PIKIMO 15, Nov 11, 2023



Multi Higgs boson signals of a modified muon Yukawa coupling

Taegyu Lee

arXiv:2311.05078

Collaboration with R. Dermisek, K. Hermanek, N. McGinnis, and S. Yoon

INDIANA UNIVERSITY BLOOMINGTON

$$\mathscr{L} \supset -y_{\mu}\bar{l}_{L}\mu_{R}H + h.c. \qquad l_{L} = (\nu_{\mu},\mu_{L})^{T} \qquad H = (G^{+},\nu + (h+iG^{0})/\sqrt{2})^{T}$$
$$\supset -m_{\mu}\bar{\mu}_{L}\mu_{R} - \frac{1}{\sqrt{2}} \lambda_{\mu\mu}^{h,\,\text{SM}} \bar{\mu}_{L}\mu_{R}h + h.c. \qquad m_{\mu} = y_{\mu}\nu \qquad \lambda_{\mu\mu}^{h,\,\text{SM}} = \frac{m_{\mu}}{\nu}$$

$$\mathscr{L} \supset -y_{\mu}\bar{l}_{L}\mu_{R}H + h.c. \qquad l_{L} = (\nu_{\mu}, \mu_{L})^{T} \qquad H = (G^{+}, \nu + (h + iG^{0})/\sqrt{2})^{T}$$
$$\supset -m_{\mu}\bar{\mu}_{L}\mu_{R} - \frac{1}{\sqrt{2}} \lambda_{\mu\mu}^{h, \text{SM}} \bar{\mu}_{L}\mu_{R}h + h.c. \qquad m_{\mu} = y_{\mu}\nu \qquad \lambda_{\mu\mu}^{h, \text{SM}} = \frac{m_{\mu}}{\nu}$$

Using the general Kappa Framework, we can express deviation in Yukawa coupling

arXiv:1209.0040. arXiv:1307.1347.

Kappa framework

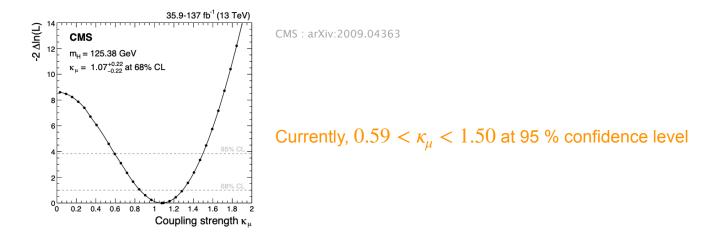
 $\lambda_{\mu\mu}^{h} = \frac{m_{\mu}}{v} \kappa_{\mu} \qquad \qquad \text{For SM, } \kappa_{\mu} = 1.$

$$\Delta \kappa_{\mu} = \kappa_{\mu} - 1$$

On the experimental side...

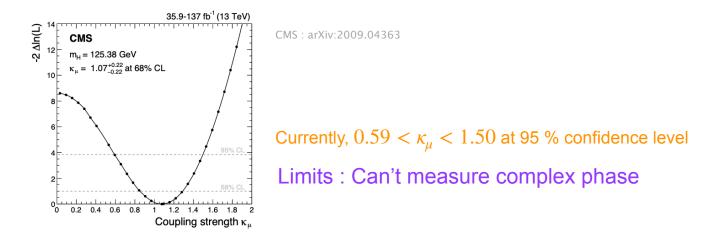


On the experimental side...



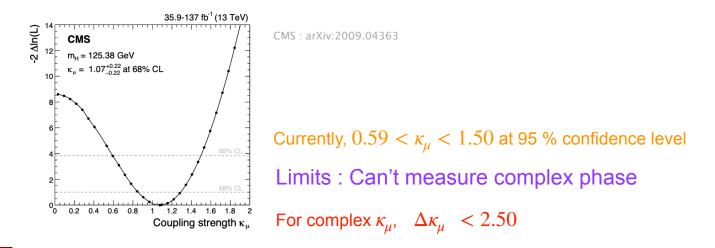


On the experimental side...





On the experimental side...



If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.



If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.

Warsaw Basis : arXiv:1008.4884

The complete set of dimension 6 operators are given in Warsaw basis. (59 independent operators)



If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.

