Gravitational transition form factors of $N \to \Delta$ through light-cone QCD sum rules

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Outline

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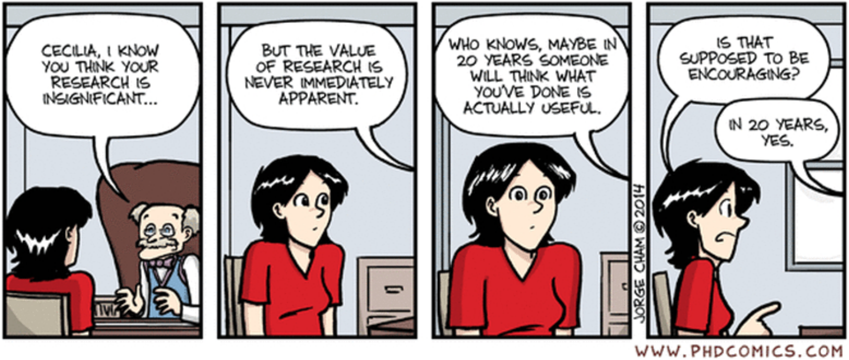
CECILIA, I KNOW YOU THINK YOUR RESEARCH IS INSIGNIFICANT...
BUT THE VALUE OF RESEARCH IS NEVER IMMEDIATELY APPARENT.
WHO KNOWS, MAYBE IN 20 YEARS SOMEONE WILL THINK WHAT YOU'VE DONE IS ACTUALLY USEFUL.
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IS THAT SUPPOSED TO BE ENCOURAGING?
IN 20 YEARS, YES.
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Motivation

- Form Factors* (FFs)? Why are they important?
- FFs describe how hadrons interact with each other and give information about the internal structure of the hadrons.
- FFs are well known that many fundamental properties hadrons, e.g., the distribution of their charge, the origin and strength of their magnetization, and their (possibly deformed) shape, can be studied on the basis of hadron form factors.
- The energy-momentum tensor or gravitational form factors (GFFs) of hadrons are basic quantities that carry valuable information on different aspects of the hadron's structure.
- The GFFs are used to calculate the pressure and energy distributions inside the hadrons as well as quantities related to its geometric shape.
- The GFFs are also sources of information on the fractions of the momenta carried by the quarks and gluons as ingredients of the hadrons. They help us know how the total angular momenta of quarks and gluons form the hadron's spin.
- They also provide knowledge on the distribution and stabilization of the strong force inside the hadrons.

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*Hofstadter et al, Rev.Mod.Phys., 1956

- FFs are non-perturbative objects.
- To study these processes, a reliable non perturbative method is needed.
- One of these methods is the light-cone QCD sum rules (LCSR)* approach.
- The LCSR method is one of the most powerful and applicable non-perturbative tools to hadron physics.
- In the LCSR, the hadronic parameters are expressed in terms of the properties of the vacuum and the distribution amplitudes.
- In the LCSR, OPE is carried out near light cone ($x^2 \simeq 0$).

*Braun et al. Z. Phys C 1989, Braun et al., Nucl.Phys. B 1989, Chernyak et al. Nucl.Phys. B 1990

- One starts with a correlation function of the form,

$\Pi_{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\langle 0|T\{J_\mu(x)J_\nu(0)\}|H(p,s)\rangle$

- The correlation functions can be expressed in terms of the properties of hadrons (hadronic side) and also in terms of the properties of the vacuum and distribution amplitudes (QCD side).
- By inserting complete sets of hadronic states, the correlation function can be written as:

$$\Pi_{\mu\nu}(p,q) = \sum_{p'} \frac{\langle 0|J_\mu|H^{'}(p')\rangle\langle H^{'}(p')|J_\nu|H(p)\rangle}{m_{H'}^2 - p'^2} + \ldots$$

- matrix elements

$\langle 0|J_\mu(0)|h^{'}(p')\rangle = \lambda_{H'}\, u^\mu(p')$

where $u^\mu(p')$ spinor and $\lambda_{H'}$ residue.

- The matrix elements $\langle H^{'}(p')|J_\nu|H(p)\rangle$ can be expressed in terms of coupling constants or form factors.

- The correlation function can also be calculated in the deep Euclidean region using OPE:

$$\Pi = \sum_d C_d(x^2) O_d(x)$$

- In the case of mass sum rules or traditional sum rules, $O_d(x)$ are local operators. After Fourier transform, the correlation function becomes:

$$\Pi = \sum_d C_d(p^2) \langle H^{'}(p') | O_d | H(p) \rangle$$

- In case of light-cone QCD sum rules, matrix elements of the form $\langle H^{'}(p') | O_d | H(p) \rangle$ are needed.
- The matrix elements are expanded around $x^2 \simeq 0$ in terms of distribution amplitudes.

