"Invariants" and Lepton Flavour

Marco Ardu , Sacha Davidson, Stephane Lavignac
IN2P3/CNRS, France
2308.16897, 2401.06214

1. intro
   - (invariants — what are they, what are they good for?)
   - LFV — what is, why is it interesting?

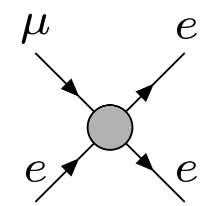
2. our bottom-up EFT to relate $\mu \leftrightarrow e$ data $\rightarrow$ (a few TeV-scale) models

3. find "invariants" (but NOT Traces)
   $$C(m_\mu) \approx \langle f|S|i\rangle \propto \Big[\Pi(\text{ SM/NP cpling matrices})\Big]_{e\mu}$$
   helpful for relating data $\leftrightarrow$ specific models

4. (?build "invariants" in EFT+SM, that allow us to understand which models are selected by data?)

Sorry to everyone my slides fail to cite ... :(

Lepton Flavour Change is Interesting!



$2\sqrt{2}G_F C$

- **LFV** $\equiv$ contact interaction changing lepton flavour
  (usually charged leptons for detection purposes, eg $\mu \to e\bar{e}e$)

- must occur, because charged leptons change flavour in ν oscillations...

  $\Rightarrow$ in m_ν models, LFV widely studied

- expt imposes bounds on coupling constants:

$$
\begin{array}{rlll}
C < 10^{-6} & \to 10^{-7} \to 10^{-8} & \to ?10^{-10} & \text{for } \mu \leftrightarrow e \\
C < 10^{-(3\to 4)} & \to 10^{-(4\to 5)} & & \text{for } \tau \leftrightarrow \ell \\
\text{current} & \text{upcoming} & \text{in planning} &
\end{array}
$$

- "upcoming" $\mu \leftrightarrow e$ start data-taking 2024-2026 $\Rightarrow$ LFV interesting now...

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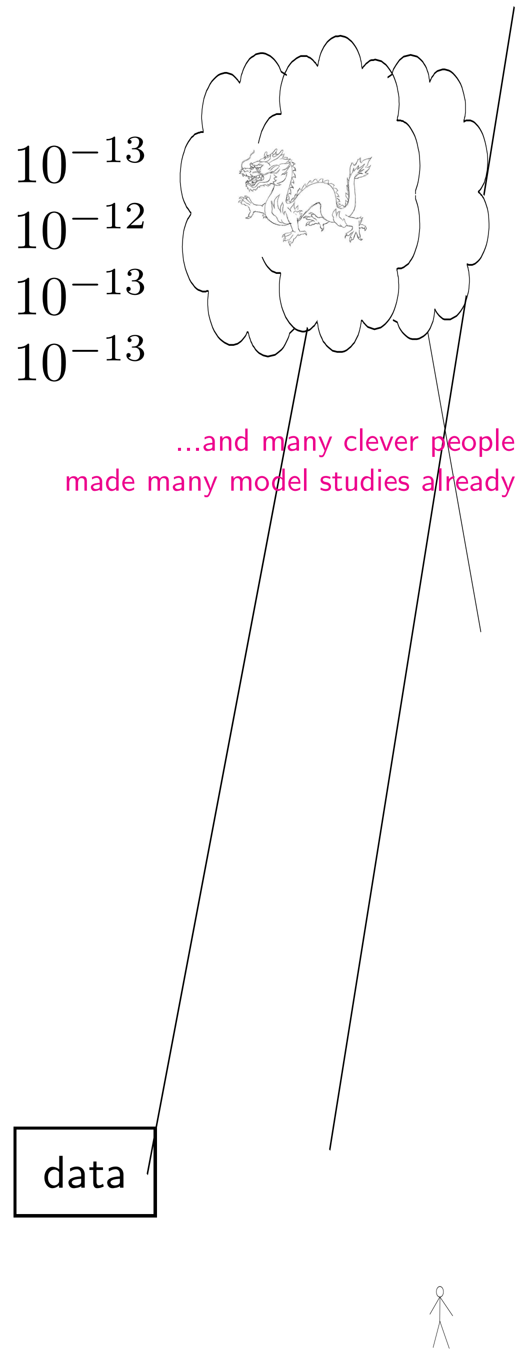
Our LFV studies of $\mu \leftrightarrow e$

1. few restrictive exptal bounds for $\mu \leftrightarrow e$:

$$\begin{cases} BR(\mu \to e\gamma) & < 3.2 \times 10^{-13} \\ BR(\mu \to e\bar{e}e) & < 1.0 \times 10^{-12} \\ BR(\mu Au \to eAu) & < 7.1 \times 10^{-13} \\ BR(\mu Ti \to eTi) & < 6.1 \times 10^{-13} \end{cases}$$

2. many models with many parameters
   $\Rightarrow$ innumerable scatter plots of model predictions for BRs
   ...but is there probability on model parameter space?

