J/ψ-pair production at LHC to study gluon TMD distributions: pushing the limits of the CS evolution formalism

12/01/2024

JELLE BOR

university of groningen / faculty of mathematics and natural sciences
van swinderen institute for particle physics and gravity

université PARIS-SACLAY | FACULTÉ DES SCIENCES D'ORSAY

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

Motivation

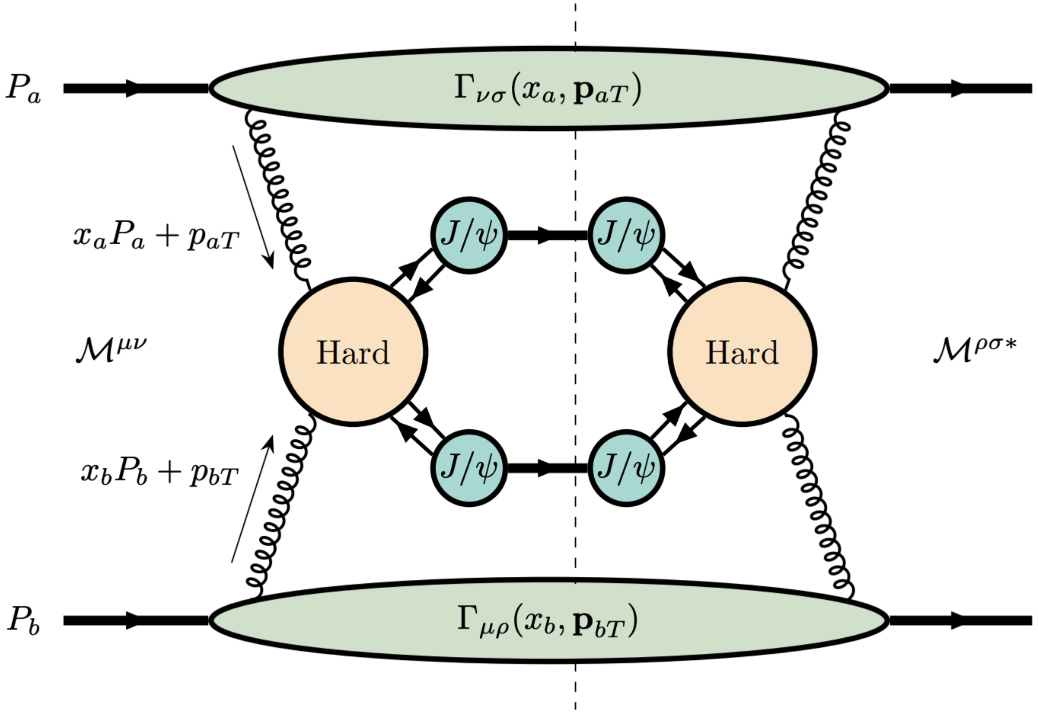
- TMDs incorporate the transverse momentum of quarks and gluons inside hadrons:
  "3D PDF"
- At small longitudinal momentum fraction x the gluons dominate, but hardly anything is known about the gluon TMDs experimentally.

- Heavy quarks are very sensitive to the gluon content of hadrons:
  - they are predominantly produced from gluons
  - not intrinsically present in hadrons at small momentum fractions.
- Furthermore, some quarkonium states, like the J/ψ, are relatively straightforward to detect and numerous events can be collected.

$\Longrightarrow$ Quarkonium production can been considered as a main tool to extract gluon TMDs

$p + p \to J/\psi + J/\psi + X$

- J/ψ-pair production gives via its P_T-spectrum and modulations access to the gluon TMDs *Lansberg et al. 2018, Scarpa et al. 2020*



$|\mathbf{q}_T| \ll \mu_H = M_{QQ}$

$x_{a,b} = \exp(\pm y_{QQ}) M_{QQ}/\sqrt{S}$

- Probe the transverse momentum of the partonic gluons via the observed quarkonia: $\boldsymbol{p}_{aT} + \boldsymbol{p}_{bT} = \boldsymbol{q}_T$
- The invariant mass M_{QQ} allows to study scale evolution of the TMDs

- Make use of CS-model in which TMD-factorization breaking effects are avoided (@ LO α_s^4)
- No TMDShF / smearing effects are expected for CS quarkonium at LO

- There are recent measurements of this process *LHCb 2023*

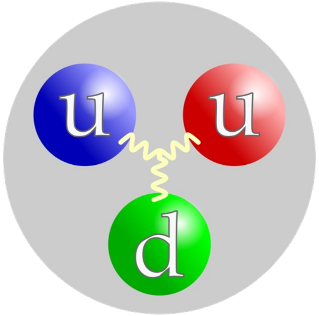
The Gluon TMD and the hadron correlator

- Unpolarized proton is parameterized by two functions at LO (twist ∼ 1/hard scale)
  - Unpolarized gluon distribution: f_1^g
  - Linearly polarized gluon distribution: $h_1^{\perp g}$

		Parent hadron polarization		
		Unpolarized	Longitudinal	Transverse
Parton polarization	U	f_1 (Number density)		$f_{1T}^{\perp}$ (Sivers)
	L		g_{1L} (Helicity)	g_{1T} (Worm-gear)
	T	$h_1^{\perp}$ (Boer-Mulders)	$h_{1L}^{\perp}$ (Worm-gear)	h_{1T} (Transversity) $h_{1T}^{\perp}$ (Pretzelosity)

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2}\left\{ -g_T^{ij} \boxed{f_1(x, \mathbf{k}_T^2)} + \left(\frac{k_T^i k_T^j}{M_h^2} + g_T^{ij} \frac{\mathbf{k}_T^2}{2M_h^2} \right) \boxed{h_1^{\perp}(x, \mathbf{k}_T^2)} \right\}$$

Mulders and Rodrigues 2001



The differential cross section at LO

$$d\sigma_{UU}^{gg} \equiv \frac{d\sigma}{dM_{QQ}\, dy_{QQ}\, d^2\mathbf{q}_T\, d\cos\theta_{CS}\, d\phi_{CS}} = \frac{\sqrt{M_{QQ}^2 - 4M_Q^2}}{(2\pi)^2 8S\, M_{QQ}^2} \Big\{ F_1 \times \mathcal{C}[f_1^g f_1^g] + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] + \{F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \times \mathcal{C}[w_3' h_1^{\perp g} f_1^g]\} \cos(2\phi_{CS}) + F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}] \cos(4\phi_{CS}) \Big\},$$

$$w_2 = \frac{1}{4M_h^4}\left[2(\mathbf{p}_{aT}\cdot\mathbf{p}_{bT})^2 - \mathbf{p}_{aT}^2\mathbf{p}_{bT}^2\right]$$

$$w_3 = \frac{1}{2M_h^2\mathbf{q}_T^2}[\mathbf{p}_{bT}^2\mathbf{q}_T^2 - 2(\mathbf{p}_{bT}\cdot\mathbf{q}_T)^2], \quad w_3' = \frac{1}{2M_h^2\mathbf{q}_T^2}[\mathbf{p}_{aT}^2\mathbf{q}_T^2 - 2(\mathbf{p}_{aT}\cdot\mathbf{q}_T)^2],$$

$$w_4 = 2\left(\frac{\mathbf{p}_{aT}\cdot\mathbf{p}_{bT}}{2M_h^2} - \frac{(\mathbf{p}_{aT}\cdot\mathbf{q}_T)(\mathbf{p}_{bT}\cdot\mathbf{q}_T)}{M_h^2\mathbf{q}_T^2}\right)^2 - \frac{\mathbf{p}_{aT}^2\mathbf{p}_{bT}^2}{4M_h^4}.$$

$$\langle\cos(2\phi_{CS})\rangle = \frac{1}{2}\frac{F_3\left(\mathcal{C}[w_3 f_1^g h_1^{\perp g}] + \mathcal{C}[w_3' h_1^{\perp g} f_1^g]\right)}{F_1\, \mathcal{C}[f_1^g f_1^g] + F_2\, \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]},$$