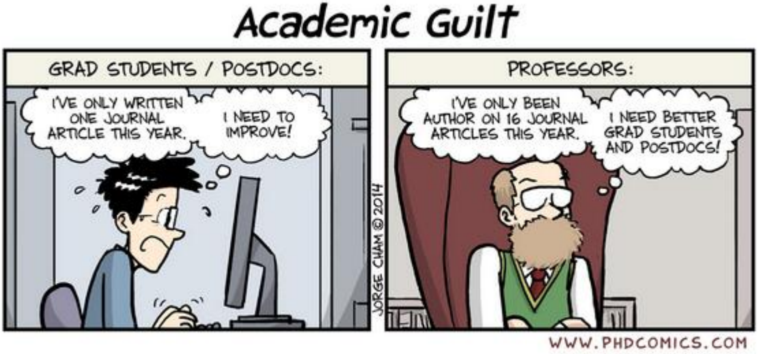
- Two expressions for the correlation function is matched using spectral representation.

$$\Pi(p^2) = \int ds \frac{\rho(s)}{s-p^2} + p^2(polynomial)$$

- To subtract the contributions of higher states and continuum, quark hadron duality is assumed:

$$\Pi^{hadron}(p^2) = \int_0^{s_0} ds\rho(s)e^{-\frac{s}{M^2}}$$

- For $p^2 > 0$, the correlation function is calculated in terms of hadronic parameters. In the deep Euclidean region, $p^2 << 0$, the correlation function is calculated using the OPE in terms of QCD degrees of freedom.
- Sum rules are obtained by matching the two representation using spectral representation.



Gravitational form factors of $N \to \Delta$ in light-cone QCD sum rules*

*UO and K. Azizi, JHEP03(2023)048, 2023

Hadronic Representation

$$\Pi_{\alpha\mu\nu}(p,q) = i \int d^4x\, e^{iqx} \langle 0 | \mathcal{T}\{J^{\Delta}_{\alpha}(0) T^{q}_{\mu\nu}(x)\} | N(p) \rangle , \tag{1}$$

In this part, we show how to correlation function is related to the physically observed hadrons. Inserting a complete set of intermediate hadronic states with the same quantum number as interpolating currents,

$$\Pi^{Had}_{\alpha\mu\nu}(p,q) = \sum_{s'} \frac{\langle 0 | J^{\Delta}_{\alpha} | \Delta(p',s') \rangle \langle \Delta(p',s') | T^{q}_{\mu\nu} | N(p,s) \rangle}{m^2_{\Delta} - p'^2} + \ldots , \tag{2}$$

where

$$\begin{aligned}
\langle \Delta(p',s') | T^{q}_{\mu\nu} | N(p,s) \rangle = \bar{u}_{\beta}(p',s') \Bigg[& F^{N\Delta}_1(Q^2) \Big\{ g_{\beta\{\mu} P_{\nu\}} + \frac{(m^2_{\Delta} - m^2_N)}{\Delta^2} g_{\mu\nu}\Delta_{\beta} - \frac{(m^2_{\Delta} - m^2_N)}{2\Delta^2} g_{\beta\{\mu}\Delta_{\nu\}} - \frac{\Delta_{\beta} P_{\{\mu}\Delta_{\nu\}}}{\Delta^2} \Big\} \\
& + \frac{F^{N\Delta}_2(Q^2)}{\bar{m}^2} \Big\{ P_{\mu}P_{\nu}\Delta_{\beta} + \frac{(m^2_{\Delta} - m^2_N)^2}{4\Delta^2} g_{\mu\nu}\Delta_{\beta} - \frac{(m^2_{\Delta} - m^2_N)}{2\Delta^2} P_{\{\mu}\Delta_{\nu\}}\Delta_{\beta} \Big\} \\
& + \frac{F^{N\Delta}_3(Q^2)}{\bar{m}^2} \Big\{ (\Delta_{\mu}\Delta_{\nu} - \Delta^2 g_{\mu\nu})\Delta_{\beta} \Big\} + F^{N\Delta}_4(Q^2)\bar{m} \Big\{ \gamma_{\{\mu} g_{\nu\}\beta} + \frac{2(m_{\Delta} + m_N)}{\Delta^2} g_{\mu\nu}\Delta_{\beta} \\
& - \frac{(m_{\Delta} + m_N)}{\Delta^2} g_{\beta\{\mu}\Delta_{\nu\}} - \frac{1}{\Delta^2}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_{\beta} \Big\} + \frac{F^{N\Delta}_5(Q^2)}{\bar{m}} \Big\{ \gamma_{\{\mu}P_{\nu\}}\Delta_{\beta} + \frac{(m^2_{\Delta} - m^2_N)(m_{\Delta} + m_N)}{\Delta^2} g_{\mu\nu}\Delta_{\beta} \\
& - \frac{(m_{\Delta} + m_N)}{\Delta^2} \Delta_{\{\mu}P_{\nu\}}\Delta_{\beta} - \frac{(m^2_{\Delta} - m^2_N)}{2\Delta^2}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_{\beta} \Big\} + \bar{C}^{N\Delta}_1(Q^2) g_{\mu\nu}\Delta_{\beta} + \frac{\bar{C}^{N\Delta}_2(Q^2)}{\bar{m}^2}\Delta_{\{\mu}P_{\nu\}}\Delta_{\beta} \\
& + \frac{\bar{C}^{N\Delta}_3(Q^2)}{\bar{m}}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_{\beta} + \bar{C}^{N\Delta}_4(Q^2) g_{\beta\{\mu}\Delta_{\nu\}} \Bigg] \gamma_5 u_N(p,s) .
\end{aligned} \tag{3}$$

