Entanglement and Symmetry in Low-energy QCD

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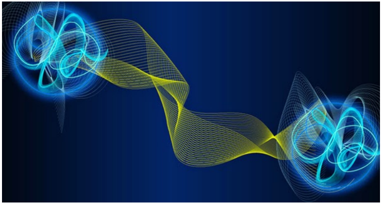
based on 2210.12085

Introduction

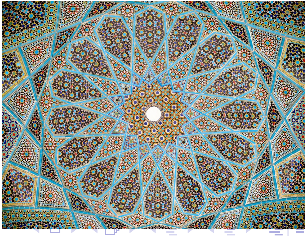
Entanglement

"characteristic trait of quantum mechanics"



Symmetry

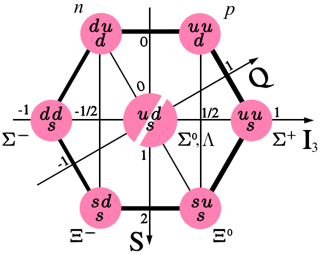
fundamental in QFT



Baryon-baryon scattering

spin-$\frac{1}{2}$ baryon octet
Flavor symmetry $SU(3)$ $(m_u = m_d = m_s)$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$



Simple EFT at very low energies.

- Only contact interactions below pion threshold.

Accidental symmetries:

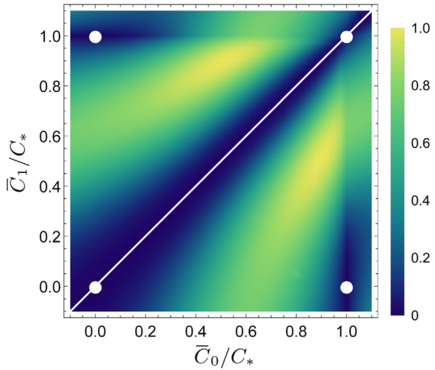
- $SU(4)$ Wigner symmetry
- $SU(2N_f)$ spin-flavor symmetry
- Schrödinger symmetry

Emergent, not symmetries in QCD

$$\begin{aligned} \mathcal{L}_{\text{LO}}^{n_f=3} = & -c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle \\ & -c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle \\ & -c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle \end{aligned}$$

First clue: nucleon-nucleon scattering

$$\mathcal{L}_{\rm LO}^{n_f=2} = -\frac{1}{2}C_S\,(N^\dagger N)^2 - \frac{1}{2}C_T\,(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$$



Beane, Kaplan, Klco and Savage [1812.03138]

First clue: nucleon-nucleon scattering

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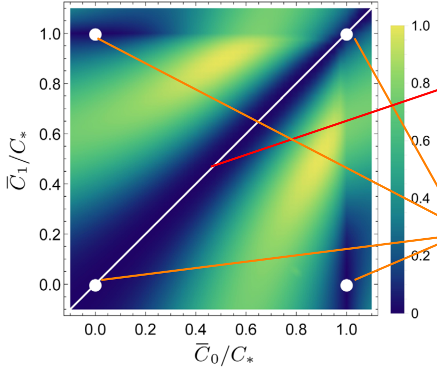


$SU(4)$ Wigner symmetry

$N = (p_\uparrow, p_\downarrow, n_\uparrow, n_\downarrow)^T$

$N \to UN\ ,\ U \in SU(4)$

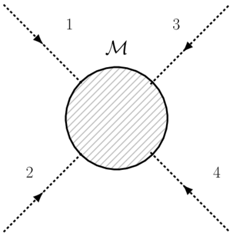
Schrödinger (conformal) symmetry

Beane, Kaplan, Klco and Savage [1812.03138]

Quantifying Entanglement

Entanglement is defined on states

- $|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle : E = 0$
- $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} : E \neq 0$



$E(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$

Entanglement of operators?

Quantifying Entanglement

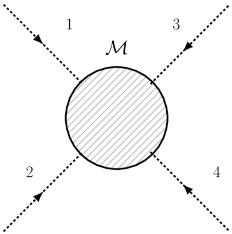
Entanglement is defined on states

- $|\uparrow\downarrow\rangle = |\uparrow\rangle \otimes |\downarrow\rangle : E = 0$
- $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2} : E \neq 0$

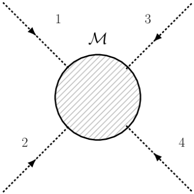
$$E(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$$

~~Entanglement of operators~~

$$E(U) = \overline{E(U\,|\psi_1\rangle \otimes |\psi_2\rangle)}$$



Minimally Entangling Operators



Minimally entangling operators:
$E(U) = 0$
product state $\xrightarrow{U}$ product state.

