Dark Dimension, Swampland and Effective Field Theory Limit

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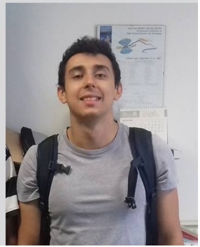
C. Branchina, VB, F. Contino, Phys.Rev.D 108 (2023) 4, 045007

C. Branchina, VB, F. Contino, A. Pernace, arXiv: 2308.16548

C. Branchina, VB, F. Contino, A. Pernace, arXiv: 2404.10068

Filippo Contino



Carlo Branchina

Arcangelo Pernace

Dark Dimension

According to the Dark Dimension Scenario we might well live in a universe with one compact extra dimension of micrometer size $\sim \text{meV}^{-1}$

Montero, Vafa, Valenzuela, JHEP 02 (2023) 022

In search for a solution to the naturalness/hierarchy problem, the idea of the possible existence of a compact dimension was put forward long ago

Arkani-Hamed, Dimopoulos, Dvali, Phys.Lett.B 429 (1998) 263

Brief recap of the DD scenario (Montero, Vafa, Valenzuela)

Combines swampland conjectures with observational data and phenomenological bounds

Based on the assumption that our universe lies in a unique (asymptotic) corner of the quantum gravity landscape, where only two kinds of light tower states are expected: towers of string excitations or of KK states

Call μ_{tow} the tower scale and Λ_{cc} the measured vacuum energy (cosmological constant times M_P^2). The application of the distance conjecture to (A)dS vacua gives

$$\mu_{tow} \sim \left| \frac{\Lambda_{cc}}{M_P^4} \right|^{\alpha} M_P \,. \quad (1)$$

Combining indications from one-loop string calculations with the Higuchi bound, α restricted to

$$\frac{1}{4} \leq \alpha \leq \frac{1}{2} \quad (2)$$

From bounds set by the non-observation of deviations from Newton's force law $\Rightarrow \mu_{tow}$ close to energy associated to the CC (neutrino scale)

$$\mu_{tow} \sim \Lambda_{cc}^{1/4} \sim \text{ meV}, \tag{3}$$

For (1) to be consistent with (3), it is necessary to take α at the lower bound of (2), $\alpha = 1/4$.

The authors then observe: "since we can describe physics above the neutrino scale with **Effective Field Theory**", the string scenario is "ruled out experimentally" $\Rightarrow$

$\Rightarrow$ The tower must be a **KK tower**, $\mu_{tow} = m_{KK}$, and (1) becomes

$$\Lambda_{cc} \sim m_{KK}^4 \tag{4}$$

The number of compact dimensions determined comparing bounds on m_{KK} from calculations of the heating of neutron stars due to the surrounding cloud of trapped KK gravitons with m_{KK} in (4). The authors conclude that there is only one extra dimension of size $\sim \mu$m

Relevant to this talk

According to the Dark Dimension scenario, above m_{KK} our universe is described by a $(4+1)$D Effective Field Theory with one compact dimension of μm size.

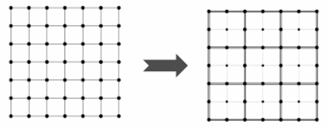
This proposal has triggered a large amount of work, where specific $(4+1)$D Effective Field Theory models that could realize the DD scenario are considered, and their phenomenological consequences studied

This talk - relation between two fundamental aspects of this proposal

String Theory side (Swampland)

Effective Field Theory limit

Wilson - Effective Field Theory paradigm



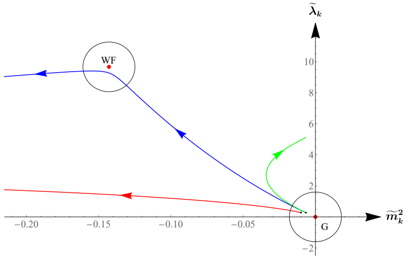
Any QFT is an Effective Field Theory

Steven Weinberg - “you may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you’ll be sorry”

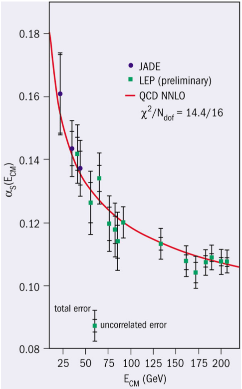
EFT & RG in any dimension: ..., $d = 3$, $d = 4$, ...

Examples

$d = 3$ dimensions : Wilson-Fisher



$d = 4$ dimensions : AF



Also for theories with $d > 4$ dimesions

in particular

Theories with compact extra dimensions: $d = 4 + n$

... Let's move then to consider theories with compact extra dimensions ...

Theories with compact extra dimensions

$$d = 4 + n$$

- Infinite tower of KK states states

$$m_n = f_n \cdot \mu_{\text{tow}}$$

- Manifest a surprising UV-softness

Towers contribute $\sim \mu_{\text{tow}}^4$

to Vacuum Energy / Effective Potential

How is it possible, Why not $\sim \Lambda^4$?

One-Loop Higgs Effective potential

Consider a (4+1)D SUSY theory on the multiply connected $\mathcal{M}^4 \times S^1$ with non-trivial boundary conditions along the compact dimension

Note that we have different R-charges q_i for the superpartners $(i = b, f)$

$$\Psi_i(x, z+2\pi R) = e^{2i\pi R q_i}\Psi_i(x, z) \;\Rightarrow\; \Psi_i(x, z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{+\infty} \psi_{i,n}(x) e^{i(\frac{n}{R}+q_i)z}$$

$\int dz\, \mathcal{S}_{(5)} \;\to\; \mathcal{S}_{(4)}$ infinite tower of $4D$ KK fields $m^2_{i,n} \propto \left(\frac{n}{R} + q_i\right)^2$

Higgs field ϕ : ϕ_0 , or $4D$ brane field , or . . .

$$M^2_{i,n}(\phi) = m^2_i(\phi) + \left(\frac{n}{R} + q_i\right)^2$$

mismatch in the masses of the superpartners

$\Rightarrow$ effective 4D non-local soft SUSY breaking

One-loop Higgs Effective Potential

$$V_{1l}^{(4)}(\phi) = \frac{1}{2}\sum_a \sum_{i_a} (-1)^{\delta_{i_a,f_a}} \sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \log\left(p^2 + M_a^2(\phi) + \left(\frac{n}{R} + q_{i_a}\right)^2\right)$$

One way of performing the calculation (not the only one)*:
Perform first the infinite sum, then integrate over p with a cutoff Λ

Delgado, Pomarol, Quiros

Each tower contributes :

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

* Other methods, "Proper time" (Antoniadis, Benakli, Quiros), "Pauli-Villars" (Contino, Pilo), Thick brane (Delgado, von Gersdor, John, Quiros), all give the same result.