Hadronic Representation

- Hadronic side of the correlation function;

$$
\begin{aligned}
\Pi^{Had}_{\alpha\mu\nu}(p,q) = {} & \frac{\lambda_\Delta}{m_\Delta^2 - p'^2}\left[-(\not{p}' + m_\Delta)\{g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2p'_\alpha p'_\beta}{3m_\Delta^2} + \frac{p'_\alpha\gamma_\beta - p'_\beta\gamma_\alpha}{3m_\Delta}\}\right] \\
& \times \left[F_1^{N\Delta}(Q^2)\Big\{ g_{\beta\{\mu}P_{\nu\}} + \frac{(m_\Delta^2 - m_N^2)}{\Delta^2} g_{\mu\nu}\Delta_\beta - \frac{(m_\Delta^2 - m_N^2)}{2\Delta^2} g_{\beta\{\mu}\Delta_{\nu\}} - \frac{\Delta_\beta P_{\{\mu}\Delta_{\nu\}}}{\Delta^2}\Big\} \right. \\
& + \frac{F_2^{N\Delta}(Q^2)}{\bar{m}^2}\Big\{ P_\mu P_\nu \Delta_\beta + \frac{(m_\Delta^2 - m_N^2)^2}{4\Delta^2} g_{\mu\nu}\Delta_\beta - \frac{(m_\Delta^2 - m_N^2)}{2\Delta^2} P_{\{\mu}\Delta_{\nu\}}\Delta_\beta \Big\} \\
& + \frac{F_3^{N\Delta}(Q^2)}{\bar{m}^2}\Big\{ (\Delta_\mu\Delta_\nu - \Delta^2 g_{\mu\nu})\Delta_\beta \Big\} \\
& + F_4^{N\Delta}(Q^2)\bar{m}\Big\{ \gamma_{\{\mu}g_{\nu\}\beta} + \frac{2(m_\Delta + m_N)}{\Delta^2} g_{\mu\nu}\Delta_\beta - \frac{(m_\Delta + m_N)}{\Delta^2} g_{\beta\{\mu}\Delta_{\nu\}} - \frac{1}{\Delta^2}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_\beta \Big\} \\
& + \frac{F_5^{N\Delta}(Q^2)}{\bar{m}}\Big\{ \gamma_{\{\mu}P_{\nu\}}\Delta_\beta + \frac{(m_\Delta^2 - m_N^2)(m_\Delta + m_N)}{\Delta^2} g_{\mu\nu}\Delta_\beta - \frac{(m_\Delta + m_N)}{\Delta^2}\Delta_{\{\mu}P_{\nu\}}\Delta_\beta \\
& - \frac{(m_\Delta^2 - m_N^2)}{2\Delta^2}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_\beta \Big\} + \bar{C}_1^{N\Delta}(Q^2) g_{\mu\nu}\Delta_\beta + \frac{\bar{C}_2^{N\Delta}(Q^2)}{\bar{m}^2}\Delta_{\{\mu}P_{\nu\}}\Delta_\beta \\
& \left. + \frac{\bar{C}_3^{N\Delta}(Q^2)}{\bar{m}}\gamma_{\{\mu}\Delta_{\nu\}}\Delta_\beta + \bar{C}_4^{N\Delta}(Q^2) g_{\beta\{\mu}\Delta_{\nu\}} \right]\gamma_5 u_N(p,s), \qquad (4)
\end{aligned}
$$

where $P = (p' + p)/2$, $\Delta = p' - p$, $\bar{m} = (m_N + m_\Delta)/2$ and $X_{\{\mu}Y_{\nu\}} = (X_\mu Y_\nu + X_\nu Y_\mu)/2$.

