QCD-Collapsed Domain Walls

QCD Phase Transition and Gravitational Wave Spectroscopy

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Based on work with Yang Bai and Ting-Kuo Chen (2306.17160))

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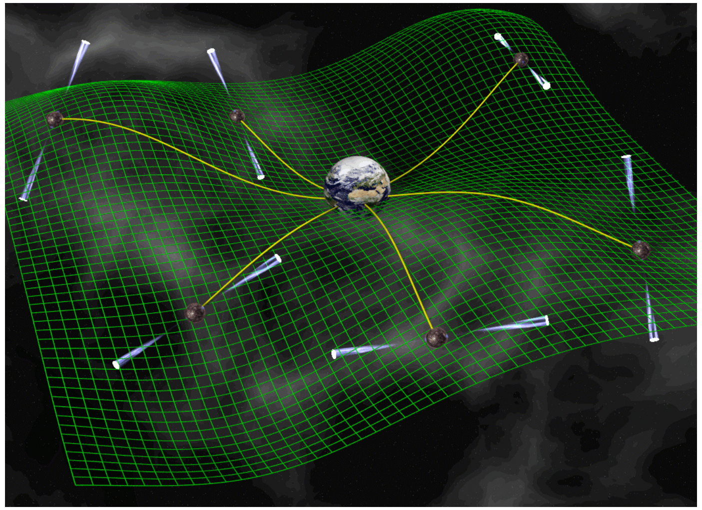
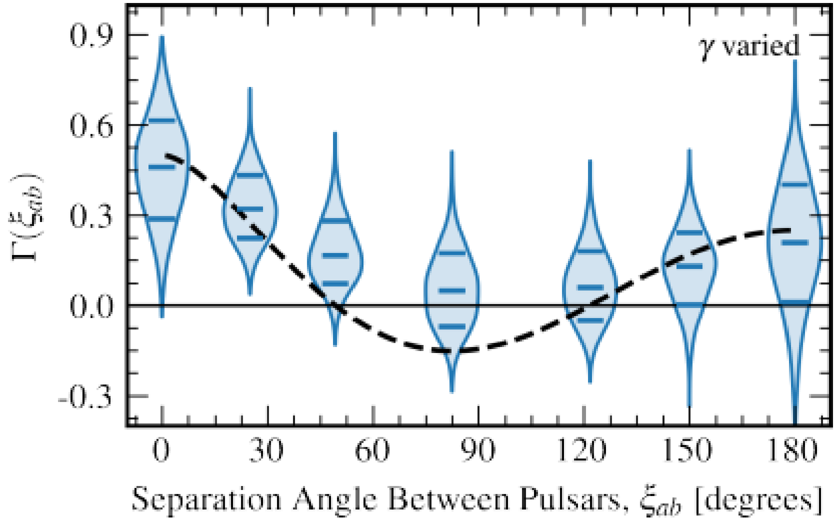
Pulsar Timing Array

- **Pulsar Timing Array** – GW passage change the time of arrival of pulses. Distance between earth and pulsar act as an arm of GW interferometer. Sensitivity of **1 nHZ – 100 nHZ**.
- **Stochastic Gravitational Wave Background** - GW background created by superposition of many independent sources, which are not individually detectable. **Isotropic** across the sky
- **Hellings–Downs correlation** – Quadrupolar nature of GW gives correlation between TOA of pulses from pulsars which depends on angular separation angle between pulsars.
- In 2023, first evidence of SGWB in 15 years of data by NANOGrav collab. Confirmed by PPTA, EPTA, InPTA, CPTA. Around 4-sigma using 15-years of observation, around 70 pulsars.