A closer look to this potential

From each tower the Higgs Potential receives the contribution

$$V_{1l}^{(4)}(\phi) = R\left(\frac{m^2\Lambda^3}{48\pi} - \frac{m^4\Lambda}{64\pi} + \frac{m^5}{60\pi}\right) - \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

- UV-sensitivity through m $\Longrightarrow$ canceled by SUSY
- No UV-sensitivity through q

$\Longrightarrow$ **Finite Higgs potential**

~ 2000

Great excitement for this result ...

- Finite Higgs effective potential!
- Finite Higgs mass!
- KK regularization

Voice "out": sum over n (tower) should be cut $\rightarrow$ UV-sensitive terms

Ghilencea, Nilles, Phys.Lett.B 507 (2001) 327

... Heated debate ...

... somehow seemed to be closed in favour of UV-insensitiveness ...

... However ...

4D Higgs Effective Potential from the 5D side

$$\mathcal{S}_{(5)} = \int dz\, d^4x \left(\frac{1}{2}\, \partial_a \widehat{\Phi}\, \partial^a \widehat{\Phi} + \partial_a \widehat{\chi}\, \partial^a \widehat{\chi}^\dagger + \frac{m_\Phi^2}{2}\, \widehat{\Phi}^2 + m_\chi^2\, \widehat{\chi}\widehat{\chi}^\dagger + \frac{\widehat{\lambda}}{4!}\, \widehat{\Phi}^4 + \frac{\widehat{g}}{2}\, \widehat{\Phi}^2 \widehat{\chi}\widehat{\chi}^\dagger \right)$$

$$\widehat{\Phi}(x, z + 2\pi R) = \widehat{\Phi}(x,z) \quad ; \quad \widehat{\chi}(x, z+2\pi R) = e^{2i\pi R\, q}\, \widehat{\chi}(x,z)$$

$$V_{1l}^{(5)}(\widehat{\Phi}) = \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\widehat{\lambda}}{2}\, \widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}} + \frac{1}{2}\mathrm{Tr}_5 \log \frac{p^2 + \left(\frac{n}{R} + q\right)^2 + m_\chi^2 + \frac{\widehat{g}}{2}\, \widehat{\Phi}^2}{p^2 + \frac{n^2}{R^2}}$$

We cannot introduce a hierarchy between the asymptotics of the different components of the loop momentum when calculating the Effective Potential

$$\text{Spherical cut} \qquad \widehat{p}^2 \leq \Lambda^2$$

$$\mathrm{Tr}_5 \equiv \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R} \right)' \equiv \frac{1}{2\pi R} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \quad ; \quad C_\Lambda^n \equiv \sqrt{\Lambda^2 - \frac{n^2}{R^2}}$$

4D Effective Potential from the 5D Effective Potential

Fourier expansion of $\widehat{\chi}(x,z)$ (similarly for $\widehat{\Phi}(x,z)$)

$$\widehat{\chi}(x,z) = \left(\sum_n \int \frac{d^4p}{(2\pi)^5 R} \right)' \widehat{\chi}_{n,p}\, e^{i\left(p\cdot x + \left(\frac{n}{R}+q\right)z\right)}$$

$$\widehat{\chi}(x,z) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \chi_n^{\Lambda}(x)\, e^{i\left(\frac{n}{R}+q\right)z}; \quad \chi_n^{\Lambda}(x) \equiv \frac{1}{\sqrt{2\pi R}} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \widehat{\chi}_{n,p}\, e^{ip\cdot x}$$

Performing the z integration $\rightarrow$ effective $4D$ theory (here is the Tr_5)

$$V_{1l}^{(4)}(\phi) = \frac{1}{2} \sum_{n=-[R\Lambda]}^{[R\Lambda]} \int^{C_\Lambda^n} \frac{d^4p}{(2\pi)^4} \left(\log \frac{p^2 + \frac{n^2}{R^2} + m_\phi^2 + \frac{\lambda}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} + \log \frac{p^2 + \left(\frac{n}{R}+q\right)^2 + m_\chi^2 + \frac{g}{2}\phi^2}{p^2 + \frac{n^2}{R^2}} \right)$$

Moreover : $\lambda \equiv \frac{\widehat{\lambda}}{2\pi R}$; $g \equiv \frac{\widehat{g}}{2\pi R}$; $\widehat{\Phi} = \frac{1}{\sqrt{2\pi R}}\phi$

$$\boxed{V_{1l}^{(4)}(\phi) = 2\pi R\, V_{1l}^{(5)}(\widehat{\Phi})}$$

... Performing the calculation ...

UV-sensitivity

$$V_{1l}^{(4)}(\phi) = \frac{5m^2 + 3q^2}{180\pi^2} R\Lambda^3 - \frac{35m^4 + 14m^2q^2 + 3q^4}{840\pi^2} R\Lambda + \frac{m^5 R}{60\pi}$$
$$- \sum_{k=1}^{\infty} \frac{e^{-2\pi k m R}(2\pi k m R(2\pi k m R + 3) + 3)\cos(2\pi k q)}{64\pi^6 k^5 R^4}$$

New q-dependent UV-sensitive terms :

- Not canceled by SUSY ! $\propto \boxed{(q_b^2 - q_f^2)}\ \Lambda^3$
- Topological origin (non-trivial boundaries)
- Absent for $q = 0$ and for $q_b = q_f$. But remember : (1) $q \neq 0$ in multiply connected space ; (2) $q_b \neq q_f$ for SUSY breaking

Dark Dimension Scenario (Montero, Vafa, Valenzuela)

Our universe is in a unique corner of the QG landscape, where only two kinds of light tower states are expected: string excitations or KK states

Basic relation $\mu_{tow} \sim \left|\frac{\Lambda_{cc}}{M_P^4}\right|^{\alpha} M_P$

Bounds, pheno, calculations $\Rightarrow$ $\mu_{tow} \sim \Lambda_{cc}^{1/4} \sim$ meV (neutrino scale)