Warsaw Basis : arXiv:1008.4884

 μ_R *

The complete set of dimension 6 operators are given in Warsaw basis. (59 independent operators)

C

' BI OOMINGT

There is only one dimension 6 SMEFT operator responsible for deviation Yukawa coupling

 $\succ H$

 $\mathcal{L} \supset -y_{\mu} \bar{l}_{L} \mu_{R} H - C_{\mu H} \bar{l}_{L} \mu_{R} H (H^{\dagger} H) + \text{h.c.} \qquad l_{L} = (\nu_{\mu}, \mu_{L})^{T}$ $M_{\mu} = y_{\mu} \nu + C_{\mu H} \nu^{3}$ $H = (G^{+}, \nu + h/\sqrt{2} + iG^{0}/\sqrt{2})^{T}$ $\lambda_{\mu\mu}^{h} = \frac{m_{\mu}}{\nu} + 2 C_{\mu H} \nu^{2}$

$$\begin{aligned} \mathscr{D} &\supset -y_{\mu} \bar{l}_{L} \mu_{R} H - C_{\mu H} \bar{l}_{L} \mu_{R} H \left(H^{\dagger} H \right) + \text{h.c.} & l_{L} = (\nu_{\mu}, \mu_{L})^{T} \\ m_{\mu} &= y_{\mu} \nu + C_{\mu H} \nu^{3} \\ \lambda_{\mu\mu}^{h} &= \frac{m_{\mu}}{\nu} + 2 C_{\mu H} \nu^{2} \\ \Delta \kappa_{\mu} &= 2 C_{\mu H} \nu^{3} / m_{\mu} \\ \end{aligned}$$

Thus we can parametrize the processes from dim 6 mass operator in terms of $\Delta \kappa_{\mu}$.

Di-Higgs, tri-Higgs : arXiv:2108.10950

	in terms of $\Delta \kappa_{\mu}$	in units of $\sigma_{\mu^+\mu^- \to hh}$
$\sigma_{\mu^+\mu^- ightarrow hh}$	$rac{9}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2$	1
$\sigma_{\mu^+\mu^- o hZ_L}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2$	$\frac{2}{9}$
$\sigma_{\mu^+\mu^- o Z_L Z_L}$	$rac{1}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2$	$\frac{1}{9}$
$\sigma_{\mu^+\mu^- \to W_L^+ W_L^-}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2$	<u>2</u> 9
	in terms of $\Delta \kappa_{\mu}$	in units of $\sigma_{\mu^+\mu^- \to hhh}$
$\sigma_{\mu^+\mu^- o hhh}$	$rac{3s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2$	1
$\sigma_{\mu^+\mu^- ightarrow hhZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2$	$\frac{1}{3}$
$\sigma_{\mu^+\mu^- ightarrow hZ_LZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2$	$\frac{1}{3}$
$\sigma_{\mu^+\mu^- o Z_L Z_L Z_L}$	$rac{3s}{2^{14}\pi^3}\left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2$	1
$\sigma_{\mu^+\mu^- \rightarrow h W^+_L W^L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2$	$\frac{2}{3}$
$\sigma_{\mu^+\mu^- \to Z_L W_L^+ W_L^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2$	$\frac{2}{3}$

Assuming muon and Higgs mass zero

- Are there other dimension 6 operators that can produce same final state?

Warsaw Basis : arXiv:1008.4884



- Are there other dimension 6 operators that can produce same final state?

Warsaw Basis : arXiv:1008.4884

$$\begin{array}{c|c} Q_{\varphi l}^{(1)} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r}) \\ Q_{\varphi l}^{(3)} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\,\varphi)(\bar{l}_{p}\tau^{I}\gamma^{\mu}l_{r}) \\ Q_{\varphi e} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{e}_{p}\gamma^{\mu}e_{r}) \end{array}$$

Derivative operators

- Are there other dimension 6 operators that can produce same final state?

Warsaw Basis : arXiv:1008.4884

$$\begin{array}{c|c} Q^{(1)}_{\varphi l} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r}) \\ Q^{(3)}_{\varphi l} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\,\varphi)(\bar{l}_{p}\tau^{I}\gamma^{\mu}l_{r}) \\ Q_{\varphi e} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{e}_{p}\gamma^{\mu}e_{r}) \end{array}$$

Derivative operators

 $\mu^+\mu^- \rightarrow W^+W^-, hZ, W^+W^-h, W^+W^-Z, Zhh, ZZZ$

- Are there other dimension 6 operators that can produce same final state?