QCD Representation

$$\Pi_{\alpha\mu\nu}(p,q) = i \int d^4 x e^{iqx} \langle 0 | \mathcal{T}\{ J^{\Delta}_{\alpha}(0) T^{q}_{\mu\nu}(x)\} | N(p)\rangle , \tag{5}$$

where

$$J^{\Delta}_{\alpha}(0) = \frac{1}{\sqrt{3}} \epsilon^{abc}[2(u^{aT}(0)C\gamma_{\alpha} d^{b}(0))u^{c}(0) + (u^{aT}(0)C\gamma_{\alpha} u^{b}(0))d^{c}(0)], \tag{6}$$

$$T_{\mu\nu}(x) = \sum_{q} T^{q}_{\mu\nu}(x) + T^{g}_{\mu\nu}(x) \tag{7}$$

with

$$T^{q}_{\mu\nu}(x) = \frac{i}{2}\Big\{ \bar{u}(x)\overleftrightarrow{D}_{\mu}\gamma_{\nu} u(x) + \bar{d}(x)\overleftrightarrow{D}_{\mu}\gamma_{\nu} d(x) + (\mu \leftrightarrow \nu)\Big\}$$
$$-ig_{\mu\nu}\Big[\bar{u}(x)(\overleftrightarrow{\not{D}} - m_u)u(x) + \bar{d}(x)\Big(\overleftrightarrow{\not{D}} - m_d\Big)d(x)\Big], \tag{8}$$

$$T^{g}_{\mu\nu}(x) = \frac{1}{4} g_{\mu\nu} F^{\alpha\beta}(x)F_{\alpha\beta}(x) - F^{\mu\alpha}(x)F^{\nu}_{\alpha}(x). \tag{9}$$

The covariant derivative, $\overleftrightarrow{D}_{\mu}(x)$, is defined as

$$\overleftrightarrow{D}_{\mu}(x) = \frac{1}{2}\Big[\overrightarrow{D}_{\mu}(x) - \overleftarrow{D}_{\mu}(x)\Big], \tag{10}$$

where

$$\overrightarrow{D}_{\mu}(x) = \overrightarrow{\partial}_{\mu}(x) - i\frac{g}{2}\lambda^{a}A^{a}_{\mu}(x), \tag{11}$$

$$\overleftarrow{D}_{\mu}(x) = \overleftarrow{\partial}_{\mu}(x) + i\frac{g}{2}\lambda^{a}A^{a}_{\mu}(x). \tag{12}$$

QCD Representation

- QCD side of the correlation function:

$$
\begin{aligned}
(\Pi^{QCD}_{\alpha\mu\nu})_{\lambda\eta}(p,q) = &-\frac{1}{8\sqrt{3}}\int d^4x e^{iqx}\Big[(C\gamma_\alpha)_{\alpha\beta}(\overleftrightarrow{D}_\mu(x)\gamma_\nu)_{\rho\sigma} + (\mu\leftrightarrow\nu)\Big] \\
&\times\Big\{4\epsilon^{abc}\langle 0|u^a_\sigma(0)u^b_\theta(x)d^c_\phi(0)|N(p,s)\rangle\Big[2\delta^\eta_\alpha\delta^\theta_\sigma\delta^\phi_\beta S_q(-x)_{\lambda\rho} \\
&+ 2\delta^\eta_\lambda\delta^\theta_\sigma\delta^\phi_\beta S_q(-x)_{\alpha\rho} + \delta^\eta_\alpha\delta^\theta_\sigma\delta^\phi_\lambda S_q(-x)_{\beta\rho} + \delta^\eta_\beta\delta^\theta_\sigma\delta^\lambda_\phi S_q(-x)_{\alpha\rho}\Big] \\
&- 4\epsilon^{abc}\langle 0|u^a_\sigma(0)u^b_\theta(0)d^c_\phi(x)|N(p,s)\rangle\Big[2\delta^\eta_\alpha\delta^\theta_\lambda\delta^\phi_\sigma S_q(-x)_{\beta\rho} \\
&+ \delta^\eta_\alpha\delta^\theta_\beta\delta^\phi_\sigma S_q(-x)_{\lambda\rho}\Big]\Big\}, \qquad (13)
\end{aligned}
$$

Where

$$
S(x) = \frac{i\not{x}}{2\pi^2x^4} - \frac{\langle q\bar{q}\rangle}{12}\left(1+\frac{m_0^2x^2}{16}\right) - ig_s\int_0^1 dv\left[\frac{\not{x}}{16\pi^2x^4}G_{\mu\nu}\sigma^{\mu\nu} - vx^\mu G_{\mu\nu}\gamma^\nu\frac{i}{4\pi^2x^2}\right].
$$

The $\langle 0|\epsilon^{abc}u^a_\sigma(0)u^b_\theta(0)d^c_\phi(x)|N(p)\rangle$ matrix element is the expression containing the distribution amplitudes of the nucleon [Braun et al., PRD 73, 2006].