In a two qubit (spin-$\frac{1}{2}$) system, only two such operators, unique up to local unitaries (in the basis of $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$),

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} = \mathbf{1}, \qquad \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} = \text{SWAP}.$$

Low and Mehen [2104.10835]

S matrix of baryon-baryon scattering

$SU(3)$ flavor symmetry $\Longrightarrow$ only 6 independent phase shifts

Charge and strangeness conservation $\Longrightarrow$ block-diagonal

$$\mathbf{8}\otimes\mathbf{8}=\mathbf{27}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{8}_S\oplus\mathbf{8}_A\oplus\mathbf{1}$$

$$S=\frac{1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}\otimes\left(\mathrm{P}_{\mathbf{27}}\,e^{2i\delta_{\mathbf{27}}}+\mathrm{P}_{\mathbf{8_S}}\,e^{2i\delta_{\mathbf{8_S}}}+\mathrm{P}_{\mathbf{1}}\,e^{2i\delta_{\mathbf{1}}}\right)$$
$$+\frac{3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}\otimes\left(\mathrm{P}_{\mathbf{10}}\,e^{2i\delta_{\mathbf{10}}}+\mathrm{P}_{\overline{\mathbf{10}}}\,e^{2i\delta_{\overline{\mathbf{10}}}}+\mathrm{P}_{\mathbf{8_A}}\,e^{2i\delta_{\mathbf{8_A}}}\right)$$

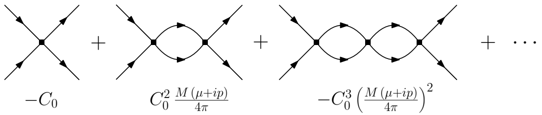
(Q,S) sectors
np
$\Sigma^-\Xi^-$
$\Sigma^+\Xi^0$
$n\Sigma^-$
$p\,\Sigma^+$
$\Xi^-\Xi^0$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$
$(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$
$(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$
$(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$
$(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$
$(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0 n,\ \Lambda\Lambda)$

Constraints on phase shifts

(Q, S) sectors	Minimal Entanglement Conditions
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$\delta_{\mathbf{27}} = \delta_{\overline{\mathbf{10}}}$ or $\delta_{\mathbf{27}} = \delta_{\overline{\mathbf{10}}} \pm \frac{\pi}{2}$
$n\Sigma^-$ $p\,\Sigma^+$ $\Xi^-\Xi^0$	$\delta_{\mathbf{27}} = \delta_{\mathbf{10}}$ or $\delta_{\mathbf{27}} = \delta_{\mathbf{10}} \pm \frac{\pi}{2}$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$\delta_{\mathbf{27}} = \delta_{\mathbf{8_S}} = \delta_{\mathbf{10}} \pm \frac{\pi}{2} = \delta_{\overline{\mathbf{10}}} \pm \frac{\pi}{2} = \delta_{\mathbf{8_A}} \pm \frac{\pi}{2}$ or $\delta_{\mathbf{27}} = \delta_{\mathbf{8_S}} = \delta_{\mathbf{10}} = \delta_{\overline{\mathbf{10}}} = \delta_{\mathbf{8_A}}$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0n,\ \Lambda\Lambda)$	$\delta_{\mathbf{27}} = \delta_{\mathbf{8_S}} = \delta_{\mathbf{1}} = \delta_{\mathbf{10}} = \delta_{\overline{\mathbf{10}}} = \delta_{\mathbf{8_A}}$ or $\delta_{\mathbf{27}} = \delta_{\mathbf{8_S}} = \delta_{\mathbf{1}} = \delta_{\mathbf{10}} \pm \frac{\pi}{2} = \delta_{\overline{\mathbf{10}}} \pm \frac{\pi}{2} = \delta_{\mathbf{8_A}} \pm \frac{\pi}{2}$

From S matrix to Lagrangian

$$\mathcal{L}_{\rm LO}^{n_f=3} = - c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle$$
$$- c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle - c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle \ .$$