...and many clever people made many model studies already



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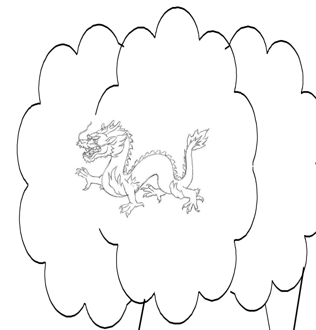

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3. we ask what the data can tell us about models:

   (a) parametrise data with cpling cts of contact interactions = operator coeffs
   (b) run the coeffs to NP scale (∼ TeV, to probe@LHC) using EFT
   (c) match to: { type II seesaw, inverse seesaw...}(LFV unsuppressed by m_ν)
   (d) enquire if models can *fill* the exptally probed ellipse at the NP scale
   = could data rule the models out?

4. ? do we learn anything *new* with our bottom-up EFT caln?

   (if we have to do scans to determine if the model sits in the ellipse, maybe not?)

"invariants" are helpful!

data

some details of that bottom-up recipe

(differences wrt top-down/models)

1. perturbative expansions of $\langle f|S|i\rangle$ different in

   (a) models: order by order in loops, *eg* exact at one-loop (in the model)
   (b) our EFT: "relevant" model loops, SM $\mathcal{O}([\alpha \log]^n)$ +a few $\alpha[\alpha \log]^n$

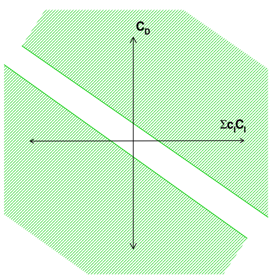
   $\Rightarrow$ we include some relevant SM effects we did not find in literature

2. data = *constraints* on 12 Wilson coefficients at exptal scale m_μ

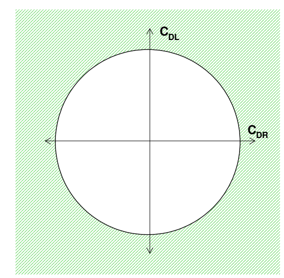
$$BR(\mu \to e\gamma) = 384\pi^2(|C_{DL}(m_\mu)|^2 + |C_{DR}(m_\mu)|^2)$$

(relativistic e_L,e_R not interfere)

$$BR(\mu A \to e_R A) \propto |c_{D,A}C_{DL} + \sum_{4f\ ops} c_{IA}C_I|^2$$



$\Rightarrow$ data = 12 (complex)-dim ellipse at origin in coefficient-space $\gg$ 4 BRs



(more details: operator bases and all that)

3. RGEs "mix" operator coefficients (and there are ~ 90 operators):

$$C_D(m_\mu) \quad = \quad C_D(\Lambda_{NP}) + \sum_I C_I(\Lambda_{NP}) \frac{\gamma_{ID}}{16\pi^2} \log + ...$$

2010.00317

$\Rightarrow$ *almost every 4-legged* $\mu \leftrightarrow e$ *operator contributes to at least one of* $\mu \to e\gamma$, $\mu \to e\bar{e}e$ *or* $\mu A \to eA$, *suppressed* $\gtrsim \mathcal{O}(10^{-3})$.

$\Rightarrow$ The 12-d ellipse is rotated in the 90-d space...so the eqn for ellipse becomes 90×90 correlation matrix. Lets rotate basis rather than ellipse, to stay in 12-d...then at Λ_{NP} match models onto the ellipse. (there are a zoo of directions whose coeffs are unconstrained by data—model predictions in those directions are irrelevant!)

4. we ask "can the data can rule the model out?" *YES*

(we do not ask what the model wants to predict...)