Warsaw Basis : arXiv:1008.4884

$$\begin{array}{c|c} Q^{(1)}_{\varphi l} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{l}_{p}\gamma^{\mu}l_{r}) \\ Q^{(3)}_{\varphi l} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}^{I}\,\varphi)(\bar{l}_{p}\tau^{I}\gamma^{\mu}l_{r}) \\ Q_{\varphi e} & (\varphi^{\dagger}i\overleftrightarrow{D}_{\mu}\,\varphi)(\bar{e}_{p}\gamma^{\mu}e_{r}) \end{array}$$

Derivative operators

$$\begin{array}{c|c} Q_{eW} & (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu} \\ Q_{eB} & (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu} \end{array}$$

 $\mu^+\mu^- \rightarrow W^+W^-, hZ, W^+W^-h, W^+W^-Z, Zhh, ZZZ$

- Are there other dimension 6 operators that can produce same final state?

Warsaw Basis : arXiv:1008.4884

$$\begin{array}{c|c} Q_{\varphi l}^{(1)} & (\varphi^{\dagger}i \overleftrightarrow{D}_{\mu} \varphi)(\bar{l}_{p} \gamma^{\mu} l_{r}) \\ Q_{\varphi l}^{(3)} & (\varphi^{\dagger}i \overleftrightarrow{D}_{\mu}^{I} \varphi)(\bar{l}_{p} \tau^{I} \gamma^{\mu} l_{r}) \\ Q_{\varphi e} & (\varphi^{\dagger}i \overleftrightarrow{D}_{\mu} \varphi)(\bar{e}_{p} \gamma^{\mu} e_{r}) \end{array}$$

$$\mu^+\mu^- \rightarrow W^+W^-, hZ, W^+W^-h, W^+W^-Z, Zhh, ZZZ$$

Derivative operators

$$\begin{array}{c|c} Q_{eW} & (\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu} \\ Q_{eB} & (\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu} \end{array}$$

 $\mu^+\mu^- \rightarrow hZ, ZZ, W^+W^-, W^+W^-h$

INDIANA UNIVERSITY BLOOMINGTON

 $\mu^+\mu^- \rightarrow hh, hhh, hZZ$ survives



 $\mu^+\mu^- \rightarrow hh, hhh, hZZ$ survives

- Is the Standard Model background small?



 $\mu^+\mu^- \rightarrow hh, hhh, hZZ$ survives

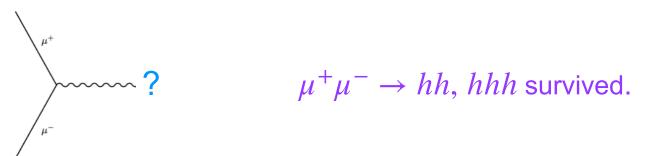
- Is the Standard Model background small?

Large SM background comes from gauge boson interaction.

 $\mu^+\mu^- \rightarrow hh, hhh, hZZ$ survives

- Is the Standard Model background small?

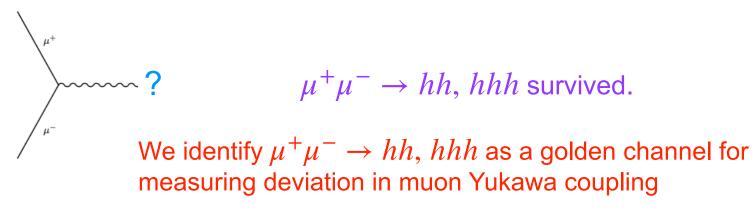
Large SM background comes from gauge boson interaction.