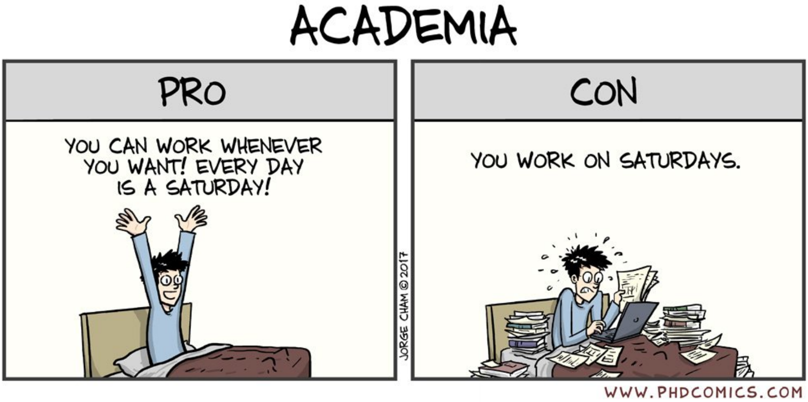
QCD Sum Rules

The required light-cone QCD sum rules for the $N \rightarrow \Delta$ GFFs are obtained by matching the coefficients of different Lorentz structures from both the hadronic and QCD representations of the correlators.

$$
\begin{aligned}
&F_1^{N\Delta}(Q^2)\,\frac{\lambda_\Delta}{(m_\Delta^2 - p'^2)} = -2\,\rho_1^{QCD}(v, y, x_1, x_2, x_3), &&F_2^{N\Delta}(Q^2)\,\frac{\lambda_\Delta(m_\Delta + m_N)}{(m_\Delta^2 - p'^2)} = \bar{m}^2\,\rho_2^{QCD}(v, y, x_1, x_2, x_3),\\
&F_3^{N\Delta}(Q^2)\,\frac{\lambda_\Delta}{(m_\Delta^2 - p'^2)} = -2\,\bar{m}^2\,\rho_3^{QCD}(v, y, x_1, x_2, x_3), &&F_4^{N\Delta}(Q^2)\,\frac{\lambda_\Delta}{(m_\Delta^2 - p'^2)} = -\frac{3}{\bar{m}}\,\rho_4^{QCD}(v, y, x_1, x_2, x_3),\\
&F_5^{N\Delta}(Q^2)\,\frac{\lambda_\Delta}{(m_\Delta^2 - p'^2)} = 2\,\bar{m}\,\rho_5^{QCD}(v, y, x_1, x_2, x_3), &&\bar{C}_1^{N\Delta}(Q^2)\,\frac{\lambda_\Delta(m_\Delta + m_N)}{(m_\Delta^2 - p'^2)} = \rho_6^{QCD}(v, y, x_1, x_2, x_3),\\
&\bar{C}_2^{N\Delta}(Q^2)\,\frac{(m_\Delta + m_N)\,\lambda_\Delta}{(m_\Delta^2 - p'^2)} = \frac{\bar{m}^2}{2}\,\rho_7^{QCD}(v, y, x_1, x_2, x_3), &&\bar{C}_3^{N\Delta}(Q^2)\,\frac{\lambda_\Delta(m_\Delta + m_N)}{(m_\Delta^2 - p'^2)} = \bar{m}\,\rho_8^{QCD}(v, y, x_1, x_2, x_3),\\
&\bar{C}_4^{N\Delta}(Q^2)\,\frac{\lambda_\Delta}{(m_N + m_\Delta)\,(m_\Delta^2 - p'^2)} = \rho_9^{QCD}(v, y, x_1, x_2, x_3). \qquad (14)
\end{aligned}
$$

We should stress that, we use the Lorentz structures $q_\mu\, g_{\alpha\nu}\, \not{q}\gamma_5$, $p'_\mu p'_\nu\, q_\alpha\, \gamma_5$, $q_\mu\, q_\nu\, q_\alpha\, \not{q}\gamma_5$, $g_{\alpha\mu}\, \gamma_\nu\, \not{q}\gamma_5$, $q_\alpha\, q_\mu\, \gamma_\nu\, \not{q}\gamma_5$, $g_{\mu\nu}\, q_\alpha\, \gamma_5$, $p'_\mu\, q_\alpha\, q_\nu\, \gamma_5$, $q_\alpha\, q_\nu\, \gamma_\mu\, \gamma_5$ and $g_{\alpha\mu}\, q_\nu\, \gamma_5$ to find the LCSR for the $N \rightarrow \Delta$ transition GFFs, $F_1^{N\Delta}(Q^2)$, $F_2^{N\Delta}(Q^2)$, $F_3^{N\Delta}(Q^2)$, $F_4^{N\Delta}(Q^2)$, $F_5^{N\Delta}(Q^2)$, $\bar{C}_1^{N\Delta}(Q^2)$, $\bar{C}_2^{N\Delta}(Q^2)$, $\bar{C}_3^{N\Delta}(Q^2)$, and $\bar{C}_4^{N\Delta}(Q^2)$, respectively.

