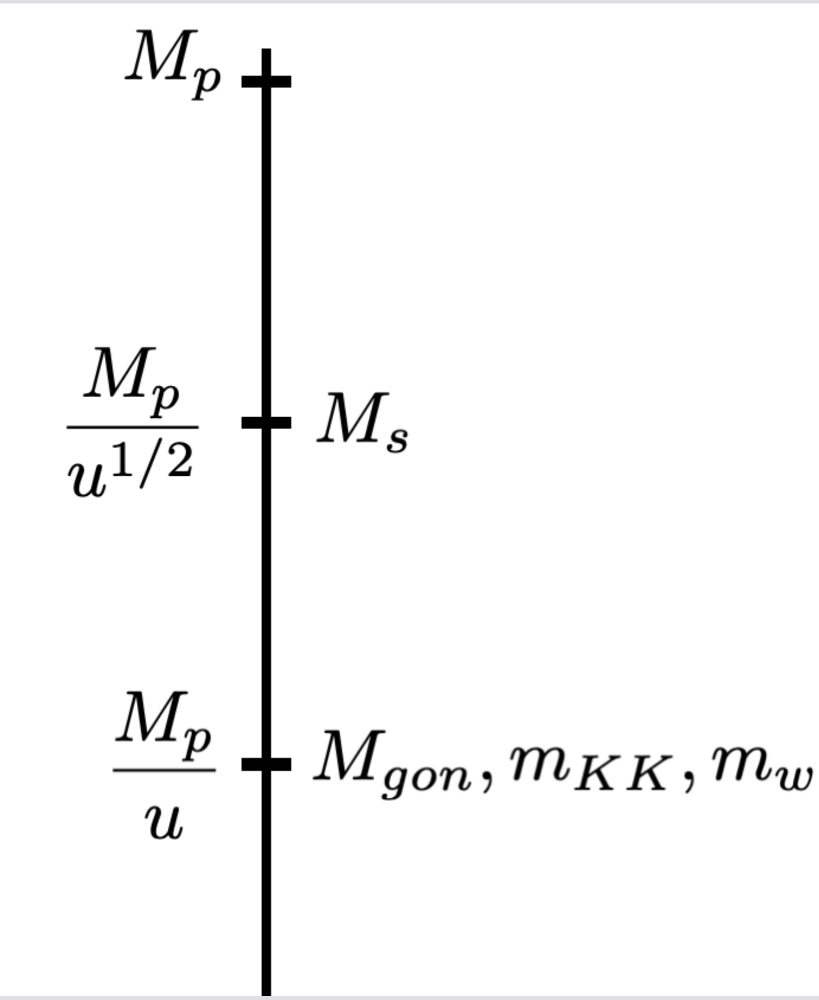
$$Gauge\ kinetic\ term\ :\ f_{Q_\nu} \simeq U \qquad so\ g_\nu \sim \frac{1}{u^{1/2}} \rightarrow 0$$

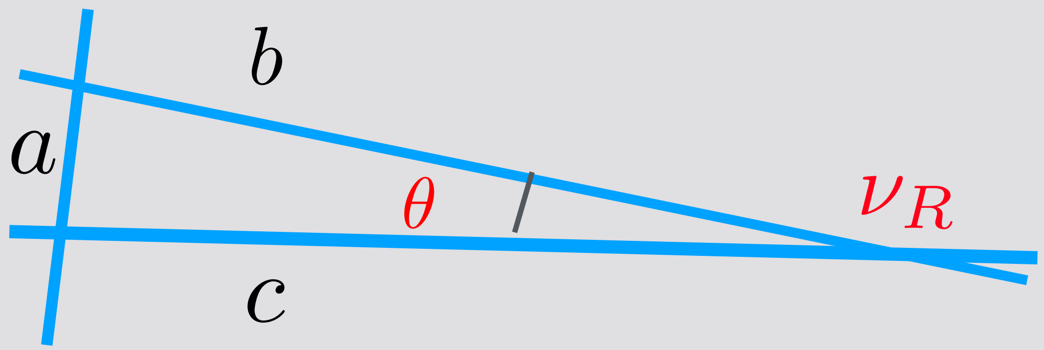


• Gonion towers

$$m_{gon_\nu} \simeq \frac{M_p}{u} \simeq g_\nu^2 M_p$$

$$M_s = e^{\phi_4} M_p \simeq \frac{M_p}{u^{1/2}} \simeq g_\nu M_p$$





Neutrino Yukawa couplings

$$Y_\nu^{ij} \simeq e^{-\phi_4} \left(\frac{m_{gon}^{\nu}}{M_p}\right)^{1/2} \left(\frac{m_{gon}^{\bar{5}}}{M_p}\right)^{1/2} \left(\frac{m_{gon}^{5_{\overline{H}}}}{M_p}\right)^{1/2} \simeq \left(\frac{m_{gon}^{\nu}}{M_p}\right)^{1/2} (\theta_{\bar{5}}\theta_{5_{\bar{H}}})^{1/4}$$