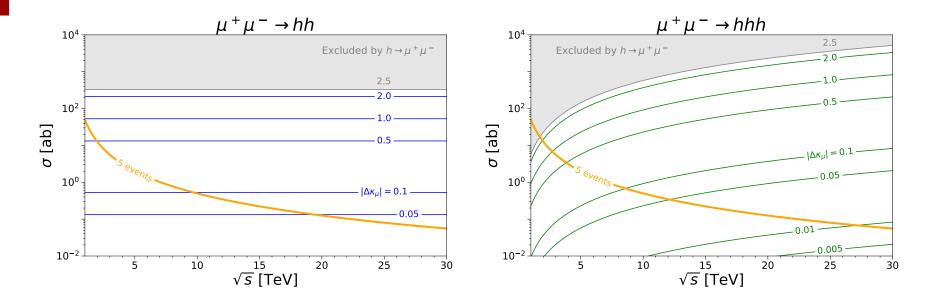


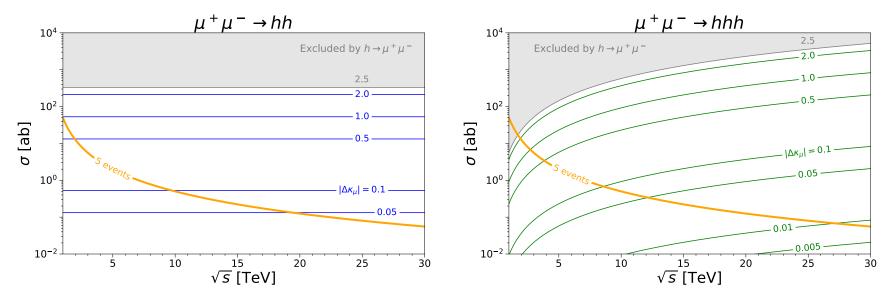
 $\mu^+\mu^- \rightarrow hh, hhh, hZZ$ survives

- Is the Standard Model background small?

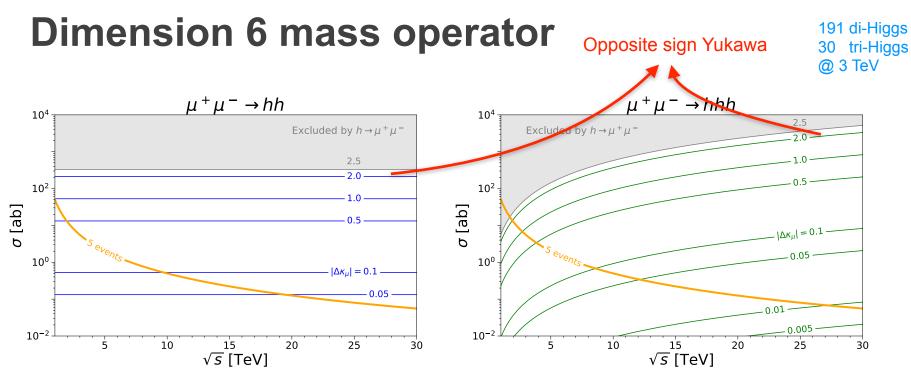
Large SM background comes from gauge boson interaction.







 $\sigma(\mu^+\mu^- \to hh)_{SM} = 1.6 \times 10^{-4}$ ab and $\sigma(\mu^+\mu^- \to hhh)_{SM} = 2.9 \times 10^{-5}$ ab at $\sqrt{s} = 3$ TeV, and falling as $1/s^2$ and 1/s at larger energies.



 $\sigma(\mu^+\mu^- \to hh)_{SM} = 1.6 \times 10^{-4}$ ab and $\sigma(\mu^+\mu^- \to hhh)_{SM} = 2.9 \times 10^{-5}$ ab at $\sqrt{s} = 3$ TeV, and falling as $1/s^2$ and 1/s at larger energies.

 $\begin{aligned} \mathscr{L} \supset &- y_{\mu} \bar{l}_{L} \mu_{R} H_{d} - C_{\mu H_{d}} \bar{l}_{L} \mu_{R} H_{d} \left(H_{d}^{\dagger} H_{d} \right) \\ &- C_{\mu H_{u}}^{(1)} \bar{l}_{L} \mu_{R} H_{d} \left(H_{u}^{\dagger} H_{u} \right) - C_{\mu H_{u}}^{(2)} \bar{l}_{L} \mu_{R} \cdot H_{u}^{\dagger} \left(H_{d} \cdot H_{u} \right) - C_{\mu H_{u}}^{(3)} \bar{l}_{L} \mu_{R} \cdot H_{u}^{\dagger} \left(H_{d}^{\dagger} \cdot H_{u}^{\dagger} \right) + \mathrm{h.c.} \end{aligned}$