$\Rightarrow$ finally: expressions for the 12 observable $C(m_\mu)$s in terms of model/SM parameters — and these are "*invariants*"

Recall about invariants

- Jarlskog: construct as Trace/Det of cpling matrices of $\mathcal{L}$, measures $\not{CP}$

$$\frac{1}{3}\mathrm{Tr}\left\{\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3\right\} = 2iJ(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2)(y_s^2 - y_d^2)$$

$$\text{with} \quad J = \mathrm{Im}\{V_{us}V_{cs}^* V_{cb} V_{ub}^*\}$$

similar ($\sim$traces) invars
for SM flavour, 2HDM, Rparity...

"invariant" under reparam. of $\mathcal{L}$ that move $\not{CP}$ around (=flavour-basis rotns)

⋆ elegant⋆, ⋆works also for non-manifest symmetries ⋆,

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- also sym.-breaking S-matrix elements are invariant, *eg* for $\not{C}\not{P}$

Brod, Gorbahn, Stamou: 1911.06822

$$\epsilon_K \propto J\Big[\mathrm{Re}\{V_{td}V_{ts}^* V_{ud}^* V_{us}\}\eta^{tt}S(x_t) + 2|V_{ud}|^2|V_{us}|^2\eta^{ut}(S(x_c) - S(x_c, x_t))\Big]$$

where $x_Q = \frac{m_Q(m_Q)}{m_W}$, η^{pq} QCD corrections, and Inami-Lim

$$S(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^2}{2(1-x_t)^3}\ln x_t \;,\quad S(x_c) - S(x_c, x_t) = x_c\left(1 - \ln\frac{x_t}{x_c}\right) + \frac{3x_c x_t}{4(1-x_t)} + \frac{3x_c x_t^2}{4(1-x_t)^2}\ln x_t \;,$$

⋆⋆ reality ⋆⋆ - functionally complicated, many parameters at different scales...

"Invariants" are useful in relating data to a chosen model

Consider two neutrino mass models at $\Lambda_{NP} \sim$ TeV

(both have LFV unsuppressed by m_ν)

Inverse Seesaw

- $\approx$ add extra heavy singlet fermion S_a, to each gen. of type I seesaw:

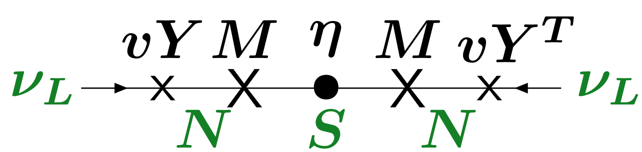
$$\delta\mathcal{L} \supset -\left(Y_\nu^{\alpha a}(\overline{\ell}_\alpha \tilde{H} N_a) + M_{ab}\overline{S}_a N_b + \frac{1}{2}\eta_{ab}\overline{S_a}S_b^c + \text{h.c}\right)$$

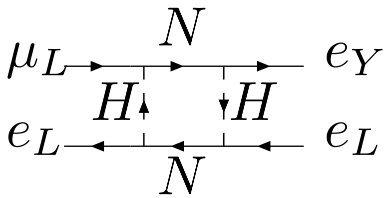
Majorana $\eta \ll M$
M is L-conserving Dirac mass for singlets

gives neutral fermion mass matrix

$$\mathcal{M}_{\nu N} \approx \overline{\left(\nu_L \;\; N^c \;\; S\right)} \begin{bmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M^T \\ 0 & M & \eta \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \\ S^c \end{pmatrix}$$

gives $m_\nu \approx m_D(M^{-1})\eta(M^T)^{-1}m_D^T$, so flavour-changing $Y_\nu^{\alpha a}$ unrelated to m_ν.





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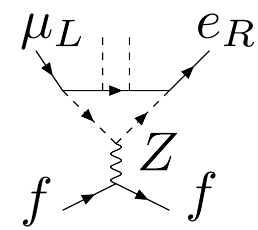

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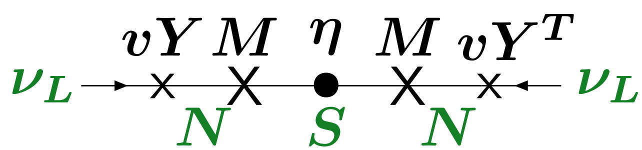


- LFV via loops with EW bosons; controlled by 2→4 "invariants" $(\Delta M : 0 \to> v)$

$$\left[Y_\nu[M^\dagger M]^{-1}Y_\nu^\dagger\right]_{e\mu} \quad , \quad \left[Y_\nu[M^\dagger M]^{-1}\log(M^\dagger M)Y_\nu^\dagger\right]_{e\mu}$$

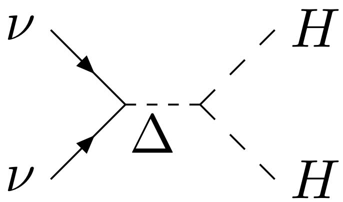
$$\left[Y_\nu f_1(M^\dagger M)Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right]_{e\mu} \quad , \quad \left[Y_\nu f_2(M^\dagger M)Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right]_{e\mu}$$

despite many parameters in Y_ν! (?no need to scan).