Authors observe: "above neutrino scale physics described with Effective Field Theory", the string scenario is "ruled out experimentally"

They conclude: light tower must be KK, $\mu_{tow} = m_{\rm KK}$

Basic relation becomes $\Lambda_{cc} \sim m_{\rm KK}^4$

Phenomenology $\Rightarrow$ only one extra dimension of size $\sim \mu$m

"Technicalities" in a $(4+1)$ D theory

Consider a $(4+1)$D theory coupled to gravity $\mathcal{S}^{(4+1)} = \mathcal{S}^{(4+1)}_{\text{grav}} + \mathcal{S}^{(4+1)}_{\text{matter}}$

$$\mathcal{S}^{(4+1)}_{\text{grav}} = \frac{1}{2\hat{\kappa}^2}\int d^4xdz\sqrt{\hat{g}}\left(\hat{\mathcal{R}} - 2\hat{\Lambda}_{cc}\right) \;\; ; \;\; \hat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi}g_{\mu\nu} - e^{2\beta\phi}A_\mu A_\nu & e^{2\beta\phi}A_\mu \\ e^{2\beta\phi}A_\nu & -e^{2\beta\phi} \end{pmatrix}$$

Integrating over the compact dimension $\Rightarrow$ 4D grav. Action

$$\mathcal{S}^{(4)}_{\text{grav}} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\times\left[\mathcal{R} - 2e^{2\alpha\phi}\hat{\Lambda}_{cc} + 2\alpha\Box\phi + \frac{(\partial\phi)^2}{2} - \frac{e^{-6\alpha\phi}}{4}F^2\right]$$

$4D$ constant κ related to $5D$ $\hat{\kappa}$: $\kappa^2 = \frac{\hat{\kappa}^2}{2\pi R}$. α and β related through $2\alpha+\beta=0$. Canonical radion kinetic term $\Rightarrow \alpha = \frac{1}{\sqrt{12}}$.

Example of matter: $5D$ scalar field $\hat{\Phi}$: $\mathcal{S}^{(4+1)}_{\text{matter}} = \int d^4xdz\,\sqrt{\hat{g}}\left(\hat{g}^{MN}\partial_M\hat{\Phi}^*\partial_N\hat{\Phi} - m^2|\hat{\Phi}|^2\right)$, with non-trivial boundary condition $\hat{\Phi}(x, z+2\pi R) = e^{2\pi iq}\,\hat{\Phi}(x,z)$. Corresponding $4D$ action

$$\mathcal{S}^{(4)}_{\text{matter}} = \int d^4x\sqrt{-g}\sum_n\left[|D\varphi_n|^2 - \left(e^{2\alpha\phi}m^2 + e^{6\alpha\phi}\frac{(n+q)^2}{R^2}\right)|\varphi_n|^2\right]$$

where $D_\mu \equiv \partial_\mu - i\left(\frac{n+q}{R}\right)A_\mu$ and $\varphi_n(x)$ are the KK modes of $\hat{\Phi}(x,z)$.

Radion dependent KK masses

From the 4D matter action (ϕ radion field)

$$\mathcal{S}^{(4)}_{\rm matter} = \int d^4x\sqrt{-g}\,\sum_n \left[|D\varphi_n|^2 - \left(e^{2\alpha\phi}m^2 + e^{6\alpha\phi}\frac{(n+q)^2}{R^2}\right)|\varphi_n|^2\right]$$

with background $g^0_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\phi =$ Constant we get the ϕ-dependent radius $R_\phi \equiv R\,e^{-3\alpha\phi}$ and mass $m^2_\phi \equiv m^2 e^{2\alpha\phi}$

The KK masses are

$$m_n^2 = m^2 e^{2\alpha\phi} + \frac{(n+q)^2}{R^2}e^{6\alpha\phi} = m_\phi^2 + \frac{(n+q)^2}{R_\phi^2}$$

From which $m^2_{\rm KK} = e^{6\alpha\phi}R^{-2} \equiv R_\phi^{-2}$

Vacuum Energy in the $(4+1)$D theory

One-loop contribution to the vacuum energy ρ_4 from a single tower

$$\rho_4 \sim (-1)^{\delta_{if}} \sum_n \int \frac{d^4 p}{(2\pi)^4} \log(p^2 + \frac{(n+q)^2}{R_\phi^2} + m_\phi^2)$$

Usually: sum over n up to infinity ; integral over p with a cutoff $\Lambda_{cut}^{(4)}$

To get the UV-insensitive result $\rho_4 \sim R_\phi^{-4} = m_{\rm KK}^4$ it is

crucial to send n to infinity while $\Lambda_{cut}^{(4)}$ is kept fixed

As for the $V_{1l}^{\rm Higgs}(\phi)$ previously considered, this mistreats the asymptotics of the 5D loop momentum of the original 5D theory

Sending $n \to \infty$ while keeping $\Lambda_{cut}^{(4)}$ fixed means that in the loop integral we are (improperly) considering first the asymptotics of the fifth component of the momentum and only later those of the other four components

Vacuum Energy in the $(4+1)$D theory

The asymptotics of all the components of $\hat{p}$ must be treated on an equal footing. This can be realized considering a 5D spherical cutoff Λ

Spherical cutoff Λ^2 for the 5D loop momentum $\hat{p}^2 \equiv (p^2 + \frac{n^2}{R_\phi^2})e^{-2\alpha\phi}$

$$\hat{p}^2 \leq \Lambda^2 \quad \Rightarrow \quad p^2 + \frac{n^2}{R_\phi^2} \leq \Lambda^2 \, e^{2\alpha\phi} \quad \left(= m_{\rm KK}^{1/3} R^{1/3} \Lambda \equiv \Lambda_\phi^2\right)$$