$$
\begin{aligned}
\rho_1^{QCD}(v, y, x_1, x_2, x_3) &= \frac{m_N^3}{2\sqrt{3}} \int_0^1 \frac{(1+x_2)}{(q-px_2)^4} dx_2 \int_0^{1-x_2} dx_1 \Big[\big(A_1^M - T_1^M - 2V_1^M\big)(x_1, x_2, 1-x_1-x_2)\Big] \\
&- \frac{m_N^3}{2\sqrt{3}} \int_0^1 \frac{(1+x_3)}{(q-px_3)^4} dx_3 \int_0^{1-x_3} dx_1 \Big[T_1^M(x_1, 1-x_1-x_3, x_3)\Big] \\
&- \frac{m_N}{4\sqrt{3}} \int_0^1 \frac{(1+x_2)}{(q-px_2)^2} dx_2 \int_0^{1-x_2} dx_1 \Big[\big(2A_1 + A_3 - 2V_1 + V_3\big)(x_1, x_2, 1-x_1-x_2)\Big] \\
&+ \frac{m_N}{4\sqrt{3}} \int_0^1 \frac{(1+x_3)}{(q-px_3)^2} dx_3 \int_0^{1-x_3} dx_1 \Big[\big(A_3 + 2T_1 + V_3\big)(x_1, 1-x_1-x_3, x_3)\Big] \\
&+ \frac{m_N^3}{4\sqrt{3}} \int_0^1 \frac{(1+v)}{(q-pv)^4} dv \int_0^v dy \int_y^1 dx_2 \int_0^{1-x_2} dx_1 \Big[\big(T_1 - T_2 - T_5 + T_6 - 2T_7 - 2T_8 \\
&+ V_1 - V_2 - V_3 - V_4 - V_5 + V_6\big)(x_1, x_2, 1-x_1-x_2)\Big] \\
&+ \frac{m_N^3}{4\sqrt{3}} \int_0^1 \frac{(1+v)}{(q-pv)^4} dv \int_0^v dy \int_y^1 dx_3 \int_0^{1-x_3} dx_1 \Big[\big(-A_1 + A_2 - A_3 - A_4 + A_5 - A_6 - 4T_1 \\
&+ 2T_2 + 2T_3 + 2T_4 + 2T_5 - 4T_6 + 2T_7 + 2T_8 + V_1 - V_2 - V_3 - V_5 + V_6\big)(x_1, 1-x_1-x_3, x_3)\Big] \\
&+ \frac{m_N}{4\sqrt{3}} \int_0^1 \frac{1}{(q-py)^2} dy \int_y^1 dx_2 \int_0^{1-x_2} dx_1 \Big[\big(-2A_1 + 2A_2 - 2A_3 - T_1 + T_2 + 2T_7 + 2V_1 - 2V_2 \\
&- 2V_3\big)(x_1, x_2, 1-x_1-x_2)\Big] \\
&+ \frac{m_N}{\sqrt{3}} \int_0^1 \frac{1}{(q-py)^2} dy \int_y^1 dx_3 \int_0^{1-x_3} dx_1 \Big[\big(T_1 + T_3 - T_7\big)(x_1, 1-x_1-x_3, x_3)\Big]. \qquad (15)
\end{aligned}
$$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Results

- The values of shape parameters of nucleon DAs in two different sets are given in Table-I. In addition, we use: $m_u = m_d = 0$, $m_N = 0.94$ GeV, $m_\Delta = 1.23$ GeV, $\lambda_\Delta = 0.038$, $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3$ GeV^3 and $m_0^2 = 0.8 \pm 0.1$ GeV^2.

	Set-I	Set-II
f_N	$(5.0 \pm 0.5) \times 10^{-3}\ GeV^2$	$(5.0 \pm 0.5) \times 10^{-3}\ GeV^2$
λ_1	$(-2.7 \pm 0.9) \times 10^{-2}\ GeV^2$	$(-2.7 \pm 0.9) \times 10^{-2}\ GeV^2$
λ_2	$(5.4 \pm 1.9) \times 10^{-2}\ GeV^2$	$(5.4 \pm 1.9) \times 10^{-2}\ GeV^2$
A_1^u	0.38 ± 0.15	0
V_1^d	0.23 ± 0.03	$1/3$
f_1^d	0.40 ± 0.05	$1/3$
f_2^d	0.22 ± 0.05	$4/15$
f_1^u	0.07 ± 0.05	$1/10$

Table-I: The numerical values of the main input parameters entering the expressions of the nucleon's DAs [Braun et al, PRD 73, 2006].