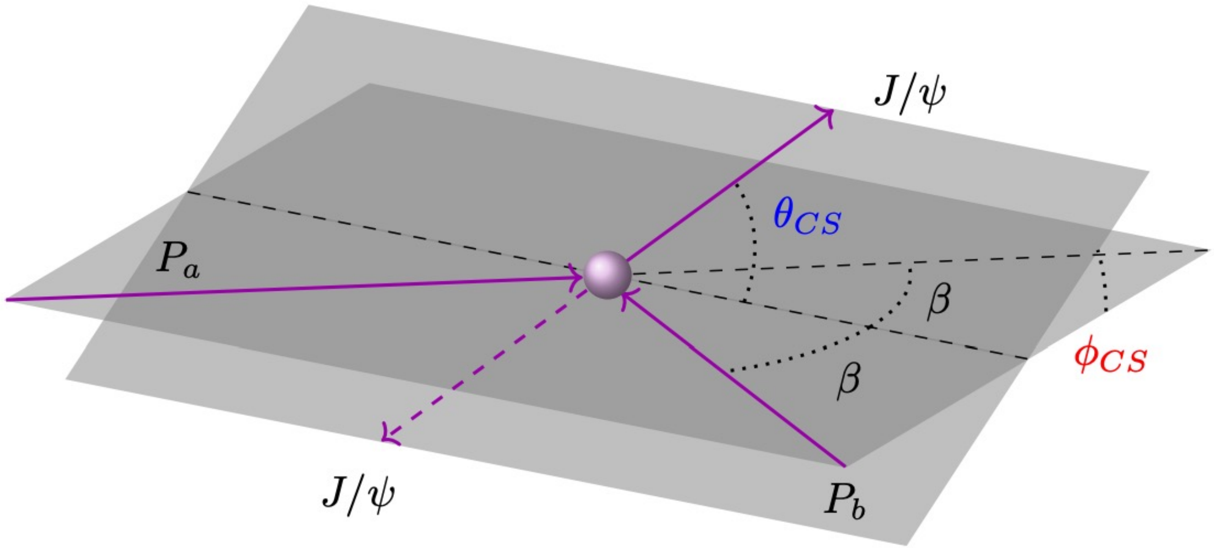
$$\langle\cos(4\phi_{CS})\rangle = \frac{1}{2}\frac{F_4\, \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1\, \mathcal{C}[f_1^g f_1^g] + F_2\, \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}.$$

$$\mathcal{C}[wfg] = \int d^2\mathbf{p}_{aT} \int d^2\mathbf{p}_{bT}\, \delta^2(\mathbf{p}_{aT} + \mathbf{p}_{bT} - \mathbf{q}_T)\, w(\mathbf{p}_{aT}, \mathbf{p}_{bT})\, f(x_a, \mathbf{p}_{aT}^2)\, g(x_b, \mathbf{p}_{bT}^2)$$

- Hard factors F_i

  Lansberg et al. 2018

- F_2 negligible when M_{QQ} is large or $|\cos\theta_{CS}| \leq 0.5$



$C[f_1^g f_1^g]$

- $C[f_1^g f_1^g]$ is a general quantity that determines the unpolarized differential cross section for any proton-proton process that are dominated by gluon-gluon fusion:
  - Higgs production *Sun et al. 2011, Boer et al. 2012*
  - $\eta_Q, \chi_{Q0}, \chi_{Q2}$ production *Boer and Pisano 2012*
  - Quarkonium + di-lepton production *Lansberg et al. 2017*
- Also, it appears next to quark-antiquark and quark-gluon contributions, where the gluon-gluon channel dominates in specific kinematic regions:
  - Higgs + jet production *Boer and Pisano 2014*
  - Di-jet production *Boer et al. 2009*
  - open heavy quark production *Boer et al. 2010, Pisano et al. 2013, Boer et al. 2016*

Introduction of evolution

- Beyond tree level, the TMDs and hard factor become scale dependent

  Collins and Soper 1981

- Implementing evolution is more easily done in impact parameter space, where convolutions become simple products

$$\frac{d\sigma}{d(\text{kinematic variables})\, d^2\mathbf{q}_T} = \int d^2\mathbf{b}_T\, e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\, \hat{W}(\mathbf{b}_T, \mu_H) + \mathcal{O}(\mathbf{q}_T^2/\mu_H^2)$$

$$\hat{W}(\mathbf{b}_T, \mu_H) = \hat{A}(\mathbf{b}_T; \zeta_A, \mu)\, \hat{B}(\mathbf{b}_T; \zeta_B, \mu)\, \mathcal{H}(\mu_H; \mu)$$

$$\boxed{\mathcal{C}[f_1^g f_1^g] = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_0(b_T q_T)\, \hat{f}_1^g(x_a, \mathbf{b}_T^2)\, \hat{f}_1^g(x_b, \mathbf{b}_T^2)}$$

$$\hat{f}_1^g(x, \mathbf{b}_T^2) \equiv \int d^2\mathbf{p}_T\, e^{i\mathbf{b}_T\cdot\mathbf{p}_T}\, f_1^g(x, \mathbf{p}_T^2)$$

The Sudakov factor and scales

- CS Evolution: $\hat{f}(x, \mathbf{b}_T^2; \zeta, \mu) = e^{-S_A(b_T, \zeta, \zeta_0, \mu, \mu_0)} \hat{f}(x, \mathbf{b}_T^2; \zeta_0, \mu_0)$

$$S_A(b_T, \zeta, \zeta_0, \mu, \mu_0) = -\frac{1}{2}\hat{K}(b_T, \mu_0) \ln \frac{\zeta}{\zeta_0} - \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'), 1) - \frac{1}{2}\gamma_K(\alpha_s(\mu')) \ln \frac{\zeta}{\mu'^2}\right]$$

- To avoid large logs in the hard factor $\mu \sim \mu_H$
- TMDs should be evaluated at their natural scale $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- Instead of choosing a low, still perturbative scale, is common to take

$$\sqrt{\zeta_0} \sim \mu_0 \sim \boxed{\mu_b \equiv b_0/b_T} = 2e^{-\gamma_E}/|\mathbf{b}_T| \qquad \boxed{\mu_b \leq \mu_H}$$

- b_T must be constrained $\boxed{b_{T,\min} = b_0/M_{QQ} \leq b_T \leq b_{T,\max} = b_0/Q_{NP}}$

- $b_{T,max}$ is the point where perturbation theory starts to fail: [0.5: 1.5] GeV^{-1}

b_T-domains and the nonperturbative Sudakov

1) $b_0/\mu_H \leq b_T$

Boer and Den Dunnen 2014

$$\mu_b \to \mu_b' = \frac{b_0}{b_T + b_0/\mu_H}$$

Collins et al. 2016

$$\mu_b \to \tilde{\mu}_b' = \frac{b_0}{\sqrt{b_T^2 + b_0^2/\mu_H^2}}$$

2) $b_T \leq b_{T,\max}$

$$b_T^*(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{T,\max})^2}}$$

Collins et al. 1984

$$\mu_b \to \tilde{\mu}_b' = \frac{b_0}{\sqrt{b_T^2 + b_0^2/M_{QQ}^2}} \to \tilde{\mu}_{b*}' = \frac{b_0}{\sqrt{b_T^{*2} + b_0^2/M_{QQ}^2}}$$

- For $b_T > b_{T,max}$:

$$\hat{W}_i(b_T, M_{QQ}) \equiv \hat{W}_i(b_T^*, M_{QQ}) e^{-S_{NP}(b_T; M_{QQ})}$$

$$S_{NP}(b_T; M_{QQ}) = \ln\left(\frac{M_{QQ}}{Q_{NP}}\right) g_K(b_T) + g_{\text{TMD}}(x_a, b_T) + g_{\text{TMD}}(x_b, b_T)$$

The convolution(s)

$$\hat{f}_1^g(x, b_T^*; \tilde{\mu}'_{b^*}) = f_1^g(x; \tilde{\mu}'_{b^*}) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\rm QCD})$$

- Perturbative TMD tail

$$\hat{h}_1^{\perp g}(x, b_T^*; \tilde{\mu}'_{b^*}) = -\frac{\alpha_s(\tilde{\mu}'_{b^*})}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1\right) \left\{ C_A\, f_1^g(x'; \tilde{\mu}'_{b^*}) + C_F \sum_{i=q,\bar{q}} f_1^i(x'; \tilde{\mu}'_{b^*}) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\rm QCD})$$

- Suppressed by α_s

$$S_A(b_T^*; M_{QQ}, \tilde{\mu}'_{b^*}) = \frac{1}{2}\frac{C_A}{\pi} \int_{\tilde{\mu}'^2_{b^*}}^{M^2_{QQ}} \frac{d\mu'^2}{\mu'^2} \left[\alpha_s(\mu') + \frac{\alpha_s(\mu')^2}{4\pi} \left\{ \left(\frac{67}{9} - \frac{\pi^2}{3} \right) N_c - \frac{20}{9} T_R n_f \right) \right\} \right] \ln \frac{M^2_{QQ}}{\mu'^2} - \frac{1}{2}\frac{C_A}{\pi} \int_{\tilde{\mu}'^2_{b^*}}^{M^2_{QQ}} \frac{d\mu'^2}{\mu'^2}\, \alpha_s(\mu')\, \beta_0$$