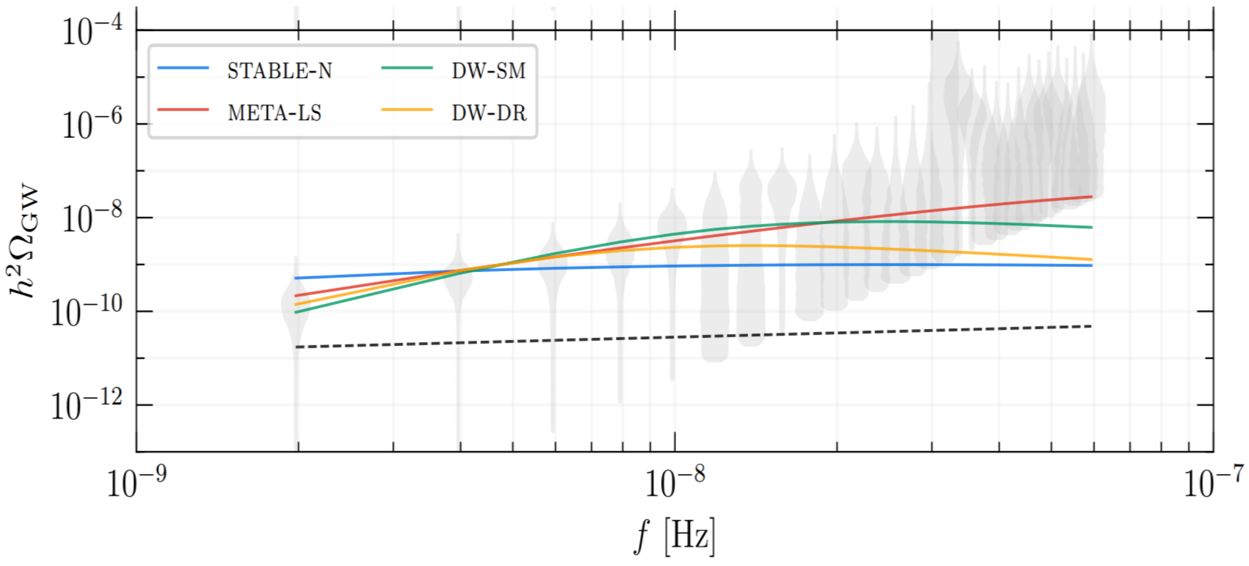


(NANOGrav collab.2023, IPTA collab., Hellings and Downs'83)

SGWB Sources

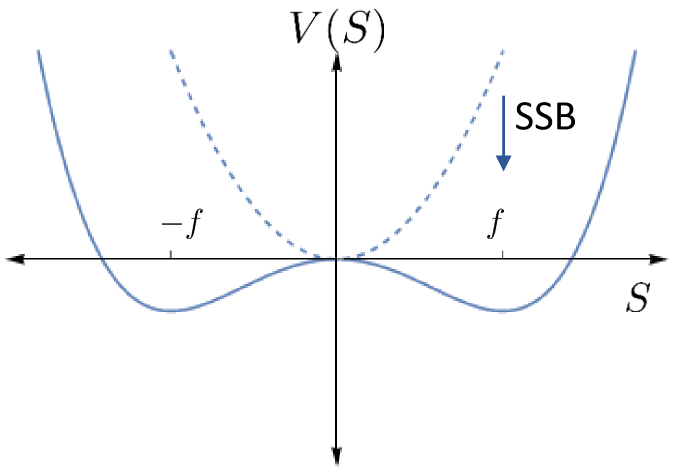
- **Sources**:
  - Supermassive Black Hole Binary mergers (SMBH BM)
  - Inflation and Primordial curvature perturbations
  - Cosmic Strings
  - Cosmic Phase Transition
  - Domain Walls

(NANOGrav collab. 2023)