 $\begin{aligned} \mathscr{L} \supset -\mathscr{I}_{\mu}I_{L}\mu_{R}H_{d} - C_{\mu}H_{d}\overline{I}_{L}\mu_{R}H_{d}\left(H_{d}^{\dagger}H_{d}\right) \\ -C_{\mu}(1)I_{L}\mu_{R}H_{d}\left(H_{u}^{\dagger}H_{u}\right) - C_{\mu}(2)I_{L}\mu_{R}\cdot H_{u}^{\dagger}\left(H_{d}\cdot H_{u}\right) - C_{\mu}(3)I_{L}\mu_{R}\cdot H_{u}^{\dagger}\left(H_{d}^{\dagger}\cdot H_{u}^{\dagger}\right) + \mathbf{h.c.} \end{aligned}$

1. Comes from new leptons with same quantum number as SM leptons 2. Leads to largest $\tan \beta$ enhancement to the processes.



 $\mathscr{L} \supset -y_{\mu}\bar{l}_{L}\mu_{R}H_{d} - C_{\mu}H_{d}\bar{l}_{L}\mu_{R}H_{d}\left(H_{d}^{\dagger}H_{d}\right)$

 $\Delta \kappa_{\mu} = 2C_{\mu H_d} v_d^3 / m_{\mu} \qquad v_d = v \cos \beta$

For each heavy Higgs in the process leads to $tan^2\beta$ enhancement to the Cross section

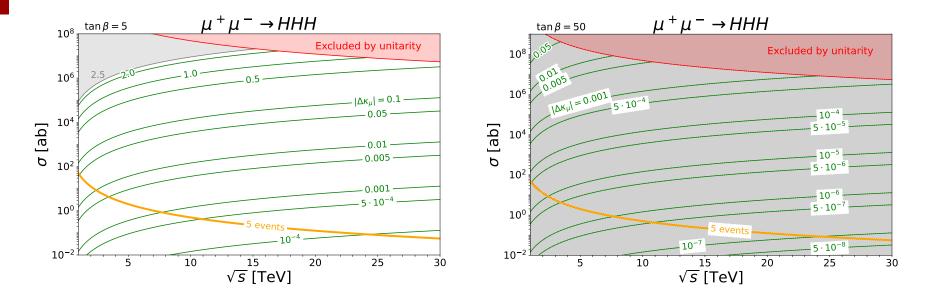
$$H_d^{\pm} = \cos\beta G^{\pm} - \sin\beta H^{\pm}$$
$$H_d^0 = v\cos\beta + \frac{1}{\sqrt{2}}(h\cos\beta + H\sin\beta) + \frac{i}{\sqrt{2}}(G\cos\beta - A\sin\beta)$$

	in terms of $\Delta \kappa_{\mu}$
$\sigma_{\mu^+\mu^- ightarrow hh}$	$rac{9}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2$
$\sigma_{\mu^+\mu^- o AA}$	$rac{1}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2 an^4eta$
$\sigma_{\mu^+\mu^- o HH}$	$rac{9}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2 an^4eta$
$\sigma_{\mu^+\mu^- ightarrow hH}$	$rac{9}{128\pi} \left(rac{m_\mu}{v^2} ight)^2 \Delta\kappa_\mu ^2 an^2 eta$
$\sigma_{\mu^+\mu^- ightarrow hA}$	$rac{1}{128\pi} \left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2 an^2 eta$
$\sigma_{\mu^+\mu^- o HA}$	$rac{1}{128\pi} \left(rac{m_\mu}{v^2} ight)^2 \Delta\kappa_\mu ^2 an^4 eta$
$\sigma_{\mu^+\mu^- ightarrow H^+H^-}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^2} ight)^2 \Delta\kappa_{\mu} ^2 an^4eta$