- operators with e_R, or scalar, have $C \propto y_e, y_\mu$ so unobservable. Remain 5Cs.
  $\Rightarrow$ model predicts 3→1 relations.

Type II seesaw — add SU(2) triplet scalar $\vec{\Delta}$, with M_Δ

$$\mathcal{L} \supset f_{\alpha\beta}\, \ell_\alpha \vec{\tau} \ell_\beta \cdot \vec{\Delta} + M_\Delta \lambda_H \, H^T \vec{\tau} H \cdot \vec{\Delta}^* + ...$$



$$[m_\nu]_{\alpha\beta} \sim \frac{[f]^*_{\alpha\beta} \lambda_H M_\Delta v^2}{M_\Delta^2} \sim 0.03 \text{ eV} \times [f]_{\alpha\beta} \frac{\lambda_H}{10^{-12}} \frac{\text{TeV}}{M_\Delta}$$

model reputed predictive, because $f_{\alpha\beta} \propto [m_\nu]_{\alpha\beta}$ (known up to $m_{min}, e^{i\phi_1}, e^{i\phi_2}$)

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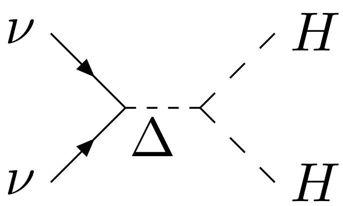

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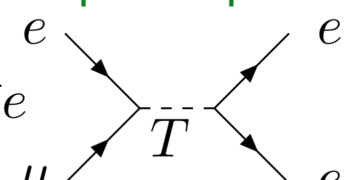
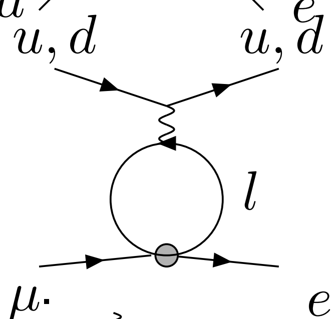
$\mu \to e\bar{e}e$



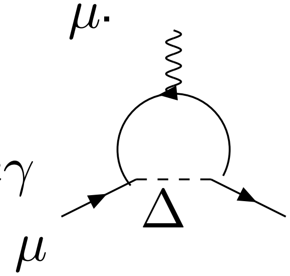
$$\sim \frac{[m_\nu^*]_{\mu e}[m_\nu]_{ee}}{2\lambda_H^2 v^2} \quad , \quad [m_\nu^*]_{\mu e}, [m_\nu]_{ee} \sim f(m_{min}, \phi_i)$$

$\mu A \to e A$



$$\sim \frac{\alpha_e}{6\pi\lambda^2 v^2} \left[m_\nu^* \log \frac{[m_e m_e^\dagger]}{M_\Delta^2} m_\nu \right]_{\mu e} , f(m_{min}, \phi_i,),$$

$\mu \to e\gamma$



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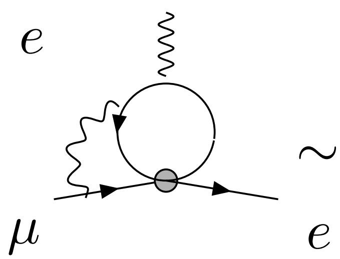


$$\sim \quad \frac{e}{128\pi^2} \left(\frac{[m_\nu^* m_\nu]_{\mu e}}{\lambda^2 v^2} + \frac{2\alpha}{\pi \lambda^2 v^2} \left[m_\nu \log \frac{[m_e m_e^\dagger]}{M_\Delta^2} m_\nu \right]_{\mu e} \right)$$

3 "invariants" determine the 5 non-negligeable Cs; 1 invar known from ν osc.