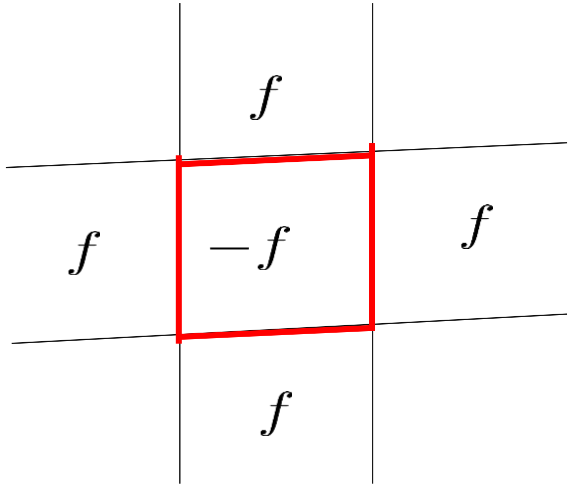


Domain Walls

- Domain walls are defects produced during spontaneous symmetry breaking of discrete $\mathbb{Z}_2$ (more generally $\mathbb{Z}_N$) symmetry
- Causally discounted patches could reside in different vacua/domains and domain walls interpolates between different domains



$$V = \frac{\lambda}{4}(S^2 - f^2)^2$$



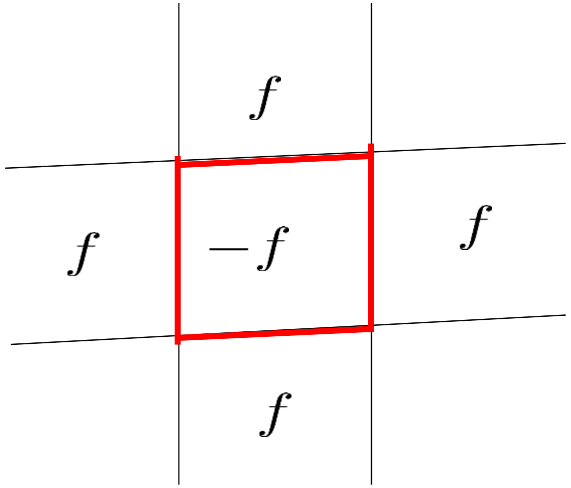
(Kibble'76)

Domain Wall Solution

- Solution to classical equation of motion which interpolates between two vacua

$$S''(z) + V'(S(z)) = 0 \text{ with BC } S(-\infty) = -f \text{ and } S(\infty) = f$$

$$S(z) = f\ \tanh[\sqrt{\frac{\lambda}{2}}\, f\, z]$$

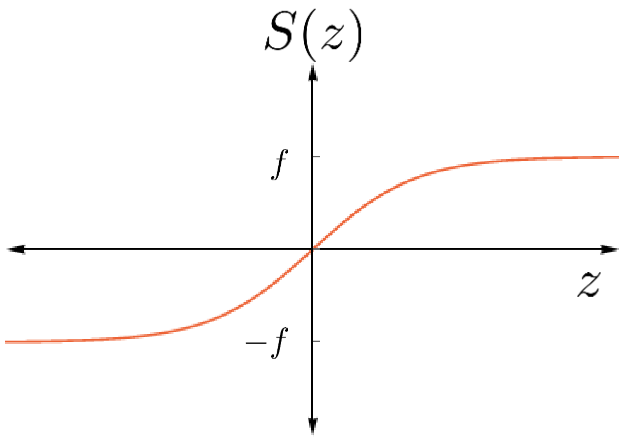


- Energy density of domain walls

$$\rho_{\rm DW} = \sigma / L$$

$$\sigma \approx \sqrt{\lambda} f^3 (\text{surface tension})$$

$$L = \text{domain size}$$



How does domain size L change during cosmic evolution?

Domain Wall Evolution

- Velocity-dependent One-Scale (VOS) model – effective description of domain wall evolution considering the surface tension and friction forces

$$\frac{dL}{dt} = HL + v^2\frac{L}{l_d} + c_w v\,,$$
$$\frac{dv}{dt} = (1-v^2)\left(\frac{k_w}{L} - \frac{v}{l_d}\right)$$

→ Scaling Solution

$$L = L_0 t, v = v_0$$

→

$$\rho_{\rm DW} = \sigma/L = \mathcal{A}\sigma/t$$
$$\mathcal{A} \approx 0.4N$$

- Domain wall dominate the energy density of universe if not annihilated as $\rho_R \sim T^4 \propto 1/t^2$