Constraints on Wilson coefficients

Flavor subspaces	Minimal Entanglement Conditions
np $\Sigma^-\Xi^-$ $\Sigma^+\Xi^0$	$c_2 = -c_6$ or $c_1 + c_5 = -\frac{2\pi}{M\mu}$, $c_2 + c_6 = \pm\frac{2\pi}{M\mu}$
$n\Sigma^-$ $p\Sigma^+$ $\Xi^-\Xi^0$	$c_1 = c_6$ or $-c_2 + c_5 = -\frac{2\pi}{M\mu}$, $c_1 - c_6 = \pm\frac{2\pi}{M\mu}$
$(p\Lambda,\ p\Sigma^0,\ n\Sigma^+)$ $(n\Lambda,\ n\Sigma^0,\ p\Sigma^-)$ $(\Sigma^-\Lambda,\ \Sigma^-\Sigma^0,\ n\,\Xi^-)$ $(\Sigma^+\Lambda,\ \Sigma^+\Sigma^0,\ p\,\Xi^0)$ $(\Sigma^-\Xi^0, \Xi^-\Sigma^0, \Xi^-\Sigma^0)$ $(\Xi^-\Sigma^+, \Xi^0\Lambda, \Xi^0\Sigma^0)$	$c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = c_6$ or $c_1 = -c_2 = -\frac{1}{2}c_3 = \frac{1}{2}c_4 = -c_5 - \frac{2\pi}{M\mu} = c_6 \pm \frac{2\pi}{M\mu}$
$(\Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0,\ \Xi^-p,\ \Xi^0 n,\ \Lambda\Lambda)$	$c_1 = c_2 = c_3 = c_4 = c_6 = 0$ or $c_1 = c_2 = c_3 = c_4 = 0, c_5 = -2\pi/M\mu, c_6 =$ $\pm 2\pi/M\mu$

Symmetries of Lagrangian: minimum in 1-d sector

$np, \Sigma^-\Xi^-, \Sigma^+\Xi^0$

$S \propto \mathbf{1} \Longleftarrow$ spin-flavor $SU(6)$

$n\Sigma^-, p\,\Sigma^+, \Xi^-\Xi^0$

$S \propto \mathbf{1} \Longleftarrow$ spin-flavor $\overline{SU(6)}$

Spin-flavor symmetry at quark level

$$SU(6): \; \left(u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow\right)^T$$

- small octet-decuplet mass difference
- magnetic moment of baryons
- Large-N_c expansion.

Baryon pairs	symmetric flavor irrep	anti-symmetric flavor irrep
np, $\Sigma^-\Xi^-$, $\Sigma^+\Xi^0$	$\mathbf{27}$	$\overline{\mathbf{10}}$
$n\Sigma^-$, $p\,\Sigma^+$, $\Xi^-\Xi^0$	$\mathbf{27}$	$\mathbf{10}$

Symmetries of Lagrangian: minimum in 3-d sector

$$S \propto \mathbf{1} \Longrightarrow \mathcal{L} = -(c_1 + c_5)\,\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle + c_1\langle B_i^\dagger B_j^\dagger\rangle\langle B_i B_j\rangle$$

Symmetries manifest when B is written in $SU(3)$ generator components.

$$\begin{aligned} B^a \equiv& \mathrm{Tr}(BT^a)\ , \qquad a = 1, \cdots, 8\ , \\ \vec{B} =& (B^1, \cdots, B^8) \\ =& \frac{1}{2}\left(\Sigma^+ + \Sigma^-, i\Sigma^+ - i\Sigma^-, p + \Xi^-, ip - i\Xi^-, n + \Xi^0, in - i\Xi^0, \sqrt{2}\Sigma^0, \sqrt{2}\Lambda\right). \end{aligned}$$

The Lagrangian becomes

$$\mathcal{L} = -2(c_1 + c_5)\left(\vec{B}_i^\dagger \cdot \vec{B}_i\right)\left(\vec{B}_j^\dagger \cdot \vec{B}_j\right) + 2c_1\left(\vec{B}_i^\dagger \cdot \vec{B}_j^\dagger\right)\left(\vec{B}_i \cdot \vec{B}_j\right),$$

$SO(8)$ symmetry

Symmetries of Lagrangian: minimum in 6-d sector

$S \propto \mathbf{1} \iff$ only c_5 nonzero.