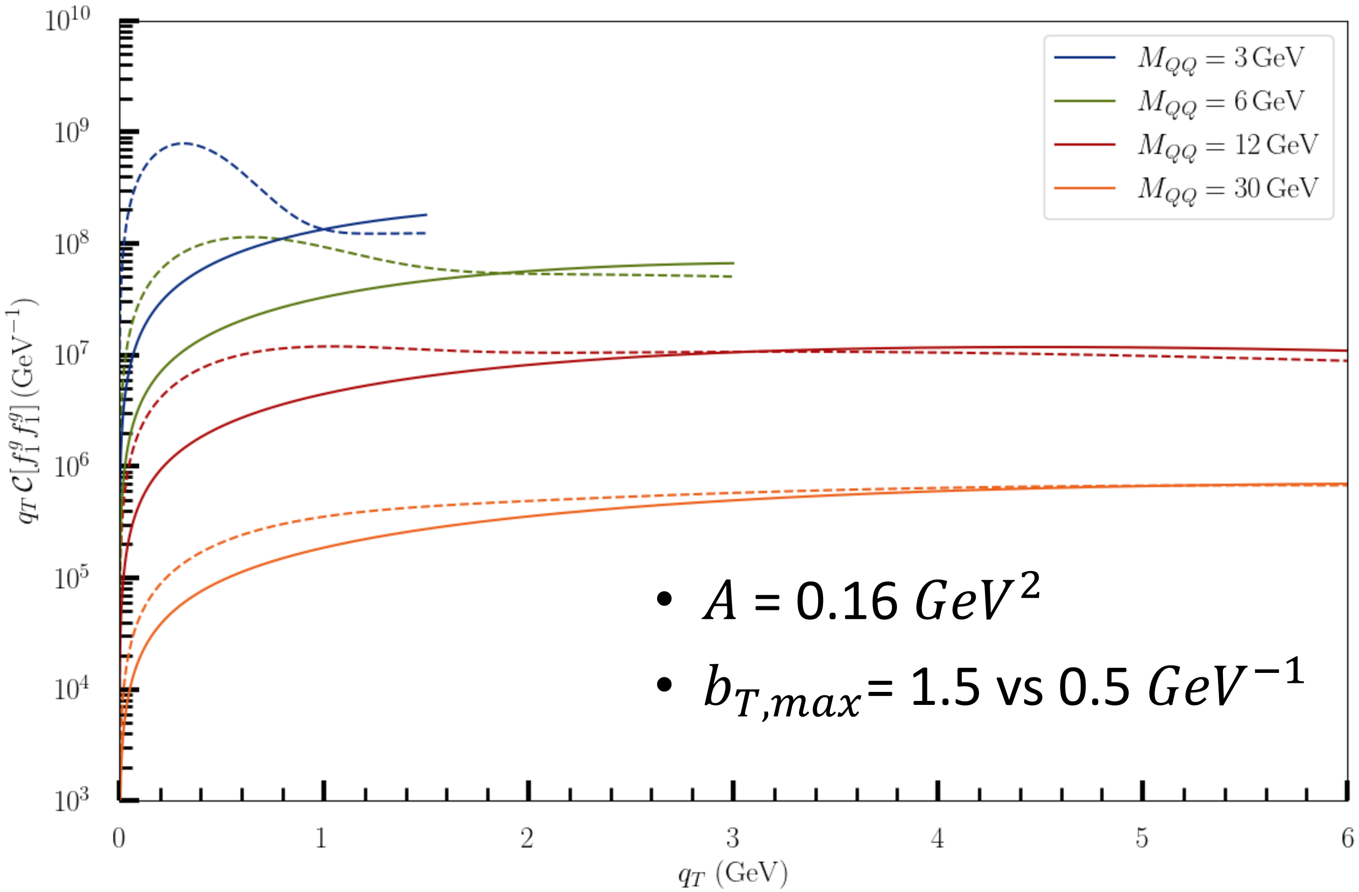
- NLL accuracy
- α_S 1-loop

$$\mathcal{C}[w_n f g](x_a, x_b, q_T; M_{QQ}) = \int_0^\infty \frac{db_T}{2\pi}\, b_T\, J_n(b_T q_T)\, e^{-2S_A(b_T^*; M_{QQ}, \tilde{\mu}'_{b^*})} e^{-S_{NP}(b_T; M_{QQ})} \times f(x_a, b_T^*; \tilde{\mu}'_{b^*})\, g(x_b, b_T^*; \tilde{\mu}'_{b^*})\,.$$

A novel nonperturbative Sudakov ?

A simple Gaussian ansatz for S_{NP} has limitations

$$\exp\left(-S_{NP}\right) = \exp\left(-A \ln(M_{QQ}/Q_{NP})\, b_T^2\right)$$



- Generates upward bump for small $b_{T,max}$ and M_{QQ} due to large contributions of the integrand at large b_T
- TMD and x independent
- Does not provide $b_{T,max}$-invariance

- Particularly relevant for quarkonia

Another problem identified …

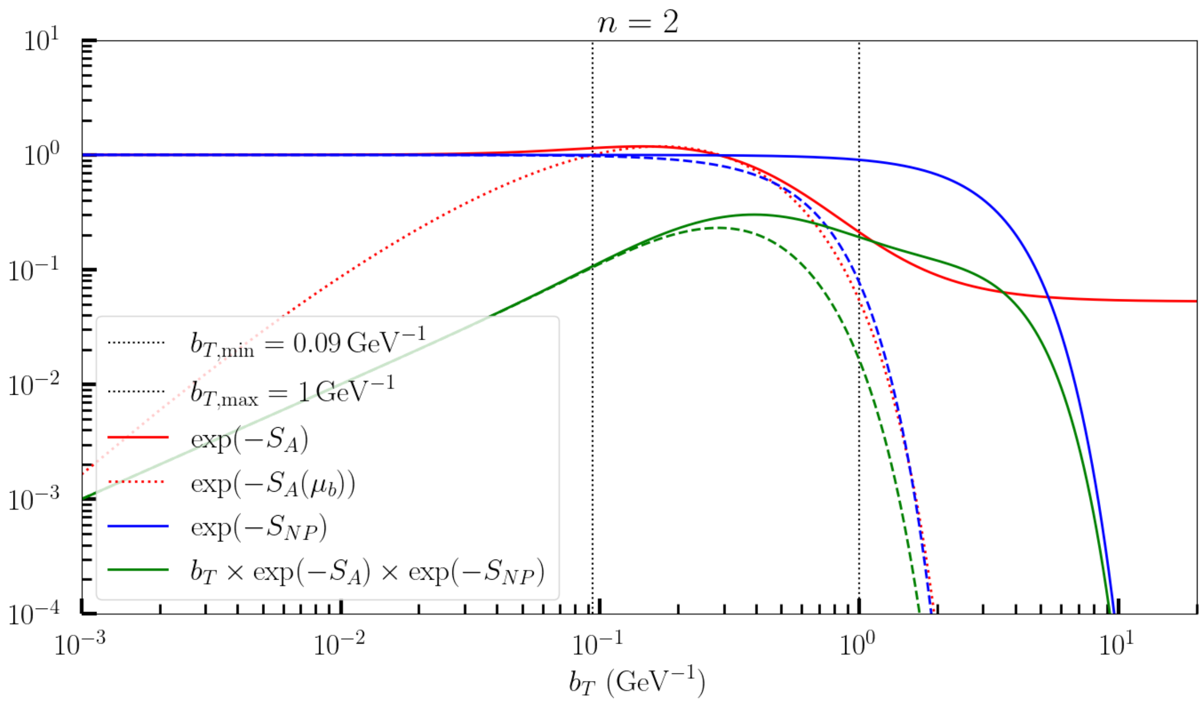
- We want to trust perturbative physics to study S_{NP} when we can