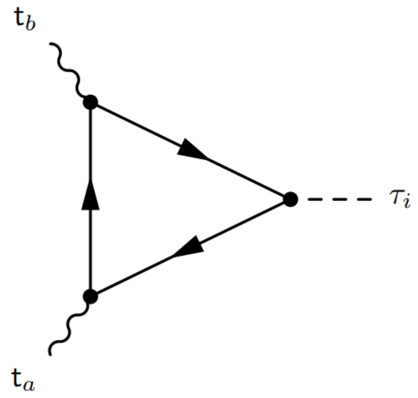
$$T_{\rm dom} \approx 45\,{\rm MeV}\left(\frac{\mathcal{A}}{0.8}\right)^{1/2}\left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^{1/2}\left(\frac{g_*(T_{\rm dom})}{10}\right)^{-1/4}$$

Domain Walls must collapse- could QCD effects trigger the collapse?

(Martins et al.'16, Bai, **MK**, Chen 2023)

QCD anomalous discrete symmetry

- To have bias between degenerate vacua we make discrete symmetry anomalous under QCD
- Add heavy vector like quarks coupled to S field and integrate out

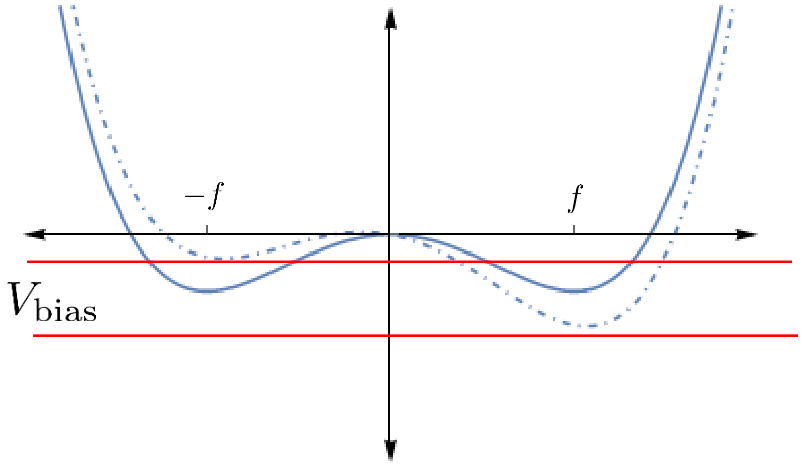


$$\mathcal{L}_{\text{Yukawa}} = yS\bar{Q}_L Q_R \text{ invariant under } \mathbb{Z}_2 \text{ as } S \to -S,\, Q_L \to -Q_L,\, Q_R \to Q_R$$

$$\mathcal{L} = -\frac{\theta}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu} \qquad \text{where } \theta = \arg(\mathrm{S})$$

- After QCD PT, chiral symmetry breaking, instanton effects generate potential for θ

$$V_{\text{bias}}(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4\, m_u\, m_d}{(m_u + m_d)^2} \sin^2\left(\frac{\theta}{2}\right)}$$

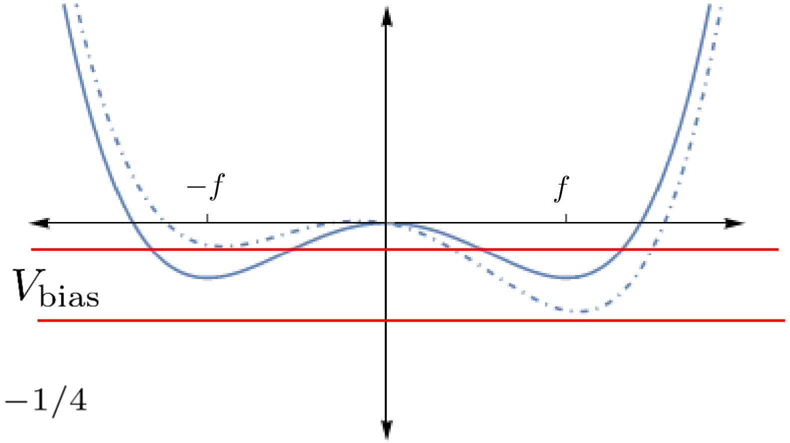


(Preskill et al.'91, Vecchia et al.' 80)