Λ_ϕ Cutoff for the rescaled momenta

Performing sum and integral in

$$\rho_4 \sim \sum_n \int \frac{d^4 p}{(2\pi)^4} \log(p^2 + \frac{(n+q)^2}{R_\phi^2} + m_\phi^2) \, .$$

with this physical cutoff $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

One-loop vacuum energy

$\Rightarrow$ Contribution to ρ_4^{1l} coming from a bulk field

$$\rho_4^{1l} = \frac{5\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} - 2}{300\pi^2} e^{2\alpha\phi} R\Lambda^5 + \frac{5m^2 + 3\frac{q^2}{R^2}e^{4\alpha\phi}}{180\pi^2} e^{2\alpha\phi} R\Lambda^3 - \frac{35m^4 + 14m^2\frac{q^2}{R^2}e^{4\alpha\phi} + 3\frac{q^4}{R^4}e^{8\alpha\phi}}{840\pi^2} e^{2\alpha\phi} R\Lambda + \frac{m^5}{60\pi} e^{2\alpha\phi} R + \frac{3\log\frac{\Lambda^2 e^{2\alpha\phi}}{\mu^2} + 2}{2880\pi^2 R^4} e^{10\alpha\phi} R\Lambda + R_4 + \mathcal{O}(\Lambda^{-1}) = 2\pi R\, e^{2\alpha\phi} \rho_5^{1l}$$

$$R_4 = -\frac{x^2 \mathrm{Li}_3\left(re^{-x}\right) + 3x\mathrm{Li}_4\left(re^{-x}\right) + 3\mathrm{Li}_5\left(re^{-x}\right) + 6\zeta(5)}{128\pi^6 R^4} e^{12\alpha\phi} + h.c.$$

$$r \equiv e^{2\pi i q} \quad , \quad x \equiv 2\pi e^{-2\alpha\phi} R\sqrt{m^2} \implies R_4 \propto \frac{e^{12\alpha\phi}}{R^4} = m_{KK}^4$$

As for $V_{1\,\mathrm{loop}}^{\mathrm{Higgs}}(\phi)$, q-depentent divergences usually missed

Look at the divergent terms in ρ_4^{1l}

For convenience take the "most divergent" term

- SUSY: $\rho_4^{1l} \sim (q_b^2 - q_f^2)\, e^{6\alpha\phi} R^{-1}\Lambda^3 = (q_b^2 - q_f^2) m_{KK}^2 R\Lambda^3$
- NON-SUSY: $\rho_4^{1l} \sim e^{2\alpha\phi} R\Lambda^5 = m_{KK}^{2/3} \left(R^{\frac{1}{3}}\Lambda\right)^5$

ρ_4^{1l} has divergences that do not disappear even in a SUSY theory

Even in the swampland scenario, that requires the light tower limit $\phi \to -\infty$, no term can overthrow these contributions
No light tower regime where $\rho_4^{1l} \sim m_{KK}^4$

What is the lesson?

In a $(4+1)$ D EFT quantum fluctuations "heavily" dress ρ_4

No automatic result $\rho_4 = \Lambda_{cc} \sim m_{KK}^4$ (**as often claimed**)

To reach $\rho_4 = \Lambda_{cc} \sim m_{KK}^4$ **fine-tuning is needed**

$\Rightarrow$ even if we believe the "swampland" conjectured

$$\rho_4 = \Lambda_{cc} \sim m_{KK}^4$$

there is an issue of **matching** between this finite result for ρ_4

and the **EFT result**

unless we resort to this fine tuning

Reactions

Soon after the appearance of our arXiv: 2308.16548, where we presented the analysis of the DD scenario ...

... we got several reactions ...

... some people happy ... others unhappy ...

Criticisms

L. A. Anchordoqui, I. Antoniadis, D. Lust, S. Lust arXiv:2309.09330

soon after our arXiv: 2308.16548

What they do?

Replacement

They present our results making the "replacement"

$$\rho_4^{1/l} \longrightarrow \Lambda_{\rm cc}$$

	Our result		How they write our result
SUSY	$\rho_4^{1/l} \sim m_{\rm KK}^2 R \Lambda^3$	$\Rightarrow$	$\Lambda_{cc} \sim m_{\rm KK}^2 R \Lambda^3$
No SUSY	$\rho_4^{1/l} \sim m_{\rm KK}^{2/3} R^{5/3} \Lambda^5$	$\Rightarrow$	$\Lambda_{cc} \sim m_{\rm KK}^{2/3} R^{5/3} \Lambda^5$

Authors take $\rho_4^{1/l}$ to coincide with the physical vacuum energy

- De facto they assume no need for fine tuning
- Opposite to what we do
- Not in itself a problem: theoretically legitimate in principle
- ... however ... **is it not possible in the present case**

This replacement has a problem

$$\Lambda_{cc} \sim m_{\rm KK}^2 R\Lambda^3 \qquad \text{and} \qquad \Lambda_{cc} \sim m_{\rm KK}^{2/3} R^{5/3}\Lambda^5 \qquad (*)$$

rather than

$$\rho_4^{1/} \sim m_{\rm KK}^2 R\Lambda^3 \qquad \text{and} \qquad \rho_4^{1/} \sim m_{\rm KK}^{2/3} R^{5/3}\Lambda^5$$

has a major consequence

> The cutoff Λ is fully determined

In fact, inserting $\Lambda_{\rm cc} \sim m_{\rm KK}^4$ in both $(*)$

$$\Rightarrow \quad R\Lambda^3 = R_\phi \Lambda_\phi^3 \sim m_{\rm KK}^2 \quad \Rightarrow \quad \Lambda_\phi^3 \sim m_{\rm KK}^3 \quad (R_\phi = m_{\rm KK}^{-1})$$

and since $\Lambda_\phi = \Lambda_{\rm SM}$ (see next slide)

$\Rightarrow \quad \Lambda_{\rm SM} \sim m_{\rm KK} \sim {\rm meV}$ **obviously not possible**

The SM lives on a brane

Standard Model cutoff $\Lambda_{\rm SM}$

Relation between the cutoff Λ of the 5D theory and the 4D cutoff $\Lambda_{\rm SM}$ of the Standard Model

$(4+1)$D theory, with compact space dimension in the shape of a circle of radius R, defined by

$$\mathcal{S} = \mathcal{S}^{(5)}_{\rm grav} + \mathcal{S}^{(5)}_{\rm mat} \quad ; \quad \mathcal{S}^{(5)}_{\rm grav} = \frac{1}{2\hat{\kappa}^2}\int d^4x dz \sqrt{\hat{g}}\,\left(\hat{\mathcal{R}} - 2\hat{\Lambda}_{cc}\right)$$

Simple example for matter action

$$\mathcal{S}^{(5)}_{\rm mat} = \int d^4x dz\,\sqrt{\hat{g}}\left(\hat{g}^{MN}\partial_M\hat{\Phi}^*\partial_N\hat{\Phi} - m^2|\hat{\Phi}|^2\right)$$

with $\hat{\Phi}$ a $(4+1)$D scalar field that obeys the boundary condition $\hat{\Phi}(x, z+2\pi R) = \hat{\Phi}(x,z)$.