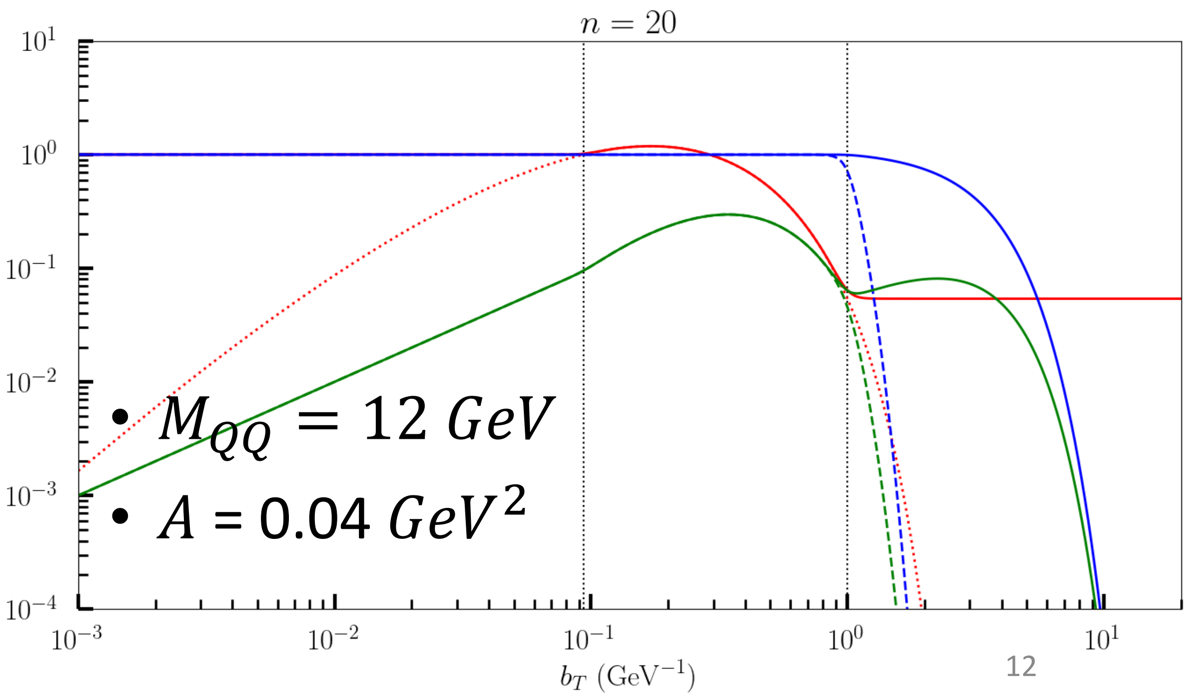
$$b_T^* = \frac{b_T}{\left(1 + (b_T/b_{T,\max})^n\right)^{1/n}}\ , \quad b_T' = \left(b_T^n + b_{T,\min}^n\right)^{1/n}$$

- Remove the ‘kink’ at the same order n

$$S_{NP}(b_T) = g\, b_T^{\dagger\, 2} \qquad b_T^{\dagger\, 2} = \left(b_T^n + b_{T,\max}^n\right)^{2/n} - b_{T,\max}^2$$

$$g_A(b_{T,\max}) = \frac{\left|\frac{\partial}{\partial b_T} S_A(b_T; M_{QQ}, \mu_b)\Big|_{b_T = b_{T,\max}}\right|}{2^{2/n}\, b_{T,\max}}$$

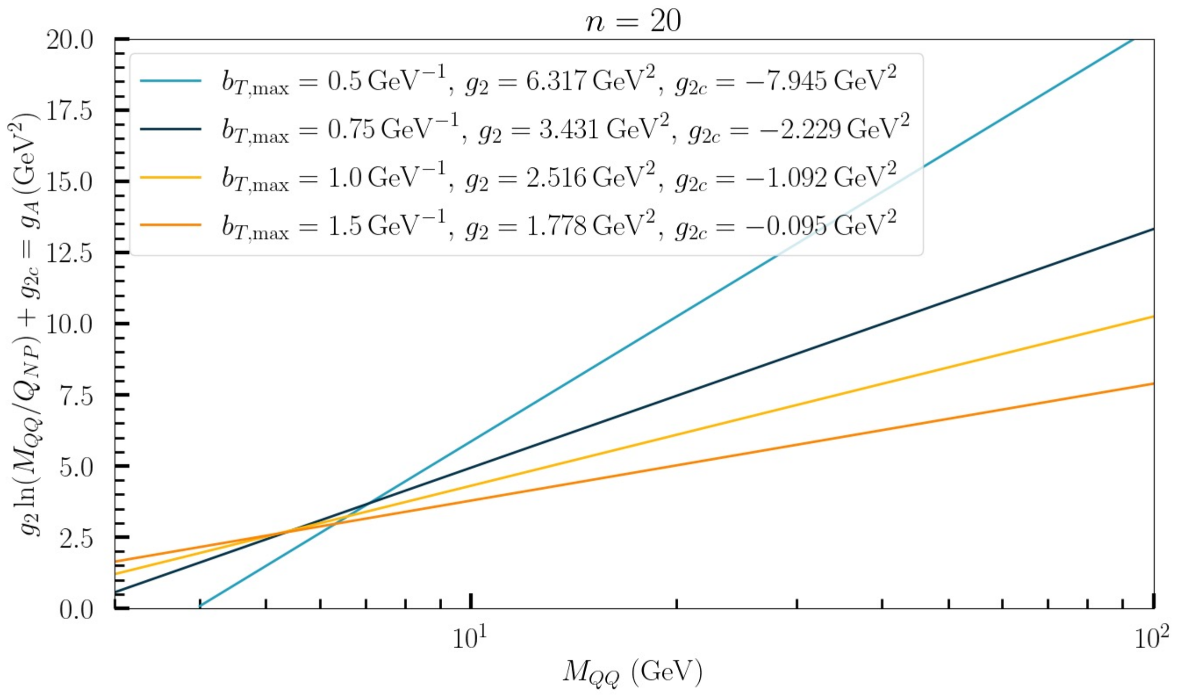


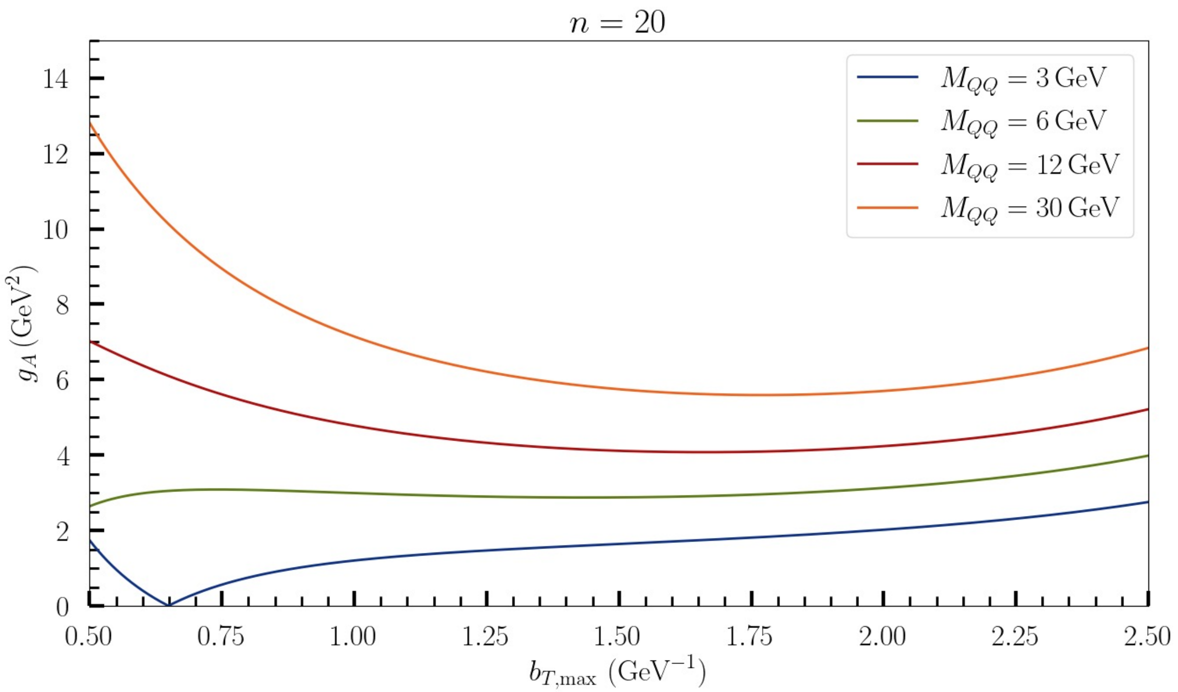


- $M_{QQ} = 12\ GeV$
- $A = 0.04\ GeV^2$

The behaviour of g_A

- Follows theoretical trend – extra term also taken into account by other fits
- Comparison with literature: for $n = 2$ and $b_{T,max}$= 1.5 GeV^{-1}, $g_A \sim$ twice as large as [Collins 2013]