Domain wall collapse

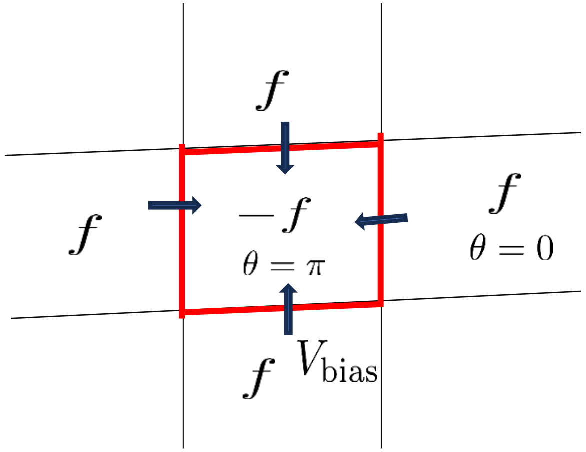
- For $\mathbb{Z}_2$, $V_{\rm bias}^{\rm max} = 0.66\, m_\pi^2\, f_\pi^2 \approx (100.4\,{\rm MeV})^4$

- The biased potential $V_{\rm bias}$ cause the DW collapse



$$T_{\rm ann} \approx 120\,{\rm MeV} \left(\frac{V_{\rm bias}}{(100\,{\rm MeV})^4}\right)^{1/2} \left(\frac{\mathcal{A}}{0.8}\right)^{-1/2} \left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^{-1/2} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{-1/4}$$

- Demanding that DW collapse before dominating the universe and after QCD PT gives upper and lower bound on σ



$$9.2 \times 10^{15}\,{\rm GeV}^3 \left(\frac{V_{\rm bias}}{(100\,{\rm MeV})^4}\right) \leq \sigma \leq 2.6 \times 10^{16}\,{\rm GeV}^3 \left(\frac{V_{\rm bias}}{(100\,{\rm MeV})^4}\right)^{1/2}$$

$$\sigma \approx 10^{16}\,{\rm GeV}^3 \text{ and } f \approx 100\,{\rm TeV}$$

(Preskill et al.'91, Vecchia et al.' 80)

GW production from DW collapse

- Collapse of DW leads to GW production with GW amplitude and frequency at production

$$Q \approx M_{\rm DW} L^2 \qquad P_{\rm GW} \sim G\,(d^3Q/dt^3)^2 \approx G\sigma^2 t^2 \qquad \rho_{\rm GW} \sim P_{\rm GW} H^{-1}/L^3 \sim G\,\sigma^2 \qquad f(t_{\rm ann}) = H(t_{\rm ann})$$

- The peak amplitude and frequency observed today

$$f_{\rm peak} = 1.1 \times 10^{-8}\,{\rm Hz} \left(\frac{g_*(T_{\rm ann})}{10}\right)^{1/2} \left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-1/3} \left(\frac{T_{\rm ann}}{100\,{\rm MeV}}\right)$$

$$\Omega_{\rm GW} h^2(t_0)\Big|_{\rm peak} = 3\times 10^{-8} \left(\frac{\tilde{\epsilon}_{\rm GW}}{0.7}\right) \left(\frac{\mathcal{A}}{0.8}\right)^2 \left(\frac{\sigma}{10^{16}\,{\rm GeV}^3}\right)^2 \left(\frac{T_{\rm ann}}{100\,{\rm MeV}}\right)^{-4} \left(\frac{g_{*s}(T_{\rm ann})}{10}\right)^{-4/3}$$

- Spectrum of GW from DW collapse:

$$\Omega_{\rm GW} \propto f^3 \text{ for } f < f_{\rm peak} \qquad \Omega_{\rm GW} \propto f^{-1} \text{ for } f > f_{\rm peak}$$