$(4+1)$D metric parametrized as

$$\hat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi}g_{\mu\nu} - e^{2\beta\phi}A_\mu A_\nu & e^{2\beta\phi}A_\mu \\ e^{2\beta\phi}A_\nu & -e^{2\beta\phi} \end{pmatrix}$$

A_μ is the graviphoton and ϕ the radion field. Considering only zero modes for $\hat{g}_{MN}$, i.e. $g_{\mu\nu}(x)$, $A_\mu(x)$ and $\phi(x)$ only depend on x. Integrating over z, the 4D gravitational action $\mathcal{S}^{(4)}_{\rm grav}$ is

$$\mathcal{S}^{(4)}_{\rm grav} = \frac{1}{2\kappa^2}\int {\rm d}^4x\,\sqrt{-g}\left[\mathcal{R} - 2e^{2\alpha\phi}\hat{\Lambda}_{cc} + 2\alpha\Box\phi + \frac{(\partial\phi)^2}{2} - \frac{e^{-6\alpha\phi}}{4}F^2\right]$$

4D constant $\kappa = M_P^2$ related to the $(4+1)$D $\hat{\kappa} = \hat{M}_P^3$ through the relation $\kappa^2 = \hat{\kappa}^2/(2\pi R)$

The fields ϕ and A_μ in the above equation are dimensionless (dimensionful fields are obtained through the redefinition $\phi \to \phi/(\sqrt{2}\kappa)$, $A_\mu \to A_\mu/(\sqrt{2}\kappa)$), and we used $2\alpha + \beta = 0$. The canonical kinetic term for the radion field is obtained taking $\alpha = 1/\sqrt{12}$.
Considering the Fourier decomposition of $\hat{\Phi}(x, z)$

$$\mathcal{S}^{(4)}_{\rm mat} = \int d^4x\sqrt{-g}\sum_n \left[|D\varphi_n|^2 - \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} m^2 + e^{\sqrt{6}\frac{\phi}{M_P}} \frac{n^2}{R^2} \right) |\varphi_n|^2 \right]$$

where $D_\mu \equiv \partial_\mu - i\,(n/R)\,A_\mu$, and $\varphi_n(x)$ are the KK modes of $\hat{\Phi}(x, z)$. Taking a constant background radion field ϕ, and the trivial background for A_μ, the metric becomes

$$\hat{g}^0_{MN} = \begin{pmatrix} e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \eta_{\mu\nu} & 0 \\ 0 & -e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \end{pmatrix}.$$

From "red above" the ϕ-dependent radius $R_\phi \equiv R\, e^{-\sqrt{\frac{3}{2}}\frac{\phi}{M_P}}$ is defined. When computing radiative corrections, the $(4+1)$D momentum $\hat{p} \equiv (p, n/R)$ is cut as

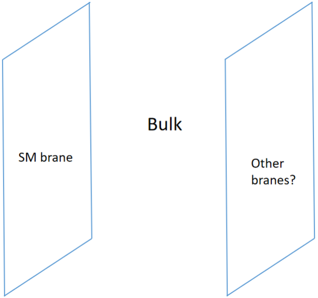
$$\widehat{p}^{\,2} = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \left(p^2 + \frac{n^2}{R^2_\phi} \right) \leq \Lambda^2 \quad \Rightarrow \quad p^2 + \frac{n^2}{R^2_\phi} \leq \Lambda^2_\phi$$

$$p^2 + \frac{n^2}{R_\phi^2} \leq \Lambda_\phi^2$$

where
$$\Lambda_\phi \equiv \Lambda \, e^{\frac{1}{\sqrt{6}} \frac{\phi}{M_P}}$$

Since p^2 is the modulus of the four-momentum on the brane, this equation tells us that Λ_ϕ is the cutoff $\Lambda_{\rm SM}$ of the SM on the brane:

$$\Lambda_{\rm SM} = \Lambda_\phi = \Lambda \, e^{\frac{1}{\sqrt{6}} \frac{\phi}{M_P}}$$



The replacement $\rho_4^{1/l} \to \Lambda_{cc}$

$\Rightarrow$ $\Lambda_{SM} \sim m_{KK} \sim \mathrm{meV}$ **no way!**

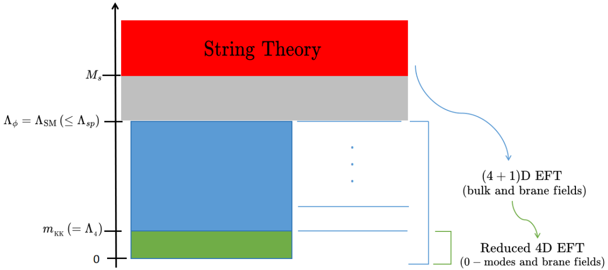
$\Rightarrow$ $\Lambda_{cc} \sim m_{KK}^2 R \Lambda^3$ and $\Lambda_{cc} \sim m_{KK}^{2/3} R^{5/3} \Lambda^5$ can be

neither written nor used to derive any relation and draw any conclusion

... but the criticisms are based on this replacement ...

On top of that

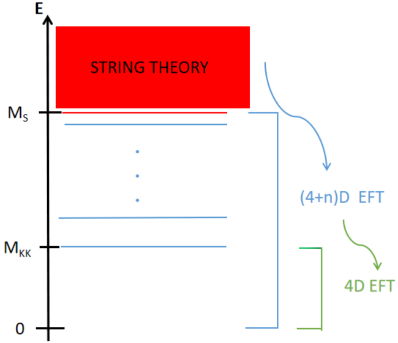
$\Lambda_\phi \sim m_{KK}$ $\Rightarrow$ no space for the $(4+1)D$ theory of the DD scenario



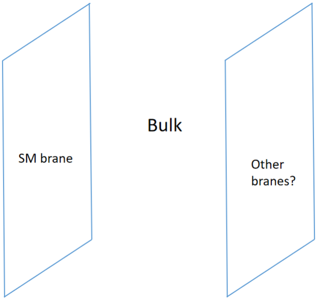
From String Theory to 4D Effective Field Theory (SM)