$\mathcal{B} = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow, \dots)^T$

$\mathcal{L} = -c_5(\mathcal{B}^\dagger\mathcal{B})^2 \iff SU(16)$ symmetry $\qquad \mathcal{B} \to U\mathcal{B}\ ,\ U \in SU(16)$

[generalization of Wigner $SU(4)$]

$S \propto$ SWAP:

$$\begin{aligned}\mathcal{L} &= -c_5\,\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\,\langle B_i^\dagger B_j\rangle\langle B_i^\dagger B_j\rangle \\ &= -2c_5\left(\vec{B}_i^\dagger\cdot\vec{B}_i\right)\left(\vec{B}_j^\dagger\cdot\vec{B}_j\right) - 2c_6\left(\vec{B}_i^\dagger\cdot\vec{B}_j\right)\left(\vec{B}_i^\dagger\cdot\vec{B}_j\right)\ .\end{aligned}$$

$SU(8)$ symmetry

Schrödinger Symmetry

The SWAP gate in each sector is achieved when $\delta = 0$ (free theory) in some channels and $\delta = \pi/2$ (unitary limit) in other channels.

The theory flows to a UV fixed point $\frac{d}{d\mu}\mu C_{R,R'} = 0$ in the unitary limit, and scattering length goes to infinity.

$\implies$ Schrödinger symmetry in these channels

Lattice Data

NPLQCD [1706.06550, 2009.12357]

	$C_{\mathbf{27}}$	$C_{\mathbf{10}}$	$C_{\overline{\mathbf{10}}}$	$C_{\mathbf{8_A}}$
natural [6]	$-16.7(2.8)$	$-50(50)$	$-11.1(2.5)$	$-7.7(1.8)$
unnatural [6]	$1.89(4)$	$1.75(6)$	$2.00(8)$	$2.17(9)$
natural [25]	-28^{+3}_{-5}	-	-29^{+3}_{-4}	-19^{+1}_{-1}
unnatural [25]	$10.0^{+0.5}_{-0.5}$	-	$11.3^{+0.5}_{-0.5}$	$12.8^{+0.5}_{-0.5}$

$m_\pi = 806$ MeV

$m_\pi = 450$ MeV

Table shows that $SU(6)$ symmetry ($C_{\mathbf{27}} = C_{\overline{\mathbf{10}}}$) holds up well in both simulations.
$SU(16)$ is present in unnatural case.

None of the symmetries is completely ruled out. Better simulation results need!

Conclusion and Outlook

We probe entanglement of the S-matrix in baryon-baryon scattering mediated by pionless EFT.

Identity and SWAP gates are realized by phase relations $\delta_R = \delta_{R'}$ and $\delta_R = 0, \delta_{R'} = \pi/2$ or $\delta_R = \pi/2, \delta_{R'} = 0$.

We find emergent symmetries of $SU(6)$, $SO(8)$, $SU(8)$ and $SU(16)$ when entanglement is minimized in different (Q, S) sectors.

Entanglement principles can be used to predict symmetries. We hope the framework can be used to study entanglement in other theories.

Thanks!

Backup slides

Minimally entangling operators in two qubit system

Any tensor product operator $U = U_A \otimes U_B$ does not entangle the spins.

The unitary operators of a two-spin system form $SU(4)$. For any $U \in SU(4)$, it has Cartan decomposition

$$U = (U_A \otimes U_B)U_d(V_A \otimes V_B), \qquad U_d = \exp(\sum_i i\beta_i \, \sigma_i \otimes \sigma_i),$$

What are the entanglement properties of U_d? The entanglement power of

$$U_d = \exp(\sum_i i\beta_i \, \sigma_i \otimes \sigma_i)$$

vanishes when $\beta_i = 0$ or $\pi/4$ for all i. [1] Given the $\pi/2$ periocity of β_i, these two solutions are all independent solutions of minimal entanglement.

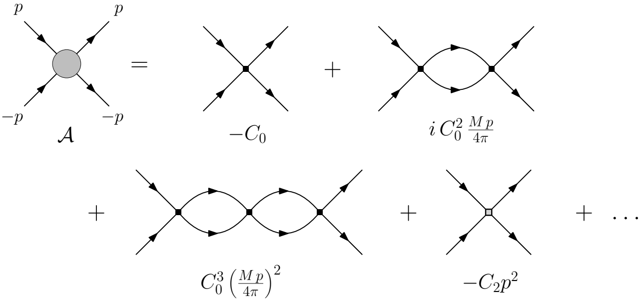
Minimally entangling operators with regard to $\beta_i = 0$ or $\pi/4$: **1** and $e^{i\frac{\pi}{4}}$ SWAP. They are unique up to local unitaries.