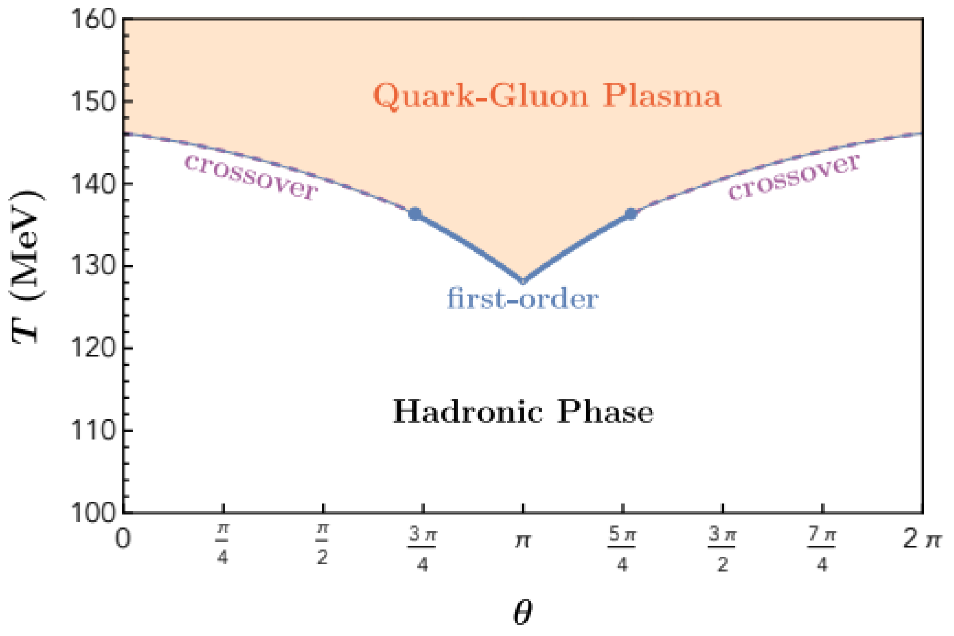
(Hiramatsu et al.'10, 13, Saikawa 17, Bai, **MK**, Chen 2023)

QCD PT in non-zero theta domains

- Lattice simulations suggest that finite temperature QCD PT is cross-over in the domains with $\theta = 0$. But is it true for $\theta \neq 0$?

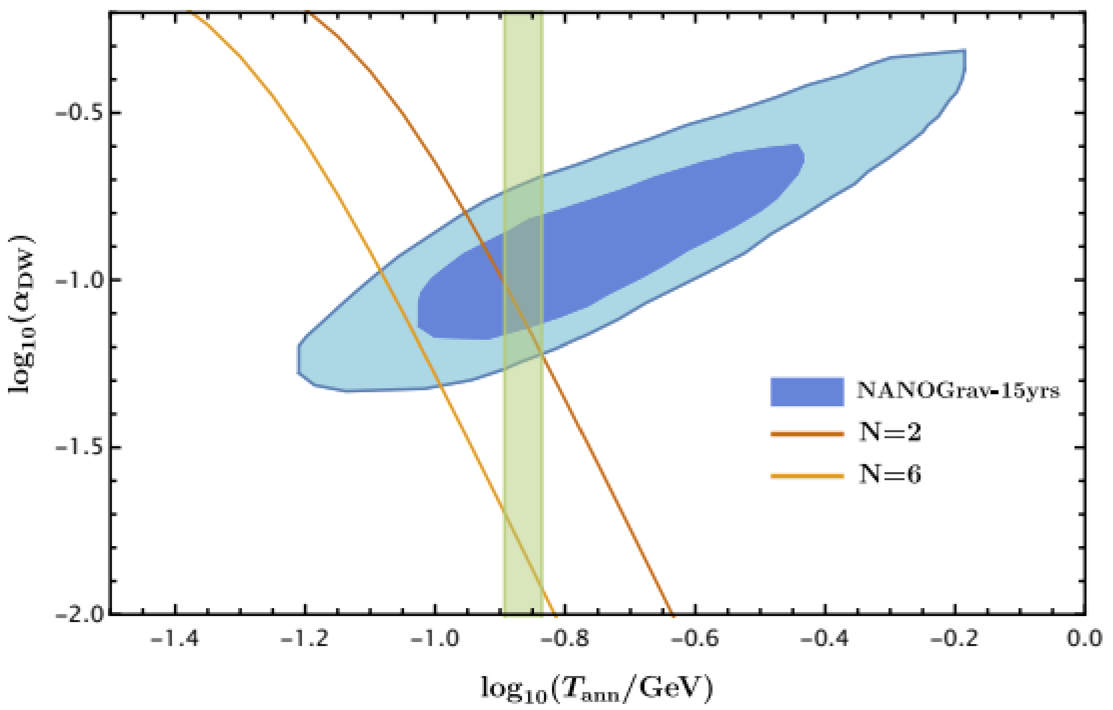
- We use Linear Sigma Model coupled to quarks (LSMq) to get a phase diagram