7 C with e_R, and scalars, are $\propto y_{e,\mu}$ because LFV in doublets

“Invariants” we met in bottom-up-EFT studies of LFV

• find op. coefficients at the exptal scale proportional to a product of NP/SM matrices, *eg*:

$$C^{e\mu dd}_{V,LR} \quad \propto \quad \left[[m^*_\nu] \ln \frac{[m_e m_e^\dagger]}{M^2_\Delta} [m_\nu] \right]_{\mu e}$$

+ combine practical realism of S-matrix element(= function of parameters with scheme and scale), with functional elegance of Lagrangian invariants

- not Traces! Element of a matrix in flavour space; eg in mass eigenstate basis.

In addition:

1) no need for model parameter scans; "invariants" are complex #s $\lesssim 1$. Simple to see which part of observable ellipse the model can, or not, fill.

2) simple to count # invariants that control $\mu \leftrightarrow e$. Find that none of our models can fill the whole ellipse!

Summary

In bottom-up EFT, we studied three NP models at the TeV (typeII + inverse seesaws+ a leptoquark), and discovered that none of them can fill the experimentally allowed ellipse in coefficient space. So upcoming observations could rule them out.

The coeffcients at the exptal scale, $C(m_\mu)$, can be expressed in terms of NP parameters at Λ_{NP} and SM parameters are various scales. They ressemble elegant Lagrangian invariants ($\sim$ Jarlskog), while remaining practical objects constructed out of scheme and scale-dependent parameters.

?Could it be possible to build invariants with the EFT + SM, that help us understand which models are selected by observations of LFV?

BackUp

if see $\mu \to e\gamma$, $\mu \to e\bar{e}e$, or $\mu A \to eA$...?can distinguish models?

...model predictions studied for decades...

EFT recipe to study this: (not scan model space—no measure)

• data is a "12-d" ellipse/box in coefficient-space (in an ideal theorist's world)

• with RGEs, can take ellipse to Λ_{NP}

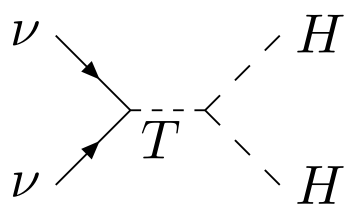
• are there parts of ellipse that a model *cannot* fill?
If yes, model can be distinguished/ruled out by $\mu \leftrightarrow e$ data.

Apply recipe:
1) type II seewaw
2) (singlet LQ for R_D^*)
3) inverse seesaw

Type II seesaw — add SU(2) triplet scalar $\vec{T}$

$$\mathcal{L} \supset \left([Y]_{\alpha\beta} \overline{\ell^c_\alpha} \varepsilon \vec{\tau} \cdot \vec{T} \ell_\beta + M_T \lambda_H \;\; H \varepsilon \vec{\tau} \cdot \vec{T}^* H + \text{h.c.} \right) + \ldots$$

get $[m_\nu]$ at tree (NB: 2 mass scales, so unclear notion of Λ_{NP}):



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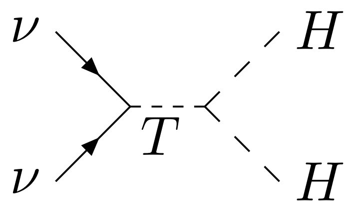

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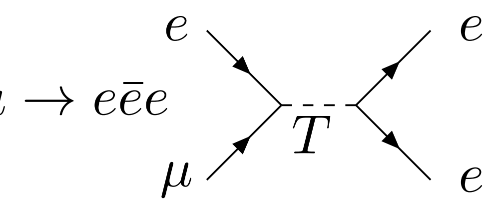
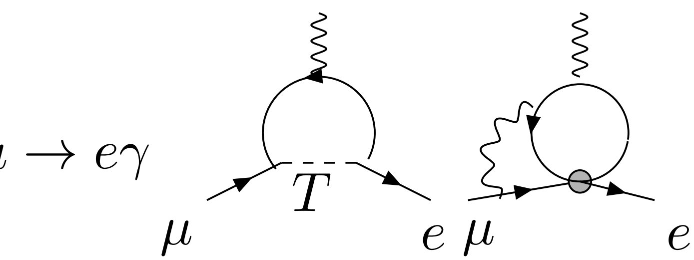
expect $\mu \to e\bar{e}e$ at tree (vanish via Majorana phases ϕ_i):