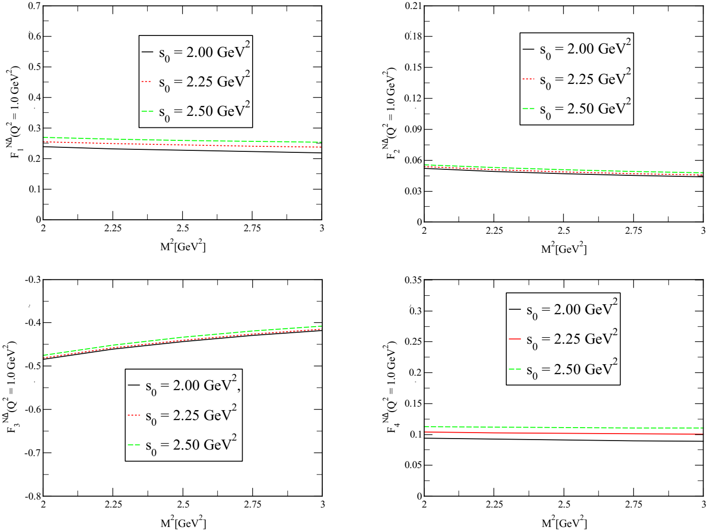


Figure-1: The dependence of the $N \to \Delta$ transition GFFs on M^2 at $Q^2 = 1.0$ GeV^2 and three fixed values of the s_0 and set-I parameters.

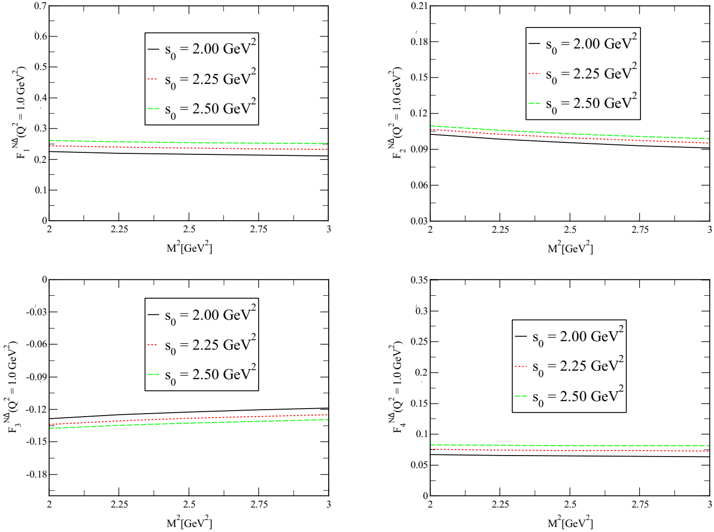


Figure-2: The dependence of the $N \to \Delta$ transition GFFs on M^2 at $Q^2 = 1.0$ GeV^2 and three fixed values of the s_0 and set-II parameters.

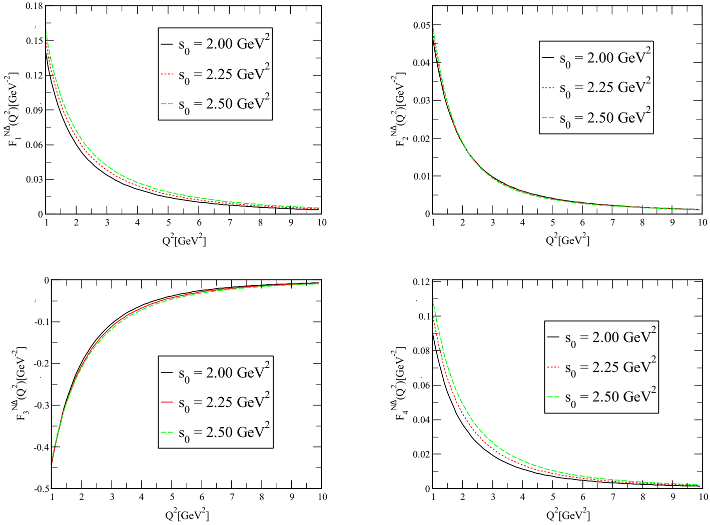


Figure-3: The dependence of the $N \to \Delta$ transition GFFs on Q^2 at fixed values of the s_0, average M^2 and set-I parameters.

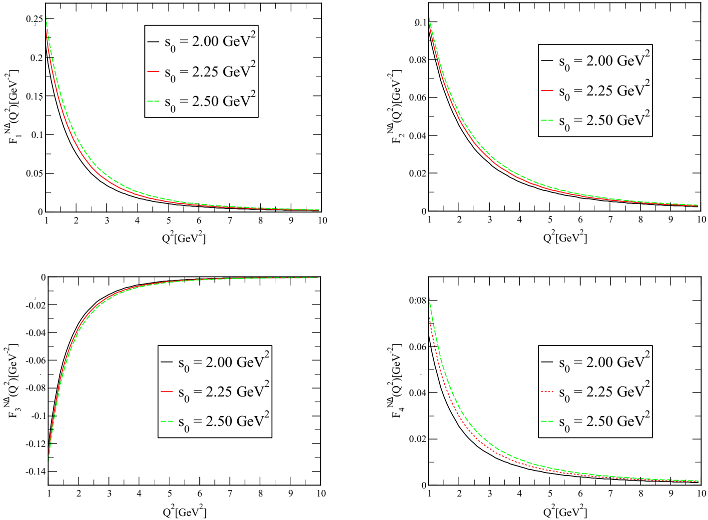


Figure-4: The dependence of the $N \to \Delta$ transition GFFs on Q^2 at fixed values of the s_0, average M^2 and set-II parameters.