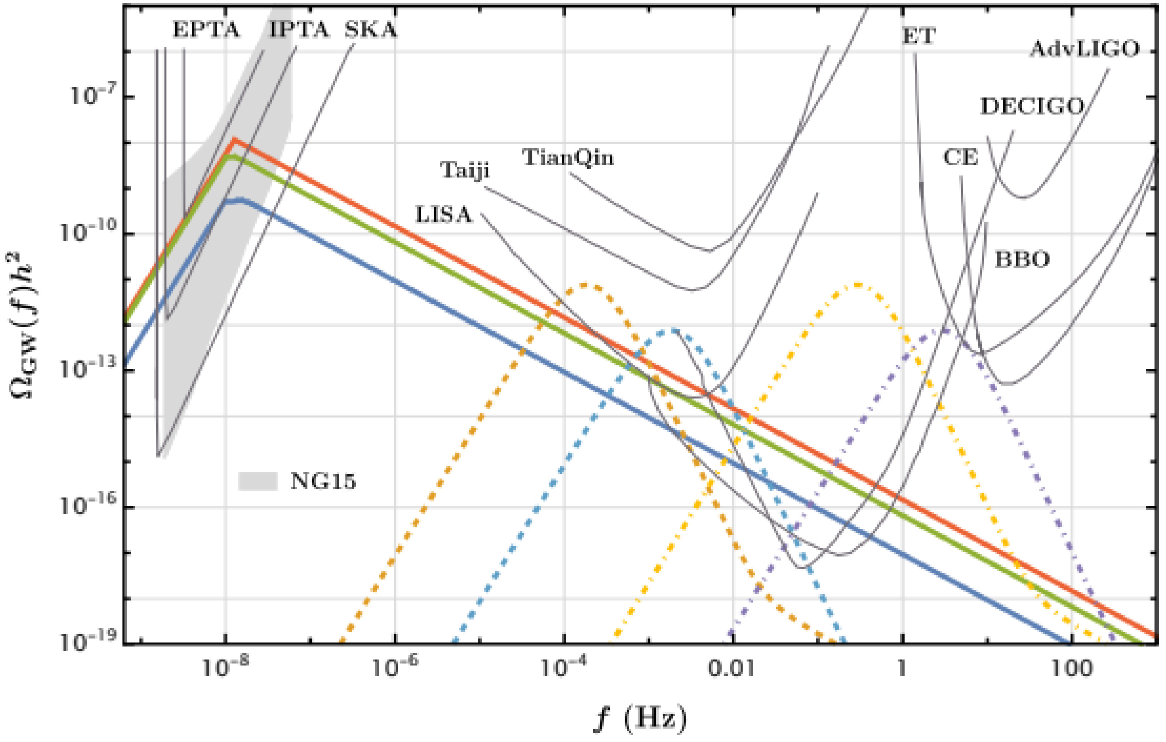
- In this model we get First order QCD PT for some $\theta \neq 0$, at finite temperature, which leads to production of GW in the LISA band



(Pisarski' 96 , Aoki et al.'06, Lenaghan et al.'2000,
Schaefer et al.' 09, Mitter et al.' 14, Bai, **MK**, Chen 2023)