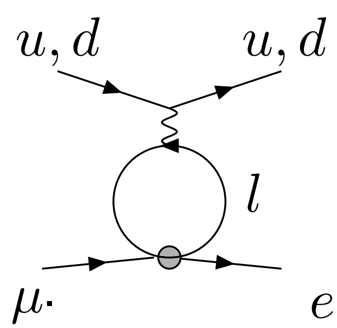


$$C^{e\mu ee}_{V,LL} \sim \frac{[Y]_{\mu e} [Y^*]_{ee} v^2}{M_T^2}$$

and $\mu \to e\gamma, \mu A \to eA$ at loop (weaker dependence on unknown model params)



$\mu A \to eA$



Type II seesaw: predictions

recall 12 (complex) operator coefficients $\left\{ \begin{array}{l} C_{DR},\ C^{e\mu ee}_{VLL},\ C^{e\mu ee}_{VLR},\ C^{e\mu ee}_{SRR},\ C_{AlightL},\ C_{AheavyR} \\ C_{DL},\ C^{e\mu ee}_{VRL},\ C^{e\mu ee}_{VRR},\ C^{e\mu ee}_{SLL},\ C_{AlightL},\ C_{AheavyR} \end{array} \right.$

- seven coefficients for LFV-involving-singlet-leptons are negligeable

(predicted by all m_ν models where NP interacts with doublets); test by polarising μ. Kuno Okada

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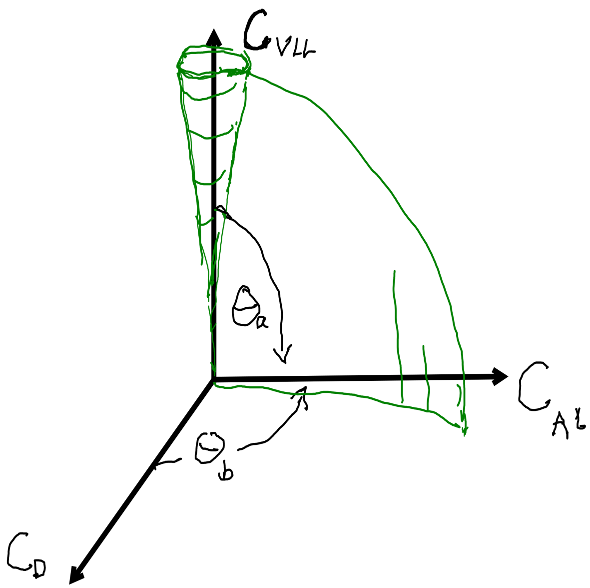
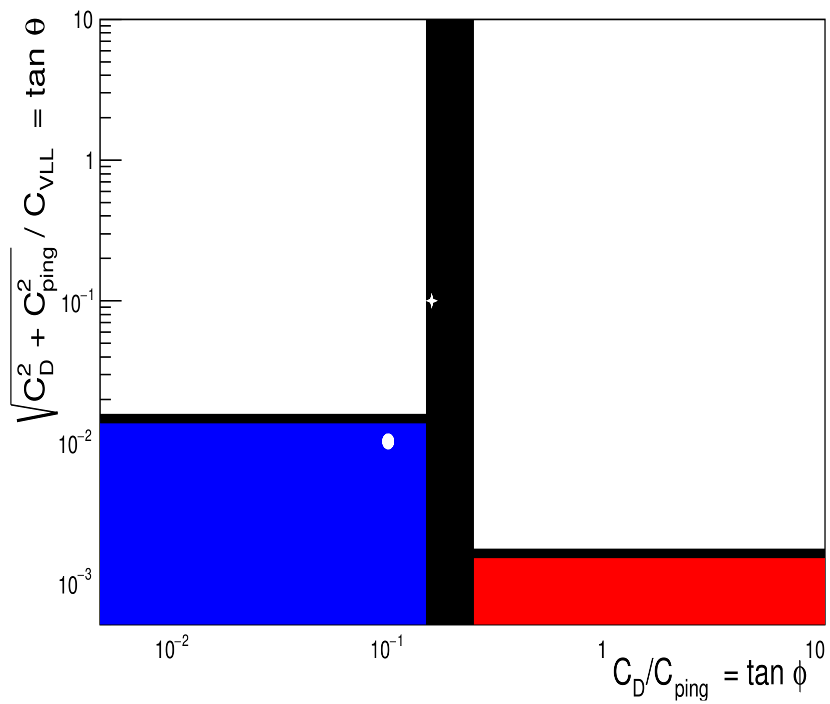
- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ or $C_{Al,L}(\mu A \to eA)$ can vanish (also any of C_{DR} for $m_\nu \gg$)

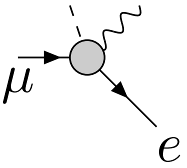
- $C^{e\mu ee}_{VLL}$ $(\mu \to e\bar{e}e)$ "naturally" large: predict $C_{DR}/C_{Al,L}$ for small $C^{e\mu ee}_{VLL}$.