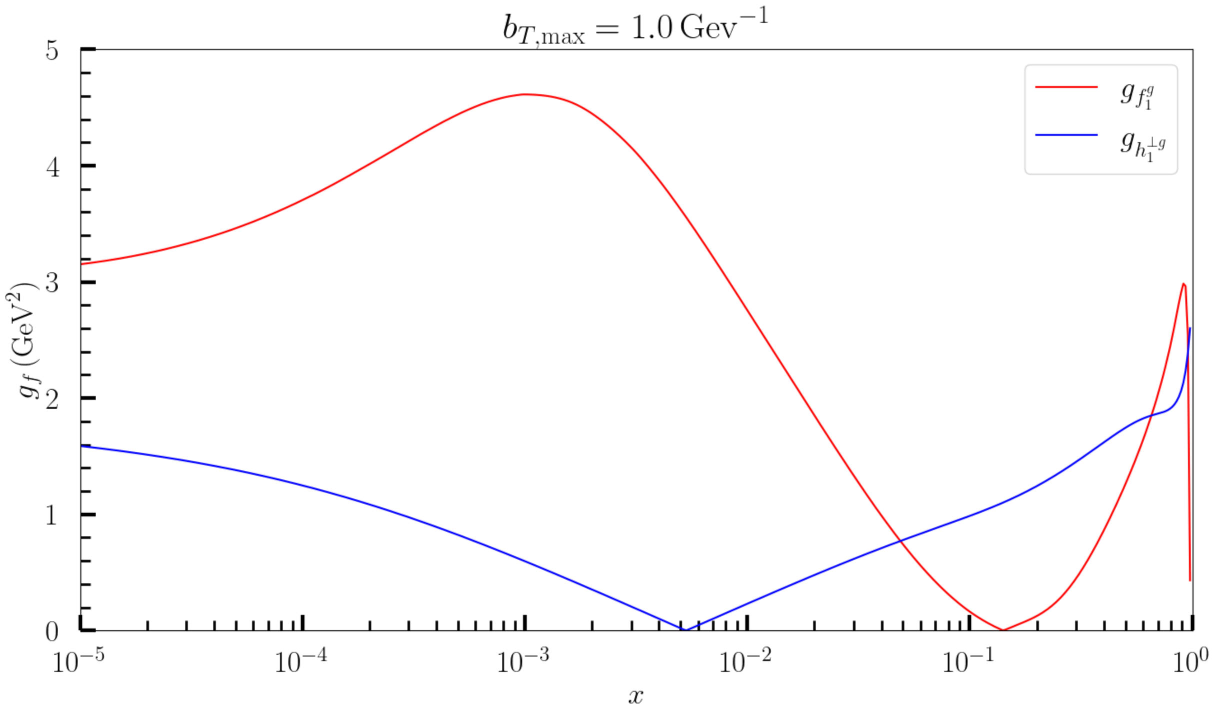


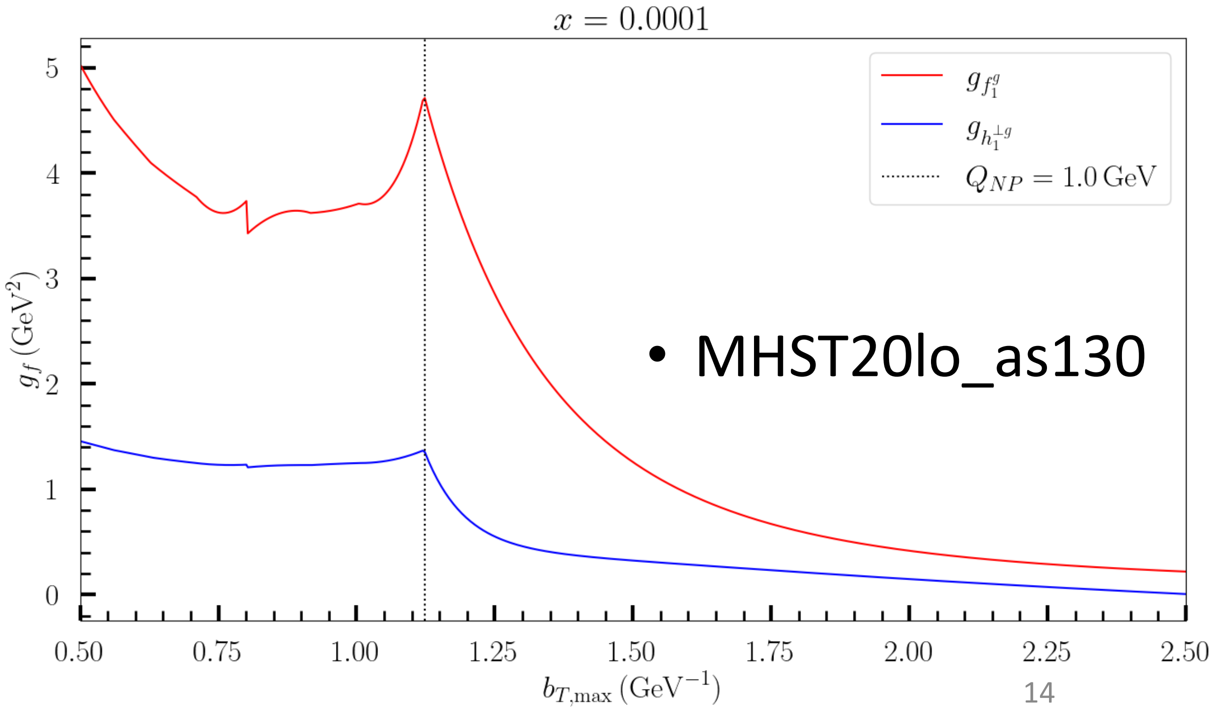
The behaviour of g_f

- Extra factor g_f needed to remove 2nd 'kink'

$$g_f(x, b_{T,\max}) = \frac{\left| \frac{\partial}{\partial b_T} \ln f(x, b_T; \mu_b)\big|_{b_T = b_{T,\max}} \right|}{2^{2/n}\, b_{T,\max}}$$

- Difficult to compare with literature: can be of same order depending on x and the kind of TMD tail
- $b_{T,max} > b_0$ dangerous





- MHST20lo_as130

The novel nonperturbative Sudakov

$$S_{NP}(x, b_{T,\max}, b_T) = \left(g_A(b_{T,\max}) + g_f(x_a, b_{T,\max}) + g_f(x_b, b_{T,\max})\right) b_T^{\dagger\,2} = g\, b_T^{\dagger\,2}$$

- Larger values g reasonable because of larger n (and smaller $b_{T,max}$)
- g can be taken lager than the found value by matching, to suppress nonperturbative physics more, but not smaller (gives back ‘kink’)

- Solves strange behaviour for small $b_{T,max}$ and M_{QQ}
- Takes into account x and TMD dependence
- Depending on the kinematics it can generate close to, but does not provide $b_{T,max}$-invariance. However, it does take $b_{T,max}$ systematically into account.