Comparison with PTA result





$$\alpha_{\rm DW} = \rho_{\rm DW}/\rho_R$$

$(N = 2, \sigma = 1 \times 10^{16}\ {\rm GeV}^3)$ (red)

$(N = 4, \sigma = 3 \times 10^{15}\ {\rm GeV}^3)$ (green)

$(N = 6, \sigma = 1 \times 10^{15}\ {\rm GeV}^3)$ (blue)

(NANOGrav collab.'23, Bai, **MK**, Chen 2023)

Summary

- Recently Stochastic Gravitational Wave background (SGWB) detected by pulsar timing array (PTA) which could be explained various sources.

- Collapse of Domain walls induced by QCD effects could lead to GW signal consistent with the data.

- Some of the domains with non-zero QCD theta angle could lead to QCD first-order phase transition and thus additional GW in LISA band.

LMSq model

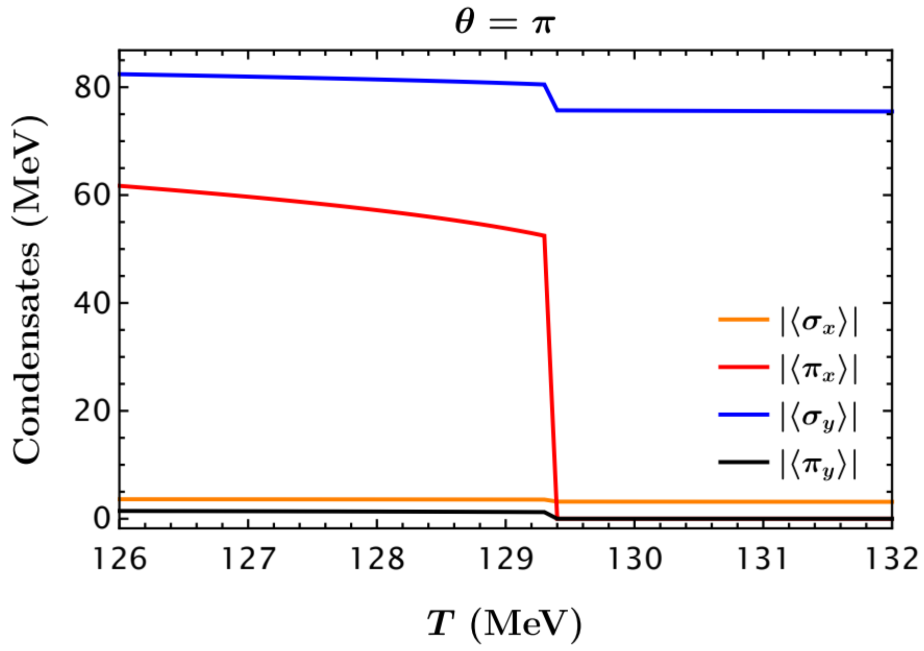
- LMSq potential at zero temperature, with $\Phi = T_a\left(\sigma_a + i\pi_a\right)$

$$V(\Phi) = \mu^2\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right) + \lambda_1\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 + \lambda_2\,\mathrm{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right]$$
$$-\frac{\kappa}{2}\left[e^{-i\theta}\det\left(\Phi\right) + e^{i\theta}\det\left(\Phi^\dagger\right)\right] - \mathrm{Tr}\left[H\left(\Phi+\Phi^\dagger\right)\right]$$

- In addition, there is a Yukawa coupling

$$\mathcal{L}_{\mathrm{Yukawa}} = \overline{q}\left[i\partial\!\!\!/ - gT_a\left(\sigma_a + i\gamma^5\pi_a\right)\right]q, \quad q = u, d, s$$

$$V_T = -\nu_q\int\frac{d^3p}{(2\pi)^3}T\log\left[1+e^{-\frac{E_q}{T}}\right] - \nu_s\int\frac{d^3p}{(2\pi)^3}T\log\left[1+e^{-\frac{E_s}{T}}\right],$$



(Lenaghan et al.'2000, Schaefer et al.' 09, Mitter et al.' 14)