	in terms of $\Delta \kappa_{\mu}$
$\sigma_{\mu^+\mu^- o hhh}$	$rac{3s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2$
$\sigma_{\mu^+\mu^- ightarrow AAA}$	$rac{3s}{2^{14}\pi^3} \left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2 \tan^6 eta$
$\sigma_{\mu^+\mu^- ightarrow HHH}$	$rac{3s}{2^{14}\pi^3} \left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2 \tan^6 eta$
$\sigma_{\mu^+\mu^- ightarrow hhH}$	$rac{9s}{2^{14}\pi^3}\left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2 an^2eta$
$\sigma_{\mu^+\mu^- o hhA}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^2eta$
$\sigma_{\mu^+\mu^- o hAA}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^4eta$
$\sigma_{\mu^+\mu^- ightarrow hHH}$	$rac{9s}{2^{14}\pi^3}\left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2 an^4eta$
$\sigma_{\mu^+\mu^- o AHH}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^6eta$
$\sigma_{\mu^+\mu^- o HAA}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^6eta$
$\sigma_{\mu^+\mu^- ightarrow hH^+H^-}$	$rac{s}{2^{13}\pi^3} \left(rac{m_{\mu}}{v^3} ight)^2 \Delta\kappa_{\mu} ^2 \tan^4 eta$
$\sigma_{\mu^+\mu^- ightarrow HH^+H^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^6eta$
$\sigma_{\mu^+\mu^- ightarrow AH^+H^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^6eta$
$\sigma_{\mu^+\mu^- o hHA}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2 \Delta\kappa_\mu ^2 an^4eta$

12

Ш



13

Summary

- Future muon collider is expected to probe deviation to muon Yukawa coupling up to 7% (0.8 %) at 10 TeV (30 TeV) from $\mu^+\mu^- \rightarrow hhh$
- Expected to measure opposite sign Yukawa even at low energy muon collider.
- 2HDM type II can be tested and if there are 2HDM type II present, we can probe deviation to muon Yukawa coupling up to the ~ 10^{-6} (10^{-8}) level at 10 TeV (30 TeV) from $\mu^+\mu^- \rightarrow HHH$



Summary

- Future muon collider is expected to probe deviation to muon Yukawa coupling up to 7% (0.8 %) at 10 TeV (30 TeV) from $\mu^+\mu^- \rightarrow hhh$
- Expected to measure opposite sign Yukawa even at low energy muon collider.
- 2HDM type II can be tested and if there are 2HDM type II present, we can probe deviation to muon Yukawa coupling up to the ~ 10^{-6} (10^{-8}) level at 10 TeV (30 TeV) from $\mu^+\mu^- \rightarrow HHH$

Thanks for listening



INDIANA UNIVERSITY BLOOMINGTON



If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.



If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.

Familiar example would be Fermi theory of weak interaction

$$\mathscr{L}_{F} = \frac{4}{\sqrt{2}} G_{F} \cdot \bar{\psi}_{L}^{(1)} \gamma^{\mu} \psi_{L}^{(2)} \cdot \bar{\psi}_{L}^{(3)} \gamma_{\mu} \psi_{L}^{(4)} \qquad G_{F} = \frac{\sqrt{2}}{8} \frac{e^{2}}{\sin^{2} \theta_{W} M_{W}^{2}}$$

If SM is considered as low energy effective theory, the effect of UV theory at some high scale shows up as a form of higher dimensional operator.

Familiar example would be Fermi theory of weak interaction

$$\mathscr{L}_{F} = \frac{4}{\sqrt{2}} G_{F} \cdot \bar{\psi}_{L}^{(1)} \gamma^{\mu} \psi_{L}^{(2)} \cdot \bar{\psi}_{L}^{(3)} \gamma_{\mu} \psi_{L}^{(4)} \qquad G_{F} = \frac{\sqrt{2}}{8} \frac{e^{2}}{\sin^{2} \theta_{W} M_{W}^{2}} \qquad \text{Scale of UV theory}$$

- Are there other dimension 6 operators that can produce same final state?
- Is the 2HDM background small?



- Are there other dimension 6 operators that can produce same final state?
- Is the 2HDM background small?

 $\mu^{+}\mu^{-} \rightarrow hh, AA, hH, HH$ $\mu^{+}\mu^{-} \rightarrow hhh, HHH, hhH, hAA, hHH, HAA$ $\mu^{+}\mu^{-} \rightarrow hAZ, HZZ$

- Are there other dimension 6 operators that can produce same final state?
- Is the 2HDM background small?