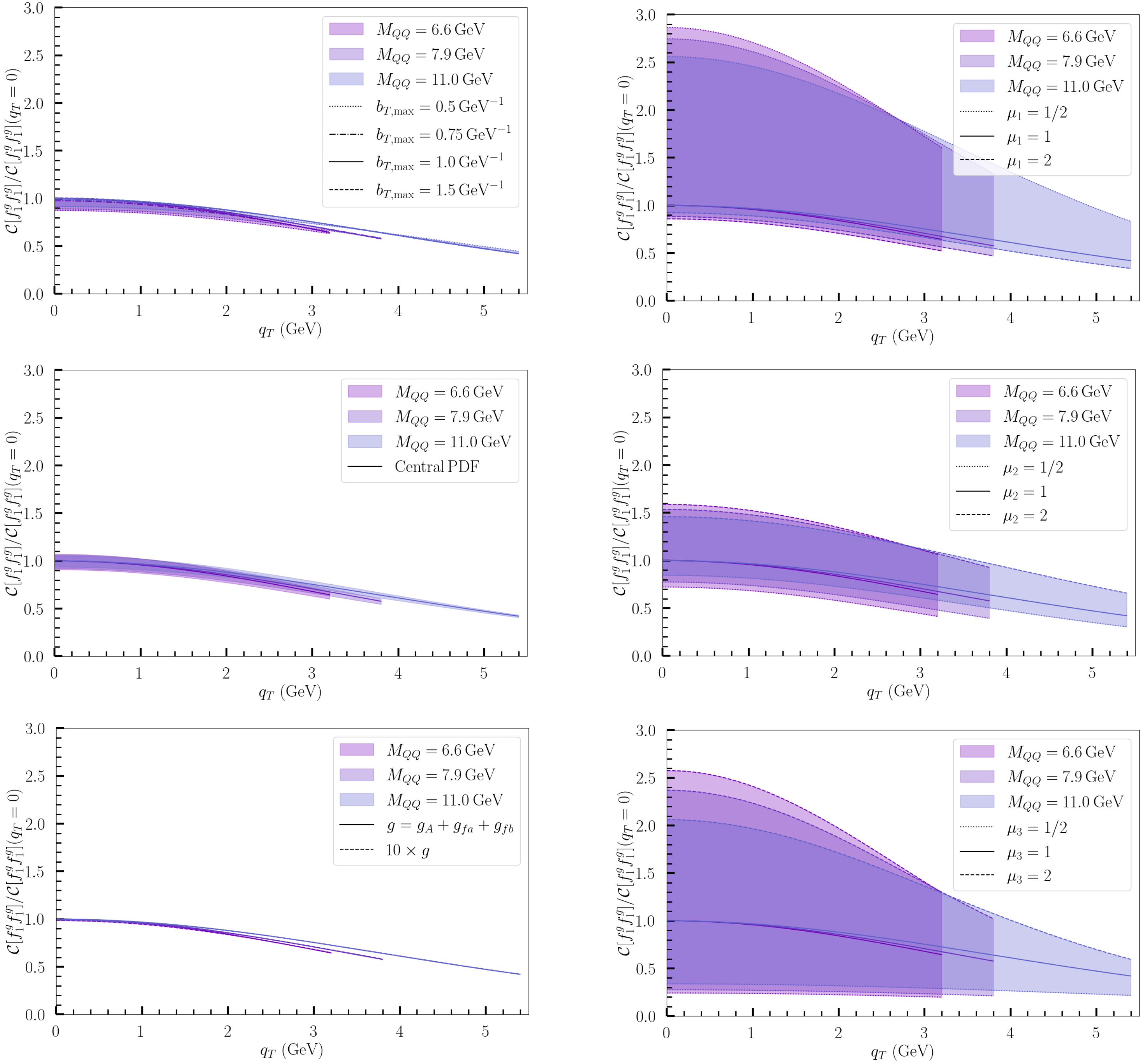
Uncertainties

- $C[f_1^g f_1^g]$ in the CS formalism has a large uncertainty from various sources
  - The PDF set uncertainty (Hessian):

$$(\Delta O)_+ = \sqrt{\sum_{j=1}^{k} \left\{ \max\left[O(S_j^+) - O(S_0), O(S_j^-) - O(S_0), 0\right] \right\}^2} \quad (\Delta O)_- = \sqrt{\sum_{j=1}^{k} \left\{ \max\left[O(S_0) - O(S_j^+), O(S_0) - O(S_j^-), 0\right] \right\}^2}$$

  - Nonperturbative physics:
    - $b_{T,max}$ variation; [0.5: 1.5] GeV^{-1}
    - g increasement; f.e. $g \rightarrow 10g$
  - Scale variation, $\mu \rightarrow a\,\mu$ with $a = [1/2 : 2]$ *Melis et al. 2015*
    - $\mu_1 = b_0/b_T$ and $\mu_2 = \mu_H = M_{QQ}$ (the lower bound of S_A and the hard scale)
    - $\mu_3 = b_0/b_T$ (in PDF of perturbative TMD tails)

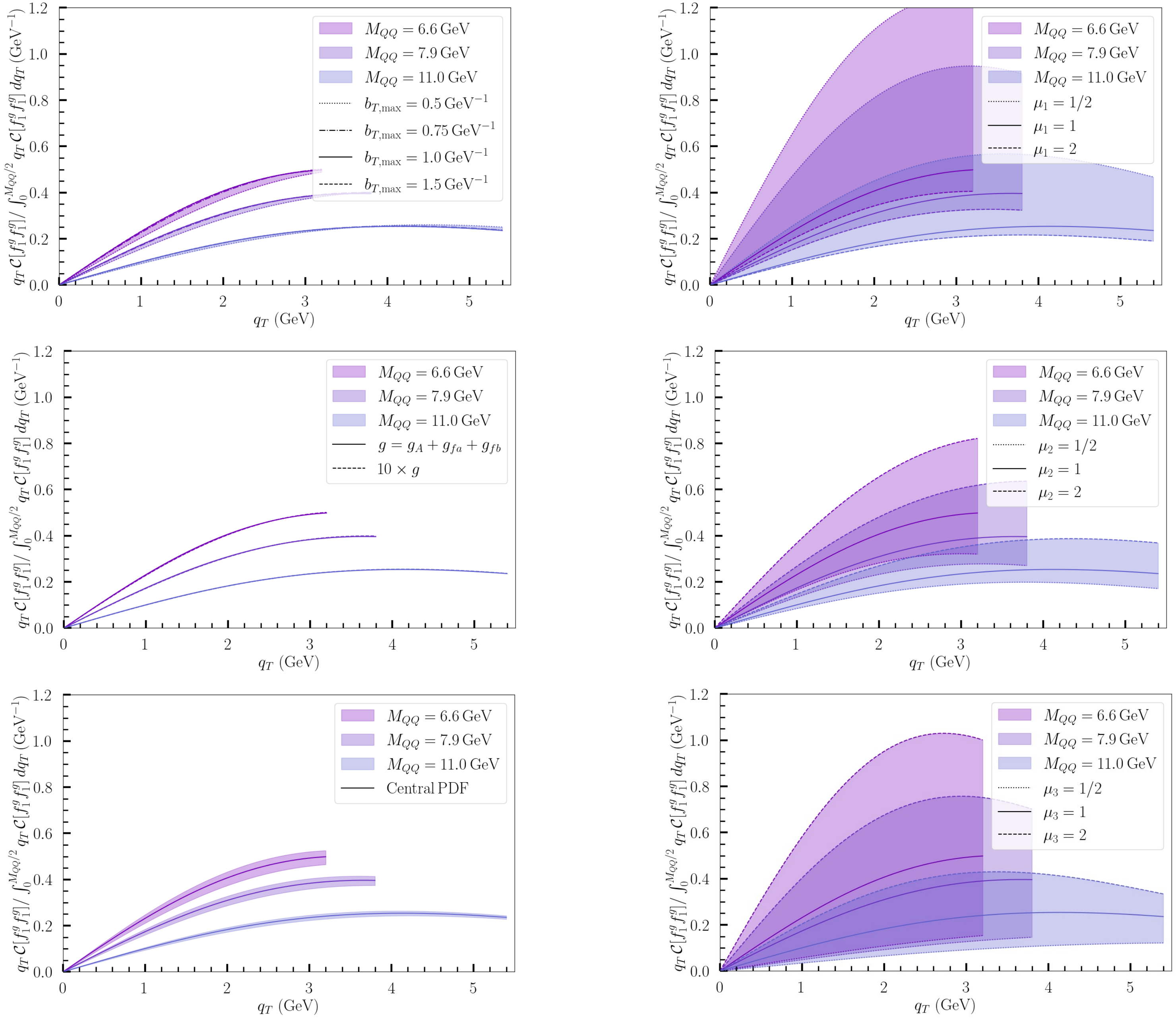
      Note:
      - μ_1 and μ_3 contain in practice the b_T-expressions (so also μ_2)
      - Scale variation alters the perturbative Sudakov (so also g_A and g_f)