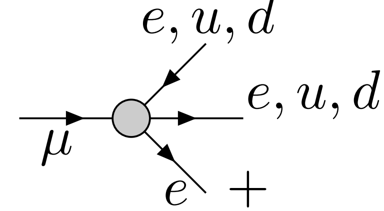


model lives in not-while areas expt can probe whole plot: $\tan\theta_{a,b} : 10^{-3} \to 10$

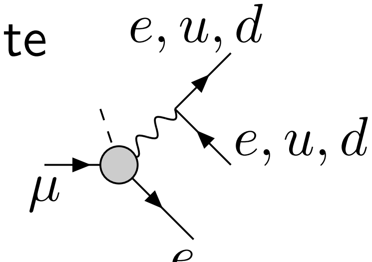
vert. axis $\sim$ loop/tree ; horiz. axis $\sim |C_D|/|C_{Al}|$



three processes: $\mu \to e$, $\Delta F_Q = 0$



- $\mu \to e\gamma$: chirality-flip, @loop in models, only dipole operators contribute



- $\mu \to e\bar{e}e$: 4lepton(V+S)+dipole ops.
  angular distributions⇒ indep constraints on 6→8 coeffs.

OkadaOkumuraShimizu

- $\mu A \to eA$ ($= \mu^-$ in $1s$ of A, turns into e^-)
  nucl. phys. ≈ WIMP scattering ⊃ SpinIndep$\sim A^2$-enhanced (+SD, neglect)
  coherent, all ops interfere ⇔ one op per target for $\{e_L, e_R\}$

- ⇒ $\mu \to e_L(e_R)$ processes, at exptal scale $\left\{ \begin{array}{l} \text{described by} \\ \text{constrain} \end{array} \right\}$ **6(+6)** operators:

$$\delta\mathcal{L}_{e_L} = \frac{1}{v^2}\Big[C_D(m_\mu \overline{e}\sigma^{\alpha\beta}P_R\mu)F_{\alpha\beta} + C_S(\overline{e}P_R\mu)(\overline{e}P_Re) + C_{VR}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Re)$$

$$+ C_{VL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Le) + C_{Alight}\mathcal{O}_{Alight} + C_{Aheavy\perp}\mathcal{O}_{Aheavy\perp}\Big]$$

$\{C\}$ are $\mathcal{O}(1)$ dimless numbers that can be measured (∃ more info than just rates)
O_{Alight} =combo of 4fermion operators probed by light targets (Al,Ti)
$O_{Aheavy\perp}$= indep. combo of 4fermion ops probed by heavy targets (Au)

What are $\mathcal{O}_{Alight}, \mathcal{O}_{Aheavy\perp}$?

$$\mathcal{O}_{Alight,X} \sim 0.7(\overline{e}P_X\mu)\Big[(\overline{u}u) + (\overline{d}d) + ...\Big] + 0.13(\overline{e}\gamma^\alpha P_X\mu)\Big[(\overline{u}\gamma_\alpha u) + (\overline{d}\gamma_\alpha d)\Big]$$

$$\mathcal{O}_{Aheavy\perp,X} \simeq (\overline{e}\gamma^\alpha P_X\mu)\Big[0.56(\overline{u}\gamma_\alpha u) + 0.8(\overline{d}\gamma_\alpha d)\Big] + ...$$

obtained by matching nucleons to quarks, then writing

$$\mathcal{O}_{Aheavy,X} = \mathcal{O}_{Alight,X} + \epsilon\mathcal{O}_{Aheavy\perp,X}$$

where ϵ calculable misalignement $\simeq 5\%$.

problem: scalar density of u quarks in $N \in \{n,p\} \simeq$ scalar density of d quarks $\Rightarrow$ with current theory uncertainties in $\mu A \to eA$, measuring C_S^n and C_S^p only allows to determine $C_S^u + C_S^d$ (but not $C_S^u - C_S^d$).

Restrict to e_L outgoing from $\mu \to e$ bilinear

take observable-motivated basis to Λ_{NP}?