 $\mu^{+}\mu^{-} \rightarrow hh, AA, hH, HH$ $\mu^{+}\mu^{-} \rightarrow hhh, HHH, hhH, hAA, hHH, HAA$ $\mu^{+}\mu^{-} \rightarrow hAZ, HZZ$ Thus, $\mu^{+}\mu^{-} \rightarrow HH$ and $\mu^{+}\mu^{-} \rightarrow HHH$ could be a golden channel for large tan β

	in terms of $\Delta \kappa_{\mu}$
$\sigma_{\mu^+\mu^- o hZ_L}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^2} ight)^2(\Delta\kappa_{\mu})^2$
$\sigma_{\mu^+\mu^- ightarrow HZ_L}$	$rac{1}{128\pi} \left(rac{m_{\mu}}{v^2} ight)^2 (\Delta\kappa_{\mu})^2 \tan^2eta$
$\sigma_{\mu^+\mu^- o AZ_L}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^{2}} ight)^{2}(\Delta\kappa_{\mu})^{2} an^{2}eta$
$\sigma_{\mu^+\mu^- ightarrow Z_L Z_L}$	$rac{1}{256\pi}\left(rac{m_{\mu}}{v^2} ight)^2(\Delta\kappa_{\mu})^2$
$\sigma_{\mu^+\mu^-\to H^+W^L/W^+_LH^-}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^{2}} ight)^{2}(\Delta\kappa_{\mu})^{2} an^{2}eta$
$\sigma_{\mu^+\mu^- o W^+_L W^L}$	$rac{1}{128\pi}\left(rac{m_{\mu}}{v^2} ight)^2(\Delta\kappa_{\mu})^2$

	in terms of $\Delta \kappa_{\mu}$
$\sigma_{\mu^+\mu^- ightarrow hhZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2$
$\sigma_{\mu^+\mu^- ightarrow hHZ_L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^- ightarrow hAZ_L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^- \to HHZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^4eta$
$\sigma_{\mu^+\mu^- o HAZ_L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^4eta$
$\sigma_{\mu^+\mu^- ightarrow AAZ_L}$	$rac{9s}{2^{14}\pi^3}\left(rac{m_{\mu}}{v^3} ight)^2 (\Delta\kappa_{\mu})^2 an^4 eta$
$\sigma_{\mu^+\mu^- ightarrow hZ_LZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2$
$\sigma_{\mu^+\mu^- ightarrow HZ_LZ_L}$	$rac{s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^- ightarrow AZ_L Z_L}$	$rac{9s}{2^{14}\pi^3} \left(rac{m_{\mu}}{v^3} ight)^2 (\Delta\kappa_{\mu})^2 an^2 eta$
$\sigma_{\mu^+\mu^- ightarrow Z_L Z_L Z_L}$	$rac{3s}{2^{14}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2$
$\sigma_{\mu^+\mu^- ightarrow h W^+_L W^L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2$
$\sigma_{\mu^+\mu^- ightarrow HW^+_LW^L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^- ightarrow AW^+_LW^L}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^-\to Z_L W_L^+ W_L^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2$
$\sigma_{\mu^+\mu^- \to hH^+W^L/hW^+_LH^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$
$\sigma_{\mu^+\mu^- \to HH^+W_L^-/HW_L^+H^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^4eta$
$\sigma_{\mu^+\mu^-\to AH^+W^L/AW^+_LH^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^4eta$
$\sigma_{\mu^+\mu^- o Z_L H^+ H^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^4eta$
$\sigma_{\mu^+\mu^-\to Z_L H^+ W^L/Z_L W^+_L H^-}$	$rac{s}{2^{13}\pi^3}\left(rac{m_\mu}{v^3} ight)^2(\Delta\kappa_\mu)^2 an^2eta$

ΠΓ



$$H_{d}^{\pm} = \cos\beta G^{\pm} - \sin\beta H^{\pm}$$

$$H_{d}^{0} = v\cos\beta + \frac{1}{\sqrt{2}}(h\cos\beta + H\sin\beta) + \frac{i}{\sqrt{2}}(G\cos\beta - A\sin\beta)$$

$$H_{u}^{0} = v\cos\beta + \frac{1}{\sqrt{2}}(h\cos\beta + H\sin\beta) + \frac{i}{\sqrt{2}}(G\cos\beta - A\sin\beta)$$
In alignment limit
$$H_{u}^{\pm} = -\sin\beta G^{\pm} - \cos\beta H^{\pm}$$

$$H_{u}^{0} = v\sin\beta + \frac{1}{\sqrt{2}}(h\sin\beta - H\cos\beta) - \frac{i}{\sqrt{2}}(G\sin\beta + A\cos\beta)$$