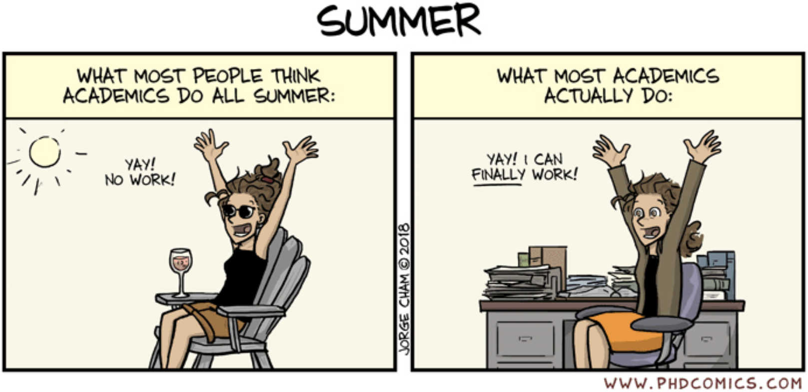
Fit function:

$$F_i^{N\Delta}(Q^2) = f(0)\left[1 + \frac{Q^2}{\mathcal{M}^2}\right]^{-\alpha}. \qquad (16)$$

	Results of set-I			Results of set-II		
Form Factors	$f(0)(\mathrm{GeV}^{-2})]$	$\mathcal{M}$(GeV)	α	$f(0)(\mathrm{GeV}^{-2})$	$\mathcal{M}$(GeV)	α
$F_1^{N\Delta}(Q^2)$	0.80 ± 0.08	1.17 ± 0.12	$2.1 - 2.3$	1.10 ± 0.20	1.18 ± 0.10	$2.8 - 3.0$
$F_2^{N\Delta}(Q^2)$	0.20 ± 0.03	1.11 ± 0.05	$2.4 - 2.6$	0.31 ± 0.04	1.26 ± 0.11	$2.2 - 2.4$
$F_3^{N\Delta}(Q^2)$	-1.57 ± 0.26	1.22 ± 0.10	$2.3 - 2.5$	-0.97 ± 0.12	1.14 ± 0.11	$3.4 - 3.6$
$F_4^{N\Delta}(Q^2)$	0.38 ± 0.06	1.17 ± 0.10	$2.4 - 2.6$	0.28 ± 0.03	1.20 ± 0.11	$2.4 - 2.6$
$F_5^{N\Delta}(Q^2)$	-0.51 ± 0.05	1.18 ± 0.09	$2.2 - 2.4$	-0.71 ± 0.11	1.10 ± 0.13	$2.2 - 2.4$
$\bar{C}_1^{N\Delta}(Q^2)$	-0.073 ± 0.003	1.24 ± 0.10	$2.4 - 2.6$	0.083 ± 0.003	1.24 ± 0.11	$2.4 - 2.6$
$\bar{C}_2^{N\Delta}(Q^2)$	0.35 ± 0.03	1.29 ± 0.05	$2.3 - 2.5$	0.63 ± 0.07	1.16 ± 0.10	$2.1 - 2.3$
$\bar{C}_3^{N\Delta}(Q^2)$	0.20 ± 0.02	1.29 ± 0.03	$1.8 - 2.0$	0.28 ± 0.03	1.19 ± 0.10	$2.7 - 2.9$
$\bar{C}_4^{N\Delta}(Q^2)$	$-$	$-$	$-$	$-$	$-$	$-$

Table-II: Numerical values of the fitting parameters for the transition GFFs of $N - \Delta$.

- We investigated the transition gravitational form factors of the $N \to \Delta$ for the first time.
- We used multipole fit functions to extrapolate the results to the small values of Q^2, $0 \leq Q^2 < 1$ GeV^2, to find the values of the gravitational form factors at static limit.
- The study of transition form factors offers a new avenue to explore the QCD structure and the dynamic properties of resonances.
- Investigation of the energy-momentum tensor current interactions of hadrons can give us valuable information about their mass and spin as well as the pressure and shear force their inside.
- Our results may be checked by other phenomenological models including the Lattice QCD as well as future related experiments.
- The direct measurement of the $N - \Delta$ gravitational form factors may not be possible with the present facilities. However, these form factors can be extracted from the $N - \Delta$ GPDs. The project of extracting the transition GPDs from the Hall-B at JLab based on data acquired with the CLAS spectrometer is ongoing.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