($L \leftrightarrow R$ not identical in SMEFT, but not worry)

1. $\mu \to e\gamma$ measures $C_{D,R}(m_\mu)$

solving RGEs gives $\vec{C}(m_\mu) = \vec{C}(m_W)\boldsymbol{G}(m_\mu, m_W)$, $\Rightarrow$ define $\vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$such that:

$$C_{DR}(m_\mu) = \vec{C}(\Lambda) \cdot \vec{v}_{\mu\to e\gamma}(m_\mu, \Lambda)$$

$$\begin{aligned}
C_{D,X}(m_\mu) \;\; = \;\; & C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\
& -\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\
& +16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\
& -8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\text{ GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)f_{TD} \\
& +16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\text{ GeV}}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right)
\end{aligned}$$

all coeffs on right side $C(m_W)$ (basis vectors rotate and change length with scale)

$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

Counting constraints in space of ∼100 operators

DKunoYamanaka

Count constraints: (write $\delta\mathcal{L} = \frac{C^{flavour}_{Lorentz,XY}}{v^n} \mathcal{O}^{flav}_{Lorentz,XY}$, $X, Y \in \{L, R\}$)

$\mu \to e\gamma$: $BR(\mu \to e\gamma) = 384\pi^2(|C_{D,L}|^2 + |C_{D,R}|^2)$ ⇒ **2 constraints**

$\mu \to e\bar{e}e$: (e relativistic ≈ chiral, neglect interference between e_L, e_R)

$$BR = \frac{|C_{S,LL}|^2}{8} + 2|C_{V,RR} + 4eC_{D,L}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,L}|^2 + |C_{V,RL} + 4eC_{D,L}|^2 + \{L \leftrightarrow R\}$$

⇒ **6 more constraints**

$\mu A \to eA$:(S^N_A,V^N_A=integral over nucleus A of N distribution×lepton wavefns, **different** for diff. A)

$$BR_{SI} \sim Z^2|V^p_A\tilde{C}^p_{V,L} + S^p_A\tilde{C}^p_{S,R} + V^n_A\tilde{C}^n_{V,L} + S^b_A\tilde{C}^n_{S,R} + D_AC_{D,R}|^2 + |L \leftrightarrow R|^2$$

$$BR_{SD} \sim |\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

SI bds on Au, Ti, (+ SD on ?Ti, Au?) ⇒ **4 + 2 more constraints**

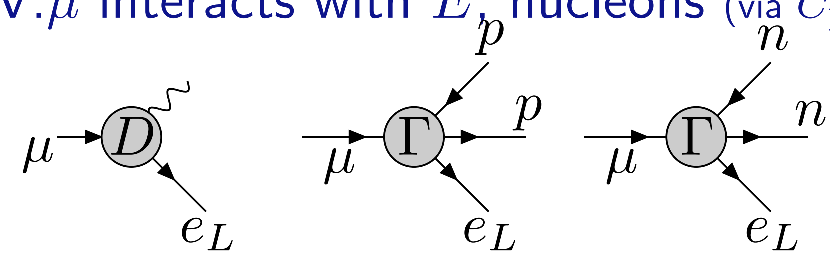
future: improved theory, 3SI+2SD targets ⇒ 6 + 4 constraints

is 12-20 constraints on ~ 100 operators a problem?

$\mu A \to eA$: most sensitive process, expt + th

- μ^- captured by Al nucleus, tumbles down to $1s$. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM: muon "capture" $\mu + p \to \nu + n$, or decay-in-orbit
- LFV:μ interacts with $\vec{E}$, nucleons (via $\tilde{C}^N_{\Gamma,X}(\bar{e}\Gamma P_X N)(\bar{N}\Gamma N)$), converts to e

($E_e \approx m_\mu$ so e_L/e_R)



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V, A, T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

KitanoKoikeOkada

$$BR_{SI} \sim \quad Z^2 |\sum ...\tilde{C}_{SI}|^2 \quad , \qquad \tilde{C}_{SI} \in \{\tilde{C}^p_V, \tilde{C}^p_S, \tilde{C}^n_V, \tilde{C}^n_S, C_D\}$$

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over $N \propto$ spin of only unpaired nucleon)

$$BR_{SD} \sim ...|\tilde{C}^N_A + 2\tilde{C}^N_T|^2$$

CiriglianoDavidsonKuno
HoferichterEtal