- $b_{T,max} = 0.5\ GeV^{-1}$
  $\rightarrow Q_{NP} = 2.25\ GeV$

$$q_T = M_{QQ}/2$$
$$b_{T,max} = 1.0\ GeV^{-1}$$
$$y_{QQ} = 3.25$$

- Uncertainties are (already!) suppressed in the normalised cross section

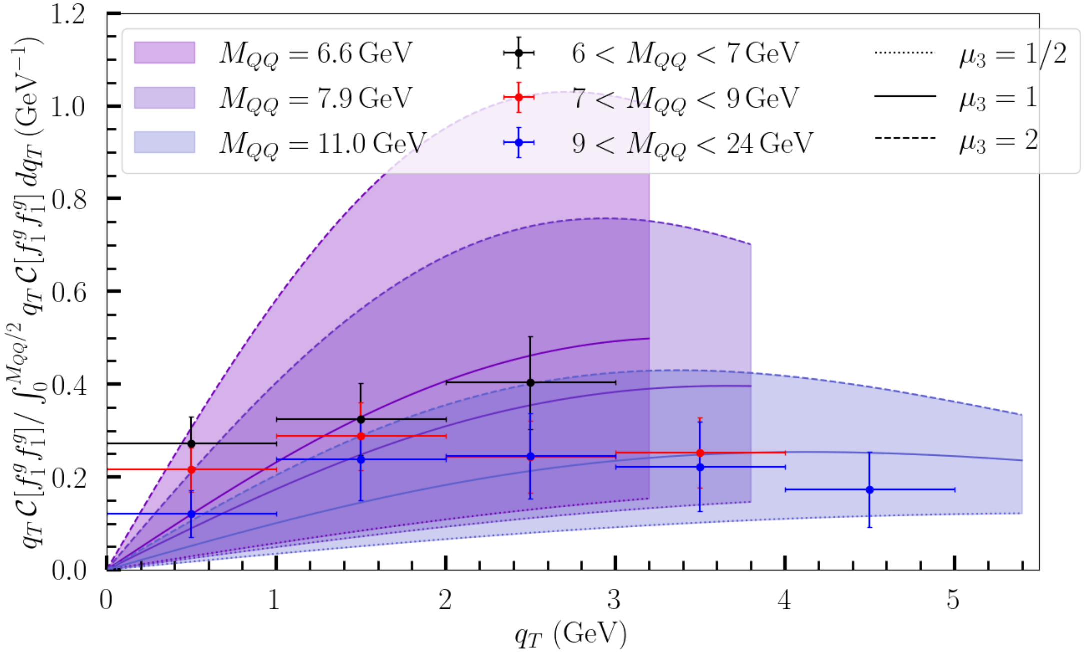


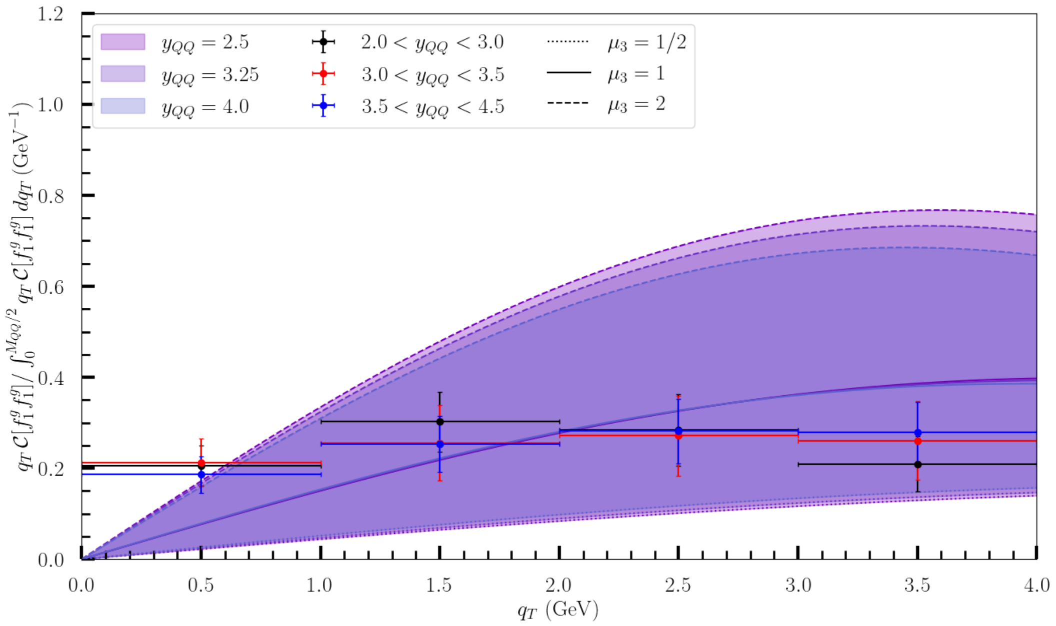
$$\frac{1}{\sigma_{UU}^{gg}}\frac{d\sigma_{UU}^{gg}}{dq_T} = \frac{q_T\,\mathcal{C}[f_1^g f_1^g]}{\int_0^{M_{QQ}/2} q_T\,\mathcal{C}[f_1^g f_1^g]\,dq_T}$$

- A similar study can be provided for $C[w_2 h_1^{\perp g} h_1^{\perp g}]$
- Lets take μ_3 …

Comparison with the new LHCb data

- Collider mode: $\sqrt{s} = 13\ TeV$

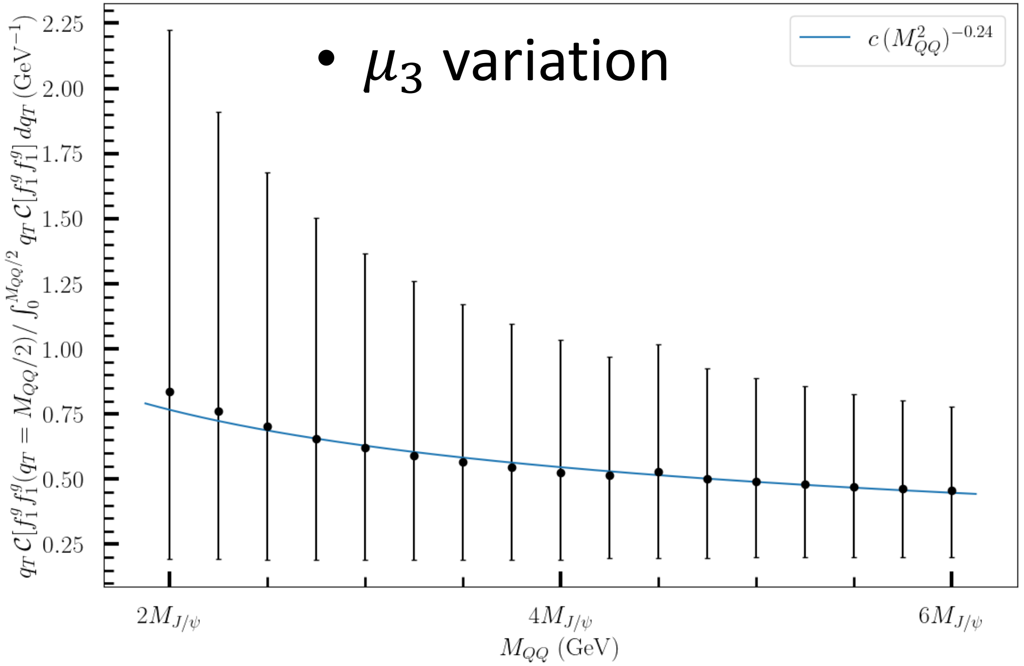


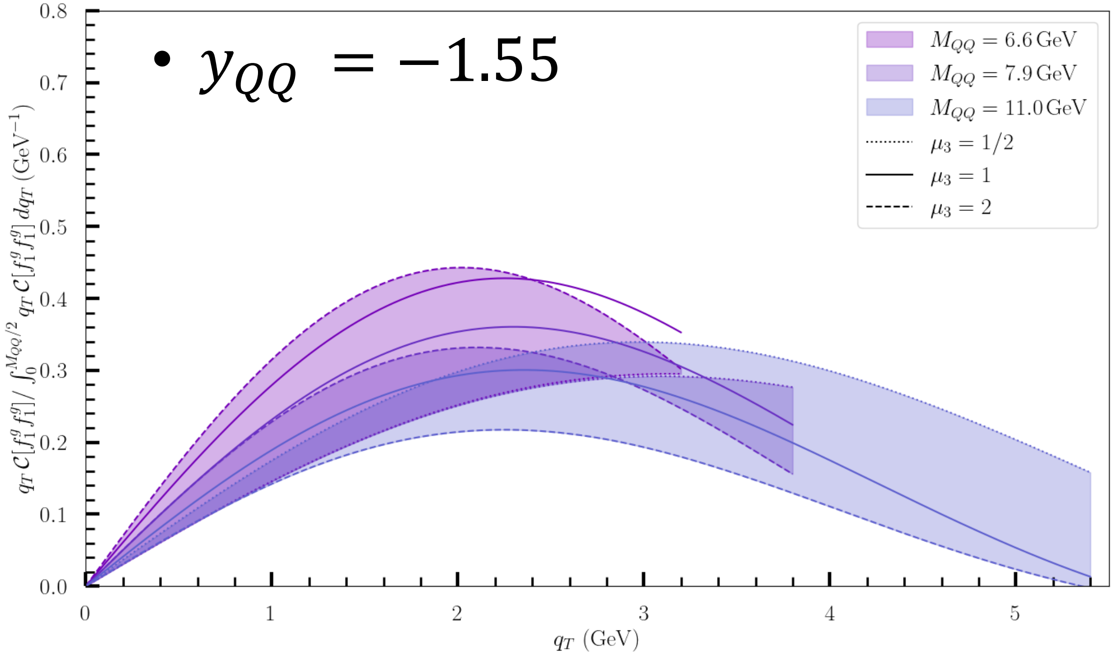


- Scale variation provides overlap between the data and the predictions for the lowest q_T and largest q_T-values