[1] Ian and Mehen, 2021. arxiv: 2104.10835

Power Counting in pionless EFT

One complexity of this theory comes from unnatural size of scattering length. It disturbs usual power counting.

Scale of pionless EFT: $\Lambda \sim m_\pi \simeq 140$ MeV



Natural scaling of Wilson coefficient with $2n$ derivatives:
$C_{2n} \ \sim\ \mathcal{O}\left(\frac{1}{M\Lambda^{2n+1}}\right)$

Unnaturalness of pionless EFT

Effective Range Expansion:
$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + \cdots = -\frac{1}{a} + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^n .$

Interaction is encoded in a, r_i which are measured in experiment.

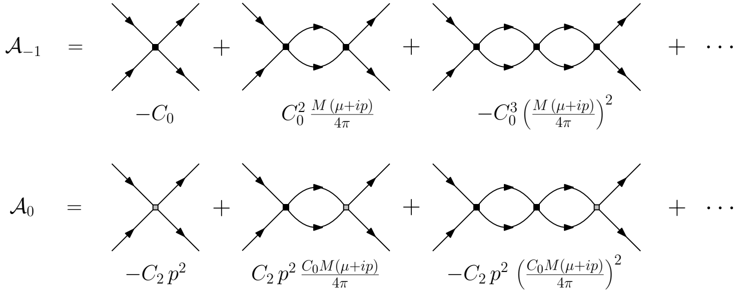
$\mathcal{A} = \frac{4\pi}{M} \frac{1}{p \cot \delta - ip} = \frac{4\pi}{M} \left[-\frac{1}{a} - ip + \frac{1}{2} r_0 p^2 + \cdots\right]^{-1} .$

Matching expansion from EFT and ERE shows $C_0 = 4\pi a/M$.

Unnaturally large scattering length due to shallow bound state.

$$a \gg 1/\Lambda \quad \Longrightarrow \quad \text{expansion breaks down when } 1/ap\Lambda$$

A fix to the problem



KSW-vK scheme subtracts extra poles and allows for a new expansion.

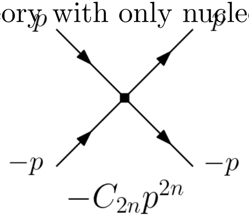
The leading order amplitude now consists of C_0 insertions to all orders.

"fine tuning" in nuclear physics

Nucleon-nucleon interactions

How does $SU(4)$ manifest in nucleon-nucleon interactions?

Nucleon-nucleon interactions of very low energy (below the threshold of pion mass) is described by an effective field theory with only nucleon field N (pionless EFT).



Non-relativistic QFT:

$$\mathcal{L}_{eff} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) \psi + C_0 (\psi^\dagger \psi)^2 + \frac{C_2}{8} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + \text{h.c.} \right] + \cdots$$

In NN scattering, two independent LO operators:

$$\mathcal{L}_{\text{LO}}^{n_f=2} = -\frac{1}{2} C_S \left(N^\dagger N \right)^2 - \frac{1}{2} C_T \left(N^\dagger \boldsymbol{\sigma} N \right) \cdot \left(N^\dagger \boldsymbol{\sigma} N \right) .$$

$$N = (p_\uparrow, p_\downarrow, n_\uparrow, n_\downarrow)^T \qquad \boldsymbol{\sigma} = (\sigma^1, \sigma^2, \sigma^3)^T$$

$SU(4)$ in nucleon-nucleon interactions

$$\mathcal{L}_{\text{LO}}^{n_f=2} = -\frac{1}{2}C_S\,(N^\dagger N)^2 - \frac{1}{2}C_T\,(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)\ .$$

C_S is $SU(4)$ invariant. C_T explicitly breaks $SU(4)$ Wigner symmetry.

Nucleon-nucleon scattering has two spin channels: $S = 0$ and $S = 1$.

$\frac{1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}$ and $\frac{3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}$ are projectors into $S = 0$ and $S = 1$ channels.

$$\bar{C}_0 = C_S - 3C_T \qquad \bar{C}_1 = C_S + C_T$$

$SU(4)$ implies $\bar{C}_0 = \bar{C}_1$.

In real life,

$$C_S = -1.2\times 10^{-4}\,\text{MeV}^{-2},\ C_T = -9.6\times 10^{-6}\,\text{MeV}^{-2}$$

$$C_T \ll C_S\ , \quad \bar{C}_0 \simeq \bar{C}_1$$

$SU(16)$ in baryon octet?

Can we generalize Wigner symmetry to three-flavor?
Eight particles $\implies$ $SU(16)$.