String theory $\Rightarrow$ EFT takes over at M_s

from M_s down to the "physical scales" : EFT heavy artilery



One last point



SM contribution to ρ_4 ... **at least** TeV^4

Summary

- Both SM and bulk give UV-sensitive contributions to ρ_4
- Below M_s, EFT takes over $\Rightarrow$ bona fide EFT calculations
- Do not mistreat the asymptotics of the loop momenta
- Correct treatment of the loop momenta asymptotics unveils the presence of UV-sensitive terms, missed in usual calculations
- No automatic matching between Swampland conjectured result for ρ^4 and EFT limit
- The UV-sensitive terms from the bulk come from non-trivial boundaries

BACK-UP SLIDES

DD relation $m_{\rm KK} \sim \left|\frac{\Lambda_{cc}}{M_P^4}\right|^{\alpha} M_P$

Comparing the DD scenario relation that I rewrite here as

$$m_{\rm KK} \sim M_P^{1-4\alpha}\,\Lambda_{cc}^{\alpha}\,, \tag{5}$$

with "our equations" (these are not our equations) also coveniently rewritten as

$$m_{\rm KK} \sim (R\Lambda^3)^{-1/2}\Lambda_{cc}^{1/2} \;\;;\;\; m_{\rm KK} \sim (R\Lambda^3)^{-5/2}\Lambda_{cc}^{3/2}\,, \tag{6}$$

the authors affirm that **"we claim"** the values $\alpha = 1/2$ and $\alpha = 3/2$

Such a claim has nothing to do with us. It results from their not allowed replacement

Higuchi bound

According to the Higuchi bound, the mass $m_{s=2}$ of a spin-2 particle in dS space and the cosmological constant satisfy the relation

$$m^2_{s=2} \geq \frac{2}{3}\frac{\Lambda_{cc}}{M_p^2} \quad \text{in the present case} \Rightarrow \Lambda_{cc} \leq \left(\frac{3}{2}M_P^2\right) m^2_{\text{KK}}$$

Comparing this latter equation with "our relations":

$$\Lambda_{cc} \sim m^2_{\text{KK}} R\Lambda^3 \qquad\qquad \Lambda_{cc} \sim m^{2/3}_{\text{KK}} R^{5/3}\Lambda^5$$

they get $\quad R\Lambda^3 \lesssim M_P^2 \qquad\qquad R\Lambda^3 \lesssim 10^{-48} M_P^2 \quad (\Lambda_{cc} \sim 10^{-120} M_P^4 \text{ used})$

- First bound: "**not too strong**" ($\Lambda \leq \hat{M}_P$, no constraint at all)
- Second bound: "**too low cutoff** "

Actually bound 2 can be rewritten as $\Lambda_\phi \lesssim 10^5 m_{\text{KK}} \sim 10^2$ eV

... Too low ... yes ... (not our fault) ... but see below ...

Higuchi bound and "Natural" choice" for R and Λ

- Authors note that "our results" depend on R and Λ
- and claim that a "natural choice" for R and Λ willmake "our results" eventually consistent with the DD scenario

"Natural" choice for R: $R = m_{\rm KK}^{-1}$

- Radion dependence missed. Correct relation: $R_\phi = Re^{-3\alpha\phi} = m_{\rm KK}^{-1}$

"Natural" choices for Λ: UV-IR mixing

- (1): $\Lambda = \Lambda_{\rm sp} \sim m_{\rm KK}^{1/3} M_P^{2/3}$, Higuchi explicitly violated

- $\Lambda_{\rm sp}$ cut on p: $\Lambda_\phi = \Lambda_{\rm sp}(\Lambda = \hat{M}_P)$ correct identification

- (2): $\Lambda = m_{\rm KK}$, everything ok, "correct choice"

- $\Lambda_\phi = m_{\rm KK}$ and not $\Lambda = m_{\rm KK}$ is what needed to have $m_{\rm KK}^4$
- Absurd again: $\Lambda_{\rm SM} \lesssim$ meV and no space for $(4+1)D$ theory

Finite temperature

Profound difference between the sums in finite T and KK

$$\rho^{1l}\ ,\ F^{1l} \sim \frac{1}{2}\sum_n \int d^d p\, \log(p^2 + m^2 + f_n)\,.$$

KK theories (ρ^{1l})

- n and p intertwined, components of $\hat{p}$
- p and n cut together: no hierarchy when including asymptotics

Finite temperature (F^{1l})

Finite for SUSY theory

- n and p not intertwined
- $\int d^3 p$: trace over quantum fluctuations
- $\sum_n$: statistical average (mixed states)
- Infinite sum: ergodicity! MUST DO: no q dependent "divergences"

$$F_T^{1l} = \frac{T}{2}\sum_{n=-\infty}^{\infty}\int d^3 p\, \log(p^2 + m^2 + f_n) - \frac{1}{2}\int d^4 p\, \log(p^2 + m^2) \sim T^4 = \text{finite}\,.$$

Casimir energy

By definition

$$\mathcal{E}_C = \rho_R - \rho_\infty$$

$$\mathcal{E}_C^{1l} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int d^4p\,\log(p^2+m^2+f_n) - \frac{1}{2}\int d^4p\,\log(p^2+m^2).$$

- Infinite sum in ρ_R (literature): $\sim$ finite T

- ρ_R and ρ_∞ have the same divergences
- $\mathcal{E}_C \sim m_{\text{KK}}^4$

-No hierarchy when including asymptotics in ρ_R (us): q-divergences

- ρ_R and ρ_∞ do not have same divergences when non-trivial boundary charges are present
- $\rho_R - \rho_\infty$ subtraction not sufficient

Calculation of the one-loop potential ($i = b, f$)

$$V_{1l}^{i}(\phi) = \frac{1}{2}\sum_{n=-L}^{L}\int^{\Lambda}\frac{d^4p}{(2\pi)^4}\log\frac{p^2+M^2+(\frac{n}{R}+q_i)^2}{p^2+\frac{n^2}{R^2}}$$

$$= \sum_{n=-L}^{L}\frac{1}{64\pi^2}\left[\Lambda^4\log\frac{\Lambda^2+M^2+\left(\frac{n}{R}+q_i\right)^2}{\Lambda^2+\frac{n^2}{R^2}} + \Lambda^2\left(M^2+\left(\frac{n}{R}+q_i\right)^2-\frac{n^2}{R^2}\right)\right.$$

$$\left. + \left(M^2+\left(\frac{n}{R}+q_i\right)^2\right)^2\log\frac{M^2+\left(\frac{n}{R}+q_i\right)^2}{\Lambda^2+M^2+\left(\frac{n}{R}+q_i\right)^2} - \frac{n^4}{R^4}\log\frac{\frac{n^2}{R^2}}{\Lambda^2+\frac{n^2}{R^2}}\right] \equiv \sum_{n=-L}^{L}F(n). \quad (7)$$