- DPS subtracted data

Hard scale dependence & fixed-target mode

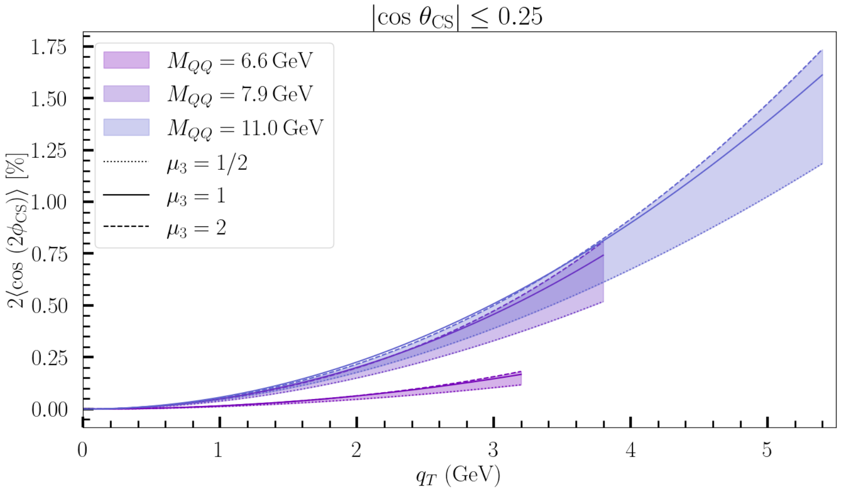


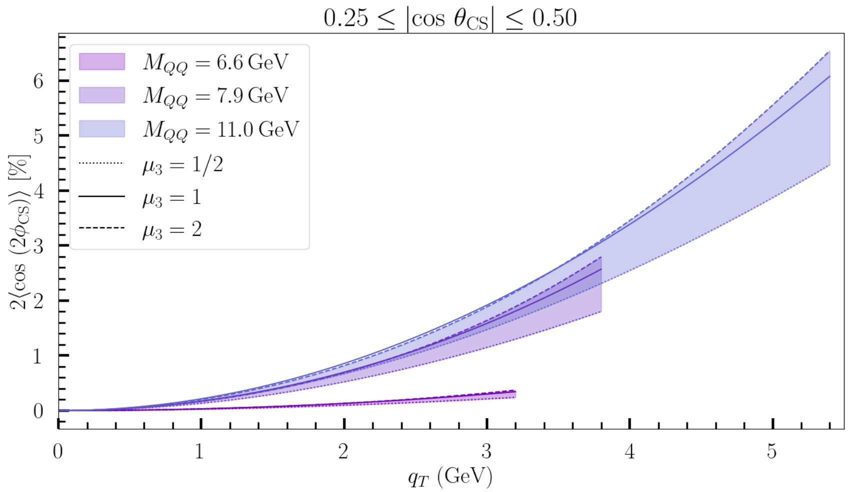


- One can also probe the evolution formalism by extracting the power law behaviour of the hard scale

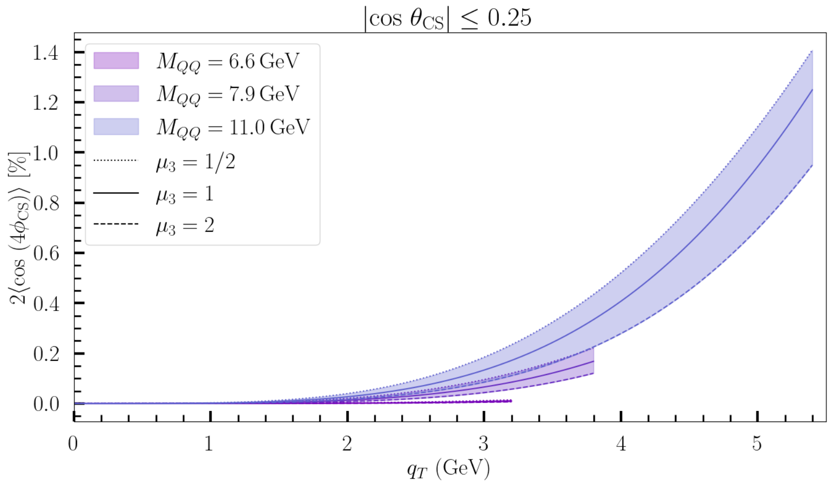
- Fixed-target mode ($\sqrt{s} = 115\ GeV$ and a rapidity shift of 4.8) provides a better possibility to observe the peak within the TMD region

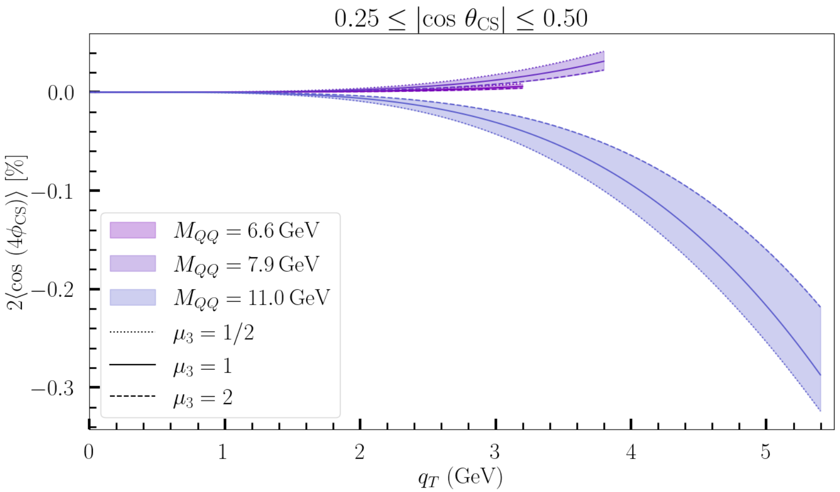
Azimuthal modulations: $C[w_3 f_1^g h_1^{\perp g}]$, $C[w_4 h_1^{\perp g} h_1^{\perp g}]$





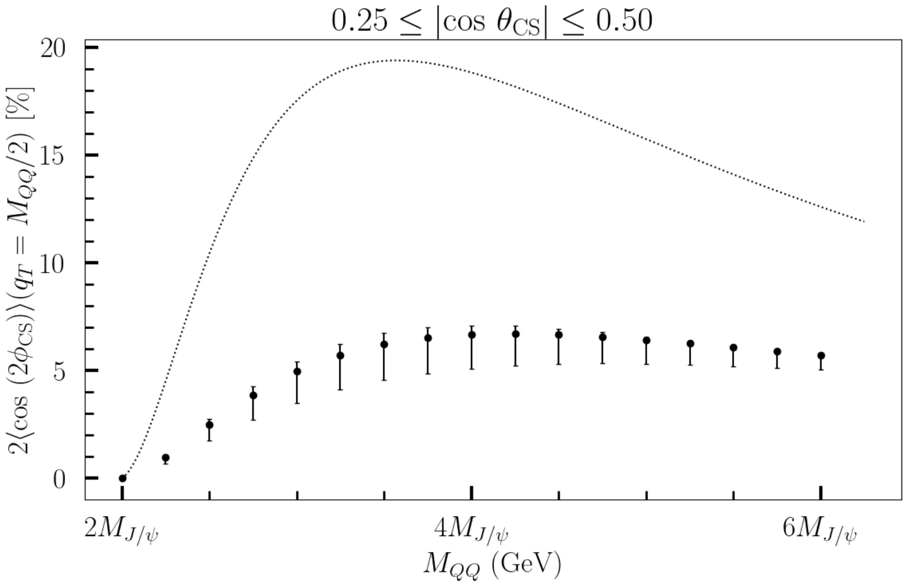
- The $\cos 2\phi_{CS}$-modulation provides a way to determine the sign of $h_1^{\perp g}$ and the convolution with w_4 can be used to extract $h_1^{\perp g}$ independently from f_1^g (when F_2 small)





- Sign flip due to F_4

Other remarks



- Behaving ‘similar’ to ‘upper bound’, but with smaller magnitude

- All convolutions decrease in magnitude when increasing $|y_{QQ}|$, however the convolutions with w_3 change significantly less such that $< \cos(2\phi_{CS}) >$ increases, while $< \cos(2\phi_{CS}) >$ decreases
- Next: combine scale variation to provide the most general predictions; find the largest and smallest contributions by all possible scale combinations

Conclusions

- We have investigated the interplay between the perturbative and nonperturbative regions that endorsed a novel nonperturbative Sudakov factor that solves problems that can arise with a simple Gaussian ansatz.

- Our predictions including scale uncertainties are agreeable with data.
- Larger M_{QQ} (f.e. di-Υ production) or future fixed-target experiments measurements at the LHC are more favourable for the normalized cross sections (such that the peak in TM can be observed more clearly), the asymmetry and for measuring the linearly polarised gluons in unpolarised protons (such that larger magnitudes can be observed).
- It might be suitable to probe the evolution formalism as well by extracting the power law behaviour of the hard scale of f.e. the normalized cross section at a specific TM.

$\Longrightarrow$ *Bor, Colpani Serri, Boer and Lansberg [in progress … 2024]*