- Thus for the heaviest neutrino:

$$Y_\nu^{(3)} \simeq \left(\frac{m_{gon,\nu}}{M_p}\right)^{1/2} \simeq g_\nu \quad \Longrightarrow \quad m_\nu^{(3)} = Y_\nu^{(3)} < |\overline{H}| > \simeq g_\nu < |\overline{H}| >$$

- Experimentally (for normal hierarchy) one has $m_\nu^{(3)} \simeq \sqrt{\Delta m_{32}^2} \simeq 5 \times 10^{-2} eV$

$$Y_\nu^3 \simeq g_\nu \simeq 6.9 \times 10^{-13}$$

- Instantons which could in principle provide a Majorana mass to ν_R negligible: $M_R \sim M_s e^{-u}$

$$Y_3^\nu \simeq g_\nu \simeq 7 \times 10^{-13}$$

- Then all scales fixed since they are determined by g_ν

  - Structure quite unique:

String Scale	SM gonions	ν_R tower	large dim	Vector boson	Gravitino
M_s	m_{gon}^{SM}	m_{gon}^{ν}	$m_{KK/W}$	M_V	$m_{3/2}$
$g_\nu M_p$	$\lesssim M_s$	$g_\nu^2 M_p$	$g_\nu^2 M_p$	$g_\nu\lvert\bar{H}\rvert - g_\nu M_p$	$\lesssim M_s^2/M_p$
700 TeV	$\lesssim$ 700 TeV	500 eV	500 eV	0.5 eV- 700 TeV	$\lesssim$ 500 eV

- Some pheno implications include :

$$*A\ tower\ of\ \nu_R - like\ gonions\ at\ scale\ \sim\ 500eV$$

$$*Two\ large\ dimensions\ at\ m_{KK} \simeq\ 500\ eV$$

$$*A\ new\ gauge\ boson\ U(1)_\nu\ with\ mass\ M_V \geq\ 0.1\ eV, coupling\ g_\nu \simeq 7 \times 10^{-13}$$

$$*String\ scale\ M_s \simeq 700\ TeV$$

$$*KK\ of\ 4\ extra\ dim\ ; KK\ replica\ SM : \lesssim\ 700\ TeV$$

$$*Neutrinos\ are\ Dirac$$

$$(Stuckelberg\ mass\ of\ U(1)_\nu\ is\ model\ dependent)$$

- Also single large dimension feasible, but difficult not to suppress normal Yukawas!

Conclusions

- Using Type IIA CY N=1 orientifolds as a laboratory:

$$Y_{abc} \longrightarrow 0 \; IS \; at \; infinite \; distance$$

- $Y_{abc} \longrightarrow 0$
  - Large volume: Decompactification to M-th
  - Large C.S: at least one tower of charged 'gonions' $and\ p = 2,4,6\ large\ dimensions$

$$Y_{abc} \sim \prod_a \left(\frac{m^a_{gon}}{M_p} \right)^{1/2}$$

- Gonions have all same charge (violate Lattice/Tower WGC) and not extremal,e.g. for a single tower of gonions

$$m_{gon} \simeq g_*^2 M_p \qquad\qquad M_s \simeq g_* M_p$$

No violation of emergent string conjecture: a p=2 gonion tower comes along with p=2 extra dimensions

- Simplest examples parametrized by a single growing c.s. field u :

$$Y \sim \frac{1}{u^r} \qquad r = 1/4, 1/2, 3/4, 1$$

- Also the gauge coupling of (sub)group felt at the intersection

$$Y \sim g_*^{2r}$$

Explains what goes wrong when $Y \to 0$

- Application to small Dirac neutrino masses: rather unique setting with

$$Y_\nu \simeq \left(\frac{m^\nu_{gon}}{M_p}\right)^{1/2} \simeq g_\nu \simeq 6.9 \times 10^{-13} \ (exp.)$$

- A number of phenomenological consequences with a lowered string scale, a tower of neutrino gonions, a possibly light and very weakly coupled vector boson and two large dimensions. Two important scales:

$$M_s \simeq Y_\nu M_p \simeq 700 \ TeV \ ; \ m_{gon,\nu} \simeq Y_\nu^2 M_p \simeq 500 \ eV$$

Thank you !!