Euler-McLaurin (EML) formula

$$V_{1l}^{i}(\phi) = \int_{-L}^{L}dx\,F(x) + \frac{F(L)+F(-L)}{2} + \sum_{k=1}^{r}\frac{B_{2k}}{(2k)!}\left(F^{(2k-1)}(L)-F^{(2k-1)}(-L)\right) + R_{2r}, \quad (8)$$

with r is an integer, B_n the Bernoulli numbers, and the rest R_{2r} is

$$R_{2r} = \sum_{k=r+1}^{\infty}\frac{B_{2k}}{(2k)!}\left(F^{(2k-1)}(L)-F^{(2k-1)}(-L)\right) = \frac{(-1)^{2r+1}}{(2r)!}\int_{-L}^{L}dx\,F^{(2r)}(x)B_{2r}(x-[x]), \quad (9)$$

$B_n(x)$ Bernoulli polynomials, $[x]$ integer part of x.

- If in (7), (8) and (9) we send $L \to \infty$ while keeping Λ fixed, we get for $V^i_{1l}(\phi)$ the usual UV-insensitive (finite) result.

- To properly take into account the asymptotics of the loop momenta $p^{(5)} = (p_1, p_2, p_3, p_4, n/R)$, we include them in (7) keeping

$$\frac{L}{R\Lambda} \quad \text{finite} \quad \text{when} \quad L, \Lambda \to \infty \,. \tag{10}$$

- From the physical meaning of the UV cuts: only values of M and q_i that fulfill the conditions

$$M^2, q_i^2 \ll \Lambda^2, L^2/R^2 \,. \tag{11}$$

- The conditions (10) and (11) are easily implemented in our calculations if we write (ξ dimensionless finite number).

$$L = \xi R \Lambda \,, \tag{12}$$

and expand each term in (8) for $M^2/\Lambda^2, q_i^2/\Lambda^2 \ll 1$. We get

$$V_{1l}(\phi) = \frac{2M^2\tan^{-1}\xi + \xi\left(\xi^2\log\frac{\xi^2}{\xi^2+1}+1\right)\left(M^2+3q_i^2\right)}{48\pi^2}\,R\Lambda^3$$
$$+\frac{\xi^2\left(M^2+3q_i^2\right)+\xi^2\left(\xi^2+1\right)\left(M^2+3q_i^2\right)\log\frac{\xi^2}{\xi^2+1}+M^2+q_i^2}{32\pi^2\left(\xi^2+1\right)}\,\Lambda^2$$
$$+\frac{\xi\,M^2\left(6q_i^2R^2+1\right)\left(\xi^2+1\right)+\xi\,q_i^2\left(q_i^2R^2+1\right)\left(3\xi^2+5\right)}{96\pi^2\left(\xi^2+1\right)^2}\,\frac{\Lambda}{R}$$
$$+\frac{\xi\log\frac{\xi^2}{\xi^2+1}\left(3R^2\left(M^2+q_i^2\right)^2+M^2+3q_i^2\right)-3M^4R^2\tan^{-1}\xi}{96\pi^2}\,\frac{\Lambda}{R}$$
$$+\frac{3\left(\xi^2+1\right)^2M^4+6\left(\xi^4+4\xi^2+3\right)M^2q_i^2+\left(3\xi^4+6\xi^2+11\right)q_i^4}{192\pi^2\left(\xi^2+1\right)^3}$$
$$+\frac{16\pi M^5R+15\log\frac{\xi^2}{\xi^2+1}\left(M^2+q_i^2\right)^2}{960\pi^2}+R_2+\mathcal{O}\left(\Lambda^{-1}\right). \tag{13}$$

To compare (13) with the usual calculations, we take limit $\xi\to\infty$, with Λ kept finite

$$V_{1l}^i(\phi)\sim\frac{R\Lambda^3M^2}{48\pi}-\frac{R\Lambda M^4}{64\pi}+\frac{RM^5}{60\pi}+\widetilde{R}_2+\mathcal{O}\left(\xi^{-1}\right). \tag{14}$$

with

$$\widetilde{R}_2\equiv\lim_{\xi\to\infty}R_2=\frac{3\zeta(5)}{64\pi^6R^4}-\frac{1}{128\pi^6R^4}\left[x^2\mathrm{Li}_3\left(r_ie^{-x}\right)+3x\mathrm{Li}_4\left(r_ie^{-x}\right)+3\mathrm{Li}_5\left(r_ie^{-x}\right)+h.c.\right].$$

Standard Model cutoff

Relation between the cutoff Λ of the $(4+1)$D theory and the 4D cutoff $\Lambda_{\rm SM}$ of the Standard Model. $(4+1)$D theory, with compact space dimension in the shape of a circle of radius R, defined by

$$\mathcal{S} = \mathcal{S}_{\rm grav} + \mathcal{S}_{\rm mat} \tag{15}$$

$$\mathcal{S}_{\rm grav} = \frac{1}{2\hat{\kappa}^2} \int d^4x dz \sqrt{\hat{g}} \left(\hat{\mathcal{R}} - 2\hat{\Lambda}_{cc} \right) \tag{16}$$

is the $(4+1)$D Einstein-Hilbert action and as an example for the matter action we take

$$\mathcal{S}_{\rm mat} = \int d^4xdz \ \sqrt{\hat{g}} \left(\hat{g}^{MN} \partial_M \hat{\Phi}^* \partial_N \hat{\Phi} - m^2 |\hat{\Phi}|^2 \right) , \tag{17}$$

with $\hat{\Phi}$ a $(4+1)$D scalar field that obeys the boundary condition $\hat{\Phi}(x, z + 2\pi R) = \hat{\Phi}(x, z)$. We indicate with x the 4D coordinates and with z the coordinate along the compact dimension. Using the signature $(+, -, -, -, -)$, the $(4+1)$D metric is parametrized as

$$\hat{g}_{MN} = \begin{pmatrix} e^{2\alpha\phi} g_{\mu\nu} - e^{2\beta\phi} A_\mu A_\nu & e^{2\beta\phi} A_\mu \\ e^{2\beta\phi} A_\nu & -e^{2\beta\phi} \end{pmatrix} \tag{18}$$