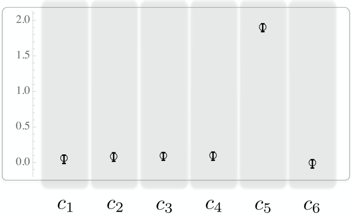
$$\mathcal{B} = (n_\uparrow, n_\downarrow, p_\uparrow, p_\downarrow, \dots)$$

Six LO operators of baryon-baryon interactions[2]

$$\mathcal{L}_{\mathrm{LO}}^{n_f=3} = -c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle - c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle\ .$$

i, j are spin indices

$SU(16)$ prediction: only c_5 nonzero.



Lattice data[3] seems to agree, but may not be the real case (unrealistic m_π)

[2] Savage and Wise, Phys.Rev.D 53 (1996), 349-354
[3] NPLQCD, Phys. Rev. D 96, 114510 (2017)

Implications of spin-flavor symmetry

At leading order, the Lagrangian has only two terms of dimension 6,

$$\mathcal{L} = -a \left(\Psi^\dagger_{\mu\nu\rho} \Psi^{\mu\nu\rho} \right)^2 - b \Psi^\dagger_{\mu\nu\sigma} \Psi^{\mu\nu\tau} \Psi^\dagger_{\rho\delta\tau} \Psi^{\rho\delta\sigma}.$$

When only spin-1/2 baryon octet is considered, the Lagrangian can be matched to the baryon interactions by writing indices in terms of the spin index and the flavor index, $(\mu, \nu, \rho) = (\alpha i, \beta j, \gamma k)$,

$$\Psi^{(\alpha i)(\beta j)(\gamma k)} = \Delta^{ijk}_{\alpha\beta\gamma} + \boxed{\frac{1}{\sqrt{18}} \left(B^i_{m,\alpha} \epsilon^{mjk} \epsilon_{\beta\gamma} + B^j_{m,\beta} \epsilon^{mki} \epsilon_{\gamma\alpha} + B^k_{m,\gamma} \epsilon^{mij} \epsilon_{\alpha\beta} \right)}.$$

.

Kaplan and Savage, Phys.Lett.B365:244-251,1996

Implications of spin-flavor symmetry

$$\mathcal{L}_{\rm LO}^{SU(6)} = -a\left(\Psi^\dagger_{\mu\nu\rho}\Psi^{\mu\nu\rho}\right)^2 - b\Psi^\dagger_{\mu\nu\sigma}\Psi^{\mu\nu\tau}\Psi^\dagger_{\rho\delta\tau}\Psi^{\rho\delta\sigma}$$

$$\begin{aligned}\mathcal{L}_{\rm LO}^{n_f=3} =& -c_1\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - c_2\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - c_3\langle B_i^\dagger B_j^\dagger B_i B_j\rangle \\ & - c_4\langle B_i^\dagger B_j^\dagger B_j B_i\rangle - c_5\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - c_6\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle\ .\end{aligned}$$

Projecting Ψ into B fields allows matching c_1-c_6 to a and b.

$$c_1 = \frac{7}{27}b \quad c_2 = \frac{1}{9}b \quad c_3 = \frac{10}{81}b \qquad c_4 = -\frac{14}{81}b \quad c_5 = a + \frac{2}{9}b \quad c_6 = -\frac{1}{9}b.$$

For $n_f = 2$ nucleon field, $SU(4)$ spin-flavor gives the same prediction as $SU(4)$ Wigner symmetry.

$$C_T = 0$$

Schrödinger Symmetry

Schrödinger symmetry is the conformal symmetry of non-relativistic QFT.

$$[J_a, J_b] = i\epsilon_{abc} J_c \ , \quad [J_a, P_b] = i\epsilon_{abc} P_c \ , \quad [J_a, K_b] = i\epsilon_{abc} K_c$$
$$[P_a, P_b] = 0 \ , \quad [K_a, K_b] = 0 \ , \quad [K_a, P_b] = iM\delta_{ab}$$
$$[H, J_a] = 0 \ , \quad [H, P_a] = 0 \ , \quad [H, K_a] = iP_a$$

Schrödinger symmetry is present when the scattering length is infinite.

This is the case in NN scattering.

$$a_0 = -24\text{fm},\ a_0^{-1} = -8.2\text{MeV} \qquad a_1 = 5.4\text{fm},\ a_1^{-1} = 36\text{MeV}$$