A_μ is the graviphoton and ϕ the radion field. Considering only zero modes for $\hat{g}_{MN}$, i.e. $g_{\mu\nu}(x)$, $A_\mu(x)$ and $\phi(x)$ only depend on x. Integrating over z, for the 4D gravitational action $\mathcal{S}^{(4)}_{\rm grav}$ we get

$$\mathcal{S}^{(4)}_{\rm grav} = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g} \left[\mathcal{R} - 2e^{2\alpha\phi} \hat{\Lambda}_{cc} + 2\alpha \Box\phi + \frac{(\partial\phi)^2}{2} - \frac{e^{-6\alpha\phi}}{4} F^2 \right] , \tag{19}$$

where the 4D constant $\kappa = M_P^2$ is related to the $(4+1)$D $\hat{\kappa} = \hat{M}_P^3$ through the relation $\kappa^2 = \hat{\kappa}^2/(2\pi R)$.

The fields ϕ and A_μ in the above equation are dimensionless (dimensionful fields are obtained through the redefinition $\phi \to \phi/(\sqrt{2}\kappa)$, $A_\mu \to A_\mu/(\sqrt{2}\kappa)$), and we used $2\alpha + \beta = 0$. The canonical kinetic term in (19) for the radion field is obtained taking $\alpha = 1/\sqrt{12}$.
Considering the Fourier decomposition of $\hat{\Phi}(x, z)$, for the $4D$ matter action (17) we have

$$\mathcal{S}^{(4)}_{\rm mat} = \int d^4x \sqrt{-g} \sum_n \left[|D\varphi_n|^2 - \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} m^2 + e^{\sqrt{6}\frac{\phi}{M_P}} \frac{n^2}{R^2} \right) |\varphi_n|^2 \right], \tag{20}$$

where $D_\mu \equiv \partial_\mu - i\,(n/R)\,A_\mu$, and $\varphi_n(x)$ are the KK modes of $\hat{\Phi}(x, z)$. Taking a constant background radion field ϕ, and the trivial background for A_μ, the metric (18) becomes

$$\hat{g}^0_{MN} = \begin{pmatrix} e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \eta_{\mu\nu} & 0 \\ 0 & -e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \end{pmatrix}. \tag{21}$$

From (20) we define the ϕ-dependent radius $R_\phi \equiv R\, e^{-\sqrt{\frac{3}{2}}\frac{\phi}{M_P}}$. With such a definition, we immediately see that, when computing radiative corrections, the $(4+1)$D momentum $\hat{p} \equiv (p, n/R)$ is cut as

$$\hat{p}^2 = e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \left(p^2 + \frac{n^2}{R^2_\phi} \right) \leq \Lambda^2. \tag{22}$$

Alternative calculations: Infinite sum + Smooth Cut

Typical argument: cut on sum → spurious "divergences" ... But ...

$$V_{1l}(\phi) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \log \left(\frac{p^2 + m^2 + \left(\frac{n}{R} + q\right)^2}{p^2 + \frac{n^2}{R^2}} \right) e^{-\frac{p^2 + \frac{n^2}{R^2}}{\Lambda^2}}$$

Same result is found

UV-sensitive terms are NOT due to the sharp cut in the sum!
They come from a careful treatment of $\hat{p}$ asymptotics

So ... why do "Proper time", "Thick brane" and "Pauli-Villars"

give UV-insensitive results ?

Secret liaison between proper time , thick brane & PV

Thick brane: $\sum_{n=-\infty}^{\infty} \int^{(\Lambda)} \frac{d^4p}{(2\pi)^4} \frac{e^{-\frac{\left(\frac{n}{R}+q\right)^2}{\Lambda^2}}}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Delgado, von Gersdor, John, Quiros

Pauli-Villars: $\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{(\Lambda R)^4}{(\Lambda R)^4+p^2+(\frac{n}{R}+q)^2} \frac{1}{p^2+m^2+\left(\frac{n}{R}+q\right)^2}$ Contino, Pilo

Proper Time: Antoniadis, Benakli

$$V_{1l}^{(4)}(\phi) = -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s} e^{-s\left(p^2+m^2+\left(\frac{n}{R}+q\right)^2\right)}$$

$$= -\sum_{n=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \Gamma\left(0, \frac{p^2+m^2+\left(\frac{n}{R}+q\right)^2}{\Lambda^2}\right)$$

Smooth cut function of $\frac{n+q}{R}$: artificial re-absorption of q

Equivalent to introduce a hierarchy between (p_1, p_2, p_3, p_4) and p_5

$\Rightarrow$ **Again : artificial wash-out of UV-sensitive terms**

Global picture: EFTs with compact dimensions

Recent argument: EFT not applicable as it requires large hierarchy between last included and first excluded KK mode Burgess, Quevedo

$\rightarrow$ Rely on usual interpretation of KK modes as massive $4D$ states

- Start: $\mathcal{S}_\Lambda^{(5)}$ w/ "Wilsonian" mode expansion $\hat{p} \in [0, \Lambda]$
- Integrating out modes in $[k, \Lambda] \rightarrow \mathcal{S}_k^{(5)}$ k Wilsonian running scale

Due to $p_5 = n/R$ discreteness, p_5 eigenmodes contribution is stepwise

- For $k < 1/R$ no p_5 eigenmodes anymore: RG evolution becomes effectively of $4D$ type

It is **only in this sense** that the $4D$ theory emerges from the $5D$ one: **by no means it has an** *infinite* **tower of states**

Cutting tower modes in Swampland program

Cut in tower typical in Swampland: Species scale Λ_{sp} (e.g. emergence proposal)

Grimm, Palti, Valenzuela

Species scale $\Lambda_{sp} = \left(M_p^2 m_{_{KK}}\right)^{\frac{1}{3}}$: dominant depend on $|\phi|/M_p \lessgtr\sim 100$

- SUSY: $\rho_4 \sim (q_b^2 - q_f^2) M_p^2 m_{_{KK}}^3 \longrightarrow \rho_4 \sim -(q_b^2 - q_f^2) m^2 M_p^{2/3} m_{_{KK}}^{7/3}$
- Non-SUSY: $\rho_4 \sim M_p^{10/3} m_{_{KK}}^{7/3} \longrightarrow \rho_4 \sim m^5 m_{_{KK}}^{2/3}$